Tutorial letter 201/2/2017

MATHEMATICS AND MATHEMATICS TEACHING

PST201F

Semester 1

Department Mathematics Education

IMPORTANT INFORMATION:

This tutorial letter contains the memorandum for Assignment 02

IMPORTANT INFORMATION:

At the end of this letter, you will find exam guidance.

BAR CODE

ASSIGNMENT 02
SOLUTIONS AND COMMENTS

Dear student

This tutorial letter contains solutions to Assignment 02. You will need to compare the solutions to your own attempts. This feedback is very important. Work through the solutions, and use the examples to prepare for the exam.

REMEMBER

NOT every question will be marked. That does not mean that the questions not marked are not important (Interpret the three NOTS in this sentence!!*). Look carefully at the marking schedule and the marks you obtained.

We are committed to help you to pass this module. Please be committed from you side to pass. Success will not come without effort.

*It means everything is important!

The solutions are placed in boxes for your convenience

PART B

Activity 1.1

Write a short paragraph on your experiences as a learner in a Mathematics class when you were at school. Write at least one good experience and one bad experience. (5)

Own ideas

Activity 1.2

Write a paragraph to describe what you think **each** of the pictures (photos) represent. Do not rush through the activity. Write as much as you can about the situation depicted in each picture. (6)

Own ideas:

Some students' opinions (students give their own impressions)

1 The teacher stands behind her table, all children listen passively to the teacher

2 Learners are engaged in a problem, where they have manipulatives (concrete apparatus which they can use to solve a problem)

3 Looks like a very old classroom set-up. One can almost smell the fear of the children – sitting on their hands!

4 Teacher listening attentively to learners

5 Learner sitting alone, grappling with mathematics – not a happy picture

6 I can feel the enthusiasm with which learners are listening to the teacher. Learners actively engaged in the lesson

Activity 1.3

3 Describe the role of the teacher and the learners in a classroom where they are doing mathematics

Use each of them in a sentence to relate them to the doing of mathematics. (4)

Here are some example

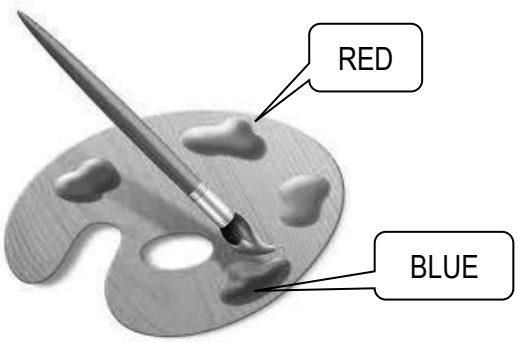
Teacher	Learner
Give appropriate tasks that promote learner thinking	Look for and discuss connections
Create environment that encourages learners to make conjectures	Consider errors as opportunities for learning
Own examples that relate to the doing of Mathematics	

Activity 1.5

Turn to page 18/42 of your prescribed textbook.

Read the instructions and answer the question:

1 *Who has the best chance of getting a red and a blue*?



If you mix blue paint with red paint, you will have purple.

Now answer the following questions:

2 *Was there a rule that you could follow to answer the question?*

There was no "rule" to be followed

3 *Did you make up your own strategy?*

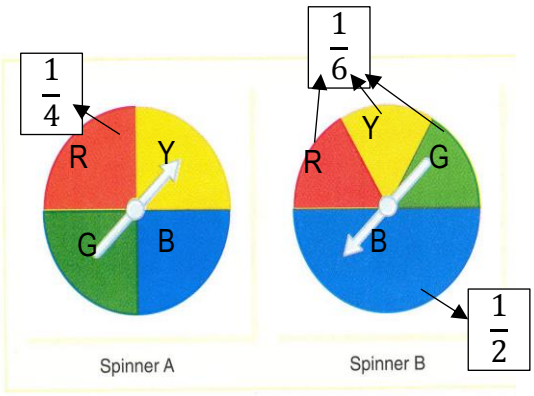
Own answer

4 *How do you feel about solving a problem without somebody giving you the "rule" to work with?* (6)

Own answer

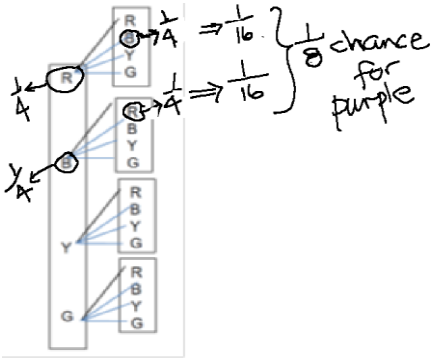
Here is the problem and some solutions

Three students are spinning to get purple with two spinners, either by spinning first red and then blue, or first blue and then red. They may choose to spin each spinner once, or one of the spinners twice. Mary chooses to spin twice on spinner A, John chooses to spin twice on spinner B, and Susan chooses to spin fist on spinner A and then on spinner B. Who has the best chance of getting a red and a blue?

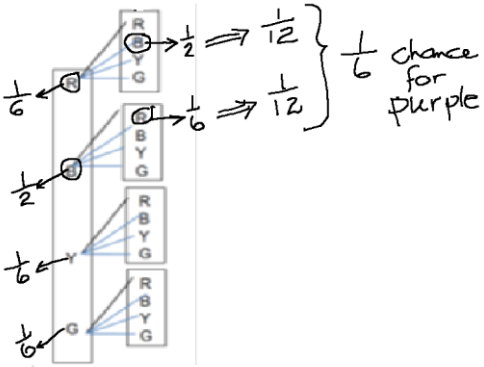


A solution

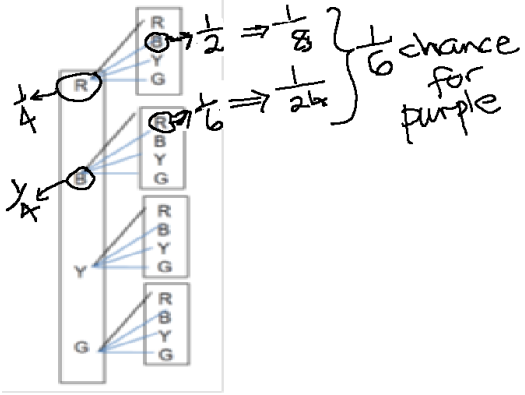
Mary Spinner A Spinner A



John Spinner B Spinner B



Susan Spinner A Spinner B



$\frac{1}{8} + \frac{1}{24}$

$= \frac{3+1}{24}$

$= \frac{1}{6}$

NB: Another strategy is explained on page 19/43-44 of the text book.

If you use a square to indicate each of the possibilities as a fraction part, then it will look as follows:

MARY

SPINNER A

RED	BLUE	GREEN	YELLOW

SPINNER A

RED
BLUE
GREEN
YELLOW

SPINNER A overlaps with itself

RR	RB	RG	RY
BR	BB	BG	BY
GR	GB	GG	GY
YR	YB	YG	YY

The square is divided into 16 parts of equal size. Two of the parts are both red and blue.

$\frac{2}{16} = \frac{1}{8}$

There is one–eighth chance of getting purple.

If you superimpose the one square (SPINNER A) over itself, you can see which parts overlap. Can you see the parts that are "purple"?

SUSAN

SPINNER A

RED	BLUE	GREEN	YELLOW

SPINNER B

BLUE
BLUE
BLUE
YELLOW
GREEN
RED

SPINNERS A AND B OVERLAPPING

BR	BB	BG	BY
BR	BB	BG	BY
BR	BB	BG	BY
YR	YB	YG	YY
GR	GB	GG	GY
RR	RB	RG	RY

The square is divided into 24 parts of equal size. Four of the parts are both red and blue.

$\frac{4}{24} = \frac{1}{6}$

There is one–sixth chance of getting purple.

If you superimpose the one square (SPINNER A) over the other (SPINNER B) you can see which parts overlap. Can You see the overlapping parts?

JOHN

If you superimpose the one square (SPINNER B) over a duplicate of itself you can see which parts overlap. Let us see which parts will be blue as well as red (BR):

SPINNER B

BLUE
BLUE
BLUE
YELLOW
GREEN
RED

SPINNER B

BLUE	BLUE	BLUE	YELLOW	GREEN	RED

SPINNER B overlaps with itself

BB	BB	BB	BY	BG	BR
BB	BB	BB	BY	BG	BR
BB	BB	BB	BY	BG	BR
YB	YB	YB	YY	YG	YR
GB	GB	GB	GY	GG	GR
RB	RB	RB	RY	RG	RR

The square is divided into 36 parts of equal size. Six of the parts are both red and blue.

$\frac{6}{36} = \frac{1}{6}\frac{4}{24} = \frac{1}{6}$

There is one–sixth chance of getting purple.

ACTIVITY 1.6 (4)

- Assimilation: It occurs when new concepts fits into an existing network of ideas. The new information expands an existing network.
- Accommodation: It takes place when the new concept does not fit into an existing network of ideas. The brain has to revamp or reorganise the network
- Disequilibrium: When new knowledge and existing knowledge do not match and the learner needs to modify the rearranged concepts and connections.
- Reflective thought: to sift through existing ideas to find those that seem related to the new knowledge.

Activity 1.7 (3)

1(a)

Michael	Romy
Uses an equal subtraction approach using known multiplication facts	Reflect the sharing context of the problem
Any other correct answer	

1(b) an algorithm is a step-by-step (finite number of steps) procedure that lead to the result.

2. Eight small boxes

Activity 1.9 (3)

Weaknesses of rote learning-your own words for the following:

- Learners do not always understand what they are learning. They will not enjoy the subject. They may perform poorly
- They forget what is taught, often because there are too many rules and algorithms to memorise. They are not able to apply their knowledge to a new situation.
- They do not know how this knowledge applies or links to the real world.
- The teacher who uses this method is more concerned about getting through the work than how and if the students are really learning.
- It is an instrumentalist form of learning. Learners are not encourage to link the new knowledge to existing knowledge.
- It is considered to be weak construction. The learner is not really building a useful network of ideas.

Activity 1.11

3 Explain what it means that understanding exists on a continuum from relational to instrumental understanding. Give an example of a mathematical concept and explain how it might be understood at different places of the continuum. (2)

See textbook and Tutorial Letter 501. Read the section on **"Relational Understanding"** in the book

Relational understanding is an understanding of what to do AND WHY. All concepts are in a connected network and when a learner wants to recall a concept, all the other related concepts are understood and can easily be recalled.

Instrumental understanding is on the opposite continuum of understanding. This is also called "doing without real understanding". The ideas are isolated, and cannot be connected (related) to other ideas. Instrumental understanding is related to the rote learning of concepts.

Give your own example to answer the question. In your discussion, amongst other things, include the use of models, finding equivalences etc. On the other hand talk about the algorithm (step-by-step procedure)

Activity 1.13 (5)

3 List at least five models (apparatus/manipulatives) that you will use in your mathematics teaching. Indicate in each case how you will use the particular model mentioned

- **Base 10 blocks** to teach place value or basic operation on whole numbers
- **Counters** to teach fractions or groupings
- **Geoboards** to teach properties of 2-Dimensonal shapes
- **Spinners** to model chance
- **Counters** to teach counting and relationships

Activity 1.14 (2)

1. Define each of the five behaviours and dispositions mentioned

Conceptual Understanding: Understanding mathematical concepts operations and relations

Procedural fluency: being able to carry out mathematical procedures flexibly, accurately and efficiently

Productive disposition: to see mathematics as sensible, useful and worthwhile.

Strategic competence: able to formulate, represent and solve mathematical problems

Adaptive reasoning: logical thought, reflection, explanation and justification

Activity 2.1 (3)

Distinguish between the following three approaches:

- **Teaching for problem-solving**: Teaching a skill that a student can later solve a problem using this skill
- **Teaching about problem-solving**: Teaching students how to solve problems. Giving them general strategies (draw a picture) for solving problems
- **Teaching through problem solving** : Learners will learn mathematical concepts through solving problems

Activity 2.6

1 What does it mean to say that a task has multiple entry points?

A problem where different strategies can be used to find a solution, is called a problem with multiple entry points

(1)

Activity 2.7

2 Use your own words to describe the teacher's actions in the *before*, *during* and *after* phases of a problem-solving lesson. USE OWN WORDS (6)

Before ⇨ Getting ready
- Activate prior knowledge
- Be sure the problem is understood
- Establish clear expectations

During ⇨ Students work
Let go!
- Notice students' mathematical thinking
- Provide appropriate support
- Provide worthwhile extensions

After ⇨ Class discussion
- Promote math community
- Listen actively
- Summarize main ideas

PART C

<u>These questions relate to Chapter 11 - 13 of the prescribed book:</u>

1 Name the three ways one can count a set of objects, and explain how these methods of counting can be used to combine concepts and written names for numbers. (6)

WAYS OF COUNTING	
Count by ones Count by groups of tens and ones Non-standard base ten	Counting by ones Counting by groups an singles Counting by tens and ones (Read on p. 205/p. 248 and explain how these methods could be used) **NB**: Think also of manipulatives that you can use for counting and explain how each can be used in counting a set of objects.

2 How can learners learn to write two- digit numbers in a way that it is connected to the base 10 meaning of ones and tens? (4)

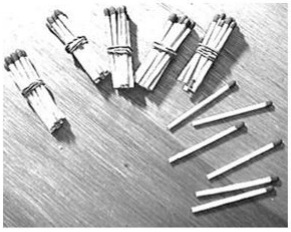
Learners can use a variety or base ten apparatus (manipulatives). They will only need units and tens concrete models. They can use the beans as illustrated in your tutorial letter 501, or matches or Dienes blocks. You can make your own Dienes blocks (see end of tutorial letter 501 for templates to cut out).The idea is that learners understand that ten ones make 10, and then the place value of the digit in the tens place means tens (or ten ones)

See tutorial letter 102 page 7

Beans – see packaging

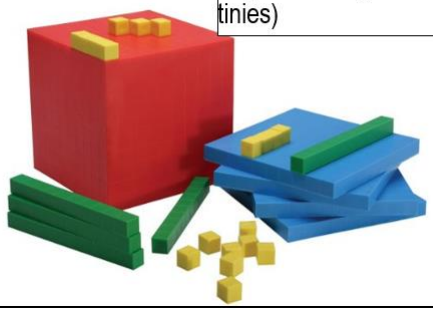


Match sticks – bundles of 10



Dienes blocks

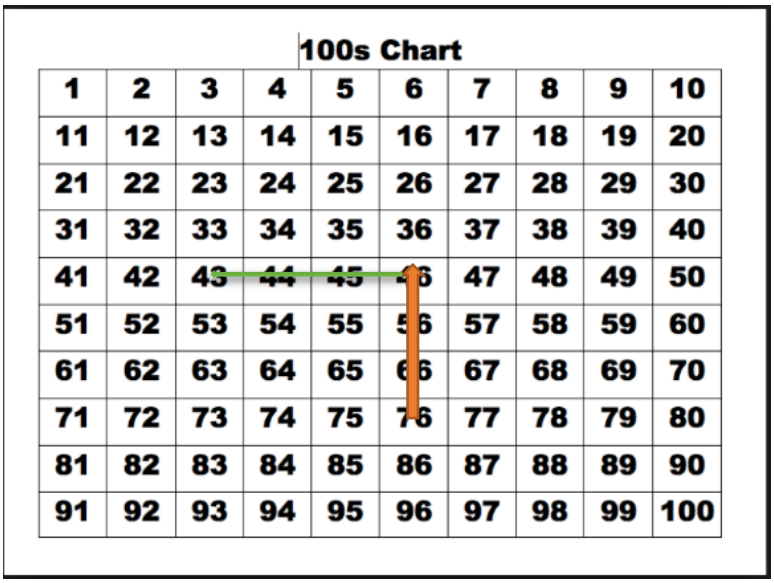
(For 2 digit numbers only use longs and tinies)



Describe in your own words

NB: You get groupable models and pre-grouped models. With groupable models, children can arrange the counters to suit their own counting but with pre-grouped models children cannot take pieces apart or group them

3 Show a way in which a hundreds chart can aid the teaching of subtraction of two digit numbers (4)



100s Chart

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- In the rows, the unit digits grow by 1 as you move from left to right.
- The last column has all unit digits as zero, but the tens digits increase by one as you move down.
- In the columns, the tens digit grows by one as you go down.
- The last number in the chart is 100

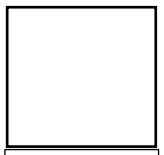
H	T	U
1	0	0

Example: $76 - 33 = 76 - 30 - 3 = 43$
Start from 76 and move three steps upwards (subtracting 3 tens) and three steps to the left (subtracting three units (ones). You will therefore land on 43 which is the answer

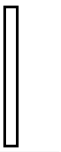
4 Draw Dienes block to show how to find the solution to:

(a) 47 + 16 (3)

Your key for Dienes block (also called base 10 blocks)



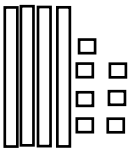
Hundreds



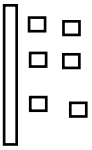
tens

units

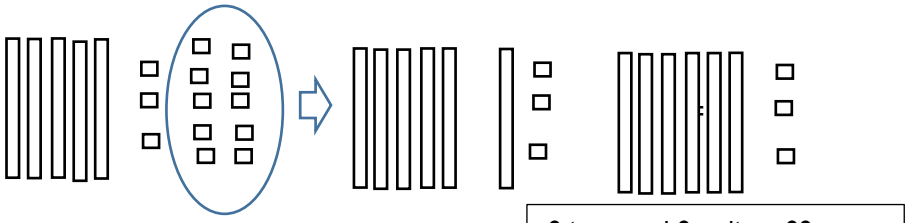
Set out 4 longs and 7 tinies



Set out 1 long and 6 tinies



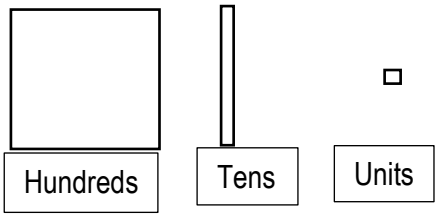
Add the tinies and exchange 10 tinies for 1 long. Add the longs



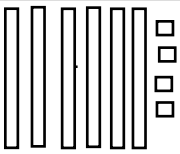
6 tens and 3 units = 63

b) 64 – 19 (3)

Your key for Dienes block (also called base 10 blocks)



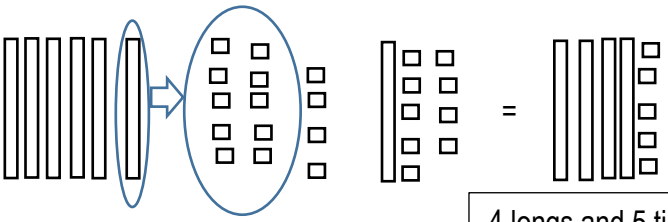
Set out 6 longs and 4 tinies



Set out 1 long and 9 tinies



Break up 1 long to ten tinies, subtract one long and nine tinies from five longs and 14 tinies



4 longs and 5 tinies = 45

5 Test the following numbers for divisibility by 6, 9, and 11. (Do not factorise)

a) 10 354 872

For 9: Add the digits: 1+0+3+5+4+8+7+2 = 30, which is not divisible by 9, so 10 354 872 is not divisible by 9 (but it is divisible by 3)

For 6: A number is divisible by 6 when both 2 and 3 can divide into it

The number is divisible by 2 because the last digit is an even number

1+0+3+5+4+8+7+2 = 30 and 3 + 0 = 3

10 354 872 is a multiple of 3. Therefore 10 354 872 it is divisible by 6

For 11

① Add every second digit: 1+3+4+7=15

② Add the other digits 0+5+8+2 = 15

③ Subtract: 15 – 15 = 0

10 354 872 is divisible by 11

b) 19 752 606

Use the same argument as in (a) to answer this question

(6)

6 The Sieve of Eratosthenes is a well-known way to find prime numbers.

6.1 Use the method given alongside the chart to find all the prime numbers between 1 and 100. (5)



METHOD

Cross out 1 (1 is not prime)

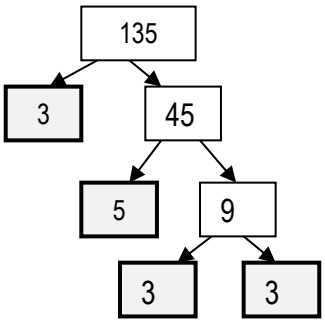
Cross out all the multiples of 2, 3, 5, 7, except these numbers themselves.

The numbers that are not crossed out will be prime.

Circle all the prime numbers

The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

6.2 Use the factor theorem to determine the prime factors of 135 (2)

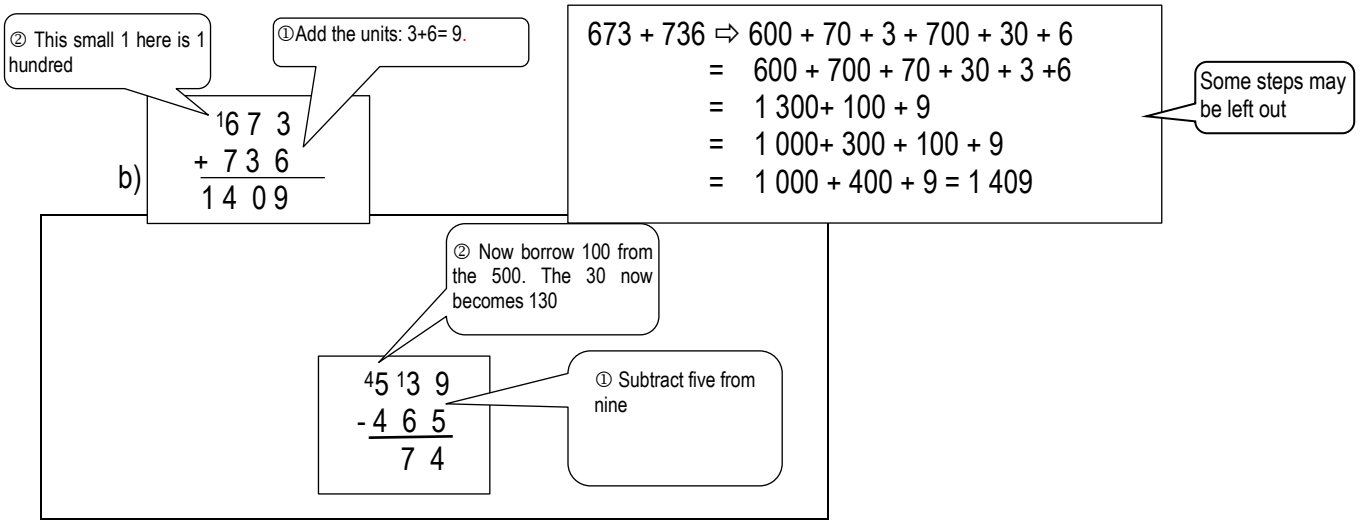


135 can be written as $3^3 \times 5$. Therefore the prime factors of 135 are 3 and 5

7 Use the vertical and horizontal algorithms to find the sum or difference of: (Explain the "borrow and carry" concepts) (4)

a) 673+736

b)



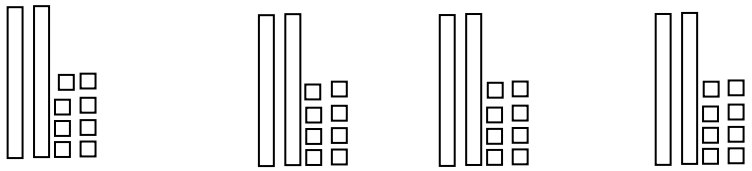
8 How are standard algorithms different from student-invented strategies? Explain the benefits of invented strategies over standard algorithms (give at least two valid points). (2)

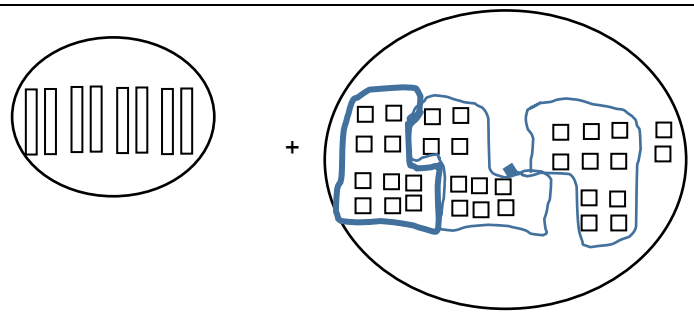
An algorithm is a rule that can be used; a procedure that is carried out which is usually learned by rote, with little understanding. Invented strategies are methods which learners produce themselves. It is usually built on concepts that the learners know already. One often finds that learners invent their own strategies in the way that they understand, and not necessarily the way in which the teacher explains.

9 Draw pictures showing how 28 × 4 could be modelled: with counters, with base – ten blocks, or with areas on base 10 grids. (Use at least two of the methods) (5)

Base 10 blocks

Set out 2 longs and 8 tinies (4 times)





= 1 Flat + 1 Long + 2 tinies

= 100 + 10 + 2

= 112

10 Use the method of compensation to show how you can make the subtraction of $468 - 39$ easier. (3)

$$468 - 39 = 468 - (39 + 1) - 1 = 468 - 40 + 1 = 428 + 1 = 429$$

11 A learner in your class says that the value of the underlined digit in 1**2** 678 is 2. How will you help the learner to overcome this misconception? (3)

Talk about the place value. Use TH H T U to explain that 2 is in the 1 000's place and therefore the value of the "2" is 2 000

PART D

These questions relate to Chapter 15 – 17 of the prescribed book:

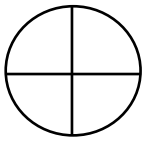
1 What does partitioning mean? Explain and illustrate (4)

Sectioning a shape into two equal-sized parts: Illustrate using your own example

2 Give examples of each of three categories of fractions: [Illustrate with diagrams]

2.1 Area model

Continuous whole (area model)



Example: A pizza divided into four equal parts

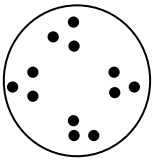


2.2 Set model

Discrete whole (set model)

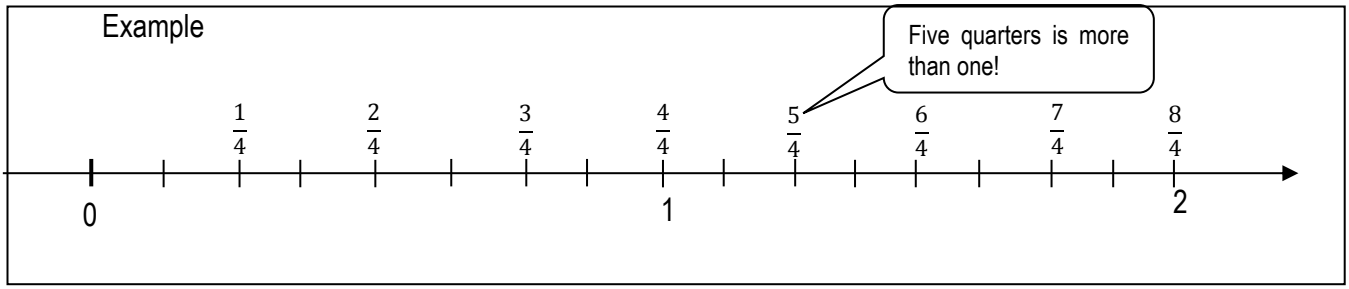
Veronica shares 12 sweets between her and 3 friends (4 people). Each friend gets one-quarter of the 12 sweets (3 sweets)

Twelve sweets divided by four people – loose objects



2.3 Length model (6)

Length model (number line) Paper strips can also be used as length models



3 In the context of choosing a "whole", explain when a "third" is not always equal to a "third". Give an example. (3)

When thinking of a fraction as a part of a whole, the part will depend on the size of the whole. For example, one third of an orange is not the same as one third of a watermelon. So when we contextualise fractions as a part of "something", this "something" has to be exactly the same. In the world of numbers, when we talk about a third, we always mean a third of ONE. (One unit)

4 Use the correct language to explain the difference between the expressions:

"It is the fifth of December"

This means the fifth day of the month of December (ordinal number).

"You are the fifth in the row"

This means if there is a row of people, you are the fifth person from the beginning (ordinal number).

"I want a fifth of the pizza"

This means if you divide a pizza in five equal parts, each part is one fifth of the whole (fraction).

What types of numbers are we dealing with in the above expressions? (4)

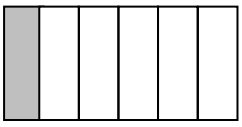
SOME NUMBERS TO NOTE

- A cardinal number tells "how many." Cardinal numbers are also known as "counting numbers," because they show quantity. Cardinal numbers are whole numbers, so fractions are not cardinal numbers.
- Ordinal numbers tell the order of things in a set—first, second, third, etc. Ordinal numbers do not show quantity. They only show rank or position.
- A nominal number names something. Nominal numbers do not show quantity or rank. They are used only to identify something (e.g. a postal code).

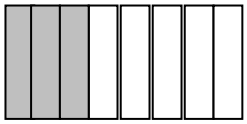
5

5.1 If the rectangle is one whole

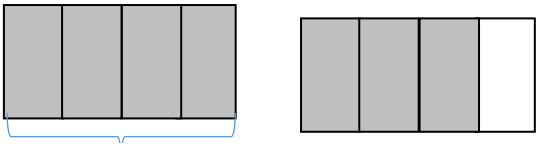
- Show one sixth



Show three eighths

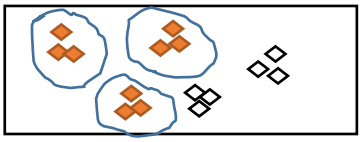


- Show seven quarters (3)

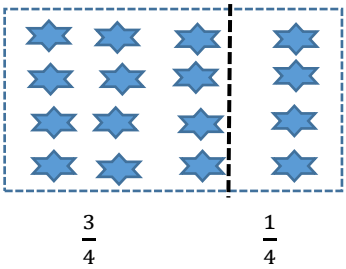


A whole

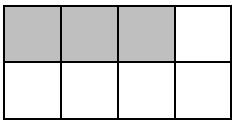
5.2 If fifteen counters are a whole set, how many are there in three-fifths of the set? Answer: __9__ (2)



5.3 If 12 counters are three fourths of a set how many counters are in full set? Answer: **16** (2)



5.4 A common error that students make is to write $\frac{3}{5}$ for the fraction of the whole represented here:

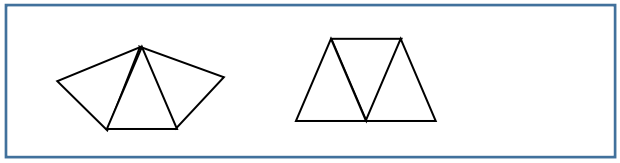


Why do you think they do this? What activity or strategy would you use to try to address this misconception? (4)

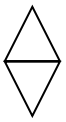
Learners think of fractions as separate numbers, in this case the shaded and the unshaded. To rectify the misconceptions that learners have, encourage them to use the correct language. The use of examples in real life situation will help learners to understand the concept (teach in context). Provide more activities where learners could differentiate between a fraction and a whole number. This should also include the difference between the numerator and the denominator.

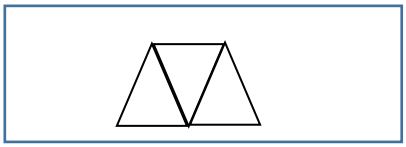
5.5 If this triangle is one third of the whole, what would the whole look like? (2)





5.6 If this figure is two-sixths of the whole, draw a shape that could be the half. (2)





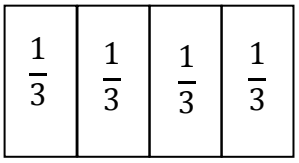
5.7 If this rectangle is four–thirds, what would the whole look like?

(2)

Whole look like:

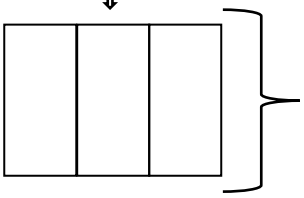
Four - thirds

First divide the whole into 4 equal parts.



Whole

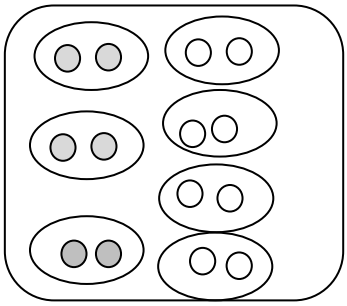
Three thirds



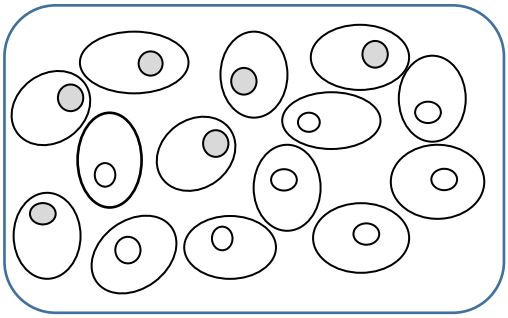
6 Use the following models to show the equivalence of the fractions $\frac{3}{7}$ *and* $\frac{6}{14}$

6.1 A set model

Choose the whole to be 14 marbles (loose objects) Can you choose another whole?



The **7** in the $\frac{3}{7}$ tells you into how many parts the whole must be divided



The 14 in the $\frac{6}{14}$ tells you into how many parts the whole must be divided

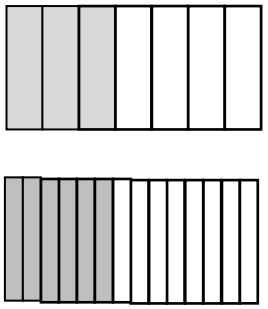
The 3 in the $\frac{3}{7}$ tells you how many parts you must choose:
Choose 3 parts of 2 marbles each, which is 6 marbles

The 6 in the $\frac{6}{14}$ tells you how many parts you must choose:
Choose 6 parts of 1 marble each, which is 6 marbles

6.2 An area model

Remember to use the same whole

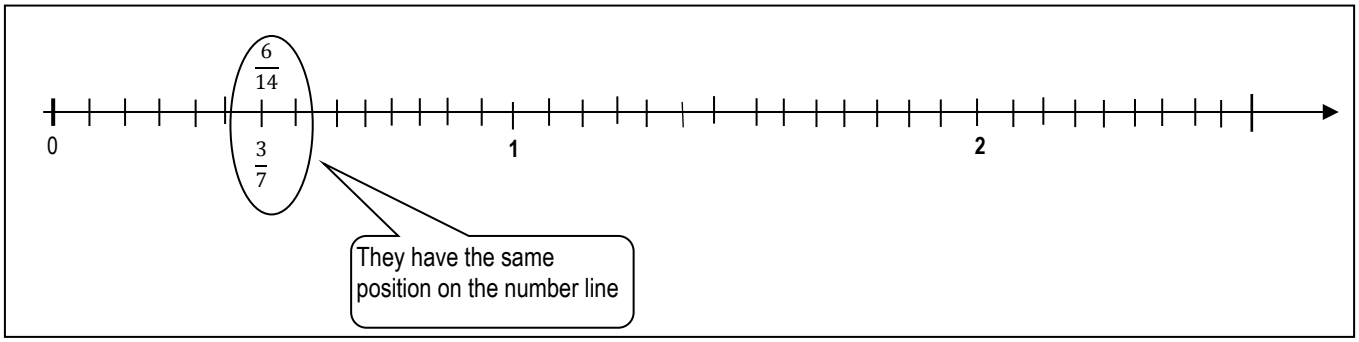
Let us choose a rectangle as the whole



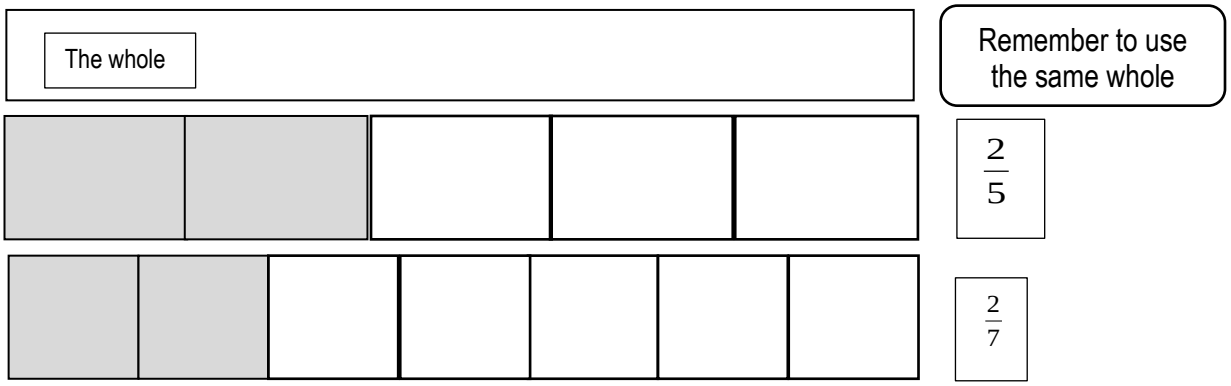
For $\frac{3}{7}$
Divide the whole into 7equal parts and shade 3

For $\frac{6}{14}$
Divide the whole into 14 equal parts and shade 6

6.3 The number line (9)



7 Compare the fractions $\frac{2}{5}; and \frac{2}{7}$ by making use of the area model/number line. (You should be able to arrange the fractions from small to big). Make sure that you choose the whole correctly, and make accurate drawings. (6)



$$\frac{2}{5} > \frac{2}{7}$$

For a number line: Choose a whole into which all the denominators can divide. Draw the number line and indicate the sevenths as well as the fifths. Use a ruler to divide the parts accurately and compare the two fractions

8 Use a standard algorithm to calculate: $5 - \frac{3}{4} - \frac{1}{2}$ (3)

① Make the denominators the same. Choose a number that 2 and 4 can divide into

② Now write all the fractions with denominators of 4 $5 = \frac{20}{4}$ $\frac{3}{4} = \frac{3}{4}$ $\frac{1}{2} = \frac{2}{4}$

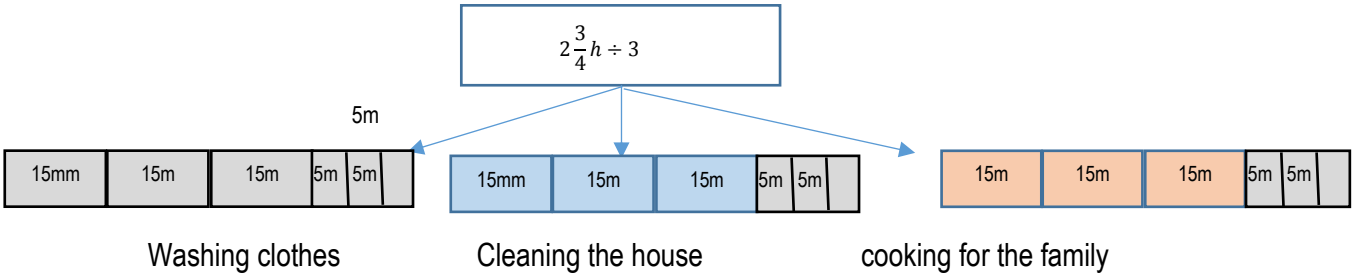
③ $5 - \frac{3}{4} - \frac{1}{2}$

$= \frac{20}{4} - \frac{3}{4} - \frac{2}{4}$

$= \frac{20-3-2}{4}$ Put over the same denominator

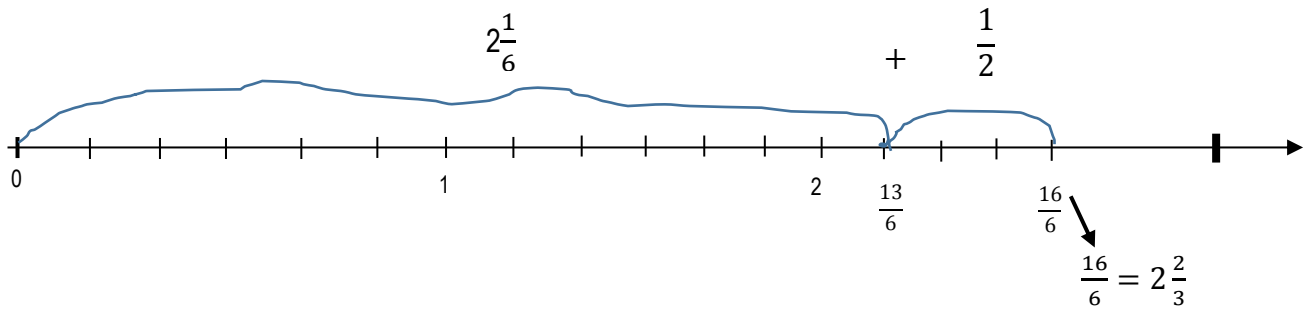
$= \frac{15}{4}$ $or\ 3\frac{3}{4}$

9 Bonolo has $2\frac{3}{4}$ hours to finish her three activities for the day (i.e. washing clothes, cleaning the house and cooking for the family). If she divides her time evenly, how many hours can she give to each activity? Illustrate the solution with drawings. (5)



Each activity will be allocated $(45 + 10)\ minutes = 55minutes = \frac{11}{12}hours$ (the remaining 30 minutes is shared amongst the three activities)

10 Use a number line to illustrate to Grade 5 learner how to add $2\frac{1}{6} + \frac{1}{2}$ .Indicate clearly the demarcations on the number line. (4)



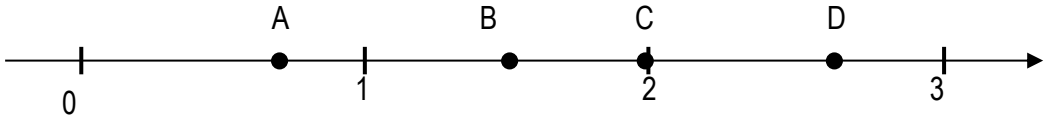
11 A learner in your class experiences difficulty with mixed numbers. Identify three misconceptions that learners might have and explain how you can prevent such misconceptions. Use examples to support your argument (6)

Possible Misconception	illustration
Subtract smaller fraction from the larger one	For $2\frac{1}{2} - 1\frac{3}{5}$: → they simplify as follows: $\frac{3}{5} - \frac{1}{2}$
Don't know what to do if one number is a whole number and the other number is a fraction	$2 - \frac{1}{4}$: →they simplify as follows: $2(\frac{2}{4} - \frac{1}{4})$
The focus is mainly on the whole number aspect	$2\frac{1}{2} - 1\frac{1}{4}$:→ $(2 - 1) = 1$ and don't know what to do with the fractional part

12 The following question was given to Grade 4 learners:

Which point on the number line shows $\frac{2}{3}$?

Draw a circle around the correct letter.



Nine percent of the learners chose A as the answer, 14% chose B, 43% chose C and 34% chose D.

12.1 What is the correct answer? (1)

Correct position of $\frac{2}{3}$ is at A (only 9% of learners had it correct)

12.2 Give an explanantion about the misconceptions learners had when they chose the incorrect answers? (3)

Possible misconception if learners chose B

Learner could have thought that the 2 indicated the whole, and saw $\frac{2}{3}$ as a part of that length

Possible misconception if learners chose C

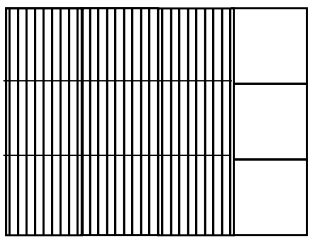
This was the most popular incorrect response. Learners mistakenly took the whole number line (from 1 to 3) as the whole, and then saw $\frac{2}{3}$ as the position of C

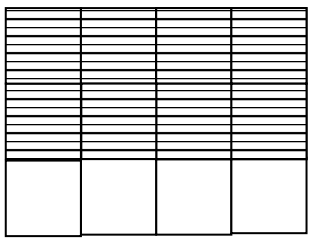
Learners who chose D, possibly thought of $\frac{2}{3}$ as a number between 2 and 3.

13 Use the grid below to illustrate: $\frac{2}{3}$ *of* $\frac{3}{4}$ (4)

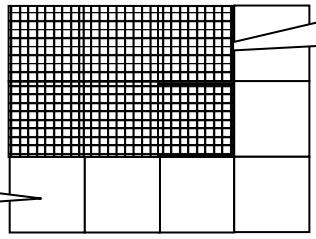
① Shade $\frac{3}{4}$ of the whole like this:

② Shade $\frac{2}{3}$ of the whole like this:





③ The section that overlaps is shaded like this:



$\frac{6}{12}$ of the whole is double shaded

This shows $\frac{2}{3}$ *of* $\frac{3}{4}$ of the whole = $\frac{1}{2}$ of the whole

Each little block is $\frac{1}{12}$ of the whole

PART E

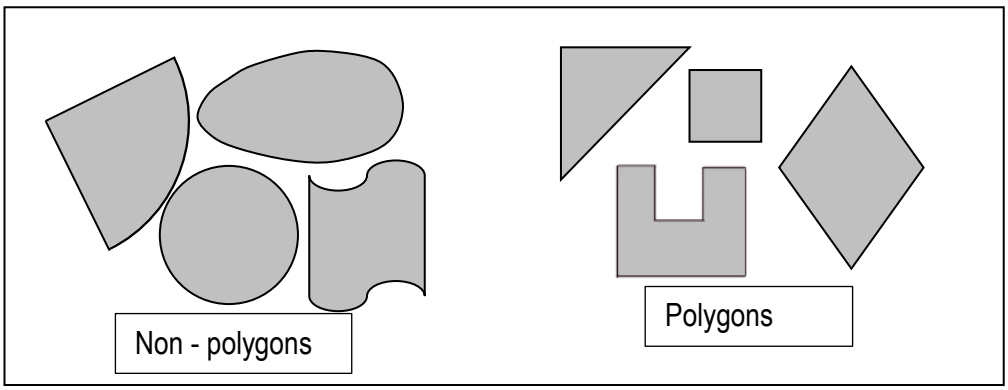
These questions relate to Chapter 20 of the prescribed book:

1 Describe in your own words the first three *van Hiele* levels of Geometric thought (level 0, 1 and 2). How do the activities that you will give learners on the three levels differ? (6)

Own words and see also tut letter 501 pages 92-93

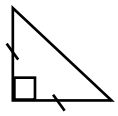
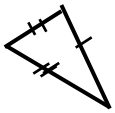
LEVEL 0 Visualization
Learners can classify shapes according to what they look like. They will see a rectangle as the same as a "door". They can distinguish between a rectangle and a square, but only by what the shape looks like and not according to their properties.
LEVEL 1 Descriptive/Analysis
Learners can describe shapes according to their properties, for example, a rectangle has equal opposite sides, and all right angles. They can group shapes according to characteristics, but do not yet know how to identify a shape if a set of properties are given. They would usually describe a shape by naming ALL the properties of that shape. For example, they will say a rectangle has equal and parallel opposite sides, equal diagonals, equal angles etc. In other words they do not yet see the minimum requirements for a shape to be identified.
LEVEL 2 Abstract/Relational/Informal Deduction
Learners at this level understand the relationships between shapes. They can now identify the minimum requirements for a shape to be identified. So they know that a rectangle is a parallelogram with one right angle, because they understand that all the other properties of a rectangle are included in that definition. They can also identify a square as a kite, because a square has all the properties of a kite.

2 Draw an assortment of different types of flat shapes that include polygons and non-polygons (at least three of each kind). Explain your classification. Why is this type of exercise necessary for learners on *van Hiele* level 0? [own explanation] (8)



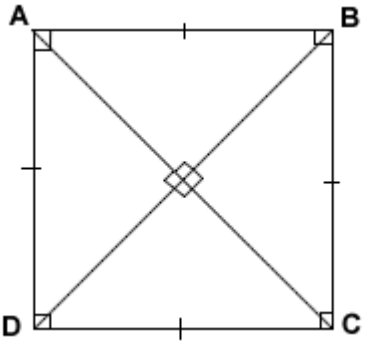
3 Draw the following triangles. You sketches must be done with a ruler and very neatly drawn.

(2)

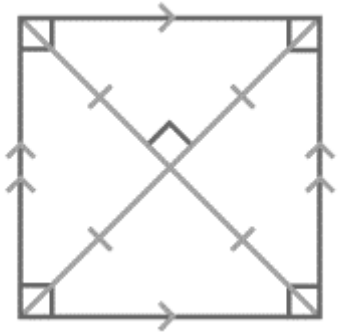
3.1 Isosceles, right angled triangle	
3.2 Scalene, acute-angled triangle	

4 Which quadrilaterals are described by the following characteristics? Make a neat drawing of each. (Do not assume properties that are not given)

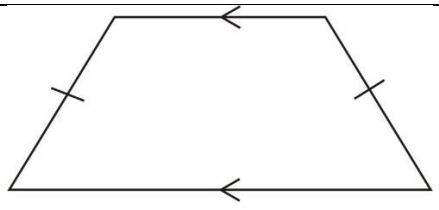
4.1 A rectangle with adjacent sides equal

Square	

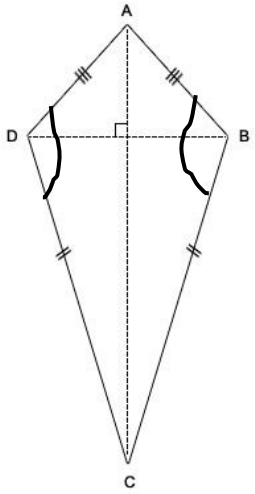
4.2 The diagonals form four isosceles right triangles

Square	

4.3 A trapezium in which the non-parallel sides are equal

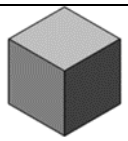
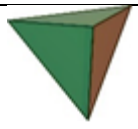
Isosceles trapezium	

4.4 One pair of opposite angles equal and the diagonals are perpendicular

Kite	

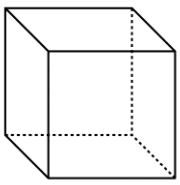
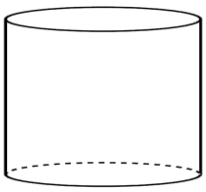
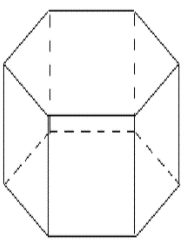
(8)

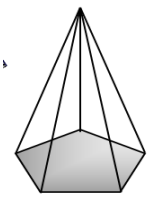
5 Classify the following shapes: polyhedrons or non-polyhedrons. Name each of the objects (a mathematical name) (10)

3D objects	Mathematical name	Polyhedron	Non polyhedron
 a	Ellipse/ovoid		✓
 b	hexahedron	✓	
 c	sphere		✓
 d	tetrahedron	✓	
 e	octahedron	✓	
 f	Cone		✓

3D objects	Mathematical name	Polyhedron	Non polyhedron
 g	Pentagonal prism	✓	
 h	cylinder		✓
 i	sphere		✓
 j	Rectangular based pyramid	✓	

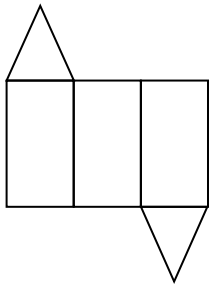
6 Draw neat diagrams of the following polyhedra, made up of: (Name the polyhedra) (8)

6.1 Regular hexahedron Cube	
6.2 Two circles and a rectangle Cylinder	
6.3 Two hexagons and rectangles Hexagonal prism	

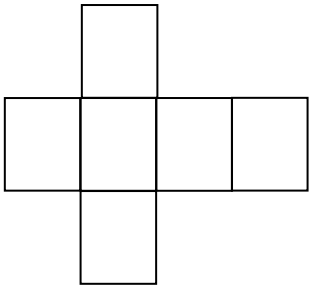
6.4 A hexahedron which is a pyramid Pentagonal pyramid	

7 Draw the nets of the following polyhedra:

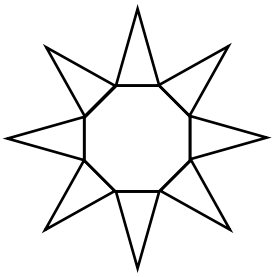
7.1 Triangular prism



7.2 Cube

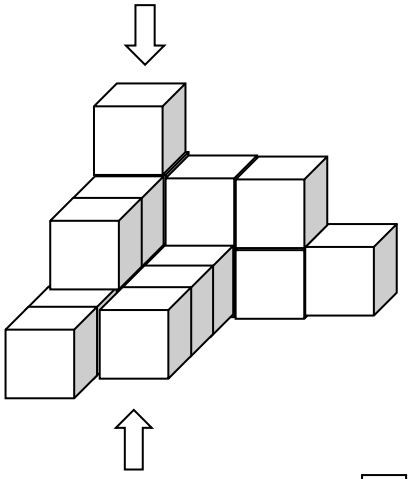


7.3 Octagonal pyramid



(6)

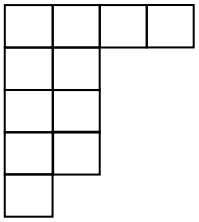
8 Draw the front, top and side view of the following structure



(4)

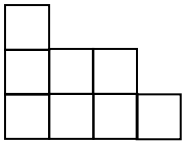


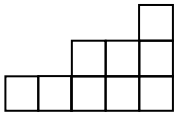
Side view



Top view

Front view



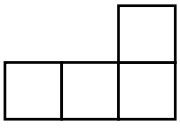


Side view

Front view

9. The side view and front view of a 3D object, built from cubes, are given. Draw the 3D object (3)

Front view



Side view

