PHYSICS GRADE 11
UNIT 5 SUMMARY

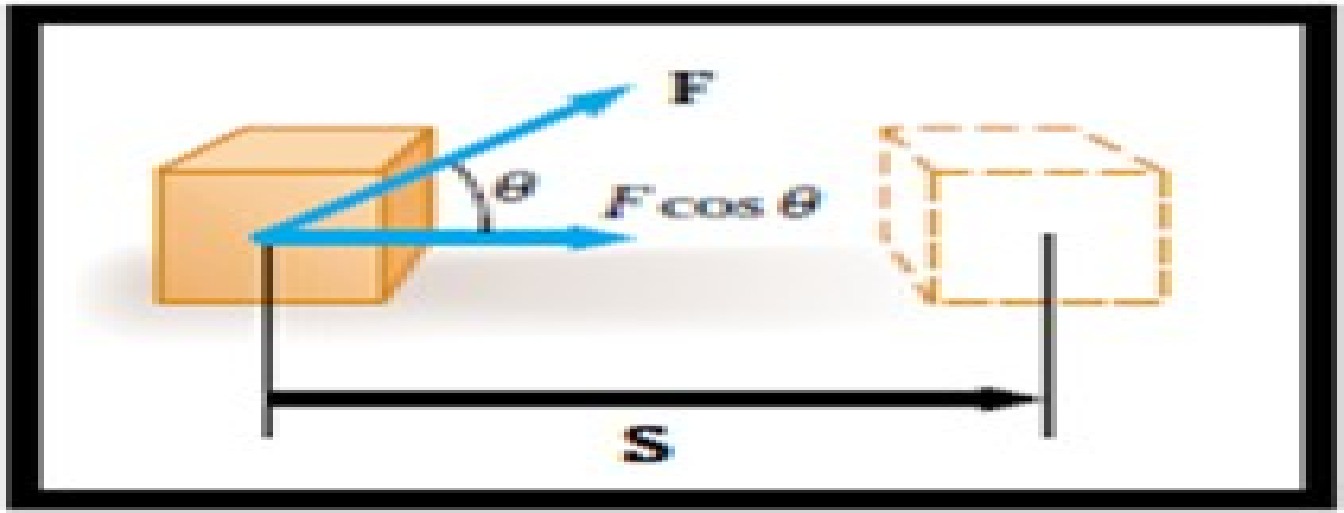
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Work, energy and power

Previous Points

Mathematical expression of mechanical work



$$W = \vec{F}.\vec{S} \quad \rightarrow \quad W = FS\cos\theta$$

$$W = FS\cos\theta$$

Work done by Constant Force

- Constant applied external force
  - Gravitational force
  - ✓ Frictional force

Work done by Variable force

- Variable applied external force
  - Spring

Work done by constant force

- Work done by Constant Applied force

$$W_F = F.S = FS\cos\theta$$

- Work done by frictional force

$$W_f = F.S = f\,S\cos 180$$

$$W_f = -f\,s$$

- Work done by Gravitational force

$$W_g = \pm mgh$$

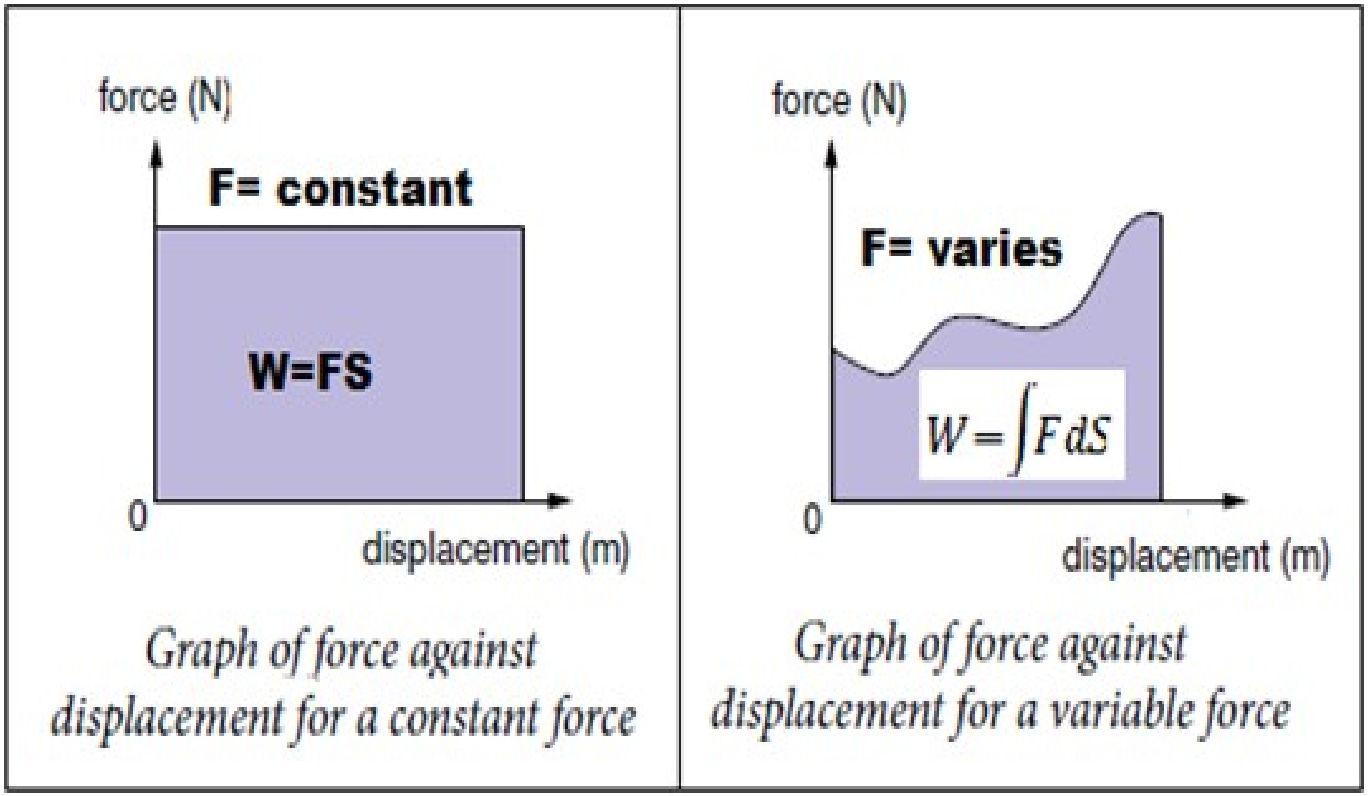
Work done by Variable force

Work done by spring force/restoring force/

$$W_S = -\frac{1}{2}kx^2$$

Graphical representation of work

- The area under this graph represents work done.

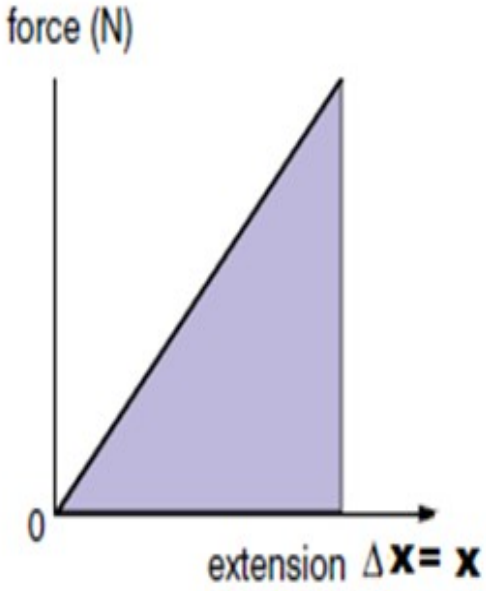


Graph of force against displacement for a constant force

Graph of force against displacement for a variable force

Work done by spring (W_S)

The work done by the spring is derived from integration as follows,



$$W = \int F dS \qquad W_S = \int F_s dx$$

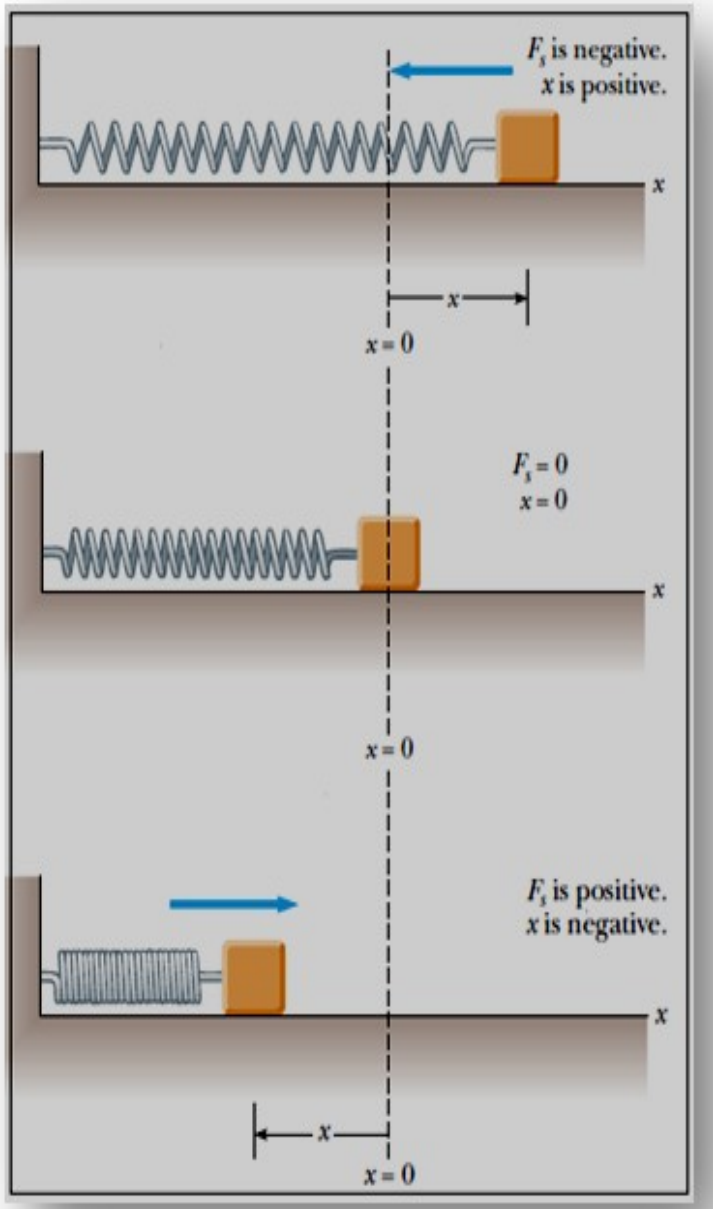
but $F_s = -kx$

$$W_S = \int (-kx) dx$$

$$\Rightarrow W_S = \int (-kx) dx = \frac{1}{2} k x^2$$

where x or Δx is the extension.

$$W_S = -\frac{1}{2} k x^2$$



Today's lesson

- Work done by conservative force and non-conservative force.
- Define Energy
- State work energy theorem and law of conservation of energy
- List the types of energy
- Define power
- Solve some problems concerning work, energy and power

Work done by conservative force and non-conservative force/***dissipative force***/

- Work done by conservative force

♥A force is conservative if its net work done for a complete path is ZERO.

♥Work done by such forces is independent of the path.

Example of such force are

Gravitational force

Elastic force like **spring force**

- Work done by non-conservative force

♥A force is non-conservative if its net work done for a complete path is DOUBLED.

♥Work done by such force depends on the path that we choose

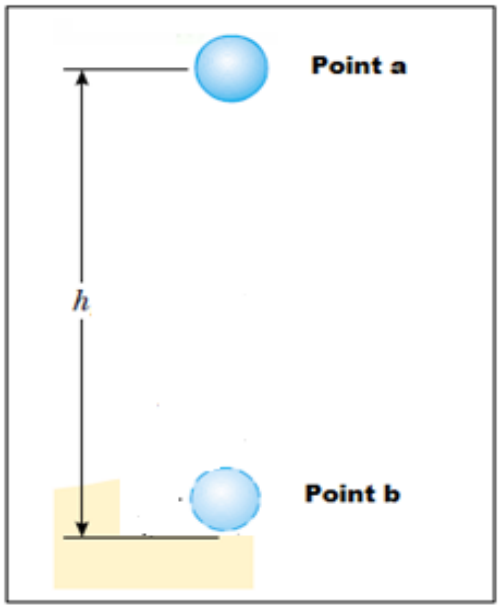
Example of such force are

- Work done by ***frictional forces***, damping force, dragging force may be considered as Conservative force.

Work done by gravitational force is zero for a complete path

Work done by potential forces like gravitational, elastic or electrostatic may be considered as Conservative force. For simplicity let us take only gravitational force.

Suppose mass (m) is displaced from point (a) to (b) and back to (a) as shown below



Work done From (a) to (b) is performed by gravity, it is positive work done since gravity and displacement are acting along the same direction.

$W_{a\to b} = Fs\cos\theta = mg\,h\cos 0$

$W_{a\to b} = mgh$

Work done From (b) to (a) is performed against gravity by some agent, though the agent performs positive work done the gravity performs negative work. since gravity and displacement are acting in opposite direction.

$W_{b\to a} = Fs\cos\theta = mgh\cos 180$

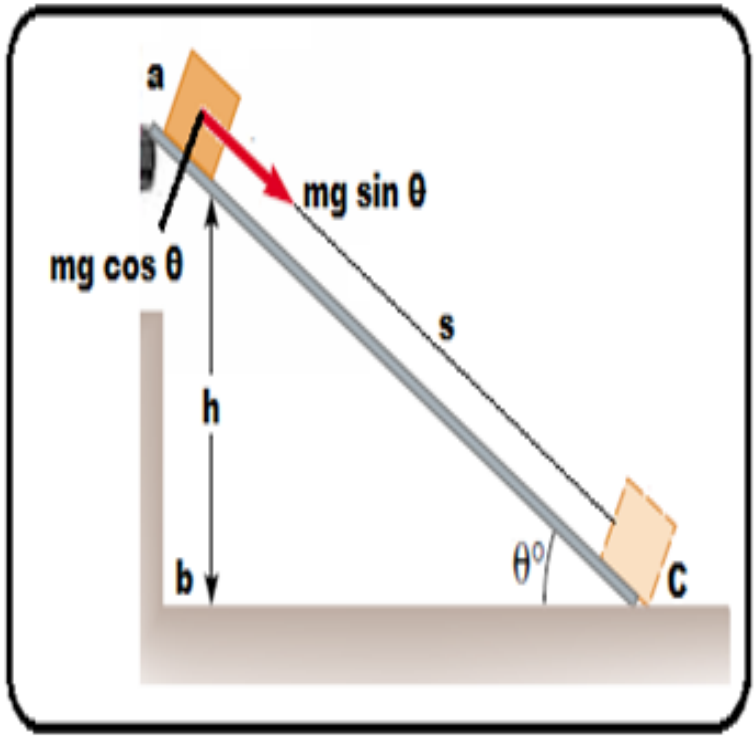
$W_{b\to a} = mg\,h(-)$

$W_{b\to a} = -mgh$

$W_{net} = W_{a\to b} + W_{b\to a} = (mg\,h) + (-mgh) \;\; = 0$

$W_{net} = 0$ *net work is zero for complate path.*

Work done by conservative force is independent of the path



Now let a mass (m) displaces from point (a) to point (b) along a path of height (h).

$W_{a\to b} = Fs\cos\theta = mgh\cos 0$

$W_{a\to b} = mgh$

$W_{a\to c} = Fs\cos\theta = Fs\cos 0 = FS$ but the force

$F = mg\sin\theta$

$W_{a\to c} = (mg\sin\theta)(S) = (mg)(S\sin\theta)$ but

$S\sin\theta = h$

$W_{a\to c} = mgh$, therefore

$W_{a\to b} = W_{a\to c}$ i.e. independent of the path.

ENERGY

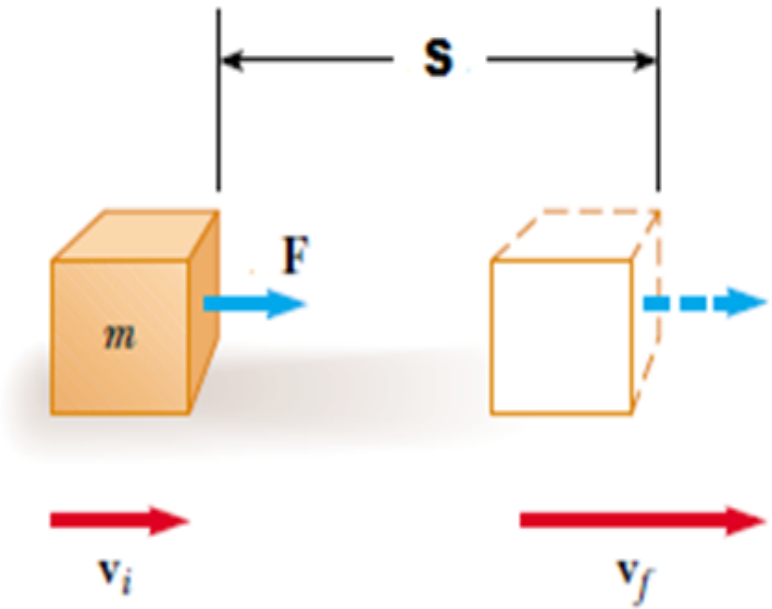
Energy is one of the abstract terms commonly used in science. It is one of the core entities of the universe. In physics energy is defined as

*" **the ability / capacity / of something which enables us to perform work ".***

This capacity /ability/ which enables us to perform work is found in different forms; even mass itself can also be as a source of energy according to Einstein equation of mass-energy relation. Here in this topic we just focus only on mechanical form of energy and this are Kinetic energy and potential energy.

i) *Kinetic Energy* (K_E)

Suppose a mass (m) is moving with initial velocity (v_i) displaces a distance of (S) and reaches a velocity of (v_f) as shown below.



$$v_f^2 = v_i^2 + 2aS \Rightarrow \quad v_f^2 - v_i^2 = 2aS$$

$\frac{v_f^2 - v_i^2}{2} = aS$ Multiplying both sides by its mass (m)

$$m\left(\frac{v_f^2 - v_i^2}{2}\right) = maS$$

$maS = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad \Rightarrow$ but $ma = F$

$FS = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \quad \Rightarrow$ since $FS = W$

$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The quantity on the right side of the above equation i.e ($1/2mv^2$) is kinetic energy.

It is a type of energy formed on a given object due to its motion. This energy enable us to perform work due to the difference in speed, it is all governed under Work- kinetic energy theorem.

Exercise

What is the kinetic energy possessed by 4 kg object as its speed is exactly 6m/s?

- Given:- m=4 kg

V=6m/s

Required:- ke=?

Solution:- $ke = \frac{1}{2} mv^2$

$ke = \frac{1}{2} (4)(6)^2$

$ke = 72\ J$

Work- kinetic energy theorem.

"In the case in which work is done on a system and the only change in the system is in its speed, the work done by the net force equals the change in kinetic energy of the system".

$$W = K_f - K_i \qquad W = \Delta k_E$$

- The work–kinetic energy theorem indicates that the speed of a particle will increase if the net work done on it is positive, because the final kinetic energy will be greater than the initial kinetic energy. The speed will decrease if the net work is negative, because the final kinetic energy will be less than the initial kinetic energy.

Exercise

What is the final speed of a 4 kg object , if its initial speed is 2 m/s; as 42 J of mechanical work is performed on it?

- Given:- m=4 kg

  Vi=2 m/s

  W= 42 J

Required:- V_f =?

Solution:- From work energy theorem

$$W = \Delta k_e \Rightarrow W = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2$$

$$W + \tfrac{1}{2} m v_i^2 = \tfrac{1}{2} m v_f^2$$

$$42 + \tfrac{1}{2} (4)(2)^2 = \tfrac{1}{2} m v_f^2$$

$$50 = \tfrac{1}{2} m v_f^2$$

$$v_f^2 = 25$$

$$v_f = 5 \text{ m/s}$$

ii) *Potential Energy (U)*

Potential energy is the other form of energy formed due to the change of position of objects.

Potential energy itself has different forms; some are gravitational potential energy, elastic potential energy, nuclear potential energy, electrical potential energy e.t.c.

(a) **Gravitational Potential Energy (GPE)**

It is potential energy formed due to the change of elevation of massive objects.

♣ Suppose a book is moving up from initial point (a) to final point (b) against gravity.

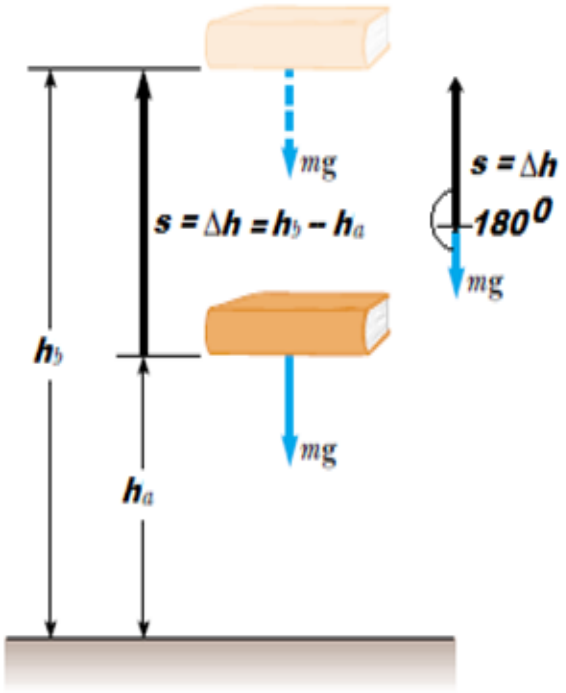
$W = F.S = Fs\cos\theta$ but $F = mg$

$S = \Delta h = h_b - h_a$ & $\theta = 180^0$

$W = F.S = Fs(\cos 180)$

$W = F.S = mg\Delta h(-)$

$W = -mg(h_b - h_a) \Rightarrow W = -(mgh_b - mgh_a)$



♣ Suppose a book is moving down from point (a) to point (b) along the direction of gravitational force.

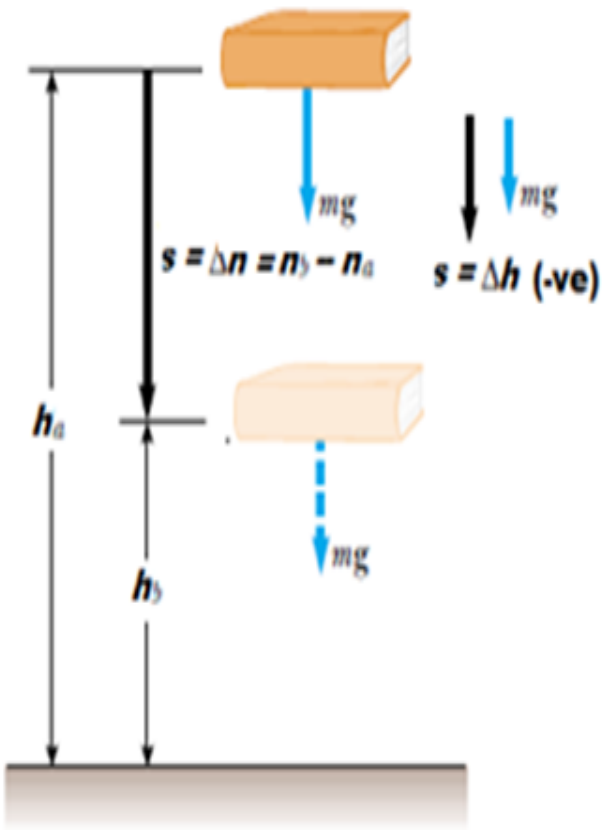
$W = F.S = Fs\cos\theta$ but $F = mg$

$s = \Delta h = h_b - h_a$ & $\theta = 0^0$

$W = F.S = Fs(\cos 0)$

$W = F.S = mg\Delta h$

$W = mg(h_b - h_a) \Rightarrow W = (mgh_b - mgh_a)$ (Since $h_b < h_a$ it is negative)



NOTE:- The quantity on the right side i.e. (mgh) is gravitational potential energy (GPE/ U_g).

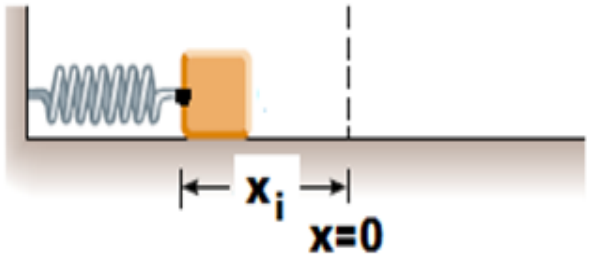
$$GPE \text{ or } U_g = mgh$$

♣ Higher elevation means higher gravitational potential energy.

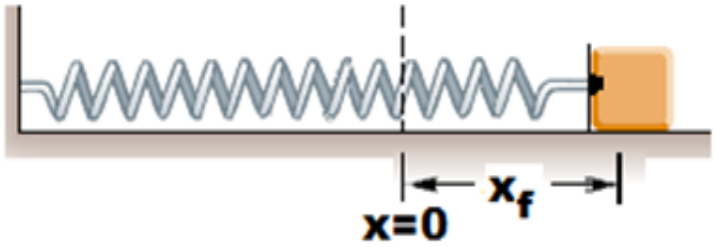
b) Elastic Potential Energy [EPE]/ U_E /

It is potential energy formed due to the change in shape of elastic materials. We usually focus on springs.

The elastic potential energy of the system can be thought of as the energy stored in the deformed spring (one that is either compressed or stretched from its equilibrium position). To visualize this, consider the Figure below, which shows a spring on a frictionless, horizontal surface. When a block is pushed against the spring and the spring is compressed a distance x, the elastic potential energy stored in the spring.



For a block-spring system, equilibrium position (x=0) is the reference point, let the block is attached with a spring of stiffness or spring constant (k), initially it is compressed to a location (x_i), then it is released and pass the equilibrium position (x=0) the block and the spring may be separated at equilibrium positions but since they are attached to each other, the spring starts to stretch beyond the equilibrium position to a location of (x_f), as shown below.



The work done by the spring in displacing the block from

x_i to x_f . $W = F_s \Delta x = F_s(x_f - x_i)$

$$W = F_{s_f} x_f - F_{s_i} x_i$$

$$F_{s_f} = -\frac{1}{2} k x_f \;\&\; F_{s_i} = -\frac{1}{2} k x_i$$

$$W = \left(-\frac{1}{2} k x_f . x_f\right) - \left(-\frac{1}{2} k x_i . x_i\right) \qquad W = -\frac{1}{2} k x_f^2 - -\frac{1}{2} k x_i^2$$

$$W = -\left(\frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2\right)$$

Summary

- Kinetic Energy

$K_e = 1/2mv^2$

$W = W_n + W_c = \Delta K_e$

$W = \Delta K_e$

$W_c = -\Delta U$

Potential Energy	
Gravitational Energy GPE= mgh	Elastic potential energy $EPE = 1/2kx^2$

EXERSISE

What is the elastic potential energy stored by a spring as it is compressed to 20 cm, where the spring is characterized by stretching 2m using 120 N ?

Given:- Δx or $x = 20\ cm = 0.2m$

Required:- EPE=?

Solution:- $EPE = \frac{1}{2} k \Delta x^2$

however the spring constant or force constant is not directly given, so we have to determine from the give characteristics

F= 120 N and x= 2m

$F \sim x \Rightarrow F = kx \Rightarrow k = F/x \Rightarrow K = 120/2 = 60 N/m$

Therefore, $EPE = \frac{1}{2} k \Delta x^2$

$EPE = \frac{1}{2} (60)(0.2)^2$

$EPE = 1.2\ J$

Conservation of Energy

Energy is always conserved where ever we go in the universe at any moment, the total form of energy is conserved, it may be converted from one form to the other. This is governed under the famous law known as law of conservation of energy. This is stated as *"Energy is neither created nor destroyed, rather it transfers from one form to another".*

Here we mainly focus on Mechanical form of energies.

"For an isolated system mechanical energy is conserved, if there is no work done by dissipative forces."

Since mechanical energy is the sum of kinetic and potential energy it can be represented as

$\Delta M_E = \Delta K_E + \Delta U$ Where

$\Delta M_E = change\ of\ mechanical\ enrgy$,

$\Delta K_E = chnage\ of\ kinetic\ energy\ and$

$\Delta U = change\ of\ potential\ enrgy.$

From work- kinetic theorem, we have $W = \Delta K_E$

where W is total work done i.e. work done by conservative and non – conservative $W_C + W_n$

$W_C + W_n = \Delta K_E$ but from work- potential energy principle $W = -\Delta U$ where W is work done by non conservative. $W_C = -\Delta U$

Therefore, $W = \Delta K_E \Rightarrow W_C + W_n = \Delta K_E \Rightarrow (-\Delta U) + W_n = \Delta K_E \Rightarrow W_n = \Delta K_E + \Delta U = \Delta M_E$

$$W_n = \Delta M_E$$

NOTE: - The work done by non-conservative (dissipative) force is equals to the change of mechanical work.

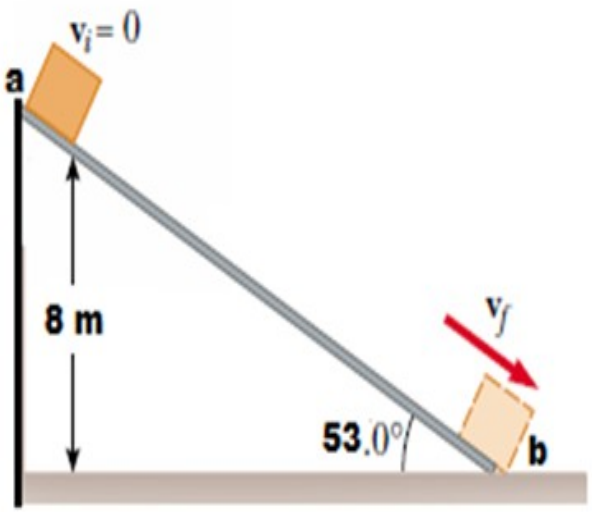
$$W_n = \Delta M_E = Me_f - Me_i \Rightarrow W_n = Me_f - Me_i \Rightarrow Me_i + W_n = Me_f$$

$$Me_i + W_n = Me_f \Rightarrow Ke_i + U_i + W_n = Ke_f + U_f$$

For the above equation if work done by non conservative force is zero mechanical energy will be conserved unless it is not conserved. i.e. $W_n = 0$

$Me_i + W_n = Me_f \Rightarrow Me_i = Me_f \Rightarrow$ $Ke_i + U_i = Ke_f + U_f$

Example



A 4 kg block is sliding over an inclined plane as shown below, what is the speed of the block as it reaches at the bottom? If

(a) the block is frictionless

(b) the block has a coefficient of friction of (0.2)

Answer

Given m= 4 kg, h=8m, $\theta = 53^0$ Req $v_f = v_b = ?$

a) the two configuration are point (a) at the top and point (b) at the bottom.

$Me_a + W_n = Me_b \Rightarrow Ke_a + U_a + W_n = Ke_b + U_b$

Since there is no frictional force, work done by non conservative force is zero.

$W_n = 0$ Therefore,

$Ke_a + U_a = Ke_b + U_b \Rightarrow \frac{1}{2}mv_a^2 + mgh_a = \frac{1}{2}mv_b^2 + mgh_b$

$m\left(\frac{1}{2}v_a^2 + gh_a\right) = m\left(\frac{1}{2}v_b^2 + gh_b\right) \Rightarrow \frac{1}{2}v_a^2 + gh_a = \frac{1}{2}v_b^2 + gh_b \Rightarrow$ but $v_a = 0,$

$h_a = h = 8m, h_b = 0$

$v_b = ? \Rightarrow (10)(8) = \frac{1}{2}v_b^2 \Rightarrow v_b = \sqrt{160}$ $v_b = 12.6\ m/s$

b) There is frictional force between a and b, the work done by the non-conservative force is due to the frictional force formed on the rough surface.

$\frac{1}{2}mv_a^2 + mgh_a + W_n = \frac{1}{2}mv_b^2 + mgh_b$ Thus $W_n = -fs$

but $f = \mu F_N \Rightarrow f = \mu mg\cos\theta \Rightarrow f = (0.2)(4)(10)\cos 53 \Rightarrow$

$f = (0.2)(4)(10)\cos 53 = 4.8N$

$W_n = -(4.8)(10) = -48\,\text{J}$

$\frac{1}{2}mv_a^2 + mgh_a + W_n = \frac{1}{2}mv_b^2 + mgh_b$ but, $v_a = 0, \quad h_a = h = 8m, h_b = 0$

$v_b = ?$

$mgh_a + W_n = \frac{1}{2}mv_b^2 \Rightarrow (4)(10)(8) + (-48) = \frac{1}{2}(4)v_b^2 \Rightarrow 2v_b^2 = 320 - 48 \Rightarrow$

$2v_b^2 = 272 \Rightarrow v_b^2 = 136 \Rightarrow$

$v_b = \sqrt{136}$ $\quad v_b = 11.7\,{}^{m}/_{s}$

Power

Power is the rate of doing work, or the work done per a unit time. It is measured in joules per second (j/s), which are also known as watts (W).

$$\bar{P} = \frac{W}{\Delta t}$$

If an external force is applied to an object (which we assume acts as a particle), and if the work done by this force in the time interval Δt is W, then the average power during this interval is defined as

Instantaneous power delivered by a force is given as

$$P = \lim_{n \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

$$P = \frac{dW}{dt}$$

but $dW = F.ds$ if F= constant

$$P = \frac{F.ds}{dt} = F.v \quad \Rightarrow \quad P = F.v$$

The SI unit of Power is joule (J), but there is also common unit known as *horsepower* (hP)

- 1hP= 746 J

Exersise

An engine raises a load of 100 kg from a mine that is 300 m deep within 2 minutes. What is the power of the engine?

- Given:- m=100kg

  h= 300 m

  t= 2 min= 2x60=120 sec

Required:- P =?

Solution:- P=w/t or P= F.V= Fv cos θ

W=mgh ⇒ W=(100)(10)(300)=300000

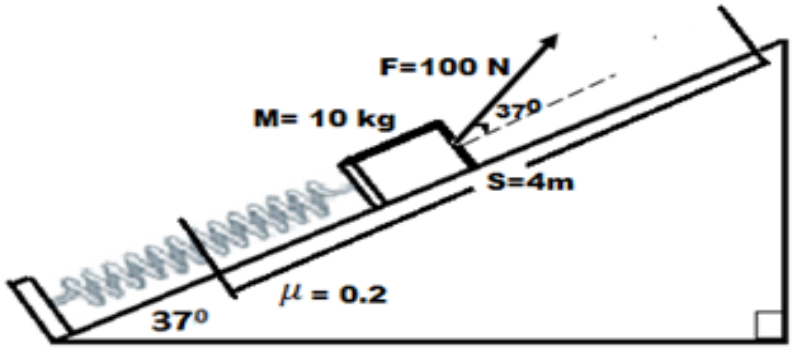
Since P=w/t ⇒ P=(300000)/(120)=2500 W

P=2500 W or

P=2.5 KW

Exercise 1

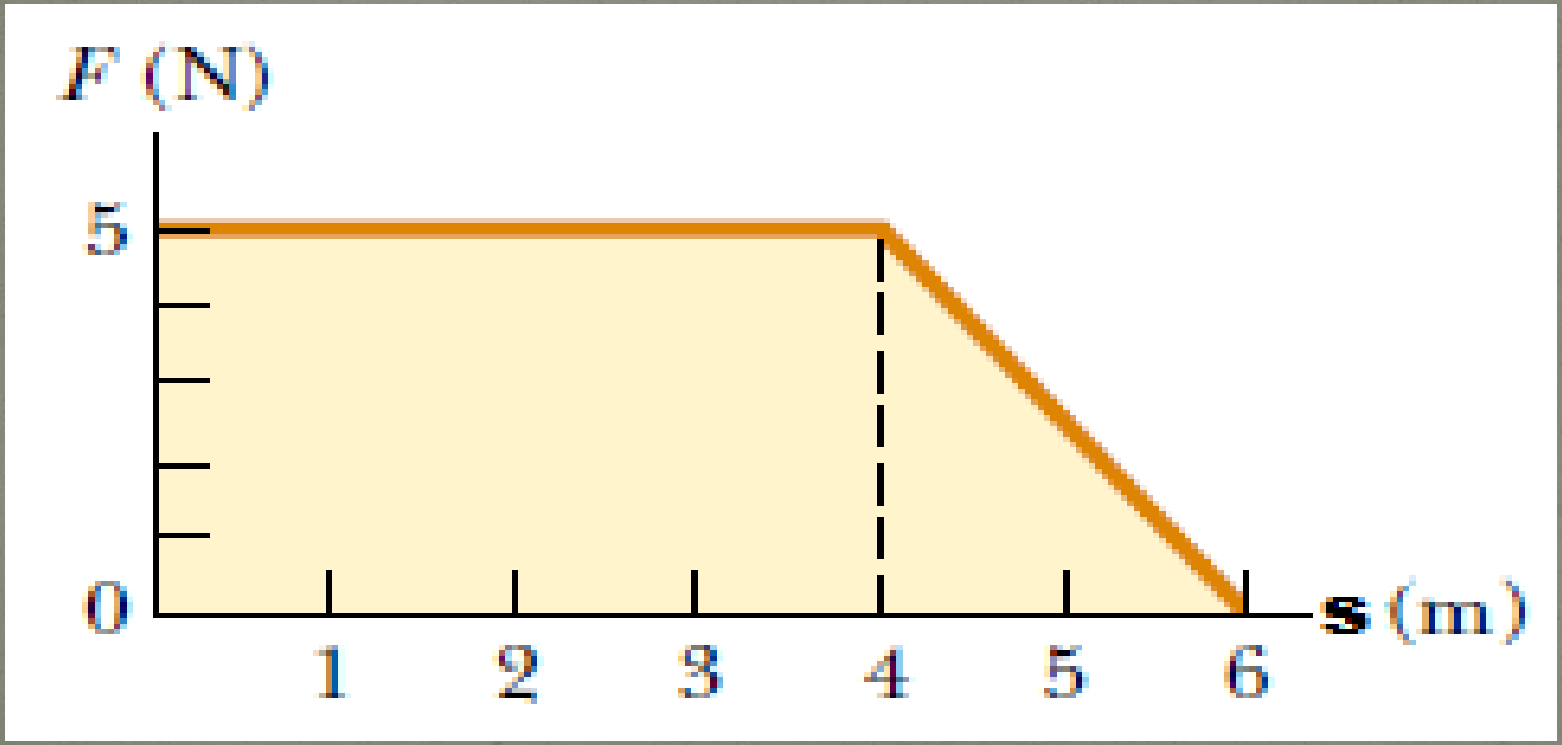
Suppose a 10 kg block which is attached with a spring of force constant 20 N/m is pulled up by 100 N forces on rough inclined plane acting at an angle of 37^0 to a displacement of 4m as shown below. Taking coefficient of friction (μ) as 0.2, compute the work done by the



(a) applied force (W_F)
(b) gravitational force (W_g)
(c) frictional force (W_f)
(d) restoring force/Spring/ (W_s)
(e) Net work done (W)

Exersise-2

What is the work done for the force-displacement graph shown below ?



3. A woman driving a 2000 kg car along a level road at 30m/s takes her foot off the gas to See how far her car will roll before it slows to a stop. She discovers that it takes 150m. What is average force of friction acting on the car?

A) 9000N

B) 6000N

C) 3000N

D) 400N