Uploaded by :



www.engzenon.com

Chapter 1

1.1

$$n_i = BT^{3/2}e^{-E_g/2kT}$$

(a) Silicon

(i) $$n_i = \left(5.23\times10^{15}\right)(250)^{3/2}\exp\left[\frac{-1.1}{2\left(86\times10^{-6}\right)(250)}\right]$$

$$= 2.067\times10^{19}\exp[-25.58]$$

$$\underline{n_i = 1.61\times10^{8}\ \text{cm}^{-3}}$$

(ii) $$n_i = \left(5.23\times10^{15}\right)(350)^{3/2}\exp\left[\frac{-1.1}{2\left(86\times10^{-6}\right)(350)}\right]$$

$$= 3.425\times10^{19}\exp[-18.27]$$

$$\underline{n_i = 3.97\times10^{11}\ \text{cm}^{-3}}$$

(b) GaAs

(i) $$n_i = \left(2.10\times10^{14}\right)(250)^{3/2}\exp\left[\frac{-1.4}{2\left(86\times10^{-6}\right)(250)}\right]$$

$$= \left(8.301\times10^{17}\right)\exp[-32.56]$$

$$\underline{n_i = 6.02\times10^{3}\ \text{cm}^{-3}}$$

(ii) $$n_i = \left(2.10\times10^{14}\right)(350)^{3/2}\exp\left[\frac{-1.4}{2\left(86\times10^{-6}\right)(350)}\right]$$

$$= \left(1.375\times10^{18}\right)\exp[-23.26]$$

$$\underline{n_i = 1.09\times10^{8}\ \text{cm}^{-3}}$$

1.2

a. $$n_i = BT^{3/2}\exp\left(\frac{-Eg}{2kT}\right)$$

$$10^{12} = 5.23\times10^{15}T^{3/2}\exp\left(\frac{-1.1}{2(86\times10^{-6})(T)}\right)$$

$$1.91\times10^{-4} = T^{3/2}\exp\left(-\frac{6.40\times10^{3}}{T}\right)$$

By trial and error, $T \approx 368$ K

b. $$n_i = 10^{9}\ \text{cm}^{-3}$$

$$10^{9} = 5.23\times10^{15}T^{3/2}\exp\left(\frac{-1.1}{2\left(86\times10^{-6}\right)(T)}\right)$$

$$1.91\times10^{-7} = T^{3/2}\exp\left(-\frac{6.40\times10^{3}}{T}\right)$$

By trial and error, $T \approx 268°$ K

1.3

Silicon

(a) $$n_i = \left(5.23\times10^{15}\right)(100)^{3/2}\exp\left[\frac{-1.1}{2\left(86\times10^{-6}\right)(100)}\right]$$

$$= \left(5.23\times10^{18}\right)\exp[-63.95]$$

$$\underline{n_i = 8.79\times10^{-10}\ \text{cm}^{-3}}$$

(b) $$n_i = \left(5.23\times10^{15}\right)(300)^{3/2}\exp\left[\frac{-1.1}{2\left(86\times10^{-6}\right)(300)}\right]$$

$$= \left(2.718\times10^{19}\right)\exp[-21.32]$$

$$\underline{n_i = 1.5\times10^{10}\ \text{cm}^{-3}}$$

(c) $$n_i = \left(5.23\times10^{15}\right)(500)^{3/2}\exp\left[\frac{-1.1}{2\left(86\times10^{-6}\right)(500)}\right]$$

$$= \left(5.847\times10^{19}\right)\exp[-12.79]$$

$$\underline{n_i = 1.63\times10^{14}\ \text{cm}^{-3}}$$

Germanium.

(a) $$n_i = \left(1.66\times10^{15}\right)(100)^{3/2}\exp\left[\frac{-0.66}{2\left(86\times10^{-6}\right)(100)}\right] = \left(1.66\times10^{18}\right)\exp[-38.37]$$

$$\underline{n_i = 35.9\ \text{cm}^{-3}}$$

(b) $$n_i = \left(1.66\times10^{15}\right)(300)^{3/2}\exp\left[\frac{-0.66}{2\left(86\times10^{-6}\right)(300)}\right] = \left(8.626\times10^{18}\right)\exp[-12.79]$$

$$\underline{n_i = 2.40\times10^{13}\ \text{cm}^{-3}}$$

(c) $$n_i = \left(1.66\times10^{15}\right)(500)^{3/2}\exp\left[\frac{-0.66}{2\left(86\times10^{-6}\right)(500)}\right] = \left(1.856\times10^{19}\right)\exp[-7.674]$$

$$\underline{n_i = 8.62\times10^{15}\ \text{cm}^{-3}}$$

1.4

(a) n-type; $n_o = 10^{15}\ \text{cm}^{-3}$; $$p_o = \frac{n_i^2}{n_o} = \frac{\left(2.4\times10^{13}\right)^2}{10^{15}} = 5.76\times10^{11}\ \text{cm}^{-3}$$

(b) n-type; $n_o = 10^{15}\ \text{cm}^{-3}$; $$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{10^{15}} = 2.25\times10^{5}\ \text{cm}^{-3}$$

1.5

(a) p-type; $p_o = 10^{16}\ \text{cm}^{-3}$; $n_o = \dfrac{n_i^2}{p_o} = \dfrac{\left(1.8\times10^6\right)^2}{10^{16}} = 3.24\times10^{-4}\ \text{cm}^{-3}$

(b) p-type; $p_o = 10^{16}\ \text{cm}^{-3}$; $n_o = \dfrac{n_i^2}{p_o} = \dfrac{\left(2.4\times10^{13}\right)^2}{10^{16}} = 5.76\times10^{10}\ \text{cm}^{-3}$

1.6

(a) *n*-type

(b) $n_o = N_d = 5\times10^{16}\ \text{cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^{3}\ \text{cm}^{-3}$$

(c) $n_o = N_d = 5\times10^{16}\ \text{cm}^{-3}$

From Problem 1.1(a)(ii) $n_i = 3.97\times10^{11}\ \text{cm}^{-3}$

$$p_o = \frac{\left(3.97\times10^{11}\right)^2}{5\times10^{16}} = 3.15\times10^{6}\ \text{cm}^{-3}$$

1.7

(a) p-type; $p_o = 5\times10^{16}\ \text{cm}^{-3}$; $n_o = \dfrac{n_i^2}{p_o} = \dfrac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^{3}\ \text{cm}^{-3}$

(b) p-type; $p_o = 5\times10^{16}\ \text{cm}^{-3}$; $n_o = \dfrac{n_i^2}{p_o} = \dfrac{\left(1.8\times10^6\right)^2}{5\times10^{16}} = 6.48\times10^{-5}\ \text{cm}^{-3}$

1.8

(a) Add boron atoms

(b) $N_a = p_o = 2\times10^{17}\ \text{cm}^{-3}$

(c) $n_o = \dfrac{n_i^2}{p_o} = \dfrac{\left(1.5\times10^{10}\right)^2}{2\times10^{17}} = 1.125\times10^{3}\ \text{cm}^{-3}$

1.9

(a) $n_o = 5\times10^{15}\ cm^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{15}} \Rightarrow p_o = 4.5\times10^{4}\ cm^{-3}$$

(b) $n_o > p_o \Rightarrow$ n-type

(c) $n_o \cong N_d = 5\times10^{15}\ cm^{-3}$

1.10

a. Add Donors

$N_d = 7\times10^{15}\ \text{cm}^{-3}$

b. Want $p_o = 10^6\ \text{cm}^{-3} = n_i^2 / N_d$

So $n_i^2 = \left(10^6\right)\left(7\times10^{15}\right) = 7\times10^{21}$

$$= B^2T^3 \exp\left(\frac{-Eg}{kT}\right)$$

$$7\times10^{21} = \left(5.23\times10^{15}\right)^2 T^3 \exp\left(\frac{-1.1}{\left(86\times10^{-6}\right)(T)}\right)$$

By trial and error, $T \approx 324°$ K

1.11

(a) $I = A\sigma\text{E} = \left(10^{-5}\right)(1.5)(10) \Rightarrow I = 0.15$ mA

(b) $I = \dfrac{A\text{E}}{\rho} \Rightarrow \text{E} = \dfrac{I\rho}{A} = \dfrac{\left(1.2\times10^{-3}\right)(0.4)}{\left(2\times10^{-4}\right)} = 2.4$ V/cm

---

1.12

$$J = \sigma\text{E} \Rightarrow \sigma = \frac{J}{\text{E}} = \frac{120}{18} = 6.67\left(\Omega - cm\right)^{-1}$$

$$\sigma \cong e\mu_n N_d \Rightarrow N_d = \frac{\sigma}{e\mu_n} = \frac{(6.67)}{\left(1.6\times10^{-19}\right)(1250)} = 3.33\times10^{16}\ \text{cm}^{-3}$$

1.13

(a) $\rho \cong \dfrac{1}{e\mu_n N_d} \Rightarrow N_d = \dfrac{1}{e\mu_n\rho} = \dfrac{1}{\left(1.6\times10^{-19}\right)(1250)(0.65)} = 7.69\times10^{15}\ \text{cm}^{-3}$

(b) $J = \dfrac{\text{E}}{\rho} \Rightarrow \text{E} = \rho J = (0.65)(160) = 104$ V/cm

---

1.14

(a) $\sigma \cong e\mu_n N_d \Rightarrow N_d = \dfrac{\sigma}{e\mu_n} = \dfrac{1.5}{\left(1.6\times10^{-19}\right)(1000)} = 9.375\times10^{15}\ \text{cm}^{-3}$

(b) $N_a = \dfrac{\sigma}{e\mu_p} = \dfrac{0.8}{\left(1.6\times10^{-19}\right)(400)} = 1.25\times10^{16}\ \text{cm}^{-3}$

1.15

(a) For n-type, $\sigma \cong e\mu_n N_d = \left(1.6\times10^{-19}\right)(8500)N_d$

For $10^{15} \le N_d \le 10^{19}\ cm^{-3} \Rightarrow 1.36 \le \sigma \le 1.36\times10^4\left(\Omega - cm\right)^{-1}$

(b) $J = \sigma E = \sigma(0.1) \Rightarrow 0.136 \le J \le 1.36\times10^3\ A/cm^2$

1.16

$D_n = (0.026)(1250) = 32.5 \text{ cm}^2/\text{s}; \quad D_p = (0.026)(450) = 11.7 \text{ cm}^2/\text{s}$

$$J_n = eD_n \frac{dn}{dx} = (1.6\times10^{-19})(32.5)\left(\frac{10^{16} - 10^{12}}{0 - 0.001}\right) = -52 \text{ A/cm}^2$$

$$J_p = -eD_p \frac{dp}{dx} = -(1.6\times10^{-19})(11.7)\left(\frac{10^{12} - 10^{16}}{0 - 0.001}\right) = -18.72 \text{ A/cm}^2$$

Total diffusion current density

$J = -52 - 18.72 = -70.7 \text{ A/cm}^2$

1.17

$$J_p = -eD_p \frac{dp}{dx}$$

$$= -eD_p (10^{15})\left(\frac{-1}{L_p}\right)\exp\left(\frac{-x}{L_p}\right)$$

$$J_p = \frac{(1.6\times10^{-19})(15)(10^{15})}{10\times10^{-4}}\exp\left(\frac{-x}{L_p}\right)$$

$$J_p = 2.4\, e^{-x/L_p}$$

(a) $x = 0 \qquad J_p = 2.4 \text{ A/cm}^2$

(b) $x = 10\ \mu\text{m} \qquad J_p = 2.4\, e^{-1} = 0.883 \text{ A/cm}^2$

(c) $x = 30\ \mu\text{m} \qquad J_p = 2.4\, e^{-3} = 0.119 \text{ A/cm}^2$

1.18

a. $N_a = 10^{17} \text{ cm}^{-3} \Rightarrow \underline{p_o = 10^{17} \text{ cm}^{-3}}$

$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8\times10^6)^2}{10^{17}} \Rightarrow \underline{n_o = 3.24\times10^{-5} \text{ cm}^{-3}}$$

b. $n = n_o + \delta n = 3.24\times10^{-5} + 10^{15} \Rightarrow \underline{n = 10^{15} \text{ cm}^{-3}}$

$p = p_o + \delta p = 10^{17} + 10^{15} \Rightarrow \underline{p = 1.01\times10^{17} \text{ cm}^{-3}}$

1.19 $V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$

(a) (i) $$V_{bi} = (0.026)\ln\left[\frac{(5\times10^{15})(5\times10^{15})}{(1.5\times10^{10})^2}\right] = 0.661 \text{ V}$$

(ii) $$V_{bi} = (0.026)\ln\left[\frac{(5\times10^{17})(10^{15})}{(1.5\times10^{10})^2}\right] = 0.739 \text{ V}$$

(iii) $$V_{bi} = (0.026)\ln\left[\frac{(10^{18})(10^{18})}{(1.5\times10^{10})^2}\right] = 0.937 \text{ V}$$

(b) (i) $$V_{bi} = (0.026)\ln\left[\frac{(5\times10^{15})(5\times10^{15})}{(1.8\times10^{6})^2}\right] = 1.13 \text{ V}$$

(ii) $$V_{bi} = (0.026)\ln\left[\frac{(5\times10^{17})(10^{15})}{(1.8\times10^{6})^2}\right] = 1.21 \text{ V}$$

(iii) $$V_{bi} = (0.026)\ln\left[\frac{(10^{18})(10^{18})}{(1.8\times10^{6})^2}\right] = 1.41 \text{ V}$$

1.20

$$V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

or

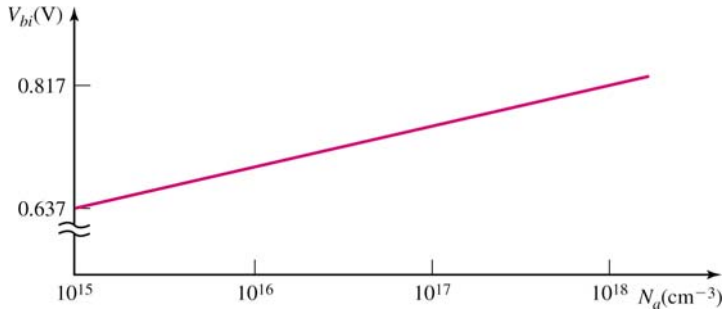
$$N_a = \frac{(n_i^2)}{N_d}\exp\left(\frac{V_{bi}}{V_T}\right) = \frac{(1.5\times10)^2}{10^{16}}\exp\left(\frac{0.712}{0.026}\right) = 1.76\times10^{16} \text{ cm}^{-3}$$

1.21

$$V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.026)\ln\left[\frac{N_a(10^{16})}{(1.5\times10^{10})^2}\right]$$

For $N_a = 10^{15}\ cm^{-3}$, $V_{bi} = 0.637\ V$

For $N_a = 10^{18}\ cm^{-3}$, $V_{bi} = 0.817\ V$



1.22

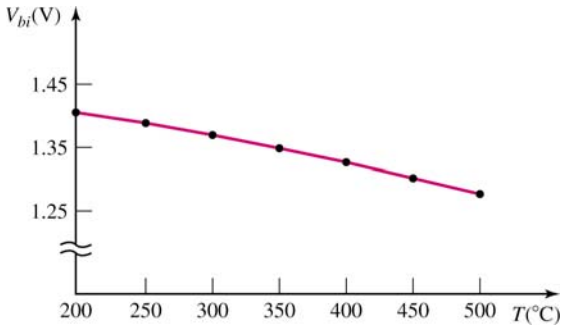
$$kT = (0.026)\left(\frac{T}{300}\right)$$

	kT	$(T)^{3/2}$
200	0.01733	2828.4
250	0.02167	3952.8
300	0.026	5196.2
350	0.03033	6547.9
400	0.03467	8000.0
450	0.0390	9545.9
500	0.04333	11,180.3

$$n_i = \left(2.1\times10^{14}\right)\left(T^{3/2}\right)\exp\left(\frac{-1.4}{2\left(86\times10^{-6}\right)(T)}\right)$$

$$V_{bi} = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

T	n_i	V_{bi}
200	1.256	1.405
250	6.02×10^3	1.389
300	1.80×10^6	1.370
350	1.09×10^8	1.349
400	2.44×10^9	1.327
450	2.80×10^{10}	1.302
500	2.00×10^{11}	1.277



1.23

$$C_j = C_{jo}\left(1+\frac{V_R}{V_{bi}}\right)^{-1/2}$$

$$V_{bi} = \left(0.026\right)\ln\left[\frac{\left(1.5\times10^{16}\right)\left(4\times10^{15}\right)}{\left(1.5\times10^{10}\right)^2}\right] = 0.684 \text{ V}$$

(a) $C_j = \left(0.4\right)\left(1+\frac{1}{0.684}\right)^{-1/2} = 0.255 \text{ pF}$

(b) $C_j = \left(0.4\right)\left(1+\frac{3}{0.684}\right)^{-1/2} = 0.172 \text{ pF}$

(c) $C_j = \left(0.4\right)\left(1+\frac{5}{0.684}\right)^{-1/2} = 0.139 \text{ pF}$

1.24

(a) $C_j = C_{jo}\left(1+\frac{V_R}{V_{bi}}\right)^{-1/2}$

For $V_R = 5\ V,\ C_j = (0.02)\left(1+\frac{5}{0.8}\right)^{-1/2} = 0.00743\ pF$

For $V_R = 1.5\ V,\ C_j = (0.02)\left(1+\frac{1.5}{0.8}\right)^{-1/2} = 0.0118\ pF$

$$C_j(avg) = \frac{0.00743 + 0.0118}{2} = 0.00962\ pF$$

$$v_C(t) = v_C(final) + \left(v_C(initial) - v_C(final)\right)e^{-t/\tau}$$

where

$$\tau = RC = RC_j(avg) = (47\times10^3)(0.00962\times10^{-12})$$

or

$$\tau = 4.52\times10^{-10}\ s$$

Then $v_C(t) = 1.5 = 0 + (5-0)e^{-t_1/\tau}$

$$\frac{5}{1.5} = e^{+t_1/\tau} \Rightarrow t_1 = \tau\ln\left(\frac{5}{1.5}\right)$$

$$\underline{t_1 = 5.44\times10^{-10}\ s}$$

(b) For $V_R = 0$ V, $C_j = C_{jo} = 0.02\ pF$

For $V_R = 3.5$ V, $C_j = (0.02)\left(1+\frac{3.5}{0.8}\right)^{-1/2} = 0.00863\ pF$

$$C_j(avg) = \frac{0.02 + 0.00863}{2} = 0.0143\ pF$$

$$\tau = RC_j(avg) = 6.72\times10^{-10}\ s$$

$$v_C(t) = v_C(final) + \left(v_C(initial) - v_C(final)\right)e^{-t/\tau}$$

$$3.5 = 5 + (0-5)e^{-t_2/\tau} = 5\left(1 - e^{-t_2/\tau}\right)$$

so that $\underline{t_2 = 8.09\times10^{-10}\ s}$

1.25

$$C_j = C_{jo}\left(1+\frac{V_R}{V_{bi}}\right)^{-1/2}; \quad V_{bi} = (0.026)\ln\left[\frac{(5\times10^{15})(10^{17})}{(1.5\times10^{10})^2}\right] = 0.739\ \text{V}$$

For $V_R = 1$ V,

$$C_j = \frac{0.60}{\sqrt{1+\frac{1}{0.739}}} = 0.391\ \text{pF}$$

For $V_R = 3$ V,

$$C_j = \frac{0.60}{\sqrt{1+\frac{3}{0.739}}} = 0.267\ \text{pF}$$

For $V_R = 5$ V,

$$C_j = \frac{0.60}{\sqrt{1+\frac{5}{0.739}}} = 0.215\ \text{pF}$$

(a) $$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.5\times10^{-3})(0.391\times10^{-12})}} \Rightarrow f_o = 6.57\ \text{MHz}$$

(b) $$f_o = \frac{1}{2\pi\sqrt{(1.5\times10^{-3})(0.267\times10^{-12})}} \Rightarrow f_o = 7.95\,MHz$$

(c) $f_o = \dfrac{1}{2\pi\sqrt{(1.5\times10^{-3})(0.215\times10^{-12})}} \Rightarrow f_o = 8.86 \text{ MHz}$

1.26

a. $I = I_S\left[\exp\left(\dfrac{V_D}{V_T}\right)-1\right] - 0.90 = \exp\left(\dfrac{V_D}{V_T}\right)-1$

$\exp\left(\dfrac{V_D}{V_T}\right) = 1 - 0.90 = 0.10$

$V_D = V_T \ln(0.10) \Rightarrow \underline{V_D = -0.0599 \text{ V}}$

b.

$$\left|\frac{I_F}{I_R}\right| = \frac{I_S}{I_S}\cdot\frac{\left[\exp\left(\dfrac{V_F}{V_T}\right)-1\right]}{\left[\exp\left(\dfrac{V_R}{V_T}\right)-1\right]} = \left|\frac{\exp\left(\dfrac{0.2}{0.026}\right)-1}{\exp\left(\dfrac{-0.2}{0.026}\right)-1}\right|$$

$$= \left|\frac{2190}{-1}\right|$$

$$\underline{\frac{I_F}{I_R} = 2190}$$

1.27 $I_D = I_S\left[\exp\left(\dfrac{V_D}{V_T}\right)-1\right]$

(a) (i) $I_D = (10^{-11})\exp\left(\dfrac{0.3}{0.026}\right) \Rightarrow 1.03\,\mu\text{A}$

(ii) $I_D = (10^{-11})\exp\left(\dfrac{0.5}{0.026}\right) \Rightarrow 2.25 \text{ mA}$

(iii) $I_D = (10^{-11})\exp\left(\dfrac{0.7}{0.026}\right) \Rightarrow 4.93 \text{ A}$

(iv) $I_D = (10^{-11})\left[\exp\left(\dfrac{-0.02}{0.026}\right)-1\right] = -5.37\times10^{-12} \text{ A}$

(v) $I_D = (10^{-11})\left[\exp\left(\dfrac{-0.20}{0.026}\right)-1\right] \cong -10^{-11} \text{ A}$

(vi) $I_D = -(10^{-11}) \text{ A}$

(b) (i) $I_D = (10^{-13})\exp\left(\dfrac{0.3}{0.026}\right) \Rightarrow 0.0103\,\mu\text{A}$

(ii) $I_D = (10^{-13})\exp\left(\dfrac{0.5}{0.026}\right) \Rightarrow 22.5\,\mu\text{A}$

(iii) $I_D = (10^{-13})\exp\left(\dfrac{0.7}{0.026}\right) \Rightarrow 49.3 \text{ mA}$

(iv) $I_D = \left(10^{-13}\right)\left[\exp\left(\frac{-0.02}{0.026}\right)-1\right] = -5.37\times10^{-14}$ A

(v) $I_D \cong -10^{-13}$ A

(vi) $I_D \cong -10^{-13}$ A

1.28 $V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$

(a) (i) $V_D = (0.026)\ln\left(\frac{10\times10^{-6}}{10^{-11}}\right) = 0.359$ V

$V_D = (0.026)\ln\left(\frac{100\times10^{-6}}{10^{-11}}\right) = 0.419$ V

$V_D = (0.026)\ln\left(\frac{10^{-3}}{10^{-11}}\right) = 0.479$ V

(ii) $-5\times10^{-12} = 10^{-11}\left[\exp\left(\frac{V_D}{0.026}\right)-1\right] \Rightarrow V_D = -0.018$ V

(b) (i) $V_D = (0.026)\ln\left(\frac{10\times10^{-6}}{10^{-13}}\right) = 0.479$ V

$V_D = (0.026)\ln\left(\frac{100\times10^{-6}}{10^{-13}}\right) = 0.539$ V

$V_D = (0.026)\ln\left(\frac{10^{-3}}{10^{-13}}\right) = 0.599$ V

(ii) $-10^{-14} = 10^{-13}\left[\exp\left(\frac{V_D}{0.026}\right)-1\right] \Rightarrow V_D = -0.00274$ V

1.29

(a) $10^{-3} = I_S \exp\left(\frac{0.7}{0.026}\right)$

$I_S = 2.03\times10^{-15}$ A

(b)

V_D	$I_D(A)(n=1)$	$I_D(A)(n=2)$
0.1	9.50×10^{-14}	1.39×10^{-14}
0.2	4.45×10^{-12}	9.50×10^{-14}
0.3	2.08×10^{-10}	6.50×10^{-13}
0.4	9.75×10^{-9}	4.45×10^{-12}
0.5	4.56×10^{-7}	3.04×10^{-11}
0.6	2.14×10^{-5}	2.08×10^{-10}
0.7	10^{-3}	1.42×10^{-9}

1.30

(a)

$I_S = 10^{-12}\ A$

V_D(v)	I_D(A)	$\log_{10} I_D$
0.10	4.68×10^{-11}	−10.3
0.20	2.19×10^{-9}	−8.66
0.30	1.03×10^{-7}	−6.99
0.40	4.80×10^{-6}	−5.32
0.50	2.25×10^{-4}	−3.65
0.60	1.05×10^{-2}	−1.98
0.70	4.93×10^{-1}	−0.307

(b)

$I_S = 10^{-14}\ A$

V_D(v)	I_D(A)	$\log_{10} I_D$
0.10	4.68×10^{-13}	−12.3
0.20	2.19×10^{-11}	−10.66
0.30	1.03×10^{-9}	−8.99
0.40	4.80×10^{-8}	−7.32
0.50	2.25×10^{-6}	−5.65
0.60	1.05×10^{-4}	−3.98
0.70	4.93×10^{-3}	−2.31

1.31

a.

$$\frac{I_{D2}}{I_{D1}} = 10 = \exp\left(\frac{V_{D2} - V_{D1}}{V_T}\right)$$

$$\Delta V_D = V_T \ln(10) \Rightarrow \underline{\Delta V_D = 59.9\ \text{mV} \approx 60\ \text{mV}}$$

b. $\Delta V_D = V_T \ln(100) \Rightarrow \underline{\Delta V_D = 119.7\ \text{mV} \approx 120\ \text{mV}}$

1.32

(a) (i) $V_D = (0.026)\ln\left(\dfrac{2}{2\times10^{-9}}\right) = 0.539\ \text{V}$

(ii) $V_D = (0.026)\ln\left(\dfrac{20}{2\times10^{-9}}\right) = 0.599\ \text{V}$

(b) (i) $I_D = \left(2\times10^{-9}\right)\exp\left(\dfrac{0.4}{0.026}\right) \Rightarrow 9.60\ \text{mA}$

(ii) $I_D = \left(2\times10^{-9}\right)\exp\left(\dfrac{0.65}{0.026}\right) \Rightarrow 144\ \text{A}$

1.33

$$V_D = V_t \ln\left(\frac{I_D}{I_S}\right) = (0.026)\ln\left(\frac{2\times10^{-3}}{5\times10^{-14}}\right) = 0.6347 \text{ V}$$

$$V_D = (0.026)\ln\left(\frac{2\times10^{-3}}{5\times10^{-12}}\right) = 0.5150 \text{ V}$$

$$\underline{0.5150 \le V_D \le 0.6347 \text{ V}}$$

1.34

(a) $1.5\times10^{-3} = I_S \exp\left(\frac{0.30}{0.026}\right) \Rightarrow I_S = 1.46\times10^{-8}\ A$

(b) (i) $I_D = \left(1.462\times10^{-8}\right)\exp\left(\frac{0.35}{0.026}\right) \Rightarrow I_D = 10.3 \text{ mA}$

(ii) $I_D = \left(1.462\times10^{-8}\right)\exp\left(\frac{0.25}{0.026}\right) \Rightarrow I_D = 0.219 \text{ mA}$

1.35

(a) $I_D = \left(10^{-22}\right)\exp\left(\frac{0.8}{0.026}\right) \Rightarrow 2.31 \text{ nA}$

$I_D = \left(10^{-22}\right)\exp\left(\frac{1.0}{0.026}\right) \Rightarrow 5.05\,\mu\text{A}$

$I_D = \left(10^{-22}\right)\exp\left(\frac{1.2}{0.026}\right) \Rightarrow 11.1 \text{ mA}$

$I_D = \left(10^{-22}\right)\left[\exp\left(\frac{-0.02}{0.026}\right) - 1\right] = -5.37\times10^{-23} \text{ A}$

For $V_D = -0.20$ V, $I_D = -10^{-22}$ A

For $V_D = -2$ V, $I_D = -10^{-22}$ A

(b)

$I_D = \left(5\times10^{-24}\right)\exp\left(\frac{0.8}{0.026}\right) \Rightarrow 115 \text{ pA}$

$I_D = \left(5\times10^{-24}\right)\exp\left(\frac{1.0}{0.026}\right) \Rightarrow 0.253\,\mu\text{A}$

$I_D = \left(5\times10^{-24}\right)\exp\left(\frac{1.2}{0.026}\right) \Rightarrow 0.554 \text{ mA}$

$I_D = \left(5\times10^{-24}\right)\left[\exp\left(\frac{-0.02}{0.026}\right) - 1\right] = -2.68\times10^{-24} \text{ A}$

For $V_D = -0.20$ V, $I_D = -5\times10^{-24}$ A

For $V_D = -2$ V, $I_D = -5\times10^{-24}$ A

1.36

I_S doubles for every 5C increase in temperature.

$I_S = 10^{-12}\ A$ at T = 300K

For $I_S = 0.5\times10^{-12}\ A \Rightarrow$ T = 295 K

For $I_S = 50\times10^{-12}\ A$, $(2)^n = 50 \Rightarrow n = 5.64$

Where n equals number of 5C increases.

Then $\Delta T = (5.64)(5) = 28.2\ K$

So $295 \le T \le 328.2\ K$

1.37

$$\frac{I_S(T)}{I_S(-55)} = 2^{\Delta T/5}, \quad \Delta T = 155^\circ\ \text{C}$$

$$\frac{I_S(100)}{I_S(-55)} = 2^{155/5} = 2.147\times10^9$$

$V_T\ @100°\text{C} \Rightarrow 373°\text{K} \Rightarrow V_T = 0.03220$

$V_T\ @-55°\text{C} \Rightarrow 216°\text{K} \Rightarrow V_T = 0.01865$

$$\frac{I_D(100)}{I_D(-55)} = (2.147\times10^9)\times\frac{\exp\left(\frac{0.6}{0.0322}\right)}{\exp\left(\frac{0.6}{0.01865}\right)}$$

$$= \frac{(2.147\times10^9)(1.237\times10^8)}{(9.374\times10^{13})}$$

$$\frac{I_D(100)}{I_D(-55)} = 2.83\times10^3$$

1.38

(a) $V_{PS} = I_D R + V_D$

$$2.8 = I_D(10^6) + V_D; \quad I_D = (5\times10^{-11})\exp\left(\frac{V_D}{0.026}\right)$$

By trial and error,

$V_D = 0.282$ V, $I_D = 2.52\,\mu$A

(b)

$I_D \cong -5\times10^{-11}$ A, $V_D = -2.8$ V

1.39

$$10 = I_D\left(2\times10^4\right)+V_D \text{ and } V_D = (0.026)\ln\left(\frac{I_D}{10^{-12}}\right)$$

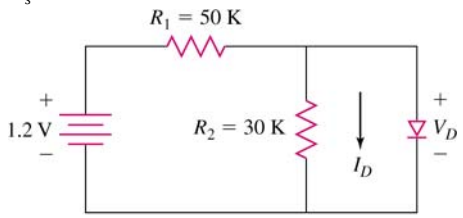
Trial and error.

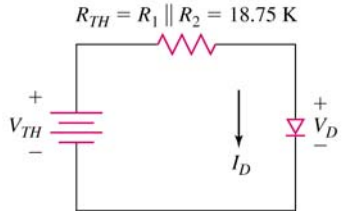
V_D(v)	I_D(A)	V_D(v)
0.50	4.75×10^{-4}	0.5194
0.517	4.7415×10^{-4}	0.5194
0.5194	4.740×10^{-4}	0.5194

$$V_D = 0.5194 \text{ V}$$
$$I_D = 0.4740 \text{ mA}$$

1.40

$$I_s = 5\times10^{-13} \text{ A}$$





$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(1.2) = \left(\frac{30}{80}\right)(1.2) = 0.45 \text{ V}$$

$$0.45 = I_D R_{TH} + V_D, \quad V_D = V_T \ln\left(\frac{I_D}{I_S}\right)$$

By trial and error:

$$I_D = 2.56\ \mu\text{A}, \quad V_D = 0.402 \text{ V}$$

1.41

(a) $I_{D1} = I_{D2} = 1 \text{ mA}$

(i) $$V_{D1} = V_{D2} = (0.026)\ln\left(\frac{10^{-3}}{10^{-13}}\right) = 0.599 \text{ V}$$

(ii) $$V_{D1} = (0.026)\ln\left(\frac{10^{-3}}{5\times10^{-14}}\right) = 0.617 \text{ V}$$

$$V_{D2} = (0.026)\ln\left(\frac{10^{-3}}{5\times10^{-13}}\right) = 0.557 \text{ V}$$

(b) $V_{D1} = V_{D2}$

(i) $$I_{D1} = I_{D2} = \frac{I_i}{2} = 0.5 \text{ mA}$$

$$V_{D1} = V_{D2} = (0.026)\ln\left(\frac{0.5\times10^{-3}}{10^{-13}}\right) = 0.581 \text{ V}$$

(ii) $$\frac{I_{D1}}{I_{D2}} = \frac{I_{S1}}{I_{S2}} = \frac{5\times10^{-14}}{5\times10^{-13}} = 0.10$$

So $I_{D1} = 0.10 I_{D2}$

$I_{D1} + I_{D2} = 1.1 I_{D2} = 1 \text{ mA}$

So $I_{D2} = 0.909 \text{ mA}, \ I_{D1} = 0.0909 \text{ mA}$

Now

$$V_{D1} = (0.026)\ln\left(\frac{0.0909\times10^{-3}}{5\times10^{-14}}\right) = 0.554 \text{ V}$$

$$V_{D2} = (0.026)\ln\left(\frac{0.909\times10^{-3}}{5\times10^{-13}}\right) = 0.554 \text{ V}$$

1.42

(a) $$I_{D3} = \left(6\times10^{-14}\right)\exp\left(\frac{0.635}{0.026}\right) \Rightarrow 2.426 \text{ mA}$$

$$I_R = \frac{0.635}{1} = 0.635 \text{ mA}$$

$$I_{D1} = I_{D2} = 2.426 + 0.635 = 3.061 \text{ mA}$$

$$V_{D1} = V_{D2} = (0.026)\ln\left(\frac{3.061\times10^{-3}}{6\times10^{-14}}\right) = 0.641 \text{ V}$$

$$V_I = 2(0.641) + 0.635 = 1.917 \text{ V}$$

(b) $I_{D3} = 2.426 \text{ mA}$

$$I_R = \frac{0.635}{0.5} = 1.27 \text{ mA}$$

$$I_{D1} = I_{D2} = 2.426 + 1.27 = 3.696 \text{ mA}$$

$$V_{D1} = V_{D2} = (0.026)\ln\left(\frac{3.696\times10^{-3}}{6\times10^{-14}}\right) = 0.6459 \text{ V}$$

$$V_I = 2(0.6459) + 0.635 = 1.927 \text{ V}$$

1.43

(a) Assume diode is conducting.

Then, $V_D = V_\gamma = 0.7\ V$

So that $I_{R2} = \dfrac{0.7}{30} \Rightarrow 23.3\ \mu A$

$$I_{R1} = \frac{1.2 - 0.7}{10} \Rightarrow 50\ \mu A$$

Then $I_D = I_{R1} - I_{R2} = 50 - 23.3$

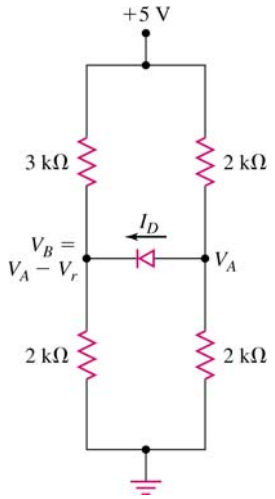
Or $\underline{I_D = 26.7\ \mu A}$

(b) Let $R_1 = 50\ k\Omega$ Diode is cutoff.

$$V_D = \frac{30}{30 + 50} \cdot (1.2) = 0.45\ V$$

Since $V_D < V_\gamma$, $I_D = 0$

1.44



At node V_A:

(1) $$\frac{5 - V_A}{2} = I_D + \frac{V_A}{2}$$

At node $V_B = V_A - V_\gamma$

(2) $$\frac{5 - (V_A - V_r)}{2} + I_D = \frac{(V_A - V_r)}{2}$$

So $$\frac{5 - (V_A - V_r)}{3} + \left[\frac{5 - V_A}{2} - \frac{V_A}{2}\right] = \frac{V_A - V_r}{2}$$

Multiply by 6:

$$10 - 2(V_A - V_r) + 15 - 6V_A = 3(V_A - V_r)$$

$$25 + 2V_r + 3V_r = 11V_A$$

(a) $V_r = 0.6$ V

$$11V_A = 25 + 5(0.6) = 28 \Rightarrow V_A = 2.545\ \text{V}$$

From (1) $I_D = \dfrac{5 - V_A}{2} - \dfrac{V_A}{2} = 2.5 - V_A \Rightarrow I_D$ Neg. $\Rightarrow \underline{I_D = 0}$

Both (a), (b) $\underline{I_D = 0}$

$V_A = 2.5,\ V_B = \dfrac{2}{5} \cdot 5 = 2\ \text{V} \Rightarrow \underline{V_D = 0.50\ \text{V}}$

1.45

(a) $V_O = I_i(1);\ I_D = 0$; for $0 \le I_i \le 0.7$ mA

$V_O = 0.7\ \text{V};\ I_D = (I_i - 0.7)\ \text{mA};$ for $I_i \ge 0.7$ mA

(b) $V_O = I_i(1);\ I_D = 0$; for $0 \le I_i \le 1.7$ mA

$V_O = 1.7\ \text{V};\ I_D = (I_i - 1.7)\ \text{mA};$ for $I_i \ge 1.7$ mA

(c) $V_O = 0.7\ \text{V};\ I_{D1} = I_i;\ I_{D2} = 0$; for $0 \le I_i \le 2$ mA

1.46

Minimum diode current for V_{PS} (min)

$I_D(\text{min}) = 2\ mA,\quad V_D = 0.7\ V$

$I_2 = \dfrac{0.7}{R_2},\quad I_1 = \dfrac{5 - 0.7}{R_1} = \dfrac{4.3}{R_1}$

We have $I_1 = I_2 + I_D$

so (1) $\dfrac{4.3}{R_1} = \dfrac{0.7}{R_2} + 2$

Maximum diode current for V_{PS} (max)

$P = I_D V_D \quad 10 = I_D(0.7) \Rightarrow I_D = 14.3\ mA$

$I_1 = I_2 + I_D$

or

(2) $\dfrac{9.3}{R_1} = \dfrac{0.7}{R_2} + 14.3$

Using Eq. (1), $\dfrac{9.3}{R_1} = \dfrac{4.3}{R_1} - 2 + 14.3 \Rightarrow \quad \underline{R_1 = 0.41\ k\Omega}$

Then $\underline{R_2 = 82.5\Omega}\quad 82.5\Omega$

1.47

(a) (i) $I = \dfrac{5 - 0.7}{20} = 0.215$ mA, $V_O = 0.7$ V

(ii) $I = \dfrac{5 - 0.6}{20} = 0.220$ mA, $V_O = 0.6$ V

(b) (i) $I = \dfrac{5 - 0.7 - (-5)}{40} = 0.2325$ mA, $V_O = (0.2325)(20) - 5 = -0.35$ V

(iii) $I = \dfrac{5 - 0.6 - (-5)}{40} = 0.235$ mA, $V_O = (0.235)(20) - 5 = -0.30$ V

(c) (i) $I = \dfrac{2-0.7-(-8)}{25} = 0.372$ mA, $V_O = 2-(0.372)(5) = 0.14$ V

(ii) $I = \dfrac{2-0.6-(-8)}{25} = 0.376$ mA, $V_O = 2-(0.376)(5) = 0.12$ V

(d) (i) $I = 0$, $V_O = -5$ V

(ii) $I = 0$, $V_O = -5$ V

1.48

(a) $I = \dfrac{5-V_O}{20}$, $I = (5\times 10^{-14})\exp\left(\dfrac{V_D}{0.026}\right)$

By trial and error, $V_D = V_O = 0.5775$ V, $I = 0.221$ mA

(b) $I = \dfrac{10-V_D}{40}$, $V_O = 5 - I(20) - V_D$

$I = 0.2355$ mA, $V_D = 0.579$ V, $V_O = -0.289$

(c) $I = \dfrac{10-V_D}{25}$, $V_O = 2 - I(5)$

$I = 0.3763$ mA, $V_D = 0.5913$ V, $V_O = 0.1185$

(d) $I = -5\times 10^{-14}$ A, $V_O \cong -5$ V

1.49

(a) Diode forward biased $V_D = 0.7$ V

$5 = (0.4)(4.7) + 0.7 + V \Rightarrow \underline{V = 2.42 \text{ V}}$

(b) $P = I \cdot V_D = (0.4)(0.7) \Rightarrow \underline{P = 0.28 \text{ m}\omega}$

1.50

(a) $I_{R2} = I_{D1} = \dfrac{0.65}{1} = \underline{0.65 \text{ mA} = I_{D1}}$

$I_{D2} = 2(0.65) = 1.30$ mA

$I_{D2} = \dfrac{V_I - 2V_r - V_0}{R_1} = \dfrac{5-3(0.65)}{R_1} = 1.30 \Rightarrow \underline{R_1 = 2.35 \text{ K}}$

(b) $I_{R2} = \dfrac{0.65}{1} = 0.65$ mA

$I_{D2} = \dfrac{8-3(0.65)}{2} \Rightarrow \underline{I_{D2} = 3.025 \text{ mA}}$

$I_{D1} = I_{D2} - I_{R2} = 3.025 - 0.65$

$\underline{I_{D1} = 2.375 \text{ mA}}$

1.51

a. $\tau_d = \dfrac{V_T}{I_{DQ}} = \dfrac{(0.026)}{1} = 0.026\ \text{k}\Omega = 26\Omega$

$i_d = 0.05 I_{DQ} = 50\ \mu\text{A}$ peak-to-peak

$v_d = i_d \tau_d = (26)(50)\ \mu\text{A} \Rightarrow \underline{v_d = 1.30\ \text{mV}\ \text{ peak-to-peak}}$

b. For $I_{DQ} = 0.1\ \text{mA} \Rightarrow \tau_d = \dfrac{(0.026)}{0.1} = 260\Omega$

$i_d = 0.05 I_{DQ} = 5\ \mu\text{A}$ peak-to-peak

$v_d = i_d \tau_d = (260)(5)\ \mu\text{V} \Rightarrow \underline{v_d = 1.30\ \text{mV}\ \text{ peak-to-peak}}$

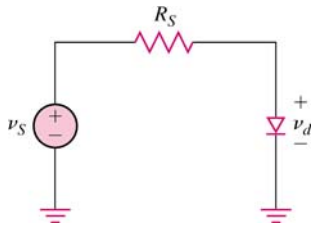
1.52

(a) $r_d = \dfrac{V_T}{I_{DQ}} = \dfrac{0.026}{0.026} = 1\ \text{k}\,\Omega$

(b) $r_d = \dfrac{0.026}{0.26} \Rightarrow 100\,\Omega$

(c) $r_d = \dfrac{0.026}{2.6} \Rightarrow 10\,\Omega$

1.53



a. diode resistance $r_d = V_T / I$

$$v_d = \left(\frac{r_d}{r_d + R_S}\right) v_S = \left(\frac{V_T / I}{\dfrac{V_T}{I} + R_S}\right) v_S$$

$$v_d = \left(\frac{V_T}{V_T + I R_S}\right) v_s = v_o$$

b. $R_S = 260\Omega$

$I = 1\ \text{mA},\ \dfrac{v_0}{v_S} = \left(\dfrac{V_T}{V_T + I R_S}\right) = \dfrac{0.026}{0.026 + (1)(0.26)} \Rightarrow \underline{\dfrac{v_0}{v_S} = 0.0909}$

$I = 0.1\ \text{mA},\ \dfrac{v_0}{v_s} = \dfrac{0.026}{0.026 + (0.1)(0.26)} \Rightarrow \underline{\dfrac{v_0}{v_S} = 0.50}$

$I = 0.01\ \text{mA}.\ \dfrac{v_0}{v_S} = \dfrac{0.026}{0.026 + (0.01)(0.26)} \Rightarrow \underline{\dfrac{v_0}{v_S} = 0.909}$

1.54

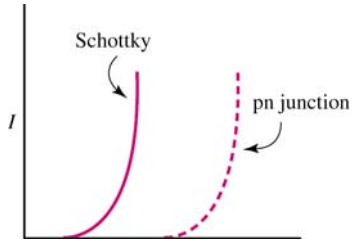
pn junction diode

$$V_D = (0.026)\ln\left(\frac{0.72\times10^{-3}}{5\times10^{-13}}\right) = 0.548 \text{ V}$$

Schottky diode

$$V_D = (0.026)\ln\left(\frac{0.72\times10^{-3}}{5\times10^{-8}}\right) = 0.249 \text{ V}$$

1.55



Schottky: $I \cong I_S \exp\left(\frac{V_a}{V_T}\right)$

$$V_a = V_T \ln\left(\frac{I}{I_S}\right) = (0.026)\ln\left(\frac{0.5\times10^{-3}}{5\times10^{-7}}\right)$$

$$= 0.1796 \text{ V}$$

Then

$$V_a \text{ of pn junction} = 0.1796 + 0.30$$

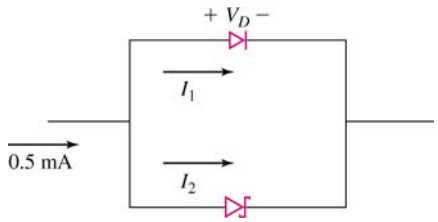
$$= 0.4796$$

$$I_S = \frac{I}{\exp\left(\frac{V_a}{V_T}\right)} = \frac{0.5\times10^{-3}}{\exp\left(\frac{0.4796}{0.026}\right)}$$

$$\underline{I_S = 4.87\times10^{-12} \text{ A}}$$

1.56

(a)



$$I_1 + I_2 = 0.5\times10^{-3}$$

$$5\times10^{-8}\ \exp\left(\frac{V_D}{V_T}\right) + 10^{-12}\ \exp\left(\frac{V_D}{V_T}\right) = 0.5\times10^{-3}$$

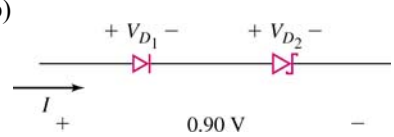
$$5.0001\times10^{-8}\ \exp\left(\frac{V_D}{V_T}\right) = 0.5\times10^{-3}$$

$$V_D = (0.026)\ln\left(\frac{0.5\times10^{-3}}{5.0001\times10^{-8}}\right) \Rightarrow \underline{V_D = 0.2395}$$

Schottky diode, $\underline{I_2 = 0.49999\ \text{mA}}$

pn junction, $\underline{I_1 = 0.00001\ \text{mA}}$

(b)



$$I = 10^{-12}\ \exp\left(\frac{V_{D1}}{V_T}\right) = 5\times10^{-8}\ \exp\left(\frac{V_{D2}}{V_T}\right)$$

$$V_{D1} + V_{D2} = 0.9$$

$$10^{-12}\ \exp\left(\frac{V_{D1}}{V_T}\right) = 5\times10^{-8}\ \exp\left(\frac{0.9 - V_{D1}}{V_T}\right)$$

$$= 5\times10^{-8}\ \exp\left(\frac{0.9}{V_T}\right)\exp\left(\frac{-V_{D1}}{V_T}\right)$$

$$\exp\left(\frac{2V_{D1}}{V_T}\right) = \left(\frac{5\times10^{-8}}{10^{-12}}\right)\ \exp\left(\frac{0.9}{0.026}\right)$$

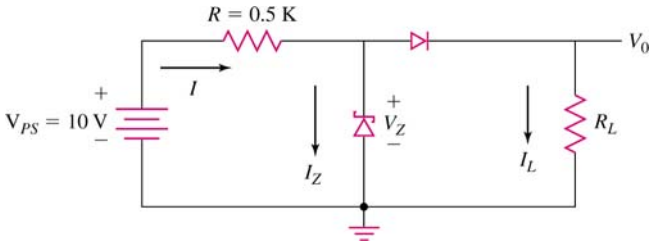
$$2V_{D1} = V_T\ \ln\left(\frac{5\times10^{-8}}{10^{-12}}\right) + 0.9 = 1.1813$$

$\underline{V_{D1} = 0.5907}$ pn junction

$\underline{V_{D2} = 0.3093}$ Schottky diode

$$I = 10^{-12}\ \exp\left(\frac{0.5907}{0.026}\right) \Rightarrow \underline{I = 7.35\ \text{mA}}$$

1.57



$V_Z = V_{Z0} = 5.6$ V at $I_Z = 0.1$ mA

$r_Z = 10\Omega$

$I_Z r_Z = (0.1)(10) = 1 \text{ mV}$

$V_{Z0} = 5.599$

a. $R_L \to \infty \Rightarrow$

$$I_Z = \frac{10 - 5.599}{R + r_Z} = \frac{4.401}{0.50 + 0.01} = 8.63 \text{ mA}$$

$$V_Z = V_{Z0} + I_Z r_Z = 5.599 + (0.00863)(10)$$

$$\underline{V_Z = V_0 = 5.685 \text{ V}}$$

b. $V_{PS} = 11 \text{ V} \Rightarrow I_Z = \dfrac{11 - 5.599}{0.51} = 10.59 \text{ mA}$

$$V_Z = V_0 = 5.599 + (0.01059)(10) = 5.7049 \text{ V}$$

$$V_{PS} = 9 \text{ V} \Rightarrow I_Z = \frac{9 - 5.599}{0.51} = 6.669 \text{ mA}$$

$$V_Z = V_0 = 5.599 + (0.006669)(10) = 5.66569 \text{ V}$$

$$\Delta V_0 = 5.7049 - 5.66569 \Rightarrow \underline{\Delta V_0 = 0.0392 \text{ V}}$$

c. $I = I_Z + I_L$

$$I_L = \frac{V_0}{R_L}, \quad I = \frac{V_{PS} - V_0}{R}, \quad I_Z = \frac{V_0 - V_{Z0}}{r_Z}$$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 5.599}{0.010} + \frac{V_0}{2}$$

$$\frac{10}{0.50} + \frac{5.599}{0.010} = V_0\left[\frac{1}{0.50} + \frac{1}{0.010} + \frac{1}{2}\right]$$

$$20.0 + 559.9 = V_0\,(102.5)$$

$$\underline{V_0 = 5.658 \text{ V}}$$

1.58

(a) $I_Z = \dfrac{10 - 6.8}{0.5} = 6.4 \text{ mA}$

$$P = I_Z V_Z = (6.4)(6.8) = 43.5 \text{ mW}$$

(b) $I_Z = (0.1)(6.4) = 0.64 \text{ mA}$

$$I_L = 6.4 - 0.64 = 5.76 \text{ mA}$$

$$I_L = \frac{V_Z}{R_L} \Rightarrow R_L = \frac{V_Z}{I_Z} = \frac{6.8}{5.76} = 1.18 \text{ k}\Omega$$

1.59

$I_Z r_Z = (0.1)(20) = 2 \text{ mV}$

$V_{Z0} = 6.8 - 0.002 = 6.798 \text{ V}$

a. $R_L = \infty$

$$I_Z = \frac{10 - 6.798}{0.5 + 0.02} \Rightarrow I_Z = 6.158 \text{ mA}$$

$$V_0 = V_Z = V_{Z0} + I_Z r_Z = 6.798 + (0.006158)(20)$$
$$\underline{V_0 = 6.921 \text{ V}}$$

b. $I = I_Z + I_L$

$$\frac{10 - V_0}{0.50} = \frac{V_0 - 6.798}{0.020} + \frac{V_0}{1}$$
$$\frac{10}{0.30} + \frac{6.798}{0.020} = V_0\left[\frac{1}{0.50} + \frac{1}{0.020} + \frac{1}{1}\right]$$
$$359.9 = V_0\ (53)$$
$$V_0 = 6.791 \text{ V}$$
$$\Delta V_0 = 6.791 - 6.921$$
$$\underline{\Delta V_0 = -0.13 \text{ V}}$$

1.60

For $V_D = 0$, $\underline{I_{SC} = 0.1 \text{ A}}$

For $I_D = 0$ $V_D = V_T \ln\left(\frac{0.2}{5\times10^{-14}} + 1\right)$

$$V_D = \underline{V_{DC} = 0.754 \text{ V}}$$

1.61 $V_D = 0, I_D = 0.2$ A

$V_D = 0.60$ V, $I_D = 0.1995$ A

$V_D = 0.65$ V, $I_D = 0.1964$ A

$V_D = 0.70$ V, $I_D = 0.1754$ A

$V_D = 0.72$ V, $I_D = 0.1468$ A

$V_D = 0.74$ V, $I_D = 0.0853$ A

$V_D = 0.7545$ V, $I_D = 0$

1.62

(a) $0.16 = 0.20 - \left(5\times10^{-14}\right)\left[\exp\left(\frac{V_D}{0.026}\right) - 1\right] \Rightarrow V_D = 0.7126$ V

(b) $P = (0.16)(0.7126) = 0.114$ W

Chapter 2

2.1

(a) For $\upsilon_I > 0.6$ V, $\upsilon_O = \left(\frac{1000}{1020}\right)(\upsilon_I - 0.6)$

For $\upsilon_I < 0.6$ V, $\upsilon_O = 0$

(b) (ii) $\upsilon_O = 0 = \left(\frac{1000}{1020}\right)[10\sin(\omega t)_1 - 0.6]$

Then $\sin(\omega t)_1 = \frac{0.6}{10} = 0.06 \Rightarrow (\omega t)_1 = 3.44° \Rightarrow 0.01911\pi$ rad

Also $(\omega t)_2 = 180 - 3.44 = 176.56° \Rightarrow 0.9809\pi$ rad

Now

$$\upsilon_O(avg) = \frac{1}{T}\int_0^T \upsilon_O(t)dt = \frac{1}{2\pi}\int_0^{2\pi}[10\sin x - 0.6]dx$$

$$= \frac{1}{2\pi}\left[-10\cos x\Big|_{0.01911\pi}^{0.9809\pi} - 0.6x\Big|_{0.01911\pi}^{0.9809\pi}\right]$$

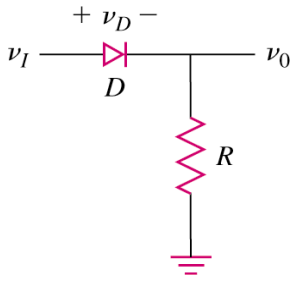
$$= \frac{1}{2\pi}[(-10)(-0.9982 - 0.9982) - 0.6(0.9809\pi - 0.01911\pi)]$$

$$\upsilon_O(avg) = 2.89 \text{ V}$$

(iii) $\upsilon_O(peak) = \left(\frac{1000}{1020}\right)\left[10\sin\left(\frac{\pi}{2}\right) - 0.6\right] = 9.2157$ V; $i_d(\max) = 9.2157$ mA

(iv) $PIV = 10$ V

2.2



$v_0 = v_I - v_D$

$v_D = V_T \ln\left(\frac{i_D}{I_S}\right)$ and $i_D = \frac{v_0}{R}$

$$v_0 = v_I - V_T \ln\left(\frac{v_0}{I_S R}\right)$$

2.3

(a) $\upsilon_S = 120\sqrt{2}\left(\frac{1}{10}\right) = 16.97$ V (peak)

$\upsilon_O(peak) = 16.27$ V

(b) $i_D(peak) = \frac{16.27}{2} = 8.14$ mA

(c) $\upsilon_O = 16.97\sin\omega t - 0.7$

$$\sin(\omega t)_1 = \frac{0.7}{16.97} = 0.04125 \Rightarrow (\omega t)_1 = 2.364°$$

$$(\omega t)_2 = 180 - 2.364 = 177.64°$$

$$\% = \left(\frac{177.64 - 2.364}{360}\right)\times 100\% = 48.7\%$$

(d)

$$\upsilon_O(avg) = \frac{1}{2\pi}\int_{0.01313\pi}^{0.9869\pi}[16.97\sin x - 0.7]dx$$

$$= \frac{1}{2\pi}\left[(-16.97)\cos x\Big|_{0.01313\pi}^{0.9869\pi} - 0.7x\Big|_{0.01313\pi}^{0.9869\pi}\right]$$

$$= \frac{1}{2\pi}[(-16.97)(-0.99915 - 0.99915) - 0.7(0.9738\pi)]$$

$$\upsilon_O(avg) = 5.06 \text{ V}$$

(e) $i_D(avg) = \frac{\upsilon_O(avg)}{2} = \frac{5.06}{2} = 2.53$ mA

2.4

(a) $\upsilon_R(t) = 15\sin\omega t - 0.7 - 9 = 15\sin\omega t - 9.7$

$$(\omega t)_1 = \sin^{-1}\left(\frac{9.7}{15}\right) = 40.29° \Rightarrow 0.2238\pi \text{ rad}$$

$$(\omega t)_2 = 180 - 40.29 = 139.71° \Rightarrow 0.7762\pi \text{ rad}$$

$$\upsilon_R(avg) = \frac{1}{2\pi}\int_{0.2238\pi}^{0.7762\pi}[15\sin x - 9.7]dx$$

$$= \frac{1}{2\pi}\left[(-15)\cos x\Big|_{0.2238\pi}^{0.7762\pi} - 9.7x\Big|_{0.2238\pi}^{0.7762\pi}\right] = \frac{1}{2\pi}[(-15)(-0.7628 - 0.7628) - 9.7(0.5523\pi)]$$

$$\upsilon_R(avg) = 0.9628 \text{ V}$$

$$i_D(avg) = 0.8 = \frac{0.9628}{R} \Rightarrow R = 1.20\,\Omega$$

(b)

$$\% = \left(\frac{139.71 - 40.29}{360}\right)\times 100\% = 27.6\%$$

2.5

(a) $i(peak) = \dfrac{\upsilon_R(peak)}{R} \Rightarrow R = \dfrac{15 - 9.7}{1.2} = 4.417\,\Omega$

(b) $\upsilon_R(t) = 15\sin\omega t - 9.7$

$(\omega t)_1 = 0.2238\pi\ ;\ (\omega t)_2 = 0.7762\pi$

$$\upsilon_R(avg) = \frac{1}{\pi}\int_{0.2238\pi}^{0.7762\pi}[15\sin x - 9.7]dx$$

Or from Problem 2.4, $\upsilon_R(avg) = 2(0.9628) = 1.9256$ V

$$i_D(avg) = \frac{\upsilon_R(avg)}{R} = \frac{1.9256}{4.417} = 0.436\text{ A}$$

(c)

$$\% = \left(\frac{139.71 - 40.29}{360}\right)\times 100\% = 27.6\%$$

2.6

(a) $\upsilon_S(peak) = 12 + 0.7 = 12.7$ V

$$\frac{N_1}{N_2} = \frac{120\sqrt{2}}{12.7} = 13.4$$

(b) $R = \dfrac{12}{0.2} = 60\,\Omega$

$$C = \frac{V_M}{2fRV_r} = \frac{12}{2(60)(60)(0.25)} \Rightarrow 6667\,\mu\text{F}$$

(c) $PIV = 2\upsilon_S(\max) - V_\gamma = 2(12.7) - 0.7 = 24.7$ V

2.7

$$v_0 = v_S - 2V_\gamma \Rightarrow v_S(\max) = v_0(\max) + 2V_\gamma$$

a. For $v_0(\max) = 25$ V $\Rightarrow v_S(\max) = 25 + 2(0.7) = 26.4$ V

$$\frac{N_1}{N_2} = \frac{160}{26.4} \Rightarrow \underline{\frac{N_1}{N_2} = 6.06}$$

b. For $v_0(\max) = 100$ V $\Rightarrow v_S(\max) = 101.4$ V

$$\frac{N_1}{N_2} = \frac{160}{101.4} \Rightarrow \underline{\frac{N_1}{N_2} = 1.58}$$

From part (a) $PIV = 2v_S(\max) - V_\gamma = 2(26.4) - 0.7$

or $\underline{PIV = 52.1\ V}$ or, from part (b) $PIV = 2(101.4) - 0.7$ or $\underline{PIV = 202.1\ V}$

2.8

(a)

$$v_s(\max) = 12 + 2(0.7) = 13.4\text{ V}$$

$$v_s(\text{rms}) = \frac{13.4}{\sqrt{2}} \Rightarrow \underline{v_s(\text{rms}) = 9.48\text{ V}}$$

(b)

$$V_r = \frac{V_M}{2f\ R_C} \Rightarrow C = \frac{V_M}{2f\ V_r R}$$

$$C = \frac{12}{2(60)(0.3)(150)} \Rightarrow \underline{C = 2222\ \mu\text{F}}$$

(c)

$$i_d,\ \text{peak} = \frac{V_M}{R}\left[1 + \pi\sqrt{\frac{2V_M}{V_r}}\right]$$

$$= \frac{12}{150}\left[1 + \pi\sqrt{\frac{2(12)}{0.3}}\right]$$

$$\underline{i_d,\ \text{peak} = 2.33\ \text{A}}$$

2.9

(a)

$$v_S(\max) = 12 + 0.7 = 12.7\ V$$

$$v_S(\text{rms}) = \frac{v_S(\max)}{\sqrt{2}} \Rightarrow \quad v_S(\text{rms}) = 8.98\ V$$

(b)

$$V_r = \frac{V_M}{fRC} \Rightarrow C = \frac{V_M}{fRV_r} = \frac{12}{(60)(150)(0.3)} \text{ or } \underline{C = 4444\ \mu F}$$

(c) For the half-wave rectifier

$$i_{D,\ \max} = \frac{V_M}{R}\left(1 + 4\pi\sqrt{\frac{V_M}{2V_r}}\right) = \frac{12}{150}\left(1 + 4\pi\sqrt{\frac{12}{2(0.3)}}\right) \text{ or } \underline{i_{D,\ \max} = 4.58\ A}$$

2.10

(a) $\upsilon_O(peak) = 10 - 0.7 = 9.3\ \text{V}$

(b) $C = \dfrac{V_M}{fRV_r} = \dfrac{9.3}{(60)(500)(0.5)} \Rightarrow 620\ \mu\text{F}$

(c) $PIV = 10 + 9.3 = 19.3\ \text{V}$

2.11

(a) $10.3 \le \upsilon_O \le 12.3\ \text{V}$

(b) $V_r = \dfrac{V_M}{fRC} = \dfrac{12.3}{(60)(1000)(350\times10^{-6})} = 0.586\ \text{V}$

$$V_r = \frac{10.3}{(60)(1000)(350\times10^{-6})} = 0.490\ \text{V}$$

So $0.490 \le V_r \le 0.586\ \text{V}$

(c) $C = \dfrac{V_M}{fRV_r} = \dfrac{12.3}{(60)(1000)(0.4)} \Rightarrow 513\ \mu\text{F}$

2.12

(a) $\upsilon_S(peak) = 8.5\left(\sqrt{2}\right) = 12.02\ \text{V}$

$|V_O|_{\max} = 12.02 - 0.7 = 11.32\ \text{V}$

(b) $C = \dfrac{V_M}{2fRV_r} = \dfrac{11.32}{2(60)(10)(0.25)} = 0.03773\ \text{F}$

(c) $\text{PIV} = 2\upsilon_S(peak) - V_\gamma = 2(12.02) - 0.7 = 23.34\ \text{V}$

2.13

(a)

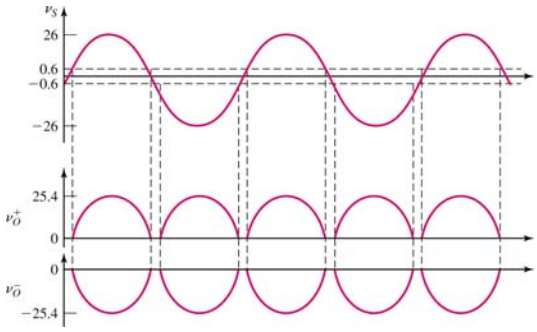
$v_s(\text{peak}) = 15 + 2(0.7) = 16.4\ \text{V}$

$v_s(\text{rms}) = \dfrac{16.4}{\sqrt{2}} = 11.6\ \text{V}$

$C = \dfrac{V_M}{2f\ RV_r} = \dfrac{15}{2(60)(125)(0.35)} = 2857\ \mu\text{F}$

(b)

2.14



2.15

(a) $\upsilon_S = 12.8\ \text{V}$

$\dfrac{N_1}{N_2} = \dfrac{120\sqrt{2}}{12.8} = 13.3$

(b) $R = \dfrac{12}{0.5} = 24\ \Omega$

$V_r = 3\% \Rightarrow V_r = (0.03)(12) = 0.36\ \text{V}$

$C = \dfrac{V_M}{2fRV_r} = \dfrac{12}{2(60)(24)(0.36)} = 0.0116\ \text{F}$

(c) $i_D(peak) = \dfrac{V_M}{R}\left(1 + \pi\sqrt{\dfrac{2V_M}{V_r}}\right) = \dfrac{12}{24}\left(1 + \pi\sqrt{\dfrac{2(12)}{0.36}}\right)$

$i_D(peak) = 13.3\ \text{A}$

(d) $i_D(avg) = \frac{1}{\pi}\sqrt{\frac{2V_r}{V_M}} \cdot \frac{V_M}{R}\left(1 + \frac{\pi}{2}\sqrt{\frac{2V_M}{V_r}}\right) = \frac{1}{\pi}\sqrt{\frac{2(0.36)}{12}}\left(\frac{12}{24}\right)\left(1 + \frac{\pi}{2}\sqrt{\frac{2(12)}{0.36}}\right)$

$i_D(avg) = 0.539\ \text{A}$

(e) $PIV = 12.8 + 12 = 24.8\ \text{V}$

2.16

(a) $\upsilon_S = 9 + 2(0.8) = 10.6\ \text{V}$

$\frac{N_1}{N_2} = \frac{120\sqrt{2}}{10.6} = 16$

(b) $R = \frac{9}{0.1} = 90\ \Omega$

$C = \frac{V_M}{2fRV_r} = \frac{9}{2(60)(90)(0.2)} \Rightarrow 4167\ \mu\text{F}$

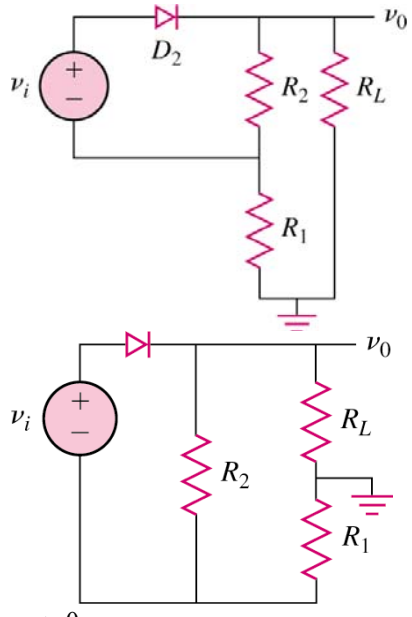
(c) $i_D(peak) = \frac{V_M}{R}\left(1 + \pi\sqrt{\frac{2V_M}{V_r}}\right) = \frac{9}{90}\left(1 + \pi\sqrt{\frac{2(9)}{0.2}}\right) = 3.08\ \text{A}$

(d) $i_D(avg) = \frac{1}{\pi}\sqrt{\frac{2V_r}{V_M}} \cdot \frac{V_M}{R}\left(1 + \frac{\pi}{2}\sqrt{\frac{2V_M}{V_r}}\right) = \frac{1}{\pi}\sqrt{\frac{2(0.2)}{9}}\left(\frac{9}{90}\right)\left(1 + \frac{\pi}{2}\sqrt{\frac{2(9)}{0.2}}\right)$

$i_D(avg) = 0.1067\ \text{A}$

(e) PIV $= \upsilon_S(\max) - V_\gamma = 10.6 - 0.8 = 9.8\ \text{V}$

2.17



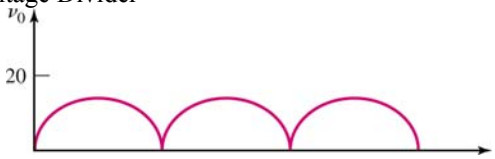
For $v_i > 0$

$V_\gamma = 0$

Voltage across $R_L + R_1 = v_i$

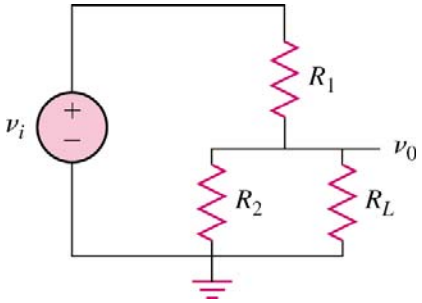
$$\Rightarrow v_0 = \left(\frac{R_L}{R_L + R_1}\right) v_i = \frac{1}{2} v_i$$

Voltage Divider



2.18

For $v_i > 0,\ (V_\gamma = 0)$



a.

$$v_0 = \left(\frac{R_2 \,||\, R_L}{R_2 \,||\, R_L + R_1}\right) |v_i|$$

$$R_2 \,||\, R_L = 2.2 \,||\, 6.8 = 1.66 \text{ k}\Omega$$

$$v_0 = \left(\frac{1.66}{1.66 + 2.2}\right) v_i = 0.43 |v_i|$$



b.

$$v_0(\text{rms}) = \frac{v_0(\text{max})}{\sqrt{2}} \Rightarrow \underline{v_0(\text{rms}) = 3.04 \text{ V}}$$

2.19

(a) $I_L = \dfrac{3.9}{4} = 0.975 \text{ mA}$

$$I_I = \frac{20 - 3.9}{12} = 1.342 \text{ mA}$$

$$I_Z = I_I - I_L = 1.342 - 0.975 = 0.367 \text{ mA}$$

$$P_Z = I_Z V_Z = (0.367)(3.9) = 1.43 \text{ mW}$$

(b) $I_L = \dfrac{3.9}{10} = 0.39 \text{ mA}$

$$I_Z = 1.342 - 0.39 = 0.952 \text{ mA}$$

$$P_Z = (0.952)(3.9) = 3.71 \text{ mW}$$

2.20

(a)

$$I_Z = \frac{40-12}{120} = 0.233 \text{ A}$$

$$P = (0.233)(12) = 2.8 \text{ W}$$

(b) $I_R = 0.233$ A, $I_L = (0.9)(0.233) = 0.21$ A

$$0.21 = \frac{12}{R_L} \Rightarrow \underline{R_L = 57.1\Omega}$$

So

(c) $$P = (0.1)(0.233)(12) \Rightarrow \underline{P = 0.28 \text{ W}}$$

2.21

(a) $P_Z = I_Z V_Z$

$$4 = I_z(15.4) \Rightarrow I_Z(\max) = 259.74 \text{ mA}$$

So $15 \le I_z \le 259.74$ mA

(b) $$I_I = \frac{60-15.4}{0.15} = 297.33 \text{ mA}$$

So $I_L(\max) = 297.33 - 15 = 282.33$ mA

$I_L(\min) = 297.33 - 259.74 = 37.59$ mA

Then $$R_L(\min) = \frac{15.4}{0.28233} = 54.55\,\Omega$$

$$R_L(\max) = \frac{15.4}{0.03759} = 410\,\Omega$$

So $54.55 \le R_L \le 410\,\Omega$

2.22

a.

$$I_I = \frac{20-10}{222} \Rightarrow \underline{I_I = 45.0 \text{ mA}}$$

$$I_L = \frac{10}{380} \Rightarrow \underline{I_L = 26.3 \text{ mA}}$$

$$I_Z = I_I - I_L \Rightarrow \underline{I_Z = 18.7 \text{ mA}}$$

b.

$$P_Z(\max) = 400 \text{ mW} \Rightarrow I_Z(\max) = \frac{400}{10} = 40 \text{ mA}$$

$$\Rightarrow I_L(\min) = I_I - I_Z(\max) = 45 - 40$$

$$\Rightarrow I_L(\min) = 5 \text{ mA} = \frac{10}{R_L}$$

$$\Rightarrow \underline{R_L = 2 \text{ k}\Omega}$$

(c) For $R_i = 175\Omega$ $\underline{I_I = 57.1 \text{ mA}}$ $\underline{I_L = 26.3 \text{ mA}}$ $\underline{I_Z = 30.8 \text{ mA}}$

$I_Z(\max) = 40 \text{ mA} \Rightarrow I_L(\min) = 57.1 - 40 = 17.1 \text{ mA}$

$$R_L = \frac{10}{17.1} \Rightarrow \underline{R_L = 585\Omega}$$

2.23

a. From Eq. (2.30)

$$I_Z(\max) = \frac{500[20-10]-50[15-10]}{15-(0.9)(10)-(0.1)(20)}$$

$$= \frac{5000-250}{4}$$

$$I_Z(\max) = 1.1875 \text{ A}$$

$$I_Z(\min) = 0.11875 \text{ A}$$

From Eq. (2.28(b)) $$R_i = \frac{20-10}{1187.5+50} \Rightarrow \underline{R_i = 8.08\Omega}$$

b.

$$P_Z = (1.1875)(10) \Rightarrow \underline{P_Z = 11.9 \text{ W}}$$

$$P_L = I_L(\max)V_0 = (0.5)(10) \Rightarrow \underline{P_L = 5 \text{ W}}$$

2.24

(a) $I_L = 0$

$$I_Z = \frac{10-5.6}{50+3} \Rightarrow 83.0 \text{ mA}$$

$$V_Z = 5.6 + (0.083)(3) = 5.85 \text{ V} = V_L$$

$$P_Z = I_Z V_Z = (0.083)(5.85) = 0.486 \text{ W}$$

(b) $$\frac{10-V_L}{50} = \frac{V_L-5.6}{3} + \frac{V_L}{200}$$

$$0.20 + 1.867 = V_L(0.02 + 0.3333 + 0.005)$$

So $V_L = 5.769 \text{ V}$

Then $$I_L = \frac{5.769}{0.2} = 28.84 \text{ mA}$$

$$I_I = \frac{10-5.769}{0.050} = 84.62 \text{ mA}$$

And $I_Z = I_I - I_L = 55.8 \text{ mA}$

$$P_Z = (0.0558)(5.769) = 0.322 \text{ W}$$

(c) $I_L = 0$

$$I_Z = \frac{12-5.6}{50+3} \Rightarrow 120.8 \text{ mA}$$

$$V_Z = V_L = 5.6 + (0.1208)(3) = 5.962 \text{ V}$$

$$P_Z = (0.1208)(5.962) = 0.72 \text{ W}$$

(d) $$\frac{12-V_L}{50} = \frac{V_L-5.6}{3} + \frac{V_L}{200}$$

$$0.24 + 1.867 = V_L(0.02 + 0.333 + 0.005)$$

So $V_L = 5.88 \text{ V}$

Then $$I_L = \frac{5.88}{0.20} = 29.4 \text{ mA}; \quad I_I = \frac{12-5.88}{0.05} = 122.4 \text{ mA}$$

$$I_Z = 122.4 - 29.4 = 93 \text{ mA}$$

$$P_Z = (0.093)(5.88) = 0.547 \text{ W}$$

2.25

(a) Set $I_Z = 10\text{ mA}$; $I_L = \dfrac{V_L}{R_L} = \dfrac{7.5}{1} = 7.5\text{ mA}$

$$I_I = 10 + 7.5 = 17.5\text{ mA}$$

$$I_I = 17.5 = \frac{12 - 7.5}{R_i} \Rightarrow R_i = 257\,\Omega$$

(b) $7.5 = V_{ZO} + (0.01)(12) \Rightarrow V_{ZO} = 7.38\text{ V}$

For $V_I = (1.1)(12) = 13.2\text{ V}$

$$\frac{13.2 - V_L}{257} = \frac{V_L - 7.38}{12} + \frac{V_L}{1000}$$

$$0.05136 + 0.615 = V_L(0.00389 + 0.0833 + 0.001) \Rightarrow V_L = 7.556\text{ V}$$

For $V_I = (0.9)(12) = 10.8\text{ V}$

$$\frac{10.8 - V_L}{257} = \frac{V_L - 7.38}{12} + \frac{V_L}{1000}$$

$$0.04202 + 0.615 = V_L(0.08819) \Rightarrow V_L = 7.450\text{ V}$$

Then, $\text{Source Reg} = \left(\dfrac{7.556 - 7.450}{13.2 - 10.8}\right) \times 100\% = 4.42\%$

(c) For $R_L = 1\,\text{k}\Omega$, $V_L = 7.50\text{ V}$

For $R_L = \infty$, $I_Z = \dfrac{12 - 7.38}{257 + 12} \Rightarrow 17.17\text{ mA}$

$$V_L = 7.38 + (0.01717)(12) = 7.586\text{ V}$$

Then, $\text{Load Reg} = \left(\dfrac{7.586 - 7.50}{7.50}\right) \times 100\% = 1.15\%$

2.26

$$\%\text{ Reg} = \frac{V_L(\max) - V_L(\min)}{V_L(nom)} \times 100\%$$

$$= \frac{V_L(nom) + I_Z(\max) r_z - \left(V_L(nom) + I_Z(\min) r_z\right)}{V_L(nom)}$$

$$= \frac{\left[I_Z(\max) - I_Z(\min)\right](3)}{6} = 0.05$$

So $I_Z(\max) - I_Z(\min) = 0.1\,A$

Now $I_L(\max) = \dfrac{6}{500} = 0.012\,A, \quad I_L(\min) = \dfrac{6}{1000} = 0.006\,A$

Now $R_i = \dfrac{V_{PS}(\min) - V_Z}{I_Z(\min) + I_L(\max)}$

or $280 = \dfrac{15 - 6}{I_Z(\min) + 0.012} \Rightarrow I_Z(\min) = 0.020\,A$

Then $I_Z(\max) = 0.1 + 0.02 = 0.12\ A$ and $R_i = \dfrac{V_{PS}(\max) - V_Z}{I_Z(\max) + I_L(\min)}$

or $280 = \dfrac{V_{PS}(\max) - 6}{0.12 + 0.006} \Rightarrow \underline{V_{PS}(\max) = 41.3\ V}$

2.27

Using Figure 2.19

a. $V_{PS} = 20 \pm 25\% \Rightarrow 15 \le V_{PS} \le 25$ V

For $V_{PS}(\min)$:

$I_I = I_Z(\min) + I_L(\max) = 5 + 20 = 25$ mA

$R_i = \dfrac{V_{PS}(\min) - V_Z}{I_I} = \dfrac{15 - 10}{25} \Rightarrow \underline{R_i = 200\Omega}$

b. For $V_{PS}(\max) \Rightarrow I_I(\max) = \dfrac{25 - 10}{R_i} \Rightarrow I_I(\max) = 75$ mA

For $I_L(\min) = 0 \Rightarrow I_Z(\max) = 75$ mA

$V_{Z0} = V_Z - I_Z r_Z = 10 - (0.025)(5) = 9.875$ V

$V_0(\max) = 9.875 + (0.075)(5) = 10.25$

$V_0(\min) = 9.875 + (0.005)(5) = 9.90$

$\underline{\Delta V_0 = 0.35\ \text{V}}$

c. $\%\ \text{Reg} = \dfrac{\Delta V_0}{V_0(\text{nom})} \times 100\% \Rightarrow \underline{\%\ \text{Reg} = 3.5\%}$

2.28

From Equation (2.28(a))

$$R_i = \frac{V_{PS}(\min) - V_Z}{I_Z(\min) + I_L(\max)} = \frac{24 - 16}{40 + 400} \quad \text{or} \quad \underline{R_i = 18.2\Omega}$$

Also $V_r = \dfrac{V_M}{2fRC} \Rightarrow C = \dfrac{V_M}{2fRV_r}$

$R \cong R_i + r_z = 18.2 + 2 = 20.2\Omega$

Then

$$C = \frac{24}{2(60)(1)(20.2)} \Rightarrow C = 9901\ \mu F$$

2.29

$V_Z = V_{Z0} + I_Z r_Z \quad V_Z(\text{nom}) = 8$ V

$8 = V_{Z0} + (0.1)(0.5) \Rightarrow V_{Z0} = 7.95$ V

$I_i = \dfrac{V_S(\max) - V_Z(\text{nom})}{R_i} = \dfrac{12 - 8}{3} = 1.333$ A

For $I_L = 0.2$ A $\Rightarrow I_Z = 1.133$ A

For $I_L = 1$ A $\Rightarrow I_Z = 0.333$ A

$$V_L(\max) = V_{Z0} + I_Z(\max) r_Z$$
$$= 7.95 + (1.133)(0.5) = 8.5165$$
$$V_L(\min) = V_{Z0} + I_Z(\min) r_Z$$
$$= 7.95 + (0.333)(0.5) = 8.1165$$
$$\Delta V_L = 0.4 \text{ V}$$
$$\% \text{ Reg} = \frac{\Delta V_L}{V_0(\text{nom})} = \frac{0.4}{8} \Rightarrow \underline{\% \text{ Reg} = 5.0\%}$$
$$V_r = \frac{V_M}{2fRC} \Rightarrow C = \frac{V_M}{2fRV_r}$$
$$R = R_i + r_z = 3 + 0.5 = 3.5\Omega$$
$$C = \frac{12}{2(60)(3.5)(0.8)} \Rightarrow \underline{C = 0.0357\ F}$$

Then

2.30

For $-6.3 \le \upsilon_I \le 3$ V, $\upsilon_O = \upsilon_I$

For $\upsilon_I > 3$ V, $I = \dfrac{\upsilon_I - 3}{1.5}$ and $\upsilon_O = \upsilon_I - I(0.5)$

$$\upsilon_O = \upsilon_I - (0.5)\left(\frac{\upsilon_I - 3}{1.5}\right) = 0.667\upsilon_I + 1.0$$

For $\upsilon_I < -6.3$ V, $I = \dfrac{\upsilon_I + 6.3}{2.5}$ and $\upsilon_O = \upsilon_I - I(0.5)$

$$\upsilon_O = \upsilon_I - (0.5)\left(\frac{\upsilon_I + 6.3}{2.5}\right) = 0.8\upsilon_I - 1.26$$

2.31

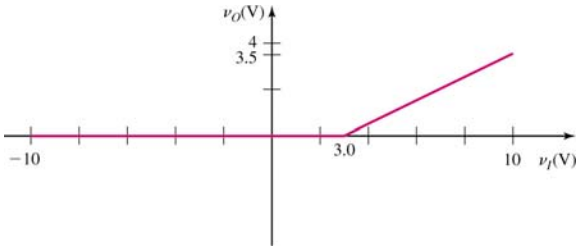
(a) For $-10 \le v_I \le 0$, both diodes are conducting $\Rightarrow v_O = 0$

For $0 \le v_I \le 3$, Zener not in breakdown, so $\underline{i_1 = 0, \quad v_O = 0}$

For $v_I > 3$ $i_1 = \dfrac{v_I - 3}{20}$ *mA*

$$v_o = \left(\frac{v_I - 3}{20}\right)(10) = \frac{1}{2}v_I - 1.5$$

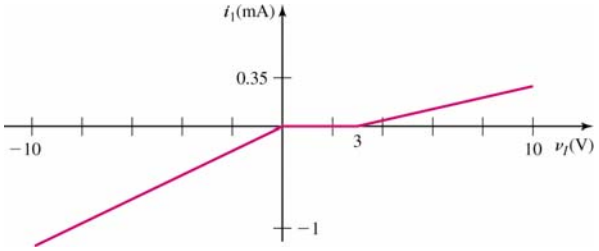
At $v_I = 10$ *V*, $v_o = 3.5$ *V*, $i_1 = 0.35$ *mA*



(b) For $v_I < 0$, both diodes forward biased

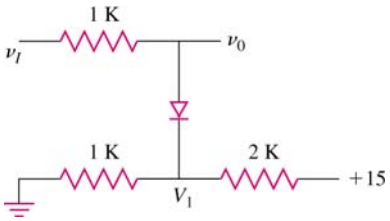
$-i_1 = \dfrac{0 - v_I}{10}$. At $v_I = -10\ V,\ i_1 = -1\ mA$

For $v_I > 3,\ i_1 = \dfrac{v_I - 3}{20}$. At $v_I = 10\ V,\ i_1 = 0.35\ mA$



2.32

(a)



$V_1 = \dfrac{1}{3} \times 15 = 5\ \text{V} \Rightarrow$ for $v_I \le 5.7,\ v_0 = v_I$

For $v_I > 5.7\ V$

$$\frac{v_I - (V_1 + 0.7)}{1} + \frac{15 - V_1}{2} = \frac{V_1}{1},\ v_0 = V_1 + 0.7$$

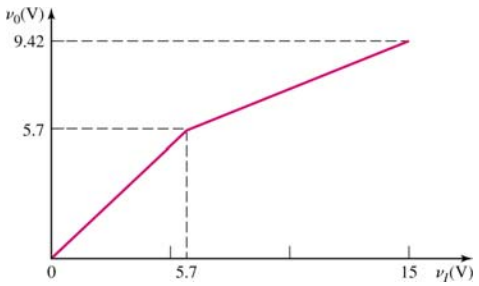
$$\frac{v_I - v_0}{1} + \frac{15 - (v_0 - 0.7)}{2} = \frac{v_0 - 0.7}{1}$$

$$\frac{v_I}{1} + \frac{15.7}{2} + \frac{0.7}{1} = v_0\left(\frac{1}{1} + \frac{1}{2} + \frac{1}{1}\right) = v_0(2.5)$$

$$v_I + 8.55 = v_0(2.5) \Rightarrow v_0 = \frac{1}{2.5}v_I + 3.42$$

$v_I = 5.7 \Rightarrow v_0 = 5.7$

$v_I = 15 \Rightarrow v_0 = 9.42$

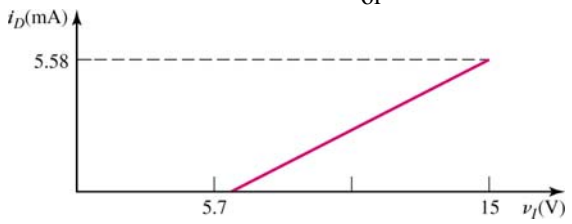


(b) $i_D = 0$ for $0 \le v_I \le 5.7$

Then for $v_I > 5.7\ V$

$$i_D = \frac{v_I - v_O}{1} = \frac{v_I - \left(\frac{v_I}{2.5} + 3.42\right)}{1} \quad \text{or} \quad i_D = \frac{0.6v_I - 3.42}{1}$$

For $v_I = 15$, $i_D = 5.58\ mA$



2.33

(a) (i) $V_B = 1.8\ \text{V}$

For $\upsilon_I \ge 1.1\ \text{V}$, $\upsilon_O = \upsilon_I$

For $\upsilon_I \le 1.1\ \text{V}$, $\upsilon_O = 1.1\ \text{V}$

(ii) $V_B = -1.8\ \text{V}$

For $\upsilon_I \ge -2.5\ \text{V}$, $\upsilon_O = \upsilon_I$

For $\upsilon_I \le -2.5\ \text{V}$, $\upsilon_O = -2.5\ \text{V}$

(b) (i) $V_B = 1.8\ \text{V}$

For $\upsilon_I \ge 2.5\ \text{V}$, $\upsilon_O = 2.5\ \text{V}$

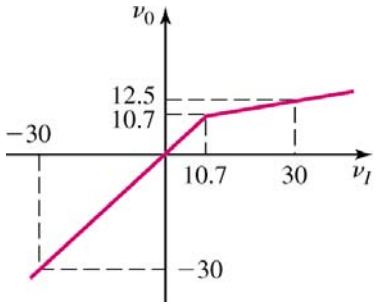
For $\upsilon_I \le 2.5\ \text{V}$, $\upsilon_O = \upsilon_I$

(ii) $V_B = -1.8\ \text{V}$

For $\upsilon_I \ge -1.1\ \text{V}$, $\upsilon_O = -1.1\ \text{V}$

For $\upsilon_I \le -1.1\ \text{V}$, $\upsilon_O = \upsilon_I$

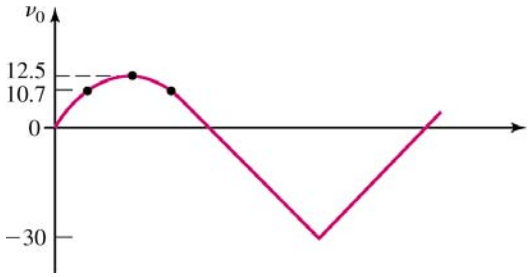
2.34



For $v_I = 30\ \text{V}$, $\text{i} = \dfrac{30 - 10.7}{100 + 10} = 0.175\ \text{A}$

$v_0 = \text{i}(10) + 10.7 = 12.5\ \text{V}$

b.



2.35

(a) (i) $V_B = 5\text{ V}$

For $v_I \geq 5.7\text{ V}$, $v_O = v_I - 5.7$

For $v_I \leq 5.7\text{ V}$, $v_O = 0$

(ii) $V_B = -5\text{ V}$

For $v_I \geq -4.3\text{ V}$, $v_O = v_I + 4.3$

For $v_I \leq -4.3\text{ V}$, $v_O = 0$

(b) (i) $V_B = 5\text{ V}$

For $v_I \geq 4.3\text{ V}$, $v_O = 0$

For $v_I \leq 4.3\text{ V}$, $v_O = v_I - 4.3$

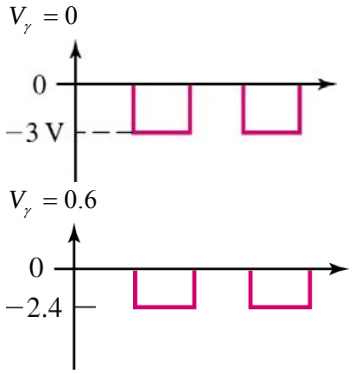
(ii) $V_B = -5\text{ V}$

For $v_I \geq -5.7\text{ V}$, $v_O = 0$

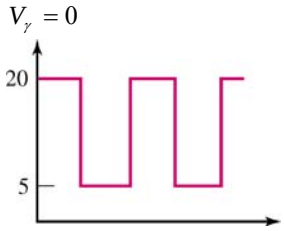
For $v_I \leq -5.7\text{ V}$, $v_O = v_I + 5.7$

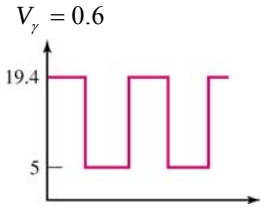
2.36

a.

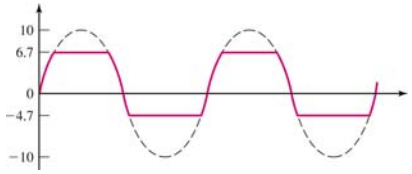


b.



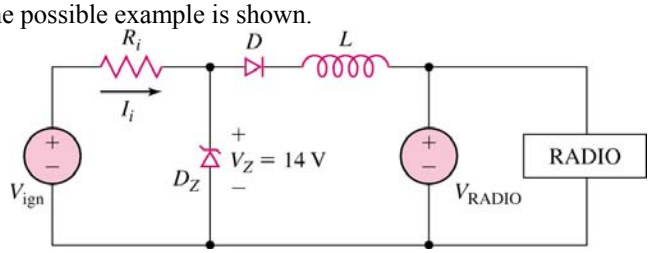


2.37



2.38

One possible example is shown.

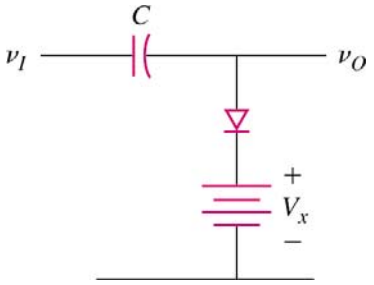


L will tend to block the transient signals
D_z will limit the voltage to +14 V and -0.7 V.
Power ratings depends on number of pulses per second and duration of pulse.

2.39

(a) Square wave between +40 V and 0.
(b) Square wave between +35 V and −5 V.
(c) Square wave between +5 V and −35 V.

2.40



a. For $V_\gamma = 0 \Rightarrow V_x = 2.7$ V

b. For $V_\gamma = 0.7\,\text{V} \Rightarrow V_x = 2.0$ V

2.41

Circuit similar to Figure 2.31(a) with $V_B = -10$ V.

2.42

In steady-state, $\upsilon_O = (10\sin\omega t + 5)$ V

2.43

(i) $V_B = 5$ V, In steady-state,

$$\upsilon_O = (10\sin\omega t - 5) \text{ V}$$

(ii) $V_B = -5$ V, In steady-state,

$$\upsilon_O = (10\sin\omega t - 15) \text{ V}$$

2.44

a.

$$I_{D1} = \frac{10 - 0.6}{9.5 + 0.5} \Rightarrow \underline{I_{D1} = 0.94 \text{ mA}} \quad \underline{I_{D2} = 0}$$

$$V_0 = I_{D1}(9.5) \Rightarrow \underline{V_0 = 8.93 \text{ V}}$$

b.

$$I_{D1} = \frac{5 - 0.6}{9.5 + 0.5} \Rightarrow \underline{I_{D1} = 0.44 \text{ mA}} \quad \underline{I_{D2} = 0}$$

$$V_0 = I_{D1}(9.5) \Rightarrow \underline{V_0 = 4.18 \text{ V}}$$

c. Same as (a)

d.

$$10 = \frac{(I)}{2}(0.5) + 0.6 + I(9.5) \Rightarrow I = 0.964 \text{ mA}$$

$$V_0 = I(9.5) \Rightarrow \underline{V_0 = 9.16 \text{ V}}$$

$$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.482 \text{ mA}}$$

2.45

a.

$$\underline{I = I_{D1} = I_{D2} = 0} \quad \underline{V_0 = 10}$$

b.

$$10 = I(9.5) + 0.6 + I(0.5) \Rightarrow \underline{I = I_{D2} = 0.94 \text{ mA}} \quad \underline{I_{D1} = 0}$$

$$V_0 = 10 - I(9.5) \Rightarrow \underline{V_0 = 1.07 \text{ V}}$$

c.

$$10 = I(9.5) + 0.6 + I(0.5) + 5 \Rightarrow \underline{I = I_{D2} = 0.44 \text{ mA}} \quad \underline{I_{D1} = 0}$$

$$V_0 = 10 - I(9.5) \Rightarrow \underline{V_0 = 5.82 \text{ V}}$$

d.

$$10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) \Rightarrow \underline{I = 0.964 \text{ mA}}$$

$$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.482 \text{ mA}}$$

$$V_0 = 10 - I(9.5) \Rightarrow \underline{V_0 = 0.842 \text{ V}}$$

2.46

a.

$$V_1 = V_2 = 0 \Rightarrow D_1,\ D_2,\ D_3, \text{ on } \underline{V_0 = 4.4 \text{ V}}$$

$$I = \frac{10 - 4.4}{9.5} \Rightarrow \quad \underline{I = 0.589 \text{ mA}}$$

$$I_{D1} = I_{D2} = \frac{4.4 - 0.6}{0.5} \Rightarrow \underline{I_{D1} = I_{D2} = 7.6 \text{ mA}}$$

$$I_{D3} = I_{D1} + I_{D2} - I = 2(7.6) - 0.589 \Rightarrow \underline{I_{D3} = 14.6 \text{ mA}}$$

b.

$$V_1 = V_2 = 5 \text{ V } D_1 \text{ and } D_2 \text{ on, } D_3 \text{ off}$$

$$10 = I(9.5) + 0.6 + \frac{I}{2}(0.5) + 5 \Rightarrow \underline{I = 0.451 \text{ mA}}$$

$$I_{D1} = I_{D2} = \frac{I}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.226 \text{ mA}}$$

$$\underline{I_{D3} = 0}$$

$$V_0 = 10 - I(9.5) = 10 - (0.451)(9.5) \Rightarrow \underline{V_0 = 5.72 \text{ V}}$$

c. $V_1 = 5$ V, $V_2 = 0$ D_1 off, D_2, D_3 on $\underline{V_0 = 4.4 \text{ V}}$

$$I = \frac{10 - 4.4}{9.5} \Rightarrow \qquad \underline{I = 0.589 \text{ mA}}$$

$$I_{D2} = \frac{4.4 - 0.6}{0.5} \Rightarrow \qquad \underline{I_{D2} = 7.6 \text{ mA}}$$

$$\underline{I_{D1} = 0}$$

$$I_{D3} = I_{D2} - I = 7.6 - 0.589 \Rightarrow \ \underline{I_{D3} = 7.01 \text{ mA}}$$

d. $V_1 = 5$ V, $V_2 = 2$ V D_1 off, D_2, D_3 on $\underline{V_0 = 4.4 \text{ V}}$

$$I = \frac{10 - 4.4}{9.5} \Rightarrow \qquad \underline{I = 0.589 \text{ mA}}$$

$$I_{D2} = \frac{4.4 - 0.6 - 2}{0.5} \Rightarrow \qquad \underline{I_{D2} = 3.6 \text{ mA}}$$

$$\underline{I_{D1} = 0}$$

$$I_{D3} = I_{D2} - I = 3.6 - 0.589 \Rightarrow \underline{I_{D3} = 3.01 \text{ mA}}$$

2.47

(a) $$V_1 = 4.4 \text{ V}, \quad I_{D1} = 0.2 = \frac{10 - 0.6 - 4.4}{R_1} \Rightarrow R_1 = 25 \text{ k}\Omega$$

$$I_{R2} = 0.2 + 0.3 = 0.5 \text{ mA}$$

$$V_2 = -0.6 \text{ V}, \quad I_{R2} = 0.5 = \frac{4.4 - (-0.6)}{R_2} \Rightarrow R_2 = 10 \text{ k}\Omega$$

$$I_{R3} = 0.5 + 0.5 = 1.0\ \text{mA}$$

$$R_3 = \frac{-0.6-(-5)}{1.0} = 4.4\ \text{k}\Omega$$

(b) Assume all diodes conducting

$$V_1 = 4.4\ \text{V},\ I_{D1} = \frac{10-0.6-4.4}{10} = 0.5\ \text{mA}$$

$$V_2 = -0.6\ \text{V},\ I_{R2} = \frac{4.4-(-0.6)}{4} = 1.25\ \text{mA}$$

Then $I_{D2} = 1.25 - 0.5 = 0.75\ \text{mA}$

$$I_{R3} = \frac{-0.6-(-5)}{2.2} = 2\ \text{mA}$$

Then $I_{D3} = 2 - 1.25 = 0.75\ \text{mA}$

(c) Diode D_2 cutoff $\Rightarrow I_{D2} = 0$

$$V_2 = -0.6\ \text{V},\ I_{D1} = \frac{10-0.6-(-0.6)}{R_1+R_2} = \frac{10}{9} = 1.11\ \text{mA}$$

$$V_1 = 10 - 0.6 - (1.11)(3) = 6.07\ \text{V}$$

$$I_{R3} = \frac{-0.6-(-5)}{2.5} = 1.76\ \text{mA}$$

Then $I_{D3} = 1.76 - 1.11 = 0.65\ \text{mA}$

(d) Diode D_3 cutoff $\Rightarrow I_{D3} = 0$

$$V_1 = 4.4\ \text{V},\ I_{D1} = \frac{10-0.6-4.4}{6} = 0.833\ \text{mA}$$

$$I_{R2} = \frac{4.4-(-5)}{R_2+R_3} = \frac{9.4}{9} = 1.044\ \text{mA}$$

$$V_2 = (1.044)(6) - 5 = 1.27\ \text{V}$$

Then $I_{D2} = 1.044 - 0.833 = 0.211\ \text{mA}$

2.48

(a) $I_{D1} = I_{D2} = 2.5\ \text{mA}$

$$I_{D2} = 2.5 = \frac{0.7-0.7-(-2)}{R} \Rightarrow R = 0.8\ \text{k}\Omega$$

(b) $I_{D1} = 0.2I_{D2}$, $I_{D1} + I_{D2} = 5$

$$0.2I_{D2} + I_{D2} = 5 \Rightarrow I_{D2} = 4.167\ \text{mA}$$

$$I_{D2} = 4.167 = \frac{2}{R} \Rightarrow R = 0.48\ \text{k}\Omega$$

(c) $I_{D1} = 5I_{D2}$, $I_{D1} + I_{D2} = 5$

$$5I_{D2} + I_{D2} = 5 \Rightarrow I_{D2} = 0.833\ \text{mA}$$

$$I_{D2} = 0.833 = \frac{2}{R} \Rightarrow R = 2.4\ \text{k}\Omega$$

2.49

(a) D_1 and D_2 on

$$5+\frac{5-0.7-V_A}{2}=\frac{V_A-0.7-(-5)}{1.1}$$

$$5+2.15-3.909=V_A\left(\frac{1}{2}+\frac{1}{1.1}\right)\Rightarrow V_A=2.30\text{ V}$$

Then $I_{D1}=\dfrac{5-0.7-2.3}{2}=1.0\text{ mA}$

$$I_{D2}=\frac{2.3-0.7-(-5)}{1.1}=6.0\text{ mA}$$

(b) D_1 cutoff, $I_{D1}=0$

$$I_{D2}=5\text{ mA},\quad V_A=0.7+(5)(2.5)-5=8.2\text{ V}$$

(c) $V_A=0$, $I_{D1}=\dfrac{5-0.7-0}{2}=2.15\text{ mA}$

Then $I_{D2}=5+2.15=7.15\text{ mA}$

$$I_{D2}=7.15=\frac{0-0.7-(-5)}{R_2}\Rightarrow R_2=0.60\text{ k}\Omega$$

2.50

(a) (i) $v_I=5\text{ V}$, D_1 and D_2 on

$$\frac{5-(v_O+0.6)}{5}+\frac{5-v_O}{5}=\frac{v_O}{0.5}+\frac{v_O-0.6}{0.5}$$

$$0.88+1.0+1.2=v_O(0.20+0.20+2.0+2.0)\Rightarrow v_O=0.7\text{ V}$$

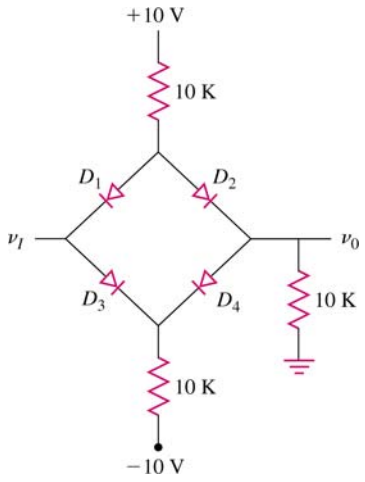
(ii) $v_I=-5\text{ V}$

$$v_O=\left(\frac{0.5}{0.5+5}\right)v_I=-0.455\text{ V}$$

(b) (i) $v_I=5\text{ V}$, $v_O=4.4\text{ V}$

(ii) $v_I=-5\text{ V}$, $v_O=-0.6\text{ V}$

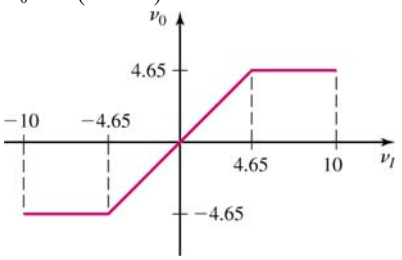
2.51



For $v_I > 0.$ when D_1 and D_4 turn off

$$I = \frac{10 - 0.7}{20} = 0.465 \text{ mA}$$

$$v_0 = I\left(10 \text{ k}\Omega\right) = 4.65 \text{ V}$$



$$\underline{v_0 = v_I \text{ for } -4.65 \le v_I \le 4.65}$$

2.52

(a) All diodes on

$$\frac{15 - V_A}{6.15} = \frac{V_A - 0.7}{2} + \frac{V_A - 0.7 - \left(-5\right)}{14} + \frac{V_A - 0.7 - \left(-10\right)}{24}$$

$$2.439 + 0.35 - 0.307 - 0.3875 = V_A\left(0.1626 + 0.50 + 0.0714 + 0.0417\right) \Rightarrow V_A = 2.70 \text{ V}$$

Then $I_{D1} = \dfrac{2.70 - 0.7}{2} = 1.0 \text{ mA}$

$$I_{D2} = \frac{2.70 - 0.7 - \left(-5\right)}{14} = 0.50 \text{ mA}$$

$$I_{D3} = \frac{2.70 - 0.7 - \left(-10\right)}{24} = 0.50 \text{ mA}$$

(b) D_1 cutoff, $I_{D1} = 0$

$$\frac{15 - V_A}{6.15} = \frac{V_A - 0.7 - \left(-5\right)}{3.3} + \frac{V_A - 0.7 - \left(-10\right)}{5.2}$$

$$2.439 - 1.303 - 1.788 = V_A\left(0.1626 + 0.303 + 0.1923\right) \Rightarrow V_A = -0.991 \text{ V}$$

Then $I_{D2} = \dfrac{-0.991 - 0.7 - \left(-5\right)}{3.3} = 1.0 \text{ mA}$

$$I_{D3} = \frac{-0.991 - 0.7 - \left(-10\right)}{5.2} = 1.60 \text{ mA}$$

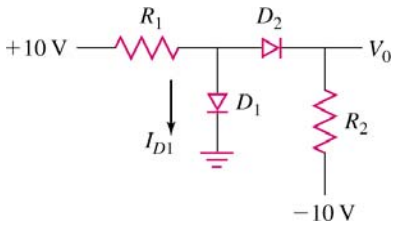
(c) D_1 and D_2 cutoff, $I_{D1} = I_{D2} = 0$

$$I_{D3} = \frac{15 - 0.7 - \left(-10\right)}{R_1 + R_4} = \frac{24.3}{6.15 + 1.32} = 3.25 \text{ mA}$$

$$V_A = 15 - \left(3.25\right)\left(6.15\right) = -5 \text{ V}$$

2.53

a.



$R_1 = 5\ \text{k}\Omega,\ R_2 = 10\ \text{k}\Omega$

D_1 and D_2 on $\Rightarrow \underline{V_0 = 0}$

$$I_{D1} = \frac{10-0.7}{5} - \frac{0-(-10)}{10} = 1.86 - 1.0$$

$$\underline{I_{D1} = 0.86\ \text{mA}}$$

b.

$R_1 = 10\ \text{k}\Omega,\ R_2 = 5\ \text{k}\Omega,\ D_1$ off, D_2 on $\underline{I_{D1} = 0}$

$$I = \frac{10-0.7-(-10)}{15} = 1.287$$

$$V_0 = IR_2 - 10 \Rightarrow \underline{V_0 = -3.57\ \text{V}}$$

2.54

If both diodes on

(a)

$$V_A = -0.7\ \text{V},\ \underline{V_O = -1.4\ \text{V}}$$

$$I_{R1} = \frac{10-(-0.7)}{10} = 1.07\ \text{mA}$$

$$I_{R2} = \frac{-1.4-(-15)}{5} = 2.72\ \text{mA}$$

$$I_{R1} + I_{D1} = I_{R2} \Rightarrow I_{D1} = 2.72 - 1.07$$

$$I_{D1} = 1.65\ \text{mA}$$

(b) D_1 off, D_2 on

$$I_{R1} = I_{R2} = \frac{10-0.7-(-15)}{5+10} = 1.62\ \text{mA}$$

$$V_O = I_{R2}R_2 - 15 = (1.62)(10) - 15 \Rightarrow V_O = 1.2\ \text{V}$$

$$V_A = 1.2 + 0.7 = 1.9\ V \Rightarrow D_1\ \text{off},$$

$$I_{D1} = 0$$

2.55

(a) D_1 on, D_2 off

$$I_{D1} = \frac{10-0.7}{10} = 0.93\ \text{mA}$$

$$V_O = -15\ \text{V}$$

(b) D_1 on, D_2 off

$$I_{D1} = \frac{10 - 0.7}{5} = 1.86 \text{ mA}$$

$$V_O = -15 \text{ V}$$

2.56

$$\frac{15 - (V_0 + 0.7)}{10} = \frac{V_0 + 0.7}{20} + \frac{V_0}{20}$$

$$\frac{15}{10} - \frac{0.7}{10} - \frac{0.7}{20} = V_0\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{20}\right) = V_0\left(\frac{4.0}{20}\right)$$

$$V_0 = 6.975 \text{ V}$$

$$I_D = \frac{V_0}{20} \Rightarrow \underline{I_D = 0.349 \text{ mA}}$$

2.57

(a) Diode is cutoff, $I_D = 0$, $V_D = 0$

$$V_A = V_B = 3 \text{ V}$$

(b) Diode is conducting, $V_D = 0.7$ V

$$\frac{5 - V_B}{10} = \frac{V_B}{10} + \frac{V_B - 0.7}{10} + \frac{V_B - 0.7 - 2}{10}$$

$$0.50 + 0.07 + 0.27 = V_B(0.10 + 0.10 + 0.10 + 0.10) \Rightarrow V_B = 2.1 \text{ V and } V_A = 1.4 \text{ V}$$

$$\frac{5 - V_B}{10} = \frac{V_B}{10} + I_D$$

So $I_D = \frac{5 - 2.1}{10} - \frac{2.1}{10} = 0.08 \text{ mA}$

(c) Diode is cutoff, $I_D = 0$

$$V_A = \frac{1}{2}(5) = 2.5 \text{ V}, \quad V_B = \frac{1}{2}(4) = 2.0 \text{ V}$$

$$V_D = 2 - 2.5 = -0.5 \text{ V}$$

(d) Diode is conducting, $V_D = 0.7$ V

$$\frac{8 - V_B}{10} = \frac{V_B}{10} + \frac{V_B - 0.7}{10} + \frac{V_B - 0.7 - 2}{10}$$

$$0.80 + 0.07 + 0.27 = V_B(0.40) \Rightarrow V_B = 2.85 \text{ V and } V_A = 2.15 \text{ V}$$

Then $I_D = \frac{8 - V_B}{10} - \frac{V_B}{10} = \frac{1}{10}[8 - 2(2.85)] = 0.23 \text{ mA}$

2.58

$v_I = 0$, D_1 off, D_2 on

$$I = \frac{10 - 2.5}{15} 0.5\,\text{mA}$$

$$v_o = 10 - (0.5)(5) \Rightarrow v_o = 7.5\,\text{V} \quad \text{for } 0 \le v_I \le 7.5\,\text{V}$$

For $v_I > 7.5\ V$, Both D_1 and D_2 on

$$\frac{v_I - v_o}{15} = \frac{v_o - 2.5}{10} + \frac{v_o - 10}{5} \quad \text{or} \quad \underline{v_I = v_o(5.5) - 33.75}$$

When $v_o = 10\ V$, D_2 turns off

$$v_I = (10)(5.5) - 33.75 = 21.25\ \text{V}$$

For $v_I > 21.25\ \text{V}$, $v_o = 10\ \text{V}$

2.59

(a) For $\upsilon_I = 0.5\ \text{V}$, $I_{D1} = I_{D2} = I_{D3} = 0$, $\upsilon_O = 0.5\ \text{V}$

(b) For $\upsilon_I = 1.5\ \text{V}$, D_1 on; $I_{D2} = I_{D3} = 0$

$$I_{D1} = \frac{1.5 - 0.7}{4 + 8} = 0.0667\ \text{mA}$$

$$\upsilon_O = 0.7 + (0.0667)(8) = 1.23\ \text{V}$$

(c) For $\upsilon_I = 3\ \text{V}$, D_1 and D_2 conducting, $I_{D3} = 0$

$$\frac{3 - \upsilon_O}{4} = \frac{\upsilon_O - 0.7}{8} + \frac{\upsilon_O - 1.7}{6}$$

$$0.75 + 0.0875 + 0.2833 = \upsilon_O(0.25 + 0.125 + 0.1667) \Rightarrow \upsilon_O = 2.069\ \text{V}$$

Then $I_{D1} = \dfrac{2.069 - 0.7}{8} = 0.171\ \text{mA}$

$$I_{D2} = \frac{2.069 - 1.7}{6} = 0.0615\ \text{mA}$$

(d) For $\upsilon_I = 5\ \text{V}$, all diodes conducting

$$\frac{5 - \upsilon_O}{4} = \frac{\upsilon_O - 0.7}{8} + \frac{\upsilon_O - 1.7}{6} + \frac{\upsilon_O - 2.7}{4}$$

$$1.25 + 0.0875 + 0.2833 + 0.675 = \upsilon_O(0.25 + 0.125 + 0.1667 + 0.25)$$

So $\upsilon_O = 2.90\ \text{V}$

Then $I_{D1} = \dfrac{2.90 - 0.7}{8} = 0.275\ \text{mA}$

$$I_{D2} = \frac{2.90 - 1.7}{6} = 0.20\ \text{mA}$$

$$I_{D3} = \frac{2.90 - 2.7}{4} = 0.05\ \text{mA}$$

2.60

(a) $I_{D2} = 0$ for $\upsilon_I < 4.5\ \text{V}$

$I_{D2} = 100\ \text{mA}$ for $\upsilon_I > 4.5\ \text{V}$

(b) $I_{D2} = 0$ for $\upsilon_I < 9\ \text{V}$

$I_{D2} = 100\ \text{mA}$ for $\upsilon_I > 9\ \text{V}$

2.61

a. $\underline{V_{01} = V_{02} = 0}$

b. $\underline{V_{01} = 4.4 \text{ V}}, \quad \underline{V_{02} = 3.8 \text{ V}}$

c. $\underline{V_{01} = 4.4 \text{ V}}, \quad \underline{V_{02} = 3.8 \text{ V}}$

Logic "1" level degrades as it goes through additional logic gates.

2.62

a. $\underline{V_{01} = V_{02} = 5 \text{ V}}$

b. $\underline{V_{01} = 0.6 \text{ V}}, \quad \underline{V_{02} = 1.2 \text{ V}}$

c. $\underline{V_{01} = 0.6 \text{ V}}, \quad \underline{V_{02} = 1.2 \text{ V}}$

Logic "0" signal degrades as it goes through additional logic gates.

2.63

$$(V_1 \; AND \; V_2) \; OR \; (V_3 \; AND \; V_4)$$

2.64

$$I = \frac{10 - 1.5 - 0.2}{R + 10} = 12 \text{ mA} = 0.012$$

$$R + 10 = \frac{8.3}{0.012} = 691.7\Omega$$

$$\underline{R = 681.7\Omega}$$

2.65

$$I = \frac{10 - 1.7 - V_I}{0.75} = 8$$

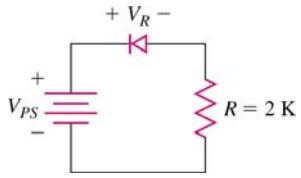
$$V_I = 10 - 1.7 - 8(0.75) \Rightarrow \underline{V_I = 2.3 \text{ V}}$$

2.66

$$R + r_f = \frac{5 - 1.7}{15} = 0.220 \text{ k}\Omega$$

$$r_f = 20\,\Omega \Rightarrow R = 200\,\Omega$$

2.67



$V_R = 1$ V, $I = 0.8$ mA

$V_{PS} = 1 + (0.8)(2)$

$\underline{V_{PS} = 2.6 \text{ V}}$

2.68

$I_{Ph} = \eta e \Phi A$

$0.6 \times 10^{-3} = (1)\left(1.6 \times 10^{-19}\right)\left(10^{17}\right) A$

$\underline{A = 3.75 \times 10^{-2} \text{ cm}^2}$

Chapter 3

3.1

$$K_n = \frac{k_n'}{2}\cdot\frac{W}{L} = \frac{120}{2}\left(\frac{10}{0.8}\right) \Rightarrow 0.75\ \text{mA/V}^2$$

(a) (i) $I_D = 0$

(ii) $I_D = (0.75)\left[2(1-0.4)(0.1)-(0.1)^2\right] \Rightarrow 82.5\ \mu\text{A}$

(iii) $I_D = (0.75)\left[2(2-0.4)(0.1)-(0.1)^2\right] = 0.2325\ \text{mA}$

(iv) $I_D = (0.75)\left[2(3-0.4)(0.1)-(0.1)^2\right] = 0.3825\ \text{mA}$

(b) (i) $I_D = 0$

(ii) $I_D = (0.75)(1-0.4)^2 = 0.27\ \text{mA}$

(iii) $I_D = (0.75)(2-0.4)^2 = 1.92\ \text{mA}$

(iv) $I_D = (0.75)(3-0.4)^2 = 5.07\ \text{mA}$

3.2

$$I_D = K_n\left[2(V_{GS}-V_{TN})V_{DS}-V_{DS}^2\right]$$
$$0.5 = K_n\left[2(0.6)V_{DS}-V_{DS}^2\right]$$
$$1.0 = K_n\left[2(1.0)V_{DS}-V_{DS}^2\right]$$

Take ratio

$$0.5 = \frac{1.2V_{DS}-V_{DS}^2}{2V_{DS}-V_{DS}^2} \Rightarrow V_{DS}-0.5V_{DS}^2 = 1.2V_{DS}-V_{DS}^2$$

or $1-0.5V_{DS} = 1.2-V_{DS}$

which yields $V_{DS} = 0.4\ \text{V}$

Then $0.5 = K_n\left[(1.2)(0.4)-(0.4)^2\right] \Rightarrow K_n = 1.56\ \text{mA/V}^2$

3.3

(a) Enhancement-mode

(b) From Graph $V_T = 1.5$ V

Now

$$0.03 = K_n(2-1.5)^2 = 0.25\,K_n \Rightarrow \quad K_n = 0.12$$
$$0.15 = K_n(3-1.5)^2 = 2.25\,K_n \qquad K_n = 0.0666$$
$$0.39 = K_n(4-1.5)^2 = 6.25\,K_n \qquad K_n = 0.0624$$
$$0.77 = K_n(5-1.5)^2 = 12.25\,K_n \qquad K_n = 0.0629$$

From last three, $K_n(\text{Avg}) = 0.0640\ \text{mA/V}^2$

$i_D(\text{sat}) = 0.0640(3.5-1.5)^2 \Rightarrow i_D(\text{sat}) = 0.256\ \text{mA}$ for $V_{GS} = 3.5$ V

(c) $i_D(\text{sat}) = 0.0640(4.5-1.5)^2 \Rightarrow i_D(\text{sat}) = 0.576\ \text{mA}$ for $V_{GS} = 4.5$ V

3.4

$V_{GS} = 0$

a. $V_{DS}(sat) = V_{GS} - V_{TN} = 0 - (-2.5) = 2.5\ V$

$V_{DS} = 0.5$ V $\Rightarrow$ Biased in nonsaturation

i. $I_D = (1.1)\left[2(0-(-2.5))(0.5) - (0.5)^2\right] \Rightarrow \underline{I_D = 2.48 \text{ mA}}$

$V_{DS} = 2.5$ V $\Rightarrow$ Biased in saturation

ii. $I_D = (1.1)(0-(-2.5))^2 \Rightarrow \underline{I_D = 6.88 \text{ mA}}$

iii. $V_{DS} = 5$ V Same as (ii) $\Rightarrow \underline{I_D = 6.88 \text{ mA}}$

b. $V_{GS} = 2$ V

$V_{DS}(\text{sat}) = 2 - (-2.5) = 4.5$ V

$V_{DS} = 0.5$ V $\Rightarrow$ Nonsaturation

i. $I_D = (1.1)\left[2(2-(-2.5))(0.5) - (0.5)^2\right] \Rightarrow \underline{I_D = 4.68 \text{ mA}}$

$V_{DS} = 2.5$ V $\Rightarrow$ Nonsaturation

ii. $I_D = (1.1)\left[2(2-(-2.5))(2.5) - (2.5)^2\right] \Rightarrow \underline{I_D = 17.9 \text{ mA}}$

$V_{DS} = 5$ V $\Rightarrow$ Saturation

iii. $I_D = (1.1)(2-(-2.5))^2 \Rightarrow \underline{I_D = 22.3 \text{ mA}}$

3.5

(a) $V_{DS}(sat) = V_{GS} - V_{TN} = 2.2 - 0.4 = 1.8$ V

$2.2 = V_{DS} > V_{DS}(sat) = 1.8 \Rightarrow$ Saturation

(b) $V_{DS}(sat) = V_{GS} - V_{TN} = 1 - 0.4 = 0.6$ V

$V_{DS} = -0.6 - (-1) = 0.4$ V $< V_{DS}(sat) = 0.6$ V $\Rightarrow$ Nonsaturation

(c) $V_{GS} = 1 - 1 = 0 \Rightarrow$ Cutoff

3.6

(a) $V_{SG} = 2.2 - 2.2 = 0 \Rightarrow$ Cutoff

(b) $V_{SG} = 2$ V, $V_{SD} = 2 - (-1) = 3$ V

$V_{SD}(sat) = V_{SG} + V_{TP} = 2 + (-0.4) = 1.6$ V

So $V_{SD} = 3 > V_{SD}(sat) = 1.6 \Rightarrow$ Saturation

(c) $V_{SG} = 2$ V, $V_{SD} = 2 - 1 = 1$ V

$V_{SD}(sat) = V_{SG} + V_{TP} = 2 + (-0.4) = 1.6$ V

So $V_{SD} = 1 < V_{SD}(sat) = 1.6 \Rightarrow$ Nonsaturation

---

3.7

$$I_D = \frac{k_n'}{2}\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$$

$$0.5 = \frac{0.12}{2}\left(\frac{W}{L}\right)[0-(-1.2)]^2 \Rightarrow \left(\frac{W}{L}\right) = 5.79$$

3.8

$$k_n' = \mu_n C_{ox} = \frac{\mu_n \in_{ox}}{t_{ox}} = \frac{(600)(3.9)(8.85\times10^{-14})}{t_{ox}} = \frac{2.071\times10^{-10}}{t_{ox}}$$

(a) 500 A $k_n' = 41.4\ \mu\text{A/V}^2$

(b) 250 $k_n' = 82.8\ \mu\text{A/V}^2$

(c) 100 $k_n' = 207\ \mu\text{A/V}^2$

(d) 50 $k_n' = 414\ \mu\text{A/V}^2$

(e) 25 $k_n' = 828\ \mu\text{A/V}^2$

3.9

(a) $K_n = \dfrac{W\mu_n \in_{ox}}{2Lt_{ox}} = \dfrac{(20\times10^{-4})(650)(3.9)(8.85\times10^{-14})}{2(0.8\times10^{-4})(200\times10^{-8})} = 1.40\ \text{mA/V}^2$

(b) $I_D = K_n(V_{GS} - V_{TN})^2 = (1.40)(2-0.4)^2$

Or $I_D = 3.58\ \text{mA}$

(c) $V_{DS}(sat) = V_{GS} - V_{TN} = 2 - 0.4 = 1.6\ \text{V}$

3.10

(a) $I_D = \dfrac{k_n'}{2}\left(\dfrac{W}{L}\right)(V_{GS} - V_{TN})^2$

$$0.6 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)(1.4-0.8)^2 \Rightarrow \left(\frac{W}{L}\right) = 27.8$$

Or $W = (27.8)(0.8) = 22.2\ \mu\text{m}$

(b) $I_D = \left(\dfrac{0.12}{2}\right)(27.8)\left[2(1.4-0.8)(0.4) - (0.4)^2\right] = 0.534\ \text{mA}$

(c) $V_{DS}(sat) = V_{GS} - V_{TN} = 1.4 - 0.8 = 0.6\ \text{V}$

3.11

$$k_n' = \mu_n C_{ox} = \frac{\mu_n \in_{ox}}{t_{ox}} = \frac{(600)(3.9)(8.85\times10^{-14})}{200\times10^{-8}}$$

$k_n' = 0.1035\ \text{mA/V}^2$

$$I_D = \frac{k_n'}{2}\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2$$

$$1.2 = \left(\frac{0.1035}{2}\right)\left(\frac{W}{L}\right)(3-0.6)^2 \Rightarrow \left(\frac{W}{L}\right) = 4.026$$

Then $W = (4.026)(0.8) = 3.22\ \mu\text{m}$

3.12

$$I_D = WC_{ox}\left(V_{GS} - V_{TN}\right)\upsilon_{sat}$$

$$C_{ox} = \frac{\in_{ox}}{t_{ox}} = \frac{(3.9)(8.85\times10^{-14})}{200\times10^{-8}} = 1.726\times10^{-7}\ \text{F/cm}$$

$$I_D = (3.22\times10^{-4})(1.726\times10^{-7})(3-0.6)(2\times10^{7})$$

$$I_D = 2.67\ \text{mA}$$

3.13

$$I_D = \frac{k_p'}{2}\left(\frac{W}{L}\right)\left(V_{SG} + V_{TP}\right)^2$$

$$0.225 = \left(\frac{0.05}{2}\right)\left(\frac{W}{L}\right)\left(2 + V_{TP}\right)^2$$

$$0.65 = \left(\frac{0.05}{2}\right)\left(\frac{W}{L}\right)\left(3 + V_{TP}\right)^2$$

Then $\sqrt{\dfrac{0.65}{0.225}} = \dfrac{3+V_{TP}}{2+V_{TP}} = 1.70 \Rightarrow V_{TP} = -0.571\ \text{V}$

And $0.225 = \left(\dfrac{0.05}{2}\right)\left(\dfrac{W}{L}\right)(2-0.571)^2 \Rightarrow \left(\dfrac{W}{L}\right) = 4.41$

3.14

$V_S = 5\ \text{V},\ V_G = 0 \Rightarrow V_{SG} = 5\ \text{V}$

$V_{TP} = -0.5\ V \Rightarrow V_{SD}(sat) = V_{SG} + V_{TP} = 5 - 0.5 = 4.5\ V$

$V_D = 0 \Rightarrow V_{SD} = 5\ \text{V} \Rightarrow$ Biased in saturation

a. $I_D = 2(5-0.5)^2 \Rightarrow \underline{I_D = 40.5\ \text{mA}}$

$V_D = 2\ \text{V} \Rightarrow V_{SD} = 3\ \text{V} \Rightarrow$ Nonsaturation

b. $I_D = 2\left[2(5-0.5)(3) - (3)^2\right] \Rightarrow \underline{I_D = 36\ \text{mA}}$

$V_D = 4\ \text{V} \Rightarrow V_{SD} = 1\ \text{V} \Rightarrow$ Nonsaturation

c. $I_D = 2\left[2(5-0.5)(1) - (1)^2\right] \Rightarrow \underline{I_D = 16\ \text{mA}}$

d. $V_D = 5\ \text{V} \Rightarrow V_{SD} = 0 \Rightarrow \underline{I_D = 0}$

3.15

(a) Enhancement-mode

(b) From Graph V_{TP} = + 0.5 V

$0.45 = k_p(2-0.5)^2 = 2.25K_p \Rightarrow$	$K_p =$	0.20
$1.25 = k_p(3-0.5)^2 = 6.25K_p$		0.20
$2.45 = k_p(4-0.5)^2 = 12.25K_p$		0.20
$4.10 = k_p(5-0.5)^2 = 20.25K_p$		0.202
	Avg $K_p =$	0.20 mA/V^2

$i_D(\text{sat}) = 0.20\ (3.5-0.5)^2 = 1.8\ \text{mA}$

(c) $i_D(\text{sat}) = 0.20\ (4.5-0.5)^2 = 3.2\ \text{mA}$

3.16

$$V_{SD}(sat) = V_{SG} + V_{TP}$$

(a) $V_{SD}(sat) = -1+2 \Rightarrow V_{SD}(sat) = 1\ V$

(b) $V_{SD}(sat) = 0+2 \Rightarrow V_{SD}(sat) = 2\ V$

(c) $V_{SD}(sat) = 1+2 \Rightarrow V_{SD}(sat) = 3\ V$

$$I_D = \frac{k_p'}{2}\cdot\frac{W}{L}\left(V_{SG}+V_{TP}\right)^2 = \frac{k_p'}{2}\cdot\frac{W}{L}\cdot\left[V_{SD}(sat)\right]^2$$

(a) $I_D = \left(\frac{0.040}{2}\right)(6)(1)^2 \Rightarrow \underline{I_D = 0.12\ mA}$

(b) $I_D = \left(\frac{0.040}{2}\right)(6)(2)^2 \Rightarrow \underline{I_D = 0.48\ mA}$

(c) $I_D = \left(\frac{0.040}{2}\right)(6)(3)^2 \Rightarrow \underline{I_D = 1.08\ mA}$

3.17

$$K_p = \frac{k_p'}{2}\left(\frac{W}{L}\right) = \frac{50}{2}\left(\frac{12}{0.8}\right) \Rightarrow K_p = 0.375\ \text{mA/V}^2$$

(a) Nonsaturation

$$I_D = (0.375)\left[2(2-0.5)(0.2)-(0.2)^2\right] = 0.21\,\text{mA}$$

(b) Nonsaturation

$$I_D = (0.375)\left[2(2-0.5)(0.8)-(0.8)^2\right] = 0.66\ \text{mA}$$

(c) Nonsaturation

$$I_D = (0.375)\left[2(2-0.5)(1.2)-(1.2)^2\right] = 0.81\,\text{mA}$$

(d) Saturation

$$I_D = (0.375)(2-0.5)^2 = 0.844\ \text{mA}$$

(e) Saturation

$$I_D = (0.375)(2-0.5)^2 = 0.844\ \text{mA}$$

3.18

$$k_p' = \mu_p C_{ox} = \frac{\mu_p \in_{ox}}{t_{0x}} = \frac{(250)(3.9)\left(8.85\times10^{-14}\right)}{t_{0x}} = \frac{8.629\times10^{-11}}{t_{0x}}$$

(a) $t_{ox} = 500\text{Å} \Rightarrow k_p' = 17.3\ \mu\text{A/V}^2$

(b) $250\text{Å} \Rightarrow k_p' = 34.5\ \mu\text{A/V}^2$

(c) $100\text{Å} \Rightarrow k_p' = 86.3\ \mu\text{A/V}^2$

(d) $50\text{Å} \Rightarrow k_p' = 173\ \mu\text{A/V}^2$

(e) $25\text{Å} \Rightarrow k_p' = 345\ \mu\text{A/V}^2$

3.19

$$C_{ox} = \frac{\in_{ox}}{t_{0x}} = \frac{(3.9)\left(8.85\times10^{-14}\right)}{500\times10^{-8}} = 6.90\times10^{-8}\ \text{F/cm}^2$$

$$k_n' = \left(\mu_n C_{ox}\right) = (675)\left(6.90\times10^{-8}\right) \Rightarrow 46.6\ \mu\text{A/V}^2$$

$$k_p' = \left(\mu_p C_{ox}\right) = (375)\left(6.90\times10^{-8}\right) \Rightarrow 25.9\ \mu\text{A/V}^2$$

PMOS:

$$I_D = \frac{k_p'}{2}\left(\frac{W}{L}\right)_p \left(V_{SG} + V_{TP}\right)^2$$

$$0.8 = \left(\frac{0.0259}{2}\right)\left(\frac{W}{L}\right)_p (5-0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_p = 3.19$$

$$L = 4\ \mu\text{m} \Rightarrow \underline{W_p = 12.8\ \mu\text{m}}$$

$$K_p = \left(\frac{0.0259}{2}\right)(3.19) \Rightarrow \underline{K_p = 41.3\ \mu\text{A/V}^2 = K_n}$$

Want $K_n = K_p$

$$\frac{k_n'}{2}\left(\frac{W}{L}\right)_N = \frac{k_p'}{2}\left(\frac{W}{L}\right)_p = 41.3$$

$$\left(\frac{46.6}{2}\right)\left(\frac{W}{L}\right)_N = 41.3 \Rightarrow \left(\frac{W}{L}\right)_N = 1.77$$

$$L = 4\ \mu\text{m} \Rightarrow \underline{W_N = 7.09\ \mu\text{m}}$$

3.20

$$V_{GS} = 2\ \text{V}, \quad I_D = (0.2)(2-1.2)^2 = 0.128\ \text{mA}$$

$$r_0 = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(0.128)} \Rightarrow \underline{r_0 = 781\ \text{k}\Omega}$$

$$V_{GS} = 4\ \text{V}, \quad I_D = (0.2)(4-1.2)^2 = 1.57\ \text{mA}$$

$$r_0 = \frac{1}{(0.01)(1.57)} \Rightarrow \underline{r_0 = 63.7\ \text{k}\Omega}$$

$$V_A = \frac{1}{\lambda} = \frac{1}{(0.01)} \Rightarrow \underline{V_A = 100\ \text{V}}$$

3.21

$$I_D = \frac{k_n'}{2}\left(\frac{W}{L}\right)\left(V_{GS} - V_{TN}\right)^2 = \left(\frac{0.12}{2}\right)(4)(2-0.5)^2 = 0.54\ \text{mA}$$

$$r_o = \frac{1}{\lambda I_{DQ}} \Rightarrow \lambda = \frac{1}{r_o I_{DQ}} = \frac{1}{\left(200\times10^3\right)\left(0.54\times10^{-3}\right)} = 0.00926\ \text{V}^{-1}$$

Then $V_A = \dfrac{1}{\lambda} = 108\ \text{V}$

3.22

$$V_{TN} = V_{TNO} + \gamma\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$$

$$\Delta V_{TN} = 2 = (0.8)\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2(0.35)}\right]$$

$$2.5 + 0.837 = \sqrt{2(0.35) + V_{SB}} \Rightarrow \underline{V_{SB} = 10.4\ V}$$

3.23

$$V_{TN} = V_{TNo} + r\left[\sqrt{2\phi_f + V_{SB}} - \sqrt{2\phi_f}\right]$$

$$= 0.75 + 0.6\left[\sqrt{2(0.37) + 3} - \sqrt{2(0.37)}\right]$$

$$= 0.75 + 0.6[1.934 - 0.860]$$

$$V_{TN} = 1.39\ \text{V}$$

$$V_{DS}(\text{sat}) = 2.5 - 1.39 = 1.11\ \text{V}$$

$$\text{Sat Region } I_D = (15)\left(\frac{0.08}{2}\right)(2.5 - 1.39)^2$$

(a) $\underline{I_D = 0.739\ \text{mA}}$

$$\text{Non-Sat } I_D = (15)\left(\frac{0.08}{2}\right)\left[2(2.5 - 1.39)(0.25) - (0.25)^2\right]$$

(b) $\underline{I_D = 0.296\ \text{mA}}$

3.24

$$\text{E}_{ox} = 6\times10^6\ \text{V/cm}$$

(a) (i) $V_G = \text{E}_{ox} t_{ox} = \left(6\times10^6\right)\left(120\times10^{-8}\right) = 7.2\ \text{V}$

(ii) $V_G = \dfrac{7.2}{3} = 2.4\ \text{V}$

(b) (i) $V_G = \left(6\times10^6\right)\left(200\times10^{-8}\right) = 12\ \text{V}$

(ii) $V_G = \dfrac{12}{3} = 4\ \text{V}$

3.25

Want $(3)(24) = \text{E}_{ox} t_{ox} = \left(6\times10^6\right) t_{ox}$

$\underline{t_{0x} = 1.2\times10^{-5}\ \text{cm} = 1200\ \text{Angstroms}}$

3.26

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD} = \left(\frac{18}{18+32}\right)(10) = 3.6 \text{ V}$$

Assume transistor biased in saturation region

$$I_D = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S} = K_n\left(V_{GS} - V_{TN}\right)^2$$

$$3.6 - V_{GS} = (0.5)(2)(V_{GS} - 0.8)^2$$

$$= V_{GS}^2 - 1.6V_{GS} + 0.64$$

$$V_{GS}^2 - 0.6V_{GS} - 2.96 = 0$$

$$V_{GS} = \frac{0.6 \pm \sqrt{(0.6)^2 + 4(2.96)}}{2} \Rightarrow \underline{V_{GS} = 2.046 \text{ V}}$$

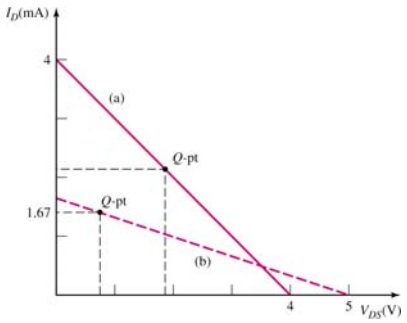
$$I_D = \frac{V_G - V_{GS}}{R_S} = \frac{3.6 - 2.046}{2} \Rightarrow \underline{I_D = 0.777 \text{ mA}}$$

$$V_{DS} = V_{DD} - I_D\left(R_D + R_S\right)$$

$$= 10 - (0.777)(4+2) \Rightarrow \underline{V_{DS} = 5.34 \text{ V}}$$

$$\underline{V_{DS} > V_{DS}(\text{sat})}$$

3.27



(a) $V_{GS} = 4 \text{ V} \quad V_{DS}(\text{sat}) = 4 - 0.8 = 3.2 \text{ V}$

If Sat $I_D = 0.25(4-0.8)^2 = 2.56$

$V_{DS} = 1.44 \quad \times$

Non-Sat

$$4 = I_D R_D + V_{DS} = K_n R_D\left[2\left(V_{GS} - V_T\right)V_{DS} - V_{DS}^2\right] + V_{DS}$$

$$4 = (0.25)(1)\left[2(4-0.8)V_{DS} - V_{DS}^2\right] + V_{DS}$$

$$4 = 2.6V_{DS} - 0.25V_{DS}^2$$

$$0.25V_{DS}^2 - 2.6V_{DS} + 4 = 0$$

$$V_{DS} = \frac{2.6 \pm \sqrt{6.76 - 4}}{2(0.25)} = 1.88 \text{ V}$$

$$I_D = \frac{4 - 1.88}{1} = 2.12 \text{ mA}$$

(b) Non-Sat region

$$5 = I_D R_D + V_{DS} = K_n R_D \left[2(V_{GS} - V_T)V_{DS} - V_{DS}^2\right] + V_{DS}$$

$$5 = (0.25)(3)\left[2(5-0.8)V_{DS} - V_{DS}^2\right] + V_{DS}$$

$$5 = 7.3V_{DS} - 0.75V_{DS}^2$$

$$0.75\ V_{DS}^2 - 7.3V_{DS} + 5 = 0$$

$$V_{DS} = \frac{7.3 \pm \sqrt{53.29 - 15}}{2(0.75)}$$

$$V_{DS} = 0.741 \text{ V}$$

$$I_D = \frac{5 - 0.741}{3} = 1.42 \text{ mA}$$

3.28

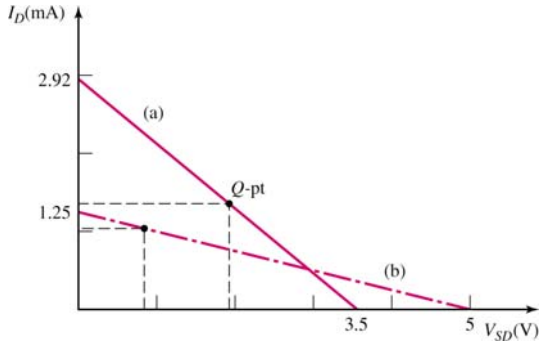
$$0.8 = \left(\frac{0.12}{2}\right)(80)(V_{GS} - 0.4)^2 \Rightarrow V_{GS} = 0.808 \text{ V}$$

$$V_{GS} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$0.80825 = \frac{1}{R_1}(200)(1.8) \Rightarrow R_1 = 445 \text{ k}\Omega$$

$$R_1 \| R_2 = R_{in} = 200 \text{ k}\Omega \Rightarrow R_2 = 363 \text{ k}\Omega$$

3.29



(a) $V_{SG} = V_{DD} = 3.5 \quad V_{SD}(\text{sat}) = 3.5 - 0.8 = 2.7 \text{ V}$

If biased in Sat region, $I_D = (0.2)(3.5 - 0.8)^2$

$= 1.46 \text{ mA}$

$V_{SD} = 3.5 - (1.46)(1.2) = 1.75 \text{ V} \quad \times$

Biased in Non-Sat Region.

$$3.5 = V_{SD} + I_D R_D = V_{SD} + K_p R_D \left[2\left(V_{SG} + V_{TP}\right) V_{SD} - V_{SD}^2 \right]$$

$$3.5 = V_{SD} + (0.2)(1.2)\left[2(3.5 - 0.8) V_{SD} - V_{SD}^2 \right]$$

$$3.5 = V_{SD} + 1.296\ V_{SD} - 0.24\ V_{SD}^2$$

$$0.24\ V_{SD}^2 - 2.296\ V_{SD} + 3.5 = 0$$

$$V_{SD} = \frac{+2.296 \pm \sqrt{5.272 - 3.36}}{2(0.24)} \text{ use} - \text{sign } \underline{V_{SD} = 1.90 \text{ V}}$$

$$I_D = (0.2)\left[2(3.5 - 0.8)(1.9) - (1.9)^2 \right] = 0.2\left[10.26 - 3.61\right]$$

$$I_D = \frac{3.5 - 1.90}{1.2} = 1.33 \text{ mA}$$

$$\underline{I_D = 1.33 \text{ mA}}$$

(b) $V_{SG} = V_{DD} = 5 \text{ V} \quad V_{SD}(\text{sat}) = 5 - 0.8 = 4.2 \text{ V}$

If Sat Region $I_D = (0.2)(5 - 0.8)^2 = 3.53 \text{ mA},\ V_{SD} < 0$

Non-Sat Region.

$$5 = V_{SD} + K_p R_D \left[2\left(V_{SG} + V_{TP}\right) V_{SD} - V_{SD}^2 \right]$$

$$5 = V_{SD} + (0.2)(4)\left[2(5 - 0.8) V_{SD} - V_{SD}^2 \right]$$

$$5 = V_{SD} + 6.72\ V_{SD} - 0.8\ V_{SD}^2$$

$$0.8\ V_{SD}^2 - 7.72\ V_{SD} + 5 = 0$$

$$V_{SD} = \frac{7.72 \pm \sqrt{59.598 - 16}}{2(0.8)} \text{ use} - \text{sign } \underline{V_{SD} = 0.698 \text{ V}}$$

$$I_D = \frac{5 - 0.698}{4} \Rightarrow \underline{I_D = 1.08 \text{ mA}}$$

3.30

$$V_G = \left(\frac{22}{22 + 8} \right)(6) - 3 = 1.40 \text{ V}$$

$$3 = K_p R_S \left(V_{SG} + V_{TP}\right)^2 + V_{SG} + V_G$$

$$3 = (0.5)(0.5)\left(V_{SG}^2 - 1.6 V_{SG} + 0.64\right) + V_{SG} + 1.40$$

$$0.25 V_{SG}^2 + 0.6 V_{SG} - 1.44 = 0 \Rightarrow V_{SG} = 1.483 \text{ V}$$

$$I_D = (0.5)(1.483 - 0.8)^2 = 0.2332 \text{ mA}$$

$$V_{SD} = 6 - (0.2332)(0.5 + 5) = 4.72 \text{ V}$$

3.31

$V_G = 0, \quad V_{SG} = V_S$

Assume saturation region

$$I_D = 0.4 = K_p\left(V_{SG} + V_{TP}\right)^2$$

$$0.4 = (0.2)(V_S - 0.8)^2$$

$$V_S = \sqrt{\frac{0.4}{0.2}} + 0.8 \Rightarrow \underline{V_S = 2.21 \text{ V}}$$

$$V_D = I_D R_D - 5 = (0.4)(5) - 5 = -3 \text{ V}$$

$$V_{SD} = V_S - V_D = 2.21 - (-3) \Rightarrow \underline{V_{SD} = 5.21 \text{ V}}$$

$$\underline{V_{SD} > V_{SD}(\text{sat})}$$

3.32

$$I_D = \frac{V_{DD} - V_{DSQ} - V_{RS}}{R_D} = \frac{3.3 - 1.6 - 0.8}{5} = 0.18 \text{ mA}$$

$$0.18 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)(0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right) = 18.75$$

$$R_S = \frac{0.8}{0.18} = 4.44 \text{ k}\Omega$$

$$V_G = 0.8 + 0.8 = 1.6 = \left(\frac{R_2}{R_1 + R_2}\right)(3.3)$$

Now $I_R = (0.05)(0.18) \Rightarrow 9 \ \mu\text{A}$

So $R_1 + R_2 = \dfrac{3.3}{0.009} = 367 \text{ k}\Omega$

Then $1.6 = \left(\dfrac{R_2}{367}\right)(3.3) \Rightarrow R_2 = 178 \text{ k}\Omega$ and $R_1 = 189 \text{ k}\Omega$

3.33

$$I_D = \frac{0.2}{1} = 0.2 \text{ mA}$$

$$V_{DS} = 1.8 - (0.2)(4 + 1) = 0.8 \text{ V}$$

Now $V_{DS}(sat) = 0.8 - 0.4 = 0.4 \text{ V}$

$$V_{DS}(sat) = V_{GS} - V_{TN} \Rightarrow 0.4 = V_{GS} - 0.4 \Rightarrow V_{GS} = 0.8 \text{ V}$$

$$I_D = \frac{k_n'}{2}\left(\frac{W}{L}\right)\left(V_{GS} - V_{TN}\right)^2$$

$$0.2 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)(0.8 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right) = 20.8$$

Now $V_G = V_{GS} + I_D R_S = 0.8 + (0.2)(1) = 1.0 \text{ V}$

$$V_G = 1 = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} = \frac{1}{R_1}(200)(1.8) \Rightarrow R_1 = 360 \text{ k}\Omega$$

$$R_1 \| R_2 = R_{in} = 200 \text{ k}\Omega \quad \Rightarrow R_2 = 450 \text{ k}\Omega$$

3.34

(a) $0.35 = \left(\frac{0.12}{2}\right)(50)(V_{GS} - 0.4)^2 \Rightarrow V_{GS} = 0.742\text{ V}$

(b) $V_{DS} = 1.8 - (0.35)(2) = 1.1\text{ V}$

$V_{DS}(sat) = V_{GS} - V_{TN} = 0.742 - 0.4 = 0.342\text{ V}$

$V_{DS} > V_{DS}(sat) \Rightarrow$ Saturation

3.35

$$V_G = \left(\frac{6}{6+14}\right)(10) - 5 = -2\text{ V}$$

$$V_G = V_{GS} + I_D R_S - 5 = V_{GS} + \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)R_S(V_{GS} - V_{TN})^2 - 5$$

$$5 - 2 = V_{GS} + \left(\frac{0.12}{2}\right)(25)(0.5)(V_{GS}^2 - 0.8V_{GS} + 0.16)$$

Or $0.75V_{GS}^2 + 0.4V_{GS} - 2.88 = 0 \Rightarrow V_{GS} = 1.71\text{ V}$

$$I_D = \left(\frac{0.12}{2}\right)(25)(1.71 - 0.4)^2 = 2.58\text{ mA}$$

$$V_{DS} = 10 - (2.58)(1.2 + 0.5) = 5.62\text{ V}$$

3.36

Let $\left(\frac{W}{L}\right) = 20$ for example, then

$$0.20 = \left(\frac{0.05}{2}\right)(20)(V_{SG} - 0.6)^2 \Rightarrow V_{SG} = 1.232\text{ V}$$

Then $V_{RS} = 1.232 = (0.2)R_S \Rightarrow R_S = 6.16\text{ k}\Omega$

$$R_D = \frac{6 - 1.232 - 3}{0.2} = 8.84\text{ k}\Omega$$

$$V_G = 3 - 1.232 - 1.232 = 0.536\text{ V}$$

$$I_R = \frac{6}{R_1 + R_2} = (0.1)(0.2) = 0.02\text{ mA}, \Rightarrow R_1 + R_2 = 300\text{ k}\Omega$$

$$V_G = 0.536 = \left(\frac{R_2}{R_1 + R_2}\right)(6) - 3 = \left(\frac{R_2}{300}\right)(6) - 3$$

Or $R_2 = 176.8\text{ k}\Omega$ and $R_1 = 123.2\text{ k}\Omega$

3.37

$$I_Q = 50 = 500(V_{GS} - 1.2)^2 \Rightarrow \underline{V_{GS} = 1.516\ V}$$

(a) (i) $V_{DS} = 5 - (-1.516) = \Rightarrow \underline{V_{DS} = 6.516\ V}$

$$I_Q = 1 = (0.5)(V_{GS} - 1.2)^2 \Rightarrow \underline{V_{GS} = 2.61\ V}$$

(ii) $V_{DS} = 5 - (-2.61) \Rightarrow \underline{V_{DS} = 7.61\ V}$

(b) (i) Same as (a) $\underline{V_{GS} = V_{DS} = 1.516\ V}$

(ii) $\underline{V_{GS} = V_{DS} = 2.61\ V}$

3.38

$$I_D = K_n\left(V_{GS} - V_{TN}\right)^2$$

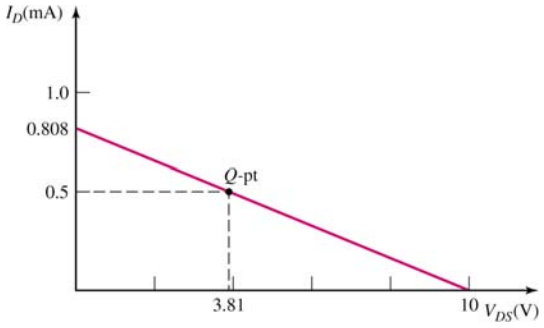
$$0.25 = (0.2)(V_{GS} - 0.6)^2$$

$$V_{GS} = \sqrt{\frac{0.25}{0.2}} + 0.6 \Rightarrow \quad V_{GS} = 1.72\ \text{V} \Rightarrow \quad \underline{V_S = -1.72\ \text{V}}$$

$$V_D = 9 - (0.25)(24) \Rightarrow \underline{V_D = 3\ \text{V}}$$

3.39

(a)



$$R_D = \frac{5-1}{0.5} \Rightarrow \underline{R_D = 8\ \text{K}}$$

$$I_{DQ} = 0.5 = 0.25(V_{GS} - 1.4)^2 \Rightarrow V_{GS} = 2.81\ \text{V}$$

$$R_S = \frac{-2.81 - (-5)}{0.5} \Rightarrow \underline{R_S = 4.38\ \text{K}}$$

(b) Let $R_D = 8.2\ \text{K},\ R_S = 4.3\ \text{K}$

Now $\dfrac{-V_{GS} - (-5)}{4.3} = I_D = 0.25(V_{GS} - 1.4)^2$

$$5 - V_{GS} = 1.075\left(V_{GS}^2 - 2.8\ V_{GS} + 1.96\right)$$

$$1.075\ V_{GS}^2 - 2.01\ V_{GS} - 2.89 = 0$$

$$V_{GS} = \frac{2.01 \pm \sqrt{4.04 + 12.427}}{2(1.075)} \Rightarrow V_{GS} = 2.82\ \text{V}$$

$$I_D = 0.25(2.82 - 1.4)^2 \Rightarrow I_D = 0.504\ \text{mA}$$

$$V_{DS} = 10 - (0.504)(8.2 + 4.3) \Rightarrow \underline{V_{DS} = 3.70\ \text{V}}$$

(c) If $R_S = 4.3 + 10\% = 4.73$ K

$$5 - V_{GS} = 1.18\left(V_{GS}^2 - 2.8V_{GS} + 1.96\right)$$

$$1.18\,V_{GS}^2 - 2.31\,V_{GS} - 2.68 = 0$$

$$V_{GS} = \frac{2.31 \pm \sqrt{5.336 + 12.65}}{2(1.18)} = 2.78 \text{ V}$$

$$I_D = (0.25)(2.78 - 1.4)^2 \Rightarrow \underline{I_D = 0.476 \text{ mA}}$$

If $R_s = 4.3 - 10\% = 3.87$ K

$$5 - V_{GS} = (0.9675)\left(V_{GS}^2 - 2.8V_{GS} + 1.96\right)$$

$$0.9675V_{GS}^2 - 1.71V_{GS} - 3.10 = 0$$

$$V_{GS} = \frac{1.71 \pm \sqrt{2.924 + 12.0}}{2(0.9675)} = 2.88 \text{ V}$$

$$I_D = (0.25)(2.88 - 1.4)^2 = \underline{0.548 \text{ mA}}$$

3.40

$$I_D = 0.10 = \frac{9 - 2.5}{R} \Rightarrow R = 65\,\text{k}\Omega$$

$$I_D = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)(V_{SG} + V_{TP})^2$$

Now $V_{SG} = V_{SD} = 2.5$ V

$$0.10 = \left(\frac{0.05}{2}\right)\left(\frac{W}{L}\right)(2.5 - 0.7)^2 \Rightarrow \left(\frac{W}{L}\right) = 1.235$$

And $W = (1.235)(0.8) = 0.988\,\mu\text{m}$

3.41

$$5 = I_{DQ}R_S + V_{SDQ} = I_{DQ}(2) + 2.5$$

$$\underline{I_{DQ} = 1.25\ mA}$$

$$I_R = \frac{10}{R_1 + R_2} = (1.25)(0.1) \Rightarrow R_1 + R_2 = 80\ k\Omega$$

$$I_{DQ} = K_p\left(V_{SG} + V_{TP}\right)^2$$

$$1.25 = 0.5(V_{SG} + 1.5)^2 \Rightarrow \sqrt{\frac{1.25}{0.5}} - 1.5 = V_{SG}$$

$$\underline{V_{SG} = 0.0811\ V}$$

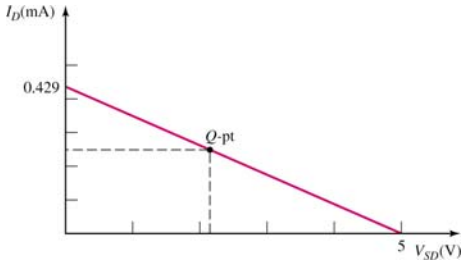
$$V_G = V_S - V_{SG} = 2.5 - 0.0811 = 2.42\ V$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5$$

$$2.42 = \left(\frac{R_2}{80}\right)(10) - 5 \Rightarrow R_2 = 59.4\ k\Omega, \quad R_1 = 20.6\ k\Omega$$

3.42

(a)



$$R_D = \frac{V_D - (-5)}{I_{DQ}} = \frac{5-2}{0.25} \Rightarrow R_D = 12 \text{ K}$$

$$I_D = \left(\frac{W}{L}\right)\left(\frac{k_p'}{2}\right)\left(V_{SG} + V_{TP}\right)^2$$

$$0.25 = (15)\left(\frac{0.035}{2}\right)\left(V_{SG} - 1.2\right)^2 \Rightarrow V_{SG} = 2.18 \text{ V}$$

$$R_S = \frac{5 - 2.18}{0.25} \Rightarrow R_S = 11.3 \text{ K}$$

$$V_{SD} = 2.18 - (-2) = 4.18 \text{ V}$$

(b)

$$k_p' = 35 + 5\% = 36.75 \ \mu\text{A/V}^2$$

$$I_D = (15)\left(\frac{0.03675}{2}\right)\left(V_{SG} - 1.2\right)^2 = \frac{5 - V_{SG}}{11.3}$$

$$3.11\left(V_{SG}^2 - 2.4V_{SG} + 1.44\right) = 5 - V_{SG}$$

$$3.11V_{SG}^2 - 6.46V_{SG} - 0.522 = 0$$

$$V_{SG} = \frac{6.46 \pm \sqrt{41.73 + 6.49}}{2(3.11)} = 2.155 \text{ V}$$

$$I_D = \frac{5 - 2.155}{11.3} = 0.252 \text{ mA}$$

$$V_{SD} = 10 - (0.252)(12 + 11.3) = 4.13 \text{ V}$$

$$k_p' = 35 - 5\% = 33.25 \ \mu\text{A/V}^2$$

$$I_D = (15)\left(\frac{0.03325}{2}\right)\left(V_{SG} - 1.2\right)^2 = \frac{5 - V_{SG}}{11.3}$$

$$2.82\left(V_{SG}^2 - 2.4V_{SG} + 1.44\right) = 5 - V_{SG}$$

$$2.82V_{SG}^2 - 5.77V_{SG} - 0.939 = 0$$

$$V_{SG} = \frac{5.77 \pm \sqrt{33.29 + 10.59}}{2(2.82)} = 2.198 \text{ V}$$

$$I_D = \frac{5 - 2.198}{11.3} = 0.248 \text{ mA}$$

$$V_{SD} = 10 - (0.248)(12 + 11.3) = 4.22 \text{ V}$$

3.43

$$I_D = \frac{-V_{SD} - (-10)}{R_D} \Rightarrow 5 = \frac{-6+10}{R_D} \Rightarrow \underline{R_D = 0.8\ \text{k}\Omega}$$

$$I_D = K_P\left(V_{SG} + V_{TP}\right)^2 \Rightarrow 5 = 3\left(V_{SG} - 1.75\right)^2$$

$$V_{SG} = \sqrt{\frac{5}{3}} + 1.75 = 3.04\ \text{V} \Rightarrow \text{V}_\text{G} = -3.04$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5 = -3.04$$

$$R_{in} = R_1 \,||\, R_2 = 80\ \text{k}\Omega$$

$$\frac{1}{R_1}\cdot(80)(10) = 5 - 3.04 \Rightarrow \underline{R_1 = 408\ \text{k}\Omega}$$

$$\frac{408R_2}{408 + R_2} = 80 \Rightarrow \underline{R_2 = 99.5\ \text{k}\Omega}$$

3.44

(a) Both M_1 and M_2 in saturation

$$I_{D1} = I_{D2}$$

$$\frac{k_n'}{2}(4)(\upsilon_I - 0.4)^2 = \frac{k_n'}{2}(1)[0 - (-0.6)]^2 \Rightarrow \upsilon_I = 0.7\ \text{V}$$

(b) (i) $\upsilon_I = 0.6\ \text{V}$; M_1 in saturation, M_2 in nonsaturation

$$I_{D1} = I_{D2}$$

$$\frac{k_n'}{2}(4)(0.6 - 0.4)^2 = \frac{k_n'}{2}(1)\left[2(0 - (-0.6))(5 - \upsilon_O) - (5 - \upsilon_O)^2\right]$$

We find $(5 - \upsilon_O)^2 - 1.2(5 - \upsilon_O) + 0.16 = 0 \Rightarrow (5 - \upsilon_O) = 0.153$

So $\upsilon_O = 4.85\ \text{V}$

(ii) $\upsilon_I = 1.5\ \text{V}$; M_1 in nonsaturation, M_2 in saturation

$$I_{D1} = I_{D2}$$

$$\frac{k_n'}{2}(4)\left[2(1.5 - 0.4)\upsilon_O - \upsilon_O^2\right] = \frac{k_n'}{2}(1)(0 - (-0.6))^2$$

We find $\upsilon_O^2 - 2.2\upsilon_O + 0.09 = 0 \Rightarrow \upsilon_O = 0.0417\ \text{V}$

3.45

M_1 in nonsaturation, M_2 in saturation

$$I_{D1} = I_{D2}$$

$$\frac{k_n'}{2}\left(\frac{W}{L}\right)_1\left[2(3 - 0.4)(0.025) - (0.025)^2\right] = \frac{k_n'}{2}(1)[0 - (-0.6)]^2$$

$$\left(\frac{W}{L}\right)_1(0.1294) = 0.36 \Rightarrow \left(\frac{W}{L}\right)_1 = 2.78$$

3.46

(a) Transistors matched

$$V_{GS1} = V_{GS2} = 2.5 \text{ V}, \quad V_O = 2.5 \text{ V}$$

$$I_D = \left(\frac{0.12}{2}\right)(30)(2.5 - 0.4)^2 = 7.938 \text{ mA}$$

(b) $I_{D1} = I_{D2}$

$$\frac{k_n'}{2}\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2 = \frac{k_n'}{2}\left(\frac{W}{L}\right)_2 (V_{GS2} - V_{TN})^2$$

$$V_{GS2} = 5 - V_{GS1}$$

Then $\sqrt{\frac{30}{15}}(V_{GS1} - 0.4) = (5 - V_{GS1} - 0.4)$

Which yields $V_{GS1} = 2.14 \text{ V}$, $V_{GS2} = V_O = 2.86 \text{ V}$

$$I_D = \left(\frac{0.12}{2}\right)(30)(2.14 - 0.4)^2 = 5.45 \text{ mA}$$

(c) $I_{D1} = I_{D2}$

$$(15)(V_{GS1} - 0.4)^2 = (30)(V_{GS2} - 0.4)^2$$

$$0.7071(V_{GS1} - 0.4) = 5 - V_{GS1} - 0.4$$

Which yields $V_{GS1} = 2.86 \text{ V}$, $V_{GS2} = V_O = 2.14 \text{ V}$

$$I_D = \left(\frac{0.12}{2}\right)(15)(2.86 - 0.4)^2 = 5.45 \text{ mA}$$

3.47

(a) $V_1 = V_{GS3} = 2.5 \text{ V}$

$$0.8 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_3 (2.5 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 3.69$$

$$V_2 = 6 = V_{GS2} + V_1 = V_{GS2} + 2.5 \Rightarrow V_{GS2} = 3.5 \text{ V}$$

$$0.8 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_2 (3.5 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 1.59$$

$$V_{GS1} = 9 - V_2 = 9 - 6 = 3 \text{ V}$$

$$0.8 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_1 (3 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 2.31$$

(b) (i) $k_n' = (120)(1.05) = 126 \ \mu \text{ A/V}^2$

No change; $V_1 = 2.5 \text{ V}$, $V_2 = 6 \text{ V}$

(ii) $k_n' = (120)(0.95) = 114 \ \mu \text{ A/V}^2$

No change; $V_1 = 2.5 \text{ V}$, $V_2 = 6 \text{ V}$

(c) $k_{n1}' = 114 \ \mu \text{ A/V}^2$, $k_{n2}' = k_{n3}' = 126 \ \mu \text{ A/V}^2$

$$I_{D2} = I_{D3}$$

$$\left(\frac{0.126}{2}\right)(1.59)(V_{GS2} - 0.6)^2 = \left(\frac{0.126}{2}\right)(3.69)(V_{GS3} - 0.6)^2$$

Now $V_{GS3} = V_1$, $V_{GS2} = V_2 - V_1$

So $(V_2 - V_1 - 0.6) = 1.523(V_1 - 0.6)$

And $V_2 = 2.523V_1 - 0.3138$

Also $I_{D1} = I_{D3}$

$$\left(\frac{0.114}{2}\right)(2.31)(V_{GS1} - 0.6)^2 = \left(\frac{0.126}{2}\right)(3.69)(V_{GS3} - 0.6)^2$$

We have $V_{GS1} = 9 - V_2 = 9 - (2.523V_1 - 0.3138) = 9.3138 - 2.523V_1$

Also $V_{GS3} = V_1$

Then $(0.13167)(9.3138 - 2.523V_1 - 0.6)^2 = (0.23247)(V_1 - 0.6)^2$

$8.7138 - 2.523V_1 = 1.3287(V_1 - 0.6)$

Which yields $V_1 = 2.469$ V and $V_2 = 5.916$ V

3.48

M_L in saturation, M_D in nonsaturation

$I_{DD} = I_{DL}$

$$\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_D\left[2(5 - 0.6)(0.15) - (0.15)^2\right] = \left(\frac{k_n'}{2}\right)(2)(5 - 0.15 - 0.6)^2$$

$$\left(\frac{W}{L}\right)_D(1.2975) = 36.125 \Rightarrow \left(\frac{W}{L}\right)_D = 27.8$$

3.49

M_L in saturation, M_D in nonsaturation

$I_{DD} = I_{DL}$

$$\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_D\left[2(5 - 0.6)(0.10) - (0.10)^2\right] = \left(\frac{k_n'}{2}\right)(2)[0 - (-1.2)]^2$$

$$\left(\frac{W}{L}\right)_D(0.87) = 2.88 \Rightarrow \left(\frac{W}{L}\right)_D = 3.31$$

3.50

(a) Transistor in nonsaturation

$$I_D = \frac{3 - 0.08}{30} = 0.09733\,\text{mA}$$

$$0.09733 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)\left[2(2.6 - 0.4)(0.08) - (0.08)^2\right]$$

$$0.09733 = \left(\frac{W}{L}\right)(0.020736) \Rightarrow \left(\frac{W}{L}\right) = 4.69$$

(b) $$0.09733 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)\left[2(3 - 0.4)(0.08) - (0.08)^2\right]$$

$$0.09733 = \left(\frac{W}{L}\right)(0.024576) \Rightarrow \left(\frac{W}{L}\right) = 3.96$$

3.51

$$5 = I_D R_D + V_\gamma + V_{DS}$$
$$5 = (12)R_D + 1.6 + 0.15 \Rightarrow R_D = 271\,\Omega$$
$$I_D = 12 = \left(\frac{0.08}{2}\right)\frac{W}{L}\left[2(5-0.6)(0.15)-(0.15)^2\right] \Rightarrow \left(\frac{W}{L}\right) = 231$$

3.52

$$5 = V_{SD} + I_D R_D + V_\gamma$$
$$5 = 0.20 + (15)R_D + 1.6 \Rightarrow R_D = 213\,\Omega$$
$$I_D = 15 = \left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)\left[2(5-0.6)(0.20)-(0.20)^2\right]$$
$$15 = \left(\frac{W}{L}\right)(0.0344) \Rightarrow \left(\frac{W}{L}\right) = 436$$

3.53

(a) $I_R = \dfrac{5-0.15}{50} = 0.097\text{ mA}$

$$I_D = \frac{0.097}{2} = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)\left[2(5-0.6)(0.15)-(0.15)^2\right]$$
$$0.0485 = \left(\frac{W}{L}\right)(0.07785) \Rightarrow \left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 0.623$$

(b) M_1 nonsaturation, M_2 cutoff

$$I_R = I_D = \frac{5-V_O}{50} = \left(\frac{0.12}{2}\right)(0.623)\left[2(5-0.6)V_O - V_O^2\right]$$

We find $1.869V_O^2 - 17.45V_O + 5 = 0 \Rightarrow V_O = 0.297\text{ V}$

3.54

(a) $V_{DS2}(sat) = 0.5 = V_{GS2} - V_{TN} = V_{GS2} - 0.4 \Rightarrow V_{GS2} = 0.9\text{ V}$

$$I_{Q1} = 125 = \left(\frac{120}{2}\right)\left(\frac{W}{L}\right)_2 (0.9-0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 8.33$$
$$V_{GS3} = V_{GS2} = 0.9\text{ V}$$
$$I_{REF1} = 225 = \left(\frac{120}{2}\right)\left(\frac{W}{L}\right)_3 (0.9-0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 15$$

M_1 and M_2 matched, so $\left(\dfrac{W}{L}\right)_1 = 8.33$

(b) $V_{GS1} = 0.9\text{ V} \Rightarrow V_{D1} = V_{DS1} - V_{GS1} = 2 - 0.9 = 1.1\text{ V}$

$$R_D = \frac{2.5-1.1}{0.125} = 11.2\,\text{k}\Omega$$

3.55

(a) $V_{SDB}(sat) = 0.8 = V_{SGB} + V_{TP} = V_{SGB} - 0.4 \Rightarrow V_{SGB} = 1.2\text{ V}$

$$I_{Q2} = 200 = \left(\frac{50}{2}\right)\left(\frac{W}{L}\right)_B (1.2-0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_B = 12.5$$

$$V_{SGC} = V_{SGB} = 1.2\text{ V}$$

$$I_{REF2} = 125 = \left(\frac{50}{2}\right)\left(\frac{W}{L}\right)_C (1.2-0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_C = 7.81$$

M_A and M_B matched, so $\left(\frac{W}{L}\right)_A = 12.5$

(b) $V_{SGA} = 1.2\text{ V}$

$$V_{DA} = V_{SGA} - V_{SDA} = 1.2 - 4 = -2.8\text{ V}$$

$$R_D = \frac{-2.8-(-5)}{0.20} = 11\,\text{k}\Omega$$

3.56

$$V_{DS2}(sat) = 0.5 = V_{GS2} - V_{TN} = V_{GS2} - 0.6 \Rightarrow V_{GS2} = 1.1\text{ V}$$

$$I_Q = 0.35 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_2 (1.1-0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 23.3$$

$$V_{GS3} = V_{GS2} = 1.1\text{ V}$$

$$I_{REF} = 0.15 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_3 (1.1-0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 10$$

$$V_{GS4} = 5 - 1.1 = 3.9\text{ V}$$

$$I_{REF} = 0.15 = \left(\frac{0.12}{2}\right)\left(\frac{W}{L}\right)_4 (3.9-0.6)^2 \Rightarrow \left(\frac{W}{L}\right)_4 = 0.23$$

M_1 and M_2 matched, so $\left(\frac{W}{L}\right)_1 = 23.3$

$V_{GS1} = 1.1\text{ V};\ V_{D1} = V_{DS1} - V_{GS1} = 3.5 - 1.1 = 2.4\text{ V}$

$$R_D = \frac{5-2.4}{0.35} = 7.43\,\text{k}\Omega$$

3.57

$$V_{DS}(\text{sat}) = V_{GS} - V_P$$

So $\underline{V_{DS} > V_{DS}(\text{sat}) = -V_P}$, $\underline{I_D = I_{DSS}}$

3.58

$$V_{DS}(\text{sat}) = V_{GS} - V_P = \underline{V_{GS} + 3 = V_{DS}(\text{sat})}$$

a. $V_{GS} = 0 \Rightarrow \underline{I_D = I_{DSS} = 6\text{ mA}}$

b. $$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 6\left(1 - \frac{-1}{-3}\right)^2 \Rightarrow \underline{I_D = 2.67\text{ mA}}$$

c. $I_D = 6\left(1-\dfrac{-2}{-3}\right)^2 \Rightarrow \underline{I_D = 0.667 \text{ mA}}$

d. $\underline{I_D = 0}$

3.59

$$I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$$

$$2.8 = I_{DSS}\left(1-\frac{1}{V_P}\right)^2$$

$$0.30 = I_{DSS}\left(1-\frac{3}{V_P}\right)^2$$

$$\frac{2.8}{0.30} = \frac{\left(1-\dfrac{1}{V_P}\right)^2}{\left(1-\dfrac{3}{V_P}\right)^2} = 9.33$$

$$\frac{\left(1-\dfrac{1}{V_P}\right)}{\left(1-\dfrac{3}{V_P}\right)} = 3.055$$

$$1-\frac{1}{V_P} = 3.055-\frac{9.165}{V_P}$$

$$\frac{8.165}{V_P} = 2.055 \Rightarrow \underline{V_P = 3.97 \text{ V}}$$

$$2.8 = I_{DSS}\left(1-\frac{1}{3.97}\right)^2 = I_{DSS}\left(0.560\right) \Rightarrow \underline{I_{DSS} = 5.0 \text{ mA}}$$

3.60

$V_S = -V_{GS}$, $V_{SD} = V_S - V_{DD}$

Want $V_{SD} \geq V_{SD}(\text{sat}) = V_P - V_{GS}$

$$V_S - V_{DD} \geq V_P - V_{GS} - V_{GS} - V_{DD} \geq V_P - V_{GS} \Rightarrow \underline{V_{DD} \leq -V_P}$$

So $\underline{V_{DD} \leq -2.5 \text{ V}}$

$$I_D = 2 = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$$

$$2 = 6\left(1-\frac{V_{GS}}{2.5}\right)^2 \Rightarrow V_{GS} = 1.06 \text{ V} \Rightarrow \underline{V_S = -1.06 \text{ V}}$$

3.61

$$I_D = K_n\left(V_{GS} - V_{TN}\right)^2$$
$$18.5 = K_n\left(0.35 - V_{TN}\right)^2$$
$$86.2 = K_n\left(0.5 - V_{TN}\right)^2$$

Then

$$\frac{18.5}{86.2} = 0.2146 = \frac{\left(0.35 - V_{TN}\right)^2}{\left(0.50 - V_{TN}\right)^2} \Rightarrow \underline{V_{TN} = 0.221\ V}$$
$$18.5 = K_n\left(0.35 - 0.221\right)^2 \Rightarrow \underline{K_n = 1.11\ mA/V^2}$$

3.62

$$I_D = K\left(V_{GS} - V_{TN}\right)^2$$
$$250 = K\left(0.75 - 0.24\right)^2 \Rightarrow \underline{K = 0.961\ mA/V^2}$$

3.63

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = \frac{V_S}{R_S} = -\frac{V_{GS}}{R_S}$$
$$10\left(1 - \frac{V_{GS}}{-5}\right)^2 = -\frac{V_{GS}}{0.2}$$
$$2\left(1 + \frac{2V_{GS}}{5} + \frac{V_{GS}^2}{25}\right) = -V_{GS}$$
$$\frac{2}{25}V_{GS}^2 + \frac{9}{5}V_{GS} + 2 = 0$$
$$2V_{GS}^2 + 45V_{GS} + 50 = 0$$
$$V_{GS} = \frac{-45 \pm \sqrt{(45)^2 - 4(2)(50)}}{2(2)} \Rightarrow \underline{V_{GS} = -1.17\ \text{V}}$$
$$I_D = -\frac{V_{GS}}{R_S} = \frac{1.17}{0.2} \Rightarrow \underline{I_D = 5.85\ \text{mA}}$$
$$V_D = 20 - (5.85)(2) = 8.3\ \text{V}$$
$$V_{DS} = V_D - V_S = 8.3 - 1.17 \Rightarrow \underline{V_{DS} = 7.13\ \text{V}}$$

3.64

$$V_{DS} = V_{DD} - V_S$$

$$8 = 10 - V_S \Rightarrow V_S = 2 \text{ V} = I_D R_S = (5) R_S \Rightarrow \underline{R_S = 0.4 \text{ k}\Omega}$$

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$5 = I_{DSS}\left(1 - \frac{-1}{V_P}\right)^2 \text{ Let } \underline{I_{DSS} = 10 \text{ mA}}$$

$$5 = 10\left(1 - \frac{-1}{V_P}\right)^2 \Rightarrow \underline{V_P = -3.41 \text{ V}}$$

$$V_G = V_{GS} + V_S = -1 + 2 = 1 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$1 = \frac{1}{R_1}(500)(10) \Rightarrow \underline{R_1 = 5 \text{ M}\Omega}$$

$$\frac{5R_2}{5 + R_2} = 0.5 \Rightarrow \underline{R_2 = 0.556 \text{ M}\Omega}$$

3.65

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$5 = 8\left(1 - \frac{V_{GS}}{4}\right)^2 \Rightarrow \underline{V_{GS} = 0.838 \text{ V}}$$

$$V_{SD} = V_{DD} - I_D\left(R_S + R_D\right)$$

$$= 20 - (5)(0.5 + 2) \Rightarrow \underline{V_{SD} = 7.5 \text{ V}}$$

$$V_S = 20 - (5)(0.5) = 17.5 \text{ V}$$

$$V_G = V_S + V_{GS} = 17.5 + 0.838 = 18.3 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$18.3 = \frac{1}{R_1}(100)(20) \Rightarrow \underline{R_1 = 109 \text{ k}\Omega}$$

$$\frac{109R_2}{109 + R_2} = 100 \Rightarrow \underline{R_2 = 1.21 \text{ M}\Omega}$$

3.66

$$I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$$

$$5 = 7\left(1-\frac{V_{GS}}{3}\right)^2 \Rightarrow \underline{V_{GS} = 0.465 \text{ V}}$$

$$V_{SD} = V_{DD} - I_D\left(R_S + R_D\right)$$

$$6 = 12 - (5)(0.3 + R_D) \Rightarrow \underline{R_D = 0.9 \text{ k}\Omega}$$

$$V_S = 12 - (5)(0.3) = 10.5 \text{ V}$$

$$V_G = V_S + V_{GS} = 10.5 + 0.465 = 10.965 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD}$$

$$10.965 = \left(\frac{R_2}{100}\right)(12) \Rightarrow \underline{R_2 = 91.4 \text{ k}\Omega} \Rightarrow \underline{R_1 = 8.6 \text{ k}\Omega}$$

3.67

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \left(\frac{60}{140 + 60}\right)(20) \Rightarrow \underline{V_G = 6 \text{ V}}$$

$$I_D = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2 = \frac{V_S}{R_S} = \frac{V_G - V_{GS}}{R_S}$$

$$(8)(2)\left(1-\frac{V_{GS}}{(-4)}\right)^2 = 6 - V_{GS}$$

$$16\left(1 + \frac{V_{GS}}{2} + \frac{V_{GS}^2}{16}\right) = 6 - V_{GS}$$

$$V_{GS}^2 + 9V_{GS} + 10 = 0$$

$$V_{GS} = \frac{-9 \pm \sqrt{(9)^2 - 4(10)}}{2} \Rightarrow \underline{V_{GS} = -1.30}$$

$$I_D = 8\left(1-\frac{(-1.30)}{(-4)}\right)^2 \Rightarrow \underline{I_D = 3.65 \text{ mA}}$$

$$V_{DS} = V_{DD} - I_D\left(R_S + R_D\right)$$
$$= 20 - (3.65)(2 + 2.7)$$

$$V_{DS} = 2.85 \text{ V}$$

$$V_{DS} > V_{DS}(\text{sat}) = V_{GS} - V_P$$
$$= -1.30 - (-4)$$
$$= 2.7 \text{ V (Yes)}$$

3.68

$$V_{DS} = V_{DD} - I_D\left(R_S + R_D\right)$$
$$5 = 12 - I_D\left(0.5 + 1\right) \Rightarrow \underline{I_D = 4.67 \text{ mA}}$$
$$V_S = I_D R_S = \left(4.67\right)\left(0.5\right) \Rightarrow V_S = 2.33 \text{ V}$$
$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \left(\frac{20}{450 + 20}\right)\left(12\right) \Rightarrow V_G = 0.511 \text{ V}$$
$$V_{GS} = V_G - V_S = 0.511 - 2.33 \Rightarrow \underline{V_{GS} = -1.82 \text{ V}}$$
$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$
$$4.67 = 10\left(1 - \frac{\left(-1.82\right)}{V_P}\right)^2 \Rightarrow \underline{V_P = -5.75 \text{ V}}$$

3.69

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2, \; V_{GS} = 0$$
$$\underline{I_D = I_{DSS} = 4 \text{ mA}}$$
$$R_D = \frac{V_{DD} - V_{DS}}{I_D} = \frac{10 - 3}{4} \Rightarrow \underline{R_D = 1.75 \text{ k}\Omega}$$

3.70

$$V_{SD} = V_{DD} - I_D R_S$$
$$10 = 20 - \left(1\right)R_S \Rightarrow \underline{R_S = 10 \text{ k}\Omega}$$
$$R_1 + R_2 = \frac{V_{DD}}{I} = \frac{20}{0.1} = 200 \text{ k}\Omega$$
$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$
$$1 = 2\left(1 - \frac{V_{GS}}{2}\right)^2 \Rightarrow \underline{V_{GS} = 0.586 \text{ V}}$$
$$V_G = V_S + V_{GS} = 10 + 0.586 = 10.586$$
$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD}$$
$$10.586 = \left(\frac{R_2}{200}\right)\left(20\right) \Rightarrow \underline{R_2 = 106 \text{ k}\Omega}$$
$$\underline{R_1 = 94 \text{ k}\Omega}$$

3.71

$$V_{DS} = V_{DD} - I_D\left(R_S + R_D\right)$$
$$2 = 3 - (0.040)(10 + R_D) \Rightarrow \underline{R_D = 15\ \text{k}\Omega}$$
$$I_D = K\left(V_{GS} - V_{TN}\right)^2$$
$$40 = 250\left(V_{GS} - 0.20\right)^2 \Rightarrow \underline{V_{GS} = 0.60\ \text{V}}$$
$$V_G = V_{GS} + V_S = 0.60 + (0.040)(10) = 1.0\ \text{V}$$
$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD}$$
$$1 = \left(\frac{R_2}{150}\right)(3) \Rightarrow \underline{R_2 = 50\ \text{k}\Omega}$$
$$\underline{R_1 = 100\ \text{k}\Omega}$$

3.72

$$V_O = 0.70\ V \Rightarrow V_{DS} = 0.70 > V_{DS}\left(sat\right) = V_{GS} - V_{TN}$$

For $0.75 - 0.15 = 0.6$

Biased in the saturation region

$$I_D = \frac{V_{DD} - V_{DS}}{R_D} = \frac{3 - 0.7}{50} \Rightarrow \underline{I_D = 46\ \mu A}$$
$$I_D = K\left(V_{GS} - V_{TN}\right)^2 \Rightarrow 46 = K\left(0.75 - 0.15\right)^2 \Rightarrow \underline{K = 128\ \mu A/V^2}$$

Chapter 4

4.1

(a) (i) $g_m = 2\sqrt{\frac{k_n'}{2}\left(\frac{W}{L}\right)I_{DQ}}$

$$0.5 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(0.5)} \Rightarrow \left(\frac{W}{L}\right) = 2.5$$

(ii) $I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left(V_{GSQ} - V_{TN}\right)^2$

$$0.5 = \left(\frac{0.1}{2}\right)(2.5)\left(V_{GSQ} - 0.4\right)^2 \Rightarrow V_{GSQ} = 2.4 \text{ V}$$

(b) (i) $0.5 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(0.15)} \Rightarrow \left(\frac{W}{L}\right) = 8.33$

(ii) $0.15 = \left(\frac{0.1}{2}\right)(8.33)\left(V_{GSQ} - 0.4\right)^2 \Rightarrow V_{GSQ} = 1.0 \text{ V}$

4.2

(a) (i) $g_m = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$

$$1.2 = 2\sqrt{\left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)(0.15)} \Rightarrow \left(\frac{W}{L}\right) = 120$$

(ii) $I_{DQ} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)\left(V_{SGQ} + V_{TP}\right)^2$

$$0.15 = \left(\frac{0.04}{2}\right)(120)\left(V_{SGQ} - 0.6\right)^2 \Rightarrow V_{SGQ} = 0.85 \text{ V}$$

(b) (i) $1.2 = 2\sqrt{\left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)(0.50)} \Rightarrow \left(\frac{W}{L}\right) = 36$

(ii) $0.50 = \left(\frac{0.04}{2}\right)(36)\left(V_{SGQ} - 0.6\right)^2 \Rightarrow V_{SGQ} = 1.43 \text{ V}$

4.3

$$I_D = K_n\left(V_{GS} - V_{TN}\right)^2\left(1 + \lambda V_{DS}\right)$$

$$\frac{I_{D1}}{I_{D2}} = \frac{1 + \lambda V_{DS1}}{1 + \lambda V_{DS2}} \Rightarrow \frac{3.4}{3.0} = \frac{1 + \lambda(10)}{1 + \lambda(5)}$$

$$3.4\left[1 + 5\lambda\right] = 3.0\left[1 + 10\lambda\right]$$

$$3.4 - 3.0 = \lambda\left(3 \cdot 10 - (3.4) \cdot 5\right) \Rightarrow \underline{\lambda = 0.0308}$$

$$r_o = \frac{\Delta V_{DS}}{\Delta I_D} = \frac{5}{0.4} = 12.5 \text{ k}\Omega$$

4.4

$$r_0 = \frac{1}{\lambda I_D}$$

$$I_D = \frac{1}{\lambda r_0} = \frac{1}{(0.012)(100)} \Rightarrow \underline{I_D = 0.833 \text{ mA}}$$

4.5

(a) $I_D = K_n (V_{GS} - V_{TN})^2 (1 + \lambda V_{DS})$

$$I_D = I_{DO}(1 + \lambda V_{DS})$$

Then $$\frac{0.258}{0.250} = \frac{(1 + 3.3\lambda)}{(1 + 1.5\lambda)}$$

Or $$1.032(1 + 1.5\lambda) = 1 + 3.3\lambda \Rightarrow \lambda = 0.01826 \text{ V}^{-1}$$

$$0.250 = I_{DO}[1 + (0.01826)(1.5)] \Rightarrow I_{DO} = 0.2433 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_{DO}} = \frac{1}{(0.01826)(0.2433)} = 225 \text{ k}\Omega$$

(b) $I_D = (0.2433)[1 + (0.01826)(5)] = 0.2655 \text{ mA}$

4.6

(a)

(i) $$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.05)} = 1000 \text{ K}$$

(ii) $$r_o = \frac{1}{(0.02)(0.5)} = 100 \text{ K}$$

(b)

(i) $$\Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1}{1000} = 0.001 \text{ mA} = 1.0 \ \mu\text{A}$$

$$\frac{\Delta I_D}{I_D} = \frac{1.0}{50} \Rightarrow 2\%$$

(ii) $$\Delta I_D = \frac{\Delta V_{DS}}{r_o} = \frac{1}{100} = 0.01 \ \Rightarrow 10 \ \mu\text{A}$$

$$\frac{\Delta I_D}{I_D} = \frac{10}{500} \Rightarrow 2\%$$

4.7

$$I_D = 1.0 \text{ mA}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.01)(1)} = 100 \text{ K}$$

4.8

(a) $V_{DD} = I_{DQ}R_D + V_{DSQ}$

$3.3 = I_{DQ}(5) + 1.5 \Rightarrow I_{DQ} = 0.36\,\text{mA}$

$$I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left(V_{GSQ} - V_{TN}\right)^2$$

$$0.36 = \left(\frac{0.1}{2}\right)(40)\left(V_{GSQ} - 0.4\right)^2 \Rightarrow V_{GSQ} = 0.824\,\text{V}$$

(b) $$K_n = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right) = \left(\frac{0.1}{2}\right)(40) = 2\ \text{mA/V}^2$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(0.36)} = 1.697\,\text{mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.025)(0.36)} = 111.1\,\text{k}\Omega$$

$$A_v = -g_m\left(r_o \| R_D\right) = -(1.697)(111.1 \| 5) = -8.12$$

4.9

(a) $A_v = -g_m R_D$

$-3.8 = -g_m(10) \Rightarrow g_m = 0.38\,\text{mA/V}$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right) I_{DQ}}$$

$$0.38 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(0.12)} \Rightarrow \left(\frac{W}{L}\right) = 6.02$$

(b) $-5 = -g_m(10) \Rightarrow g_m = 0.50\,\text{mA/V}$

$$0.50 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(0.12)} \Rightarrow \left(\frac{W}{L}\right) = 10.4$$

4.10

(a) $V_{DD} = I_{DQ}R_D + V_{DSQ}$

$5 = (0.5)R_D + 3 \Rightarrow R_D = 4\,\text{k}\Omega$

$$I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left(V_{GSQ} - V_{TN}\right)^2$$

$$0.5 = \left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)(1.2 - 0.6)^2 \Rightarrow \left(\frac{W}{L}\right) = 34.7$$

(b) $$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right) I_{DQ}} = 2\sqrt{\left(\frac{0.08}{2}\right)(34.7)(0.5)} = 1.666\,\text{mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(0.5)} = 133.3\,\text{k}\Omega$$

(c) $A_v = -g_m\left(r_o \| R_D\right) = -(1.666)(133.3 \| 4) = -6.47$

4.11

$$K_n v_{gs}^2 = K_n\left[V_{gs}\sin\omega t\right]^2 = K_n V_{gs}^2 \sin^2\omega t$$

$$\sin^2\omega t = \frac{1}{2}\left[1-\cos 2\omega t\right]$$

$$\text{So } K_n v_{gs}^2 = \frac{K_n V_{gs}^2}{2}\left[1-\cos 2\omega t\right]$$

Ratio of signal at 2ω to that at ω: $\dfrac{\frac{K_n V_{gs}^2}{2}\cdot\cos 2\omega t}{2K_n\left(V_{GSQ}-V_{TN}\right)V_{gs}\cdot\sin\omega t}$

The coefficient of this expression is then: $\underline{\dfrac{V_{gs}}{4\left(V_{GSQ}-V_{TN}\right)}}$

4.12

$$0.01 = \frac{V_{gs}}{4\left(V_{GSQ}-V_{TN}\right)}$$

$$\text{So } V_{gs} = (0.01)(4)(3-1) \Rightarrow \underline{V_{gs} = 0.08 \text{ V}}$$

4.13

(a) $$V_{GS} = \left(\frac{R_2}{R_1+R_2}\right)\cdot V_{DD} = \left(\frac{60}{60+240}\right)(3.3) = 0.66 \text{ V}$$

$$I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GS}-V_{TN})^2 = \left(\frac{0.1}{2}\right)(80)(0.66-0.4)^2 = 0.270 \text{ mA}$$

$$V_{DSQ} = V_{DD} - I_{DQ}R_D = 3.3-(0.270)(8) = 1.14 \text{ V}$$

(b) $$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{DQ}} = 2\sqrt{\left(\frac{0.1}{2}\right)(80)(0.270)} = 2.078 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.27)} = 185 \text{ k}\Omega$$

(c) $$A_v = -g_m\left(r_o \| R_D\right)\left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{Si}}\right)$$

We find $r_o \| R_D = 185 \| 8 = 7.668 \text{ k}\Omega$

$R_1 \| R_2 = 60 \| 240 = 48 \text{ k}\Omega$

So $$A_v = -(2.078)(7.668)\left(\frac{48}{48+2}\right) = -15.3$$

4.14

$$A_v = -g_m\left(r_0 \| R_D\right)$$

$$-10 = -g_m\left(100 \| 5\right) \Rightarrow \underline{g_m = 2.1 \text{ mA/V}}$$

4.15

(a) $V_G = \left(\frac{R_2}{R_1+R_2}\right)\cdot V_{DD} = \left(\frac{175}{175+225}\right)(5) = 2.1875 \text{ V}$

$2.1875 = V_{GS} + I_D R_S = V_{GS} + K_n R_S \left(V_{GS} - V_{TN}\right)^2$

$2.1875 = V_{GS} + (1)(1)\left(V_{GS}^2 - 1.6V_{GS} + 0.64\right)$

or $V_{GS}^2 - 0.6V_{GS} - 1.5475 = 0 \Rightarrow V_{GS} = 1.58 \text{ V}$

$I_{DQ} = K_n\left(V_{GS} - V_{TN}\right)^2 = (1)(1.58 - 0.8)^2 = 0.608 \text{ mA}$

$V_{DSQ} = V_{DD} - I_{DQ}\left(R_S + R_D\right) = 5 - (0.608)(1+4) = 1.96 \text{ V}$

(b) $A_\upsilon = \frac{-g_m R_D}{1 + g_m R_S}$

$g_m = 2\sqrt{(1)(0.608)} = 1.56 \text{ mA/V}$

$A_\upsilon = \frac{-(1.56)(4)}{1+(1.56)(1)} = -2.44$

(c) $A_\upsilon = \frac{-g_m\left(R_D \| R_L\right)}{1+g_m R_S} = \frac{-(1.56)\left(R_D \| R_L\right)}{1+(1.56)(1)} = -0.6094\left(R_D \| R_L\right)$

$-(0.75)(2.44) = -(0.6094)\left(R_D \| R_L\right) \Rightarrow R_D \| R_L = 3.0 \text{ k}\Omega$

$4 \| R_L = 3 \Rightarrow R_L = 12 \text{ k}\Omega$

4.16

(a) $V_{DSQ} = V_{DD} - I_{DQ}\left(R_S + R_D\right)$

$5 = 12 - (2)\left(R_S + R_D\right) \Rightarrow R_S + R_D = 3.5 \text{ k}\Omega$

$R_S = 0.5 \text{ k}\Omega$, then $R_D = 3 \text{ k}\Omega$

$I_{DQ} = K_n\left(V_{GSQ} - V_{TN}\right)^2$

$2 = 1.5\left(V_{GSQ} - 1.2\right)^2 \Rightarrow V_{GSQ} = 2.355 \text{ V}$

$V_G = V_{GSQ} + I_{DQ} R_S = 2.355 + (2)(0.5) = 3.355 \text{ V}$

$V_G = \left(\frac{R_2}{R_1+R_2}\right)\cdot V_{DD} = \frac{1}{R_1}\cdot R_{in} \cdot V_{DD}$

$3.355 = \frac{1}{R_1}(250)(12) \Rightarrow R_1 = 894 \text{ k}\Omega$

$R_1 \| R_2 = 894 \| R_2 = 250 \Rightarrow R_2 = 347 \text{ k}\Omega$

(b) $g_m = 2\sqrt{(1.5)(2)} = 3.464 \text{ mA/V}$

$A_\upsilon = \frac{-g_m\left(R_D \| R_L\right)}{1+g_m R_S} = \frac{-(3.464)(3 \| 10)}{1+(3.464)(0.5)} = -2.93$

4.17

(a) From Problem 4.16;

$R_S = 0.5 \text{ k}\Omega$, $R_D = 3 \text{ k}\Omega$,

$R_1 = 894 \text{ k}\Omega$, $R_2 = 347 \text{ k}\Omega$

(b) $g_m = 2\sqrt{(1.5)(2)} = 3.464$ mA/V

$A_v = -g_m\left(R_D \| R_L\right) = -(3.464)(3\|10) = -7.99$

4.18

(a)

$A_v = -g_m R_D$

$-15 = -2R_D \Rightarrow \underline{R_D = 7.5 \text{ K}}$

(b)

$$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

$$-5 = \frac{-(2)(7.5)}{1+(2)R_S} \Rightarrow \underline{R_S = 1 \text{ K}}$$

4.19

(a) $$A_v = \frac{-g_m R_D}{1 + g_m R_S}$$

(1) $$-8 = \frac{-g_m R_D}{1 + g_m (1)}$$

(2) $-16 = -g_m R_D$

$$8 = \frac{16}{1+g_m(1)} \Rightarrow \quad \underline{g_m = 1 \text{ mA/V}}$$

Then $\underline{R_D = 16 \text{ K}}$

$$A_v = -10 = \frac{-(1)(16)}{1+(1)R_S}$$

(b) $\underline{R_S = 0.6 \text{ K}}$

4.20

(a) $$I_{DQ} = I_Q = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left(V_{GSQ} - V_{TN}\right)^2$$

$$0.5 = \left(\frac{0.1}{2}\right)(50)\left(V_{GSQ} - 0.8\right)^2 \Rightarrow V_{GSQ} = 1.247 \text{ V}$$

$$V_{DSQ} = V^+ - I_{DQ}R_D - \left(-V_{GSQ}\right) = 5 - (0.5)(6) + 1.247 = 3.25 \text{ V}$$

(b) $$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{DQ}} = 2\sqrt{\left(\frac{0.1}{2}\right)(50)(0.5)} = 2.236 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$$

$A_v = -g_m\left(r_o \| R_D\right) = -(2.236)(100\|6) = -12.7$

(c) $A_v = -g_m\left(r_o \| R_D \| R_L\right) = -(2.236)(100\|6\|20) = -9.86$

(d) $A_v = -g_m\left(r_o \| R_D \| R_L\right) = -(2.236)(100\|6\|6) = -6.51$

4.21

(a) $V_{DSQ} = 5-(-5)-I_{DQ}\left(R_S + R_D\right)$

$5.5 = 10-(0.1)(R_S + R_D) \Rightarrow R_S + R_D = 45\ \text{k}\Omega$

$I_{DQ} = K_n\left(V_{GSQ} - V_{TN}\right)^2$

$(0.1) = (0.85)(V_{GSQ} - 0.8)^2 \Rightarrow V_{GSQ} = 1.143\ \text{V}$

$V_{GS} + I_D R_S = 5$

$1.143 + (0.1)R_S = 5 \Rightarrow R_S = 38.6\ \text{k}\Omega$

$\Rightarrow R_D = 6.43\ \text{k}\Omega$

(b) $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.85)(0.1)} = 0.583\ \text{mA/V}$

$r_o = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0.02)(0.1)} = 500\ \text{k}\Omega$

(c) $A_v = -g_m\left(R_D \| r_o \| R_L\right) = -(0.583)(6.43\|500\|40) = -3.19$

4.22

(a) $V_{DSQ} = V_{DD} - I_{DQ}\left(R_S + R_D\right)$

$2 = 3.3-(0.5)(R_S + R_D) \Rightarrow R_S + R_D = 2.6\ \text{k}\Omega$

$I_{DQ} = K_n\left(V_{GSQ} - V_{TN}\right)^2$

$0.5 = 2\left(V_{GSQ} - (-0.8)\right)^2 \Rightarrow V_{GSQ} = -0.3\ \text{V}$

$V_{GSQ} + I_{DQ} R_S = 0$

$-0.3 + (0.5)R_S = 0 \Rightarrow R_S = 0.6\ \text{k}\Omega$

$\Rightarrow R_D = 2\ \text{k}\Omega$

(b) $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(0.5)} = 2\ \text{mA/V}$

$A_v = \dfrac{-g_m\left(R_D \| R_L\right)}{1 + g_m R_S} = \dfrac{-(2)(2\|10)}{1+(2)(0.6)} = -1.52$

4.23

(a) $V_{DD} = I_{DQ} R_D + V_{DSQ} + I_{DQ} R_S$ and $V_{GS} + I_{DQ} R_S = 0$

Then $V_{DD} = K_n R_D\left(V_{GS} - V_{TN}\right)^2 + V_{DSQ} - V_{GS}$

$5 = (2)(2)(V_{GS} + 0.8)^2 + 2.5 - V_{GS}$

Which yields

$4V_{GS}^2 + 5.4V_{GS} + 0.06 = 0$, $\Rightarrow V_{GS} = -0.0112\ \text{V}$

and $I_{DQ} = 2(-0.0112 + 0.8)^2 = 1.244\ \text{mA}$

$5 = (1.244)(2) + 2.5 + (1.244)R_S \Rightarrow R_S = 8.99\,\Omega \cong 9\,\Omega$

(b) $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(1.244)} = 3.155 \text{ mA/V}$

$$A_v = \frac{-g_m(R_D \| R_L)}{1 + g_m R_S} = \frac{-(3.155)(2\|2)}{1+(3.155)(0.009)} = -3.07$$

4.24

a.

$$5 = I_{DQ}R_S + V_{SDQ} + I_{DQ}R_D - 5$$
$$5 = I_{DQ}R_S + 6 + I_{DQ}(10) - 5$$
$$\underline{I_{DQ} = \frac{4}{R_S + 10}}$$

1. $V_S = V_{SDQ} + I_{DQ}R_D - 5 = V_{SGQ}$
2. $1 + I_{DQ}(10) = V_{SGQ}$
3. $I_{DQ} = K_p(V_{SGQ} - 2)^2$

Choose $\underline{R_S = 10 \text{ k}\Omega} \Rightarrow I_{DQ} = \frac{4}{20} = 0.20 \text{ mA}$

$V_{SGQ} = 1 + (0.2)(10) = 3 \text{ V}$

$0.20 = K_P(3-2)^2 \Rightarrow \underline{K_P = 0.20 \text{ mA/V}^2}$

b.

$I_{DQ} = (0.20)(3-2)^2 = 0.20 \text{ mA}$

$g_m = 2\sqrt{K_P I_{DQ}} = 2\sqrt{(0.2)(0.2)} = 0.4 \text{ mA/V}$

$A_v = -g_m(R_D \| R_L) = -(0.4)(10 \| 10) \Rightarrow \underline{A_v = -2.0}$

c.

Choose $\underline{R_S = 20 \text{ k}\Omega} \Rightarrow I_{DQ} = \frac{4}{30} = 0.133 \text{ mA}$

$V_{SGQ} = 1 + (0.133)(10) = 2.33 \text{ V}$

$0.133 = K_p(2.33-2)^2 \Rightarrow \underline{K_p = 1.22 \text{ mA/V}^2}$

$g_m = 2\sqrt{(1.22)(0.133)} = 0.806 \text{ mA/V}$

$A_v = -(0.806)(10\|10) \Rightarrow \underline{A_v = -4.03}$

A larger gain can be achieved.

4.25

(a)

$$I_{DQ} = K_p\left(V_{SGQ} + V_{TP}\right)^2$$

$$0.25 = 0.8\left(V_{SGQ} - 0.5\right)^2$$

$$V_{SGQ} = 1.059 \text{ V}$$

$$R_S = \frac{3 - 1.059}{0.25} \Rightarrow \underline{R_S = 7.76 \text{ K}}$$

$$V_D = V_S - V_{SDQ} = 1.059 - 1.5 = -0.441 \text{ V}$$

$$R_D = \frac{-0.441 - (-3)}{0.25} \Rightarrow \underline{R_D = 10.2 \text{ K}}$$

(b)

$$A_v = -g_m\left(R_D \| R_L\right)$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.8)(0.25)} = 0.8944 \text{ mA/V}$$

$$A_v = -(0.8944)(10.2 \| 2)$$

$$A_v = -1.50$$

(c)

$$\Delta V_O = \Delta I\left(R_D \| R_L\right) = 0.25(10.2 \| 2) = 0.418$$

So $\underline{\Delta V_O = 0.836 \text{ peak-to-peak}}$

4.26

$$I_{DQ} = K_n\left(V_{GSQ} - V_{TN}\right)^2$$

$$g_m = 2\sqrt{K_n I_{DQ}}$$

$$2.2 = 2\sqrt{K_n(6)} \Rightarrow \underline{K_n = 0.202\ mA/V^2}$$

$$6 = 0.202\left(2.8 - V_{TN}\right)^2 \Rightarrow \underline{V_{TN} = -2.65\ V}$$

$$V_{DSQ} = 18 - I_{DQ}\left(R_S + R_D\right)$$

$$R_S + R_D = \frac{18-10}{6} = 1.33\ \text{k}\Omega \Rightarrow R_S = 1.33 - R_D$$

$$A_v = -\frac{g_m\left(R_D \| R_L\right)}{1 + g_m R_S}$$

$$-1 = \frac{-2.2\left(\dfrac{R_D \cdot 1}{R_D + 1}\right)}{1 + (2.2)(1.33 - R_D)}$$

$$1 + 2.93 - 2.2R_D = \frac{2.2R_D}{1 + R_D}$$

$$(3.93 - 2.2R_D)(1 + R_D) = 2.2R_D$$

$$3.93 + 1.73R_D - 2.2R_D^2 = 2.2R_D$$

$$2.2R_D^2 + 0.47R_D - 3.93 = 0$$

$$R_D = \frac{-0.47 + \sqrt{(0.47)^2 + 4(2.2)(3.93)}}{2(2.2)} \Rightarrow \underline{R_D = 1.23\ \text{k}\Omega,} \qquad \underline{R_S = 0.10\ \text{k}\Omega}$$

$$V_G = V_{GS} + V_S = 2.8 + (6)(0.1) = 3.4\ \text{V}$$

$$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} = \frac{1}{R_1}(100)(18) = 3.4 \Rightarrow \underline{R_1 = 529\ \text{k}\Omega}$$

$$\frac{529R_2}{529 + R_2} = 100 \Rightarrow \underline{R_2 = 123\ \text{k}\Omega}$$

4.27

(a) $V_S = V_{SGQ} = V_{SDQ} + I_{DQ}R_D - 9$

$$V_{SGQ} = 5 + I_{DQ}(4) - 9 = K_p(4)\left(V_{SGQ} + V_{TP}\right)^2 - 4$$

$$V_{SGQ} = 8\left(V_{SGQ}^2 - 2.4V_{SGQ} + 1.44\right) - 4$$

Or $8V_{SGQ}^2 - 20.2V_{SGQ} + 7.52 = 0 \Rightarrow V_{SGQ} = 2.071\ \text{V}$

$$I_{DQ} = I_Q = K_p\left(V_{SGQ} + V_{TP}\right)^2 = 2(2.071 - 1.2)^2 = 1.518\ \text{mA}$$

(b) $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2)(1.518)} = 3.485\ \text{mA/V}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.03)(1.518)} = 22\ \text{k}\Omega$$

$$A_v = -g_m\left(R_D \| r_o\right) = -(3.485)(4 \| 22) = -11.8$$

(c) $A_v = -g_m\left(R_D \| r_o \| R_L\right) = -(3.485)(4 \| 22 \| 8) = -8.29$

4.28

(a)

$$I_{DQ} = K_p\left(V_{SGQ} + V_{TP}\right)^2$$

$$0.5 = 0.25\left(V_{SGQ} + 0.8\right)^2$$

$$V_{SGQ} = 0.614\ V = V_S$$

$$R_S = \frac{10 - 0.614}{0.5} \Rightarrow \underline{R_S = 18.8\ \text{K}}$$

$$V_D = V_S - V_{SDQ} = 0.614 - 3 = -2.386\ \text{V}$$

$$R_D = \frac{-2.386 - (-10)}{0.5} \Rightarrow \underline{R_D = 15.2\ \text{K}}$$

(b)

$$A_v = -g_m R_D$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.25)(0.5)} = 0.7071\ \text{mA/V}$$

$$A_v = -(0.7071)(15.2)$$

$$\underline{A_v = -10.7}$$

4.29

$$A_v = -g_m\left(R_D \| R_L\right)$$

$$V_{DSQ} = V_{DD} - I_{DQ}\left(R_S + R_D\right)$$

$$10 = 20 - (1)\left(R_S + R_D\right) \Rightarrow R_S + R_D = 10\ \text{k}\Omega$$

Let $\underline{R_D = 8\ \text{k}\Omega}$, $\underline{R_S = 2\ \text{k}\Omega}$

$$A_v = -10 = -g_m\left(8 \| 20\right)$$

$$g_m = 1.75\,\text{mA/V} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{K_n (1)} \Rightarrow K_n = 0.766\ \text{mA/V}^2$$

$$V_S = I_{DQ} R_S = (1)(2) = 2\ \text{V}$$

$$I_{DQ} = K_n\left(V_{GS} - V_{TN}\right)^2 \Rightarrow 1 = 0.766\left(V_{GS} - 2\right)^2 \Rightarrow V_{GS} = 3.14\ \text{V}$$

$$V_G = V_{GS} + V_S = 3.14 + 2 = 5.14$$

$$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \Rightarrow \frac{1}{R_1}(200)(20) = 5.14 \Rightarrow \underline{R_1 = 778\ \text{k}\Omega}$$

$$\frac{778 R_2}{778 + R_2} = 200 \Rightarrow \underline{R_2 = 269\ \text{k}\Omega}$$

4.30

(a) $$A_v = \frac{g_m r_o}{1 + g_m r_o} = \frac{(5)(100)}{1 + (5)(100)} = 0.998$$

$$R_o = \frac{1}{g_m} \| r_o = \frac{1}{5} \| 100 = 0.2 \| 100 \Rightarrow R_o \cong 200\,\Omega$$

(b) $A_\upsilon = \dfrac{g_m\left(r_o \| R_S\right)}{1+g_m\left(r_o \| R_S\right)}$

We have $r_o \| R_S = 100 \| 5 = 4.762 \ \text{k}\Omega$

$$A_\upsilon = \frac{(5)(4.762)}{1+(5)(4.762)} = 0.960$$

4.31

$$A_v = \frac{g_m\left(R_L \| r_o\right)}{1+g_m\left(R_L \| r_o\right)}$$

$$0.98 = \frac{g_m r_o}{1+g_m r_o} \Rightarrow g_m r_o = 49$$

$$\text{Also } 0.49 = \frac{g_m\left(R_L \| r_o\right)}{1+g_m\left(R_L \| r_o\right)} = \frac{g_m\left(\dfrac{R_L r_o}{R_L + r_o}\right)}{1+g_m\left(\dfrac{R_L r_o}{R_L + r_o}\right)}$$

$$0.49 = \frac{g_m\left(R_L r_o\right)}{R_L + r_o + g_m\left(R_L r_o\right)}$$

$$0.49 = \frac{(49)(1)}{1+r_o+(49)(1)} = \frac{49}{50+r_o}$$

$\underline{r_o = 50 \ \text{K}}$

$\underline{g_m = 0.98 \ \text{mA/V}}$

4.32

(a)

$$A_v = \frac{g_m r_o}{1+g_m r_o} = \frac{(2)(25)}{1+(2)(25)}$$

$$A_v = 0.98$$

$$R_o = \frac{1}{g_m} \Big\| r_o = \frac{1}{2} \Big\| 25 = 0.5 \| 25$$

$\underline{R_o = 0.49 \ \text{K}}$

(b)

$$A_v = \frac{g_m\left(r_o \| R_L\right)}{1+g_m\left(r_o \| R_L\right)} = \frac{2(25 \| 2)}{1+2(25 \| 2)} = \frac{2(1.852)}{1+2(1.852)}$$

$\underline{A_v = 0.787}$

4.33

(a) $I_{DQ} = \dfrac{0-(-1.5)}{10} = 0.15\,\text{mA}$

$$I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$$

$$0.15 = \left(\frac{0.1}{2}\right)(80)(V_{GSQ} - 0.4)^2 \Rightarrow V_{GSQ} = 0.594\ \text{V}$$

(b) $g_m = 2\sqrt{\left(\dfrac{k_n'}{2}\right)\left(\dfrac{W}{L}\right)I_{DQ}} = 2\sqrt{\left(\dfrac{0.1}{2}\right)(80)(0.15)} = 1.549\,\text{mA/V}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.15)} = 333\,\text{k}\Omega$$

We find $r_o \| R_S = 333 \| 10 = 9.708\ \text{k}\Omega$

Then $A_v = \dfrac{g_m(r_o \| R_S)}{1 + g_m(r_o \| R_S)} = \dfrac{(1.549)(9.708)}{1 + (1.549)(9.708)} = 0.938$

(c) $R_o = \dfrac{1}{g_m} \| r_o \| R_S = \dfrac{1}{1.549} \| 333 \| 10 = 0.6456 \| 9.708$

or $R_o = 605\,\Omega$

4.34

(a) $I_{DQ} = \dfrac{V_{DD} - V_{DSQ}}{R_S} = \dfrac{2.5 - 1.25}{0.5} = 2.5\,\text{mA}$

(b) $A_v = \dfrac{g_m R_S}{1 + g_m R_S}$

$$0.85 = \frac{g_m(0.5)}{1 + g_m(0.5)} \Rightarrow g_m = 11.33\,\text{mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$$

$$11.33 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(2.5)} \Rightarrow \left(\frac{W}{L}\right) = 257$$

(c) $$I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GSQ} - V_{TN})^2$$

$$2.5 = \left(\frac{0.1}{2}\right)(257)(V_{GSQ} - 0.6)^2 \Rightarrow V_{GSQ} = 1.041\ \text{V}$$

$$V_{IQ} = V_{GSQ} + V_O = 1.041 + 1.25 = 2.291\ \text{V}$$

4.35

(a) $P = I_Q V_{DD} \Rightarrow 2.5 = I_Q(2.5) \Rightarrow I_Q = 1\text{ mA}$

(b) $R_o = \dfrac{1}{g_m} \| r_o$

$r_o = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0.02)(1)} = 50\text{ k}\Omega$

So $0.5 = \dfrac{1}{g_m} \| 50 \Rightarrow g_m = 1.98\text{ mA/V}$

$g_m = 2\sqrt{\left(\dfrac{k_n'}{2}\right)\left(\dfrac{W}{L}\right) I_{DQ}}$

$1.98 = 2\sqrt{\left(\dfrac{0.1}{2}\right)\left(\dfrac{W}{L}\right)(1)} \Rightarrow \left(\dfrac{W}{L}\right) = 19.6$

(c) $A_v = \dfrac{g_m r_o}{1 + g_m r_o} = \dfrac{(1.98)(50)}{1 + (1.98)(50)} = 0.990$

(d) $g_m = 2\sqrt{\left(\dfrac{0.1}{2}\right)(100)(1)} = 4.472\text{ mA/V}$

$R_o = \dfrac{1}{g_m} \| r_o = \dfrac{1}{4.472} \| 50 = 0.2236 \| 50 \Rightarrow R_o = 223\,\Omega$

4.36

(a) $V_G = \left(\dfrac{R_2}{R_1 + R_2}\right) \cdot V_{DD} = \left(\dfrac{350}{350 + 850}\right)(10) = 2.917\text{ V}$

$10 = I_{DQ} R_S + V_{SGQ} + V_G$

Now $K_p = \left(\dfrac{k_p'}{2}\right)\left(\dfrac{W}{L}\right) = \left(\dfrac{0.04}{2}\right)(80) = 1.6\text{ mA/V}^2$

So $10 = (1.6)(4)(V_{SGQ} - 1.2)^2 + V_{SGQ} + 2.917$

We find $6.4V_{SGQ}^2 - 14.36V_{SGQ} + 2.133 = 0 \Rightarrow V_{SGQ} = 2.084\text{ V}$

$I_{DQ} = 1.6(2.084 - 1.2)^2 = 1.25\text{ mA}$

$V_{SDQ} = 10 - (1.25)(4) = 5\text{ V}$

(b) $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(1.6)(1.25)} = 2.828\text{ mA/V}$

$r_o = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0.05)(1.25)} = 16\text{ k}\Omega$

$r_o \| R_S \| R_L = 16 \| 4 \| 4 = 1.778\text{ k}\Omega$

$A_v = \dfrac{g_m (r_o \| R_S \| R_L)}{1 + g_m (r_o \| R_S \| R_L)} = \dfrac{(2.828)(1.778)}{1 + (2.828)(1.778)} = 0.834$

(c) $A_g = \dfrac{i_o}{v_i} = \dfrac{i_o}{v_o} \cdot \dfrac{v_o}{v_i} = \dfrac{1}{R_L} \cdot \dfrac{v_o}{v_i} = \dfrac{1}{4}(0.834) = 0.2085\text{ mA/V}$

(d) $R_o = \dfrac{1}{g_m}\|R_S\|r_o = \dfrac{1}{2.828}\|4\|16 = 0.3536\|3.2$

$R_o = 318\,\Omega$

4.37

(a) (i) $K_n = \dfrac{k_n'}{2}\cdot\dfrac{W}{L} = \left(\dfrac{0.1}{2}\right)(20) = 1\,\text{mA/V}^2$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(5)} = 4.472\ \text{mA/V}$

$r_o = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0.02)(5)} = 10\,\text{k}\Omega$

$r_o\|R_L = 10\|4 = 2.857\ \text{k}\Omega$

$A_v = \dfrac{g_m(r_o\|R_L)}{1+g_m(r_o\|R_L)} = \dfrac{(4.472)(2.857)}{1+(4.472)(2.857)} = 0.927$

(ii) $R_o = \dfrac{1}{g_m}\|r_o = \dfrac{1}{4.472}\|10$

$R_o = 219\,\Omega$

(b) (i) $g_m = 2\sqrt{(1)(2)} = 2.828\,\text{mA/V}$

$r_o = \dfrac{1}{(0.02)(2)} = 25\,\text{k}\Omega$

$r_o\|R_L = 25\|4 = 3.448\ \text{k}\Omega$

$A_v = \dfrac{(2.828)(3.448)}{1+(2.828)(3.448)} = 0.907$

(ii) $R_o = \dfrac{1}{g_m}\|r_o = \dfrac{1}{2.828}\|25$

$R_o = 349\,\Omega$

4.38

a.

$A_v = \dfrac{g_m R_L}{1+g_m R_L} \Rightarrow 0.95 = \dfrac{g_m(4)}{1+g_m(4)}$

$0.95 = 4(1-0.95)g_m \Rightarrow g_m = 4.75\ \text{mA/V}$

$g_m = 2\sqrt{\left(\dfrac{1}{2}\mu_n C_{ox}\right)\left(\dfrac{W}{L}\right)I_Q}$

$4.75 = 2\sqrt{(0.030)\left(\dfrac{W}{L}\right)(4)} \Rightarrow \underline{\dfrac{W}{L} = 47.0}$

$g_m = 2\sqrt{\left(\dfrac{1}{2}\mu_n C_{ox}\right)\left(\dfrac{W}{L}\right)I_Q}$

b. $4.75 = 2\sqrt{(0.030)(60)I_Q} \Rightarrow \underline{I_Q = 3.13\ \text{mA}}$

4.39

$$I_{DQ} = K_n\left(V_{GS} - V_{TN}\right)^2$$

$$5 = 5\left(V_{GS} + 2\right)^2 \Rightarrow V_{GS} = -1\text{ V} \Rightarrow V_S = -V_{GS} = 1\text{ V}$$

$$I_{DQ} = \frac{V_S - (-5)}{R_S} \Rightarrow R_S = \frac{1+5}{5} \Rightarrow \underline{R_S = 1.2\text{ k}\Omega}$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(5)(5)} = 10\ mA/V$$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20\text{ k}\Omega$$

$$A_v = \frac{g_m\left(r_0 \| R_S \| R_L\right)}{1 + g_m\left(r_0 \| R_S \| R_L\right)}$$

$$= \frac{(10)(20\|1.2\|2)}{1+(10)(20\|1.2\|2)} \Rightarrow \underline{A_v = 0.878}$$

$$R_0 = \frac{1}{g_m} \| r_0 \| R_S = \frac{1}{10} \| 20 \| 1.2 \Rightarrow \underline{R_o = 91.9\ \Omega}$$

4.40

(a) $V_S = I_{DQ}R_S - 5 = (5)(1) - 5 = 0 \Rightarrow V_{GS} = 0$

$$I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left(V_{GSQ} - V_{TN}\right)^2$$

$$5 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)[0-(-2)]^2 \Rightarrow \left(\frac{W}{L}\right) = 25$$

(b) $g_m = 2\sqrt{\left(\frac{0.1}{2}\right)(25)(5)} = 5\text{ mA/V}$

$$r_o = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20\text{ k}\Omega$$

$$r_o \| R_S = 20\|1 = 0.9524\text{ k}\Omega$$

$$A_v = \frac{g_m\left(r_o \| R_S\right)}{1 + g_m\left(r_o \| R_S\right)} = \frac{(5)(0.9524)}{1+(5)(0.9524)} = 0.826$$

(c) $R_o = \frac{1}{g_m} \| r_o \| R_S = \frac{1}{5} \| 20 \| 1 \Rightarrow R_o = 165\,\Omega$

(d) $r_o \| R_S \| R_L = 20\|1\|2 = 0.6452\text{ k}\Omega$

$$A_v = \frac{g_m\left(r_o \| R_S \| R_L\right)}{1 + g_m\left(r_o \| R_S \| R_L\right)} = \frac{(5)(0.6452)}{1+(5)(0.6452)} = 0.763$$

4.41

$$R_0 = \frac{1}{g_m} \| R_S$$

Output resistance determined primarily by g_m

Set $\dfrac{1}{g_m} = 0.2\ \text{k}\Omega \Rightarrow g_m = 5\ \text{mA/V}$

$g_m = 2\sqrt{K_n I_{DQ}} \Rightarrow 5 = 2\sqrt{(4) I_{DQ}} \Rightarrow I_{DQ} = 1.56\ \text{mA}$

$I_{DQ} = K_n \left(V_{GS} - V_{TN}\right)^2$

$1.56 = 4\left(V_{GS} + 2\right)^2$

$V_{GS} = -1.38\ \text{V}, \quad V_S = -V_{GS} = 1.38\ \text{V}$

$R_S = \dfrac{1.38 - (-5)}{1.56} \Rightarrow \underline{R_S = 4.09\ \text{k}\Omega}$

$A_v = \dfrac{g_m \left(R_S \| R_L\right)}{1 + g_m \left(R_S \| R_L\right)} = \dfrac{5\left(4.09 \| 2\right)}{1 + 5\left(4.09 \| 2\right)} \Rightarrow \underline{A_v = 0.870}$

4.42

(a) $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(5)(10)} = 14.14\ \text{mA/V}$

$r_o = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0.01)(10)} = 10\,\text{k}\,\Omega$

$A_v = \dfrac{g_m r_o}{1 + g_m r_o} = \dfrac{(14.14)(10)}{1 + (14.14)(10)} = 0.993$

(b) $R_o = \dfrac{1}{g_m} \| r_o = \dfrac{1}{14.14} \| 10 = 0.07072 \| 10 \Rightarrow R_o = 70.2\,\Omega$

(c) $A_v = \dfrac{g_m \left(r_o \| R_L\right)}{1 + g_m \left(r_o \| R_L\right)}$

$0.90 = \dfrac{(14.14)\left(r_o \| R_L\right)}{1 + (14.14)\left(r_o \| R_L\right)} \Rightarrow \left(r_o \| R_L\right) = 0.6365\ \text{k}\,\Omega$

$10 \| R_L = 0.6365 \Rightarrow R_L = 680\,\Omega$

4.43

$\left|\Delta i_D\right| = I_{DQ} = \dfrac{-1}{R_S \| R_L} \cdot \Delta v_0$

$\Delta v_0 = -I_{DQ} \cdot R_S \| R_L = -\dfrac{I_{DQ} \cdot R_S R_L}{R_S + R_L}$

$v_0\,(\min) = -\dfrac{I_{DQ} R_S}{1 + \dfrac{R_S}{R_L}}$

$A_v = \dfrac{g_m \left(R_S \| R_L\right)}{1 + g_m \left(R_S \| R_L\right)} = \dfrac{v_0}{v_i}$

$v_i = \dfrac{-I_{DQ} \left(R_S \| R_L\right)\left[1 + g_m \left(R_S \| R_L\right)\right]}{g_m \left(R_S \| R_L\right)}$

$v_i\,(\min) = -\dfrac{I_{DQ}}{g_m}\left[1 + g_m \left(R_S \| R_L\right)\right]$

4.44

(a)

$$V_{DD} = V_{DSQ} + I_{DQ}R_S$$

$$3 = 1.5 + (0.25)R_S \Rightarrow \underline{R_S = 6\text{ K}}$$

$$V_S = 1.5\text{ V}$$

$$I_{DQ} = K_n\left(V_{GSQ} - V_{TN}\right)^2$$

$$0.25 = 0.5\left(V_{GSQ} - 0.4\right)^2$$

$$V_{GSQ} = 1.107\text{ V}$$

$$V_G = V_{GSQ} + V_S + 1.107 + 1.5 = 2.607\text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \frac{1}{R_1} - R_L - V_{DD}$$

$$2.607 = \frac{1}{R_1}(300)(3) \Rightarrow \underline{R_1 = 345.2\text{ K}} \Rightarrow \underline{R_2 = 2291\text{ K}}$$

(b)

$$A_v = \frac{g_m R_S}{1 + g_m R_S} \quad g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.25)} = 0.7071\text{ mA/V}$$

$$A_v = \frac{(0.7071)(6)}{1 + (0.7071)(6)} \Rightarrow \underline{A_v = 0.809}$$

$$R_o = \frac{1}{g_m}\left\| R_S = \frac{1}{(0.7071)}\right\| 6 = 1.414 \,\|\, 6$$

$$\underline{R_o = 1.14\text{ K}}$$

4.45

$$R_i = \frac{1}{g_m} = 0.5 \Rightarrow g_m = \frac{1}{0.5} = 2\text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{DQ}}$$

$$2 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow \left(\frac{W}{L}\right) = 80$$

$$A_v = g_m R_D$$

$$20 = (2)R_D \Rightarrow R_D = 10\text{ k}\Omega$$

4.46

(a) $R_o = R_D = 500\ \Omega$

(b) $V_{GSQ} = 1.2\text{ V}$

$$I_{DQ} = \frac{V_{DD} - V_{DS}}{R_D} = \frac{2.2 - (V_{DS}(sat) + 0.3)}{0.5} = \frac{2.2 - (1.2 - 0.4 + 0.3)}{0.5}$$

$$I_{DQ} = 2.2\text{ mA}$$

(c) $I_{DQ} = K_n\left(V_{GSQ} - V_{TN}\right)^2$

$2.2 = K_n(1.2-0.4)^2 \Rightarrow K_n = 3.438 \text{ mA/V}^2$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3.438)(2.2)} = 5.5 \text{ mA/V}$

$R_i = \dfrac{1}{g_m} = \dfrac{1}{5.5} \Rightarrow R_i = 182\,\Omega$

(d) $A_\upsilon = g_m R_D = (5.5)(0.5) = 2.75$

4.47

$K_n = \left(\dfrac{k_n'}{2}\right)\left(\dfrac{W}{L}\right) = \left(\dfrac{0.1}{2}\right)(80) = 4 \text{ mA/V}^2$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(0.5)} = 2.828 \text{ mA/V}$

$A_\upsilon = g_m R_D = (2.828)(4) = 11.3$

$R_i = 10 \left\| \dfrac{1}{g_m} = 10 \right\| \dfrac{1}{2.828} = 10 \| 0.3536 \Rightarrow R_i = 342\,\Omega$

4.48

a.

$V_{GS} + I_{DQ} R_S = 5$

$I_{DQ} = \dfrac{5 - V_{GS}}{R_S} = K_n\left(V_{GS} - V_{TN}\right)^2$

$5 - V_{GS} = (10)(3)\left(V_{GS}^2 - 2V_{GS} + 1\right)$

$30V_{GS}^2 - 59V_{GS} + 25 = 0$

$V_{GS} = \dfrac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow \underline{V_{GS} = 1.35 \text{ V}}$

$I_{DQ} = (3)(1.35-1)^2 \Rightarrow \underline{I_{DQ} = 0.365 \text{ mA}}$

$V_{DSQ} = 10 - (0.365)(5+10) \Rightarrow \underline{V_{DSQ} = 4.53 \text{ V}}$

b.

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3)(0.365)} \Rightarrow \underline{g_m = 2.093\ mA/V}$

$r_0 = \dfrac{1}{\lambda I_{DQ}} = \dfrac{1}{(0)(0.365)} \Rightarrow \underline{r_0 = \infty}$

c. $A_v = g_m\left(R_D \| R_L\right) = (2.093)(5\|4) \Rightarrow \underline{A_v = 4.65}$

4.49

a.

$$I_{DQ} = K_p\left(V_{SG} + V_{TP}\right)^2$$

$$0.75 = (0.5)(V_{SG} - 1)^2 \Rightarrow V_{SG} = 2.225 \text{ V}$$

$$5 = I_{DQ}R_S + V_{SG} \Rightarrow R_S = \frac{5 - 2.225}{0.75} \Rightarrow \underline{R_S = 3.70 \text{ k}\Omega}$$

$$V_{SDQ} = 10 - I_{DQ}\left(R_S + R_D\right)$$

$$6 = 10 - (0.75)(3.70 + R_D) \Rightarrow \underline{R_D = 1.63 \text{ k}\Omega}$$

b.

$$R_i = \frac{1}{g_m}$$

$$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.5)(0.75)} = 1.225 \ mA/V$$

$$R_i = \frac{1}{1.225} \Rightarrow \underline{R_i = 0.816 \text{ k}\Omega}$$

$$R_o = R_D \Rightarrow \underline{R_o = 1.63 \text{ k}\Omega}$$

c.

$$i_0 = \left(\frac{R_D}{R_D + R_L}\right)\left(\frac{R_S}{R_S + [1/g_m]}\right) \cdot i_i$$

$$i_0 = \left(\frac{1.63}{1.63 + 2}\right)\left(\frac{3.70}{3.70 + 0.816}\right) i_i$$

$$i_0 = 0.368 i_i = \underline{i_0 = 1.84 \sin \omega t \left(\mu A\right)}$$

$$v_0 = i_0 R_L = (1.84)(2)\sin \omega t \Rightarrow \underline{v_0 = 3.68 \sin \omega t \left(\text{mV}\right)}$$

4.50

(a) $V_O = \left(V_{DS}\left(sat\right) + 0.25\right) - V_{GS}$

$$V_O = V_{GS} - V_{TN} + 0.25 - V_{GS} = -0.4 + 0.25 = -0.15 \text{ V}$$

$$R_D = \frac{1.8 - (-0.15)}{2} = 0.975 \text{ k}\Omega$$

(b) $A_\upsilon = g_m R_D$

$$6 = g_m(0.975) \Rightarrow g_m = 6.154 \text{ mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right) I_{DQ}}$$

$$6.154 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(2)} \Rightarrow \left(\frac{W}{L}\right) = 94.7$$

(c) $I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left(V_{GSQ} - V_{TN}\right)^2$

$$2 = \left(\frac{0.1}{2}\right)(94.7)\left(V_{GSQ} - 0.4\right)^2 \Rightarrow V_{GSQ} = 1.05 \text{ V}$$

4.51

(a) $I_{DQ} = I_Q = K_n\left(V_{GSQ} - V_{TN}\right)^2$

$2 = 4\left(V_{GSQ} - 0.6\right)^2 \Rightarrow V_{GSQ} = 1.307 \text{ V}$

$V_D = V_{DSQ} - V_{GSQ} = 3.5 - 1.307 = 2.193 \text{ V}$

$R_D = \dfrac{3.3 - 2.193}{2} = 0.554 \text{ k}\Omega$

(b) $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(2)} = 5.657 \text{ mA/V}$

$R_i = \dfrac{1}{g_m} = \dfrac{1}{5.657} \Rightarrow R_i = 177\,\Omega$

(c) $A_\upsilon = g_m\left(R_D \| R_L\right) = (5.657)(0.554 \| 4) = 2.75$

4.52

(a) $I_{DQ} = K_p\left(V_{SGQ} + V_{TP}\right)^2$

$1.2 = 2.5\left(V_{SGQ} - 0.8\right)^2 \Rightarrow V_{SGQ} = 1.493 \text{ V}$

$R_S = \dfrac{V^+ - V_{SGQ}}{I_{DQ}} = \dfrac{3.3 - 1.493}{1.2} = 1.51 \text{ k}\Omega$

$V_{SDQ} = 6.6 - I_{DQ}\left(R_S + R_D\right) \Rightarrow 3 = 6.6 - 1.2(1.51 + R_D) \Rightarrow R_D = 1.49 \text{ k}\Omega$

(b) $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(2.5)(1.2)} = 3.464 \text{ mA/V}$

$A_\upsilon = g_m\left(R_D \| R_L\right) = (3.464)(1.49 \| 4) = 3.76$

4.53

(a) $|A_\upsilon| = \sqrt{\dfrac{K_D}{K_L}} = \sqrt{\dfrac{(W/L)_D}{(W/L)_L}} = 5$

So $\left(\dfrac{W}{L}\right)_D = 25$

From Example 4.11,

$\upsilon_{GSDt} = \dfrac{(3.3 - 0.6) + (0.6)(1 + 5)}{1 + 5} = 1.05 \text{ V}$

$\upsilon_{GSDQ} = \dfrac{1.05 - 0.6}{2} + 0.6 = 0.825 \text{ V}$

(b) $I_{DQ} = \left(\dfrac{k_n'}{2}\right)\left(\dfrac{W}{L}\right)_D \left(V_{GSDQ} - V_{TN}\right)^2 = \left(\dfrac{0.1}{2}\right)(25)(0.825 - 0.6)^2$

$I_{DQ} = 0.0633 \text{ mA}$

Now $I_{DD} = I_{DL}$

$\left(\dfrac{W}{L}\right)_D \left(V_{GSDQ} - V_{TN}\right)^2 = \left(\dfrac{W}{L}\right)_L \left(V_{GSLQ} - V_{TN}\right)^2$

$\sqrt{\dfrac{25}{1}}\left(V_{GSDQ} - V_{TN}\right) = V_{DD} - V_O - V_{TN}$

$5(0.825 - 0.6) = 3.3 - V_O - 0.6$

Or $V_{DSDQ} = V_O = 1.575 \text{ V}$

4.54

(a) Neglect λ in dc analysis.

Transition points:

For point B,

$$V_{OtB} = V_{DD} - |V_{TNL}| = 5 - 0.8 = 4.2 \text{ V}$$

For point A,

$$I_{DD} = I_{DL}$$

$$K_{nD}\left(V_{GSDQ} - V_{TND}\right)^2 = K_{nL}\left(V_{GSLQ} - V_{TNL}\right)^2$$

$$(1.2)\left(V_{GSDQ} - 0.6\right)^2 = (0.2)[0 - (-0.8)]^2$$

So $V_{GSDQ} = \sqrt{\dfrac{0.2}{1.2}}(0.8) + 0.6 = 0.9266 \text{ V}$

Then $V_{OtA} = V_{GSDQ} - V_{TND} = 0.9266 - 0.6 = 0.3266 \text{ V}$

For point A: $V_{OtA} = 0.3266 \text{ V}$, $V_{GSDQ} = 0.9266 \text{ V}$

For point B: $V_{OtB} = 4.2 \text{ V}$, $V_{GSDQ} = 0.9266 \text{ V}$

(b) $V_{GSDQ} = 0.9266 \text{ V}$,

$$V_{DSDQ} = \frac{4.2 - 0.3266}{2} + 0.3266 = 2.2633 \text{ V}$$

(c) $$I_{DQ} = K_{nD}\left(V_{GSDQ} - V_{TND}\right)^2 = (1.2)(0.9266 - 0.6)^2 = 0.128 \text{ mA}$$

(d) $$A_v = -g_{mD}\left(r_{oD} \| r_{oL}\right)$$

$$r_{oD} = r_{oL} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.128)} = 390.6 \text{ k}\Omega$$

$$g_{mD} = 2\sqrt{K_{nD} I_{DQ}} = 2\sqrt{(1.2)(0.128)} = 0.7838 \text{ mA/V}$$

$$A_v = -(0.7838)(390.6 \| 390.6) = -153$$

4.55

(a) $V_{TN} = 0.6 \text{ V}$

$$I_D = K_n\left(V_{DS} - V_{TN}\right)^2$$

$$0.5 = K_n(1.5 - 0.6)^2 \Rightarrow K_n = 0.6173 \text{ mA/V}^2$$

$$\frac{1}{r} = \frac{dI_D}{dV_{DS}} = 2K_n\left(V_{DS} - V_{TN}\right)$$

Then $$r = \frac{1}{2K_n\left(V_{DS} - V_{TN}\right)} = \frac{1}{2(0.6173)(1.5 - 0.6)} \Rightarrow r = 900\,\Omega$$

(b) $$I_D = (0.6173)(3 - 0.6)^2 = 3.56 \text{ mA}$$

$$r = \frac{1}{2(0.6173)(3 - 0.6)} \Rightarrow r = 337\,\Omega$$

4.56

a.

$$I_{DQ} = K_{nD}\left(V_{GS} - V_{TND}\right)^2 = (0.5)\left(0-(-1)\right)^2$$

$$\underline{I_{DQ} = 0.5\ mA}$$

$$I_{DQ} = K_{nL}\left(V_{GSL} - V_{TNL}\right)^2 = K_{nL}\left(V_{DD} - V_O - V_{TNL}\right)^2$$

$$0.5 = 0.030\left(10 - V_0 - 1\right)^2$$

$$\sqrt{\frac{0.5}{0.030}} = 9 - V_0 \Rightarrow \underline{V_0 = 4.92\ \text{V}}$$

b.

$$I_{DD} = I_{DL}$$

$$K_{nD}\left(V_i - V_{TND}\right)^2 = K_{nL}\left(V_{DD} - V_o - V_{TNL}\right)^2$$

$$\sqrt{\frac{K_{nD}}{K_{nL}}}\left(V_i - V_{TND}\right) = V_{DD} - V_o - V_{TNL}$$

$$V_o = V_{DD} - V_{TNL} - \sqrt{\frac{K_{nD}}{K_{nL}}}\left(V_i - V_{TND}\right)$$

$$A_v = \frac{dV_o}{dV_i} = -\sqrt{\frac{K_{nD}}{K_{nL}}} = -\sqrt{\frac{(W/L)_D}{(W/L)_L}}$$

$$A_v = -\sqrt{\frac{500}{30}} \Rightarrow \underline{A_v = -4.08}$$

4.57

(a)

$$I_{DQ} = K_L\left(V_{GSL} - V_{TNL}\right)^2 = K_L\left(V_{DSL} - V_{TNL}\right)^2$$

$$I_D = (0.1)(4-1)^2 = 0.9\ mA$$

$$I_{DQ} = K_D\left(V_{GSD} - V_{TND}\right)^2$$

$$0.9 = (1)\left(V_{GSD} - 1\right)^2 \Rightarrow V_{GSD} = 1.95\ \text{V}$$

$$V_{GG} = V_{GSD} + V_{DSL} = 1.95 + 4 \Rightarrow \underline{V_{GG} = 5.95\ \text{V}}$$

b.

$$I_{DD} = I_{DL}$$

$$K_D\left(V_{GSD} - V_{TND}\right)^2 = K_L\left(V_{GSL} - V_{TNL}\right)^2$$

$$\sqrt{\frac{K_D}{K_L}}\left(V_{GG} + V_i - V_o - V_{TND}\right) = V_o - V_{TNL}$$

$$V_o\left(1 + \sqrt{\frac{K_D}{K_L}}\right) = \sqrt{\frac{K_D}{K_L}}\left(V_{GG} + V_i - V_{TND}\right) + V_{TNL}$$

$$A_v = \frac{dV_o}{dV_i} = \frac{\sqrt{K_D/K_L}}{1+\sqrt{K_D/K_L}} \Rightarrow \quad \underline{A_v = \frac{1}{1+\sqrt{K_L/K_D}}}$$

(c) From Problem 4.55.

$$R_{LD} = \frac{1}{2K_L\left(V_{DSL} - V_{TNL}\right)}$$

$$= \frac{1}{2(0.1)(4-1)} = 1.67\ k\Omega$$

…

$$g_m = 2\sqrt{K_D I_{DQ}} = 2\sqrt{(1)(0.9)} = 1.90\ mA/V$$

$$A_v = \frac{g_m\left(R_{LD} \| R_L\right)}{1 + g_m\left(R_{LD} \| R_L\right)} = \frac{(1.90)(1.67 \| 4)}{1 + (1.90)(1.67 \| 4)} \Rightarrow \underline{A_v = 0.691}$$

4.58

a. From Problem 4.57.

$$A_v = \frac{g_m\left(R_{LD} \| R_L\right)}{1 + g_m\left(R_{LD} \| R_L\right)} = \frac{(1.90)(1.67 \| 10)}{1 + (1.90)(1.67 \| 10)}$$

$$\underline{A_v = 0.731}$$

b.

$$R_0 = \frac{1}{g_m} \left\| R_{LD} = \frac{1}{1.90} \right\| 1.67 = 0.526 \| 1.67$$

$$\underline{R_0 = 0.40\ \text{k}\Omega}$$

4.59

(a) $A_v = -g_{mD}\left(r_{oD} \| r_{oL}\right)$

$$r_{oD} = \frac{1}{\lambda_D I_{DQ}} = \frac{1}{(0.02)(0.5)} = 100\ \text{k}\Omega$$

$$r_{oL} = \frac{1}{\lambda_L I_{DQ}} = \frac{1}{(0.04)(0.5)} = 50\ \text{k}\Omega$$

$$r_{oD} \| r_{oL} = 100 \| 50 = 33.33\ \text{k}\Omega$$

Then $-40 = -g_{mD}(33.33) \Rightarrow g_{mD} = 1.20\ \text{mA/V}$

$$g_{mD} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_D \left(I_{DQ}\right)}$$

$$1.20 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_D (0.5)} \Rightarrow \left(\frac{W}{L}\right)_D = 14.4$$

(b) $I_{DQ} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_L \left(V_{SGQ} + V_{TP}\right)^2$

$$0.5 = \left(\frac{0.04}{2}\right)(50)\left(V_{SGQ} - 0.4\right)^2 \Rightarrow V_{SGQ} = 1.107\ \text{V}$$

$$V_{SGQ} = V^+ - V_B$$

$$1.107 = 2.5 - V_B \Rightarrow V_B = 1.393\ \text{V}$$

(c) $I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_D \left(V_{GSDQ} - V_{TN}\right)^2$

$$0.5 = \left(\frac{0.1}{2}\right)(14.4)\left(V_{GSDQ} - 0.4\right)^2 \Rightarrow V_{GSDQ} = 1.233 \text{ V}$$

4.60

(a) (i) $r_{oD} = \frac{1}{\lambda_D I_{DQ}} = \frac{1}{(0.04)(0.25)} = 100 \text{ k}\Omega$

$$r_{oL} = \frac{1}{\lambda_L I_{DQ}} = \frac{1}{(0.02)(0.25)} = 200 \text{ k}\Omega$$

$$r_{oD} \| r_{oL} = 100 \| 200 = 66.67 \text{ k}\Omega$$

$$A_v = -g_{mD}\left(r_{oD} \| r_{oL}\right)$$

$$-25 = -g_{mD}(66.67) \Rightarrow g_{mD} = 0.375 \text{ mA/V}$$

$$g_{mD} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_D I_{DQ}}$$

$$0.375 = 2\sqrt{\left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)_D (0.25)} \Rightarrow \left(\frac{W}{L}\right)_D = 7.03$$

(ii) $I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L \left(V_B - V_{TN}\right)^2$

$$0.25 = \left(\frac{0.1}{2}\right)(10)\left(V_B - 0.4\right)^2 \Rightarrow V_B = 1.107 \text{ V}$$

(iii) $I_{DQ} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_D \left(V_{SGDQ} + V_{TP}\right)^2$

$$0.25 = \left(\frac{0.04}{2}\right)(7.03)\left(V_{SGDQ} - 0.6\right)^2 \Rightarrow V_{SGDQ} = 1.933 \text{ V}$$

(b) (i) $r_{oD} = \frac{1}{\lambda_D I_{DQ}} = \frac{1}{(0.04)(0.1)} = 250 \text{ k}\Omega$

$$r_{oL} = \frac{1}{\lambda_L I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$r_{oD} \| r_{oL} = 250 \| 500 = 166.7 \text{ k}\Omega$$

$$A_v = -g_{mD}\left(r_{oD} \| r_{oL}\right)$$

$$-25 = -g_{mD}(166.7) \Rightarrow g_{mD} = 0.15 \text{ mA/V}$$

$$g_{mD} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_D I_{DQ}}$$

$$0.15 = 2\sqrt{\left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)_D (0.1)} \Rightarrow \left(\frac{W}{L}\right)_D = 2.81$$

(ii) $I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L (V_B - V_{TN})^2$

$$0.1 = \left(\frac{0.1}{2}\right)(10)(V_B - 0.4)^2 \Rightarrow V_B = 0.874 \text{ V}$$

(iii) $I_{DQ} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_D (V_{SGDQ} + V_{TP})^2$

$$0.1 = \left(\frac{0.04}{2}\right)(2.81)(V_{SGDQ} - 0.6)^2 \Rightarrow V_{SGDQ} = 1.934 \text{ V}$$

4.61

$$K_{n1} = \left(\frac{85}{2}\right)(50) \Rightarrow 2.125 \text{ mA/V}^2$$

$$g_{m1} = 2\sqrt{K_{n1} I_{D1}} = 2\sqrt{(2.125)(0.1)} = 0.9220$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.05)(0.1)} = 200 \text{ K}$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.075)(0.1)} = 133.3 \text{ K}$$

$$A_v = -g_{m1}(r_{o1} \| r_{o2}) = -(0.922)(200 \| 133.3)$$

$$\underline{A_v = -73.7}$$

4.62

$$K_{p1} = \frac{k_p'}{2}\left(\frac{w}{L}\right) = \left(\frac{40}{2}\right)(50) \Rightarrow 1.0 \text{ mA/V}^2$$

$$g_{m1} = 2\sqrt{K_{p1} I_{D1}} = 2\sqrt{(1)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.075)(0.1)} = 133.3 \text{ K}$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.05)(0.1)} = 200 \text{ K}$$

$$A_v = -g_{m1}(r_{o1} \| r_{o2}) = -(0.6325)(133.3 \| 200)$$

$$\underline{A_v = -50.6}$$

4.63

(a) $I_{DD} = I_{DL}$

$$K_{nD}(V_{GSD} - V_{TND})^2 = K_{nL}(V_{GSL} - V_{TNL})^2$$

$$\sqrt{\frac{2}{0.5}}(V_I - V_O - 0.4) = V_O - 0.4$$

$$2V_I - 0.8 = 3V_O - 0.4$$

$$V_O = \left(\frac{2}{3}\right)V_I - \left(\frac{1}{3}\right)(0.4)$$

For $V_I = 0.8$ V, $V_O = 0.4$ V

For $V_I = 2.5$ V, $V_O = 1.533$ V

(b) $I_D = K_{nL}(V_{GSL} - V_{TNL})^2 = (0.5)(V_O - 0.4)^2$

$$I_D = (0.5)\left[\left(\frac{2}{3}\right)V_I - \left(\frac{1}{3}\right)(0.4) - 0.4\right]^2 = (0.5)\left[\left(\frac{2}{3}\right)V_I - 0.533\right]^2$$

For $V_I = 0.8$ V, $I_D = 0$; For $V_I = 2.5$ V, $I_D = 0.642$ mA

(c) From (a), voltage gain = constant = 2/3 = 0.667

4.64

(a) $V_{SD}(sat) = V_{SG} + V_{TP} = (2.5 - 1) - 0.4 = 1.1$ V

$V_{SD} = V^+ - V_O \Rightarrow V_O(\max) = V^+ - V_{SD}(sat) = 2.5 - 1.1 = 1.4$ V

(b) $$I_D = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_L (V_{SGL} + V_{TP})^2 = \left(\frac{0.04}{2}\right)(5)(2.5 - 1 - 0.4)^2 = 0.121 \text{ mA}$$

(c) $$I_D = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_D (V_{SGD} + V_{TP})^2$$

$$0.121 = \left(\frac{0.04}{2}\right)(50)(V_{SGD} - 0.4)^2 \Rightarrow V_{SGD} = 0.748 \text{ V}$$

(d) $$g_{mD} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_D I_D} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)(0.121)} = 0.6957 \text{ mA/V}$$

$$r_{oD} = r_{oL} = \frac{1}{\lambda I_D} = \frac{1}{(0.025)(0.121)} = 330.6 \text{ k}\Omega$$

$$A_v = \frac{g_{mD}(r_{oD} \| r_{oL})}{1 + g_{mD}(r_{oD} \| r_{oL})} = \frac{(0.6957)(165.3)}{1 + (0.6957)(165.3)} = 0.9914$$

4.65

(a) $I_{DQ} = K_n(V_{GSDQ} - V_{TN})^2$

$1 = 2(V_{GSDQ} - 0.6)^2 \Rightarrow V_{GSDQ} = 1.307$ V

$I_{DQ} = K_p(V_{SGLQ} + V_{TP})^2$

$1 = 0.5(V_{SGLQ} - 0.6)^2 \Rightarrow V_{SGLQ} = 2.014$ V

$V_O = 3.3 - 2.014 = 1.286$ V

$V_{DSDQ} = V_O - V_S = 1.286 - (-1.307) = 2.593$ V

(b) $I_d = I_l$

$g_{mD} V_i = g_{mL} V_o$

$$A_v = \frac{V_o}{V_i} = \frac{g_{mD}}{g_{mL}} = \sqrt{\frac{K_n}{K_p}}$$

(c) $$A_v = \sqrt{\frac{2}{0.5}} = 2$$

4.66

(a)

$$K_{n1} = \left(\frac{85}{2}\right)(50) \Rightarrow 2.125 \text{ mA/V}^2$$

$$g_{m1} = 2\sqrt{K_{n1} I_{D1}} = 2\sqrt{(2.125)(0.1)} = 0.922 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.05)(0.1)} = 200 \text{ K}$$

$$r_{o2} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.075)(0.1)} = 133.3 \text{ K}$$

(b)

$$R_{i1} = \frac{1}{g_{m1}} = \frac{1}{0.922} = 1.085 \text{ K}$$

$$V_{gs1} = -\left(\frac{R_{i1}}{R_{i1} + 0.050}\right) V_i = -\left(\frac{1.085}{1.085 + 0.050}\right) V_i = -0.956 V_i$$

$$A_v = -g_{m1}\left(r_{o1} \| r_{o2}\right) \cdot \frac{V_{gs1}}{V_i} = +(0.956)(0.922)(200)(133.3)$$

$$\underline{A_v = 70.5}$$

(c)

$$R_i = 0.05 + \frac{1}{g_{m1}} = 0.05 + \frac{1}{0.922} \Rightarrow \underline{R_i = 1.135 \text{ K}}$$

(d)

$$R_o \approx r_{o1} \| r_{o2} = 200 \| 133.7 \Rightarrow R_o \approx 80 \text{ K}$$

4.67

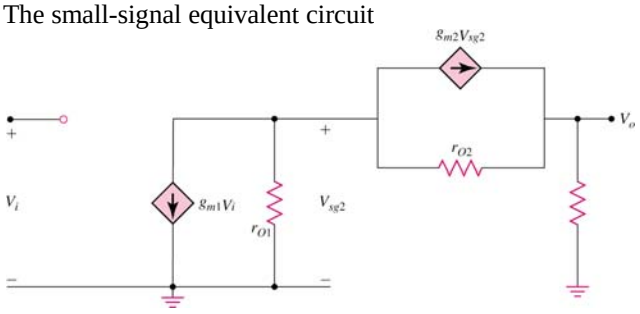
(a)

$$g_{m1} = 2\sqrt{K_n I_{D1}} = 2\sqrt{(2)(0.1)} = 0.8944 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{(2)(0.1)} = 0.8944 \text{ mA/V}$$

$$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.1)} = 100 \text{ K}$$

(b) The small-signal equivalent circuit



(1)

$$g_{m1} V_i + \frac{V_{sg2}}{r_{o1}} + g_{m2} V_{sg2} + \frac{V_{sg2} - V_o}{r_{o2}} = 0$$

(2)

$$\frac{V_o}{r_o}+\frac{V_o-V_{sg2}}{r_{o2}}=g_{m2}V_{sg2}$$

$$V_o\left(\frac{1}{r_o}+\frac{1}{r_{o2}}\right)=V_{sg2}\left(\frac{1}{r_{o2}}+g_{m2}\right)$$

$$V_o\left(\frac{1}{50}+\frac{1}{100}\right)=V_{sg2}\left(\frac{1}{100}+0.8944\right)\Rightarrow V_{sg2}=V_o\left(0.03317\right)$$

(1)

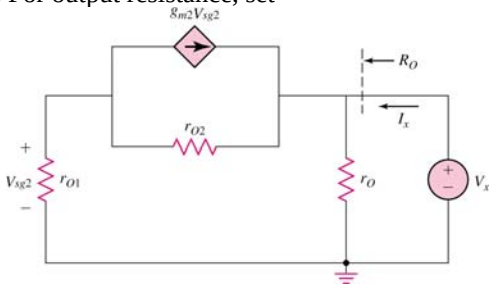
$$g_{m1}V_i+V_{sg2}\left(\frac{1}{r_{o1}}+g_{m2}+\frac{1}{r_{o2}}\right)=\frac{V_o}{r_{o2}}$$

$$0.8944\ V_i+V_o\left(0.03317\right)\left(\frac{1}{100}+0.8944+\frac{1}{100}\right)=\frac{V_o}{100}$$

$$0.8944\ V_i=V_o\left(0.01-0.03033\right)$$

$$\frac{V_o}{V_i}=-44$$

(c) For output resistance, set $V_i=0.$



(1) $$g_{m2}V_{sg2}+I_x=\frac{V_x}{r_o}+\frac{V_x-V_{sg2}}{r_{o2}}$$

(2) $$\frac{V_{sg2}}{r_{o1}}+g_{m2}V_{sg2}+\frac{V_{sg2}-V_x}{r_{o2}}=0$$

(2)

$$V_{sg2}\left(\frac{1}{r_{o1}}+g_{m2}+\frac{1}{r_{o2}}\right)=\frac{V_x}{r_{o2}}$$

$$V_{sg2}\left(\frac{1}{100}+0.8944+\frac{1}{100}\right)=\frac{V_x}{100}$$

$$V_{sg2}=V_x\left(0.010936\right)$$

(1)

$$I_x=V_x\left(\frac{1}{r_o}+\frac{1}{r_{o2}}\right)-V_{sg2}\left(\frac{1}{r_{o2}}+g_{m2}\right)$$

$$I_x=V_x\left(\frac{1}{50}+\frac{1}{100}\right)-V_x\left(0.010936\right)\left(\frac{1}{100}+0.8944\right)$$

$$I_x=V_x\left(0.03-0.0098905\right)$$

$$R_o=\frac{V_x}{I_x}=49.7\ \text{K}$$

4.68

(a) $I_{DQ1} = K_{n1}\left(V_{GSQ1} - V_{TN1}\right)^2$

$0.2 = 0.2\left(V_{GSQ1} - 0.6\right)^2 \Rightarrow V_{GSQ1} = 1.6 \text{ V}$

$I_{DQ2} = K_{p2}\left(V_{SGQ2} + V_{TP2}\right)^2$

$0.5 = 1.0\left(V_{SGQ2} - 0.6\right)^2 \Rightarrow V_{SGQ2} = 1.307 \text{ V}$

$R_{S1} = \dfrac{0.6}{0.2} = 3 \text{ k}\Omega$

$V_{D1} = 0.6 + 2 = 2.6 \text{ V}$

$R_{D1} = \dfrac{5 - 2.6}{0.2} = 12 \text{ k}\Omega$

$V_{G1} = V_{GSQ1} + 0.6 = 1.6 + 0.6 = 2.2 \text{ V}$

$V_{G1} = 2.2 = \left(\dfrac{R_2}{R_1 + R_2}\right)(5) = \dfrac{1}{R_1} \cdot R_{in} \cdot (5) = \dfrac{1}{R_1}(400)(5)$

Or $R_1 = 909 \text{ k}\Omega$ and $R_1 \| R_2 = R_{in} = 400 \text{ k}\Omega \Rightarrow R_2 = 714 \text{ k}\Omega$

$V_{S2} = V_{D1} + V_{SGQ2} = 2.6 + 1.307 = 3.907 \text{ V}$

$R_{S2} = \dfrac{5 - 3.907}{0.5} = 2.19 \text{ k}\Omega$

$V_{D2} = V_{S2} - 3 = 3.907 - 3 = 0.907 \text{ V}$

$R_{D2} = \dfrac{0.907}{0.5} = 1.81 \text{ k}\Omega$

(b) $g_{m1} = 2\sqrt{K_{n1} I_{DQ1}} = 2\sqrt{(0.2)(0.2)} = 0.4 \text{ mA/V}$

$g_{m2} = 2\sqrt{K_{p2} I_{DQ2}} = 2\sqrt{(1)(0.5)} = 1.414 \text{ mA/V}$

$A_v = (-g_{m1} R_{D1})(-g_{m2} R_{D2}) = g_{m1} g_{m2} R_{D1} R_{D2}$

$A_v = (0.4)(1.414)(12)(1.81) = 12.3$

4.69

(a) $I_{DQ1} = K_{n1}\left(V_{GS1} - V_{TN1}\right)^2$

$0.1 = 0.2\left(V_{GS1} - 0.6\right)^2 \Rightarrow V_{GSQ1} = 1.307 \text{ V}$

$I_{DQ2} = K_{p2}\left(V_{SGQ2} + V_{TP}\right)^2$

$0.25 = 1.0\left(V_{SGQ2} - 0.6\right)^2 \Rightarrow V_{SGQ2} = 1.10 \text{ V}$

$V_{G1} = V_{GSQ1} + I_{DQ1} R_{S1} = 1.307 + (0.1)(1) = 1.407 \text{ V}$

$V_{G1} = \left(\dfrac{R_2}{R_1 + R_2}\right) \cdot V_{DD} = \dfrac{1}{R_1} \cdot R_{in} \cdot V_{DD}$

$1.407 = \dfrac{1}{R_1}(250)(3.3) \Rightarrow R_1 = 586 \text{ k}\Omega$

$R_1 \| R_2 = R_{in} = 250 \text{ k}\Omega \Rightarrow R_2 = 436 \text{ k}\Omega$

$V_{D1} = I_{DQ1} R_{S1} + V_{DSQ1} = (0.1)(1) + 1.2 = 1.3 \text{ V}$

$R_{D1} = \dfrac{3.3 - 1.3}{0.1} = 20 \text{ k}\Omega$

$$V_{S2} = V_{D1} + V_{SGQ2} = 1.3 + 1.1 = 2.4 \text{ V}$$

$$R_{S2} = \frac{3.3 - 2.4}{0.25} = 3.6 \text{ k}\Omega$$

$$V_{D2} = V_{S2} - V_{SDQ2} = 2.4 - 1.8 = 0.6 \text{ V}$$

$$R_{D2} = \frac{0.6}{0.25} = 2.4 \text{ k}\Omega$$

(b) $$g_{m1} = 2\sqrt{K_{n1} I_{DQ1}} = 2\sqrt{(0.2)(0.1)} = 0.2828 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{p2} I_{DQ2}} = 2\sqrt{(1)(0.25)} = 1.0 \text{ mA/V}$$

$$A_v = g_{m1} g_{m2} R_{D1} R_{D2} = (0.2828)(1.0)(20)(2.4) = 13.6$$

4.70

$$K_{p1} = \left(\frac{0.04}{2}\right)(20) = 0.4 \text{ mA/V}^2$$

$$K_{n2} = \left(\frac{0.1}{2}\right)(80) = 4.0 \text{ mA/V}^2$$

(a) $$R_{S1} = \frac{0.6}{0.1} = 6 \text{ k}\Omega$$

$$V_{D1} = 1.8 - 0.6 - 1 = 0.2 \text{ V}$$

$$R_{D1} = \frac{0.2 - (-1.8)}{0.1} = 20 \text{ k}\Omega$$

$$I_{DQ1} = K_{p1}\left(V_{SGQ1} + V_{TP}\right)^2$$

$$0.1 = 0.4\left(V_{SGQ1} - 0.4\right)^2 \Rightarrow V_{SGQ1} = 0.90 \text{ V}$$

$$I_{DQ2} = K_{n2}\left(V_{GSQ2} - V_{TN}\right)^2$$

$$0.3 = 4\left(V_{GSQ2} - 0.4\right)^2 \Rightarrow V_{GSQ2} = 0.6739 \text{ V}$$

$$V_{G1} = 1.8 - 0.6 - V_{SGQ1} = 1.8 - 0.6 - 0.9 = 0.3 \text{ V}$$

$$V_{G1} = \left(\frac{R_2}{R_1 + R_2}\right)(3.6) - 1.8$$

$$0.3 = \frac{1}{R_1}(200)(3.6) - 1.8 \Rightarrow R_1 = 343 \text{ k}\Omega$$

$$R_1 \| R_2 = 200 \text{ k}\Omega \Rightarrow R_2 = 480 \text{ k}\Omega$$

$$V_{D1} = 1.8 - 0.6 - 1.0 = 0.2 \text{ V}$$

$$V_{S2} = V_{D1} - V_{GSQ2} = 0.2 - 0.6739 = -0.4739 \text{ V}$$

$$R_{S2} = \frac{-0.4739 - (-1.8)}{0.3} = 4.42 \text{ k}\Omega$$

(b) $$A_v = \left(\frac{-g_{m1} R_{D1}}{1 + g_{m1} R_{S1}}\right)\left(\frac{g_{m2} R_{S2}}{1 + g_{m2} R_{S2}}\right)$$

$$g_{m1} = 2\sqrt{K_{p1} I_{DQ1}} = 2\sqrt{(0.4)(0.1)} = 0.4 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_{n2} I_{DQ2}} = 2\sqrt{(4)(0.3)} = 2.191 \text{ mA/V}$$

$$A_v = \frac{-(0.4)(20)}{1 + (0.4)(6)} \cdot \frac{(2.191)(4.42)}{1 + (2.191)(4.42)} = -2.13$$

(c) $R_o = \dfrac{1}{g_{m2}} \| R_{S2} = \dfrac{1}{2.191} \| 4.42 = 0.4564 \| 4.42 \Rightarrow R_o = 414\,\Omega$

4.71

(a)

$$I_{DQ1} = \frac{10 - V_{GS1}}{R_{S2}} = K_{n1}\left(V_{GS1} - V_{TN1}\right)^2$$

$$10 - V_{GS1} = (4)(10)\left(V_{GS1}^2 - 4V_{GS1} + 4\right)$$

$$40V_{GS1}^2 - 159V_{GS1} + 150 = 0$$

$$V_{GS1} = \frac{159 \pm \sqrt{(159)^2 - 4(40)(150)}}{2(40)} \Rightarrow V_{GS1} = 2.435 \text{ V}$$

$$I_{DQ1} = (4)(2.435 - 2)^2 \Rightarrow \underline{I_{DQ1} = 0.757 \text{ mA}}$$

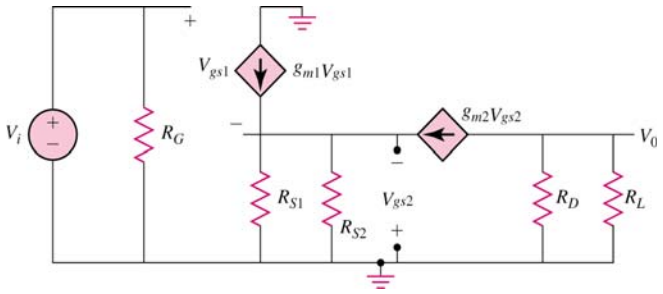
$$V_{DSQ1} = 20 - (0.757)(10) \Rightarrow \underline{V_{DSQ1} = 12.4 \text{ V}}$$

Also $\underline{I_{DQ2} = 0.757 \text{ mA}}$

$$V_{DSQ2} = 20 - (0.757)(10 + 5) \Rightarrow \underline{V_{DSQ2} = 8.65 \text{ V}}$$

(b) $$g_{m1} = g_{m2} = 2\sqrt{KI_{DQ}} = 2\sqrt{(4)(0.757)} \Rightarrow \underline{g_{m1} = g_{m2} = 3.48\ mA/V}$$

c.



$$V_0 = -\left(g_{m2}V_{gs2}\right)\left(R_D \| R_L\right)$$

$$V_{gs2} = \left(-g_{m1}V_{gs1} - g_{m2}V_{gs2}\right)\left(R_{S1} \| R_{S2}\right)$$

$$V_i = V_{gs1} - V_{gs2} \Rightarrow V_{gs1} = V_i + V_{gs2}$$

$$V_{gs2} + g_{m2}V_{gs2}\left(R_{S1} \| R_{S2}\right) = -g_{m1}\left(V_i + V_{gs2}\right)\left(R_{S1} \| R_{S2}\right)$$

$$V_{gs2} + g_{m2}V_{gs2}\left(R_{S1} \| R_{S2}\right) + g_{m1}V_{gs2}\left(R_{S1} \| R_{s2}\right) = -g_{m1}V_i\left(R_{S1} \| R_{S2}\right)$$

$$V_{gs2} = \frac{-g_{m1}V_i\left(R_{S1} \| R_{S2}\right)}{1 + g_{m2}\left(R_{S1} \| R_{S2}\right) + g_{m1}\left(R_{S1} \| R_{S2}\right)}$$

$$A_v = \frac{V_0}{V_i} = \frac{g_{m1}g_{m2}\left(R_{S1} \| R_{S2}\right)\left(R_D \| R_L\right)}{1 + \left(g_{m1} + g_{m2}\right)\left(R_{S1} \| R_{S2}\right)}$$

$$A_v = \frac{(3.48)^2\,(10 \| 10)(5 \| 2)}{1 + (3.48 + 3.48)(10 \| 10)} \Rightarrow \underline{A_v = 2.42}$$

4.72

a.

$I_{DQ} = 3 \text{ mA}$

$V_{S1} = I_{DQ}R_S - 5 = (3)(1.2) - 5 = -1.4 \text{ V}$

$I_{DQ} = K_1(V_{GS} - V_{TN})^2$

$3 = 2(V_{GS} - 1)^2 \Rightarrow V_{GS} = 2.225 \text{ V}$

$V_{G1} = V_{GS} + V_{S1} = 2.225 - 1.4 = 0.825 \text{ V}$

$$V_{G1} = \left(\frac{R_3}{R_1 + R_2 + R_3}\right)(5) \Rightarrow 0.825 = \left(\frac{R_3}{500}\right)(5) \Rightarrow \underline{R_3 = 82.5 \text{ k}\Omega}$$

$V_{D1} = V_{S1} + V_{DSQ1} = -1.4 + 2.5 = 1.1 \text{ V}$

$V_{G2} = V_{D1} + V_{GS} = 1.1 + 2.225 = 3.325 \text{ V}$

$$V_{G2} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)(5) \Rightarrow 3.325 = \left(\frac{R_2 + R_3}{500}\right)(5)$$

$R_2 + R_3 = 332.5 \Rightarrow \underline{R_2 = 250 \text{ k}\Omega}$

$R_1 = 500 - 250 - 82.5 \Rightarrow \underline{R_1 = 167.5 \text{ k}\Omega}$

$V_{D2} = V_{D1} + V_{DSQ2} = 1.1 + 2.5 = 3.6 \text{ V}$

$$R_D = \frac{5 - 3.6}{3} \Rightarrow \underline{R_D = 0.467 \text{ k}\Omega}$$

b.

$A_v = -g_{m1}R_D$

$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(2)(3)} = 4.90 \text{ } mA/V$

$A_v = -(4.90)(0.467) \Rightarrow \underline{A_v = -2.29}$

4.73

a.

$V_{S1} = I_{DQ}R_S - 10 = (5)(2) - 10 \Rightarrow V_{S1} = 0$

$I_{DQ} = K_1(V_{GS1} - V_{TN})^2$

$5 = 4(V_{GS1} - 1.5)^2 \Rightarrow V_{GS1} = 2.618 \text{ V}$

$V_{G1} = V_{GS1} + V_{S1} = 2.618 \text{ V} = IR_3 = (0.1)R_3 \Rightarrow \underline{R_3 = 26.2 \text{ k}\Omega}$

$V_{D1} = V_{S1} + V_{DSQ1} = 0 + 3.5 = 3.5 \text{ V}$

$V_{G2} = V_{D1} + V_{GS} = 3.5 + 2.62 = 6.12 \text{ V}$

$= (0.1)(R_2 + R_3)$

$R_2 + R_3 = 61.2 \text{ k}\Omega \Rightarrow \underline{R_2 = 35 \text{ k}\Omega}$

$V_{D2} = V_{D1} + V_{DSQ2} = 3.5 + 3.5 = 7.0 \text{ V}$

$$R_D = \frac{10 - 7}{5} \Rightarrow \underline{R_D = 0.6 \text{ k}\Omega}$$

$$R_1 = \frac{10 - 6.12}{0.1} \Rightarrow \underline{R_1 = 38.8 \text{ k}\Omega}$$

b.

$$A_v = -g_{m1}R_D$$
$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(4)(5)} = 8.944\ mA/V$$
$$A_v = -(8.944)(0.6) \Rightarrow \underline{A_v = -5.37}$$

4.74

a.

$$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$
$$4 = 6\left(1 - \frac{V_{GS}}{(-3)}\right)^2$$
$$V_{GS} = (-3)\left[1 - \sqrt{\frac{4}{6}}\right] \Rightarrow \underline{V_{GS} = -0.551\ \text{V}}$$
$$V_{DSQ} = V_{DD} - I_{DQ}R_D$$
$$6 = 10 - (4)R_D \Rightarrow \underline{R_D = 1\ \text{k}\Omega}$$

b.

$$g_m = \frac{2I_{DSS}}{(-V_P)}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(6)}{3}\left(1 - \frac{-0.551}{-3}\right) \Rightarrow \underline{g_m = 3.265\ \text{mA/V}}$$
$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(4)} \Rightarrow \underline{r_0 = 25\ \text{k}\Omega}$$

c. $$A_v = -g_m(r_o \| R_D) = -(3.265)(25\|1) \Rightarrow A_v = -3.14$$

4.75

$$V_{GS} + I_{DQ}\left(R_{S1} + R_{S2}\right) = 0$$

$$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$V_{GS} + I_{DSS}\left(R_{S1} + R_{S2}\right)\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 0$$

$$V_{GS} + (2)(0.1 + 0.25)\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 0$$

$$V_{GS} + 0.7\left(1 - \frac{2_{VGS}}{(-2)} + \frac{V_{GS}^2}{(-2)^2}\right) = 0$$

$$0.175V_{GS}^2 + 1.7V_{GS} + 0.7 = 0$$

$$V_{GS} = \frac{-1.7 \pm \sqrt{(1.7)^2 - 4(0.175)(0.7)}}{2(0.175)} \Rightarrow V_{GS} = -0.4314 \text{ V}$$

$$g_m = \frac{2I_{DSS}}{-V_P}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(2)}{2}\left(1 - \frac{-0.431}{-2}\right) \Rightarrow \underline{g_m = 1.569 \text{ mA/V}}$$

$$A_v = \frac{-g_m\left(R_D \| R_L\right)}{1 + g_m R_{S1}} = \frac{-(1.569)(8\|4)}{1 + (1.569)(0.1)} \Rightarrow \underline{A_v = -3.62}$$

$$A_i = \frac{i_0}{i_i} = \frac{(v_0 / R_L)}{(v_i / R_G)} = \frac{v_0}{v_i} \cdot \frac{R_G}{R_L} = (-3.62)\left(\frac{50}{4}\right) \Rightarrow \underline{A_i = -45.2}$$

4.76

$$I_{DQ} = \frac{I_{DSS}}{2} = 4 \text{ mA}$$

$$V_{DSQ} = \frac{V_{DD}}{2} = 10 \text{ V}$$

$$V_{DSQ} = V_{DD} - I_{DQ}\left(R_S + R_D\right)$$

$$10 = 20 - (4)\left(R_S + R_D\right) \Rightarrow R_S + R_D = 2.5 \text{ k}\Omega$$

$$V_S = 2 \text{ V} = I_{DQ}R_S = 4R_S \Rightarrow \underline{R_S = 0.5 \text{ k}\Omega}, \quad \underline{R_D = 2.0 \text{ k}\Omega}$$

$$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$4 = 8\left(1 - \frac{V_{GS}}{(-4.2)}\right)^2 \Rightarrow V_{GS} = (-4.2)\left(1 - \sqrt{\frac{4}{8}}\right) \Rightarrow V_{GS} = -1.23 \text{ V}$$

$$V_G = V_S + V_{GS} = 2 - 1.23$$

$$V_G = 0.77 \text{ V} = \left(\frac{R_2}{R_1 + R_2}\right)(20) = \left(\frac{R_2}{100}\right)(20) \Rightarrow \underline{R_2 = 3.85 \text{ k}\Omega}, \quad \underline{R_1 = 96.2 \text{ K}\Omega}$$

4.77

a.

$$I_{DQ} = \frac{I_{DSS}}{2} = 5 \text{ mA}$$

$$V_{DSQ} = \frac{V_{DD}}{2} = \frac{12}{2} = 6 \text{ V}$$

$$R_S = \frac{12-6}{5} \Rightarrow \underline{R_S = 1.2 \text{ k}\Omega}$$

$$I_{DQ} = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$5 = 10\left(1 - \frac{V_{GS}}{(-5)}\right)^2 \Rightarrow V_{GS} = (-5)\left(1 - \sqrt{\frac{5}{10}}\right) \Rightarrow V_{GS} = -1.464 \text{ V}$$

$$V_G = V_S + V_{GS} = 6 - 1.464 = 4.536 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD} = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD}$$

$$4.536 = \frac{1}{R_1}(100)(12) \Rightarrow \underline{R_1 = 265 \text{ k}\Omega}$$

$$\frac{265R_2}{265 + R_2} = 100 \Rightarrow \underline{R_2 = 161 \text{ k}\Omega}$$

b.

$$g_m = \frac{2I_{DSS}}{(-V_P)}\left(1 - \frac{V_{GS}}{V_P}\right) = \frac{2(10)}{5}\left(1 - \frac{-1.46}{-5}\right) \Rightarrow g_m = 2.83 \text{ mA/V}$$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.01)(5)} = 20 \text{ k}\Omega$$

$$A_v = \frac{g_m\left(r_0 \| R_s \| R_L\right)}{1 + g_m\left(r_0 \| R_S \| R_L\right)}$$

$$A_v = \frac{(2.83)\left(20 \| 1.2 \| 0.5\right)}{1 + (2.83)\left(20 \| 1.2 \| 0.5\right)} \Rightarrow \underline{A_v = 0.495}$$

$$R_0 = \frac{1}{g_m} \left\| R_S = \frac{1}{2.83} \right\| 1.2 = 0.353 \| 1.2 \Rightarrow \underline{R_0 = 0.273 \text{ k}\Omega}$$

4.78

a.

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD} = \left(\frac{110}{110+90}\right)(10) = 5.5 \text{ V}$$

$$I_{DQ} = \frac{10-(V_G - V_{GS})}{R_S} = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$$

$$10 - 5.5 + V_{GS} = (2)(5)\left(1-\frac{V_{GS}}{1.75}\right)^2$$

$$4.5 + V_{GS} = 10\left(1 - 1.143V_{GS} + 0.3265V_{GS}^2\right)$$

$$3.265V_{Gs}^2 - 12.43V_{GS} + 5.5 = 0$$

$$V_{GS} = \frac{12.43 \pm \sqrt{(12.43)^2 - 4(3.265)(5.5)}}{2(3.265)} \Rightarrow V_{GS} = 0.511 \text{ V}$$

$$I_{DQ} = (2)\left(1-\frac{0.511}{1.75}\right)^2 \Rightarrow \underline{I_{DQ} = 1.00 \text{ mA}}$$

$$V_{SDQ} = 10 - (1.00)(5) \Rightarrow \underline{V_{SDQ} = 5.0 \text{ V}}$$

b.

$$g_m = \frac{2I_{DSS}}{V_P}\left(1-\frac{V_{GS}}{V_P}\right) = \frac{2(2)}{1.75}\left(1-\frac{0.511}{1.75}\right) \Rightarrow g_m = 1.618 \text{ mA/V}$$

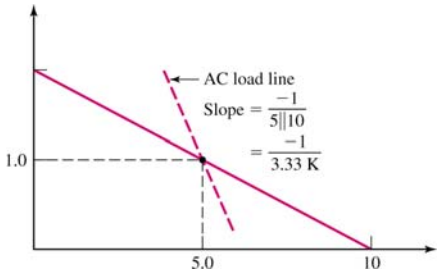
$$A_v = \frac{g_m(R_S \| R_L)}{1 + g_m(R_S \| R_L)} = \frac{(1.618)(5\|10)}{1+(1.618)(5\|10)} \Rightarrow \underline{A_v = 0.844}$$

$$A_i = \frac{i_0}{i_i} = \frac{(v_0/R_L)}{(v_i/R_i)} = A_v \cdot \left(\frac{R_i}{R_L}\right)$$

$$R_i = R_1 \| R_2 = 90 \| 110 = 49.5 \text{ k}\Omega$$

$$A_i = (0.844)\left(\frac{49.5}{10}\right) \Rightarrow \underline{A_i = 4.18}$$

c.



$$\Delta i_d = 1.0 \text{ mA}$$

$$v_{sd} = (3.33)(1.0) = 3.33 \text{ V}$$

Maximum swing in output voltage = 6.66 V peak-to-peak

4.79

$$I_{DQ} = I_{DSS}\left(1-\frac{V_{GS}}{V_P}\right)^2$$

$$4 = 8\left(1-\frac{V_{GS}}{4}\right)^2 \Rightarrow V_{GS} = 4\left(1-\sqrt{\frac{4}{8}}\right) \Rightarrow V_{GS} = 1.17 \text{ V}$$

$$V_{SDQ} = V_{DD} - I_{DQ}\left(R_S + R_D\right)$$

$$7.5 = 20 - 4\left(R_S + R_D\right) \Rightarrow R_S + R_D = 3.125 \text{ k}\Omega$$

$$g_m = \frac{2I_{DSS}}{V_P}\left(1-\frac{V_{GS}}{V_P}\right) = \frac{2(8)}{4}\left(1-\frac{1.17}{4}\right) \Rightarrow g_m = 2.83 \text{ mA/V}$$

$$R_S = 3.125 - R_D$$

$$A_v = \frac{-g_m R_D}{1+g_m R_S}$$

$$-3\left(1+g_m R_S\right) = -g_m R_D$$

$$3\left[1+(2.83)\left(3.125-R_D\right)\right] = (2.83)R_D$$

$$9.844 - 2.83R_D = 0.9433R_D \Rightarrow \underline{R_D = 2.61 \text{ k}\Omega} \quad \underline{R_S = 0.516 \text{ k}\Omega}$$

$$V_S = 20 - (4)(0.516) \Rightarrow V_S = 17.94 \text{ V}$$

$$V_G = V_S - V_{GS} = 17.94 - 1.17 = 16.77 \text{ V}$$

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD} = \left(\frac{R_2}{400}\right)(20) \Rightarrow \underline{R_2 = 335 \text{ k}\Omega.} \quad \underline{R_1 = 65 \text{ k}\Omega}$$

Chapter 5

5.1

(a) $i_E = (1+\beta) i_B \Rightarrow 1+\beta = \dfrac{325}{2.8} = 116 \Rightarrow \beta = 115$

$$\alpha = \frac{\beta}{1+\beta} = \frac{115}{116} = 0.9914$$

$$i_C = i_E - i_B = 325 - 2.8 = 322\ \mu\text{A}$$

(b) $1+\beta = \dfrac{1.80}{0.020} = 90 \Rightarrow \beta = 89$

$$\alpha = \frac{89}{90} = 0.9889$$

$$i_C = 1.80 - 0.02 = 1.78\ \text{mA}$$

5.2

(a) $\alpha = \dfrac{i_C}{i_E} = \dfrac{726}{732} = 0.9918$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.9918}{1-0.9918} = 121$$

$$i_B = i_E - i_C = 732 - 726 = 6\ \mu\text{A}$$

(b) $\alpha = \dfrac{2.902}{2.961} = 0.9801$

$$\beta = \frac{\alpha}{1-\alpha} = \frac{0.980074}{1-0.980074} = 49.19$$

$$i_B = 2.961 - 2.902 \Rightarrow i_B = 59\ \mu\text{A}$$

5.3

(a) For $\beta = 110$: $\alpha = \dfrac{\beta}{1+\beta} = \dfrac{110}{111} = 0.99099$

For $\beta = 180$: $\alpha = \dfrac{180}{181} = 0.99448$

$$\underline{0.99099 \le \alpha \le 0.99448}$$

(b) $I_C = \beta I_B = 110(50\ \mu\text{A}) \Rightarrow I_C = 5.50\ mA$

or $I_C = 180(50\ \mu\text{A}) \Rightarrow I_C = 9.00\ mA$

so $\underline{5.50 \le I_C \le 9.0\ mA}$

5.4

(a) $i_B = \dfrac{i_E}{1+\beta} = \dfrac{1.25}{151} \Rightarrow i_B = 8.28\,\mu\text{A}$

$i_C = \left(\dfrac{\beta}{1+\beta}\right)\cdot i_E = \left(\dfrac{150}{151}\right)(1.25) = 1.242\text{ mA}$

$\alpha = \dfrac{150}{151} = 0.9934$

(b) $i_B = \dfrac{4.52}{81} \Rightarrow i_B = 55.8\,\mu\text{A}$

$i_C = \left(\dfrac{80}{81}\right)(4.52) = 4.46\text{ mA}$

$\alpha = \dfrac{80}{81} = 0.9877$

5.5

(a)

α	$\beta = \dfrac{\alpha}{1-\alpha}$
0.9	9
0.95	19
0.98	49
0.99	99
0.995	199
0.999	999

(b)

β	$\alpha = \dfrac{\beta}{1+\beta}$
20	0.9524
50	0.9804
100	0.9901
150	0.9934
220	0.9955
400	0.9975

5.6

(a) $I_B = \dfrac{I_E}{1+\beta} = \dfrac{1.2}{81} \Rightarrow I_B = 14.8\,\mu\text{A}$

$I_C = \left(\dfrac{\beta}{1+\beta}\right)\cdot I_E = \left(\dfrac{80}{81}\right)(1.2) = 1.185\text{ mA}$

$\alpha = \dfrac{80}{81} = 0.9877$

$V_C = 5 - (1.185)(2) = 2.63\text{ V}$

(b) $I_B = \dfrac{0.8}{81} \Rightarrow I_B = 9.88\,\mu\text{A}$

$I_C = \left(\dfrac{80}{81}\right)(0.8) = 0.790\text{ mA}$

$\alpha = 0.9877$

$V_C = 5 - (0.790)(2) = 3.42\text{ V}$

(c) (i) $I_B = \dfrac{1.2}{121} \Rightarrow I_B = 9.92\,\mu\text{A}$

$I_C = \left(\dfrac{120}{121}\right)(1.2) = 1.19\,\text{mA}$

$\alpha = \dfrac{120}{121} = 0.9917$

$V_C = 5 - (1.19)(2) = 2.62\text{ V}$

(ii) $I_B = \dfrac{0.8}{121} \Rightarrow I_B = 6.61\,\mu\text{A}$

$I_C = \left(\dfrac{120}{121}\right)(0.8) = 0.7934\text{ mA}$

$\alpha = 0.9917$

$V_C = 5 - (0.7934)(2) = 3.41\text{ V}$

5.7

$$I_E = I_{Eo}\exp\left(\frac{V_{BE}}{V_T}\right)$$

$$0.80\times10^{-3} = \left(5\times10^{-14}\right)\exp\left(\frac{V_{BE}}{V_T}\right)$$

Then $V_{BE} = (0.026)\ln\left(\dfrac{0.80\times10^{-3}}{5\times10^{-14}}\right) = 0.6109\text{ V}$

$\beta = \dfrac{\alpha}{1-\alpha} = \dfrac{0.9910}{1-0.9910} = 110$

$I_C = \alpha I_E = (0.9910)(0.80) = 0.7928\,\text{mA}$

$I_B = \dfrac{I_E}{1+\beta} = \dfrac{0.80}{111} \Rightarrow I_B = 7.21\,\mu\text{A}$

$V_C = 5 - I_C R_C = 5 - (0.7928)(2) = 3.41\,\text{V}$

5.8

$$I_B = \frac{0.75}{61} \Rightarrow 12.3\ \mu\text{A}$$

$$I_C = (0.75)\left(\frac{60}{61}\right) = 0.738\ \text{mA}$$

$$\alpha = \frac{60}{61} = 0.9836$$

$$V_C = I_C R_C - 10 = (0.738)(5) - 10$$

(a) $V_C = -6.31\ \text{V}$

$$I_B = \frac{1.5}{61} \Rightarrow 24.6\ \mu\text{A}$$

$$I_C = (1.5)\left(\frac{60}{61}\right) = 1.475\ \text{mA}$$

$$\alpha = \left(\frac{60}{61}\right) = 0.9836$$

(b) $V_C = (1.475)(5) - 10 \Rightarrow V_C = -2.625\ \text{V}$

(c) Yes, $V_C < 0$ in both cases so that B-C junction is reverse biased.

5.9

(a) $V_C = I_C(5) - 10$

$$-1.2 = I_C(5) - 10 \Rightarrow I_C = 1.76\ \text{mA}$$

$$I_E = \frac{I_C}{\alpha} = \frac{1.76}{0.986} = 1.785\ \text{mA}$$

$$I_B = I_E - I_C = 1.785 - 1.76 \Rightarrow I_B = 25\ \mu\text{A}$$

(b) $I_E = I_{Eo} \exp\left(\frac{V_{EB}}{V_T}\right)$

$$1.785 \times 10^{-3} = 2 \times 10^{-15} \exp\left(\frac{V_{EB}}{V_T}\right)$$

$$V_{EB} = (0.026)\ln\left(\frac{1.785 \times 10^{-3}}{2 \times 10^{-15}}\right) = 0.7154\ \text{V}$$

5.10

$$i_C = I_S \exp\left(\frac{\upsilon_{BE}}{V_T}\right) = (5 \times 10^{-15})\exp\left(\frac{0.615}{0.026}\right)$$

$$i_C = 93.7\ \mu\text{A}$$

$$i_B = \frac{93.7}{125} = 0.7495\ \mu\text{A}$$

$$i_E = (126)(0.7495) = 94.44\ \mu\text{A}$$

5.11

Device 1: $i_E = I_{Eo1}e^{v_{EB}/V_T} \Rightarrow 0.5\times10^{-3} = I_{Eo1}e^{0.650/0.026}$

So that

$$\underline{I_{EO1} = 6.94\times10^{-15}\ A}$$

Device 2: $12.2\times10^{-3} = I_{Eo2}e^{0.650/0.026}$

Or

$$\underline{I_{Eo2} = 1.69\times10^{-13}\ A}$$

$$\text{Ratio of areas} = \frac{I_{Eo2}}{I_{Eo1}} = \frac{1.69\times10^{-13}}{6.94\times10^{-15}} \Rightarrow \underline{\text{Ratio} = 24.4}$$

5.12

For transistor A:

$$\upsilon_{BE}(A) = V_T\ \ln\left(\frac{I_C}{I_{SA}}\right) = (0.026)\ln\left(\frac{275\times10^{-6}}{8\times10^{-16}}\right) = 0.6906\ \text{V}$$

For transistor B:

$$I_{SB} = 4I_{SA} = 4\left(8\times10^{-16}\right) = 3.2\times10^{-15}\ \text{A}$$

$$\upsilon_{BE}(B) = (0.026)\ln\left(\frac{275\times10^{-6}}{3.2\times10^{-15}}\right) = 0.6546\ \text{V}$$

5.13

(a) $i_C = I_{Co}\left(1+\dfrac{\upsilon_{CE}}{V_A}\right)$

$$0.6 = I_{Co}\left(1+\frac{2}{80}\right) \Rightarrow I_{Co} = 0.58537\ \text{mA}$$

At $\upsilon_{CE} = 5\ \text{V}$

$$i_C = (0.58537)\left(1+\frac{5}{80}\right) = 0.622\ \text{mA}$$

(b) $r_o = \dfrac{\Delta\upsilon_{CE}}{\Delta i_C} = \dfrac{5-2}{0.621956-0.60} \Rightarrow r_o = 137\ \text{k}\Omega$

5.14

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}} = \frac{60}{\sqrt[3]{100}}$$

$$\underline{BV_{CE0} = 12.9\ \text{V}}$$

5.15

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}}$$

$$56 = \frac{220}{\sqrt[3]{\beta}} \Rightarrow \sqrt[3]{\beta} = \frac{220}{56} = 3.93$$

$$\underline{\beta = 60.6}$$

5.16

$$BV_{CE0} = \frac{BV_{CB0}}{\sqrt[3]{\beta}}$$

$$BV_{CB0} = \left(BV_{CE0}\right)\sqrt[3]{\beta} = (50)\sqrt[3]{50}$$

$$\underline{BV_{CB0} = 184 \text{ V}}$$

5.17

(a)

$$I_E = \frac{-0.7-(-10)}{5} = 1.86 \text{ mA}$$

$$I_C = (1.86)\left(\frac{75}{76}\right) = 1.836 \text{ mA}$$

$$V_C = -0.7 + 4 = 3.3 \text{ V}$$

$$R_C = \frac{10-3.3}{1.836} \Rightarrow R_C = 3.65 \text{ K}$$

(b)

$$I_B = \frac{0.5}{76} = 0.00658 \text{ mA}$$

$$V_B = I_B R_B = (0.00658)(25) \Rightarrow V_B = 0.164 \text{ V}$$

$$I_C = (0.5)\left(\frac{75}{76}\right) = \underline{0.493 \text{ mA}}$$

$$R_C = \frac{-1-(-5)}{0.493} \Rightarrow R_C = 8.11 \text{ K}$$

(c)

$$O = \frac{I_E}{76}(10) + 0.7 + I_E(4) - 8$$

$$7.3 = I_E(4 + 0.132) \Rightarrow I_E = 1.767 \text{ mA}$$

$$I_C = (1.767)\left(\frac{75}{76}\right) = \underline{1.744 \text{ mA}}$$

$$V_{CE} = 8 - (1.744)(4) - \left[(1.767)(4) - 8\right]$$

$$= 16 - 6.972 - 7.068 \Rightarrow V_{CE} = 1.96 \text{ V}$$

(d)

$$5 = I_E(10) + \left(\frac{I_E}{76}\right)(20) + 0.7 + I_E(2) \qquad = I_E(10 + 0.263 + 2) + 0.7$$

$$I_E = 0.3506 \text{ mA} \Rightarrow \underline{I_B = 4.61 \ \mu\text{A}} \qquad V_C = 5 - (0.3506)(10)$$

$$\underline{V_C = 1.49 \text{ V}}$$

5.18

For Fig. P5.17(a), $R_E = 5 + 5\% = 5.25\,\text{k}\Omega$

$$I_E = \frac{-0.7-(-10)}{5.25} = 1.77\text{ mA}$$

$$I_C = 1.75\text{ mA}$$

$$R_C = \frac{10-3.3}{1.75} = 3.83\text{ K}$$

$$R_E = 5 - 5\% = 4.75\text{ K}$$

$$I_E = \frac{-0.7-(-10)}{4.75} = 1.96\text{ mA}$$

$$I_C = 1.93\text{ mA}$$

$$R_C = \frac{10-3.3}{1.93} = 3.47\text{ K}$$

So $1.75 \le I_C \le 1.93$ mA $\quad 3.47 \le R_C \le 3.83$ K

For Fig. P5.17(c), $R_E = 4 + 5\% = 4.2\,\text{k}\Omega$

$$I_B = \frac{8-0.7}{10+(76)(4.2)} = 0.0222\text{ mA} \quad \underline{I_C = 1.66\text{ mA}}$$

$$I_E = 1.69\text{ mA}$$

$$V_{CE} = 16-(1.66)(4)-(1.69)(4.2)$$
$$= 16 - 6.64 - 7.098 \Rightarrow \quad \underline{V_{CE} = 2.26\text{ V}}$$

$$R_E = 4 - 5\% = 3.8\text{ K}$$

$$I_B = \frac{8-0.7}{10+(76)(3.8)} = 0.0244 \quad I_C = 1.83\text{ mA}$$

$$I_E = 1.86\text{ mA}$$

$$V_{CE} = 16-(1.83)(4)-(1.86)(3.8)$$
$$= 16 - 7.32 - 7.068$$

$$V_{CE} = 1.61\text{ V}$$

So $1.66 \le I_C \le 1.83$ mA $\quad 1.61 \le V_{CE} \le 2.26$ V

5.19

(a) $V_{CC} = I_C R_C + V_{CE}$

$$I_C = \frac{2.5-1.1}{4} = 0.35\,\text{mA}$$

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)$$

$$V_{BE} = V_{BB} = (0.026)\ln\left(\frac{0.35\times10^{-3}}{5\times10^{-16}}\right) = 0.7091\text{ V}$$

(b) $I_E = \dfrac{V_{CC} - V_{CE}}{R_E} = \dfrac{2.5 - 1.1}{2} = 0.7 \text{ mA}$

$$I_C = \left(\frac{\beta}{1+\beta}\right) \cdot I_E = \left(\frac{90}{91}\right)(0.70) = 0.6923 \text{ mA}$$

$$V_{BE} = (0.026)\ln\left(\frac{0.6923 \times 10^{-3}}{5 \times 10^{-16}}\right) = 0.7269 \text{ V}$$

$$V_{BB} = V_{BE} + I_E R_E = 0.7269 + (0.7)(2) = 2.127 \text{ V}$$

5.20

(a) $I_C = 0$, $V_{CE} = 2 \text{ V}$

(b) $I_C = \beta I_B = (120)(2) \Rightarrow I_C = 0.24 \text{ mA}$

$$V_{CE} = 2 - (0.24)(4) = 1.04 \text{ V}$$

(c) $I_E = \dfrac{1.4 - 0.7}{2} = 0.35 \text{ mA}$

$$I_C = \left(\frac{120}{121}\right)(0.35) = 0.3471 \text{ mA}$$

$$V_{CE} = 2 - (0.3471)(4) - (0.35)(2) = -0.088 \text{ V} \text{ - Not possible}$$

Transistor in Saturation

$V_{CE} = 0.2 \text{ V}$

$V_E = 0.7 \text{ V} \Rightarrow V_C = 0.9 \text{ V}$

$$I_C = \frac{2 - 0.9}{4} = 0.275 \text{ mA}$$

5.21

(a) $I_E = \dfrac{2 - (0.7 + 0.2)}{1.5} = 0.7333 \text{ mA}$

$$I_C = \left(\frac{\beta}{1+\beta}\right) \cdot I_E = \left(\frac{120}{121}\right)(0.7333) = 0.7273 \text{ mA}$$

$V_{EC} = V_E = 0.9 \text{ V}$

(b) $I_C = \beta I_B = (120)(15) \Rightarrow I_C = 1.8 \text{ mA}$ - Not possible

Transistor in saturation

$V_{EC} = 0.2 \text{ V}$

$$I_E = \frac{2 - 0.2}{1.5} = 1.2 \text{ mA}$$

$$I_C = I_E - I_B = 1.2 - 0.015 = 1.185 \text{ mA}$$

(c) Transistor cutoff

$I_C = 0$, $V_{EC} = 2 \text{ V}$

5.22

(a) $I_{BQ} = \dfrac{V_{BB} - V_{BE}(on)}{R_B} \Rightarrow R_B = \dfrac{1.3 - 0.7}{0.005} = 120\ \text{k}\Omega$

$I_{CQ} = \beta I_{BQ} = (100)(0.005) = 0.5\ \text{mA}$

$R_C = \dfrac{3 - 1.5}{0.5} = 3\ \text{k}\Omega$

(b) For $\beta = 75$, $I_{CQ} = (75)(0.005) = 0.375\ \text{mA}$

$V_{CE} = 3 - (0.375)(3) = 1.875\ \text{V}$

For $\beta = 125$, $I_{CQ} = (125)(0.005) = 0.625\ \text{mA}$

$V_{CE} = 3 - (0.625)(3) = 1.125\ \text{V}$

So $1.125 \le V_{CE} \le 1.875\ \text{V}$

5.23

(a)

$V_B = -I_B R_B \Rightarrow I_B = \dfrac{-V_B}{R_B} = \dfrac{-(-1)}{500}$

$I_B = 2.0\ \mu\text{A}$

$V_E = -1 - 0.7 = -1.7\ \text{V}$

$I_E = \dfrac{V_E - (-3)}{R_E} = \dfrac{-1.7 + 3}{4.8} = 0.2708\ \text{mA}$

$\dfrac{I_E}{I_B} = (1 + \beta) = \dfrac{0.2708}{0.002} = 135.4 \Rightarrow \underline{\beta = 134.4}$

$\alpha = \dfrac{\beta}{1 + \beta} \Rightarrow \underline{\alpha = 0.9926}$

$I_C = \beta I_B \Rightarrow \underline{I_C = 0.269\ \text{mA}}$

$V_{CE} = 3 - V_E = 3 - (-1.7) \Rightarrow \underline{V_{CE} = 4.7\ \text{V}}$

(b)

$I_E = \dfrac{5 - 4}{2} \Rightarrow I_E = 0.5\ \text{mA}$

$4 = 0.7 + I_B R_B + (I_B + I_C) R_C - 5$

$I_B + I_C = I_E$

$I_B + I_C = I_E$

$4 = 0.7 + I_B(100) + (0.5)(8) - 5$

$I_B = 0.043 \Rightarrow \dfrac{I_E}{I_B} = (1 + \beta) = \dfrac{0.5}{0.043} = 11.63$

$\underline{\beta = 10.63}$, $\alpha = \dfrac{\beta}{1 + \beta} \Rightarrow \underline{\alpha = 0.9140}$

5.24

(a) $I_C = \dfrac{5 - V_B}{10}$, $I_E = \dfrac{(V_B - 0.7) + 5}{3}$, $I_C = \left(\dfrac{\beta}{1+\beta}\right) \cdot I_E = \left(\dfrac{90}{91}\right) \cdot I_E$

Then $\dfrac{5 - V_B}{10} = \left(\dfrac{90}{91}\right)\left(\dfrac{V_B + 4.3}{3}\right) \Rightarrow V_B = -2.136 \text{ V}$

$$I_E = \frac{-2.136 - 0.7 + 5}{3} = 0.721 \text{ mA}$$

(b) $V_{CE} = 10 - I_C(10) - I_E(3)$

$$2 = 10 - I_E\left[3 + \left(\frac{90}{91}\right)(10)\right] = 10 - I_E(12.89)$$

Then $I_E = 0.6206 \text{ mA}$

And $V_B = 0.7 + (0.6206)(3) - 5 = -2.438 \text{ V}$

5.25

(a) $I_E = \dfrac{3.3 - 0.85}{10} = 0.245 \text{ mA}$

$$I_B = \frac{0.85 - 0.7}{50} \Rightarrow I_B = 3\,\mu\text{A}$$

$$I_C = I_E - I_B = 0.245 - 0.003 = 0.242 \text{ mA}$$

$$\beta = \frac{I_C}{I_B} = \frac{0.242}{0.003} = 80.67$$

$$\alpha = \frac{\beta}{1+\beta} = \frac{80.67}{81.67} = 0.9878$$

$$V_C = (0.242)(10) - 3.3 = -0.88 \text{ V}$$

$$V_{EC} = 0.85 - (-0.88) = 1.73 \text{ V}$$

(b) $\beta = (80.67)(1.10) = 88.73$

$$I_B = \frac{V_E - 0.7}{50}, \quad I_E = \frac{3.3 - V_E}{10} = (89.73)\left(\frac{V_E - 0.7}{50}\right), \quad \Rightarrow V_E = 0.8371 \text{ V}$$

$$I_E = \frac{3.3 - 0.8371}{10} = 0.2463 \text{ mA}$$

$$I_C = \left(\frac{88.73}{89.73}\right)(0.2463) = 0.2435 \text{ mA}$$

$$V_C = (0.2435)(10) - 3.3 = -0.8645 \text{ V}$$

$$V_{EC} = 0.8371 - (-0.8645) = 1.70 \text{ V}$$

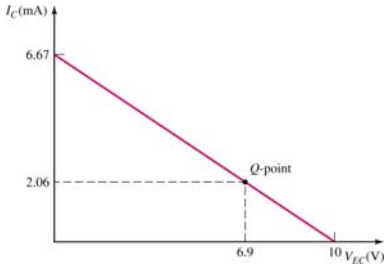
5.26

$$I_B = \frac{5 - 0.7}{250} \Rightarrow 17.2\ \mu\text{A}$$

$$I_C = (120)(0.0172) = 2.064 \text{ mA}$$

$$V_C = (2.064)(1.5) - 5 = -1.90 \text{ V}$$

$$V_{EC} = 5 - (-1.90) \Rightarrow \underline{V_{EC} = 6.90 \text{ V}}$$



5.27

$$I_C = \left(\frac{50}{51}\right)(1) = 0.98\ mA$$

$$V_C = I_C R_C - 9 = (0.98)(4.7) - 9 \text{ or } \underline{V_C = -4.39\ V}$$

$$I_B = \frac{1}{51} = 0.0196\ mA$$

$$V_E = I_B R_B + V_{EB}(on) = (0.0196)(50) + 0.7 \text{ or } \underline{V_E = 1.68\ V}$$

5.28

$$I_C = \left(\frac{50}{51}\right)(0.5) = 0.49\ mA,\ I_B = \frac{0.5}{51} = 0.0098\ mA$$

$$V_E = I_B R_B + V_{EB}(on) = (0.0098)(50) + 0.7 \text{ or } V_E = 1.19\ V$$

$$V_C = I_C R_C - 9 = (0.49)(4.7) - 9 = -6.70\ V$$

$$\text{Then } V_{EC} = V_E - V_C = 1.19 - (-6.7) == 7.89\ V$$

$$P_Q = I_C V_{EC} + I_B V_{EB} = (0.49)(7.89) + (0.0098)(0.7) \text{ or } \underline{P_Q = 3.87\ mW}$$

$$\text{Power Dissipated} = P_S = I_Q(9 - V_E) = (0.5)(9 - 1.19)$$

Or $\underline{P_S = 3.91\ mW}$

5.29

$$I_{E1} = I_{E2} = \frac{I}{2} \Rightarrow \underline{I_{E1} = I_{E2} = 0.5\ \text{mA}}$$

$$I_{C1} = I_{C2} \approx 0.5\ \text{mA}$$

$$V_{C1} = V_{C2} = 5 - (0.5)(4) \Rightarrow \underline{V_{C1} = V_{C2} = 3\ \text{V}}$$

5.30

$$R_E = 0 \quad I_B = \frac{2 - 0.7}{R_B} = \frac{1.3}{R_B}$$

$$I_C = (80)\left(\frac{1.3}{R_B}\right) = \frac{5 - 2}{R_C} = 0.8 \Rightarrow R_C = 3.75\ \text{K}$$

(a) $R_B = 130\ \text{K}$

$R_E = 1\text{ K} \quad I_B = \dfrac{0.8}{80} = 0.010\text{ mA} \quad I_E = 0.8\left(\dfrac{81}{80}\right) = 0.81\text{ mA}$

$2 = (0.010)(R_B) + 0.7 + (0.81)(1) \Rightarrow \quad \underline{R_B = 49\text{ K}}$

(b) $5 = (0.8)R_C + 2 + (0.81)(1) \Rightarrow \quad \underline{R_C = 2.74\text{ K}}$

(c) For part (a) $I_B = \dfrac{2 - 0.7}{130} = 0.01\text{ mA}$

$I_C = (120)(0.01) \Rightarrow \underline{I_C = 1.20\text{ mA}}$

$V_{CE} = 5 - (1.2)(3.75) \Rightarrow \underline{V_{CE} = 0.5\text{ V}}$

For part (b) $2 = I_B(49) + 0.7 + (121)I_B(1)$

$I_B = 0.00765\text{ mA}, I_E = 0.925\text{ mA}, I_C = 0.918\text{ mA}$

$V_{CE} = 5 - (0.918)(2.74) - (0.925)(1) \Rightarrow V_{CE} = 1.56\text{ V}$

Including R_E result in smaller changes in Q-point values.

5.31

(a) $R_C = \dfrac{V_{CC} - V_{CEQ}}{I_{CQ}} = \dfrac{9 - 4.5}{0.25} = 18\,\text{k}\Omega$

$I_{BQ} = \dfrac{0.25}{80} \Rightarrow I_{BQ} = 3.125\,\mu\text{ A}$

$R_B = \dfrac{9 - 0.7}{0.003125} \Rightarrow R_B = 2.656\,\text{M}\Omega$

(b) $I_{CQ} = (120)(0.003125) = 0.375\text{ mA}$

$V_{CEQ} = 9 - (0.375)(18) = 2.25\text{ V}$

5.32

(a) $I_C = I_E = 0\,, \quad V_C = 6\text{ V}$

(b) $I_E = \dfrac{0.9 - 0.7}{1} = 0.2\text{ mA}, \quad I_C = \left(\dfrac{150}{151}\right)(0.2) = 0.1987\text{ mA}$

$V_C = 6 - (0.1987)(10) = 4.013\text{ V}$

(c) $I_E = \dfrac{1.5 - 0.7}{1} = 0.8\,\text{mA}$

Transistor in saturation

$V_C = 1.5 - 0.7 + 0.2 = 1\,\text{V}$

$I_C = \dfrac{6 - 1}{10} = 0.5\,\text{mA}$

(d) $I_E = \dfrac{2.2 - 0.7}{1} = 1.5\text{ mA}$

$V_C = 2.2 - 0.7 + 0.2 = 1.7\text{ V}$

$I_C = \dfrac{6 - 1.7}{10} = 0.43\text{ mA}$

5.33

$V_{BB} = 0.$

$$\text{Cutoff } V_0 = \left(\frac{R_L}{R_C + R_L}\right)V_{CC} = \left(\frac{10}{10+5}\right)(5)$$

a. $\underline{V_0 = 3.33 \text{ V}}$

$V_{BB} = 1 \text{ V}$

$$I_B = \frac{1 - 0.7}{50} \Rightarrow 6 \ \mu\text{A}$$

$$I_C = \beta I_B = (75)(6) \Rightarrow I_C = 0.45 \text{ mA}$$

$$\frac{5 - V_0}{5} = I_C + \frac{V_0}{10}$$

b. $$1 - 0.45 = V_0\left(\frac{1}{5} + \frac{1}{10}\right) \Rightarrow \underline{V_0 = 1.83 \text{ V}}$$

c. Transistor in saturation $\underline{V_0 = V_{CE}(\text{sat}) = 0.2 \text{ V}}$

5.34

(a) $\beta = 100$

$$I_Q = 0.1 \ mA \quad I_C = \left(\frac{100}{101}\right)(0.1) = 0.0990 \ mA$$

(i) $V_O = 5 - (0.099)(5) \Rightarrow \underline{V_O = 4.505 \ V}$

$$I_Q = 0.5 \ mA \quad I_C = \left(\frac{100}{101}\right)(0.5) = 0.495 \ mA$$

(ii) $V_O = 5 - (0.495)(5) \Rightarrow \quad \underline{V_O = 2.525 \ V}$

$I_Q = 2$ mA Transistor is in saturation

(iii) $V_O = -V_{BE}(sat) + V_{CE}(sat) = -0.7 + 0.2 \Rightarrow \underline{V_O = -0.5 \text{ V}}$

(b) $\beta = 150$

$$I_Q = 0.1 \ mA \quad I_C = \left(\frac{150}{151}\right)(0.1) = 0.09934 \ mA$$

$V_O = 5 - (0.09934)(5) \Rightarrow \quad \underline{V_O = 4.503 \ V}$

(i) $$\% \text{ change} = \frac{4.503 - 4.505}{4.503} \times 100\% = \underline{-0.044\%}$$

$$I_Q = 0.5 \ mA \quad I_C = \left(\frac{150}{151}\right)(0.5) = 0.4967 \ mA$$

$V_O = 5 - (0.4967)(5) \Rightarrow \underline{V_O = 2.517 \ V}$

(ii) $$\% \text{ change} = \frac{2.517 - 2.525}{2.525} \times 100\% = \underline{-0.32\%}$$

$I_Q = 2 \ mA$ Transistor in saturation

(iii) $\underline{V_o = -8.5 \text{ V}}$ $\underline{\text{No change}}$

5.35

(a) $I_C = \dfrac{5-4}{5} = 0.20\ \text{mA}$

$$I_Q = I_E = \left(\frac{121}{120}\right)(0.20) = 0.2017\ \text{mA}$$

(b) $I_C = \dfrac{5-2}{5} = 0.60\ \text{mA}$

$$I_Q = I_E = \left(\frac{121}{120}\right)(0.60) = 0.605\ \text{mA}$$

(c) $I_C = \dfrac{5-0}{5} = 1.0\ \text{mA}$

$$I_Q = I_E = \left(\frac{121}{120}\right)(1.0) = 1.008\ \text{mA}$$

5.36

For $I_Q = 0,$ then $\underline{P_Q = 0}$

For $I_Q = 0.5\ mA, \quad I_C = \left(\dfrac{50}{51}\right)(0.5) = 0.49\ mA$

$$I_B = \frac{0.5}{51} = 0.0098\ mA,\ V_B = 0.490\ V,\ V_E = 1.19\ V$$

$$V_C = (0.49)(4.7) - 9 = -6.70\ V \Rightarrow V_{EC} = 7.89\ V$$

$$P \cong I_C V_{EC} = (0.49)(7.89) \Rightarrow \quad \underline{P = 3.87\ mW}$$

For $I_Q = 1.0\ mA,$ Using the same calculations as above, we find $P = 5.95\ mW$

For $I_Q = 1.5\ mA, \quad P = 6.26\ mW$

For $I_Q = 2\ mA, \quad P = 4.80\ mW$

For $I_Q = 2.5\ mA, \quad P = 1.57\ mW$

For $I_Q = 3\ mA,$ Transistor is in saturation.

$$0.7 + I_B(50) = 0.2 + I_C(4.7) - 9$$

$$I_E = I_Q = I_B + I_C \Rightarrow I_B = 3 - I_C$$

Then, $0.7 + (3 - I_C)(50) = 0.2 + I_C(4.7) - 9$

Which yields $I_C = 2.916\ mA$ and $I_B = 0.084\ mA$

$$P = I_B V_{EB} + I_C V_{EC} = (0.084)(0.7) + (2.916)(0.2) \text{ or } P = 0.642\ mW$$

5.37

$$I_E = \frac{V_{EE} - V_{EB}(\text{on})}{R_E} = \frac{9 - 0.7}{4} \Rightarrow \underline{I_E = 2.075\ \text{mA}}$$

$$I_C = \alpha I_E = (0.9920)(2.075) \Rightarrow \underline{I_C = 2.06\ \text{mA}}$$

$$V_{BC} + I_C R_C = V_{CC}$$

$$V_{BC} = 9 - (2.06)(2.2) \Rightarrow \underline{V_{BC} = 4.47\ \text{V}}$$

5.38

(a) $I_C = \dfrac{12-6}{2.2} = 2.727 \text{ mA}, \quad I_B = \dfrac{2.727}{80} = 0.03409 \text{ mA}$

$$I_{R2} = \frac{0.7-(-12)}{100} = 0.127 \text{ mA}$$

$$I_{R1} = I_B + I_{R2} = 0.1611 \text{ mA}$$

$$V_I = I_{R1}R_1 + 0.7 = (0.1611)(15) + 0.7 = 3.12 \text{ V}$$

(b) For $V_{CEQ} = 9$ V, $I_C = \dfrac{12-9}{2.2} = 1.364 \text{ mA}, \quad I_B = 0.01705 \text{ mA}$

$$I_{R1} = 0.01705 + 0.127 = 0.14405 \text{ mA}$$

$$V_I = (0.14405)(15) + 0.7 = 2.86 \text{ V}$$

For $V_{CEQ} = 3$ V, $I_C = \dfrac{12-3}{2.2} = 4.0909 \text{ mA}, \quad I_B = 0.05114 \text{ mA}$

$$I_{R1} = 0.05114 + 0.127 = 0.1781 \text{ mA}$$

$$V_I = (0.1781)(15) + 0.7 = 3.37 \text{ V}$$

So $2.86 \le V_I \le 3.37$ V

5.39

For $V_{CE} = 4.5$

$$I_{CQ} = \frac{5-4.5}{1} = 0.5 \text{ mA}$$

$$I_{BQ} = \frac{0.5}{25} = 0.02 \text{ mA}$$

$$I_{R2} = \frac{0.7-(-5)}{100} = 0.057 \text{ mA}$$

$$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.02 = 0.077 \text{ mA}$$

$$V_1 = I_{R1}R_1 + V_{BE}(\text{on}) = (0.077)(15) + 0.7 = 1.86 \text{ V}$$

For $V_{CE} = 1.0$

$$I_{CQ} = \frac{5-1}{1} = 4 \text{ mA}$$

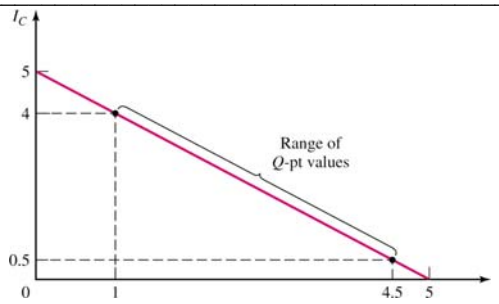
$$I_{BQ} = \frac{4}{25} = 0.16 \text{ mA}$$

$$I_{R2} = 0.057 \text{ mA}$$

$$I_{R1} = I_{R2} + I_{BQ} = 0.057 + 0.16 = 0.217 \text{ mA}$$

$$V_1 = (0.217)(15) + 0.7 \Rightarrow 3.96 \text{ V}$$

So $1.86 \le V_1 \le 3.96$ V



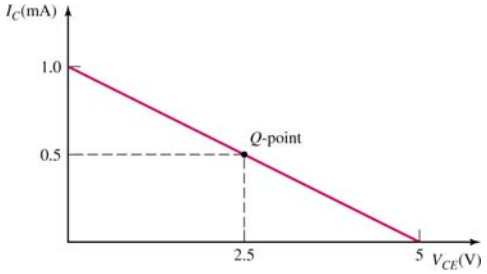
5.40

$$R_C = \frac{5-2.5}{0.5} = 5 \text{ K}$$

$$I_B = \frac{0.5}{120} = 0.00417 \text{ mA}$$

$$R_B = \frac{5-0.7}{0.00417} = 1032 \text{ K}$$

(a)



Choose $R_C = 5.1$ K

(b) $R_B = 1 \text{ M}\Omega$

For $R_B = 1 \text{ M}\Omega + 10\% = 1.1$ M, $R_C = 5.1 \text{ k} + 10\% = 5.61$ K

$$I_{BQ} = \frac{5-0.7}{1.1} = 3.91 \ \mu\text{A} \Rightarrow I_{CQ} = 0.469 \text{ mA}$$

$$V_{CEQ} = 2.37 \text{ V}$$

$R_B = 1 \text{ M}\Omega + 10\% = 1.1\text{M}$, $R_C = 5.1 \ K - 10\% = 4.59$ K

$I_{BQ} = 3.91 \ \mu\text{A} \Rightarrow I_{CQ} = 0.469$ mA

$V_{CEQ} = 2.85$ V

$R_B = 1 \text{ M}\Omega - 10\% = 0.90 \text{ M}\Omega$ $\quad R_C = 5.1 \text{ k} + 10\% = 5.61$ K

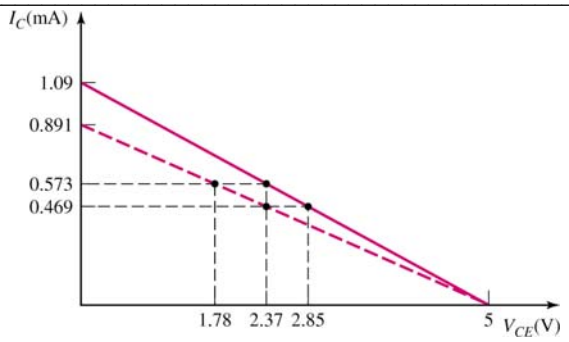
$$I_{BQ} = \frac{5-0.7}{0.90} = 4.78 \ \mu\text{A} \Rightarrow I_C = 0.573 \text{ mA}$$

$V_{CEQ} = 1.78$ V

$R_B = 1 \text{ M}\Omega - 10\% = 0.90 \text{ M}\Omega$ $\quad R_C = 5.1 \text{ k} - 10\% = 4.59$ K

$I_{BQ} = 4.78 \ \mu\text{A} \Rightarrow I_C = 0.573$ mA

$V_{CEQ} = 2.37$ V



5.41

$$V_{E2} = 5 - V_{BE2} \qquad V_{E1} = 5 - V_{BE1}$$

$$V_O = V_{E2} - V_{E1} = (5 - V_{BE2}) - (5 - V_{BE1})$$

$$V_O = V_{BE1} - V_{BE2}$$

We have $V_{BE1} = V_E \ln\left(\frac{I_{E1}}{I_{EO}}\right)$

$$V_{BE2} = V_T \ln\left(\frac{I_{E2}}{I_{EO}}\right)$$

$$V_O = V_T\left[\ln\left(\frac{I_{E1}}{I_{EO}}\right) - \ln\left(\frac{I_{E2}}{I_{EO}}\right)\right]$$

$$V_O = V_T \ln\left(\frac{I_{E1}}{I_{E2}}\right) = V_T \ln\left(\frac{10_I}{I}\right)$$

$$\boxed{V_O = \frac{kT}{e}\ln(10)}$$

5.42

(a) (i) $I_C = \frac{5-4}{4} = 0.25\,\text{mA},\ I_B = \frac{0.25}{120} = 0.002083\,\text{mA}$

$$V_I = (0.002083)(200) + 0.7 = 1.117\,\text{V}$$

(ii) $I_C = 0.25\,\text{mA},\ I_E = \left(\frac{121}{120}\right)(0.25) = 0.252\,\text{mA}$

$$V_I = (0.002083)(200) + 0.7 + (0.252)(1) = 1.369\,\text{V}$$

(b) (i) $I_C = \frac{5-2.5}{4} = 0.625\,\text{mA},\ I_B = \frac{0.625}{120} = 0.005208\,\text{mA}$

$$V_I = (0.005208)(200) + 0.7 = 1.742\,\text{V}$$

(ii) $I_E = \left(\frac{121}{120}\right)(0.625) = 0.6302\,\text{mA}$

$$V_I = 1.742 + (0.6302)(1) = 2.372\,\text{V}$$

(c) Transistor biased in saturation

$$3.5 = I_B(200) + 0.7 + I_E(1)$$

$$I_C = \frac{5 - V_O}{4},\quad I_E = \frac{V_O - 0.2}{1}$$

$$I_B = I_E - I_C = \frac{V_O - 0.2}{1} - \left(\frac{5 - V_O}{4}\right) = V_O(1.25) - 1.45$$

Then $3.5 = [V_O(1.25) - 1.45](200) + 0.7 + (V_O - 0.2)$

$V_O = 1.167$ V

5.43

For $4.3 \le V_I \le 5$ Q is cutoff $I_C = 0$

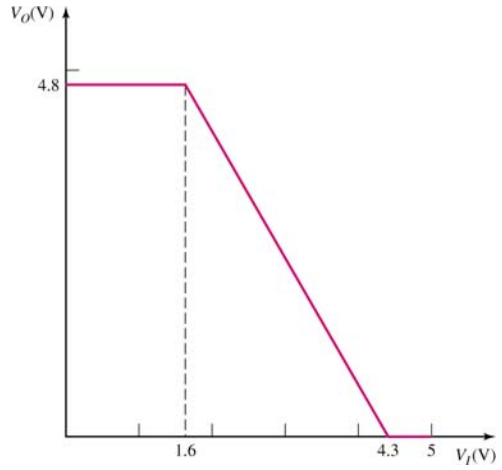
$V_O = 0$

If Q reaches saturation, $V_O = 4.8$

$$I_C = \frac{4.8}{4} = 1.2 \text{ mA}$$

$$I_B = \frac{1.2}{80} = 0.015 = \frac{5 - 0.7 - V_I}{180} \Rightarrow V_I = 1.6$$

So $V_I \le 1.6,\ V_O = 4.8$



5.44

(a) For $V_I \geq 4.3$, Q is off and $V_O = 0$

$$5 = \left(\frac{101}{100}\right) I_C(1) + 0.2 + I_C(4) \Rightarrow I_C = 0.958 \text{ mA}$$

When transistor enters saturation,

$V_O = 3.832$ V

$I_B = 0.00958$ mA

$$5 = \left(\frac{101}{100}\right)(0.958)(1) + 0.7 + (0.00958)(180) + V_I$$

$V_I = 5 - 0.7 - 0.9676 - 1.7244 \Rightarrow V_I = 1.61$ V

For $V_I = 0$, transistor in saturation

$5 = I_E(1) + 0.2 + I_C(4) \Rightarrow 5 = I_C(1) + I_B(1) + 0.2 + I_C(4)$

$5 = I_E(1) + 0.7 + I_B(180)$ $5 = I_C(1) + I_B(1) + 0.7 + I_B(180)$

$I_E = I_C + I_B$

$4.8 = 5I_C + I_B(1)$

$4.3 = 1I_C + 181I_B$

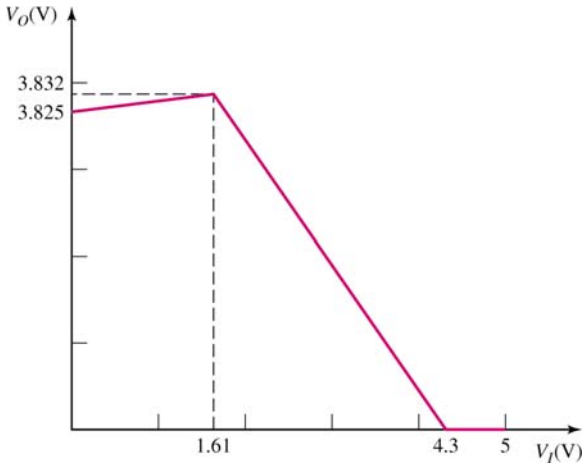
$I_B = 4.8 - 5I_C$

$4.3 = I_C + (181)(4.8 - 5I_C)$

$904I_C = 864.5$

$I_C = 0.956$ mA

$V_O = 3.825$ V



5.45

$$I_C = \frac{V_{CC} - V_{CE}(sat)}{R_C} = \frac{5 - 0.2}{0.2} = 24 \text{ mA}$$

$$\frac{I_C}{I_B} = 20 \Rightarrow I_B = \frac{24}{20} = 1.2 \text{ mA}$$

$$I_B = \frac{V_I - V_{BE}(on)}{R_B} \Rightarrow R_B = \frac{5 - 0.7}{1.2} = 3.58 \text{ k}\Omega$$

5.46

(a) $V_I = 0\,,\ \Rightarrow I_B = I_C = I_E = 0,\ V_O = 0$

(b) $V_I = 2.5$ V,

$$V_I = I_B R_B + V_{BE}(on) + I_E R_E\,,\ \text{and}\ \ I_E = (1+\beta) I_B$$

Then $I_B = \dfrac{V_I - V_{BE}(on)}{R_B + (1+\beta)R_E} = \dfrac{2.5 - 0.7}{10 + (51)(0.5)} \Rightarrow I_B = 50.7\ \mu\text{A}$

$$I_C = (50)(0.0507) = 2.535\ \text{mA},\ \ I_E = (51)(0.0507) = 2.586\ \text{mA}$$

$$V_O = I_E R_E = (2.586)(0.5) = 1.293\ \text{V}$$

(c) $V_I = 5$ V, Transistor in saturation

$$V_O = 2.8\ \text{V},\ \ I_E = \frac{2.8}{0.5} = 5.6\ \text{mA}$$

$$I_B = \frac{V_I - V_{BE}(on) - V_O}{R_B} = \frac{5 - 0.7 - 2.8}{10} = 0.15\ \text{mA}$$

$$I_C = I_E - I_B = 5.6 - 0.15 = 5.45\ \text{mA}$$

5.47

$$I_C = \frac{V_O}{R_C} = \frac{8.8}{0.5} = 17.6\ \text{mA}$$

$$\frac{I_C}{I_B} = 25 \Rightarrow I_B = \frac{17.6}{25} = 0.704\ \text{mA}$$

$$I_B = \frac{9 - V_{EB}(on) - V_I}{R_B} \Rightarrow R_B = \frac{9 - 0.7 - 5}{0.704} = 4.69\ \text{k}\Omega$$

5.48

(a) $I_{CQ} = \dfrac{3 - 1.6}{2} = 0.7\ \text{mA},\ \ I_{BQ} = \dfrac{0.7}{120} \Rightarrow I_{BQ} = 5.833\ \mu\text{A}$

$$R_B = \frac{V_{BB} - V_{BE}(on)}{I_{BQ}} = \frac{1 - 0.7}{0.005833} = 51.4\ \text{k}\Omega$$

(b) $\Delta V_O = 3.3 - 0.2 = 2.8$ V, peak-to-peak

(c) $\dfrac{\Delta V_O}{\Delta V_I} = \dfrac{3 - 1.6}{0.7 - 1.0} = A_\upsilon = -4.67$

(d) $\Delta V_I(\max) = \dfrac{\Delta V_O(\max)}{|A_\upsilon|} = \dfrac{2.8}{4.667} = 0.6$ V

So $\upsilon_i = 0.6$ V, peak-to-peak

5.49

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.15}{120} \Rightarrow I_{BQ} = 1.25\ \mu\text{A}$$

$$I_{EQ} = I_{CQ}\left(\frac{1+\beta}{\beta}\right) = (0.15)\left(\frac{121}{120}\right) = 0.15125\ \text{mA}$$

We have $R_{TH} = 200\ \text{k}\Omega$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E = (0.00125)(200) + 0.7 + (0.15125)(2) = 1.2525 \text{ V}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)\cdot V_{CC} = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC}$$

So $1.2525 = \frac{1}{R_1}(200)(2.5)$

$$\Rightarrow R_1 = 399\,\text{k}\Omega \quad \text{and} \quad R_2 = 401\,\text{k}\Omega$$

$$V_{CEQ} = 2.5 - I_{CQ}R_C - I_{EQ}R_E = 2.5 - (0.15)(6) - (0.15125)(2) = 1.30 \text{ V}$$

5.50

$$I_{EQ} = \left(\frac{1+\beta}{\beta}\right)\cdot I_{CQ} = \left(\frac{151}{150}\right)(0.20) = 0.2013 \text{ mA}, \quad I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.20}{150} \Rightarrow I_{BQ} = 1.33\,\mu\text{A}$$

$$V_{CC} = I_{CQ}R_C + V_{CEQ} + I_{EQ}R_E$$

$$2.5 = (0.20)R_C + 1.5 + (0.2013)(1) \Rightarrow R_C = 4\,\text{k}\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E = (0.00133)(120) + 0.7 + (0.2013)(1) = 1.061 \text{ V}$$

$$V_{TH} = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} \Rightarrow 1.061 = \frac{1}{R_1}(120)(2.5)$$

So $R_1 = 283\,\text{k}\Omega$ and $R_2 = 208\,\text{k}\Omega$

5.51

$$R_{TH} = R_1 \| R_2 = 20 \| 15 = 8.57\,\text{k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)\cdot V_{CC} = \left(\frac{15}{15+20}\right)(10) = 4.29 \text{ V}$$

$$V_{CC} = I_{EQ}R_E + V_{EB}(on) + \frac{I_{EQ}}{1+\beta}\cdot R_{TH} + V_{TH}$$

$$10 = I_{EQ}(1) + 0.7 + I_{EQ}\left(\frac{8.57}{101}\right) + 4.29$$

Then $I_{EQ} = \dfrac{10 - 0.7 - 4.29}{1 + \dfrac{8.57}{101}} = \dfrac{5.01}{1.085} \Rightarrow I_{EQ} = 4.62 \text{ mA}$

$$V_B = \frac{I_{EQ}}{1+\beta}\cdot R_{TH} + V_{TH} = \left(\frac{4.62}{101}\right)(8.57) + 4.29 \Rightarrow V_B = 4.68 \text{ V}$$

5.52

(a)

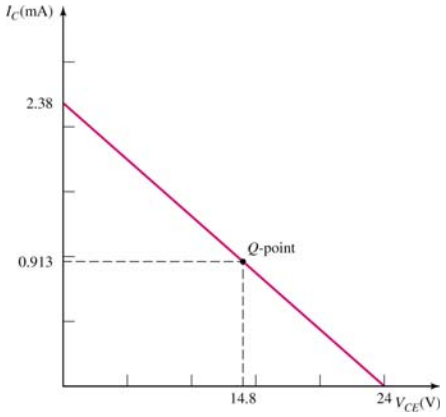
$$R_{TH} = 42 \| 58 = 24.36\,\text{k}\Omega$$

$$V_{TH} = \left(\frac{42}{100}\right)(24) = 10.08 \text{ V}$$

$$I_{BQ} = \frac{10.08 - 0.7}{24.36 + (126)(10)} = \frac{9.38}{1284.36} \Rightarrow I_{BQ} = 7.30\,\mu\text{A}$$

$$I_{CQ} = 0.913 \text{ mA}, \quad I_{EQ} = 0.9202 \text{ mA}$$

$$V_{CEQ} = 14.8 \text{ V}$$



(b)

$R_1 + 5\% = 60.9,\ R_2 + 5\% = 44.1 \qquad R_{TH} = 25.58\ \text{K} \qquad V_{TH} = 10.08$

$$I_{BQ} = \frac{10.08 - 0.7}{25.58 + 126(10)} = \frac{9.38}{1285.58} \Rightarrow 7.30\ \mu\text{A}$$

$I_{CQ} = 0.912\ \text{mA} \qquad I_{EQ} = 0.919$

$V_{CEQ} = 14.81$

$R_1 + 5\% = 60.9,\ R_2 - 5\% = 39.90 \qquad R_{TH} = 24.11\ \text{K} \qquad V_{TH} = 9.50$

$$I_{BQ} = \frac{9.50 - 0.7}{24.11 + (126)(10)} = \frac{8.8}{1284.11} = 6.85\ \mu\text{A}$$

$I_{CQ} = 0.857\ \text{mA} \qquad I_{EQ} = 0.8635\ \text{mA}$

$V_{CEQ} = 15.37\ \text{V}$

$R_1 - 5\% = 55.1\ \text{K} \qquad R_2 + 5\% = 44.1\ \text{K} \qquad R_{TH} = 24.50\ \text{K} \qquad V_{TH} = 10.67\ \text{V}$

$$I_{BQ} = \frac{10.67 - 0.7}{24.50 + 1260} = \frac{9.97}{1284.5} = 7.76\ \mu\text{A}$$

$I_{CQ} = 0.970\ \text{mA} \qquad I_{EQ} = 0.978\ \text{mA}$

$V_{CEQ} = 14.22\ \text{V}$

$R_1 - 5\% = 55.1\ \text{K} \qquad R_2 - 5\% = 39.90 \qquad R_{TH} = 23.14\ \text{K} \qquad V_{TH} = 10.08$

$$I_{BQ} = \frac{10.08 - 0.7}{23.14 + 1260} = \frac{9.38}{1283.14} = 7.31\ \mu\text{A}$$

$I_{CQ} = 0.914\ \text{mA} \qquad I_{EQ} = 0.9211\ \text{mA}$

$V_{CEQ} = 14.79\ \text{V}$

So we have $0.857 \le I_{CQ} \le 0.970\ \text{mA}$

$14.22 \le V_{CEQ} \le 15.37\ \text{V}$

5.53

(a) $R_{TH} = R_1 \| R_2 = 96 \| 24 = 19.2\ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{24}{24 + 96}\right)(9) = 1.80\ \text{V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1+\beta)R_E} = \frac{1.80 - 0.7}{19.2 + (81)(1)} \Rightarrow I_{BQ} = 10.98\ \mu\text{A}$$

$$I_{CQ} = \beta I_{BQ} = (80)(0.01098) = 0.8782 \text{ mA}, \quad I_{EQ} = (81)(0.01098) = 0.8892 \text{ mA}$$
$$V_{CEQ} = 9 - (0.8782)(5.25) - (0.8892)(1) = 3.50 \text{ V}$$

(b) $$I_{BQ} = \frac{1.80 - 0.7}{19.2 + (121)(1)} \Rightarrow I_{BQ} = 7.846\,\mu\text{A}$$
$$I_{CQ} = (120)(0.007846) = 0.9415 \text{ mA}, \quad I_{EQ} = (121)(0.007846) = 0.9494 \text{ mA}$$
$$V_{CEQ} = 9 - (0.9415)(5.25) - (0.9494)(1) = 3.108 \text{ V}$$

For I_{CQ}: $\left(\frac{0.9415 - 0.8782}{0.8782}\right) \times 100\% = 7.21\%$

For V_{CEQ}: $\left(\frac{3.108 - 3.500}{3.500}\right) \times 100\% = -11.2\%$

5.54

(a)

$$I_{CQ} \cong I_{EQ} = 0.4\ mA$$
$$R_C = \frac{3}{0.4} \Rightarrow R_C = 7.5\ k\Omega; \quad R_E = \frac{3}{0.4} \Rightarrow R_E = 7.5\ k\Omega$$
$$R_1 + R_2 \cong \frac{9}{(0.2)(0.4)} = 112.5\ k\Omega$$
$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta) I_{BQ} R_E$$
$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(112.5 - R_2)R_2}{112.5}, \ I_{BQ} = \frac{0.4}{100} = 0.004\ mA$$
$$R_2\left(\frac{9}{112.5}\right) = (0.004)\left[\frac{(112.5 - R_2)R_2}{112.5}\right] + 0.7 + (101)(0.004)(7.5)$$

We obtain $R_2(0.08) = 0.004R_2 - 3.56 \times 10^{-5} R_2^2 + 3.73$

From this quadratic, we find $R_2 = 48\ k\Omega \Rightarrow \quad R_1 = 64.5\ k\Omega$

(b) Standard resistor values:

Set $R_E = R_C = 7.5\,\text{k}\Omega$ and $R_1 = 62\,\text{k}\Omega$, $R_2 = 47\,\text{k}\Omega$

Now $R_{TH} = R_1 \| R_2 = 62 \| 47 = 26.7\,\text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = \left(\frac{47}{47 + 62}\right)(9) = 3.88\ V$$
$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta) I_{BQ} R_E$$
$$\text{So } I_{BQ} = \frac{3.88 - 0.7}{26.7 + (101)(7.5)} = 0.00406\ mA$$
$$\text{Then } I_{CQ} = 0.406\ mA$$
$$V_{RC} = V_{RE} = (0.406)(7.5) = 3.05\ V$$

5.55

(a)

$$R_{TH} = R_1 \| R_2 = 12 \| 2 = 1.714\ \text{k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5 = \left(\frac{2}{14}\right)(10) - 5 \Rightarrow V_{TH} = -3.571\ \text{V}$$

(b)

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(\text{on}) + (1+\beta) I_{BQ} R_E - 5$$

$$-3.57 = I_{BQ}(1.714) + 0.7 + (101) I_{BQ}(0.5) - 5$$

$$I_{BQ} = \frac{5 - 0.7 - 3.571}{1.714 + (101)(0.5)} = \frac{0.729}{52.21} \Rightarrow 13.96\ \mu\text{A}$$

$$I_{CQ} = 1.396\ \text{mA},\ I_{EQ} = 1.410\ \text{mA}$$

$$V_{CEQ} = 10 - (1.396)(5) - (1.41)(0.5) \Rightarrow V_{CEQ} = 2.32\ \text{V}$$

(d)

$$R_E = 0.5 + 5\% = 0.525\ \text{K} \qquad R_C = 5 + 5\% = 5.25\ \text{K}$$

$$I_{BQ} = \frac{0.729}{1.714 + (101)(0.525)} \Rightarrow 13.32\ \mu\text{A}$$

$$\underline{I_{CQ} = 1.332\ \text{mA}} \qquad I_{EQ} = 1.345\ \text{mA}$$

$$V_{CEQ} = 10 - (1.332)(5.25) - (1.345)(0.525)$$

$$= 10 - 6.993 - 0.7061 \Rightarrow \underline{V_{CEQ} = 2.30\ \text{V}}$$

$$R_E = 0.5 + 5\% = 0.525\ \text{K} \qquad R_C = 5 - 5\% = 4.75\ \text{K}$$

$$\underline{I_{CQ} = 1.332\ \text{mA}} \qquad I_{EQ} = 1.345\ \text{mA}$$

$$V_{CEQ} = 10 - (1.332)(4.75) - (1.345)(0.525)$$

$$= 10 - 6.327 - 0.7061 \Rightarrow \underline{V_{CEQ} = 2.97\ \text{V}}$$

$$R_E = 0.5 - 5\% = 0.475\ \text{K} \qquad R_C = 5 + 5\% = 5.25\ \text{K}$$

$$I_{BQ} = \frac{0.729}{1.714 + (101)(0.475)} \Rightarrow 14.67\ \mu\text{A}$$

$$\underline{I_{CQ} = 1.467\ \text{mA}} \qquad I_{EQ} = 1.482\ \text{mA}$$

$$V_{CEQ} = 10 - (1.467)(5.25) - (1.482)(0.475)$$

$$= 10 - 7.70175 - 0.70395 \Rightarrow \underline{V_{CEQ} = 1.59\ \text{V}}$$

$$R_E = 0.5 - 5\% = 0.475\ \text{K} \qquad R_C = 5 - 5\% = 4.75\ \text{K}$$

$$\underline{I_{CQ} = 1.467\ \text{mA}} \qquad I_{EQ} = 1.482\ \text{mA}$$

$$V_{CEQ} = 10 - (1.467)(4.75) - (1.482)(0.475)$$

$$= 10 - 6.96825 - 0.70395 \Rightarrow \underline{V_{CEQ} = 2.33\ \text{V}}$$

5.56

(a) $R_{TH} = R_1 \| R_2 = 40 \| 40 = 20\ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V^+ = \left(\frac{40}{40 + 40}\right)(2.5) = 1.25\ \text{V}$$

$$V^+ = I_{EQ} R_E + V_{EB}(on) + I_{BQ} R_{TH} + V_{TH}$$

So $I_{BQ} = \dfrac{V^+ - V_{EB}(on) - V_{TH}}{R_{TH} + (1+\beta)R_E} = \dfrac{2.5 - 0.7 - 1.25}{20 + (91)(0.7)} \Rightarrow I_{BQ} = 6.57\ \mu\text{A}$

$I_{CQ} = 0.5914\ \text{mA}, \quad I_{EQ} = 0.5980\ \text{mA}$

$V_{ECQ} = 2.5 - (0.5914)(1.6) - (0.5980)(0.7) = 1.135\ \text{V}$

(b) $I_{BQ} = \dfrac{2.5 - 0.7 - 1.25}{20 + (151)(0.7)} \Rightarrow I_{BQ} = 4.375\ \mu\text{A}$

$I_{CQ} = 0.6563\ \text{mA}, \quad I_{EQ} = 0.6607\ \text{mA}$

$V_{ECQ} = 2.5 - (0.6563)(1.6) - (0.6607)(0.7) = 0.9874\ \text{V}$

For I_{CQ}: $\left(\dfrac{0.6563 - 0.5914}{0.5914}\right) \times 100\% = 10.97\%$

For V_{ECQ}: $\left(\dfrac{0.9874 - 1.135}{1.135}\right) \times 100\% = -13.0\%$

5.57

(a)

$R_{TH} = 36 \| 68 = 23.5\ \text{k}\Omega$

$V_{TH} = \left(\dfrac{36}{36 + 68}\right)(10) = 3.46\ \text{V}$

$I_{BQ} = \dfrac{3.46 - 0.7}{23.5 + (51)(30)} = 0.00178\ \text{mA}$

$I_{CQ} = 0.0888\ \text{mA}, \quad I_{EQ} = 0.0906\ \text{mA}$

$V_{CEQ} = 10 - (0.0888)(42) - (0.0906)(30) \Rightarrow V_{CEQ} = 3.55\ \text{V}$

(b)

$R_1 = 22.7, R_2 = 12\ \text{K}, \ R_C = 14\ \text{K}, \ R_E = 10\ \text{K}$

$R_{TH} = 7.85\ \text{k} \qquad V_{TH} = 3.46$

$I_{BQ} = \dfrac{3.46 - 0.7}{7.85 + (51)(10)} = 0.00533\ \text{mA}$

$I_{CQ} = 0.266\ \text{mA} \qquad I_{EQ} = 0.272\ \text{mA}$

$V_{CE} = 10 - (0.266)(14) - (0.272)(10)$

$\underline{V_{CE} = 3.56\ \text{V}}$

5.58

(a)

$$R_{TH} = 36\|68 = 23.5\ \text{k}\Omega\ ;\ V_{TH} = \left(\frac{68}{36+68}\right)(10) - 5 = 1.54\ \text{V}$$

$$5 = (51)I_{BQ}(30) + 0.7 + I_{BQ}(23.5) + 1.54$$

$$I_{BQ} = \frac{2.76}{1553.5} = 1.78\ \mu\text{A} \Rightarrow \quad I_{CQ} = 0.0888\ \text{mA}$$

$$I_{EQ} = 0.0906\ \text{mA}$$

$$V_{ECQ} = 10 - (0.0906)(30) - (0.0888)(42)$$

$$= 10 - 2.718 - 3.7296 \Rightarrow \quad \underline{V_{ECQ} = 3.55\ \text{V}}$$

(b)

$$R_{TH} = 12\|22.7 = 7.85\ \text{k}\Omega$$

$$V_{TH} = 1.54\ \text{V},\quad R_E = 10\ \text{k}\Omega,\quad R_C = 14\ \text{k}\Omega$$

$$5 = (51)I_{BQ}(10) + 0.7 + I_{BQ}(7.85) + 1.54$$

$$I_{BQ} = \frac{2.76}{517.85} \Rightarrow 5.33\ \mu\text{A},\quad I_{CQ} = 0.266\ \text{mA},\quad I_{EQ} = 0.272\ \text{mA}$$

$$V_{ECQ} = 10 - (0.272)(10) - (0.266)(14) = 3.56\ \text{V}$$

5.59

(a)

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(0.5) = 5.05\ k\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.8}{100} = 0.008\ mA$$

Then $\frac{1}{R_1}(5.05)(10) = (0.008)(5.05) + 0.7 + (101)(0.008)(0.5)$

or $\underline{R_1 = 44.1\ k\Omega},\quad \frac{44.1R_2}{44.1+R_2} = 5.05 \Rightarrow \quad \underline{R_2 = 5.70\ k\Omega}$

Now $I_{EQ} = \left(\frac{101}{100}\right)(0.8) = 0.808\ mA$

$$V_{CC} = I_{CQ}R_C + V_{CEQ} + I_{EQ}R_E$$

$$10 = (0.8)R_C + 5 + (0.808)(0.5)$$

$$\underline{R_C = 5.75\ k\Omega}$$

(b) For $75 \le \beta \le 150$

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(V_{CC}) = \left(\frac{5.7}{5.7+44.1}\right)(10) = 1.145\ V$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$$

$$\beta = 75,\ I_{BQ} = \frac{1.145 - 0.7}{5.05 + (76)(0.5)} = 0.0103\ mA$$

For

Then $I_{CQ} = (75)(0.0103) = 0.775\ mA$

$$\beta = 150,\ I_{BQ} = \frac{1.145 - 0.7}{5.05 + (151)(0.5)} = 0.00552\ mA$$

For

Then $I_{CQ} = 0.829\ mA$

$$\%\ \text{Change} = \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.829 - 0.775}{0.80} \times 100\% \Rightarrow \underline{\%\ \text{Change} = 6.75\%}$$

(c) For $R_E = 1\ k\Omega$

$$R_{TH} = (0.1)(101)(1) = 10.1\ k\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(10.1)(10) = (0.008)(10.1) + 0.7 + (101)(0.008)(1)$$

which yields $\underline{R_1 = 63.6\ k\Omega}$

And $$\frac{63.6R_2}{63.6 + R_2} = 10.1 \Rightarrow \underline{R_2 = 12.0\ k\Omega}$$

Now $$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = \left(\frac{12}{12 + 63.6}\right)(10) = 1.587\ V$$

$$\beta = 75,\ I_{BQ} = \frac{1.587 - 0.7}{10.1 + (76)(1)} = 0.0103\ mA$$

For

So $I_{CQ} = 0.773\ mA$

$$\beta = 150,\ I_{BQ} = \frac{1.587 - 0.7}{10.1 + (151)(1)} = 0.00551\ mA$$

For

Then $I_{CQ} = 0.826\ mA$

$$\%\ \text{Change} = \frac{\Delta I_{CQ}}{I_{CQ}} = \frac{0.826 - 0.773}{0.8} \times 100\% \Rightarrow \underline{\%\ \text{Change} = 6.63\%}$$

5.60

$$V_{CC} \cong I_{CQ}(R_C + R_E) + V_{CEQ}$$

$$10 = (0.8)(R_C + R_E) + 5 \Rightarrow R_C + R_E = 6.25\ k\Omega$$

Let $R_E = 0.875\ k\Omega$

Then, for bias stable $R_{TH} = (0.1)(121)(0.875) = 10.6\ k\Omega$

$$I_{BQ} = \frac{0.8}{120} = 0.00667\ mA$$

$$\frac{1}{R_1}(10.6)(10) = (0.00667)(10.6) + 0.7 + (121)(0.00667)(0.875)$$

So $R_1 = 71.8\ k\Omega$ and $$\frac{71.8R_2}{71.8 + R_2} = 10.6 \Rightarrow R_2 = 12.4\ k\Omega$$

Then $I_R \cong \dfrac{10}{71.8+12.4} = 0.119\ mA$

This is close to the design specification.

5.61

$$I_{CQ} \approx I_{EQ} \Rightarrow V_{CEQ} = V_{CC} - I_{CQ}\left(R_C + R_E\right)$$

$$6 = 12 - I_{CQ}\left(2+0.2\right)$$

$$\underline{I_{CQ} = 2.73\ \text{mA}}, \quad \underline{I_{BQ} = 0.0218\ \text{mA}}$$

$$V_{CEQ} = 6\ \text{V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 6$$

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)(12) - 6, \quad R_{TH} = R_1 \| R_2$$

Bias stable $\Rightarrow R_{TH} = (0.1)(1+\beta)R_E = (0.1)(126)(0.2) = 2.52\ \text{k}\Omega$

$$V_{TH} = \left(\frac{1}{R_1}\right)(R_{TH})(12) - 6$$

$$\frac{1}{R_1}(2.52)(12) - 6 = (0.0218)(2.52) + 0.7 + (126)(0.0218)(0.2) - 6$$

$$\frac{1}{R_1}(30.24) = 0.7549 + 0.5494$$

$$\underline{R_1 = 23.2\ \text{k}\Omega}, \quad \frac{23.2R_2}{23.2+R_2} = 2.52$$

$$\underline{R_2 = 2.83\ \text{k}\Omega}$$

5.62

(a) $R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(0.2) = 2.42\ \text{k}\Omega$

$$V_{CEQ} = 6 - I_{CQ}\left[R_C + \left(\frac{1+\beta}{\beta}\right)R_E\right] = 6 - I_{CQ}(2.202)$$

$$2.8 = 6 - I_{CQ}(2.202) \Rightarrow I_{CQ} = 1.453\ \text{mA}$$

Then $I_{EQ} = 1.465\ \text{mA}, \quad I_{BQ} = 12.11\ \mu\text{A}$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E - 3 = (0.01211)(2.42) + 0.7 + (1.465)(0.2) - 3$$

Then $V_{TH} = -1.978\ \text{V} = \dfrac{1}{R_1}\cdot R_{TH}(6) - 3 = \dfrac{1}{R_1}(2.42)(6) - 3$

Which yields $R_1 = 14.2\ \text{k}\Omega$ and $R_2 = 2.92\ \text{k}\Omega$

(b) For $R_1 = (1.05)(14.2) = 14.91\ \text{k}\Omega$

$R_2 = (0.95)(2.92) = 2.774\ \text{k}\Omega$

$R_{TH} = R_1 \| R_2 = 2.34\ \text{k}\Omega$

$$V_{TH} = \left(\frac{2.774}{2.774+14.91}\right)(6) - 3 = -2.059\ \text{V}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 3$$

So $I_{BQ} = \dfrac{3-2.059-0.7}{2.34+(121)(0.2)} \Rightarrow I_{BQ} = 9.08\,\mu\text{A}$

$I_{CQ} = 1.090\text{ mA},\quad I_{EQ} = 1.099\text{ mA}$

$V_{CEQ} = 6-(1.09)(2)-(1.099)(0.2) = 3.60\text{ V}$

For $R_1 = (0.95)(14.2) = 13.49\text{ k}\Omega$

$R_2 = (1.05)(2.92) = 3.066\text{ k}\Omega$

$R_{TH} = R_1 \| R_2 = 2.50\text{ k}\Omega$

$V_{TH} = \left(\dfrac{3.066}{3.066+13.49}\right)(6)-3 = -1.889\text{ V}$

$I_{BQ} = \dfrac{3-1.889-0.7}{2.50+(121)(0.2)} \Rightarrow I_{BQ} = 15.39\,\mu\text{A}$

$I_{CQ} = 1.847\text{ mA},\quad I_{EQ} = 1.863\text{ mA}$

$V_{CEQ} = 6-(1.847)(2)-(1.863)(0.2) = 1.933\text{ V}$

So $1.09 \le I_{CQ} \le 1.847\text{ mA}$

$1.933 \le V_{CEQ} \le 3.60\text{ V}$

5.63

$V_{CEQ} \cong V_{CC} - I_{CQ}(R_C + R_E)$

$5 = 12-3(R_C+R_E) \Rightarrow R_C + R_E = 2.33\ k\Omega$

Let $\underline{R_E = 0.333\ k\Omega}$ and $\underline{R_C = 2\ k\Omega}$

Nominal value of $\beta = 100$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(0.333) = 3.36\ k\Omega$

$I_{BQ} = \dfrac{3}{100} = 0.03\,mA$

$V_{TH} = \dfrac{1}{R_1}\cdot R_{TH}\cdot(12)-6 = \dfrac{1}{R_1}(3.36)(12)-6$

Then $V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 6$

$\dfrac{1}{R_1}(3.36)(12)-6 = (0.03)(3.36)+0.7+(101)(0.03)(0.333)-6$

which yields $\underline{R_1 = 22.3\ k\Omega}$ and $\underline{R_2 = 3.96\ k\Omega}$

Now $V_{TH} = \left(\dfrac{R_2}{R_1+R_2}\right)(12)-6 = \left(\dfrac{3.96}{3.96+22.3}\right)(12)-6$ or $V_{TH} = -4.19\ V$

For $\beta = 75,\ V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 6$

$I_{BQ} = \dfrac{V_{TH}+6-0.7}{R_{TH}+(1+\beta)R_E} = \dfrac{-4.19+6-0.7}{3.36+(76)(0.333)} = 0.0387\ mA \Rightarrow \underline{I_C = 2.90\ mA}$

For $\beta = 150,\ I_{BQ} = \dfrac{-4.19+6-0.7}{3.36+(151)(0.333)} = 0.0207\ mA$

Then $\underline{I_C = 3.10\ mA}$

Specifications are met.

5.64

(a) $V^+ = V_{ECQ} + I_{CQ}\left[R_C + \left(\frac{1+\beta}{\beta}\right)R_E\right]$

$3.3 = 1.5 + I_{CQ}(3.022) \Rightarrow I_{CQ} = 0.5956$ mA

$I_{BQ} = 6.618\ \mu$ A, $I_{EQ} = 0.6022$ mA

$V^+ = I_{EQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$

$3.3 = (0.6022)(2) + 0.7 + (0.006618)(2.4) + V_{TH}$

So $V_{TH} = 1.380\text{ V} = \frac{1}{R_1}\cdot R_{TH}\cdot V^+ = \frac{1}{R_1}(2.4)(3.3)$

Which yields $R_1 = 5.74\text{ k}\Omega$ and $R_2 = 4.12\text{ k}\Omega$

(b) $I_{BQ} = \frac{V^+ - V_{EB}(on) - V_{TH}}{R_{TH} + (1+\beta)R_E} = \frac{3.3 - 0.7 - 1.38}{2.4 + (131)(2)} \Rightarrow I_{BQ} = 4.61\ \mu$ A

$I_{CQ} = 0.60$ mA, $I_{EQ} = 0.6045$ mA

$V_{ECQ} = 3.3 - (0.60)(1) - (0.6045)(2) = 1.49$ V

5.65

$I_{CQ} = 4.8\text{ mA} \rightarrow I_{EQ} = 4.84\text{ mA}$

$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$

$6 = 18 - (4.8)(2) - (4.84)R_E \Rightarrow \underline{R_E = 0.496\text{ k}\Omega}$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(0.496) = 6.0\text{ k}\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E$

$I_{BQ} = 0.040$ mA

$V_{TH} = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = \frac{1}{R_1}(6.0)(18)$

$\frac{1}{R_1}(6.0)(18) = (0.04)(6.0) + 0.70 + (121)(0.04)(0.496)$

$\frac{1}{R_1}(108) = 3.34$

$\underline{R_1 = 32.3\text{ k}\Omega}$, $\frac{32.3R_2}{32.3 + R_2} = 6.0$

$\underline{R_2 = 7.37\text{ k}\Omega}$

5.66

For $I_{EQ} \cong I_{CQ}$, $R_C + R_E = \frac{V_{CC} - V_{CEQ}}{I_{CQ}} = \frac{2.5 - 1.6}{0.2} = 4.5\ \text{k}\Omega$

So $R_E = 0.5\text{ k}\Omega$

For $\beta = 100$

$R_{TH} = (0.1)(101)(0.5) = 5.05\text{ k}\Omega$

$I_{BQ} = \frac{0.2}{100} \Rightarrow I_{BQ} = 2\ \mu$ A, $I_{EQ} = 0.202$ mA

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E = (0.002)(5.05) + 0.7 + (0.202)(0.5) = 0.8111$ V

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} \Rightarrow 0.8111 = \frac{1}{R_1}(5.05)(2.5)$$

So $R_1 = 15.6\,\text{k}\Omega$ and $R_2 = 7.47\,\text{k}\Omega$

For $\beta = 80$,

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1+\beta)R_E} = \frac{0.8111 - 0.7}{5.05 + (81)(0.5)} \Rightarrow I_{BQ} = 2.439\,\mu\text{A}, \quad I_{CQ} = 0.1951\text{ mA}$$

For $\beta = 120$,

$$I_{BQ} = \frac{0.8111 - 0.7}{5.05 + (121)(0.5)} \Rightarrow I_{BQ} = 1.695\,\mu\text{A}, \quad I_{CQ} = 0.2034\text{ mA}$$

Design is valid

5.67

$$I_{CQ} = 1\text{ mA} \rightarrow I_{EQ} = 1.017\text{ mA}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E$$

$$5 = 15 - (1)(5) - (1.017)R_E \Rightarrow \underline{R_E = 4.92\text{ k}\Omega}$$

Bias stable: $R_{TH} = (0.1)(1+\beta)R_E = (0.1)(61)(4.92) = 30.0\text{ k}\Omega$

$$I_{BQ} = \frac{1}{60} = 0.0167\text{ mA}$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + I_{EQ}R_E$$

$$\frac{1}{R_1}(30.0)(15) = (0.0167)(30.0) + 0.70 + (1.017)(4.92)$$

$$\frac{1}{R_1}(448.5) = 6.197$$

$$\underline{R_1 = 72.5\text{ k}\Omega}, \quad \frac{72.5R_2}{72.5 + R_2} = 30.0$$

$$\underline{R_2 = 51.2\text{ k}\Omega}$$

Check: For $\beta = 45$

$$V_{TH} = \left(\frac{51.2}{51.2 + 72.5}\right)(15) = 6.21\text{V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{6.21 - 0.7}{30 + (46)(4.92)} = 0.0215\text{ mA}$$

$$\underline{I_{CQ} = 0.967\text{ mA}}, \quad \frac{\Delta I_C}{I_C} = 3.27\%$$

Check: For $\beta = 75$

$$I_{BQ} = \frac{6.21 - 0.7}{30.0 + (76)(4.92)} = 0.0136\text{ mA}$$

$$\underline{I_{CQ} = 1.023\text{ mA}}, \quad \frac{\Delta I_C}{I_C} = 2.31\%$$

Design criterion is satisfied.

5.68

(a)

$$V_{CC} \cong I_{CQ}\left(R_C + R_E\right) + V_{CEQ}$$
$$3 = (0.1)(5R_E + R_E) + 1.4 \Rightarrow \underline{R_E = 2.67\ k\Omega}$$
$$\underline{R_C = 13.3\ k\Omega}, \quad I_{BQ} = \frac{100}{120} = 0.833\ \mu A$$
$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(2.67) = 32.3\,k\Omega$$
$$V_{TH} = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = \frac{1}{R_1}(32.3)(3)$$
$$= I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$$
$$= (0.000833)(32.3) + 0.7 + (121)(0.000833)(2.67)$$

which gives $\underline{R_1 = 97.3\ k\Omega}$, and $\underline{R_2 = 48.4\ k\Omega}$

(b)

$$I_R \cong \frac{3}{R_1 + R_2} = \frac{3}{97.3 + 48.4} \Rightarrow 20.6\ \mu A$$
$$I_{CQ} = 100\ \mu A$$
$$P = (I_{CQ} + I_R)V_{CC} = (100 + 20.6)(3)$$

or $\underline{P = 362\ \mu W}$

5.69

$$I_E = \frac{5 - V_E}{R_E} = \frac{5}{3} = 1.67\ \text{mA}$$
$$R_{TH} = R_1 \,||\, R_2 = (0.1)(1+\beta)R_E = (0.1)(101)(3) = 30.3\ \text{k}\Omega$$
$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(4) - 2 = \frac{1}{R_1}\cdot R_{TH}\cdot(4) - 2$$
$$I_{BQ} = \frac{I_{EQ}}{1+\beta} = 0.0165\ \text{mA}$$
$$5 = I_{EQ}R_E + V_{EB}(\text{on}) + I_B R_{TH} + V_{TH}$$
$$5 = (1.67)(3) + 0.7 + (0.0165)(30.3) + \frac{1}{R_1}(30.3)(4) - 2$$
$$0.80 = \frac{1}{R_1}(30.3)(4) \Rightarrow \underline{R_1 = 152\ \text{k}\Omega}$$
$$\frac{152R_2}{152 + R_2} = 30.3 \Rightarrow \underline{R_2 = 37.8\ \text{k}\Omega}$$

5.70

a. $R_{TH} = R_1 \| R_2 = 10 \| 20 = 6.67\ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5 = \left(\frac{20}{20+10}\right)(10) - 5 = 1.67\ \text{V}$$

b.

$$10 = (1+\beta) I_{BQ} R_E + V_{EB}(\text{on}) + I_{BQ} R_{TH} + V_{TH}$$

$$I_{BQ} = \frac{10 - 0.7 - 1.67}{6.67 + (61)(2)} = \frac{7.63}{128.7} \Rightarrow \underline{I_{BQ} = 0.0593\ \text{mA}}$$

$\underline{I_{CQ} = 3.56\ \text{mA}}$, $I_{EQ} = 3.62\ \text{mA}$

$$V_E = 10 - I_{EQ} R_E = 10 - (3.62)(2)$$

$$\underline{V_E = 2.76\ \text{V}}$$

$$V_C = I_{CQ} R_C - 10 = (3.56)(2.2) - 10$$

$$\underline{V_C = -2.17\ \text{V}}$$

5.71

$$V^+ - V^- \cong I_{CQ}(R_C + R_E) + V_{ECQ}$$

$$20 = (0.5)(R_C + R_E) + 8 \Rightarrow (R_C + R_E) = 24\ k\Omega$$

Let $\underline{R_E = 10\ k\Omega}$ then $\underline{R_C = 14\ k\Omega}$

Let $\beta = 60$ from previous problem.

$$R_{TH} = (0.1)(1+\beta) R_E = (0.1)(61)(10)$$

Or $R_{TH} = 61\ k\Omega$

$$I_{BQ} = \frac{0.5}{60} = 0.00833\ mA$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5 = \frac{1}{R_1} \cdot R_{TH} \cdot 10 - 5$$

Now $10 = (1+\beta) I_{BQ} R_E + V_{EB}(on) + I_{BQ} R_{TH} + V_{TH}$

$$10 = (61)(0.00833)(10) + 0.7 + (0.00833)(61) + \frac{1}{R_1}(61)(10) - 5$$

Then $\underline{R_1 = 70.0\ k\Omega}$ and $\underline{R_2 = 474\ k\Omega}$

$$I_R \cong \frac{10}{R_1 + R_2} = \frac{10}{70 + 474} \Rightarrow 18.4\ \mu A$$

So the $40\ \mu A$ current limit is met.

5.72

(a) $$V_{ECQ} = (V^+ - V^-) - I_{CQ}\left[R_C + \left(\frac{1+\beta}{\beta}\right) \cdot R_E\right]$$

$$2.7 = 5 - (0.15)\left[R_C + \left(\frac{81}{80}\right)(2)\right] \Rightarrow R_C = 13.3\ \text{k}\Omega$$

$$I_{EQ} = 0.1519\ \text{mA}, \quad I_{BQ} = 1.875\ \mu\ \text{A}$$

$$R_{TH} = (0.1)(81)(2) = 16.2\ \text{k}\Omega$$

$$V^+ = I_{EQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$$
$$2.5 = (0.1519)(2) + 0.7 + (0.001875)(16.2) + V_{TH} \quad \Rightarrow V_{TH} = 1.466\text{ V}$$
$$V_{TH} = \frac{1}{R_1}\cdot R_{TH}(5) - 2.5 \Rightarrow 1.466 = \frac{1}{R_1}(16.2)(5) - 2.5$$

So $R_1 = 20.4\,\text{k}\Omega$ and $R_2 = 78.7\,\text{k}\Omega$

(b) For $\beta = 60$

$$I_{BQ} = \frac{V^+ - V_{TH} - V_{EB}(on)}{R_{TH} + (1+\beta)R_E} = \frac{2.5 - 1.466 - 0.7}{16.2 + (61)(2)} \Rightarrow I_{BQ} = 2.417\,\mu\text{A}$$
$$I_{CQ} = 0.145\text{ mA}, \quad I_{EQ} = 0.1474\text{ mA}$$
$$V_{ECQ} = 5 - (0.145)(13.3) - (0.1474)(2) = 2.777\text{ V}$$

For $\beta = 100$

$$I_{BQ} = \frac{2.5 - 1.466 - 0.7}{16.2 + (101)(2)} \Rightarrow I_{BQ} = 1.531\,\mu\text{A}$$
$$I_{CQ} = 0.1531\text{ mA}, \quad I_{EQ} = 0.1546\text{ mA}$$
$$V_{ECQ} = 5 - (0.1531)(13.3) - (0.1546)(2) = 2.655\text{ V}$$

For I_{CQ}: $\left(\frac{0.1531 - 0.145}{0.15}\right)\times 100\% = 5.4\%$

For V_{ECQ}: $\left(\frac{2.655 - 2.777}{2.70}\right)\times 100\% = -4.52\%$

5.73

a.

$$R_{TH} = 500 \| 500 \| 70 = 250 \| 70 = 54.7\,\text{k}\Omega$$
$$\frac{5 - V_{TH}}{500} + \frac{3 - V_{TH}}{500} = \frac{V_{TH} - (-5)}{70}$$
$$\frac{5}{500} + \frac{3}{500} - \frac{5}{70} = V_{TH}\left(\frac{1}{500} + \frac{1}{500} + \frac{1}{70}\right) \Rightarrow -0.0554 = V_{TH}(0.0183)$$
$$V_{TH} = -3.03\text{ V}$$

b.

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on}) - (-5)}{R_{TH} + (1+\beta)R_E}$$
$$= \frac{-3.03 - 0.7 + 5}{54.7 + (101)(5)}$$
$$I_{BQ} = 0.00227\text{ mA}$$
$$\underline{I_{CQ} = 0.227\text{ mA}}, \quad I_{EQ} = 0.229$$
$$V_{CEQ} = 20 - (0.227)(50) - (0.229)(5)$$
$$\underline{V_{CEQ} = 7.51\text{ V}}$$

5.74

$$R_E \cong \frac{V_E}{I_{CQ}} = \frac{1.5}{0.8} = 1.87\ \text{k}\Omega$$

$$10 = (0.8)R_C + 4 + 1.5 \Rightarrow R_C = 5.63\ \text{k}\Omega$$

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(1.87) = 22.6\ \text{k}\Omega$$

$$I_{BQ} = \frac{0.8}{120} \Rightarrow I_{BQ} = 6.67\ \mu\text{A}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC}$$

$$(0.00667)(22.6) + 0.7 + (0.807)(1.87) = \frac{1}{R_1}(22.6)(10)$$

which yields $R_1 = 95.8\ \text{k}\Omega$ and $R_2 = 29.6\ \text{k}\Omega$

5.75

$$I_{CQ} = 50\ \mu\text{A},\ I_{BQ} = 0.625\ \mu\text{A},\ I_{EQ} = 50.6\ \mu\text{A}$$

(a)

$$R_E = \frac{1}{0.0506} = 19.8\ \text{K}$$

$$5 = (0.050)R_C + 5 + (0.0506)(19.8) - 5$$

$$R_C = 80\ \text{K}$$

$R_{TH} = R_1 \| R_2$, Design bias stable circuit

$$R_{TH} = (0.1)(51)(19.8) = 101\ \text{k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5 = \frac{1}{R_1}\cdot R_{TH}\cdot(10) - 5$$

So $\dfrac{1}{R_1}(101)(10) - 5 = I_{BQ}(101) + 0.7 + (0.0506)(19.8) - 5$

$$\frac{1}{R_1}(1010) = 0.0631 + 0.7 + 1$$

$$R_1 = 573\ \text{K} \qquad \frac{573\ R_2}{573 + R_2} = 101$$

$$R_2 = 123\ \text{K}$$

(b)

$$R_{TH} = 101\ \text{K},\ V_{TH} = -3.23\ \text{V}$$

$$V_{TH} = I_{BQ}R_{TH} + 0.7 + (121)(19.8)I_{BQ} - 5$$

$$1.07 = I_{BQ}(101 + 2395.8) \Rightarrow I_{BQ} = 0.429\ \mu\text{A}$$

$$I_{CQ} = 0.0514\ \text{mA},\ I_{EQ} = 0.0519\ \text{mA}$$

$$V_{CEQ} = 10 - (0.0514)(80) - (0.0519)(19.8)$$

$$= 10 - 4.11 - 1.03 \Rightarrow \underline{V_{CEQ} = 4.86\ \text{V}}$$

5.76

(a) $R_E \cong \dfrac{0.7}{0.5} = 1.4\ \text{k}\Omega$, $I_{BQ} = \dfrac{0.5}{120} \Rightarrow I_{BQ} = 4.167\ \mu\text{A}$

$V_{CEQ} = 6 - I_{CQ}R_C - 0.7$

$2.5 = 6 - (0.5)R_C - 0.7 \Rightarrow R_C = 5.6\ \text{k}\Omega$

$R_{TH} = (0.1)(121)(1.4) = 16.9\ \text{k}\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 3$

$V_{TH} = (0.004167)(16.9) + 0.7 + (121)(0.004167)(1.4) - 3 = -1.5237\ \text{V}$

$V_{TH} = \dfrac{1}{R_1} \cdot R_{TH}(6) - 3 \Rightarrow -1.5237 = \dfrac{1}{R_1}(16.9)(6) - 3$

which yields $R_1 = 68.7\ \text{k}\Omega$ and $R_2 = 22.4\ \text{k}\Omega$

(b) For standard resistor values:

Let $R_E = 1.5\ \text{k}\Omega$, $R_C = 5.6\ \text{k}\Omega$, $R_1 = 68\ \text{k}\Omega$, $R_2 = 22\ \text{k}\Omega$

$R_{TH} = R_1 \| R_2 = 68 \| 22 = 16.62\ \text{k}\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1 + R_2}\right)(6) - 3 = \left(\dfrac{22}{22+68}\right)(6) - 3 = -1.533\ \text{V}$

$I_{BQ} = \dfrac{3 + V_{TH} - V_{BE}(on)}{R_{TH} + (1+\beta)R_E} = \dfrac{3 - 1.533 - 0.7}{16.62 + (121)(1.5)} \Rightarrow I_{BQ} = 3.87\ \mu\text{A}$

$I_{CQ} = 0.4646\ \text{mA}$, $I_{EQ} = 0.4684\ \text{mA}$

$V_{CEQ} = 6 - (0.4646)(5.6) - (0.4684)(1.5) = 2.70\ \text{V}$

5.77

(a) $R_E \cong \dfrac{0.7}{0.1} = 7\ \text{k}\Omega$, $R_C + R_E = \dfrac{6-3}{0.1} = 30\ \text{k}\Omega$, $\Rightarrow R_C = 23\ \text{k}\Omega$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(111)(7) = 77.7\ \text{k}\Omega$

$I_{BQ} = \dfrac{0.1}{110} \Rightarrow I_{BQ} = 0.909\ \mu\text{A}$, $I_{EQ} = 0.1009\ \text{mA}$

$V^+ = I_{EQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$

$3 = (0.1009)(7) + 0.7 + (0.000909)(77.7) + V_{TH}$, $\Rightarrow V_{TH} = 1.523\ \text{V}$

$V_{TH} = \dfrac{1}{R_1} \cdot R_{TH}(6) - 3 \Rightarrow 1.523 = \dfrac{1}{R_1}(77.7)(6) - 3$

which yields $R_1 = 103\ \text{k}\Omega$ and $R_2 = 316\ \text{k}\Omega$

$I_{BQ} = \dfrac{V^+ - V_{EB}(on) - V_{TH}}{R_{TH} + (1+\beta)R_E} = \dfrac{3 - 0.7 - 1.523}{77.7 + (151)(7)} \Rightarrow I_{BQ} = 0.685\ \mu\text{A}$

$I_{CQ} = 0.1027\ \text{mA}$, $I_{EQ} = 0.1034\ \text{mA}$

$V_{ECQ} = 6 - (0.1027)(23) - (0.1034)(7) = 2.914\ \text{V}$

5.78

(a) $V_{ECQ} = 18 - I_{CQ}R_C - V_{RE}$

$6 = 18 - (1.2)R_C - 1.5 \Rightarrow R_C = 8.75\,\text{k}\Omega$

$I_{EQ} = \left(\frac{76}{75}\right)(1.2) = 1.216\,\text{mA}, \quad I_{BQ} = \frac{1.2}{75} \Rightarrow I_{BQ} = 16\,\mu\,\text{A}$

$R_E = \frac{1.5}{1.216} = 1.234\,\text{k}\Omega$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(76)(1.234) = 9.375\,\text{k}\Omega$

$V^+ = I_{EQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$

$9 = (1.216)(1.234) + 0.7 + (0.016)(9.375) + V_{TH} \Rightarrow V_{TH} = 6.65\ \text{V}$

$V_{TH} = \frac{1}{R_1}(R_{TH})(18) - 9 \Rightarrow 6.65 = \frac{1}{R_1}(9.375)(18) - 9$

which yields $R_1 = 10.78\,\text{k}\Omega$ and $R_2 = 71.8\,\text{k}\Omega$

Set $R_C = 9.1\,\text{k}\Omega$, $R_E = 1.2\,\text{k}\Omega$, $R_1 = 11\,\text{k}\Omega$, $R_2 = 68\,\text{k}\Omega$

$R_{TH} = R_1 \| R_2 = 11 \| 68 = 9.47\,\text{k}\Omega$

$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(18) - 9 = \left(\frac{68}{68+11}\right)(18) - 9 = 6.494\ \text{V}$

$I_{BQ} = \frac{V^+ - V_{EB}(on) - V_{TH}}{R_{TH} + (1+\beta)R_E} = \frac{9 - 0.7 - 6.494}{9.47 + (76)(1.2)} \Rightarrow I_{BQ} = 17.94\,\mu\,\text{A}$

$I_{CQ} = 1.345\,\text{mA}, \quad I_{EQ} = 1.363\,\text{mA}$

$V_{ECQ} = 18 - (1.345)(9.1) - (1.363)(1.2) = 4.12\ \text{V}$

5.79

$R_{TH} = R_1 \| R_2 = 100 \| 40 = 28.6\,\text{k}\Omega$

$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) = \left(\frac{40}{40+100}\right)(10) = 2.86\ \text{V}$

$I_{B1} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1+\beta)R_{E1}} = \frac{2.86 - 0.7}{28.6 + (121)(1)}$

$I_{B1} = 0.0144\,\text{mA}, \ I_{C1} = 1.73\,\text{mA}, \ I_{E1} = 1.75\,\text{mA}$

$\frac{10 - V_{B2}}{3} = I_{C1} + I_{B2}$

$I_{E2} = \frac{V_{B2} - V_{BE}(\text{on}) - (-10)}{5}$

$\frac{10 - V_{B2}}{3} = I_{C1} + \frac{V_{B2} - 0.7 + 10}{(121)(5)}$

$\frac{10}{3} - 1.73 - \frac{9.3}{605} = V_{B2}\left(\frac{1}{3} + \frac{1}{(121)(5)}\right)$

$1.588 = V_{B2}(0.335) \Rightarrow V_{B2} = 4.74\ \text{V}$

$$I_{E2} = \frac{4.74 - 0.7 - (-10)}{5} \Rightarrow \underline{I_{E2} = 2.808 \text{ mA}}$$

$$\underline{I_{B2} = 0.0232 \text{ mA}}$$

$$\underline{I_{C2} = 2.785 \text{ mA}}$$

$$V_{CEQ1} = 4.74 - (1.75)(1) \Rightarrow \underline{V_{CEQ1} = 2.99 \text{ V}}$$

$$V_{CEQ2} = 10 - (4.74 - 0.7) \Rightarrow \underline{V_{CEQ2} = 5.96 \text{ V}}$$

5.80

$$V_{E1} = -0.7$$

$$I_{R1} = \frac{-0.7 - (-5)}{20} = 0.215 \text{ mA}$$

$$V_{E2} = -0.7 - 0.7 = -1.4$$

$$I_{E2} = \frac{-1.4 - (-5)}{1} \Rightarrow \quad \underline{I_{E2} = 3.6 \text{ mA}}$$

$$\underline{I_{B2} = 0.0444 \text{ mA}}$$

$$\underline{I_{C2} = 3.56 \text{ mA}}$$

$$I_{E1} = I_{R1} + I_{B2} = 0.215 + 0.0444$$

$$\underline{I_{E1} = 0.259 \text{ mA}}$$

$$\underline{I_{B1} = 0.00320 \text{ mA}}$$

$$\underline{I_{C1} = 0.256 \text{ mA}}$$

5.81

$$V_{B1} = V_{RE} + V_{BE}(on) = 0.5 + 0.7 = 1.2 \text{ V}$$

$$R_3 = \frac{1.2}{0.020} = 60 \text{ k}\Omega$$

$$R_E = \frac{0.5}{0.2} = 2.5 \text{ k}\Omega$$

$$V_{B2} = V_{BE}(on) + V_{CE} + V_{RE} = 0.7 + 1.2 + 0.5 = 2.4 \text{ V}$$

$$R_2 = \frac{V_{B2} - V_{B1}}{I_{R2}} = \frac{2.4 - 1.2}{0.020} = 60 \text{ k}\Omega$$

$$R_1 = \frac{V^+ - V_{B2}}{I_{R1}} = \frac{5 - 2.4}{0.020} = 130 \text{ k}\Omega$$

$$V_{C2} = 2V_{CE} + V_{RE} = 2(1.2) + 0.5 = 2.9 \text{ V}$$

$$R_C = \frac{V^+ - V_{C2}}{I_{C2}} = \frac{5 - 2.9}{0.20} = 10.5 \text{ k}\Omega$$

5.82

$$R_{TH} = 40 \| 80 = 26.67\,\text{k}\Omega$$

$$V_{TH} = \left(\frac{40}{40+80}\right)(9) = 3\text{ V}$$

$$I_{B1} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1+\beta_n)R_{E1}} = \frac{3-0.7}{26.67 + (121)(2)} \Rightarrow I_{B1} = 8.56\,\mu\text{A}$$

$$I_{C1} = 1.027\text{ mA},\quad I_{E1} = 1.036\text{ mA}$$

$$\frac{9 - V_{C1}}{2} + I_{B2} = I_{C1}$$

$$9 = I_{E2}(0.1) + V_{EB}(on) + V_{C1} \Rightarrow I_{B2} = \frac{9 - 0.7 - V_{C1}}{(1+\beta_p)(0.1)} = \frac{8.3 - V_{C1}}{8.1}$$

So $$\frac{9 - V_{C1}}{2} + \frac{8.3 - V_{C1}}{8.1} = 1.027 \Rightarrow V_{C1} = 7.214\text{ V}$$

$$I_{B2} = \frac{8.3 - 7.214}{8.1} = 0.134\text{ mA}$$

$$I_{C2} = 10.73\text{ mA},\quad I_{E2} = 10.86\text{ mA}$$

$$V_{E1} = I_{E1}R_{E1} = (1.036)(2) = 2.072\text{ V}$$

$$V_{CE1} = V_{C1} - V_{E1} = 7.214 - 2.072 = 5.14\text{ V}$$

$$V_{EC2} = 9 - (10.86)(0.1) - (10.73)(0.2) = 5.77\text{ V}$$

5.83

$$R_{TH} = R_1 \| R_2 = 50 \| 100 = 33.3\,\text{k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5 = \left(\frac{100}{100+50}\right)(10) - 5 = 1.67\text{ V}$$

$$5 = I_{E1}R_{E1} + V_{EB}(on) + I_{B1}R_{TH} + V_{TH}$$

$$I_{E1} = \left(\frac{101}{100}\right)(0.8) = 0.808\text{ mA}$$

$$I_{B1} = 0.008\text{ mA}$$

$$5 = (0.808)R_{E1} + 0.7 + (0.008)(33.3) + 1.67$$

$$R_{E1} = 2.93\text{ k}\Omega$$

$$V_{E1} = 5 - (0.808)(2.93) = 2.63\text{ V}$$

$$V_{C1} = V_{E1} - V_{ECQ1} = 2.63 - 3.5 = -0.87\text{ V}$$

$$V_{E2} = -0.87 - 0.70 = -1.57\text{ V}$$

$$I_{E2} = \frac{-1.57 - (-5)}{R_{E2}} = 0.808 \Rightarrow \underline{R_{E2} = 4.25\,\text{k}\Omega}$$

$$V_{CEQ2} = 4 \Rightarrow V_{C2} = -1.57 + 4 = 2.43\text{ V}$$

$$R_{C2} = \frac{5 - 2.43}{0.8} \Rightarrow \underline{R_{C2} = 3.21\,\text{k}\Omega}$$

$$I_{RC1} = I_{C1} - I_{B2} = 0.8 - 0.008 = 0.792 \text{ mA}$$

$$R_{C1} = \frac{-0.87 - (-5)}{0.792} \Rightarrow \underline{R_{C1} = 5.21 \text{ k}\Omega}$$

Chapter 6

6.1

(a) (i) $g_m = \dfrac{I_{CQ}}{V_T} = \dfrac{0.5}{0.026} = 19.23 \text{ mA/V}$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(180)(0.026)}{0.5} = 9.36 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{150}{0.5} = 300 \text{ k}\Omega$$

(ii) $g_m = \dfrac{2}{0.026} = 76.92 \text{ mA/V}$

$$r_\pi = \frac{(180)(0.026)}{2} = 2.34 \text{ k}\Omega$$

$$r_o = \frac{150}{2} = 75 \text{ k}\Omega$$

(b) (i) $g_m = \dfrac{0.25}{0.026} = 9.615 \text{ mA/V}$

$$r_\pi = \frac{(80)(0.026)}{0.25} = 8.32 \text{ k}\Omega$$

$$r_o = \frac{100}{0.25} = 400 \text{ k}\Omega$$

(ii) $g_m = \dfrac{0.08}{0.026} = 3.077 \text{ mA/V}$

$$r_\pi = \frac{(80)(0.026)}{0.08} = 26 \text{ k}\Omega$$

$$r_o = \frac{100}{0.08} = 1250 \text{ k}\Omega$$

6.2

(a) $g_m = \dfrac{I_{CQ}}{V_T} \Rightarrow I_{CQ} = (95)(0.026) = 2.47 \text{ mA}$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(125)(0.026)}{2.47} = 1.32 \text{ k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{200}{2.47} = 81 \text{ k}\Omega$$

(b) $I_{CQ} = g_m V_T = (120)(0.026) = 3.12 \text{ mA}$

$\beta = g_m r_\pi = (120)(1.2) = 144$

6.3

$$g_m = \frac{0.8}{0.026} = 30.77 \text{ mA/V}; \quad g_m = \frac{1.2}{0.026} = 46.15 \text{ mA/V}$$

So $30.77 \le g_m \le 46.15 \text{ mA/V}$

$$r_\pi = \frac{(180)(0.026)}{0.8} = 5.85\ \text{k}\Omega\ ;\quad r_\pi = \frac{(90)(0.026)}{1.2} = 1.95\ \text{k}\Omega$$

So $1.95 \le r_\pi \le 5.85\ \text{k}\Omega$

6.4

(a) $i_C = I_{CQ} + g_m \upsilon_{be} = 0.12 + \left(\frac{0.12}{0.026}\right)\left(5\times10^{-3}\sin\omega t\right)$

$$i_C = 0.12 + 0.0231\sin\omega t\ \text{(mA)}$$

$$\upsilon_{CE} = V_{CEQ} - i_C R_C = [3.3 - (0.12)(15)] - (15)(0.0231\sin\omega t)$$

$$\upsilon_{CE} = 1.5 - 0.346\sin\omega t\ \text{(V)}$$

(b) $A_\upsilon = \frac{\upsilon_{ce}}{\upsilon_{be}} = \frac{-0.346}{0.005} = -69.2$

6.5

(a) $I_{CQ} = \frac{\beta(V_{BB} - V_{BE}(on))}{R_B} = \frac{(120)(1.10 - 0.7)}{110} = 0.436\ \text{mA}$

$$g_m = \frac{0.436}{0.026} = 16.78\ \text{mA/V}$$

$$r_\pi = \frac{(120)(0.026)}{0.436} = 7.15\ \text{k}\Omega$$

$$r_o = \frac{80}{0.436} = 183\ \text{k}\Omega$$

(b) $A_\upsilon = -g_m(R_C \| r_o)\left(\frac{r_\pi}{r_\pi + R_B}\right) = -(16.78)(4\|183)\left(\frac{7.15}{7.15+110}\right)$

$$A_\upsilon = -4.0$$

(c) $\upsilon_s = \frac{\upsilon_o}{A_\upsilon} = \frac{0.5\sin(100t)}{-4} = -0.125\sin(100t)\ \text{(V)}$

6.6

a.

$$r_\pi = 5.4 = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{I_{CQ}} \Rightarrow I_{CQ} = 0.578\ \text{mA}$$

$$V_{CEQ} = \frac{1}{2}V_{CC} = \frac{1}{2}(5) = 2.5\ \text{V}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C \Rightarrow 2.5 = 5.0 - (0.578)R_C \Rightarrow \underline{R_C = 4.33\ \text{k}\Omega}$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.578}{120} = 0.00482\ \text{mA}$$

$$V_{BB} = I_{BQ}R_B + V_{BE}(\text{on})$$

$$= (0.00482)(25) + 0.70 \Rightarrow \underline{V_{BB} = 0.820\ \text{V}}$$

b.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.578} = 5.40\ \text{k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.578}{0.026} = 22.2\ \text{mA/V}$$

$$r_0 = \frac{V_A}{I_{CQ}} = \frac{100}{0.578} = 173\ \text{k}\Omega$$

$$V_0 = -g_m\left(r_0 \| R_C\right)V_\pi,\ V_\pi = \left(\frac{r_\pi}{r_\pi + R_B}\right)V_S$$

$$A_v = -g_m\left(\frac{r_\pi}{r_\pi + R_B}\right)\left(r_0 \| R_C\right) = -\frac{\beta\left(r_0 \| R_C\right)}{r_\pi + R_B}$$

$$A_v = -\frac{(120)\left[173 \| 4.33\right]}{5.40 + 25} = -\frac{(120)(4.22)}{30.4} \Rightarrow \underline{A_v = -16.7}$$

6.7

$$r_\pi = \frac{(120)(0.026)}{0.5} = 6.24\ \text{k}\Omega$$

(a) $R_i = R_B + r_\pi = 50 + 6.24 = 56.24\ \text{k}\Omega$

(b) $R_i = R_B \| r_\pi = 100 \| 6.24 = 5.87\ \text{k}\Omega$

(c) $R_i = r_\pi = 6.24\ \text{k}\Omega$

6.8

$$r_o = \frac{V_A}{I_{CQ}} = \frac{80}{0.2} = 400\ \text{k}\Omega$$

(a) $R_o = R_C \| r_o = 4 \| 400 = 3.96\ \text{k}\Omega$

(b) $R_o = R_C \| r_o = 10 \| 400 = 9.76\ \text{k}\Omega$

(c) $R_o = R_C \| R_L \| r_o = 10 \| 5 \| 400 = 3.333 \| 400 = 3.31\ \text{k}\Omega$

6.9

$$I_{CQ} = \frac{10 - 4}{4} = 1.5\ \text{mA}$$

$$I_{BQ} = \frac{1.5}{100} = 0.015\ \text{mA}$$

$$r_\pi = \frac{(100)(0.026)}{1.5} = 1.73\ \text{K}$$

$$i_b = \frac{v_{be}}{r_\pi} = \frac{5\sin\omega t\,(\text{mV})}{1.73\ \text{k}\Omega} = 2.89\sin\omega t\,(\mu\text{A})$$

So

$$i_B(t) = I_{BQ} + i_b(t) = 15 + 2.89\sin\omega t \ (\mu\text{ A})$$
$$i_{C1}(t) = \beta i_B(t) = 1.5 + 0.289\sin\omega t \ (\text{mA})$$
$$\upsilon_{C1}(t) = 10 - i_{C1}(t)R_C = 10 - [1.5 + 0.289\sin\omega t](4) \ (\text{V})$$
$$\upsilon_{C1}(t) = 4 - 1.156\sin\omega t \quad (\text{V})$$
$$A_\upsilon = \frac{\upsilon_c(t)}{\upsilon_{be}(t)} = \frac{-1.156}{0.005} \Rightarrow A_\upsilon = -231$$

6.10

a.

$$V_{ECQ} = \frac{1}{2}V_{CC} = 5 \text{ V}$$
$$V_{ECQ} = 10 - I_{CQ}R_C \Rightarrow 5 = 10 - (0.5)R_C \Rightarrow \underline{R_C = 10 \text{ k}\Omega}$$
$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.5}{100} = 0.005$$
$$V_{EB}(\text{on}) + I_{BQ}R_B = V_{BB} = (0.70) + (0.005)(50) \Rightarrow \underline{V_{BB} = 0.95 \text{ V}}$$

b.

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.5}{0.026} \Rightarrow \underline{g_m = 19.2 \text{ mA/V}}$$
$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.5} \Rightarrow \underline{r_\pi = 5.2 \text{ k}\Omega}$$
$$r_0 = \frac{V_A}{I_{CQ}} = \frac{\infty}{0.5} \Rightarrow \underline{r_0 = \infty}$$
$$A_v = -\frac{\beta R_C}{r_\pi + R_B} = -\frac{(100)(10)}{5.2 + 50} \Rightarrow \underline{A_v = -18.1}$$

c.

6.11

$$v_o = 1.2\sin\omega t(\text{V})$$
$$i_C(t)R_C + v_o = 0 \Rightarrow i_C(t) = \frac{-1.2\sin\omega t}{2}$$
$$i_C(t) = -0.60\sin\omega t(\text{mA})$$
$$i_b(t) = \frac{i_C(t)}{\beta} = -6\sin\omega t(\mu\text{A})$$
$$v_{be}(t) = i_b(t)\cdot r_\pi \quad g_m r_\pi = \beta$$
$$r_\pi = \frac{100}{50} = 2 \text{ K}$$
$$v_{be}(t) = -12\sin\omega t(\text{mV})$$

6.12

a.

$$I_{CQ} \approx I_{EQ}$$

$$V_{CEQ} = 5 = 10 - I_{CQ}\left(R_C + R_E\right)$$
$$= 10 - I_{CQ}\left(1.2 + 0.2\right)$$

$$I_{CQ} = 3.57 \text{ mA}$$

$$I_{BQ} = \frac{3.57}{150} = 0.0238 \text{ mA}$$

$$R_1 \| R_2 = R_{TH} = (0.1)(1+\beta)R_E$$
$$= (0.1)(151)(0.2) = 3.02 \text{ k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E - 5$$

$$\frac{1}{R_1}(3.02)(10) - 5 = (0.0238)(3.02) + 0.7 + (151)(0.0238)(0.2) - 5$$

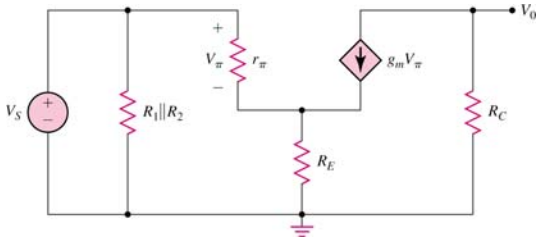
$$\frac{1}{R_1}(30.2) = 1.50 \Rightarrow R_1 = 20.1\ k\Omega$$

$$\frac{20.1R_2}{20.1 + R_2} = 3.02 \Rightarrow \underline{R_2 = 3.55 \text{ k}\Omega}$$

b.

$$r_\pi = \frac{(150)(0.026)}{3.57} = 1.09 \text{ k}\Omega$$

$$g_m = \frac{3.57}{0.026} = 137 \text{ mA/V}$$



$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E} = \frac{-(150)(1.2)}{1.09 + (151)(0.2)} \Rightarrow A_v = -5.75$$

6.13

(a) $R_{TH} = R_1 \| R_2 = 33 \| 50 = 19.88 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{50}{50 + 33}\right)(3.3) = 1.988 \text{ V}$$

$$V_{CC} = I_{BQ}(1+\beta)R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$$

Then $I_{BQ} = \dfrac{3.3 - 0.7 - 1.988}{19.88 + (101)(1)} = 0.005063 \text{ mA}$

$I_{CQ} = 0.506 \text{ mA}$; $I_{EQ} = 0.511 \text{ mA}$

$$V_{ECQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E = 3.3 - (0.506)(2) - (0.511)(1)$$
$$V_{ECQ} = 1.78 \text{ V}$$

(b) $$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.506} = 5.14 \text{ k}\Omega$$

$$A_\upsilon = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E} = \frac{-(100)(2)}{5.14 + (101)(1)} = -1.884$$

(c) $$R_1 = (1.05)(33) = 34.65 \text{ k}\Omega$$
$$R_2 = (0.95)(50) = 47.5 \text{ k}\Omega$$
$$R_{TH} = R_1 \| R_2 = 34.65 \| 47.5 = 20.03 \text{ k}\Omega$$
$$V_{TH} = \left(\frac{47.5}{47.5 + 34.65}\right)(3.3) = 1.908 \text{ V}$$
$$I_{CQ} = (100)\left(\frac{3.3 - 0.7 - 1.908}{20.03 + (101)(1)}\right) = 0.5718 \text{ mA}$$
$$r_\pi = \frac{(100)(0.026)}{0.5718} = 4.547 \text{ k}\Omega$$
$$A_\upsilon = \frac{-(100)(2)}{4.547 + (101)(1)} = -1.895$$

Also $$R_1 = (0.95)(33) = 31.35 \text{ k}\Omega$$
$$R_2 = (1.05)(50) = 52.5 \text{ k}\Omega$$
$$R_{TH} = 31.35 \| 52.5 = 19.63 \text{ k}\Omega$$
$$V_{TH} = \left(\frac{52.5}{52.5 + 31.35}\right)(3.3) = 2.066 \text{ V}$$
$$I_{CQ} = (100)\left(\frac{3.3 - 0.7 - 2.066}{19.63 + (101)(1)}\right) = 0.4427 \text{ mA}$$
$$r_\pi = \frac{(100)(0.026)}{0.4427} = 5.873 \text{ k}\Omega$$
$$A_\upsilon = \frac{-(100)(2)}{5.873 + (101)(1)} = -1.871$$

So $$1.871 \le |A_\upsilon| \le 1.895$$

6.14

(a)

$$V_{CC} = \left(\frac{1+\beta}{\beta}\right) I_{CQ} R_E + V_{ECQ} + I_{CQ} R_C$$

$$12 = \left(\frac{101}{100}\right) I_{CQ}(1) + 6 + I_{CQ}(2)$$

so that $\underline{I_{CQ} = 1.99 \ mA}$

$$I_{BQ} = \frac{1.99}{100} = 0.0199 \ mA$$

$$R_{TH} = (0.1)(1+\beta) R_E = (0.1)(101)(1) = 10.1 \ k\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(10.1)(12)$$

$$V_{CC} = (1+\beta) I_{BQ} R_E + V_{EB}(on) + I_{BQ} R_{TH} + V_{TH}$$

$$12 = (101)(0.0199)(1) + 0.7 + (0.0199)(10.1) + \frac{121.2}{R_1}$$

which yields $\underline{R_1 = 13.3 \ k\Omega}$ and $\underline{R_2 = 41.6 \ k\Omega}$

(b) $$A_\upsilon = \frac{-\beta R_C}{r_\pi + (1+\beta) R_E} = \frac{-(100)(2)}{1.31 + (101)(1)} \Rightarrow A_\upsilon = -1.95$$

6.15

$$I_{CQ} = 0.25 \text{ mA}, \ I_{EQ} = 0.2525 \text{ mA}$$

$$I_{BQ} = 0.0025 \text{ mA}$$

$$I_{BQ} R_B + V_{BE}(\text{on}) + I_{EQ}(R_S + R_E) - 5 = 0$$

$$(0.0025)(50) + 0.7 + (0.2525)(0.1 + R_E) = 5$$

$$\underline{R_E = 16.4 \text{ k}\Omega}$$

$$V_E = -(0.0025)(50) - 0.7 = -0.825 \text{ V}$$

$$V_C = V_{CEQ} + V_E = 3 - 0.825 = 2.175 \text{ V}$$

$$R_C = \frac{5 - 2.175}{0.25} \Rightarrow \underline{R_C = 11.3 \text{ k}\Omega}$$

$$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta) R_S}$$

$$r_\pi = \frac{(100)(0.026)}{0.25} = 10.4 \text{ k}\Omega$$

$$A_v = \frac{-(100)(11.3)}{10.4 + (101)(0.1)} \Rightarrow \underline{A_v = -55.1}$$

$$R_i = R_B \left\| \left[r_\pi + (1+\beta) R_S \right] \right.$$

$$= 50 \left\| \left[10.4 + (101)(0.1) \right] \right.$$

$$R_i = 50 \| 20.5 \Rightarrow \underline{R_i = 14.5 \text{ k}\Omega}$$

6.16

(a) $I_{CQ} \cong \dfrac{V_{CC} - V_{CEQ}}{R_C + R_E} = \dfrac{9 - 5.20}{2.2 + 2} = 0.905 \text{ mA}$

$I_{BQ} \cong 0.00754 \text{ mA}; \quad I_{EQ} \cong 0.9123 \text{ mA}$

$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(2) = 24.2 \text{ k}\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$

$= (0.00754)(24.2) + 0.7 + (0.9123)(2) = 2.707 \text{ V}$

Now $V_{TH} = 2.707 = \dfrac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \dfrac{1}{R_1}(24.2)(9)$

Or $R_1 = 80.5 \text{ k}\Omega$ and then $R_2 = 34.6 \text{ k}\Omega$

(b) $r_\pi = \dfrac{(120)(0.026)}{0.905} = 3.448 \text{ k}\Omega$

$g_m = \dfrac{0.905}{0.026} = 34.81 \text{ mA/V}$

$r_o = \dfrac{100}{0.905} = 110 \text{ k}\Omega$

$V_\pi = i_s(R_1 \| R_2 \| r_\pi) = i_s(80.5 \| 34.6 \| 3.448)$

$V_\pi = i_s(3.02)$

$\upsilon_o = -g_m V_\pi (r_o \| R_C \| R_L)$

Then $R_m = \dfrac{\upsilon_o}{i_s} = -g_m(3.02)(110 \| 2.2 \| 1) = -(34.81)(3.02)(110 \| 0.6875)$

Or $R_m = -71.8 \text{ V/mA}$

(c) For $\beta = 100$,

$I_{BQ} = \dfrac{2.707 - 0.7}{24.2 + (101)(2)} = 0.008873 \text{ mA}$

$I_{CQ} = 0.8873 \text{ mA}$

$r_\pi = \dfrac{(100)(0.026)}{0.8873} = 2.93 \text{ k}\Omega, \quad g_m = \dfrac{0.8873}{0.026} = 34.13 \text{ mA/V}, \quad r_o = \dfrac{100}{0.8873} = 113 \text{ k}\Omega$

$V_\pi = i_s(R_{TH} \| r_\pi) = i_s(24.2 \| 2.93) = i_s(2.614)$

$R_m = \dfrac{\upsilon_o}{i_s} = -(34.13)(2.614)(113 \| 0.6875) = -61.0 \text{ V/mA}$

For $\beta = 150$,

$I_{CQ} = (150)\left(\dfrac{2.707 - 0.7}{24.2 + (151)(2)}\right) = 0.923 \text{ mA}$

$r_\pi = 4.225 \text{ k}\Omega, \quad g_m = 35.5 \text{ mA/V}, \quad r_o = 108 \text{ k}\Omega$

$V_\pi = i_s(24.2 \| 4.225) = i_s(3.597)$

$R_m = \dfrac{\upsilon_o}{i_s} = -(35.5)(3.597)(108 \| 0.6875) = -87.2 \text{ V/mA}$

So $61.0 \le |R_m| \le 87.2 \text{ V/mA}$

6.17

(a) (i) $I_{BQ} = \dfrac{0.8}{81} = 0.009877\text{ mA}, \quad I_{CQ} = 0.790\text{ mA}$

$$R_B = \frac{0.2}{0.009877} = 20.25\text{ k}\Omega$$

$$R_C = \frac{-3-(-5)}{0.79} = 2.53\text{ k}\Omega$$

(ii) $g_m = \dfrac{0.79}{0.026} = 30.38\text{ mA/V}, \quad r_o = \dfrac{100}{0.79} = 127\text{ k}\Omega$

$$r_o \| R_C = 127 \| 2.53 = 2.48\text{ k}\Omega$$

$$i_o = -g_m \upsilon_s \left(\frac{2.48}{2.48+4}\right)$$

$$G_f = \frac{i_o}{\upsilon_s} = -(30.38)\left(\frac{2.48}{2.48+4}\right) = -11.63\text{ mA/V}$$

(b) (i) $I_{BQ} = \dfrac{0.8}{121} = 0.00661\text{ mA}, \quad I_{CQ} = 0.7934\text{ mA}$

$$R_B = \frac{0.2}{0.00661} = 30.3\text{ k}\Omega$$

$$R_C = \frac{-3-(-5)}{0.7934} = 2.52\text{ k}\Omega$$

(ii) $g_m = \dfrac{0.7934}{0.026} = 30.52\text{ mA/V}, \quad r_o = \dfrac{80}{0.7934} = 101\text{ k}\Omega$

$$r_o \| R_C = 101 \| 2.52 = 2.459\text{ k}\Omega$$

$$G_f = -(30.52)\left(\frac{2.459}{2.459+4}\right) = -11.62\text{ mA/V}$$

6.18

(a) (i) $I_{BQ} = \dfrac{0.25}{120} = 0.00208\text{ mA}, \quad I_{EQ} = 0.252\text{ mA}$

$$5 = (0.00208)(2.5) + 0.7 + (0.252)R_E \quad \Rightarrow R_E = 17.0\text{ k}\Omega$$

$$V_{CEQ} = 3 = 10 - (0.252)(17) - (0.25)R_C \quad \Rightarrow R_C = 10.9\text{ k}\Omega$$

(ii) $g_m = \dfrac{0.25}{0.026} = 9.615\text{ mA/V}, \quad r_\pi = \dfrac{(120)(0.026)}{0.25} = 12.48\text{ k}\Omega$

$$A_\upsilon = -g_m\left(R_C \| R_L\right)\left(\frac{r_\pi}{r_\pi + R_S}\right) = -(9.615)(10.9 \| 5)\left(\frac{12.48}{12.48+2.5}\right) = -27.5$$

(iii) $\upsilon_o = -(27.49)\left(5\times10^{-3}\sin\omega t\right) = -0.137\sin\omega t\ \text{(V)}$

(b) (i) $5 = 0.7 + (0.252)R_E \Rightarrow R_E = 17.1\text{ k}\Omega$

$$V_{CEQ} = 3 = 10 - (0.252)(17.06) - (0.25)R_C \Rightarrow R_C = 10.8\text{ k}\Omega$$

(ii) $g_m = 9.615\text{ mA/V}, \quad r_\pi = 12.48\text{ k}\Omega$

$$A_\upsilon = -g_m\left(R_C \| R_L\right) = -(9.615)(10.8 \| 5) = -32.9$$

(iii) $\upsilon_o = -(32.86)\left(5\times10^{-3}\sin\omega t\right) = -0.164\sin\omega t\ \text{(V)}$

6.19

(a) (i) $I_{BQ} = \dfrac{5-0.7}{2.5+(81)(10)} = 0.005292 \text{ mA}, \quad \Rightarrow I_{CQ} = 0.4234 \text{ mA}$

$$g_m = \frac{0.4234}{0.026} = 16.28 \text{ mA/V}, \; r_\pi = \frac{(80)(0.026)}{0.4234} = 4.91 \text{ k}\Omega$$

$$A_\upsilon = -g_m\left(R_C \| R_L\right)\left(\frac{r_\pi}{r_\pi + R_S}\right) = -(16.28)(5\|5)\left(\frac{4.91}{4.91+2.5}\right) = -26.97$$

$$i_o = \frac{\upsilon_o}{R_L} = \frac{A_\upsilon \upsilon_s}{R_L}$$

or $\quad G_f = \dfrac{i_o}{\upsilon_s} = \dfrac{-26.97}{5} = -5.39 \text{ mA/V}$

(ii) $\upsilon_o = -(26.97)\left(4\times10^{-3}\sin\omega t\right) = -0.108\sin\omega t \;\; (\text{V})$

$$i_o = \left(-5.39\times10^{-3}\right)\left(4\times10^{-3}\sin\omega t\right) \Rightarrow -21.6\sin\omega t \;\; (\mu\text{A})$$

(b) (i) $I_{CQ} = (120)\left(\dfrac{5-0.7}{2.5+(121)(10)}\right) = 0.4256 \text{ mA}$

$$g_m = \frac{0.4256}{0.026} = 16.37 \text{ mA/V}, \; r_\pi = \frac{(120)(0.026)}{0.4256} = 7.33 \text{ k}\Omega$$

$$A_\upsilon = -(16.37)(5\|5)\left(\frac{7.33}{7.33+2.5}\right) = -30.5$$

$$G_f = \frac{-30.5}{5} = -6.1 \text{ mA/V}$$

(ii) $\upsilon_o = -(30.5)\left(4\times10^{-3}\sin\omega t\right) = -0.122\sin\omega t \;\; (\text{V})$

$$i_o = -24.4\sin\omega t \;\; (\mu\text{A})$$

6.20

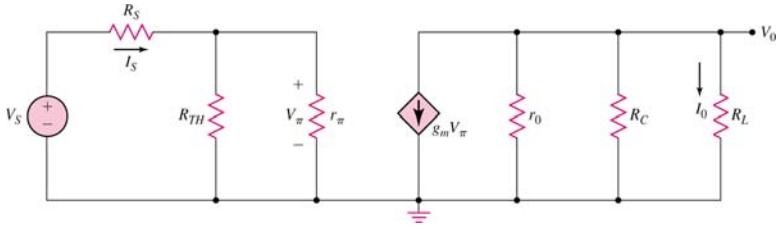
$$R_{TH} = R_1 \| R_2 = 27\|15 = 9.64 \text{ K}$$

$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{15}{15+27}\right)(9) = 3.214 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{3.214-0.7}{9.64+(101)(1.2)} = \frac{2.514}{130.84}$$

$$I_{BQ} = 0.0192 \text{ mA} \quad I_{CQ} = 1.9214 \text{ mA}$$

$$g_m = \frac{1.92}{0.026} = 73.9 \text{ mA/V} \quad r_\pi = \frac{(100)(0.026)}{1.92} = 1.35 \text{ K}$$



$$r_o = \frac{100}{1.92} = 52.1 \text{ K}$$

$$V_o = -g_m V_\pi \left(r_0 \| R_C \| R_L \right) \quad V_\pi = \left(\frac{r_\pi \| R_{TH}}{r_\pi \| R_{TH} + R_S} \right) V_S$$

$$r_\pi \| R_{TH} = 1.35 \| 9.64 = 1.184 \text{ K}$$

$$V_\pi = \left(\frac{1.184}{1.184 + 10} \right) V_S$$

$$= 0.1059 V_S$$

$$A_v = -(73.9)(0.1059)\left(52.1 \| 2.2 \| 2\right)$$

$$= -(73.9)(0.1059)\left(52.1 \| 1.0476\right)$$

$$= -(73.9)(0.1059)(1.027)$$

$$A_v = -8.04$$

$$A_I = \frac{I_o}{I_S} = \frac{-g_m V_\pi \left(\dfrac{r_o \| R_C}{r_o \| R_C + R_L} \right)}{\dfrac{V_\pi}{R_{TH} \| r_\pi}}$$

$$A_I = -g_m \left(R_{TH} \| r_\pi \right) \left(\frac{r_o \| R_C}{r_o \| R_C + R_L} \right)$$

$$r_o \| R_C = 52.1 \| 2.2 = 2.11 \text{ K}$$

$$R_{TH} \| r_\pi = 9.64 \| 1.35 = 1.184 \text{ K}$$

$$A_I = -(73.9)(1.184)\left(\frac{2.11}{2.11 + 2} \right)$$

$$\underline{A_I = -44.9}$$

$$R_i = R_{TH} \| r_\pi = 9.64 \| 1.35$$

$$\underline{R_i = 1.184 \text{ K}}$$

6.21

a.

$$I_E = 0.35 \text{ mA}, \quad I_B = \frac{0.35}{101} = 0.00347 \text{ mA}$$

$$V_B = -I_B R_B = -(0.00347)(10) \Rightarrow V_B = -0.0347 \text{ V}$$

$$V_E = V_B - V_{BE}(on) \Rightarrow V_E = -0.735 \text{ V}$$

b.

$$V_C = V_{CEQ} + V_E = 3.5 - 0.735 = 2.77 \text{ V}$$

$$I_C = \left(\frac{\beta}{1 + \beta} \right) \cdot I_E = \left(\frac{100}{101} \right)(0.35) = 0.347 \text{ mA}$$

$$R_C = \frac{V^+ - V_C}{I_C} = \frac{5 - 2.77}{0.347} \Rightarrow R_C = 6.43 \text{ k}\Omega$$

(c)

$$A_\upsilon = -g_m\left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\right)\left(R_C \| r_o\right)$$

$$g_m = \frac{0.347}{0.026} = 13.3 \text{ mA/V}, \quad r_o = \frac{100}{0.347} = 288 \text{ k}\Omega$$

$$r_\pi = \frac{(100)(0.026)}{0.347} = 7.49 \text{ k}\Omega$$

$$R_B \| r_\pi = 10 \| 7.49 = 4.28 \text{ k}\Omega$$

$$A_\upsilon = -(13.3)\left(\frac{4.28}{4.28 + 0.1}\right)(6.43 \| 288) \Rightarrow A_\upsilon = -81.7$$

d.

$$A_\upsilon = -g_m\left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\right)\left(R_C \| r_o\right)$$

$$R_B \| r_\pi = 10 \| 7.49 = 4.28 \text{ k}\Omega$$

$$A_\upsilon = -(13.3)\left(\frac{4.28}{4.28 + 0.5}\right)(6.43 \| 288) \Rightarrow A_\upsilon = -74.9$$

6.22

a.

$$R_{TH} = R_1 \| R_2 = 6 \| 1.5 = 1.2 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V^+ = \left(\frac{1.5}{1.5 + 6}\right)(5) = 1.0 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(\text{on})}{R_{TH} + (1+\beta)R_E} = \frac{1.0 - 0.7}{1.2 + (181)(0.1)} = 0.0155 \text{ mA}$$

$$\underline{I_{CQ} = 2.80 \text{ mA}}, \; I_{EQ} = 2.81$$

$$V_{CEQ} = V^+ - I_{CQ}R_C - I_{EQ}R_E$$
$$= 5 - (2.8)(1) - (2.81)(0.1) \Rightarrow \underline{V_{CEQ} = 1.92 \text{ V}}$$

b.

$$r_\pi = \frac{(180)(0.026)}{2.80} = 1.67 \text{ k}\Omega$$

$$g_m = \frac{2.80}{0.026} = 108 \text{ mA/V}$$

(c)

$$A_\upsilon = g_m\left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S}\right)\left(R_C \| R_L\right)$$

$$R_1 \| R_2 \| r_\pi = 6 \| 1.5 \| 1.67 = 0.698 \text{ k}\Omega$$

$$A_\upsilon = (108)\left(\frac{0.698}{0.698 + 0.2}\right)(1 \| 1.2) = 45.8$$

6.23

a.

$$9 = I_{EQ}R_E + V_{EB}(\text{on}) + I_{BQ}R_S$$

$$I_{EQ} = 0.75 \text{ mA},\ I_{BQ} = \frac{0.75}{81} = 0.00926 \text{ mA}$$

$$I_{CQ} = 0.741 \text{ mA}$$

$$9 = (0.75)R_E + 0.7 + (0.00926)(2) \Rightarrow \underline{R_E = 11.0 \text{ k}\Omega}$$

b.

$$V_E = 9 - (0.75)(11) = 0.75 \text{ V}$$

$$V_C = V_E - V_{ECQ} = 0.75 - 7 = -6.25 \text{ V}$$

$$R_C = \frac{V_C - (-9)}{I_{CQ}} = \frac{9 - 6.25}{0.741} \Rightarrow \underline{R_C = 3.71 \text{ k}\Omega}$$

c.

$$A_\upsilon = -g_m\left(\frac{r_\pi}{r_\pi + R_S}\right)(R_C \| R_L \| r_o)$$

$$r_\pi = \frac{(80)(0.026)}{0.741} = 2.81 \text{ k}\,\Omega$$

$$r_o = \frac{80}{0.741} = 108 \text{ k}\,\Omega$$

$$A_\upsilon = \frac{-80}{2.81 + 2}(3.71 \| 10 \| 108)$$

$$A_\upsilon = -43.9$$

d.

$$R_i = R_S + r_\pi = 2 + 2.81 \Rightarrow R_i = 4.81 \text{ k}\,\Omega$$

6.24

(a) For $\beta = 80$, $I_{CQ} = (80)\left[\frac{4 - 0.7}{5 + (81)(5)}\right] = 0.6439 \text{ mA}$

$$g_m = \frac{0.6439}{0.026} = 24.77 \text{ mA/V},\quad r_\pi = \frac{(80)(0.026)}{0.6439} = 3.23 \text{ k}\,\Omega,\quad r_o = \frac{50}{0.6439} = 77.7 \text{ k}\,\Omega$$

$$V_\pi = \frac{r_\pi \| R_B}{r_\pi \| R_B + R_S} \cdot \upsilon_s = \frac{(3.23 \| 5)}{(3.23 \| 5) + 1} \cdot \upsilon_s = (0.6624)\upsilon_s$$

$$A_\upsilon = -g_m \cdot \frac{V_\pi}{\upsilon_s} \cdot (r_o \| R_C \| R_L) = -(24.77)(0.6624)(77.7 \| 4 \| 4) = -32$$

For $\beta = 120$, $I_{CQ} = (120)\left[\frac{4 - 0.7}{5 + (121)(5)}\right] = 0.6492 \text{ mA}$

$$g_m = 24.97 \text{ mA/V},\quad r_\pi = 4.806 \text{ k}\,\Omega,\quad r_o = 77 \text{ k}\,\Omega$$

$$V_\pi = \frac{(4.806 \| 5)}{(4.806 \| 5) + 1} \cdot \upsilon_s = (0.710)\upsilon_s$$

$$A_\upsilon = -(24.97)(0.710)(77 \| 4 \| 4) = -34.6$$

So $32 \le |A_\upsilon| \le 34.6$

(b) $R_i = R_B \| r_\pi$

For $\beta = 80$, $R_i = 5 \| 3.23 = 1.96\ \text{k}\Omega$

For $\beta = 120$, $R_i = 5 \| 4.806 = 2.45\ \text{k}\Omega$

So $1.96 \le R_i \le 2.45\ \text{k}\Omega$

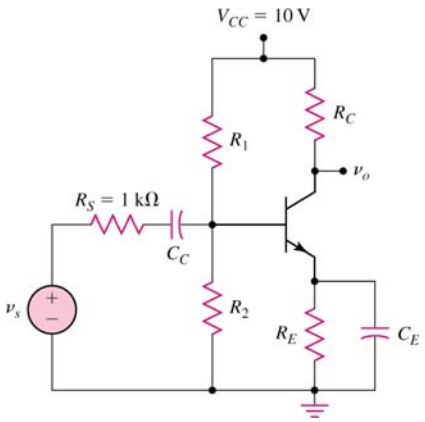
(c) $R_o = R_C \| r_o$

For $\beta = 80$, $R_o = 4 \| 77.7 = 3.804\ \text{k}\Omega$

For $\beta = 120$, $R_o = 4 \| 77 = 3.802\ \text{k}\Omega$

So $3.802 \le R_o \le 3.804\ \text{k}\Omega$

6.25



Assume an npn transistor with $\beta = 100$ and $V_A = \infty$. Let $V_{CC} = 10$ V.

$$|A_v| = \frac{0.5}{0.01} = 50$$

Bias at $I_{CQ} = 1\ mA$ and let $R_E = 1\ k\Omega$

For a bias stable circuit

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(101)(1) = 10.1\ \text{k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(10.1)(10) = \frac{101}{R_1}$$

$$I_{BQ} = \frac{1}{100} = 0.01\ \text{mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$$

$$\frac{101}{R_1} = (0.01)(10.1) + 0.7 + (101)(0.01)(1)$$

which yields $R_1 = 55.8\ \text{k}\Omega$, $R_2 = 12.3\ \text{k}\Omega$

Now

$$r_\pi = \frac{(100)(0.026)}{1} = 2.6\ \text{k}\Omega$$

$$g_m = \frac{1}{0.026} = 38.46\ \text{mA/V}$$

$$V_o = -g_m V_\pi R_C$$

where $V_\pi = \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S}\right) \cdot V_s = \left(\frac{10.1 \| 2.6}{10.1 \| 2.6 + 1}\right) \cdot V_s = 0.674 V_s$

Then $A_v = \frac{V_o}{V_s} = -(0.674) g_m R_C = -(0.674)(38.46) R_C = -50$

which yields $\underline{R_C = 1.93\ k\Omega}$

With this R_C, the dc bias is OK.

Finish Design, Set $R_C = 2\,\text{k}\Omega$, $R_E = 1\,\text{k}\Omega$

$R_1 = 56\,\text{k}\Omega$, $R_2 = 12\,\text{k}\Omega$

$R_{TH} = R_1 \| R_2 = 9.88$ K

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) V_{CC} = \left(\frac{12}{12 + 56}\right)(10) = 1.765 \text{ V}$$

$$I_{BQ} = \frac{1.765 - 0.7}{9.88 + (101)(1)} = 9.60\ \mu\text{A}$$

$I_{CQ} = 0.9605$ mA

$$r_\pi = \frac{(100)(0.026)}{0.9605} = 2.707 \text{ K} \quad g_m = \frac{0.9605}{0.026} = 36.94$$

$R_{TH} \| r_\pi = 2.125$ K

$$V_\pi = \left(\frac{R_{TH} \| r_\pi}{R_{TH} \| r_\pi + R_S}\right) \cdot V_i = \left(\frac{2.125}{2.125 + 1}\right) \cdot V_i = (0.680) \cdot V_i$$

$A_\upsilon = -(0.680) g_m R_C = -(0.680)(36.94)(2) = -50.2$

Design specification met.

6.26

a.

$$I_{BQ} = \frac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$$

$\underline{I_{CQ} = 1.69 \text{ mA}}$, $I_{EQ} = 1.71$ mA

$V_{CEQ} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$

$\underline{V_{CEQ} = 5.38 \text{ V}}$

b.

$$g_m = \frac{1.69}{0.026} = 65 \text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{1.69} = 1.54\,\text{k}\Omega \text{ , } \quad r_o = \infty$$

(c)

$$A_v = \frac{-\beta (R_C \| R_L)}{r_\pi + (1 + \beta) R_E} \cdot \frac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S}$$

$R_{ib} = r_\pi + (1 + \beta) R_E = 1.54 + (101)(3) = 304.5\ k\Omega$

$R_B \| R_{ib} = 10 \| 304.5 = 9.68\ k\Omega$

Then

$$A_\upsilon = \frac{-(100)(6.8\|6.8)}{1.54+(101)(3)}\cdot\left(\frac{9.68}{9.68+0.5}\right) = -1.06$$

$$i_o = \left(\frac{R_C}{R_C+R_L}\right)(-\beta i_b)$$

$$i_b = \left(\frac{R_B}{R_B+r_\pi+(1+\beta)R_E}\right)\cdot i_s$$

$$A_i = -(\beta)\left(\frac{R_C}{R_C+R_L}\right)\left(\frac{R_B}{R_B+r_\pi+(1+\beta)R_E}\right)$$

$$= -(100)\left(\frac{6.8}{6.8+6.8}\right)\left(\frac{10}{10+1.54+(101)(3)}\right) \Rightarrow A_i = -1.59$$

(d) $R_{is} = R_S + R_B\|R_{ib} = 0.5+10\|304.5 = 10.2\ k\Omega$

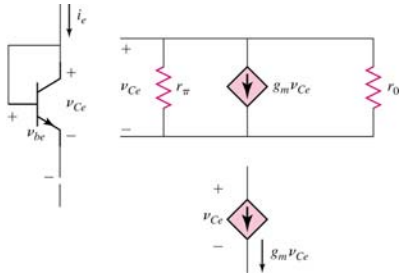
(e)

$$A_\upsilon = \frac{-\beta(R_C\|R_L)}{r_\pi+(1+\beta)R_E} = \frac{-(100)(6.8\|6.8)}{1.54+(101)(3)}$$

$A_\upsilon = -1.12$

$A_i = -1.59$, same as part (c)

6.27



$$r = \frac{\upsilon_{ce}}{g_m\upsilon_{ce}} = \frac{1}{g_m}$$

So $r_e = r_\pi\left\|\frac{1}{g_m}\right\|r_o$

6.28

(a) Set $\frac{R_C}{R_E} = 12.5 \Rightarrow R_C = 12.5R_E$

Set $V_{CEQ} = 1.5 \cong 3.3 - I_{CQ}(R_C+R_E) = 3.3 - I_{CQ}(13.5R_E)$

Set $I_{CQ} = 0.1\,\text{mA}, \Rightarrow R_E = 1.33\,\text{k}\Omega$ and $R_C = 16.7\,\text{k}\Omega$

We find $I_{BQ} = 0.833\,\mu\text{A}$

Set $R_{TH} = (0.1)(1+\beta)R_E = (0.1)(121)(1.33) = 16.1\,\text{k}\Omega$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E = (0.000833)(16.1)+0.7+(121)(0.000833)(1.33)$

$$V_{TH} = 0.8475 = \frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = \frac{1}{R_1}(16.1)(3.3)$$

So $R_1 = 62.7\ \text{k}\Omega$, $R_2 = 21.7\ \text{k}\Omega$

Actual gain:

$$r_\pi = \frac{(120)(0.026)}{0.1} = 31.2\ \text{k}\Omega$$

$$R_{ib} = r_\pi + (1+\beta)R_E = 31.2 + (121)(1.33) = 192.1\ \text{k}\Omega$$

$$R_i = R_{TH}\,\|\,R_{ib} = 16.1\|192.1 = 14.85\ \text{k}\Omega$$

Then $$A_v = \frac{-(120)(16.7)}{31.2+(121)(1.33)}\cdot\left(\frac{14.85}{14.85+0.1}\right) = -10.4$$

(b) $$R_{is} = R_S + R_i = 0.1 + 14.85 = 14.95\ \text{k}\Omega$$

$$R_o = R_C = 16.7\ \text{k}\Omega$$

6.29

Need a voltage gain of $\frac{100}{5} = 20.$

Assume a sign inversion from a common-emitter is not important. Use the configuration for Figure 6.31.

Let $R_S = 0.$ Need an input resistance of

$$R_i = \frac{5\times10^{-3}}{0.2\times10^{-6}} = 25\times10^3 \Rightarrow R_i = 25\ \text{k}\Omega$$

$R_i = R_{TH}\,\|\,R_{ib}$, Let $R_{TH} = 50\ \text{k}\Omega$, then $R_{ib} = 50\ \text{k}\Omega$

$$R_{ib} = r_\pi + (1+\beta)R_E \cong (1+\beta)R_E$$

For $\beta = 100$, $$R_E \cong \frac{R_{ib}}{1+\beta} = \frac{50}{101} = 0.495\ \text{k}\Omega$$

Let $R_E = 0.5\ \text{k}\Omega$, $V_{CC} = 10$ V, $I_{CQ} = 0.2$ mA

Then $I_{BQ} = \frac{0.2}{100} = 0.002\ mA$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$$

$$\frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = \frac{1}{R_1}(50)(10) = (0.002)(50) + 0.7 + (101)(0.002)(0.5)$$

which yields $R_1 = 555\ k\Omega$ and $R_2 = 55\ k\Omega$

Now $$A_v = \frac{-\beta R_C}{r_\pi + (1+\beta)R_E}, \quad r_\pi = \frac{(100)(0.026)}{0.2} = 13\ k\Omega$$

So

$$-20 = \frac{-(100)R_C}{13+(101)(0.5)} \Rightarrow \underline{R_C = 12.7\ k\Omega}$$

[Note: $I_{CQ}R_C = (0.2)(12.7) = 2.54\ V.$ So dc biasing is OK.]

6.30

Set $\dfrac{R_C}{R_E} = 15 \Rightarrow R_C = 15R_E$

Set $R_C = 5\ \text{k}\Omega$, then $R_E = 0.333\ \text{k}\Omega$

Set $I_{CQ} = 0.5\ \text{mA}$, then $V_{ECQ} \cong 5 - (0.5)(5.333) = 2.33\ \text{V}$

$r_\pi = 4.68\ \text{k}\Omega$, $R_{ib} = r_\pi + (1+\beta)R_E = 4.68 + (91)(0.333) = 35\ \text{k}\Omega$

Set $R_i = 22\ \text{k}\Omega = R_{TH} \| R_{ib} = R_{TH} \| 35 \Rightarrow R_{TH} \cong 60\ \text{k}\Omega$

Now $I_{BQ} = 0.00556\ \text{mA}$, $I_{EQ} = 0.506\ \text{mA}$

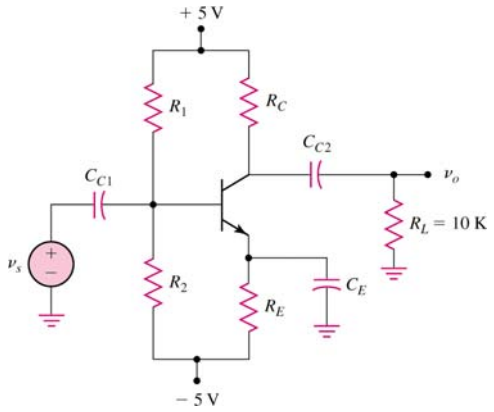
$V_{CC} = I_{EQ}R_E + V_{EB}(on) + I_{BQ}R_{TH} + V_{TH}$

$5 = (0.506)(0.333) + 0.7 + (0.00556)(60) + V_{TH}$

$V_{TH} = 3.798 = \dfrac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \dfrac{1}{R_1}(60)(5)$

So that $R_1 = 79\ \text{k}\Omega$ and $R_2 = 249\ \text{k}\Omega$

6.31



$\beta = 120$

Let $I_{CQ} = 0.35\ \text{mA}$, $I_{EQ} = 0.353\ \text{mA}$

$I_{BQ} = 0.00292\ \text{mA}$

Let $R_E = 2\ \text{k}\Omega$. For $V_{CEQ} = 4\ \text{V} \Rightarrow 10 = 4 + (0.35)R_C + (0.353)(2)$

$\underline{R_C = 15.1\ \text{k}\Omega}$, $r_\pi = \dfrac{(120)(0.026)}{0.35} = 8.91\ \text{k}\Omega$

$A_v = \dfrac{-\beta(R_C \| R_L)}{r_\pi} = -\dfrac{(120)(15.1 \| 10)}{8.91}$

$\underline{A_v = -81.0}$

For bias stable circuit:

$R_1 \| R_2 = R_{TH} = (0.1)(1+\beta)R_E$

$= (0.1)(121)(2) = 24.2\ \text{k}\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1 + R_2}\right)(10) - 5 = \dfrac{1}{R_1} \cdot R_{TH} \cdot (10) - 5$

$V_{TH} = I_{BQ}R_{TH} + V_{BE}(\text{on}) + (1+\beta)I_{BQ}R_E - 5$

$$\frac{1}{R_1}(24.2)(10)-5=(0.00292)(24.2)+0.7+(121)(0.00292)(2)-5$$

$$\frac{1}{R_1}(242)=1.477,\quad \underline{R_1=164\ \text{k}\Omega}$$

$$\frac{164R_2}{164+R_2}=24.2\Rightarrow \underline{R_2=28.4\ \text{k}\Omega}$$

$$\frac{10}{164+28.4}=0.052,\quad 0.35+0.052=\underline{0.402\ \text{mA}}$$

So bias current specification is met.

6.32

$$R_{TH}=R_1\|R_2=33\|50=19.88\ \text{k}\Omega$$

$$V_{TH}=\left(\frac{R_2}{R_1+R_2}\right)\cdot V_{CC}=\left(\frac{50}{50+33}\right)(3.3)=1.988\ \text{V}$$

$$3.3=(101)I_{BQ}(1)+0.7+I_{BQ}(19.88)+1.988$$

So $I_{BQ}=0.005063\,\text{mA},\ I_{CQ}=0.5063\,\text{mA},\ I_{EQ}=0.5114\ \text{mA}$

$$V_{ECQ}=3.3-(0.5114)(1)-(0.5063)(2)=1.776\ \text{V}$$

Then $\Delta V_{EC}=1.776-0.5=1.276\ \text{V}$, or $\Delta V_{EC}=3-1.776=1.224\ \text{V}$

So $\Delta V_{EC}=2(1.224)=2.448\ \text{V}$ peak-to-peak

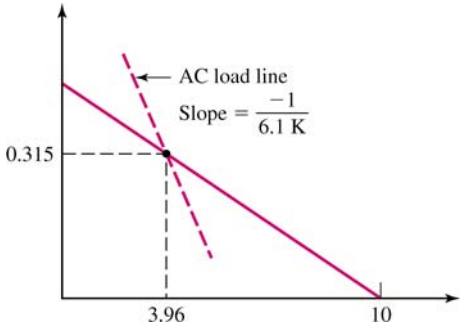
6.33

$$I_{BQ}=\frac{5-0.7}{50+(101)(0.1+12.9)}=0.00315\ \text{mA}$$

$$\underline{I_{CQ}=0.315\ \text{mA}},\ I_{EQ}=0.319\ \text{mA}$$

$$V_{CEQ}=(5+5)-(0.315)(6)-(0.319)(13)$$

$$V_{CEQ}=3.96\ \text{V}$$



$$\Delta i_C=-\frac{1}{6.1}\Delta v_{eC}$$

For $\Delta i_C=0.315-0.05=0.265\Rightarrow|\Delta v_{EC}|=1.62$

$$v_{EC}(\text{min})=3.96-1.62=2.34$$

Output signal swing determined by current:

Max. output swing $= \underline{3.24 \text{ V peak-to-peak}}$

6.34

(a) $5 = (81)I_{BQ}(10) + 0.7 + I_{BQ}(2.5)$

So that $I_{BQ} = 0.005292$ mA, $I_{CQ} = 0.4234$ mA, $I_{EQ} = 0.4287$ mA

Now $V_{ECQ} = 10 - (0.4234)(5) - (0.4287)(10) = 3.60$ V

$\Delta V_{EC} = \Delta I_C (R_C \| R_L) = \Delta I_C (2.5)$

For $\Delta I_C = 0.4234 \Rightarrow \Delta V_{EC} = 1.06$ V

So that $\Delta V_{EC} = 2(1.06) = 2.12$ V peak-to-peak

(b) $\Delta I_C = 2(0.4234) = 0.847$ mA peak-to-peak

6.35

$I_{EQ} = 0.8$ mA, $I_{CQ} = 0.790$ mA, $I_{BQ} = 0.009877$ mA

$V_E = 0.7 + (0.009877)(20) = 0.898$ V

$V_C = (0.79)(2.5) - 5 = -3.025$ V

Then $V_{ECQ} = V_E - V_C = 3.923$ V

$\Delta V_{EC} = \Delta I_C (R_C \| R_L) = \Delta I_C (2.5 \| 4) = \Delta I_C (1.538)$

For $\Delta I_C = 0.79 - 0.08 = 0.71$ mA, then $\Delta V_{EC} = (0.71)(1.538) = 1.09$ V

So, $\Delta I_C = 2(0.71) = 1.42$ mA peak-to-peak,

$$\Delta i_o = \left(\frac{2.5}{2.5+4}\right)\Delta I_C = 0.546 \text{ mA peak-to-peak}$$

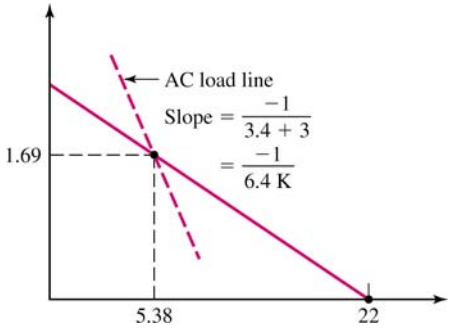
6.36

$$I_{BQ} = \frac{6 - 0.7}{10 + (101)(3)} = 0.0169 \text{ mA}$$

$\underline{I_{CQ} = 1.69 \text{ mA}}$, $I_{EQ} = 1.71$ mA

$V_{CEQ} = (16 + 6) - (1.69)(6.8) - (1.71)(3)$

$V_{CEQ} = 5.38$ V



$$\Delta i_c = -\frac{1}{6.4}\Delta v_{ce}$$

For $\upsilon_{ce}(\min) = 1\text{ V},\ \Delta\upsilon_{ce} = 5.38 - 1 = 4.38\text{ V} \Rightarrow |\Delta i_c| = \dfrac{4.38}{6.4} = 0.684\text{ mA}$

Output swing limited by voltage:

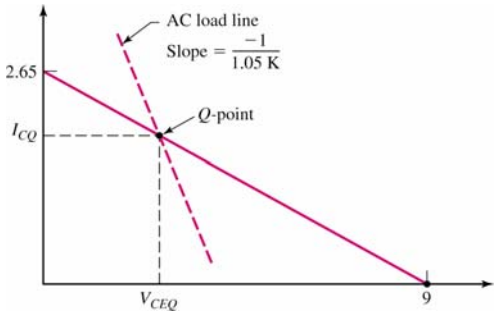
Δv_{ce} = Max. swing in output voltage

$= \underline{8.76\text{ V peak-to-peak}}$

$\Delta i_0 = \dfrac{1}{2}\Delta i_C \Rightarrow \Delta i_0 = 0.342\text{ mA}$

or $\Delta i_0 = 0.684\text{ mA}$ (peak-to-peak)

6.37



$r_o = \dfrac{100}{I_{CQ}}$

Neglect r_o as 1st approx.: dc load line $V_{CE} = 9 - I_C(3.4)$

$\Delta I_C = I_{CQ} - 0.1$

$\Delta V_{CE} = V_{CEQ} - 1$

Also $\Delta V_{CE} = \Delta I_C\left(R_C \| R_L\right) = \Delta I_C(1.05)$

Or $V_{CEQ} - 1 = \left(I_{CQ} - 0.1\right)(1.05)$

Substituting the expression for the dc load line.

$\left[9 - I_{CQ}(3.4) - 1\right] = \left(I_{CQ} - 0.1\right)(1.05)$

$8.105 = I_{CQ}(4.45) \Rightarrow I_{CQ} = 1.821\text{ mA}$

$V_{CEQ} = 2.81\text{ V}$

$I_{BQ} = \dfrac{1.821}{100} = 0.01821$

$R_{TH} = (0.1)(101)(1.2) = 12.12\text{ K}$

$V_{TH} = \dfrac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \dfrac{1}{R_1}(12.12)(9) = (0.01821)(12.12) + 0.7 + (101)(0.01821)(1.2)$

$= 0.2207 + 0.7 + 2.20705$

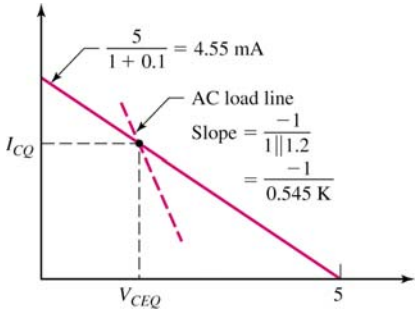
$R_1 = 34.9\text{ K}$

$R_2 = 18.6\text{ K}$

$\dfrac{34.9R_2}{34.9 + R_2} = 12.12$

6.38

dc load line



For maximum symmetrical swing

$$\Delta i_C = I_{CQ} - 0.25$$

$$\Delta v_{CE} = V_{CEQ} - 0.5 \text{ and } |\Delta i_C| = \frac{1}{0.545\ \text{k}\Omega} \cdot |\Delta v_{CE}|$$

$$I_{CQ} - 0.25 = \frac{V_{CEQ} - 0.5}{0.545}$$

$$V_{CEQ} = 5 - I_{CQ}(1.1)$$

$$0.545\left(I_{CQ} - 0.25\right) = \left[5 - I_{CQ}(1.1)\right] - 0.5$$

$$(0.545 + 1.1) I_{CQ} = 5 - 0.5 + 0.136$$

$$\underline{I_{CQ} = 2.82\ \text{mA}}, \qquad I_{BQ} = 0.0157\ \text{mA}$$

$$R_{TH} = R_1 \| R_2 = (0.1)(1+\beta) R_E$$

$$= (0.1)(181)(0.1) = 1.81\ \text{k}\Omega$$

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V^+ = I_{BQ} R_{TH} + V_{BE}(on) + (1+\beta) I_{BQ} R_E$$

$$\frac{1}{R_1}(1.81)(5) = (0.0157)(1.81) + 0.7 + (181)(0.0157)(0.1)$$

$$\frac{1}{R_1}(9.05) = 1.013 \Rightarrow \underline{R_1 = 8.93\ \text{k}\Omega}$$

$$\frac{8.93 R_2}{8.93 + R_2} = 1.81 \Rightarrow \underline{R_2 = 2.27\ \text{k}\Omega}$$

6.39

$$I_{CQ} = 0.647\ \text{mA}, \quad V_{CEQ} = 10 - (0.647)(9) = 4.18\ \text{V}$$

$$\Delta i_c = I_{CQ} = 0.647\ \text{mA}$$

So $\Delta \upsilon_{ce} = \Delta i_c (4 \| 4) = (0.647)(2) = 1.294\ \text{V}$

Voltage swing is well within the voltage specification. Then $\Delta \upsilon_{ce} = 2(1.294) = 2.59\ \text{V}$ peak-to-peak.

6.40

(a) $A_v = \dfrac{(1+\beta)R_E}{r_\pi + (1+\beta)R_E}$

$$0.92 = \frac{(121)(0.5)}{r_\pi + (121)(0.5)} \Rightarrow r_\pi = 5.261 = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{I_{CQ}} \Rightarrow I_{CQ} = 0.593\,\text{mA}$$

(b) $r_o = \dfrac{V_A}{I_{CQ}} = \dfrac{20}{0.593} = 33.7\,\text{k}\Omega$

$$A_v = \frac{(1+\beta)(r_o \| R_E)}{r_\pi + (1+\beta)(r_o \| R_E)}$$

$$r_o \| R_E = 33.7 \| 0.5 = 0.4927\,\text{k}\Omega$$

Then $A_v = \dfrac{(121)(0.4927)}{5.261 + (121)(0.4927)} = 0.919$

(c) For (a), $R_o = R_E \left\| \dfrac{r_\pi}{1+\beta} = 0.5 \right\| \dfrac{5.261}{121} = 0.5 \| 0.04348 \Rightarrow R_o = 40\,\Omega$

For (b), $R_o = R_E \| r_o \left\| \dfrac{r_\pi}{1+\beta} = 0.04 \right\| 33.7 \Rightarrow R_o = 39.95\,\Omega$

6.41

(a) $R_{ib} = r_\pi + (1+\beta)R_E$

$$A_v = \frac{(1+\beta)R_E}{r_\pi + (1+\beta)R_E} = \frac{(1+\beta)R_E}{R_{ib}}$$

$$R_E = \frac{(A_v)(R_{ib})}{1+\beta} = \frac{(0.95)(50)}{81} = 0.586\,\text{k}\Omega$$

$$50 = r_\pi + (81)(0.586) \Rightarrow r_\pi = 2.5\,\text{k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} \Rightarrow I_{CQ} = \frac{(80)(0.026)}{2.5} = 0.832\,\text{mA}$$

(b) $R_o = R_E \left\| \dfrac{r_\pi}{1+\beta} = 0.586 \right\| \dfrac{2.5}{81} = 0.586 \| 0.03086$

$$R_o = 29.3\,\Omega$$

6.42

(a) $g_m = 76.92\,\text{mA/V}$, $r_\pi = 1.04\,\text{k}\Omega$, $r_o = 25\,\text{k}\Omega$

$$A_v = \frac{(1+\beta)r_o}{R_S + r_\pi + (1+\beta)r_o} = \frac{(81)(25)}{1 + 1.04 + (81)(25)} = 0.999$$

$$R_i = R_S + r_\pi + (1+\beta)r_o = 1 + 1.04 + (81)(25) \Rightarrow R_i = 2.027\,\text{M}\Omega$$

$$R_o = r_o \left\| \frac{r_\pi + R_S}{1+\beta} = 25 \right\| \left(\frac{1.04 + 1}{81} \right) = 25 \| 0.0252 \Rightarrow R_o = 25.2\,\Omega$$

(b) $r_\pi = 10.4\,\text{k}\Omega$, $r_o = 250\,\text{k}\Omega$

$$A_v = \frac{(81)(250)}{1 + 10.4 + (81)(250)} = 0.9994$$

$$R_i = 1 + 10.4 + (81)(250) \Rightarrow R_i = 20.26\ \text{M}\Omega$$

$$R_o = 250 \left\| \left(\frac{10.4 + 1}{81} \right) \right. = 250 \| 0.1407 \Rightarrow R_o = 141\ \Omega$$

6.43

(a) $$R_o = \frac{r_\pi + R_S}{1 + \beta}$$

$$0.015 = \frac{r_\pi + 0.5}{121} \Rightarrow r_\pi = 1.315\ \text{k}\Omega$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} \Rightarrow I_{CQ} = \frac{(120)(0.026)}{1.315} = 2.37\ \text{mA}$$

(b) $$r_o = \frac{50}{2.373} = 21.07\ \text{k}\Omega$$

$$A_\upsilon = \frac{(1+\beta) r_o}{R_S + r_\pi + (1+\beta) r_o} = \frac{(121)(21.07)}{0.5 + 1.315 + (121)(21.07)} = 0.9993$$

6.44

a.

$$R_{TH} = R_1 \| R_2 = 10 \| 10 = 5\ \text{k}\Omega$$

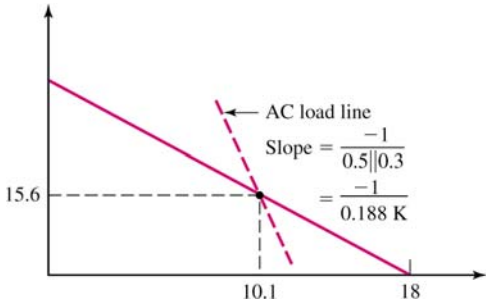
$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right)(18) - 9 = \left(\frac{10}{10 + 10} \right)(18) - 9 = 0$$

$$I_{BQ} = \frac{0 - 0.7 - (-9)}{5 + (181)(0.5)} = 0.0869\ \text{mA}$$

$$I_{CQ} = 15.6\ \text{mA}, \quad I_{EQ} = 15.7\ \text{mA}$$

$$V_{CEQ} = 18 - (15.7)(0.5) = 10.1\ \text{V}$$

b.



c.

$$r_\pi = \frac{(180)(0.026)}{15.6} = 0.30\ \text{k}\Omega$$

$$A_\upsilon = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \left(\frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S} \right)$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L) = 0.30 + (181)(0.5 \| 0.3) = 34.2\ \text{k}\Omega$$
$$R_1 \| R_2 \| R_{ib} = 5 \| 34.2 = 4.36\ \text{k}\Omega$$
$$A_\upsilon = \frac{(181)(0.5 \| 0.3)}{0.3 + (181)(0.5 \| 0.3)} \cdot \left(\frac{4.36}{4.36+1}\right) \Rightarrow A_\upsilon = 0.806$$

d.

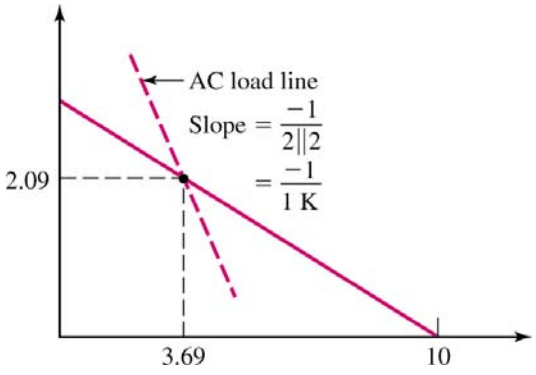
$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$$
$$R_{ib} = 0.30 + (181)(0.188) = 34.3\ \text{k}\Omega$$
$$R_o = R_E \left\| \left(\frac{r_\pi + R_1 \| R_2 \| R_S}{1+\beta}\right) = 0.5 \right\| \left(\frac{0.3 + 5 \| 1}{181}\right) \Rightarrow R_o = 6.18\,\Omega$$

6.45

a.

$$R_{TH} = R_1 \| R_2 = 10 \| 10 = 5\ \text{k}\Omega$$
$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(-10) = -5\ \text{V}$$
$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + (1+\beta) I_{BQ} R_E - 10$$
$$I_{BQ} = \frac{-5 - 0.7 - (-10)}{5 + (121)(2)} = 0.0174\ \text{mA}$$
$$I_{CQ} = 2.09\ \text{mA}, \quad I_{EQ} = 2.11\ \text{mA}$$
$$V_{CEQ} = 10 - (2.09)(1) - (2.11)(2) = 3.69\ \text{V}$$

b.



c.

$$r_\pi = \frac{(120)(0.026)}{2.09} = 1.49\ \text{k}\Omega$$
$$A_v = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \left(\frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S}\right)$$
$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L) = 1.49 + (121)(2 \| 2)$$
$$R_{ib} = 122.5\ k\Omega, \qquad R_1 \| R_2 \| R_{ib} = 5 \| 122.5 = 4.80\ k\Omega$$
$$A_v = \frac{(121)(2 \| 2)}{1.49 + (121)(2 \| 2)} \cdot \left(\frac{4.80}{4.80+5}\right) \Rightarrow \underline{A_v = 0.484}$$

d.

$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$$
$$R_{ib} = 1.49 + (121)(2\|2) \Rightarrow R_{ib} = 122\ \text{k}\Omega$$
$$R_o = R_E \left\| \left(\frac{r_\pi + R_1 \| R_2 \| R_S}{1+\beta} \right) = 2 \right\| \left(\frac{1.49 + 5\|5}{121} \right) \Rightarrow R_o = 32.4\ \Omega$$

6.46

(a) $R_{TH} = R_1 \| R_2 = 585 \| 135 = 109.7\ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2} \right) \cdot V_{CC} = \left(\frac{135}{135+585} \right)(3.3) = 0.61875\ \text{V}$$
$$3.3 = (1+\beta) I_{BQ} R_E + V_{EB}(on) + I_{BQ} R_{TH} + V_{TH}$$
$$I_{BQ} = \frac{3.3 - 0.7 - 0.61875}{109.7 + (91)(12)} = 0.001649\ \text{mA}$$

Then $I_{CQ} = 0.1484\ \text{mA}$, $I_{EQ} = 0.150\ \text{mA}$

$$V_{ECQ} = 3.3 - (0.15)(12) = 1.5\ \text{V}$$

(c) $r_\pi = \dfrac{(90)(0.026)}{0.1484} = 15.77\ \text{k}\Omega$, $r_o = \dfrac{60}{0.1484} = 404\ \text{k}\Omega$

$$A_\upsilon = \frac{(1+\beta)(r_o \| R_E \| R_L)}{r_\pi + (1+\beta)(r_o \| R_E \| R_L)}$$
$$r_o \| R_E \| R_L = 404 \| 12 \| 4 = 2.978\ \text{k}\Omega$$
$$A_\upsilon = \frac{(91)(2.978)}{15.77 + (91)(2.978)} = 0.945$$
$$R_{ib} = r_\pi + (1+\beta)(r_o \| R_E \| R_L) = 15.77 + (91)(2.978) = 286.8\ \text{k}\Omega$$
$$A_i = (1+\beta)\left(\frac{R_{TH}}{R_{TH} + R_{ib}} \right)\left(\frac{r_o \| R_E}{r_o \| R_E + R_L} \right) = (91)\left(\frac{109.7}{109.7 + 286.8} \right)\left(\frac{404 \| 12}{404 \| 12 + 4} \right) = 18.7$$

(d) $R_{ib} = 286.8\ \text{k}\Omega$

$$R_o = R_E \| r_o \left\| \frac{r_\pi}{1+\beta} = 12 \right\| 404 \left\| \frac{15.77}{91} \Rightarrow R_o = 171\Omega \right.$$

6.47

(a)

$$I_{BQ} = \frac{0.5}{81} = 0.00617\ mA$$
$$V_B = I_{BQ} R_B = (0.00617)(10) \Rightarrow \underline{V_B = 0.0617\ V}$$
$$V_E = V_B + 0.7 \Rightarrow \underline{V_E = 0.7617\ V}$$

(b)

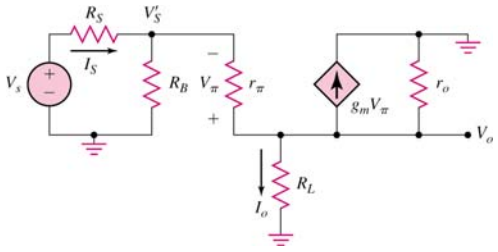
$$I_{CQ} = (0.5)\left(\frac{80}{81}\right) = 0.494\ mA$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.494}{0.026} \Rightarrow \underline{g_m = 19\ mA/V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(80)(0.026)}{0.494} \Rightarrow \underline{r_\pi = 4.21\ k\Omega}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{150}{0.494} \Rightarrow \underline{r_o = 304\ k\Omega}$$

(c)



For $R_S = 0$

$$V_o = -\left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right)(R_L \| r_o)$$

so that $V_\pi = \dfrac{-V_o}{\left(\dfrac{1+\beta}{r_\pi}\right)(R_L \| r_o)}$

Now $V_s + V_\pi = V_o$

$$V_s = V_o - V_\pi = V_o + \frac{V_o}{\left(\dfrac{1+\beta}{r_\pi}\right)(R_L \| r_o)}$$

We find

$$A_v = \frac{V_o}{V_s} = \frac{(1+\beta)(R_L \| r_o)}{r_\pi + (1+\beta)(R_L \| r_o)} = \frac{(81)(0.5 \| 304)}{4.21 + (81)(0.5 \| 304)}$$

$$A_v = 0.906$$

$$R_{ib} = r_\pi + (1+\beta)(R_L \| r_o) \cong 4.21 + (81)(0.5) = 44.7\ \text{k}\,\Omega$$

$$I_b = \left(\frac{R_B}{R_B + R_{ib}}\right) \cdot I_s \quad \text{and} \quad I_o = \left(\frac{r_o}{r_o + R_L}\right)(1+\beta) I_b$$

Then

$$A_i = \frac{I_o}{I_s} = (1+\beta)\left(\frac{R_B}{R_B + R_{ib}}\right)\left(\frac{r_o}{r_o + R_L}\right)$$

$$A_i \cong (81)\left(\frac{10}{10 + 44.7}\right)(1) \Rightarrow \underline{A_i = 14.8}$$

(d)

$$V_s' = \left(\frac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S}\right) \cdot V_s = \left(\frac{10 \| 44.7}{10 \| 44.7 + 2}\right) \cdot V_s = (0.803) V_s$$

Then $A_\upsilon = (0.803)(0.906) \Rightarrow A_\upsilon = 0.728$

$A_i = 14.8$ (unchanged)

6.48

(a)

$$I_{CQ} = 1.98\,\text{mA}, \quad r_\pi = \frac{(100)(0.026)}{1.98} = 1.313\,\text{k}\Omega$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{1.98} = 50.5\,\text{k}\Omega$$

$$R_o = \frac{r_\pi + R_S}{1+\beta} \| r_o = \frac{1.31 + 10}{101} \| 50.5 \Rightarrow R_o = 112\,\Omega$$

(b) From Equation (6.68)

$$A_v = \frac{(1+\beta)(r_o \| R_L)}{r_\pi + (1+\beta)(r_o \| R_L)} \quad r_o = \frac{100}{1.98} = 50.5\ \text{K}$$

(i)

$$R_L = 0.5\ \text{K}$$

$$A_v = \frac{(101)(50.5 \| 0.5)}{1.31 + (101)(50.5 \| 0.5)}$$

$$A_v = \frac{(101)(0.4951)}{1.31 + (101)(0.4951)} \Rightarrow A_v = 0.974$$

(ii)

$$R_L = 5\ \text{K} \qquad r_o \| R_L = 50.5 \| 5 = 4.5495$$

$$A_v = \frac{(101)(4.55)}{1.31 + (101)(4.55)} \Rightarrow A_v = 0.997$$

6.49

(a) $\dfrac{5 - 0.7}{3.3} = I_{EQ} = 1.303\ \text{mA}, \quad I_{CQ} = 1.29\ \text{mA}$

$$V_{ECQ} = 0.7 - (-5) = 5.7\ \text{V}$$

(b) $r_\pi = \dfrac{(110)(0.026)}{1.29} = 2.217\,\text{k}\Omega\,, \quad r_o = \dfrac{50}{1.29} = 38.76\,\text{k}\Omega$

$$A_\upsilon = \frac{(1+\beta)(r_o \| R_E \| R_L)}{r_\pi + (1+\beta)(r_o \| R_E \| R_L)}$$

$$r_o \| R_E \| R_L = 38.76 \| 3.3 \| 1 = 0.7525\ \text{k}\Omega$$

$$A_\upsilon = \frac{(111)(0.7525)}{2.217 + (111)(0.7525)} = 0.974$$

$$R_{ib} = r_\pi + (1+\beta)(r_o \| R_E \| R_L) = 2.217 + (111)(0.7525) = 85.7\,\text{k}\Omega$$

$$R_o = \frac{r_\pi}{1+\beta}\|R_E\|r_o = \frac{2.217}{111}\|3.3\|38.76 \Rightarrow R_o = 19.8\,\Omega$$

(c) $i_s(t) = \dfrac{\upsilon_s(t)}{R_{ib}} = \dfrac{2.8\sin\omega t}{85.7} \Rightarrow i_s(t) = 32.7\sin\omega t \;\; (\mu\text{ A})$

$$i_o(t) = (1+\beta)\left(\frac{R_E\|r_o}{R_E\|r_o + R_L}\right)\cdot i_s(t) = (111)\left(\frac{3.041}{3.041+1}\right)(32.7\sin\omega t)$$

or $i_o(t) = 2.73\sin\omega t$ (mA)

$\upsilon_o(t) = i_o(t)R_L = 2.73\sin\omega t$ (V)

$\upsilon_{eb}(t) = -i_s(t)r_\pi = -(32.7\sin\omega t)(2.217)$

$\upsilon_{eb}(t) = -72.5\sin\omega t$ (mV)

6.50

a.

$I_{EQ} = 1$ mA, $V_{CEQ} = V_{CC} - I_{EQ}R_E$

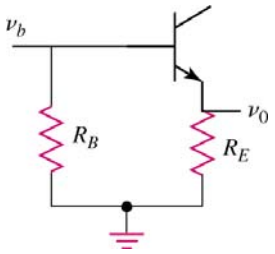
$5 = 10 - (1)(R_E) \Rightarrow \underline{R_E = 5\text{ k}\Omega}$

$$I_{BQ} = \frac{1}{101} = 0.0099\ mA$$

$10 = I_{BQ}R_B + V_{BE}(\text{on}) + I_{EQ}R_E$

$10 = (0.0099)R_B + 0.7 + (1)(5) \Rightarrow \underline{R_B = 434\text{ k}\Omega}$

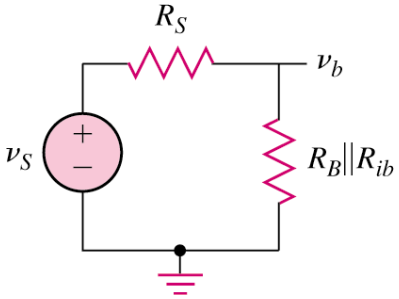
b.



$$r_\pi = \frac{(100)(0.026)}{0.99} = 2.63\text{ k}\Omega$$

$$\frac{v_0}{v_b} = \frac{(1+\beta)R_E}{r_\pi + (1+\beta)R_E} = \frac{(101)(5)}{2.63 + (101)(5)} = 0.995$$

$$\Rightarrow v_b = \frac{v_0}{0.995} = \frac{4}{0.995} \Rightarrow \underline{v_b = 4.02\text{ V peak-to-peak at base}}$$



$$R_{ib} = r_\pi + (1+\beta) R_E = 508 \text{ k}\Omega$$
$$R_B \| R_{ib} = 434 \| 508 = 234 \text{ k}\Omega$$
$$v_b = \frac{R_B \| R_{ib}}{R_B \| R_{ib} + R_S} \cdot v_S = \frac{234 v_S}{234 + 0.7} = \frac{234}{234.7} v_S$$
$$v_b = 0.997 v_S \Rightarrow v_S = \frac{4.02}{0.997} \Rightarrow v_S = 4.03 \text{ V peak-to-peak}$$

c.

$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$$
$$R_{ib} = 2.63 + (101)(5 \| 1) = 86.8 \text{ k}\Omega$$
$$R_B \| R_{ib} = 434 \| 86.8 = 72.3 \text{ k}\Omega$$
$$\upsilon_b = \left(\frac{72.3}{72.3 + 0.7}\right) \cdot \upsilon_s = 0.99 \upsilon_s = (0.99)(4.03)$$
$$\upsilon_b = 3.99 \text{ V peak-to-peak}$$
$$\upsilon_o = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \upsilon_b$$
$$= \frac{(101)(0.833)}{2.63 + (101)(0.833)} \cdot (3.99)$$
$$\upsilon_o = 3.87 \text{ V peak-to-peak}$$

6.51

$$R_{TH} = R_1 \| R_2 = 40 \| 60 = 24 \text{ k}\Omega$$
$$V_{TH} = \left(\frac{60}{60 + 40}\right)(10) = 6 \text{ V}$$

For $\beta = 75$, $I_{BQ} = \dfrac{6 - 0.7}{24 + (76)(5)} = 0.0131 \text{ mA}$

$$I_{CQ} = 0.984 \text{ mA}$$

For $\beta = 150$, $I_{BQ} = \dfrac{6 - 0.7}{24 + (151)(5)} = 0.00680 \text{ mA}$

$$I_{CQ} = 1.02 \text{ mA}$$

For $\beta = 75$, $r_\pi = \dfrac{(75)(0.026)}{0.984} = 1.98 \text{ k}\Omega$

For $\beta = 150$, $r_\pi = \dfrac{(150)(0.026)}{1.02} = 3.82 \text{ k}\Omega$

For $\beta = 75$, $R_{ib} = r_\pi + (1+\beta)(R_E \| R_L) = 65.3 \text{ k}\Omega$

For $\beta = 150$, $R_{ib} = 130 \text{ k}\Omega$

$$A_\upsilon = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \frac{R_1 \| R_2 \| R_{ib}}{R_1 \| R_2 \| R_{ib} + R_S}$$

For $\beta = 75$, $R_1 \| R_2 \| R_{ib} = 40 \| 60 \| 65.3 = 17.5 \text{ k}\Omega$

$$A_\upsilon = \frac{(76)(0.833)}{1.98 + (76)(0.833)} \cdot \frac{17.5}{17.5 + 4} = 0.789$$

For $\beta = 150$, $R_1 \| R_2 \| R_{ib} = 40 \| 60 \| 130 = 20.3\,\text{k}\Omega$

$$A_v = \frac{(151)(0.833)}{3.82 + (151)(0.833)} \cdot \frac{20.3}{20.3 + 4} = 0.811$$

So $0.789 \le A_v \le 0.811$

$$A_i = (1+\beta)\left(\frac{R_E}{R_E + R_L}\right)\left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right)$$

$\beta = 75$

$$A_i = (76)\left(\frac{5}{5+1}\right)\left(\frac{24}{24+65.3}\right) \Rightarrow \underline{A_i = 17.0}$$

$\beta = 150$

$$A_i = (151)\left(\frac{5}{6}\right)\left(\frac{24}{24+130}\right) \Rightarrow \underline{A_i = 19.6}$$

$\underline{17.0 \le A_i \le 19.6}$

6.52

(a)

$$9 = \left(\frac{I_E}{1+\beta}\right)(100) + V_{BE}(\text{on}) + I_E R_E$$

$$I_E = \frac{9 - 0.7}{\left(\frac{100}{1+\beta}\right) + R_E}$$

$\beta = 50 \quad I_E = \dfrac{8.3}{\left(\frac{100}{51}\right)+1} = 2.803$ mA

$\beta = 200 \quad I_E = \dfrac{8.3}{\left(\frac{100}{201}\right)+1} = 5.543$ mA

$2.80 \le I_E \le 5.54$ mA

$V_E = I_E R_E, \quad \beta = 50, \quad V_E = 2.80$ V

$\beta = 200, \quad V_E = 5.54$ V

(b) $\beta = 50, \quad I_{CQ} = 2.748$ mA, $\quad r_\pi = 0.473$ K

$\beta = 200, \quad I_{CQ} = 5.515$ mA, $\quad r_\pi = 0.943$ K

$$R_i = R_B \| [r_\pi + (1+\beta)(R_E \| R_L)]$$

$\beta = 50$, $\quad R_i = 100 \| [0.473 + (51)(1 \| 1)] = 100 \| 25.97 = 20.6\,\text{k}\Omega$

$\beta = 200$, $\quad R_i = 100 \| [0.943 + (201)(1 \| 1)] = 100 \| 101.4 = 50.3\,\text{k}\Omega$

From Fig. (6.68)

$$A_v = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \left(\frac{R_i}{R_i + R_S}\right)$$

$$= \frac{(51)(1\|1)}{0.473 + (51)(1\|1)} \cdot \left(\frac{20.6}{20.6+10}\right)$$

$$\beta = 50 \Rightarrow A_v = 0.661$$

$$\beta = 200 \Rightarrow A_v = \frac{(201)(1\|1)}{0.943 + (201)(1\|1)}\left(\frac{50.3}{50.3+10}\right)$$

$$A_v = 0.826$$

6.53

$$V_o = (1+\beta) I_b R_L$$

$$I_b = \frac{V_s}{r_\pi + (1+\beta) R_L}$$

$$\text{so } A_v = \frac{(1+\beta) R_L}{r_\pi + (1+\beta) R_L}$$

For $\beta = 100,\ R_L = 0.5\ k\Omega$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2\ k\Omega$$

$$\text{Then } A_v(\min) = \frac{(101)(0.5)}{5.2 + (101)(0.5)} = 0.9066$$

Then $\beta = 180,\ R_L = 500\ k\Omega$

$$r_\pi = \frac{(180)(0.026)}{0.5} = 9.36\ k\Omega$$

$$\text{Then } A_v(\max) = \frac{(181)(500)}{9.36 + (181)(500)} = 0.9999$$

6.54

(a) $V_{CEQ} = V_{CC} - I_{EQ} R_E$

Let $V_{CEQ} = 2.5$ V, then $2.5 = 5 - I_{EQ}(0.5) \Rightarrow I_{EQ} \cong I_{CQ} = 5$ mA, $I_{BQ} = 0.04167$ mA

$$r_\pi = \frac{(120)(0.026)}{5} = 0.624\ \text{k}\Omega$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L) = 0.624 + (121)(0.5\|0.5) = 30.87\ \text{k}\Omega$$

$$A_i = (1+\beta)\left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right)\left(\frac{R_E}{R_E + R_L}\right)$$

$$10 = (121)\left(\frac{R_{TH}}{R_{TH} + 30.87}\right)\left(\frac{0.5}{0.5+0.5}\right) \Rightarrow R_{TH} = 6.113\ \text{k}\Omega$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + I_{EQ} R_E = (0.04167)(6.113) + 0.7 + (5)(0.5) = 3.455\ \text{V}$$

$$V_{TH} = 3.455 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(6.113)(5)$$

So $R_1 = 8.85\ \text{k}\Omega$ and $R_2 = 19.8\ \text{k}\Omega$

$$R_o = R_E \left\| \frac{r_\pi}{1+\beta} = 0.5 \right\| \frac{0.624}{121} \Rightarrow R_o = 5.10\,\Omega$$

(b) $$A_i = (1+\beta)\left(\frac{R_{TH}}{R_{TH}+R_{ib}}\right)\left(\frac{R_E}{R_E+R_L}\right)$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L) = 0.624 + (121)(0.5\|2) = 49.02\ \text{k}\Omega$$

$$A_i = (121)\left(\frac{6.113}{6.113+49.02}\right)\left(\frac{0.5}{0.5+2}\right) = 2.68$$

6.55

$R_i = R_{TH} \| R_{ib}$ where $R_{ib} = r_\pi + (1+\beta)R_E$

$V_{CEQ} = 3.5, \quad I_{CQ}\dfrac{5-3.5}{2} = 0.75\ mA$

$$r_\pi = \frac{(120)(0.026)}{0.75} = 4.16\ k\Omega$$

$$R_{ib} = 4.16 + (121)(2) = 246\ k\Omega$$

Then $R_i = 120 = R_{TH} \| 246 \Rightarrow R_{TH} = 234\ k\Omega$

$$I_{BQ} = \frac{0.75}{120} = 0.00625\ mA$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E$$

$$\frac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = \frac{1}{R_1}(234)(5) = (0.00625)(234) + 0.7 + (121)(0.00625)(2)$$

which yields $\underline{R_1 = 318\ k\Omega}$ and $\underline{R_2 = 886\ k\Omega}$

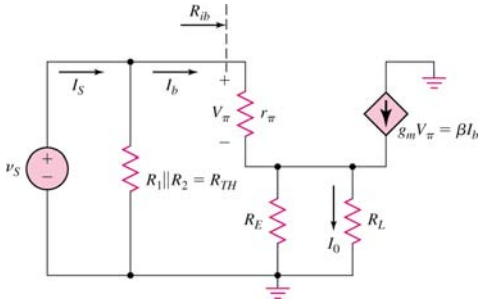
6.56

a.

Let $R_E = 24\ \Omega$ and $V_{CEQ} = \frac{1}{2}V_{CC} = 12\ \text{V} \Rightarrow I_{EQ} = \dfrac{12}{24} = 0.5\ \text{A}$

$I_{CQ} = 0.493\ \text{A},\ I_{BQ} = 6.58\ \text{mA}$

$$r_\pi = \frac{(75)(0.026)}{0.493} = 3.96\ \Omega$$



$$I_0 = (1+\beta)I_b\left(\frac{R_E}{R_E+R_L}\right)$$

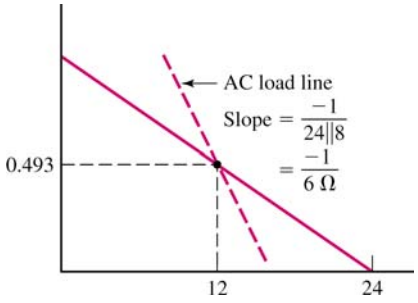
$$I_b = I_S\left(\frac{R_{TH}}{R_{TH}+R_{ib}}\right)$$

$$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$$
$$= 3.96 + (76)(24\|8) \Rightarrow R_{ib} = 460\ \Omega$$
$$A_i = \frac{I_0}{I_S} = (1+\beta)\left(\frac{R_E}{R_E + R_L}\right)\left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right)$$
$$8 = (76)\left(\frac{24}{24+8}\right)\left(\frac{R_{TH}}{R_{TH} + 460}\right)$$
$$0.140 = \frac{R_{TH}}{R_{TH} + 460} \Rightarrow \underline{R_{TH} = 74.9\ \Omega} \text{ (Minimum value)}$$

dc analysis:

$$V_{TH} = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$
$$= I_{BQ} R_{TH} + V_{BE}(\text{on}) + I_{EQ} R_E$$
$$\frac{1}{R_1}(74.9)(24) = (0.00658)(74.9) + 0.70 + (0.5)(24)$$
$$= 13.19$$
$$\underline{R_1 = 136\ \Omega}, \quad \frac{136 R_2}{136 + R_2} = 74.9 \Rightarrow \underline{R_2 = 167\ \Omega}$$

b.



$$\Delta i_C = -\frac{1}{6}\Delta v_{ce}$$

For $\Delta i_C = 0.493 \Rightarrow |\Delta v_{ce}| = (0.493)(6) \Rightarrow$ Max. swing in output voltage for this design

$= \underline{5.92 \text{ V peak-to-peak}}$

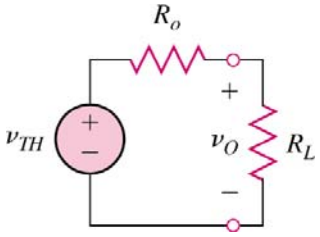
c.

$$R_0 = \frac{r_\pi}{1+\beta} \left\| R_E = \frac{3.96}{76} \right\| 24 = 0.0521 \| 24 \Rightarrow \underline{R_0 = 52\ \text{m}\Omega}$$

6.57

The output of the emitter follower is

$$v_o = \left(\frac{R_L}{R_L + R_o}\right) \cdot v_{TH}$$



For v_O to be within 5% for a range of R_L, we have

$$\frac{R_L(\min)}{R_L(\min) + R_o} = (0.95)\frac{R_L(\max)}{R_L(\max) + R_o}$$

$$\frac{4}{4 + R_o} = (0.95)\frac{10}{10 + R_o} \text{ which yields } R_o = 0.364\ k\Omega$$

We have $R_o = \left(\dfrac{r_\pi + R_1 \| R_2 \| R_S}{1+\beta}\right) \| R_E \| r_o$

The first term dominates

Let $R_1 \| R_2 \| R_S \cong R_S$, then

$$R_o \cong \frac{r_\pi + R_S}{1+\beta} \Rightarrow 0.364 = \frac{r_\pi + 4}{1+\beta}$$

or $$0.364 = \frac{r_\pi}{1+\beta} + \frac{4}{1+\beta} = \frac{\beta V_T}{I_{CQ}(1+\beta)} + \frac{4}{1+\beta}$$

$$0.364 \cong \frac{V_T}{I_{CQ}} + \frac{4}{1+\beta}$$

The factor $\dfrac{4}{1+\beta}$ is in the range of $\dfrac{4}{91} = 0.044$ to $\dfrac{4}{131} = 0.0305$. We can set $R_o \cong 0.32 = \dfrac{V_T}{I_{CQ}}$

Or $I_{CQ} = 0.08125\ mA$. To take into account other factors, set $I_{CQ} = 0.15\ mA$,

$$I_{BQ} = \frac{0.15}{110} = 0.00136\ mA$$

For $V_{CEQ} \cong 5\ V$, set $R_E = \dfrac{5}{0.15} = 33.3\ k\Omega$

Design a bias stable circuit.

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(10) - 5 = \frac{1}{R_1}(R_{TH})(10) - 5$$

$$R_{TH} = (0.1)(1+\beta)R_E = (0.1)(111)(33.3) = 370\ k\Omega$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + (1+\beta)I_{BQ}R_E - 5$$

So $\frac{1}{R_1}(370)(10)-5=(0.00136)(370)+0.7+(111)(0.00136)(33.3)-5$

which yields $\underline{R_1 = 594\ k\Omega}$ and $\underline{R_2 = 981\ k\Omega}$

Now $A_v = \dfrac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \cdot \left(\dfrac{R_{TH} \| R_{ib}}{R_{TH} \| R_{ib} + R_S} \right)$

$R_{ib} = r_\pi + (1+\beta)(R_E \| R_L)$ and $r_\pi = \dfrac{\beta V_T}{I_{CQ}}$

$\underline{\text{For } \beta = 90,\ R_L = 4\ k\Omega,}$

$r_\pi = 15.6\ \text{k}\Omega\ ,\quad R_{ib} = 340.6\ \text{k}\Omega$

$$A_v = \frac{(91)(33.3 \| 4)}{15.6 + (91)(33.3 \| 4)} \cdot \frac{370 \| 340.6}{370 \| 340.6 + 4} \Rightarrow A_v = 0.9332$$

$\underline{\text{For } \beta = 90,\ R_L = 10\ k\Omega}$

$R_{ib} = 715.4\ \text{k}\Omega$

$$A_v = \frac{(91)(33.3 \| 10)}{15.6 + (91)(33.3 \| 10)} \cdot \frac{370 \| 715.4}{370 \| 715.4 + 4} \Rightarrow A_v = 0.9625$$

$\underline{\text{For } \beta = 130,\ R_L = 4\ k\Omega}$

$r_\pi = 22.5\ \text{k}\Omega\ ,\quad R_{ib} = 490\ \text{k}\Omega$

$$A_v = \frac{(131)(33.3 \| 4)}{22.5 + (131)(33.3 \| 4)} \cdot \frac{370 \| 490}{370 \| 490 + 4} \Rightarrow A_v = 0.9360$$

$\underline{\text{For } \beta = 130,\ R_L = 10\ k\Omega}$

$R_{ib} = 1030\ \text{k}\Omega$

$$A_v = \frac{(131)(33.3 \| 10)}{22.5 + (131)(33.3 \| 10)} \cdot \frac{370 \| 1030}{370 \| 1030 + 4} \Rightarrow A_v = 0.9645$$

Now $v_O(\min) = |A_v(\min)| . v_S = 3.73 \sin \omega t$

$v_O(\max) = |A_v(\max)| . v_S = 3.86 \sin \omega t$

$$\frac{\Delta v_O}{v_O} = 3.5\%$$

6.58

$P_{AVG} = i_L^2(rms) R_L \Rightarrow 1 = i_L^2(rms)(12)$

so $i_L(rms) = 0.289\ A \Rightarrow i_L(peak) = \sqrt{2}(0.289)$

$i_L(peak) = 0.409\ A$

$v_L(peak) = i_L(peak) \cdot R_L = (0.409)(12) = 4.91\ V$

Need a gain of $\dfrac{4.91}{5} = 0.982$

With $R_S = 10\ k\Omega,$ we will not be able to meet this voltage gain requirement. Need to insert a buffer or an op-amp voltage follower (see Chapter 9) between R_S and C_{C1}.

Set $I_{EQ} = 0.5A,\ V_{CEQ} = \frac{1}{3}\left(12-(-12)\right) = 8\ V$

$24 = I_{EQ}R_E + V_{CEQ} = (0.5)R_E + 8 \Rightarrow \underline{R_E = 32\ \Omega}$

Let $\beta = 50,\ I_{CQ} = \frac{50}{51}(0.5) = 0.49\ A$

$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(50)(0.026)}{0.49} = 2.65\ \Omega$

$R_{ib} = r_\pi + (1+\beta)\left(R_E \| R_L\right) = 2.65 + (51)(32\|12)$

$R_{ib} = 448\ \Omega$

$A_v = \frac{(1+\beta)\left(R_E \| R_L\right)}{r_\pi + (1+\beta)\left(R_E \| R_L\right)} = \frac{(51)(32\|12)}{2.65 + (51)(32\|12)} = 0.994$

So gain requirement has been met.

$I_{BQ} = \frac{0.49}{50} = 0.0098\ A = 9.8\ mA$

Let $I_R \cong \frac{24}{R_1 + R_2} \cong 10I_B = 98\ mA$

So that $R_1 + R_2 = 245\ \Omega$

$V_{TH} = \frac{R_2}{R_1 + R_2}(24) - 12 = I_{BQ}R_{TH} + V_{BE}\ (on) + I_{EQ}R_E - 12$

$\left(\frac{R_2}{245}\right)(24) = \frac{(0.0098)R_1R_2}{245} + 0.7 + (0.5)(32)$

Now $R_1 = 245 - R_2$

So we obtain

$4\times10^{-5}R_2^2 + 0.0882R_2 - 16.7 = 0$ which yields $R_2 = 175\ \Omega$ and $R_1 = 70\ \Omega$

6.59

(a) $g_m = \frac{1}{0.026} = 38.46$ mA/V

$A_v = g_m R_C = (38.46)(2) = 76.9$

(b) $A_i = \frac{\beta}{1+\beta} = \frac{120}{121} = 0.9917$

(c) $R_i = \frac{r_\pi}{1+\beta}$, $r_\pi = \frac{(120)(0.026)}{1} = 3.12\ \text{k}\Omega$

$R_i = \frac{3.12}{121} \Rightarrow R_i = 25.8\ \Omega$

(d) $R_o = R_C = 2\ \text{k}\Omega$

6.60

(a) $I_{CQ} = \left(\frac{80}{81}\right)(2) = 1.975$ mA

$g_m = \frac{1.975}{0.026} = 75.97$ mA/V, $r_\pi = \frac{(80)(0.026)}{1.975} = 1.053\ \text{k}\Omega$

$$A_\upsilon = g_m\left(R_C \| R_L\right) = (75.97)(1.5 \| 2.5) = 71.2$$

(b) $$A_i = \left(\frac{\beta}{1+\beta}\right)\left(\frac{R_C}{R_C + R_L}\right) = \left(\frac{80}{81}\right)\left(\frac{1.5}{1.5+2.5}\right) = 0.370$$

(c) $$R_i = \frac{r_\pi}{1+\beta} = \frac{1.053}{81} \Rightarrow R_i = 13\,\Omega$$

(d) $$r_o = \frac{80}{1.975} = 40.5\ \text{k}\,\Omega$$

(i) $$R_{oc} = r_o = 40.5\ \text{k}\,\Omega$$

(ii) $$R_o = r_o \| R_C \| R_L = 40.5 \| 1.5 \| 2.5 = 0.916\ \text{k}\,\Omega$$

6.61

(a) $$I_{CQ} = \left(\frac{110}{111}\right)(0.5) = 0.4955\ \text{mA}$$

$$g_m = \frac{0.4955}{0.026} = 19.06\ \text{mA/V}, \quad r_\pi = \frac{(110)(0.026)}{0.4955} = 5.77\ \text{k}\,\Omega$$

$$A_\upsilon = g_m\left(\frac{R_C}{R_S}\right)\left[\frac{r_\pi}{1+\beta} \| R_S\right] = (19.06)\left(\frac{4}{1}\right)\left[\frac{5.77}{111} \| 1\right] = (76.24)[0.05198 \| 1]$$

$$A_\upsilon = 3.77$$

(b) $$A_i = \frac{\beta}{1+\beta} = \frac{110}{111} = 0.991$$

(c) $$R_i = R_S + \frac{r_\pi}{1+\beta} = 1 + \frac{5.77}{111} = 1.052\ \text{k}\,\Omega$$

(d) $$R_o = R_C = 4\ \text{k}\,\Omega$$

6.62

(a) $$I_{R2} = \frac{0.7}{2.8} = 0.25\ \text{mA}$$

$$I_{CQ} \cong 1.50 - 0.25 = 1.25\ \text{mA}$$

$$V_C = (0.25)(5 + 2.8) = 1.95\ \text{V} = V_{CEQ}$$

(b) $$g_m = \frac{1.25}{0.026} = 48.08\ \text{mA/V}, \quad r_\pi = \frac{(120)(0.026)}{1.25} = 2.5\ \text{k}\,\Omega$$

$$A_\upsilon = g_m\left(R_1 \| R_L\right) = (48.08)(5 \| 10) = 160.3$$

(c) $$A_i = \left(\frac{\beta}{1+\beta}\right)\left(\frac{R_1}{R_1 + R_L}\right) = \left(\frac{120}{121}\right)\left(\frac{5}{5+10}\right) = 0.3306$$

6.63

(a)

$$I_{EQ} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}$$

$$\underline{I_{CQ} = 0.921 \text{ mA}}$$

$$V_{ECQ} = 20 - (0.93)(10) - (0.921)(5)$$

$$\underline{V_{ECQ} = 6.10 \text{ V}}$$

(b)

$$g_m = \frac{0.921}{0.026} = 35.42 \text{ mA/V}$$

$$A_v = g_m (R_C \| R_L) = (35.42)(5 \| 50)$$

$$\underline{A_v = 161}$$

6.64

(a) $I_{EQ} = 0.93 \text{ mA}, \quad I_{CQ} = 0.921 \text{ mA}$

$$V_{ECQ} = 6.10 \text{ V}$$

(b) $$g_m = \frac{0.921}{0.026} = 35.42 \text{ mA/V} \quad r_\pi = 2.82 \text{ K}$$

From Eq. 6.90

$$A_v = g_m \frac{(R_C \| R_L)}{R_S} \left[\frac{r_\pi}{1+\beta} \| R_E \| R_S \right]$$

$$= \frac{(35.42)(50 \| 5)}{0.1} \left[\frac{2.82}{101} \| 10 \| 0.1 \right]$$

$$A_v = \frac{(35.42)(4.545)}{0.1} [0.0218]$$

$$\underline{A_v = 35.1}$$

6.65

(a) $I_{EQ} = 1 \text{ mA}, \quad I_{CQ} = 0.9917 \text{ mA}$

$$V_C = 5 - (0.9917)(2) = 3.017 \text{ V}$$

$$V_E = -0.7 \text{ V}$$

$$\underline{V_{CEQ} = 3.72 \text{ V}}$$

(b)

$$A_v = g_m (R_C \| R_L)$$

$$g_m = \frac{0.9917}{0.026} = 38.14 \text{ mA/V}$$

$$A_v = (38.14)(2 \| 10) \Rightarrow \underline{A_v = 63.6}$$

6.66

a. Emitter current

$$I_{EQ} = I_{CC} = 0.5 \text{ mA}$$

$$I_{BQ} = \frac{0.5}{101} = 0.00495 \text{ mA}$$

$$V_E = I_{EQ}R_E = (0.5)(1) \Rightarrow \underline{V_E = 0.5 \text{ V}}$$

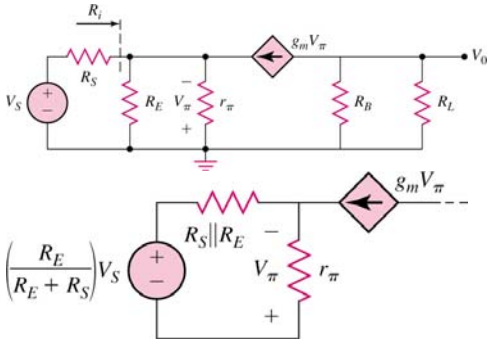
$$V_B = V_E + V_{BE}(\text{on}) = 0.5 + 0.7 \Rightarrow \underline{V_B = 1.20 \text{ V}}$$

$$V_C = V_B + I_{BQ}R_B = 1.20 + (0.00495)(100) \Rightarrow \underline{V_C = 1.7 \text{ V}}$$

b.

$$r_\pi = \frac{(100)(0.026)}{(100)(0.00495)} = 5.25 \text{ k}\Omega$$

$$g_m = \frac{(100)(0.00495)}{0.026} = 19.0 \text{ mA/V}$$



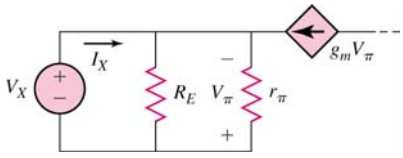
$$V_o = -g_m V_\pi (R_B \| R_L)$$

$$V_\pi = \frac{-(R_E \| R_{ie})}{R_E \| R_{ie} + R_S} \cdot V_s = -(0.4971)V_s$$

$$V_o = (19)(0.4971)V_s(100 \| 1)$$

$$A_\upsilon = 9.37$$

c.



$$I_X = \frac{V_X}{R_E} + \frac{V_X}{r_\pi} - g_m V_\pi, \quad V_\pi = -V_X$$

$$\frac{I_X}{V_X} = \frac{1}{R_i} = \frac{1}{R_E} + \frac{1}{r_\pi} + g_m$$

or $$R_i = R_E \| r_\pi \left\| \frac{1}{g_m} = 1 \| 5.253 \right\| \frac{1}{19} = 0.84 \| 0.05252$$

$$R_i = 49.4 \, \Omega$$

6.67

(a) $R_{TH} = R_1 \| R_2 = 150 \| 50 = 37.5 \ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{50}{50+150}\right)(9) = 2.25 \text{ V}$$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + (1+\beta) I_{BQ} R_E$$

or $I_{BQ} = \dfrac{2.25 - 0.7}{37.5 + (126)(3)} = 0.00373 \text{ mA}$, $I_{CQ} = 0.4663 \text{ mA}$, $I_{EQ} = 0.470 \text{ mA}$

$$V_{CEQ} = 9 - (0.4663)(6) - (0.470)(3) = 4.79 \text{ V}$$

(b) $g_m = \dfrac{0.4663}{0.026} = 17.93 \text{ mA/V}$, $r_\pi = \dfrac{(125)(0.026)}{0.4663} = 6.97 \text{ k}\Omega$

$$R_m = \frac{\upsilon_o}{i_s} = \left(\frac{R_S \| R_E}{R_S \| R_E + R_{ie}}\right)\left(\frac{\beta}{1+\beta}\right)(R_C \| R_L)$$

We find $R_S \| R_E = 100 \| 3 = 2.913 \ \text{k}\Omega$

$$R_{ie} = \frac{r_\pi + R_{TH}}{1+\beta} = \frac{6.97 + 37.5}{126} = 0.3529 \text{ k}\Omega$$

$$R_m = \left(\frac{2.913}{2.913 + 0.3529}\right)\left(\frac{125}{126}\right)(6 \| 4) = 2.12 \text{ V/mA}$$

(c) $\upsilon_s = i_s\left(R_S \| R_E \| R_{ie}\right) = i_s\left(100 \| 3 \| 0.3529\right) = i_s(0.3148)$

So $i_s = \dfrac{\upsilon_s}{0.3148}$

Then $A_\upsilon = \dfrac{\upsilon_o}{\upsilon_s} = \dfrac{\upsilon_o}{i_s} \cdot \dfrac{i_s}{\upsilon_s} = \dfrac{2.12}{0.3148} = 6.73$

6.68

(a) $V_{CEQ} \cong V_{CC} - I_{CQ}(R_C + R_E)$

Let $V_{CEQ} = 2.5 \text{ V}$ and $I_{CQ} = 0.25 \text{ mA}$

Then $2.5 = 5 - (0.25)(R_C + 0.5) \Rightarrow R_C = 9.5 \ \text{k}\Omega$

$$\upsilon_o = i_c(R_C \| R_L) = \left(\frac{\beta}{1+\beta}\right) \cdot i_e \cdot (R_C \| R_L)$$

$$i_e = \frac{\upsilon_s}{R_{ie}} = \frac{\upsilon_s}{\left(\dfrac{r_\pi + R_{TH}}{1+\beta}\right)}$$

$$A_\upsilon = \frac{\upsilon_o}{\upsilon_s} = \left(\frac{\beta}{r_\pi + R_{TH}}\right)(R_C \| R_L)$$

Now $r_\pi = \dfrac{(100)(0.026)}{0.25} = 10.4 \ \text{k}\Omega$

$$A_\upsilon = 25 = \left(\frac{100}{10.4 + R_{TH}}\right)(9.5 \| 12) = \frac{530.2}{10.4 + R_{TH}} \Rightarrow R_{TH} = 10.81 \ \text{k}\Omega$$

Also $I_{BQ} = 0.0025 \text{ mA}$

$$V_{TH} = I_{BQ} R_{TH} + V_{BE}(on) + (1+\beta) I_{BQ} R_E = (0.0025)(10.81) + 0.7 + (101)(0.0025)(0.5)$$

$$V_{TH} = 0.853275 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1}(10.81)(5)$$

We obtain $R_1 = 63.3\,\text{k}\Omega$ and $R_2 = 13\,\text{k}\Omega$

(b) From part (a), $I_{CQ} = 0.25\,\text{mA}$, $V_{CEQ} = 2.5\,\text{V}$

(c) $A_v = g_m\left(R_C \| R_L\right)$

$$g_m = \frac{0.25}{0.026} = 9.615\ \text{mA/V}$$

$$A_v = (9.615)(9.5 \| 12) = 51$$

6.69

(a)

$$I_{CQ} = \left(\frac{60}{61}\right)(1) \Rightarrow I_{CQ} = 0.984\ \text{mA}$$

$$V_{CEQ} = I_{BQ}R_B + V_{BE}\,(on) = \left(\frac{1}{61}\right)(100) + 0.7$$

$$\underline{V_{CEQ} = 2.34\ \text{V}}$$

(b)

$$A_v = g_m \frac{\left(R_B \| R_L\right)}{R_S}\left[\frac{r_\pi}{1+\beta} \| R_S\right]$$

$$g_m = \frac{0.984}{0.026} = 37.85\ \text{mA/V}$$

$$r_\pi = 1.59\ \text{K}$$

$$A_v = \frac{(37.85)(100 \| 2)}{0.05}\left[\frac{1.59}{61} \| 0.05\right]$$

$$= 1484\left[0.0261 \| 0.05\right]$$

$$A_v = 25.4$$

6.70

$i_s(peak) = 2.5\ \mu\text{A},\ V_o(peak) = 5\ \text{mV}$

So we need $R_m = \dfrac{\upsilon_o}{i_s} = \dfrac{5\times10^{-3}}{2.5\times10^{-6}} = 2\times10^3\ \Omega \Rightarrow R_m = 2\ \text{k}\Omega$

We have

$$\frac{V_o}{I_s} = \left(\frac{\beta}{1+\beta}\right)(R_C \square R_L)\left(\frac{R_S \| R_E}{R_S \| R_E + R_{ie}}\right)$$

Let $R_C = 4\ k\Omega,\ R_L = 5\ k\Omega,\ R_E = 2\ k\Omega$

Now $\beta = 120,$ so we have

$$2 = \left(\frac{120}{121}\right)(4\|5)\left(\frac{R_S \| R_E}{R_S \| R_E + R_{ie}}\right) = 2.204\left(\frac{R_S \| R_E}{R_S \| R_E + R_{ie}}\right)$$

Then $\dfrac{R_S \| R_E}{R_S \| R_E + R_e} = 0.9075$

$R_S \| R_E = 50\|2 = 1.923\ k\Omega,$ so that $R_{ie} = 0.196\ k\Omega$

Assume $V_{CEQ} = 3\ V$

$V_{CC} \cong I_{CQ}(R_C + R_E) + V_{CEQ}$

$5 = I_{CQ}(4+2) + 3 \Rightarrow I_{CQ} = 0.333\ mA$

$r_\pi = \dfrac{(120)(0.026)}{0.333} = 9.37\ k\Omega$

$R_{ie} = \dfrac{r_\pi + R_{TH}}{1+\beta} \Rightarrow 0.196 = \dfrac{9.37 + R_{TH}}{121}$

which yields $R_{TH} = 14.35\ k\Omega$

Now $V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E$

$I_{BQ} = \dfrac{1}{120} = 0.00833\ mA,\ I_{EQ} = \left(\dfrac{121}{120}\right)(1) = 1.008\ mA$

$V_{TH} = \dfrac{1}{R_1}\cdot R_{TH}\cdot V_{CC} = \dfrac{1}{R_1}(14.35)(5) = (0.00833)(14.35) + 0.7 + (1.008)(2)$

which yields $R_1 = 25.3\ k\Omega$

and $R_2 = 33.2\ k\Omega$

6.71

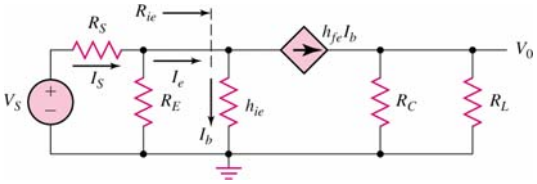
a.

$$I_{EQ} = \frac{20 - 0.7}{10} = 1.93\ \text{mA}$$

$$\underline{I_{CQ} = 1.91\ \text{mA}}$$

$$V_{ECQ} = V_{CC} + V_{EB}(\text{on}) - I_C R_C$$
$$= 25 + 0.7 - (1.91)(6.5) \Rightarrow \underline{V_{ECQ} = 13.3\ \text{V}}$$

b.



Neglect effect h_{oe}

Assume

$$2.45 \le h_{ie} \le 3.7 \text{ k}\Omega$$

$$80 \le h_{fe} \le 120$$

$$V_o = (h_{fe} I_b)(R_C \| R_L)$$

$$R_{ie} = \frac{h_{ie}}{1+h_{fe}}, \quad I_e = \left(\frac{R_E}{R_E + R_{ie}}\right) \cdot I_s$$

$$I_b = \left(\frac{I_e}{1+h_{fe}}\right), \quad I_s = \frac{V_s}{R_S + (R_E \| R_{ie})}$$

$$A_\upsilon = \left(\frac{h_{fe}}{1+h_{fe}}\right)(R_C \| R_L)\left(\frac{R_E}{R_E + R_{ie}}\right)\left(\frac{1}{R_S + R_E \| R_{ie}}\right)$$

High gain device: $h_{ie} = 3.7 \text{ k}\Omega,\ h_{fe} = 120$

$$R_{ie} = \frac{3.7}{121} = 0.0306 \text{ k}\Omega$$

$$R_E \| R_{ie} = 10 \| 0.0306 = 0.0305 \text{ k}\Omega$$

$$A_\upsilon = \left(\frac{120}{121}\right)(6.5 \| 5)\left(\frac{10}{10+0.0306}\right)\left(\frac{1}{1+0.0305}\right) \Rightarrow A_\upsilon = 2.711$$

Low gain device: $h_{ie} = 2.45 \text{ k}\Omega,\ h_{fe} = 80$

$$R_{ie} = \frac{2.45}{81} = 0.03025 \text{ k}\Omega$$

$$R_E \| R_{ie} = 10 \| 0.03025 = 0.0302 \text{ k}\Omega$$

$$A_\upsilon = \left(\frac{80}{81}\right)(6.5 \| 5)\left(\frac{10}{10+0.03025}\right)\left(\frac{1}{1+0.0302}\right) \Rightarrow A_\upsilon = 2.70$$

$\underline{2.70 \le A_v \le 2.71}$ So $A_\upsilon \cong$ constant

c.

$$R_i = R_E \| R_{ie}$$

We found $\underline{0.0302 \le R_i \le 0.0305 \text{ k}\Omega}$

Neglecting h_{oe}, $\underline{R_o = R_C = 6.5 \text{ k}\Omega}$

6.72

a. Small-signal voltage gain

$$A_v = g_m\left(R_C \| R_L\right) \Rightarrow 25 = g_m\left(R_C \| 1\right)$$

For $V_{ECQ} = 3\text{ V}$, $V_C = -V_{ECQ} + V_{EB}(on) = -3 + 0.7 = -2.3\text{ V}$

$$V_{CC} - I_{CQ}R_C + V_C = 0 \Rightarrow I_{CQ} = \frac{5 - 2.3}{R_C} = \frac{2.7}{R_C}$$

For $I_{CQ} = 1\text{ mA}$, $R_C = 2.7\text{ k}\Omega$

$$g_m = \frac{1}{0.026} = 38.5\text{ mA/V}$$

$$A_v = (38.5)(2.7 \| 1) = 28.1$$

Design criterion satisfied and V_{ECQ} satisfied.

$$I_E = \left(\frac{101}{100}\right)(1) = 1.01\text{ mA}$$

$$V_{EE} = I_E R_E + V_{EB}(\text{on}) \Rightarrow R_E = \frac{5 - 0.7}{1.01} \Rightarrow \underline{R_E = 4.26\text{ k}\Omega}$$

b.

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1} \Rightarrow \underline{r_\pi = 2.6\text{ k}\Omega},\ \underline{g_m = 38.5\text{ mA/V}},\ \underline{r_o = \infty}$$

6.73

(a) $$g_{m1} = g_{m2} = \frac{1}{0.026} = 38.46\text{ mA/V}$$

$$r_{\pi 1} = \frac{(120)(0.026)}{1} = 3.12\text{ k}\Omega\text{ ,}\quad r_{\pi 2} = \frac{(80)(0.026)}{1} = 2.08\text{ k}\Omega$$

$$A_{v1} = \frac{V_{o1}}{V_i} = \frac{(1+\beta_1)\left(R_E \left\| \dfrac{r_{\pi 2}}{1+\beta_2}\right.\right)}{r_{\pi 1} + (1+\beta_1)\left(R_E \left\| \dfrac{r_{\pi 2}}{1+\beta_2}\right.\right)}$$

We find $$R_E \left\| \frac{r_{\pi 2}}{1+\beta_2} = 1 \right\| \frac{2.08}{81} = 0.02504\text{ k}\Omega$$

Then $$A_{v1} = \frac{(121)(0.02504)}{3.12 + (121)(0.02504)} = 0.4927$$

(b) $$A_{v2} = \frac{V_{o2}}{V_{o1}} = g_{m2}R_C = (38.46)(4) = 153.8$$

(c) $$A_v = A_{v1} \cdot A_{v2} = (0.4927)(153.8) = 75.8$$

6.74

(a) $$g_{m1} = \frac{0.5}{0.026} = 19.23\text{ mA/V},\quad g_{m2} = \frac{2}{0.026} = 76.92\text{ mA/V}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{0.5} = 5.2\text{ k}\Omega\text{ ,}\quad r_{\pi 2} = \frac{(100)(0.026)}{2} = 1.3\text{ k}\Omega$$

$$A_{v1} = \frac{V_{o1}}{V_i} = -g_{m1}\left(R_{C1} \| R_{ib2}\right)$$

where $R_{ib2} = r_{\pi 2} + (1+\beta_2)R_{E2} = 1.3 + (101)(4) = 405.3\ \text{k}\Omega$

$$A_{v1} = -(19.23)(4 \| 405.3) = -76.17$$

(b) $$A_{v2} = \frac{V_{o2}}{V_{o1}} = \frac{(1+\beta_2)(R_{E2})}{r_{\pi 2} + (1+\beta_2)R_{E2}} = \frac{(101)(4)}{1.3 + (101)(4)} = 0.9968$$

(c) $$A_v = A_{v1} \cdot A_{v2} = (-76.17)(0.9968) = -75.93$$

6.75

a.

$$V_{TH1} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{20}{20+80}\right)(10) = 2.0\ \text{V}$$

$$R_{TH1} = R_1 \| R_2 = 20 \| 80 = 16\ \text{k}\Omega$$

$$I_{B1} = \frac{2 - 0.7}{16 + (101)(1)} = 0.0111\ \text{mA}$$

$$I_{C1} = 1.11\ \text{mA};\quad g_{m1} = \frac{1.11}{0.026} = 42.74\ \text{mA/V}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{1.11} = 2.34\ \text{k}\Omega$$

$$r_{o1} = \frac{\infty}{1.11} = \infty$$

$$V_{TH2} = \left(\frac{R_4}{R_3 + R_4}\right) \cdot V_{CC} = \left(\frac{15}{15+85}\right)(10) = 1.50\ \text{V}$$

$$R_{TH2} = R_3 \| R_4 = 15 \| 85 = 12.75\ \text{k}\Omega$$

$$I_{B2} = \frac{1.50 - 0.70}{12.75 + (101)(0.5)} = 0.01265\ \text{mA}$$

$$I_{C2} = 1.265\ \text{mA} \Rightarrow g_{m2} = \frac{1.265}{0.026} \Rightarrow \underline{g_{m2} = 48.65\ \text{mA/V}}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.26} \Rightarrow \underline{r_{\pi 2} = 2.06\ \text{k}\Omega}$$

$$\underline{r_{02} = \infty}$$

b.

$$A_{v1} = -g_{m1}R_{C1} = -(42.7)(2) \Rightarrow \underline{A_{v1} = -85.48}$$

$$A_{v2} = -g_{m2}\left(R_{C2} \| R_L\right) = -(48.5)(4 \| 4) \Rightarrow \underline{A_{v2} = -97.3}$$

c. Input resistance of 2nd stage

$$R_{i2} = R_3 \| R_4 \| r_{\pi 2} = 15 \| 85 \| 2.06 = 12.75 \| 2.06$$

$$R_{i2} = 1.773\ \text{k}\Omega$$

$$A'_{v1} = -g_{m1}\left(R_{C1} \| R_{i2}\right) = -(42.7)(2 \| 1.773)$$

$$A'_{v1} = -40.17$$

Overall gain: $A_v = (-40.17)(-97.3) \Rightarrow \underline{A_v = 3909}$

If we had $A_{v1} \cdot A_{v2} = (-85.48)(-97.3) = 8317$

Loading effect reduces overall gain

6.76

a.

$$V_{TH1} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{12.7}{12.7 + 67.3}\right)(12) = 1.905 \text{ V}$$

$$R_{TH1} = R_1 \| R_2 = 12.7 \| 67.3 = 10.68 \text{ k}\Omega$$

$$I_{B1} = \frac{1.905 - 0.7}{10.68 + (121)(2)} = 0.00477 \text{ mA}$$

$$I_{C1} = 0.572 \text{ mA}$$

$$g_{m1} = \frac{0.572}{0.026} = 22 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(120)(0.026)}{0.572} = 5.45 \text{ k}\Omega$$

$$r_{o1} = \frac{\infty}{0.572} = \infty$$

$$V_{TH2} = \left(\frac{R_4}{R_3 + R_4}\right) \cdot V_{CC} = \left(\frac{45}{45 + 15}\right)(12) = 9.0 \text{ V}$$

$$R_{TH2} = R_3 \| R_4 = 15 \| 45 = 11.25 \text{ k}\Omega$$

$$I_{B2} = \frac{9.0 - 0.7}{11.25 + (121)(1.6)} = 0.0405 \text{ mA}$$

$$I_{C2} = 4.86 \text{ mA}$$

$$g_{m2} = \frac{4.86}{0.026} = 187 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(120)(0.026)}{4.86} = 0.642 \text{ k}\Omega, \quad r_{o2} = \infty$$

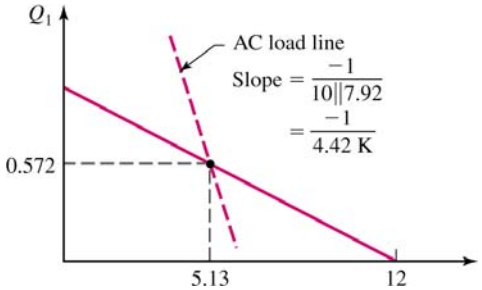
b.

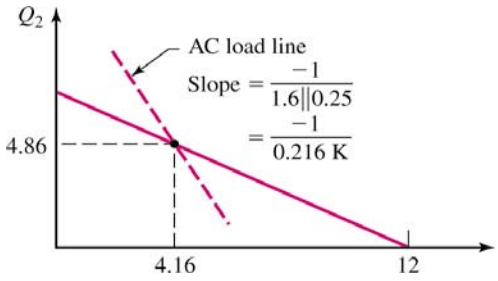
$$I_{E1} = 0.577 \text{ mA}$$

$$V_{CEQ1} = 12 - (0.572)(10) - (0.577)(2) \Rightarrow \underline{V_{CEQ1} = 5.13 \text{ V}}$$

$$I_{E2} = 4.90$$

$$V_{CEQ2} = 12 - (4.90)(1.6) \Rightarrow \underline{V_{CEQ2} = 4.16 \text{ V}}$$





$$R_{i2} = R_3\|R_4\|R_{ib}$$
$$R_{ib} = r_{\pi 2} + (1+\beta)(R_{E2}\|R_L)$$
$$= 0.642 + (121)(1.6\|0.25) = 26.8\ \text{k}\Omega$$
$$R_{i2} = 15\|45\|26.8 = 7.92\ \text{k}\Omega$$

c.

$$A_{v1} = -g_{m1}(R_{C1}\|R_{i2}) = -(22)(10\|7.92) \Rightarrow A_{v1} = -97.2$$
$$A_{v2} = \frac{(1+\beta)(R_{E2}\|R_L)}{r_{\pi 2} + (1+\beta)(R_{E2}\|R_L)} = \frac{(121)(0.216)}{0.642 + (121)(0.216)}$$
$$A_{v2} = 0.976$$

Overall gain, $A_v = -(97.2)(0.976) = -94.9$

d.

$$R_{iS} = R_1\|R_2\|r_{\pi 1} = 67.3\|12.7\|5.45 = 3.61\ \text{k}\Omega$$
$$R_o = \frac{r_{\pi 2} + R_S}{1+\beta}\|R_{E2}$$

where $R_S = R_3\|R_3\|R_{C1} = 15\|45\|10 = 5.29\ \text{k}\Omega$

$$R_o = \frac{0.642 + 5.29}{121}\|1.6 = 0.049\|1.6 \Rightarrow R_o = 47.6\ \Omega$$

e.

$$\Delta i_C = \frac{-1}{0.216\ \text{k}\Omega}\cdot\Delta v_{ce},\ \Delta i_C = 4.86$$
$$|\Delta v_{ce}| = (4.86)(0.216) = 1.05\ \text{V}$$

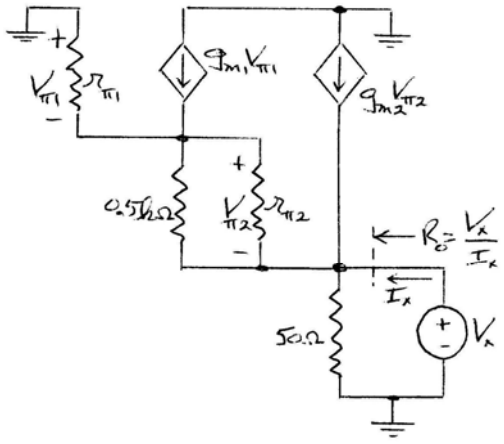
Max. output voltage swing = 2.10 V peak-to-peak

6.77

(a) $I_{R2} = \dfrac{5 - 2(0.7)}{0.050} = 72\ \text{mA},\ I_{R1} = \dfrac{0.7}{0.5} = 1.4\ \text{mA}$

$$I_{E2} = 72 - 1.4 = 70.6\ \text{mA},\ I_{C2} = \left(\frac{80}{81}\right)(70.6) = 69.73\ \text{mA},\ I_{B2} = \frac{70.6}{81} = 0.8716\ \text{mA}$$
$$I_{E1} = 1.4 + 0.8716 = 2.2716\ \text{mA},\ I_{C1} = \left(\frac{120}{121}\right)(2.2716) = 2.253\ \text{mA}$$

(b)



$$g_{m1} = \frac{2.253}{0.026} = 86.65 \text{ mA/V}, \quad g_{m2} = \frac{69.73}{0.026} = 2681.9 \text{ mA/V}$$

$$r_{\pi1} = \frac{(120)(0.026)}{2.253} = 1.385 \text{ k}\Omega, \quad r_{\pi2} = \frac{(80)(0.026)}{69.73} = 0.02983 \text{ k}\Omega$$

$$V_s = V_{\pi1} + V_{\pi2} + V_o$$

$$V_o = \left[\left(\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1}\right) + g_{m2}V_{\pi2}\right](0.05)$$

$$V_{\pi2} = \left(\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1}\right)(0.5 \| r_{\pi2}) = V_{\pi1}\left(\frac{1}{1.385} + 86.65\right)(0.5 \| 0.02983) = V_{\pi1}(2.4595)$$

$$V_o = \left[V_{\pi1}\left(\frac{1}{1.385} + 86.65\right) + (2681.9)V_{\pi1}(2.4595)\right](0.05) = V_{\pi1}(334.175)$$

$$V_s = V_{\pi1} + V_{\pi2} + V_o = V_{\pi1} + V_{\pi1}(2.4595) + V_o$$

So $V_{\pi1} = (V_s - V_o)(0.28906)$

And $V_o = (334.175)(V_s - V_o)(0.28906) = 96.596(V_s - V_o)$

$$A_v = \frac{V_o}{V_s} = 0.990$$

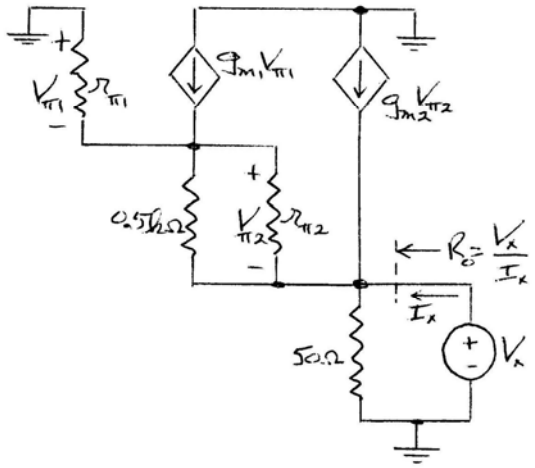
(c) For R_{ib}:

We have $V_o = (0.989754)V_s$

$$V_{\pi1} = (V_s - V_o)(0.28906) = V_s(1 - 0.98975)(0.28906) = V_s(0.0029618)$$

$$R_{ib} = \frac{V_s}{I_s} = \frac{V_s}{\left(V_{\pi1} / r_{\pi1}\right)} = \frac{r_{\pi1}}{0.0029618} \Rightarrow R_{ib} = 467.6 \text{ k}\Omega$$

For R_o :



(1) $V_{\pi 1} + V_{\pi 2} + V_x = 0$

(2) $I_x + g_{m2}V_{\pi 2} + \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right) = \frac{V_x}{0.05}$

We had $V_{\pi 2} = V_{\pi 1}(2.4595)$

(1) $V_{\pi 1} + V_{\pi 1}(2.4595) + V_x = 0 \Rightarrow V_{\pi 1} = -V_x(0.28906)$

(2) $I_x + g_{m2}V_{\pi 1}(2.4595) + V_{\pi 1}\left(\frac{1}{r_{\pi 1}} + g_{m1}\right) = \frac{V_x}{0.05}$

$$I_x + V_{\pi 1}\left[(2681.9)(2.4595) + \left(\frac{1}{1.385} + 86.65\right)\right] = \frac{V_x}{0.05}$$

$$I_x - V_x(0.28906)[6683.5] = \frac{V_x}{0.05}$$

$$R_o = \frac{V_x}{I_x} = 0.512\ \Omega$$

6.78

a.

$$R_{TH} = R_1 \| R_2 = 335 \| 125 = 91.0\ \text{k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC}$$

$$= \left(\frac{125}{125 + 335}\right)(10) = 2.717\ \text{V}$$

$$V_{TH} = I_{B1}R_{TH} + V_{BE1} + V_{BE2} + I_{E2}R_{E2}$$

$$I_{E2} = (1+\beta)I_{E1} = (1+\beta)^2 I_{B1}$$

$$I_{B1} = \frac{2.717 - 1.40}{91.0 + (101)^2(1)} \Rightarrow I_{B1} = 0.128\ \mu\text{A}$$

$$\underline{I_{C1} = 12.8\ \mu\text{A}}$$

$$I_{C2} = \beta I_{E1} = \beta(1+\beta) I_{B1} = (100)(101)(0.128\ \mu\text{A})$$

$$\underline{I_{C2} = 1.29\ \text{mA},\ I_{E2} = 1.31\ \text{mA}}$$

$$I_{RC} = I_{C2} + I_{C1} = 1.29 + 0.0128 = 1.30\ \text{mA}$$

$$V_C = 10 - I_{RC} R_C = 10 - (1.30)(2.2) = 7.14\ \text{V}$$

$$V_E = I_{E2} R_{E2} = (1.30)(1) = 1.30\ \text{V}$$

$$V_{CE2} = 7.14 - 1.30 = 5.84\ \text{V}$$

$$V_{CE1} = V_{CE2} - V_{BE2} = 5.84 - 0.7$$

$$V_{CE1} = 5.14\ \text{V}$$

Summary:

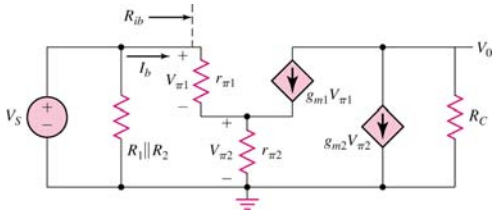
$$I_{C1} = 12.8\ \mu\text{A} \qquad I_{C2} = 1.29\ \text{mA}$$

$$V_{CE1} = 5.14\ \text{V} \qquad V_{CE2} = 5.84\ \text{V}$$

b.

$$g_{m1} = \frac{0.0128}{0.026} = 0.492\ \text{mA/V}$$

$$g_{m2} = \frac{1.292}{0.026} = 49.7\ \text{mA/V}$$



$$V_0 = -(g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2})R_C$$

$$V_S = V_{\pi 1} + V_{\pi 2}, \quad V_{\pi 1} = V_S - V_{\pi 2}$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right) r_{\pi 2}$$

$$V_{\pi 2} = V_{\pi 1}\left(\frac{1+\beta}{r_{\pi 1}}\right) r_{\pi 2}$$

$$V_0 = -\left[g_{m1}(V_S - V_{\pi 2}) + g_{m2}V_{\pi 2}\right] R_C$$

$$V_0 = -\left[g_{m1}V_S + (g_{m2} - g_{m1})V_{\pi 2}\right] R_C$$

$$V_{\pi 2} = (V_S - V_{\pi 2})(1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)$$

$$V_{\pi 2}\left[1 + (1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)\right] = V_S(1+\beta)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)$$

$$V_0 = -\left\{ g_{m1}V_S + \left(g_{m2} - g_{m1}\right)\cdot \frac{V_S\left(1+\beta\right)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)}{1+\left(1+\beta\right)\left(\frac{r_{\pi 2}}{r_{\pi 1}}\right)} \right\} R_C$$

$$A_v = \frac{V_0}{V_S}$$

$$= -\left\{ \left(0.492\right) + \frac{\left(49.7 - 0.492\right)\left(101\right)\left(\frac{2.01}{203}\right)}{1+\left(101\right)\left(\frac{2.01}{203}\right)} \right\} 2.2$$

$$A_v = -55.2$$

c.

$$R_{is} = R_1 \| R_2 \| R_{ib}$$

$$R_{ib} = r_{\pi 1} + \left(1+\beta\right) r_{\pi 2}$$

$$= 203 + \left(101\right)\left(2.01\right) = 406\ \text{k}\Omega$$

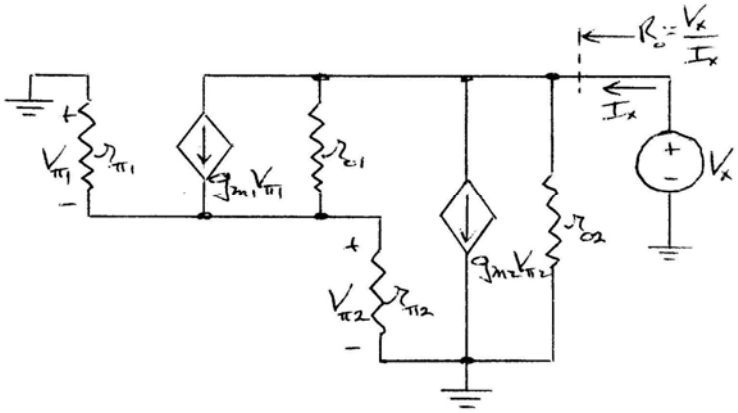
$$R_{is} = 91 \| 406 = \underline{74.3\ \text{k}\Omega = R_{is}}$$

$$\underline{R_0 = R_C = 2.2\ \text{k}\Omega}$$

6.79

$$I_{E1} = I_{Bias} + I_{B2} = I_{Bias} + \frac{I_{C2}}{\beta_2}$$

$$I_{C1} = \left(\frac{\beta_1}{1+\beta_1}\right)\left[I_{Bias} + \frac{I_{C2}}{\beta_2} \right]$$



(a) $I_{C2} = I_{Bias} = 1\ \text{mA},\quad I_{C1} = \left(\frac{120}{121}\right)\left[1 + \frac{1}{80}\right] = 1.004\ \text{mA}$

$g_{m1} = 38.62\ \text{mA/V},\quad r_{\pi 1} = 3.108\ \text{k}\Omega,\quad r_{o1} = 79.68\ \text{k}\Omega$

$g_{m2} = 38.46\ \text{mA/V},\quad r_{\pi 2} = 2.08\ \text{k}\Omega,\quad r_{o2} = 50\ \text{k}\Omega$

(1) $I_x = \dfrac{V_x}{r_{o2}} + g_{m2}V_{\pi 2} + \dfrac{V_x - V_{\pi 2}}{r_{o1}} + g_{m1}V_{\pi 1}$

(2) $V_{\pi 2} = \left[\dfrac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} + \dfrac{V_x - V_{\pi 2}}{r_{o1}}\right]\cdot r_{\pi 2}$

(3) $V_{\pi 1} + V_{\pi 2} = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$

Then (1) $I_x = \dfrac{V_x}{r_{o2}} + g_{m2}V_{\pi 2} + \dfrac{V_x - V_{\pi 2}}{r_{o1}} - g_{m1}V_{\pi 2}$

$$I_x = V_x\left(\frac{1}{r_{o2}} + \frac{1}{r_{o1}}\right) + V_{\pi 2}\left(g_{m2} - \frac{1}{r_{o1}} - g_{m1}\right)$$

(2) $V_{\pi 2} = -V_{\pi 2}\left(\dfrac{1}{r_{\pi 1}} + g_{m1} + \dfrac{1}{r_{o1}}\right)\cdot r_{\pi 2} + V_x\left(\dfrac{r_{\pi 2}}{r_{o1}}\right)$

$$V_{\pi 2}\left[1 + \left(\frac{1}{r_{\pi 1}} + g_{m1} + \frac{1}{r_{o1}}\right)\cdot r_{\pi 2}\right] = V_x\left(\frac{r_{\pi_2}}{r_{o1}}\right)$$

Now (2) $V_{\pi 2}\left[1 + \left(\dfrac{1}{3.108} + 38.62 + \dfrac{1}{79.68}\right)(2.08)\right] = V_x\left(\dfrac{2.08}{79.68}\right)$

$$V_{\pi 2} = V_x(0.00031825)$$

Then (1) $I_x = V_x\left(\dfrac{1}{50} + \dfrac{1}{79.68}\right) + V_x(0.00031825)\left(38.46 - \dfrac{1}{79.68} - 38.62\right) = V_x(0.32495)$

$$R_o = \frac{V_x}{I_x} = 30.77\ \text{k}\Omega$$

(b) $I_{C2} = 1\ \text{mA},\ I_{Bias} = 0.2\ \text{mA},\ I_{C1} = \left(\dfrac{120}{121}\right)\left(0.2 + \dfrac{1}{80}\right) = 0.2107\ \text{mA}$

$g_{m1} = 8.104\ \text{mA/V},\ r_{\pi 1} = 14.81\ \text{k}\Omega,\ r_{o1} = 379.7\ \text{k}\Omega$

$g_{m2} = 38.46\ \text{mA/V},\ r_{\pi 2} = 2.08\ \text{k}\Omega,\ r_{o2} = 50\ \text{k}\Omega$

Now (2) $V_{\pi 2}\left[1 + \left(\dfrac{1}{14.81} + 8.104 + \dfrac{1}{379.7}\right)(2.08)\right] = V_x\left(\dfrac{2.08}{379.7}\right)$

$$V_{\pi 2} = V_x(0.0003043)$$

(1) $I_x = V_x\left(\dfrac{1}{50} + \dfrac{1}{379.7}\right) + V_x(0.0003043)\left(38.46 - \dfrac{1}{379.7} - 8.104\right) = V_x(0.031867)$

$$R_o = \frac{V_x}{I_x} = 31.38\ \text{k}\Omega$$

(c) $I_{C2} = 2\ \text{mA},\ I_{Bias} = 0,\ I_{C1} = \left(\dfrac{120}{121}\right)\left(\dfrac{2}{80}\right) = 0.02479\ \text{mA}$

$g_{m1} = 0.9536\ \text{mA/V},\ r_{\pi 1} = 125.9\ \text{k}\Omega,\ r_{o1} = 3327\ \text{k}\Omega$

$g_{m2} = 76.92\ \text{mA/V},\ r_{\pi 2} = 1.04\ \text{k}\Omega,\ r_{o2} = 25\ \text{k}\Omega$

Now (2) $V_{\pi 2}\left[1 + \left(\dfrac{1}{125.9} + 0.9536 + \dfrac{1}{3327}\right)(1.04)\right] = V_x\left(\dfrac{1.04}{3327}\right)$

$$V_{\pi 2} = V_x(0.00015627)$$

(1) $I_x = V_x\left(\dfrac{1}{25} + \dfrac{1}{3327}\right) + V_x(0.00015627)\left(76.92 - \dfrac{1}{3327} - 0.9536\right) = V_x(0.05217)$

$$R_o = \frac{V_x}{I_x} = 19.17\ \text{k}\Omega$$

6.80

(a) $R_{TH} = R_1 \| R_2 = 250 \| 75 = 57.69\ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{75}{75 + 250}\right)(5) = 1.1538\ \text{V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1+\beta)R_E} = \frac{1.1538 - 0.7}{57.69 + (121)(0.6)} = 0.003483\ \text{mA}$$

$$\Rightarrow I_{CQ} = 0.418\ \text{mA},\quad I_{EQ} = 0.4214\ \text{mA}$$

$$V_{CEQ} = 5 - (0.418)(5.6) - (0.4214)(0.6) = 2.406\ \text{V}$$

For Q: $P_Q = I_{CQ} V_{CEQ} = (0.418)(2.406) = 1.01\ \text{mW}$

For R_C: $P_{RC} = I_{CQ}^2 R_C = (0.418)^2 (5.6) = 0.978\ \text{mW}$

For R_E: $P_{RE} = I_{EQ}^2 R_E = (0.4214)^2 (0.6) = 0.107\ \text{mW}$

(b) $R_{ib} = r_\pi + (1+\beta)R_E$

$$r_\pi = \frac{(120)(0.026)}{0.418} = 7.464\ \text{k}\Omega$$

$$R_{ib} = 7.464 + (121)(0.6) = 80.06\ \text{k}\Omega$$

$$R_i = R_{TH} \| R_{ib} = 57.69 \| 80.06 = 33.53\ \text{k}\Omega$$

$$R_{iS} = R_S + R_i = 0.5 + 33.53 = 34.03\ \text{k}\Omega$$

$$i_s = \frac{\upsilon_s}{R_{iS}};\quad i_b = \left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right) \cdot i_s = \left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right)\left(\frac{\upsilon_s}{R_{iS}}\right)$$

$$i_c = \beta\left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right)\left(\frac{\upsilon_s}{R_{iS}}\right);\quad i_e = (1+\beta)\left(\frac{R_{TH}}{R_{TH} + R_{ib}}\right)\left(\frac{\upsilon_s}{R_{iS}}\right)$$

Now $i_c = (120)\left(\frac{57.69}{57.69 + 80.06}\right)\left(\frac{0.1\sin\omega t}{34.03}\right) = 0.1477\sin\omega t\ \ (\text{mA})$

$$i_e = 0.1489\sin\omega t\ \ (\text{mA})$$

For R_C: $\overline{P}_{RC} = I_{CQ}^2 R_C + \frac{1}{2} I_c^2 R_C = 0.978 + \frac{1}{2}(0.1477)^2 (5.6) = 1.039\ \text{mW}$

For R_E: $\overline{P}_{RE} = I_{EQ}^2 R_E + \frac{1}{2} I_e^2 R_E = 0.107 + \frac{1}{2}(0.1489)^2 (0.6) = 0.1137\ \text{mW}$

For Q: $\overline{P}_Q = I_{CQ} V_{CEQ} - \frac{1}{2} I_c^2 R_C - \frac{1}{2} I_e^2 R_E = 1.01 - 0.0611 - 0.00665 = 0.942\ \text{mW}$

6.81

(a) $I_{BQ} R_B + V_{BE}(on) + (1+\beta) I_{BQ} R_E + V^- = 0$

$$I_{BQ} = \frac{5 - 0.7}{100 + (121)(4)} = 0.007363\ \text{mA};\quad I_{CQ} = 0.8836\ \text{mA},\quad I_{EQ} = 0.8909\ \text{mA}$$

$$V_{CEQ} = 10 - (0.8836)(4) - (0.8909)(4) = 2.902\ \text{V}$$

For Q: $P_Q = I_{CQ} V_{CEQ} = (0.8836)(2.902) = 2.564\ \text{mW}$

For R_C: $P_{RC} = I_{CQ}^2 R_C = (0.8836)^2(4) = 3.123$ mW

For R_E: $P_{RE} = I_{EQ}^2 R_E = (0.8909)^2(4) = 3.175$ mW

(b) $\Delta V_{CE} = \Delta I_C(R_C) = (0.8836)(4) = 3.534$ V - Not possible

$$\Delta V_{CE} = 2.902 - 0.5 = 2.402 \text{ V}$$

So $\Delta I_C = \dfrac{2.402}{4} = 0.6005$ mA

$$\overline{p}_{RC} = \frac{1}{2}(\Delta I_C)^2 R_C = \frac{1}{2}(0.6005)^2(4) = 0.721 \text{ mW}$$

6.82

a.

$$I_{BQ} = \frac{10 - 0.7}{50 + (151)(10)} = 0.00596 \text{ mA}$$

$$I_{CQ} = 0.894 \text{ mA}, \quad I_{EQ} = 0.90 \text{ mA}$$

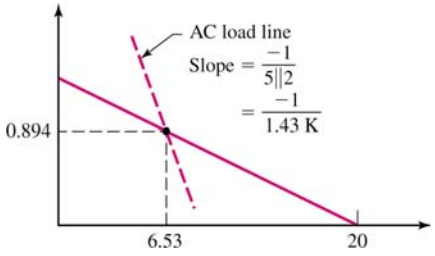
$$V_{ECQ} = 20 - (0.894)(5) - (0.90)(10) \Rightarrow V_{ECQ} = 6.53 \text{ V}$$

$$P_Q \cong I_{CQ} V_{ECQ} = (0.894)(6.53) \Rightarrow \underline{P_Q = 5.84 \text{ mW}}$$

$$P_{RC} \cong I_{CQ}^2 R_C = (0.894)^2(5) \Rightarrow \underline{P_{RC} = 4.0 \text{ mW}}$$

$$P_{RE} \cong I_{EQ}^2 R_E = (0.90)^2(10) \Rightarrow \underline{P_{RE} = 8.1 \text{ mW}}$$

b.



$$\Delta i_C = \frac{-1}{1.43 \text{ k}\Omega} \cdot \Delta v_{ec}$$

$$\Delta i_C = 0.894 \Rightarrow |\Delta v_{ec}| = (0.894)(1.43) = 1.28 \text{ V}$$

$$\Delta i_0 = \left(\frac{5}{5+2}\right)\Delta i_C = 0.639 \text{ mA}$$

$$\overline{P_{RL}} = \frac{1}{2}(0.639)^2(2) \Rightarrow \underline{\overline{P_{RL}} = 0.408 \text{ mW}}$$

$$\overline{P_{RC}} = \frac{1}{2} \cdot (0.894 - 0.639)^2(5) \Rightarrow \underline{\overline{P_{RC}} = 0.163 \text{ mW}}$$

$$\underline{\overline{P_{RE}} = 0}$$

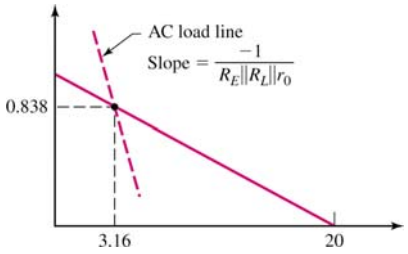
$$\overline{P_Q} = 5.84 - 0.408 - 0.163 \Rightarrow \underline{\overline{P_Q} = 5.27 \text{ mW}}$$

6.83

$$I_{BQ} = \frac{10 - 0.70}{100 + (101)(10)} = 0.00838 \text{ mA}$$

$$I_{CQ} = 0.838 \text{ mA}, \quad I_{EQ} = 0.846 \text{ mA}$$

$$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10) \Rightarrow V_{CEQ} = 3.16 \text{ V}$$



$$r_0 = \frac{100}{0.838} = 119 \text{ k}\Omega$$

Neglecting base currents:

a.

$$R_L = 1 \text{ k}\Omega$$

$$\text{slope} = \frac{-1}{10 \| 1 \| 119} = \frac{-1}{0.902 \text{ k}\Omega}$$

$$\Delta i_C = \frac{-1}{0.902 \text{ k}\Omega} \cdot \Delta V_{ce}$$

$$\Delta i_C = 0.838 \Rightarrow |\Delta V_{ce}| = (0.902)(0.838) = 0.756 \text{ V}$$

$$\overline{P_{RL}} = \frac{1}{2}\frac{(0.756)^2}{1} \Rightarrow \underline{\overline{P_{RL}} = 0.286 \text{ mW}}$$

b.

$$R_L = 10 \text{ k}\Omega$$

$$\text{slope} = \frac{-1}{10 \| 10 \| 119} = \frac{-1}{4.80}$$

For $\Delta i_C = 0.838 \Rightarrow |\Delta v_{ce}| = (0.838)(4.80) = 4.02$

Max. swing determined by voltage $\overline{P_{RL}} = \frac{1}{2}\frac{(3.16)^2}{10} \Rightarrow \underline{\overline{P_{RL}} = 0.499 \text{ mW}}$

6.84

a.

$$I_{BQ} = \frac{10 - 0.7}{100 + (101)(10)} = 0.00838 \text{ mA}$$

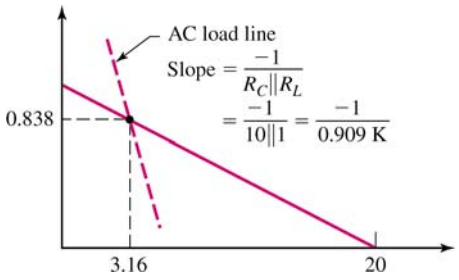
$$I_{CQ} = 0.838 \text{ mA}, \quad I_{EQ} = 0.846 \text{ mA}$$

$$V_{CEQ} = 20 - (0.838)(10) - (0.846)(10) \Rightarrow V_{CEQ} = 3.16 \text{ V}$$

$$P_Q \cong I_{CQ}V_{CEQ} = (0.838)(3.16) \Rightarrow \underline{P_Q = 2.65 \text{ mW}}$$

$$P_{RC} \cong I_{CQ}^2 R_C = (0.838)^2 (10) \Rightarrow \underline{P_{RC} = 7.02 \text{ mW}}$$

b.



$$\Delta i_C = \frac{-1}{0.909 \text{ k}\Omega} \cdot \Delta v_{ce}$$

For $\Delta i_C = 0.838 \Rightarrow |\Delta v_{ce}| = (0.909)(0.838) = 0.762$ V

$$\Delta i_0 = \left(\frac{R_C}{R_C + R_L}\right)\Delta i_C = \left(\frac{10}{10+1}\right)\Delta i_C = 0.762 \text{ mA}$$

$$\overline{P_{RL}} = \frac{1}{2}(0.762)^2 (1) \Rightarrow \underline{\overline{P_{RL}} = 0.290 \text{ mW}}$$

$$\overline{P_{RC}} = \frac{1}{2} \cdot (0.838 - 0.762)^2 (10) \Rightarrow \underline{\overline{P_{RC}} = 0.0289 \text{ mW}}$$

$$\overline{P_Q} = 2.65 - 0.290 - 0.0289 \Rightarrow \underline{\overline{P_Q} = 2.33 \text{ mW}}$$

Chapter 7

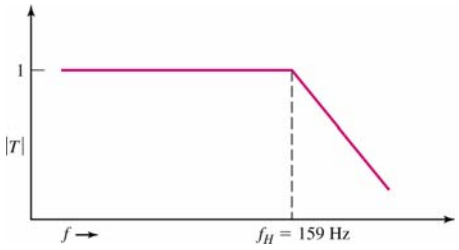
7.1

a.

$$T(s)=\frac{V_0(s)}{V_i(s)}=\frac{1/(sC_1)}{[1/(sC_1)]+R_1}$$

$$\underline{T(s)=\frac{1}{1+sR_1C_1}}$$

b.



$$f_H=\frac{1}{2\pi R_1C_1}=\frac{1}{2\pi(10^3)(10^{-6})}\Rightarrow \underline{f_H=159\text{ Hz}}$$

c.

$$V_0(s)=V_i(s)\cdot\frac{1}{1+sR_1C_1}$$

For a step function $V_i(s)=\frac{1}{s}$

$$V_0(s)=\frac{1}{s}\cdot\frac{1}{1+sR_1C_1}=\frac{K_1}{s}+\frac{K_2}{1+sR_1C_1}$$

$$=\frac{K_1(1+sR_1C_1)+K_2s}{s(1+sR_1C_1)}$$

$$=\frac{K_1+s(K_1R_1C_1+K_2)}{s(1+sR_1C_1)}$$

$K_2=-K_1R_1C_1$ and $K_1=1$

$$V_0(s)=\frac{1}{s}+\frac{-R_1C_1}{1+sR_1C_1}$$

$$=\frac{1}{s}-\frac{1}{\frac{1}{R_1C_1}+s}$$

$$\underline{v_0(t)=1-e^{-t/R_1C_1}}$$

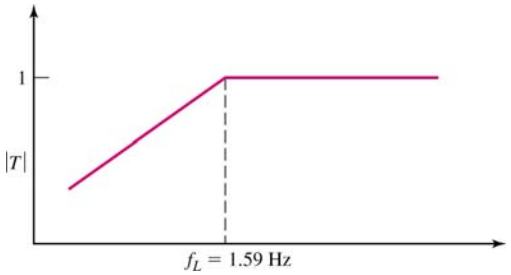
7.2

a.

$$T(s)=\frac{V_0(s)}{V_i(s)}=\frac{R_2}{R_2+[1/(sC_2)]}$$

$$\underline{T(s)=\frac{sR_2C_2}{1+sR_2C_2}}$$

b.



$$f_L = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi\left(10^4\right)\left(10\times10^{-6}\right)} \Rightarrow \underline{f_L = 1.59 \text{ Hz}}$$

c.

$$V_0(s) = V_i(s)\cdot\frac{sR_2C_2}{1+sR_2C_2}$$

$$V_i(s) = \frac{1}{s}$$

$$V_0(s) = \frac{R_2C_2}{1+sR_2C_2} = \frac{1}{s+\dfrac{1}{R_2C_2}}$$

$$\underline{v_0(t) = e^{-t/R_2C_2}}$$

7.3

(a) $$T(s) = \frac{V_o}{V_i} = \frac{R_2 \left\| \dfrac{1}{sC_2} \right.}{R_2 \left\| \dfrac{1}{sC_2} \right. + R_1}$$

Now $$R_2 \left\| \frac{1}{sC_2} \right. = \frac{R_2\left(\dfrac{1}{sC_2}\right)}{R_2 + \dfrac{1}{sC_2}} = \frac{R_2}{1+sR_2C_2}$$

Then $$T(s) = \frac{\dfrac{R_2}{1+sR_2C_2}}{\dfrac{R_2}{1+sR_2C_2} + R_1} = \frac{R_2}{R_1 + R_2 + sR_1R_2C_2}$$

$$T(s) = \left(\frac{R_2}{R_1+R_2}\right)\cdot\frac{1}{1+s\left(R_1 \| R_2\right)C_2}$$

(b) $$\tau = \left(R_1 \| R_2\right)C_2 = \left(10 \| 20\right)\times10^3 \times10\times10^{-6} \Rightarrow \tau = 66.7 \text{ ms}$$

(c) $$f = \frac{1}{2\pi\tau} = \frac{1}{2\pi\left(66.7\times10^{-3}\right)} = 2.39 \text{ Hz}$$

7.4

a.

$$\tau_S = (R_i + R_P)C_S = (30+10)\times 10^3 \times (10\times 10^{-6}) \Rightarrow \tau_S = 0.40 \text{ s}$$

$$\tau_P = (R_i \| R_P)C_P = (30\| 10)\times 10^3 \times (50\times 10^{-12}) \Rightarrow \tau_P = 0.375\ \mu\text{s}$$

b.

$$f_L = \frac{1}{2\pi\tau_S} = \frac{1}{2\pi(0.4)} \Rightarrow f_L = 0.398 \text{ Hz}$$

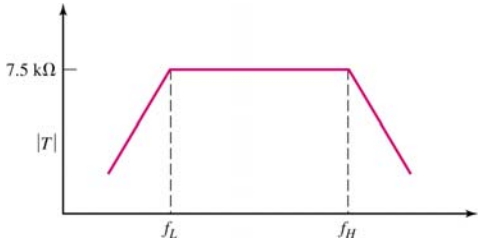
$$f_H = \frac{1}{2\pi\tau_P} = \frac{1}{2\pi(0.375\times 10^{-6})} \Rightarrow f_H = 424 \text{ kHz}$$

At midband. $C_S \rightarrow$ short, $C_P \rightarrow$ open

$$V_o = I_i (R_i \| R_P)$$

$$T(s) = R_i \| R_P = 30\| 10 \Rightarrow \underline{T(s) = 7.5\ k\Omega}$$

c.



7.5

(a) $$\frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1} = \frac{20}{20+10} = 0.667$$

(b) $$\frac{V_o}{V_i} = 1$$

(c) $$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_2}{R_2 + R_1 \left\| \frac{1}{sC_1} \right.} = \frac{R_2}{R_2 + \frac{R_1}{1+sR_1C_1}}$$

$$T(s) = \frac{R_2(1+sR_1C_1)}{R_1 + R_2 + sR_1R_2C_1} = \left(\frac{R_2}{R_1+R_2}\right)\cdot\frac{(1+sR_1C_1)}{[1+s(R_1\| R_2)C_1]}$$

We have $K = \dfrac{R_2}{R_1+R_2}$, $\tau_A = R_1C_1$, $\tau_B = (R_1\| R_2)C_1$

7.6

a.

$$T(s)=\frac{V_o(s)}{V_i(s)}=\frac{R_P\left\|\frac{1}{sC_P}\right.}{R_P\left\|\frac{1}{sC_P}+\left(R_S+\frac{1}{sC_S}\right)\right.}$$

$$R_P\left\|\frac{1}{sC_P}\right.=\frac{R_P\cdot\frac{1}{sC_P}}{R_P+\frac{1}{sC_P}}=\frac{R_P}{1+sR_PC_P}$$

Then

$$T(s)=\frac{R_P}{R_P+\left(R_S+\frac{1}{sC_S}\right)(1+sR_PC_P)}$$

$$=\frac{R_P}{R_P+R_S+\frac{R_PC_P}{C_S}+\frac{1}{sC_S}+sR_SR_PC_P}$$

$$T(s)=\left(\frac{R_P}{R_P+R_S}\right)\times\left(1/\left[1+\frac{R_P}{R_P+R_S}\cdot\frac{C_P}{C_S}+\frac{1}{s(R_S+R_P)C_S}+\frac{sR_PR_S}{R_S+R_P}\cdot C_P\right]\right)$$

b.

$$T(s)=\left(\frac{10}{10+10}\right)\times\left(1/\left[1+\frac{10}{20}\cdot\frac{10^{-11}}{10^{-6}}+\frac{1}{s\left(2\times10^4\right)\cdot10^{-6}}+s\left(5\times10^3\right)\cdot10^{-11}\right]\right)$$

$$\cong\frac{1}{2}\cdot\frac{1}{1+\frac{1}{s(0.02)}+s\left(5\times10^{-8}\right)}$$

$s=j\omega$

$$T(j\omega)=\frac{1}{2}\cdot\frac{1}{1+j\left[\omega\left(5\times10^{-8}\right)-\frac{1}{\omega(0.02)}\right]}$$

For $\omega_L=\frac{1}{(R_S+R_R)C_S}=\frac{1}{\left(2\times10^4\right)\left(10^{-6}\right)}=50$

$$T(j\omega)=\frac{1}{2}\cdot\frac{1}{1+j\left[(50)\left(5\times10^{-8}\right)-\frac{1}{(50)(0.02)}\right]}$$

$$\approx\frac{1}{2}\cdot\frac{1}{1-j}\Rightarrow\underline{|T(j\omega)|=\frac{1}{2}\cdot\frac{1}{\sqrt{2}}}$$

For

$$\omega_H = \frac{1}{(R_S \| R_P)C_P} = \frac{1}{(5\times10^3)(10^{-11})} = 2\times10^7$$

$$T(j\omega) = \frac{1}{2}\cdot\frac{1}{1+j\left[(2\times10^7)(5\times10^{-8}) - \frac{1}{(2\times10^7)(0.02)}\right]}$$

$$T(j\omega) \cong \frac{1}{2}\cdot\frac{1}{1+j} \Rightarrow |T(j\omega)| = \frac{1}{2}\cdot\frac{1}{\sqrt{2}}$$

In each case, $|T(j\omega)| = \frac{1}{\sqrt{2}}\cdot\frac{R_P}{R_S + R_P}$

c.

$R_S = R_P = 10\ \text{k}\Omega, \quad C_S = C_P = 0.1\ \mu\text{F}$

$$T(s) = \frac{1}{2}\cdot\left(1/\left[1+\frac{1}{2}\cdot\frac{1}{1}+\frac{1}{s\left(2\times10^4\left(10^{-7}\right)\right)}+s\left(5\times10^3\right)\left(10^{-7}\right)\right]\right)$$

$s = j\omega$

$$T(j\omega) = \frac{1}{2}\cdot\frac{1}{1+\frac{1}{2}+j\left[\omega\left(5\times10^{-4}\right) - \frac{1}{\omega\left(2\times10^{-3}\right)}\right]}$$

For $\omega = \frac{1}{(2\times10^4)(10^{-7})} = 500$

$$T(j\omega) = \frac{1}{2}\cdot\frac{1}{1.5+j\left[(500)\left(5\times10^{-4}\right) - \frac{1}{(500)\left(2\times10^{-3}\right)}\right]}$$

$$= \frac{1}{2}\cdot\frac{1}{1.5-j(0.75)} \Rightarrow \underline{|T(j\omega)| = 0.298}$$

For $\omega = \frac{1}{(5\times10^3)(10^{-7})} = 2\times10^3$

$$T(j\omega) = \frac{1}{2}\cdot\left\{1/\left(1.5+j\left[\left(2\times10^3\right)\left(5\times10^{-4}\right) - \frac{1}{\left(2\times10^3\right)\left(2\times10^{-3}\right)}\right]\right)\right\}$$

$$= \frac{1}{2}\cdot\frac{1}{1.5+j(0.75)} \Rightarrow \underline{|T(j\omega)| = 0.298}$$

In each case, $|T(j\omega)| < \frac{1}{\sqrt{2}}\cdot\frac{R_P}{R_P + R_S}$

7.7

(a) $$|T| = \frac{1}{\left[\sqrt{1+\left(\frac{f}{f_T}\right)^2}\right]^3}$$

At $f = f_T$, $|T| = \frac{1}{\left(\sqrt{2}\right)^3} = 0.35355$

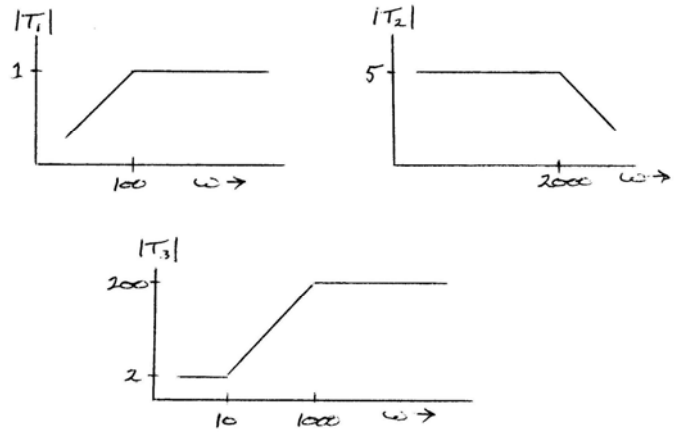
Or $|T|_{dB} = 20\log_{10}(0.35355) = -9.03$ dB

$$\phi = -\tan^{-1}\left(\frac{f}{f_F}\right)^3 = -3\tan^{-1}\left(\frac{f}{f_T}\right) = -3\tan^{-1}(1) = -135°$$

(b) Slope $= 3(-6) = -18$ dB/octave $= -60$ dB/decade

$\phi = 3(-90) = -270°$

7.8



7.9

(a) (ii) $\omega_1 = 1$ rad/s; $\omega_2 = 10$ rad/s; $\omega_3 = 100$ rad/s; $\omega_4 = 1000$ rad/s

(iii) $|T(0)| = 10$

(iv) $|T(\infty)| = 10$

(b) (ii) $\omega = 5$ rad/s

(iii) $|T(0)| = 0$

(iv) $|T(\infty)| = \dfrac{8}{(0.2)^2} = 200$

7.10

(a) $$T(j\omega) = 5\left(\frac{j\frac{\omega}{10^2}}{1+j\frac{\omega}{10^2}}\right)\left(\frac{1}{1+j\frac{\omega}{5\times10^4}}\right)$$

or $$T(j\omega) = 2.5\times10^5\left(\frac{j\omega}{10^2+j\omega}\right)\left(\frac{1}{5\times10^4+j\omega}\right)$$

(b) $$|T| = \frac{5\left(\frac{\omega}{10^2}\right)}{\sqrt{1+\left(\frac{\omega}{10^2}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{\omega}{5\times 10^4}\right)^2}}$$

(i) At $\omega = 50$ rad/s

$$|T| = \frac{5\left(\frac{50}{100}\right)}{\sqrt{1+\left(\frac{50}{100}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{50}{5\times 10^4}\right)^2}} = 2.236$$

(ii) At $\omega = 150$ rad/s

$$|T| = \frac{5\left(\frac{150}{100}\right)}{\sqrt{1+\left(\frac{150}{100}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{150}{5\times 10^4}\right)^2}} = 4.16$$

(iii) At $\omega = 10^5$

$$|T| = \frac{5\left(\frac{10^5}{10^2}\right)}{\sqrt{1+\left(\frac{10^5}{10^2}\right)^2}} \cdot \frac{1}{\sqrt{1+\left(\frac{10^5}{5\times 10^4}\right)^2}} = 2.236$$

7.11

a.

$$V_0 = -g_m V_\pi R_L \quad V_\pi = \left(\frac{r_\pi}{r_\pi + R_S}\right) V_i$$

$$|T| = g_m R_L \left(\frac{r_\pi}{r_\pi + R_S}\right) = (29)(6)\left(\frac{5.2}{5.2+0.5}\right)$$

$$\underline{|T_{\text{midband}}| = 159}$$

b.

$$\tau_S = (R_S + r_\pi) C_C$$

$$f_L = \frac{1}{2\pi\tau_S} \Rightarrow \tau_S = \frac{1}{2\pi f_L} = \frac{1}{2\pi(30)} \Rightarrow \tau_S = 5.31\,\text{ms}, \text{ Open-Circuit}$$

$$\tau_P = \frac{1}{2\pi f_H} = \frac{1}{2\pi(480\times 10^3)} \Rightarrow \tau_P = 0.332\,\mu\text{s}, \text{ Short-Circuit}$$

c.

$$C_C = \frac{\tau_S}{(R_S + r_\pi)} = \frac{5.31\times 10^{-3}}{(0.5+5.2)\times 10^3} \Rightarrow C_C = 0.932\ \mu\text{F}$$

$$\tau_P = R_L C_L$$

$$C_L = \frac{\tau_P}{R_L} = \frac{0.332\times 10^{-6}}{6\times 10^3} \Rightarrow C_L = 55.3\,\text{pF}$$

7.12

(a) $\dfrac{V_o}{V_i}=\dfrac{R_2+R_3}{R_1+R_2+R_3}=\dfrac{10+40}{10+10=40}=0.833$

(b) $\dfrac{V_o}{V_i}=\dfrac{R_2}{R_1+R_2}=\dfrac{10}{10+10}=0.50$

(c) $R_3\left\|\dfrac{1}{sC}\right.=\dfrac{R_3\left(\dfrac{1}{sC}\right)}{R_3+\dfrac{1}{sC}}=\dfrac{R_3}{1+sR_3C}$

$$T(s)=\frac{V_o(s)}{V_i(s)}=\frac{R_2+\dfrac{R_3}{1+sR_3C}}{R_1+R_2+\dfrac{R_3}{1+sR_3C}}=\frac{R_2+R_3+sR_2R_3C}{R_1+R_2+R_3+s(R_1+R_2)R_3C}$$

or $$T(s)=\left(\frac{R_2+R_3}{R_1+R_2+R_3}\right)\cdot\frac{\left[1+s(R_2\|R_3)C\right]}{\left[1+s((R_1+R_2)\|R_3)C\right]}$$

where $K=\dfrac{R_2+R_3}{R_1+R_2+R_3}$, $\tau_A=(R_2\|R_3)C$, $\tau_B=((R_1+R_2)\|R_3)C$

7.13 Computer Analysis

7.14

(a) $|A_v|_{\max}=g_mR_D$, $g_m=2\sqrt{K_nI_{DQ}}=2\sqrt{(0.4)(0.8)}=1.131$ mA/V

$|A_v|_{\max}=(1.131)(1)=1.13$

(b) $f_H=\dfrac{1}{2\pi R_DC_L}=\dfrac{1}{2\pi(10^3)(10^{-12})}$

$f_H=BW=159$ MHz

7.15

(a) $f_H=\dfrac{1}{2\pi R_CC_L}\Rightarrow R_C=\dfrac{1}{2\pi f_HC_L}=\dfrac{1}{2\pi(800\times10^6)(0.08\times10^{-12})}$

or $R_C=2.49\,\text{k}\Omega$

(b) $I_{CQ}=\dfrac{V_{CC}-V_{CEQ}}{R_C}=\dfrac{2.5-1.25}{2.487}=0.503\,\text{mA}$

(c) $|A_v|_{\max}=g_mR_C$, $g_m=\dfrac{0.5026}{0.026}=19.33$ mA/V

$|A_v|_{\max}=(19.33)(2.487)=48.1$

7.16

(a) $$T(s)=\frac{V_o(s)}{V_i(s)}=-g_m\left(r_o\left\|\frac{1}{sC_L}\right.\right)=-g_m\left(\frac{r_o\cdot\frac{1}{sC_L}}{r_o+\frac{1}{sC_L}}\right)=-g_m\,r_o\left(\frac{1}{1+sr_oC_L}\right)$$

(b) $$g_m=2\sqrt{K_nI_{DQ}}=2\sqrt{(0.05)(0.1)}=0.1414\ \text{mA/V}$$

$$r_o=\frac{1}{\lambda I_{DQ}}=\frac{1}{(0.01)(0.1)}=1000\ \text{k}\Omega$$

$$|A_v|_{\max}=g_mr_o=(0.1414)(1000)=141.4$$

(c) $$f_H=BW=\frac{1}{2\pi r_oC_L}=\frac{1}{2\pi(10^6)(0.5\times10^{-12})}\Rightarrow f_H=318\ \text{kHz}$$

7.17

a.

$$R_{TH}=R_1\|R_2=10\|1.5=1.304\ \text{k}\Omega$$

$$V_{TH}=\left(\frac{R_2}{R_1+R_2}\right)V_{CC}=\left(\frac{1.5}{1.5+10}\right)(12)=1.565\ \text{V}$$

$$I_{BQ}=\frac{1.565-0.7}{1.30+(101)(0.1)}=0.0759\ \text{mA}$$

$$I_{CQ}=7.585\ \text{mA}$$

$$r_\pi=\frac{(100)(0.026)}{7.59}=0.343\ \text{k}\Omega$$

$$g_m=\frac{7.59}{0.026}=292\ \text{mA/V}$$

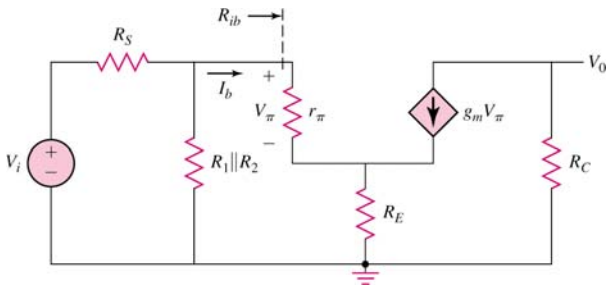
$$R_i=R_1\|R_2\|[r_\pi+(1+\beta)R_E]$$
$$=10\|1.5\|[0.343+(101)(0.1)]$$
$$=1.30\|10.44\Rightarrow R_i=1.159\ \text{k}\Omega$$

$$\tau=(R_S+R_i)C_C=(0.5+1.16)\times10^3\times(0.1\times10^{-6})$$

$$\tau=1.659\times10^{-4}\ \text{s}$$

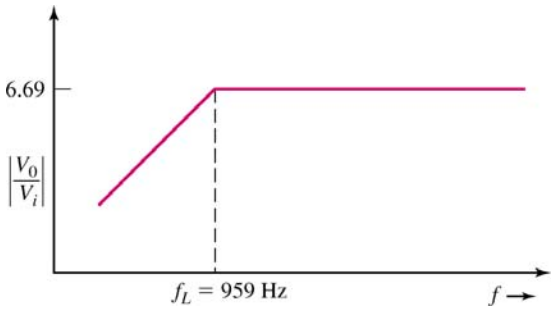
$$f_L=\frac{1}{2\pi\tau}=\frac{1}{2\pi(1.66\times10^{-4})}\Rightarrow f_L=959\ \text{Hz}$$

b.



$$V_0 = -(\beta I_b) R_C$$
$$R_{1b} = r_\pi + (1+\beta) R_E$$
$$= 0.343 + (101)(0.1) = 10.44 \text{ k}\Omega$$
$$I_b = \left(\frac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right) I_i$$
$$= \left(\frac{1.30}{1.30 + 10.4} \right) I_i = (0.111) I_i$$
$$I_i = \frac{V_i}{R_S + R_1 \| R_2 \| R_{ib}}$$
$$= \frac{V_i}{0.5 + (1.3) \| (10.44)}$$
$$I_i = \frac{V_i}{1.659}$$
$$\left| \frac{V_0}{V_i} \right| = \frac{\beta R_C (0.111)}{1.659} \Rightarrow \left| \frac{V_0}{V_i} \right|_{\text{midband}} = \frac{(100)(1)(0.111)}{1.659} \Rightarrow \underline{\left| \frac{V_0}{V_i} \right|_{\text{midband}} = 6.69}$$

c.



7.18

(a) $V_{DSQ} = V_{DD} - I_{DQ}(R_D + R_S)$

$3.2 = 9 - (0.8)(R_D + 0.5) \Rightarrow R_D = 6.75 \text{ k}\Omega$

$I_{DQ} = K_n (V_{GSQ} - V_{TN})^2$

$0.8 = 0.5(V_{GSQ} - 1.2)^2 \Rightarrow V_{GSQ} = 2.465 \text{ V}$

$V_G = (0.8)(0.5) + 2.465 = 2.865 \text{ V}$

$V_G = \frac{1}{R_1} \cdot R_{in} \cdot V_{DD} \Rightarrow 2.865 = \frac{1}{R_1}(160)(9)$

which yields $R_1 = 503 \text{ k}\Omega$ and $R_2 = 235 \text{ k}\Omega$

(b) $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.8)} = 1.265 \text{ mA/V}$

$A_v = \frac{-g_m R_D}{1 + g_m R_S} = \frac{-(1.265)(6.75)}{1 + (1.265)(0.5)} = -5.23$

(c) $f_L = \frac{1}{2\pi R_{in} C_C} \Rightarrow C_C = \frac{1}{2\pi f_L R_{in}} = \frac{1}{2\pi(16)(160 \times 10^3)} \Rightarrow C_C = 0.06217 \ \mu\text{F}$

$\tau_S = R_{in} C_C = (160 \times 10^3)(0.06217 \times 10^{-6}) = 9.947 \times 10^{-3} \text{ s}$

$$|A_v| = 5.23\left|\frac{s\tau_S}{1+s\tau_S}\right| = (5.23)\left[\frac{\left(\frac{f}{f_L}\right)}{\sqrt{1+\left(\frac{f}{f_L}\right)^2}}\right]$$

(i) For $f = 5$ Hz,

$$|A_v| = (5.23)\left[\frac{\frac{5}{16}}{\sqrt{1+\left(\frac{5}{16}\right)^2}}\right] = 1.56$$

(ii) For $f = 14$ Hz,

$$|A_v| = (5.23)\left[\frac{\frac{14}{16}}{\sqrt{1+\left(\frac{14}{16}\right)^2}}\right] = 3.44$$

(iii) For $f = 25$ Hz,

$$|A_v| = (5.23)\left[\frac{\frac{25}{16}}{\sqrt{1+\left(\frac{25}{16}\right)^2}}\right] = 4.405$$

7.19

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2 \Rightarrow V_{GS} = \sqrt{\frac{I_{DQ}}{K_n}} + V_{TN} = \sqrt{\frac{1}{0.5}} + 1 = 2.414\ V$$

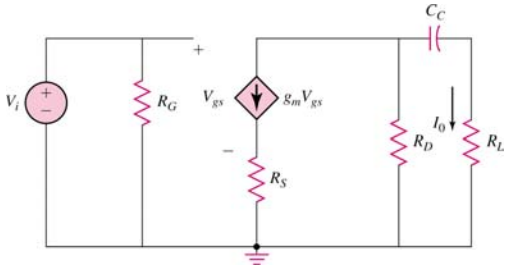
$$V_S = -2.414\ \text{V}$$

$$R_S = \frac{-2.414 - (-5)}{1} \Rightarrow \underline{R_S = 2.59\ \text{k}\Omega}$$

$$V_D = V_{DSQ} + V_S = 3 - 2.414 = 0.586\ \text{V}$$

$$R_D = \frac{5 - 0.59}{1} \Rightarrow R_D = 4.41\ \text{k}\Omega$$

b.



$$I_0 = -\left(g_m V_{gs}\right)\left(\frac{R_D}{R_D + R_L + \dfrac{1}{sC_C}}\right)$$

$$V_{gs} = \frac{V_i}{1 + g_m R_S}$$

$$\frac{I_0(s)}{V_i(s)} = \frac{-g_m}{1 + g_m R_S} \cdot R_D\left[\frac{sC_C}{1 + s(R_D + R_L)C_C}\right]$$

$$T(s) = \frac{I_0(s)}{V_i(s)}$$

$$= \frac{-g_m R_D}{1 + g_m R_S} \cdot \frac{1}{R_D + R_L} \cdot \frac{s(R_D + R_L)C_C}{1 + s(R_D + R_L)C_C}$$

c.

$$f_L = \frac{1}{2\pi\tau_L} \Rightarrow \tau_L = \frac{1}{2\pi f_L} = \frac{1}{2\pi(10)} \Rightarrow \tau_L = 15.92 \text{ ms}$$

$$\tau_L = (R_D + R_L)C_C \Rightarrow C_C = \frac{\tau_L}{R_D + R_L} = \frac{15.9 \times 10^{-3}}{(4.41 + 4) \times 10^3} \Rightarrow C_C = 1.89\ \mu\text{ F}$$

7.20

a.

$$\frac{9 - V_{SG}}{R_S} = I_D = K_P\left(V_{SG} + V_{TP}\right)^2$$

$$9 - V_{SG} = (0.5)(12)\left(V_{SG}^2 - 4V_{SG} + 4\right)$$

$$6V_{SG}^2 - 23V_{SG} + 15 = 0$$

$$V_{SG} = \frac{23 \pm \sqrt{(23)^2 - 4(6)(15)}}{2(6)} \Rightarrow V_{SG} = 3 \text{ V}$$

$$g_m = 2K_P\left(V_{SG} + V_{TP}\right) = 2(0.5)(3 - 2) \Rightarrow g_m = 1 \text{ mA/V}$$

$$R_o = \frac{1}{g_m} \Big\| R_S = 1 \| 12 \Rightarrow \underline{R_o = 0.923 \text{ k}\Omega}$$

b. $\tau = (R_o + R_L)C_C$

$$f_L = \frac{1}{2\pi\tau} \Rightarrow \tau = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} \Rightarrow \tau = 7.96 \text{ ms}$$

c. $$C_C = \frac{\tau}{R_o + R_L} = \frac{7.96 \times 10^{-3}}{(0.923 + 10) \times 10^3} \Rightarrow C_C = 0.729\ \ \mu\text{ F}$$

7.21

(a) $$R_{TH} = (0.1)(1 + \beta)R_E = (0.1)(121)(4) = 48.4 \text{ k}\Omega$$

$$I_{BQ} = \frac{I_{EQ}}{1 + \beta} = \frac{1.5}{121} = 0.012397 \text{ mA}$$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE}(on) + I_{EQ}R_E = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC}$$

so $\frac{1}{R_1}(48.4)(12) = (0.012397)(48.4) + 0.7 + (1.5)(4)$

which yields $R_1 = 79.6\,\text{k}\Omega$ and $R_2 = 124\,\text{k}\Omega$

(b) $I_{CQ} = \left(\frac{120}{121}\right)(1.5) = 1.488\,\text{mA}$

$$r_\pi = \frac{(120)(0.026)}{1.488} = 2.097\,\text{k}\Omega, \quad r_o = \frac{50}{1.488} = 33.6\,\text{k}\Omega$$

$$A_v = \frac{(1+\beta)(r_o \| R_E \| R_L)}{r_\pi + (1+\beta)(r_o \| R_E \| R_L)}$$

Now $r_o \| R_E \| R_L = 33.6 \| 4 \| 4 = 1.888\,\text{k}\Omega$

$$A_v = \frac{(121)(1.888)}{2.097 + (121)(1.888)} = 0.991$$

(c) $R_o = R_E \| r_o \left\| \frac{r_\pi}{1+\beta} = 4 \right\| 33.6 \left\| \frac{2.097}{121} \Rightarrow R_o = 17.25\,\Omega \right.$

(d) $f_L = \frac{1}{2\pi(R_o + R_L)C_{C2}} = \frac{1}{2\pi(17.25 + 4000)(2\times 10^{-6})}$

$f_L = 19.8\,\text{Hz}$

7.22

(a) $V_o(s) = -g_m\left(r_o \left\| R_D \right\| \frac{1}{sC_L}\right) \cdot V_{gs} = -g_m \left[\frac{\left(r_o \| R_D\right)\left(\frac{1}{sC_L}\right)}{r_o \left\| R_D + \frac{1}{sC_L}\right.}\right] \cdot V_{gs} = -g_m\left[\frac{r_o \| R_D}{1 + s(r_o \| R_D)C_L}\right] \cdot V_{gs}$

$$V_{gs} = \frac{\left(\frac{1}{sC_i}\right)(V_i(s))}{\frac{1}{sC_i} + R_{Si}} = \frac{V_i(s)}{1 + sR_{Si}C_i}$$

$$T(s) = \frac{V_o(s)}{V_i(s)} = -g_m\left(\frac{1}{1+sR_{Si}C_i}\right)\left(\frac{r_o \| R_D}{1 + s(r_o \| R_D)C_L}\right)$$

(b) $\tau = R_{Si}C_i$

(c) $\tau = (r_o \| R_D)C_L$

7.23

(a) $$\frac{V_{gs}}{V_i} = \frac{-\left(\frac{1}{g_m} \left\| \frac{1}{sC_i}\right.\right)}{\left(\frac{1}{g_m} \left\| \frac{1}{sC_i}\right.\right) + R_S}$$

Now $\left(\frac{1}{g_m}\middle\|\frac{1}{sC_i}\right)=\frac{\left(\frac{1}{g_m}\right)\left(\frac{1}{sC_i}\right)}{\frac{1}{g_m}+\frac{1}{sC_i}}=\frac{\frac{1}{g_m}}{1+s\left(\frac{1}{g_m}\right)C_i}$

So $\frac{V_{gs}}{V_i}=\frac{-\frac{1}{g_m}}{\frac{1}{g_m}+R_S\left(1+s\left(\frac{1}{g_m}\right)C_i\right)}=\left(\frac{-\frac{1}{g_m}}{\frac{1}{g_m}+R_S}\right)\cdot\frac{1}{\left[1+s\left(\frac{1}{g_m}\middle\|R_S\right)C_i\right]}$

We have

$$V_o=-g_mV_{gs}\left[\frac{R_D}{R_D+R_L+\frac{1}{sC_C}}\right]\cdot R_L=-g_mV_{gs}\left[\frac{R_DR_L(sC_C)}{1+s(R_D+R_L)C_C}\right]$$

$$V_o=-g_mV_{gs}\left(\frac{R_DR_L}{R_D+R_L}\right)\left[\frac{s(R_D+R_L)C_C}{1+s(R_D+R_L)C_C}\right]$$

Then $T(s)=\frac{V_o(s)}{V_i(s)}=\frac{+g_m(R_D\|R_L)}{1+g_mR_S}\cdot\frac{1}{\left[1+s\left(\frac{1}{g_m}\middle\|R_S\right)C_i\right]}\cdot\left[\frac{s(R_D+R_L)C_C}{1+s(R_D+R_L)C_C}\right]$

(b) $\tau=\left(\frac{1}{g_m}\middle\|R_S\right)C_i$

(c) $\tau=(R_D+R_L)C_C$

7.24

(a)

$$I_{EQ}=\frac{5-0.7}{4}=1.075\text{ mA}\quad I_{CQ}=1.064\text{ mA}$$

$$V_{CEQ}=10-(1.064)(2)-(1.075)(4)$$

$$V_{CEQ}=3.57\text{ V}$$

$$g_m=\frac{I_{CQ}}{V_T}=\frac{1.064}{0.026}=40.92\text{ mA/V}$$

$$r_\pi=\frac{\beta V_T}{I_{CQ}}=\frac{(100)(0.026)}{1.064}=2.44\text{ K}$$

(b)

For C_{C1}; $R_{eq1}=R_S+R_E\left\|\frac{r_\pi}{1+\beta}=200+4000\right\|\frac{2440}{101}$

$$R_{eq1}=224.0\,\Omega;\quad \tau_1=R_{eq1}C_{C1}=1.053\,\text{ms}$$

For C_{C2}; $R_{eq2}=R_C+R_L=2+47=49\,\text{k}\Omega$

$$\tau_2=R_{eq2}C_{C2}=49\text{ ms}$$

(c)
$$f_1 = \frac{1}{2\pi\tau_1} = \frac{1}{2\pi\left(1.053\times10^{-3}\right)} \Rightarrow \underline{f_1 = 151\text{ Hz}}$$

7.25

(a)
$$\tau_H = \left(R_C \| R_L\right) C_L = \left(2\|47\right)\times10^3 \times 10\times10^{-12}$$
$$= 1.918\times10^{-8}\text{s}$$
$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi\left(1.918\times10^{-8}\right)} \Rightarrow \underline{f_H = 8.30\text{ MHz}}$$

(b)
$$\frac{1}{\sqrt{1+\left(2\pi\tau_H f\right)^2}} = 0.1$$
$$\left(\frac{1}{0.1}\right)^2 = 100 = 1+\left(2\pi\tau_H f\right)^2$$
$$f = \frac{\sqrt{99}}{2\pi\tau_H} = \frac{\sqrt{99}}{2\pi\left(1.918\times10^{-8}\right)}$$
$$f = 82.6\text{ MHz}$$

7.26

(a)
$$\frac{5-V_{SG}}{R_1} = K_P\left(V_{SG}+V_{TP}\right)^2$$
$$5-V_{SG} = (1)(1.2)\left(V_{SG}-1.5\right)^2 = (1.2)\left(V_{SG}^2-3V_{SG}+2.25\right)$$
$$1.2V_{SG}^2 - 2.6V_{SG} - 2.3 = 0 \Rightarrow \underline{V_{SG} = 2.84\,V}$$
$$\underline{I_{DQ} = 1.8\ mA}$$
$$V_{SDQ} = 10-(1.8)(1.2+1.2) \Rightarrow \underline{V_{SDQ} = 5.68\,V}$$
$$g_m = 2\sqrt{K_P I_{DQ}} = 2\sqrt{(1)(1.8)} = 2.683\ mA/V$$
$$r_o = \infty$$

(b)
$$R_{is} = \frac{1}{g_m} = \frac{1}{2.68} = 0.3727\ k\Omega$$
$$R_i = 1.2\|0.373 = 0.284\ k\Omega$$
For C_{C1}, $\tau_{s1} = \left(284+200\right)\left(4.7\times10^{-6}\right) = 2.27\ ms$
For C_{C2}, $\tau_{s2} = \left(1.2x10^3+50\times10^3\right)\left(10^{-6}\right) = 51.2\ ms$

(c)
C_{C2} dominates,

$$f_{3-dB} = \frac{1}{2\pi\tau_{s2}} = \frac{1}{2\pi\left(51.2\times10^{-3}\right)} = 3.1\ Hz$$

7.27

Assume $V_{TN} = 1V, \quad k_n' = 80\mu A/V^2, \quad \lambda = 0$

Neglecting $R_{Si} = 200\Omega$, Midband gain is:

$$|A_v| = g_m R_D$$

Let $I_{DQ} = 0.2\ mA, V_{DSQ} = 5V$

Then $R_D = \dfrac{9-5}{0.2} \Rightarrow \underline{R_D = 20\ k\Omega}$

We need $g_m = \dfrac{|A_v|}{R_D} = \dfrac{10}{20} = 0.5$ mA/V and $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{\left(\dfrac{k_n'}{2}\right)\left(\dfrac{W}{L}\right)I_{DQ}}$

or $0.5 = 2\sqrt{\left(\dfrac{0.080}{2}\right)\left(\dfrac{W}{L}\right)(0.2)} \Rightarrow \underline{\dfrac{W}{L} = 7.81}$

Let

$$R_1 + R_2 = \frac{9}{(0.2)I_{DQ}} = \frac{9}{(0.2)(0.2)} = 225\ k\Omega$$

$$I_{DQ} = 0.2 = \left(\frac{0.080}{2}\right)(7.81)(V_{GS}-1)^2 \Rightarrow V_{GS} = 1.80 = \left(\frac{R_2}{R_1+R_2}\right)(9) = \left(\frac{R_2}{225}\right)(9) \Rightarrow$$

$$\underline{R_2 = 45\ k\Omega,\ R_1 = 180\ k\Omega}$$

$$R_{TH} = R_1 \| R_2 = 180 \| 45 = 36\ k\Omega$$

$$\tau_1 = \frac{1}{2\pi f_1} = \frac{1}{2\pi(200)} = 7.958\times10^{-4}\ s = (R_{Si} + R_{TH})C_C \text{ or } C_C = \frac{7.96\times10^{-4}}{\left(200+36\times10^3\right)} \Rightarrow$$

$$C_C = 0.022\ \mu F$$

$$\tau_2 = \frac{1}{2\pi f_2} = \frac{1}{2\pi\left(3x10^3\right)} = 5.305\times10^{-5}\ s = R_D C_L \text{ or } C_L = \frac{5.31\times10^{-5}}{20\times10^3} \Rightarrow \underline{C_L = 2.65\ nF}$$

7.28

$$I_{BQ} = \frac{10 - 0.7}{430 + (201)(2.5)} = 0.00997\ mA$$
$$I_{CQ} = (200) I_{BQ} = 1.995\ mA$$
$$r_\pi = \frac{(200)(0.026)}{1.99} = 2.61\ k\Omega$$
$$R_{ib} = 2.61 + (201)(2.5) = 505\ k\Omega$$
$$\tau_s = \frac{1}{2\pi f_L} = \frac{1}{2\pi(15)} = 0.0106\ s$$
$$= R_{eq} C_C = \left(0.5 + 505 \| 430\right) \times 10^3 C_C = 232.7 \times 10^3 C_C$$

Or $\underline{C_C = 4.55 \times 10^{-8} F \Rightarrow 45.5\ nF}$

7.29

(a) $V^+ = V_{CEQ} + I_{EQ} R_E$

$$3.3 = 1.8 + (0.25) R_E \Rightarrow R_E = 6\ \text{k}\Omega$$
$$I_{BQ} = \frac{0.25}{121} = 0.002066\ \text{mA}$$
$$V^+ = I_{BQ} R_B + V_{BE}(on) + I_{EQ} R_E$$
$$3.3 = (0.002066)(R_B) + 0.7 + (0.25)(6) \Rightarrow R_B = 532\ \text{k}\Omega$$

(b) $I_{CQ} = \left(\frac{120}{121}\right)(0.25) = 0.2479\ \text{mA}, \quad r_\pi = \frac{(120)(0.026)}{0.2479} = 12.59\ \text{k}\Omega$

$$R_{ib} = r_\pi + (1+\beta) R_E = 12.59 + (121)(6) = 738.6\ \text{k}\Omega$$
$$R_i = R_B \| R_{ib} = 532 \| 738.6 = 309.25\ \text{k}\Omega$$
$$\tau_S = \frac{1}{2\pi f_L} = \frac{1}{2\pi(20)} = 0.007958 = (R_S + R_i) C_C$$

so $C_C = \dfrac{0.007958}{(0.1 + 309.25) \times 10^3} \Rightarrow C_C = 0.0257\ \mu\text{F}$

(c) For $R_S << R_B$,

$$A_\upsilon \cong \frac{(1+\beta) R_E}{r_\pi + (1+\beta) R_E} = \frac{(121)(6)}{12.59 + (121)(6)} = 0.983$$

7.30

$$R_{TH} = R_1 \| R_2 = 1.2 \| 1.2 = 0.6\ k\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(V_{CC}) = \left(\frac{1.2}{1.2+1.2}\right)(5) = 2.5\ V$$

$$I_{BQ} = \frac{2.5 - 0.7}{0.6 + (101)(0.05)} = 0.319\ mA$$

$$I_{CQ} = 31.9\ mA$$

$$r_\pi = \frac{(100)(0.026)}{31.9} = 0.0815\ k\Omega$$

$\tau_{C_{C1}} >> \tau_{C_{C2}}$ and $f = \dfrac{1}{2\pi\tau}$ so that $f_{3-dB}(C_{C1}) << f_{3-dB}(C_{C2})$

Then, for $f_{3-dB}(C_{C1}) \Rightarrow C_{C2}$ acts as an open and for $f_{3-dB}(C_{C2}) \Rightarrow C_{C1}$ acts as a short circuit.

$f_{3-dB}(C_{C2}) = 25\ Hz = \dfrac{1}{2\pi\tau_2}$, so that $\tau_2 = \dfrac{1}{2\pi(25)} = 0.006366\ s = R_{eq}C_{C2}$

where

$$R_{eq} = R_L + R_E \left\| \left(\frac{r_\pi + R_1 \| R_2 \| R_S}{1+\beta} \right) \right.$$

$$= 10 + 50 \left\| \left(\frac{81.5 + 600 \| 300}{101} \right) \right. = 10 + 50 \| 2.787 \Rightarrow$$

$$R_{eq} = 12.64\ \Omega \Rightarrow C_{C2} = \frac{0.00637}{12.6} \Rightarrow \underline{C_{C2} = 504\ \mu F}$$

$R_{ib} = r_\pi + (1+\beta)R_E$ Assume C_{C2} an open

$$R_{ib} = 81.5 + (101)(50) = 5132\ \Omega$$

$$\tau_1 = (100)\tau_2 = (100)(0.006366) = 0.6366\ s = R_{eq1}C_{C1}$$

$$R_{eq1} = R_S + R_{TH} \| R_{ib} = 300 + 600 \| 5132 = 837.2\ \Omega$$

So $C_{C1} = \dfrac{0.6366}{837.2} \Rightarrow \underline{C_{C1} = 760\ \mu F}$

7.31

From Problem 7.30 $R_{TH} = 0.6$ K, $I_{CQ} = 31.9$ mA, $r_\pi = 81.5\ \Omega$

$\tau_{C2} >> \tau_{C1}$ and $f = \dfrac{1}{2\pi\tau}$ so $f_{3-dB}(C_{C2}) << f_{3-dB}(C_{C1})$

Then $f_{3-dB}(C_{C2}) \Rightarrow C_{C1}$ acts as an open circuit and for $f_{3-dB}(C_{C1}) \Rightarrow C_{C2}$ acts as a short circuit.

$$f_{3-dB}\left(C_{C1}\right)=20\text{ Hz}=\frac{1}{2\pi\tau_{C1}}\Rightarrow\tau_{C1}=0.007958\text{ s}$$

$$R_{ib}=r_\pi+\left(1+\beta\right)\left(R_E\left\|R_L\right.\right)=81.5+\left(101\right)\left(50\left\|10\right.\right)=923.2\ \Omega$$

$$\tau_{C1}\Rightarrow R_{eq1}=R_S+R_{TH}\left\|R_{ib}\right.=300+600\left\|923.2\right.=663.7\ \Omega$$

$$C_{C1}=\frac{0.007958}{663.7}\Rightarrow C_{C1}=12\ \mu\text{F}$$

$$\tau_{C2}=100\tau_{C1}=0.7958\text{ s}$$

$$R_{eq2}=R_L+R_E\left\|\left(\frac{r_\pi+R_{TH}}{1+\beta}\right)\right.=10+50\left\|\left(\frac{81.5+600}{101}\right)\right.$$

$$R_{eq2}=10+50\left\|6.748\right.=15.95\ \Omega$$

$$C_{C2}=\frac{0.7958}{15.95}\Rightarrow\underline{C_{C2}=0.050\text{ F}}$$

7.32

(a) $I_{EQ}=0.2\text{ mA},\ I_{BQ}=\dfrac{0.2}{121}=0.001653\text{ mA},\ \ I_{CQ}=\left(\dfrac{120}{121}\right)\left(0.2\right)=0.1983\text{ mA}$

$$V_E=-\left(I_{BQ}R_i+V_{BE}\left(on\right)\right)=-\left[\left(0.001653\right)\left(10\right)+0.7\right]=-0.7165\text{ V}$$

$$V_C=V_E+V_{CEQ}=-0.7165+2.2=1.483\text{ V}$$

$$R_C=\frac{3-1.483}{0.1983}=7.65\text{ k}\Omega$$

(b) $r_\pi=\dfrac{\left(120\right)\left(0.026\right)}{0.1983}=15.73\text{ k}\Omega,\ \ g_m=\dfrac{0.1983}{0.026}=7.627\text{ mA/V}$

$$A_\upsilon=-g_m\left(R_C\left\|R_L\right.\right)\left(\frac{r_\pi}{r_\pi+R_i}\right)=-\left(7.627\right)\left(7.65\left\|20\right.\right)\left(\frac{15.73}{15.73+10}\right)=-25.8$$

(c) For C_C: $\tau_C=\left(R_C+R_L\right)C_C$

$$f_C=\frac{1}{2\pi\tau_C}=\frac{1}{2\pi\left(R_C+R_L\right)C_C}$$

For C_E: $\tau_E=\left(\dfrac{r_\pi+R_i}{1+\beta}\right)C_E\Rightarrow f_E=\dfrac{1}{2\pi\tau_E}$

(d) $f_E=10=\dfrac{1}{2\pi\tau_E}\Rightarrow\tau_E=0.015915\text{ s}$

$$0.015915=\left(\frac{15.73+10}{121}\right)\times10^3\times C_E\Rightarrow C_E=74.8\ \mu\text{F}$$

$$f_C=50=\frac{1}{2\pi\tau_C}\Rightarrow\tau_C=0.003183\text{ s}$$

$$0.003183=\left(7.65+20\right)\times10^3\times C_C\Rightarrow C_C=0.115\ \mu\text{F}$$

7.33

a.

$$I_D = K_n\left(V_{GS} - V_{TN}\right)^2$$

$$V_{GS} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{0.5}{0.5}} + 0.8 = 1.8\ V$$

$$R_S = \frac{-V_{GS} - (-5)}{0.5} = \frac{5 - 1.8}{0.5} \Rightarrow \underline{R_S = 6.4\ \text{k}\Omega}$$

$$V_D = V_{DSQ} + V_S = 4 - 1.8 = 2.2\ \text{V}$$

$$R_D = \frac{5 - 2.2}{0.5} \Rightarrow \underline{R_D = 5.6\ \text{k}\Omega}$$

(b)

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.5)} = 1\ \text{mA/V}$$

$$\tau_A = R_S C_S = \left(6.4\times10^3\right)\left(5\times10^{-6}\right)$$

$$= 3.2\times10^{-2}\ \text{s}$$

$$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi\left(3.2\times10^{-2}\right)} \Rightarrow f_A = 4.97\ \text{Hz}$$

$$\tau_B = \left(\frac{R_S}{1 + g_m R_S}\right) C_S = \left[\frac{6.4\times10^3}{1 + (1)(6.4)}\right]\left(5\times10^{-6}\right)$$

$$= 4.32\times10^{-3}\ \text{s}$$

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi\left(4.32\times10^{-3}\right)} \Rightarrow f_B = 36.8\ \text{Hz}$$

c.

$$|A_v| = \frac{g_m R_D\left(1 + sR_S C_S\right)}{\left(1 + g_m R_S\right)\left[1 + s\left(\frac{R_S}{1 + g_m R_S}\right) C_S\right]}$$

As R_S becomes large

$$|A_v| \to \frac{g_m R_D\left(sR_S C_S\right)}{\left(g_m R_S\right)\left[1 + s\left(\frac{R_S}{g_m R_S}\right) C_S\right]}$$

$$A_v = \frac{\left(g_m R_D\right)\left[s\left(\frac{1}{g_m}\right) C_S\right]}{1 + s\left(\frac{1}{g_m}\right) C_S}$$

The corner frequency $f_B = \dfrac{1}{2\pi\left(1/g_m\right) C_S}$ and the corresponding $\underline{f_A \to 0}$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.5)(0.5)} = 1\ mA/V$$

$$f_B = \frac{1}{2\pi\left(\frac{1}{10^{-3}}\right)\left(5\times10^{-6}\right)} \Rightarrow \underline{f_B = 31.8\ \text{Hz}}$$

7.34

(a) (i) $T_1(s)=\dfrac{V_{o1}}{V_i}=-g_{m1}\left(r_o\left\|R_D\right\|\dfrac{1}{sC_L}\right)=-g_{m1}\left[\dfrac{(r_o\|R_D)\left(\dfrac{1}{sC_L}\right)}{(r_o\|R_D)+\left(\dfrac{1}{sC_L}\right)}\right]$

$$T_1(s)=-g_{m1}(r_o\|R_D)\cdot\frac{1}{[1+s(r_o\|R_D)C_L]}$$

(ii) $T_2(s)=\dfrac{V_o}{V_{o1}}=-g_{m2}(r_o\|R_D)\cdot\dfrac{1}{[1+s(r_o\|R_D)C_L]}$

(iii) $T(s)=\dfrac{V_o}{V_i}=g_{m1}g_{m2}(r_o\|R_D)^2\cdot\dfrac{1}{[1+s(r_o\|R_D)C_L]^2}$

(b) (i) $f_{3-dB}=\dfrac{1}{2\pi(r_o\|R_D)C_L}$

Now $r_o=\dfrac{1}{(0.02)(0.5)}=100\,\text{k}\Omega$, $r_o\|R_D=100\|5=4.762\ \text{k}\Omega$

$$f_{3-dB}=\frac{1}{2\pi(4.762\times10^3)(12\times10^{-12})}\Rightarrow f_{3-dB}=2.785\,\text{MHz}$$

(ii) $f_{3-dB}=2.785$ MHz

(iii) Want $\left\{\dfrac{1}{\sqrt{1+[(2\pi f)(r_o\|R_D)C_L]^2}}\right\}^2=\dfrac{1}{\sqrt{2}}$

So $\dfrac{1}{1+\left(\dfrac{f}{2.785\times10^6}\right)^2}=\dfrac{1}{\sqrt{2}}=0.7071$

$$\left(\frac{f}{2.785\times10^6}\right)^2=\frac{1}{0.7071}-1=0.4142$$

which yields $f=1.792$ MHz

7.35

a. Expression for the voltage gain is the same as Equation (7.59) with $R_S=0$.

b.

$$\tau_A=R_EC_E$$

$$\tau_B=\frac{R_Er_\pi C_E}{r_\pi+(1+\beta)R_E}$$

7.36

(a) $I_{EQ} = \left(\frac{91}{90}\right)(0.15) = 0.1517 \text{ mA}$

$$R_E = \frac{3 - 0.7}{0.1517} = 15.16 \text{ k}\Omega$$

$$V_C = 0.7 - V_{ECQ} = 0.7 - 2.2 = -1.5 \text{ V}$$

$$R_C = \frac{-1.5 - (-3)}{0.15} = 10 \text{ k}\Omega$$

(b) $g_m = \frac{0.15}{0.026} = 5.769 \text{ mA/V}$

$$A_\upsilon = -g_m R_C = -(5.769)(10) = -57.7$$

(c) $\tau_A = R_E C_E = (15.16 \times 10^3)(3 \times 10^{-6}) = 4.548 \times 10^{-2} \text{ s}$

$$f_A = \frac{1}{2\pi\tau_A} = \frac{1}{2\pi(4.548 \times 10^{-2})} = 3.5 \text{ Hz}$$

$$\tau_B = \frac{R_E r_\pi C_E}{r_\pi + (1+\beta)R_E}, \quad \text{where} \quad r_\pi = \frac{(90)(0.026)}{0.15} = 15.6 \text{ k}\Omega$$

$$\tau_B = \frac{(15.16 \times 10^3)(15.6 \times 10^3)(3 \times 10^{-6})}{15.6 \times 10^3 + (91)(15.16 \times 10^3)} = 5.085 \times 10^{-4} \text{ s}$$

$$f_B = \frac{1}{2\pi\tau_B} = \frac{1}{2\pi(5.085 \times 10^{-4})} = 313 \text{ Hz}$$

7.37

(a) $I_{EQ} = \frac{10 - 0.7}{10} = 0.93 \text{ mA}, \quad I_{CQ} = \left(\frac{90}{91}\right)(0.93) = 0.9198 \text{ mA}$

$$g_m = \frac{0.9198}{0.026} = 35.38 \text{ mA/V}$$

$$A_\upsilon = g_m (R_C \| R_L) = (35.38)(5 \| 10) = 118$$

(b) $f = \frac{1}{2\pi\tau} = \frac{1}{2\pi(R_C \| R_L)C_L} = \frac{1}{2\pi(5 \| 10) \times 10^3 \times (3 \times 10^{-12})}$

$$f = 15.9 \text{ MHz}$$

7.38

(a) $I_{DQ} = K_p (V_{SGQ} + V_{TP})^2$

$$0.2 = 0.1(V_{SGQ} - 0.6)^2 \Rightarrow V_{SGQ} = 2.014 \text{ V}$$

$$R_S = \frac{3 - 2.014}{0.2} = 4.93 \text{ k}\Omega$$

$$V_D = V_{SGQ} - V_{SDQ} = 2.014 - 1.9 = 0.114 \text{ V}$$

$$R_D = \frac{0.114 - (-3)}{0.2} = 15.6 \text{ k}\Omega$$

(b) $f_H = \dfrac{1}{2\pi(R_D \| R_L)C_L}$

or $C_L = \dfrac{1}{2\pi(15.6 \| 20)\times 10^3 \times 4\times 10^6} \Rightarrow C_L = 4.54 \text{ pF}$

7.39

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)V_{DD} = \left(\frac{166}{166+234}\right)(10)$$
$$= 4.15 \text{ V}$$

$$I_D = \frac{V_G - V_{GS}}{R_S} = K_n\left(V_{GS} - V_{TN}\right)^2$$

$$4.15 - V_{GS} = (0.5)(0.5)\left(V_{GS}^2 - 4V_{GS} + 4\right)$$

$$0.25V_{GS}^2 - 3.15 = 0 \Rightarrow V_{GS} = 3.55 \text{ V}$$

$$g_m = 2K_n\left(V_{GS} - V_{TN}\right) = 2(0.5)(3.55-2)$$

$$g_m = 1.55 \; mA/V$$

$$R_0 = R_S \left\| \frac{1}{g_m} = 0.5 \right\| \frac{1}{1.55} = 0.5 \| 0.645$$

$$R_0 = 0.282 \text{ k}\Omega$$

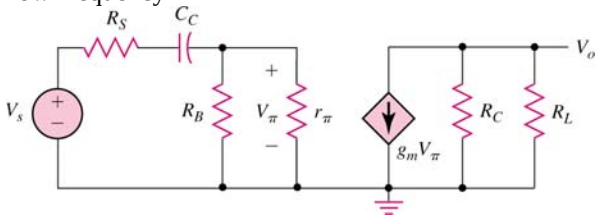
$$\tau = (R_o \| R_L)C_L \quad \text{and} \quad f_H = \frac{1}{2\pi\tau}$$

$$\text{BW} \cong f_H = 5 \text{ MHz} \Rightarrow \tau = \frac{1}{2\pi(5\times 10^6)} = 3.18\times 10^{-8} \text{ s}$$

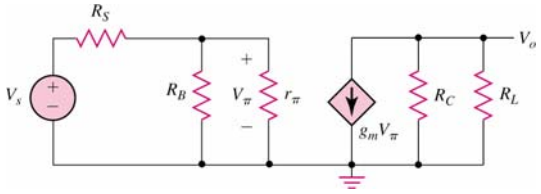
$$C_L = \frac{\tau}{R_o \| R_L} = \frac{3.18\times 10^{-8}}{(0.282 \| 4)\times 10^3} \Rightarrow C_L = 121 \text{ pF}$$

7.40

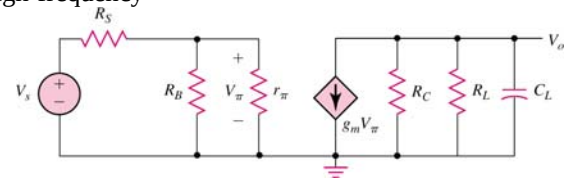
(a) Low-frequency



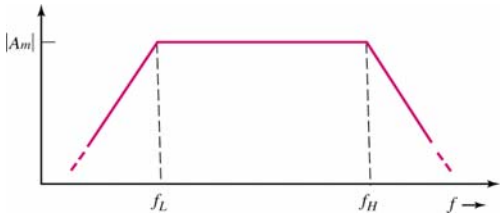
Mid-Band



High-frequency



(b)



(c)

$$I_{BQ} = \frac{12-0.7}{1\,M\Omega} = 11.3\ \mu A$$

$$I_{CQ} = 1.13\ mA$$

$$r_\pi = \frac{(100)(0.026)}{1.13} = 2.3\ k\Omega$$

$$g_m = \frac{1.13}{0.026} = 43.46\ mA/V$$

$$A_m = \frac{V_o}{V_s}(midband) = -g_m(R_C \| R_L)\left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_S}\right)$$

$$= -(43.46)(5.1 \| 500)\left(\frac{1000 \| 2.3}{1000 \| 2.3 + 1}\right)$$

$$= -(43.46)(5.05)\left(\frac{2.29}{2.29+1}\right) \Rightarrow |A_m| = 153$$

$$|A_m|_{dB} = 43.7\ \text{dB}$$

$$f_L = \frac{1}{2\pi\tau_L}, \quad \tau_L = (R_S + R_B \| r_\pi)C_C = (1 + 1000 \| 2.3)\times 10^3 \times (10\times 10^{-6})$$

$$\Rightarrow \tau_L = 3.29\times 10^{-2}\ \text{s}, \quad f_L = 4.83\ \text{Hz}$$

$$f_H = \frac{1}{2\pi\tau_H}, \quad \tau_L = (R_C \| R_L)C_L = (5.1 \| 500)\times 10^3 \times (10\times 10^{-12})$$

$$\Rightarrow \tau_H = 5.05\times 10^{-8}\ \text{s}, \quad f_H = 3.15\ \text{MHz}$$

7.41

(a) $$A_\upsilon = -g_m\left(R_D \| R_L \| \frac{1}{sC_L}\right) = -g_m\left[\frac{(R_D \| R_L)\cdot\frac{1}{sC_L}}{(R_D \| R_L) + \frac{1}{sC_L}}\right]$$

$$A_\upsilon = -g_m(R_D \| R_L)\left[\frac{1}{1 + s(R_D \| R_L)C_L}\right]$$

(b) $$\tau = (R_D \| R_L)C_L$$

(c) $5 = I_D R_S + V_{SG} = K_p R_S (V_{SG} + V_{TP})^2 + V_{SG}$

$5 = (0.25)(3.2)(V_{SG} - 2)^2 + V_{SG}$

We find $0.8V_{SG}^2 - 2.2V_{SG} - 1.8 = 0 \Rightarrow V_{SG} = 3.41\,\text{V}$

$I_{DQ} = (0.25)(3.41 - 2)^2 = 0.497\,\text{mA}$

$\tau = (10 \| 20) \times 10^3 \times 10 \times 10^{-12} = 6.67 \times 10^{-8}\,\text{s}$

$f_H = \dfrac{1}{2\pi\tau} = \dfrac{1}{2\pi(6.67 \times 10^{-8})} \Rightarrow f_H = 2.39\,\text{MHz}$

$g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.25)(0.497)} = 0.705\,\text{mA/V}$

$A_v = -g_m (R_D \| R_L) = -(0.705)(10 \| 20) = -4.7$

7.42 Computer Analysis

7.43 Computer Analysis

7.44 Computer Analysis

7.45

$g_m = \dfrac{0.25}{0.026} = 9.615\,\text{mA/V}$

$f_\beta = \dfrac{f_T}{\beta_o} = \dfrac{4 \times 10^9}{120} \Rightarrow f_\beta = 33.3\,\text{MHz}$

$f_T = \dfrac{g_m}{2\pi(C_\pi + C_\mu)} \Rightarrow C_\pi + C_\mu = \dfrac{g_m}{2\pi f_T} = \dfrac{9.615 \times 10^{-3}}{2\pi(4 \times 10^9)}$

or $C_\pi + C_\mu = 0.3826\,\text{pF}$

Then $C_\pi = 0.3826 - 0.08 = 0.303\,\text{pF}$

7.46

(a) $f_\beta = \dfrac{f_T}{\beta_o} = \dfrac{2 \times 10^9}{120} \Rightarrow f_\beta = 16.67\,\text{MHz}$

$g_m = \dfrac{0.4}{0.026} = 15.38\,\text{mA/V}$

$C_\pi + C_\mu = \dfrac{g_m}{2\pi f_T} = \dfrac{15.38 \times 10^{-3}}{2\pi(2 \times 10^9)}$

$C_\pi + C_\mu = 1.224\,\text{pF}, \quad C_\pi = 1.224 - 0.075 = 1.15\,\text{pF}$

(b) $|h_{fe}| = \dfrac{\beta_o}{\sqrt{1 + \left(\dfrac{f}{f_\beta}\right)^2}}$

(i)At $f = 10\,\text{MHz}$, $\left|h_{fe}\right| = \dfrac{120}{\sqrt{1+\left(\dfrac{10}{16.67}\right)^2}} = 103$

(ii)At $f = 20\,\text{MHz}$, $\left|h_{fe}\right| = \dfrac{120}{\sqrt{1+\left(\dfrac{20}{16.67}\right)^2}} = 76.8$

(iii) At $f = 50\,\text{MHz}$, $\left|h_{fe}\right| = \dfrac{120}{\sqrt{1+\left(\dfrac{50}{16.67}\right)^2}} = 38.0$

7.47

(a) $f_\beta = \dfrac{f_T}{\beta_o} = \dfrac{540\times10^6}{120} \Rightarrow f_\beta = 4.5\,\text{MHz}$

$g_m = \dfrac{0.2}{0.026} = 7.692\,\text{mA/V}$

$C_\pi + C_\mu = \dfrac{g_m}{2\pi f_T} = \dfrac{7.692\times10^{-3}}{2\pi\left(540\times10^6\right)} \Rightarrow C_\pi + C_\mu = 2.267\,\text{pF}$

$C_\pi = 2.267 - 0.4 = 1.87\,\text{pF}$

(b) $g_m = \dfrac{0.8}{0.026} = 30.77\,\text{mA/V}$

$f_T = \dfrac{g_m}{2\pi\left(C_\pi + C_\mu\right)} = \dfrac{30.77\times10^{-3}}{2\pi\left(2.267\times10^{-12}\right)} \Rightarrow f_T = 2.16\,\text{GHz}$

$f_\beta = \dfrac{2.16\times10^9}{120} \Rightarrow f_\beta = 18.0\,\text{MHz}$

7.48

(a)

$V_0 = -g_m V_\pi R_L$ where

$$V_\pi = \frac{r_\pi \left\| \dfrac{1}{sC_1} \right.}{r_\pi \left\| \dfrac{1}{sC_1} \right. + r_b} \cdot V_i = \frac{\dfrac{r_\pi}{1+sr_\pi C_1}}{\dfrac{r_\pi}{1+sr_\pi C_1} + r_b} \cdot V_i$$

$$= \frac{r_\pi}{r_\pi + r_b + sr_b r_\pi C_1} \cdot V_i = \left(\frac{r_\pi}{r_\pi + r_b}\right)\left(\frac{1}{1+s\left(r_b \left\| r_\pi\right.\right)C_1}\right) \cdot V_i$$

$$A_v(s) = \frac{V_0(s)}{V_i(s)} = -g_m R_L \left(\frac{r_\pi}{r_\pi + r_b}\right)\left(\frac{1}{1+s\left(r_b \left\| r_\pi\right.\right)C_1}\right)$$

So

(b) Midband gain: $r_\pi = \dfrac{(100)(0.026)}{1} = 2.6\ k\Omega,\ g_m = \dfrac{1}{0.026} = 38.46\ mA/V$

(i) For $r_b = 100\ \Omega$

$$A_{v1} = -(38.46)(4)\left(\frac{2.6}{2.6+0.1}\right) \Rightarrow \underline{A_{v1} = -148.1}$$

(ii) For $r_b = 500\ \Omega$

$$A_{v2} = -(38.46)(4)\left(\frac{2.6}{2.6+0.5}\right) \Rightarrow \underline{A_{v2} = -129.0}$$

(c) $f_{3-dB} = \dfrac{1}{2\pi\tau}, \quad \tau = (r_b \| r_\pi) C_1$

(i) For $r_b = 100\ \Omega$

$$\tau_1 = (0.1 \| 2.6) \times 10^3 \left(2.2 \times 10^{-12}\right) = 2.12 \times 10^{-10}\ s \Rightarrow \underline{f_{3-db} = 751\ MHz}$$

(ii) For $r_b = 500\ \Omega$

$$\tau_2 = (0.5 \| 2.6) \times 10^3 \left(2.2 \times 10^{-12}\right) = 9.23 \times 10^{-10}\ s\ \underline{f_{3-dB} = 173\ MHz}$$

7.49

(b) $f = 10\ \text{kHz} = 10^4$

$$Z_i = 200 + \frac{2500\left(1 - j\left(10^4\right)\left(1.333 \times 10^{-6}\right)\right)}{1 + \left(10^4\right)^2 \left(1.333 \times 10^{-6}\right)^2}$$

$$= 200 + 2500 - j33.3 = 2700 - j33.3$$

(c) $f = 100\ \text{kHz} = 10^5$

$$Z_i = 200 + \frac{2500\left(1 - j\left(10^5\right)\left(1.333 \times 10^{-6}\right)\right)}{1 + \left(10^5\right)^2 \left(1.333 \times 10^{-6}\right)^2}$$

$$Z_i = 200 + 2456 - j327 = 2656 - j327$$

(d) $f = 1\ \text{MHz} = 10^6$

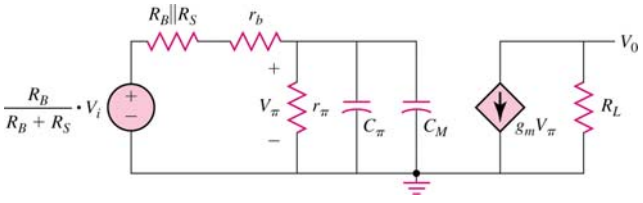
$$Z_i = 200 + \frac{2500\left(1 - j\left(10^6\right)\left(1.333 \times 10^{-6}\right)\right)}{1 + \left(10^6\right)^2 \left(1.333 \times 10^{-6}\right)^2}$$

$$Z_i = 200 + 900 - j1200 = 1100 - j1200$$

7.50

a. $\underline{C_M = C_\mu (1 + g_m R_L)}$

b.



$V_0 = -g_m V_\pi R_L \quad$ Let $C_\pi + C_M = C_i$

$$V_\pi = \frac{r_\pi \left\| \frac{1}{sC_i} \right.}{r_\pi \left\| \frac{1}{sC_1} \right. + R_B \| R_S + r_b} \cdot \left(\frac{R_B}{R_B + R_S} \right) V_i$$

$$A_\upsilon(s) = \frac{V_o(s)}{V_i(s)}$$

$$= -g_m R_L \left(\frac{R_B}{R_B + R_S} \right) \left[\frac{\dfrac{r_\pi \cdot \dfrac{1}{sC_i}}{r_\pi + \dfrac{1}{sC_i}}}{\dfrac{r_\pi \cdot \dfrac{1}{sC_i}}{r_\pi + \dfrac{1}{sC_i}} + R_B \| R_S + r_b} \right]$$

$$= -g_m R_L \left(\frac{R_B}{R_B + R_S} \right) \cdot \left[\frac{r_\pi}{r_\pi + (1 + sr_\pi C_i)(R_B \| R_S + r_b)} \right]$$

Let $R_{eq} = (R_B \| R_S + r_b)$

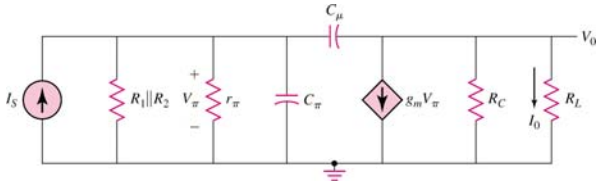
$$A_v(s) = -\beta R_L \left(\frac{R_B}{R_B + R_S} \right) \times \left[\frac{1}{(r_\pi + R_{eq})\left[1 + s(r_\pi \| R_{eq}) C_i\right]} \right]$$

$$A_v(s) = \frac{-\beta R_L}{r_\pi + R_{eq}} \cdot \left(\frac{R_B}{R_B + R_S} \right) \cdot \frac{1}{1 + s(r_\pi \| R_{eq}) C_i}$$

c. $$f_H = \frac{1}{2\pi (r_\pi \| R_{eq}) C_i}$$

7.51

High Freq. $\Rightarrow C_{C1},\ C_{C2},\ C_E \rightarrow$ short circuits



$$g_m = \frac{I_{CQ}}{V_T} = \frac{5}{0.026} = 192.3 \text{ mA/V}$$

$$f_T = \frac{g_m}{2\pi (C_\pi + C_\mu)} \Rightarrow 250 \times 10^6 = \frac{192 \times 10^{-3}}{2\pi (C_\pi + C_\mu)}$$

$C_\pi + C_\mu = 122.4$ pF $\Rightarrow C_\mu = 5$ pF, $C_\pi = 117.4$ pF

$$C_M = C_\mu\left(1+g_m\left(R_C\|R_L\right)\right)$$
$$= 5\left[1+(192.3)(1\|1)\right] \Rightarrow C_M = 485.8 \text{ pF}$$
$$C_i = C_\pi + C_M = 117 + 485 = 603 \text{ pF}$$
$$r_\pi = \frac{(200)(0.026)}{5} = 1.04 \text{ k}\Omega$$
$$R_{eq} = R_1\|R_2\|r_\pi = 5\|1.04 = 0.861 \text{ k}\Omega$$
$$\tau = R_{eq}\cdot C_i = \left(0.861\times10^3\right)\left(603\times10^{-12}\right)$$
$$= 5.19\times10^{-7} \text{ s}$$
$$f = \frac{1}{2\pi\tau} = \frac{1}{2\pi\left(5.19\times10^{-7}\right)} \Rightarrow f = 307 \text{ kHz}$$

7.52

$$R_{TH} = R_1\|R_2 = 60\|5.5 = 5.04 \text{ k}\Omega$$
$$V_{TH} = \left(\frac{R_2}{R_1+R_2}\right)V_{CC} = \left(\frac{5.5}{5.5+60}\right)(15) = 1.26 \text{ V}$$
$$I_{BQ} = \frac{1.26-0.7}{5.04+(101)(0.2)} = 0.0222 \text{ mA}$$
$$I_{CQ} = 2.22 \text{ mA}$$
$$r_\pi = \frac{(100)(0.026)}{2.22} = 1.17 \text{ k}\Omega$$
$$g_m = \frac{2.22}{0.026} = 85.4 \text{ mA/V}$$

Lower 3 – *dB* frequency:

$$\tau_L = R_{eq}\cdot C_{C1}$$
$$R_{eq} = R_S + R_1\|R_2\|r_\pi$$
$$= 2 + 60\|5.5\|1.17 = 2.95 \text{ k}\Omega$$
$$\tau_L = \left(2.95\times10^3\right)\left(0.1\times10^{-6}\right) = 2.95\times10^{-4} \text{ s}$$
$$f_L = \frac{1}{2\pi\tau_L} = \frac{1}{2\pi\left(2.95\times10^{-4}\right)} \Rightarrow f_L = 540 \text{ Hz}$$

Upper 3 – *dB* frequency:

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)} \Rightarrow 400\times10^6 = \frac{85.4\times10^{-3}}{2\pi\left(C_\pi + C_\mu\right)}$$
$$C_\pi + C_\mu = 34 \text{ pF};\quad C_\mu = 2 \text{ pF};\quad C_\pi = 32 \text{ pF}$$
$$C_M = C_\mu\left(1+g_m R_C\right) = 2\left[1+(85.4)(4)\right] \Rightarrow C_M = 685 \text{ pF}$$
$$C_i = C_\pi + C_M = 32 + 685 = 717 \text{ pF}$$
$$R_{eq} = R_S\|R_1\|R_2\|r_\pi = 2\|60\|5.5\|1.17 \Rightarrow R_{eq} = 0.644 \text{ k}\Omega$$
$$\tau = R_{eq}\cdot C_i = \left(0.644\times10^3\right)\left(717\times10^{-12}\right)$$
$$= 4.62\times10^{-7} \text{ s}$$
$$f_H = \frac{1}{2\pi\tau} \Rightarrow f_H = 344 \text{ kHz}$$

7.53

$$R_{TH} = R_1 \| R_2 = 600 \| 55 = 50.38 \text{ K}$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)(15) = \left(\frac{55}{600 + 55}\right)(15) = 1.2595 \text{ V}$$

$$I_{BQ} = \frac{1.26 - 0.7}{50.4 + (101)(2)} = 0.00222 \text{ mA}$$

$$I_{CQ} = 0.2217 \text{ mA}$$

$$r_\pi = \frac{(100)(0.026)}{0.222} = 11.73 \text{ K}$$

$$g_m = \frac{0.2217}{0.026} = 8.527 \text{ mA/V}$$

Lower – 3dB Freq

$$\tau_L = \mathrm{R}_{eq1} C_{c1};\ R_{eq1} = R_S + R_{TH} \| r_\pi$$
$$= 0.50 + 50.38 \| 11.73 = 10.0 \text{ K}$$

$$\tau_L = \left(10 \times 10^3\right)\left(0.1 \times 10^{-6}\right) = 10^{-3} \text{ s}$$

$$f_L = \frac{1}{2\pi\tau_L} = \frac{1}{2\pi\left(10^{-3}\right)} \Rightarrow f_L = 159 \text{ Hz}$$

Upper – 3dB Freq

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)} = \frac{8.527 \times 10^{-3}}{2\pi\left(C_\pi + 2\right) \times 10^{-12}} = 400 \times 10^6$$

$$C_\pi + C_\mu = 3.393 \text{ pF} \Rightarrow C_\pi = 1.393 \text{ pF}$$

$$C_M = C_\mu\left(1 + g_m R_C\right) = 2\left[1 + (8.527)(40)\right] = 684 \text{ pF}$$

$$C_T = C_\pi + C_M = 1.393 + 684 = 685.4 \text{ pF}$$

$$R_{eq2} = R_S \| R_{TH} \| r_\pi = 0.5 \| 50.38 \| 11.73$$
$$= 50.38 \| 0.480 = 0.4750 \text{ K}$$

$$\tau_H = \mathrm{R}_{eq2} . C_T = \left(0.4750 \times 10^3\right)\left(685.4 \times 10^{-12}\right)$$
$$= 3.256 \times 10^{-7} \text{ s}$$

$$f_H = \frac{1}{2\pi\tau_H} = \frac{1}{2\pi\left(3.256 \times 10^{-7}\right)} \Rightarrow f_H = 489 \text{ KHz}$$

7.54

(a) $R_{TH} = R_1 \| R_2 = 33 \| 22 = 13.2 \ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{22}{22+33}\right)(5) = 2 \ \text{V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1+\beta)R_E} = \frac{2 - 0.7}{13.2 + (151)(4)} = 0.002106 \ \text{mA}$$

$$I_{CQ} = 0.3159 \ \text{mA}, \quad I_{EQ} = 0.3180 \ \text{mA}$$

$$V_{CEQ} = 5 - (0.3159)(5) - (0.3180)(4) = 2.15 \ \text{V}$$

(b) $$f_\beta = \frac{f_T}{\beta_o} = \frac{800 \times 10^6}{150} \Rightarrow f_\beta = 5.33 \ \text{MHz}$$

$$g_m = \frac{0.3159}{0.026} = 12.15 \ \text{mA/V}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{12.15 \times 10^{-3}}{2\pi(800 \times 10^6)} \Rightarrow C_\pi + C_\mu = 2.417 \ \text{pF}$$

$$C_\pi = 2.417 - 0.45 = 1.97 \ \text{pF}$$

$$C_M = C_\mu[1 + g_m R_C] = (0.45)[1 + (12.15)(5)] = 27.79 \ \text{pF}$$

(c) $$r_\pi = \frac{(150)(0.026)}{0.3159} = 12.35 \ \text{k}\Omega, \quad R_{TH} \| r_\pi = 13.2 \| 12.35 = 6.38 \ \text{k}\Omega$$

$$f_{3-dB} = \frac{1}{2\pi(R_{TH} \| r_\pi)(C_\pi + C_M)} = \frac{1}{2\pi(6.38 \times 10^3)(1.97 + 27.79) \times 10^{-12}}$$

$$f_{3-dB} = 838 \ \text{kHz}$$

7.55

$$g_{m1} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right) I_{DQ}} = 2\sqrt{\left(\frac{0.08}{2}\right)\left(\frac{4}{0.8}\right)(0.6)} = 0.6928 \ \text{mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{0.6928 \times 10^{-3}}{2\pi(50 + 10) \times 10^{-15}} \Rightarrow f_T = 1.84 \ \text{GHz}$$

7.56

$$I_{DQ} = K_n(V_{GS} - V_{TN})^2$$

$$0.12 = K_n(0.2)^2 \Rightarrow K_n = 3 \ \text{mA/V}^2$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(3)(0.12)} = 1.2 \ \text{mA/V}$$

$$f_T = \frac{g_m}{2\pi(C_{gs} + C_{gd})} = \frac{1.2 \times 10^{-3}}{2\pi(40 + 10) \times 10^{-15}} \Rightarrow f_T = 3.82 \ \text{GHz}$$

7.57

(a) $$g_m = 2\sqrt{(1.5)(0.05)} = 0.5477 \ \text{mA/V}$$

$$f_T = \frac{0.5477 \times 10^{-3}}{2\pi(60 + 10) \times 10^{-15}} \Rightarrow f_T = 1.25 \ \text{GHz}$$

(b) $g_m = 2\sqrt{(1.5)(0.3)} = 1.342$ mA/V

$$f_T = \frac{1.342\times10^{-3}}{2\pi(60+10)\times10^{-15}} \Rightarrow f_T = 3.05\,\text{GHz}$$

(c) $$3\times10^9 = \frac{g_m}{2\pi(60+10)\times10^{-15}} \Rightarrow g_m = 1.319\text{ mA/V}$$

$$g_m = 2\sqrt{K_n I_{DQ}} \Rightarrow I_{DQ} = \frac{1}{K_n}\left(\frac{g_m}{2}\right)^2 = \frac{1}{1.5}\left(\frac{1.319}{2}\right)^2 = 0.29\,\text{mA}$$

(d) $g_m = 2\sqrt{(1.5)(0.25)} = 1.225$ mA/V

$$2.5\times10^9 = \frac{1.225\times10^{-3}}{2\pi(C_{gs}+8)\times10^{-15}} \Rightarrow C_{gs} = 70\,\text{fF}$$

7.58

$$f_T = \frac{g_m}{2\pi(C_{gs}+C_{gd})}$$

$$C_{gs}+C_{gd} = WLC_{ox}$$

$$g_m = 2K_n(V_{GS}-V_{TN}) = 2\left(\frac{W}{L}\right)\left(\frac{\mu_n C_{ox}}{2}\right)(V_{GS}-V_{TN})$$

Then $$f_T = \frac{\left(\frac{W}{L}\right)(\mu_n C_{ox})(V_{GS}-V_{TN})}{2\pi WLC_{ox}}$$

$$f_T = \frac{\mu_n(V_{GS}-V_{TN})}{2\pi L^2}$$

(a) $$f_T = \frac{450(0.5)}{2\pi(1.2\times10^{-4})^2} \Rightarrow f_T = 2.49\text{ GHz}$$

(b) $$f_T = \frac{450(0.5)}{2\pi(0.18\times10^{-4})} \Rightarrow f_T = 111\,\text{GHz}$$

7.59

(a) $C_M = C_{gd}\left[1+g_m(r_o \| R_D)\right] = (12)\left[1+(3)(120\|10)\right] = 344.3$ fF

(b) $$f_{3-dB} = \frac{1}{2\pi\tau}$$

$$\tau = r_i(C_{gs}+C_M) = (10^4)(80+344.3)\times10^{-15} = 4.243\times10^{-9}\,\text{s}$$

$$f_{3-dB} = \frac{1}{2\pi(4.243\times10^{-9})} \Rightarrow f_{3-dB} = 37.5\,\text{MHz}$$

7.60

$$f_T = \frac{g_m}{2\pi\left(C_{gsT} + C_{gdT}\right)} \quad \text{(Eq. (7.97))}$$

Let $C_{gdT} = 0$ and $C_{gsT} = \left(\frac{2}{3}\right)(WLC_{ox})$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{\left(\frac{\mu_n C_{ox}}{2}\right)\left[\frac{W}{L}\right] I_D}$$

$$\text{So } f_T = \frac{2\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right) I_D}}{2\pi\left(\frac{2}{3}\right)(W\ LC_{ox})}$$

$$= \frac{3}{2\pi L} \cdot \frac{\sqrt{\left(\frac{1}{2}\mu_n C_{ox}\right)\left(\frac{W}{L}\right) I_D}}{W\ C_{ox}}$$

$$\underline{f_T = \frac{3}{2\pi L} \cdot \sqrt{\frac{\mu_n I_D}{2W\ C_{ox} L}}}$$

7.61

(a) $$K_n = \left(\frac{\mu_n C_{ox}}{2}\right)\left(\frac{W}{L}\right) = \frac{(400)\left(6.9\times10^{-9}\right)}{2}(8) \Rightarrow K_n = 1.104 \text{ mA/V}^2$$

$$I_D = K_n\left(V_{GS} - V_{TN}\right)^2 = (1.104)(3-0.4)^2 = 7.463 \text{ mA}$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(1.104)(7.463)} = 5.741 \text{ mA/V}$$

$$g'_m = \frac{g_m}{1 + g_m r_s}$$

$$(0.8) g_m = \frac{g_m}{1 + g_m r_s} \Rightarrow g_m r_s = \frac{1}{0.8} - 1 = 0.25$$

$$r_s = \frac{0.25}{5.741} \Rightarrow r_s = 43.5\,\Omega$$

(b) $$I_D = (1.104)(1-0.4)^2 = 0.3974 \text{ mA}$$

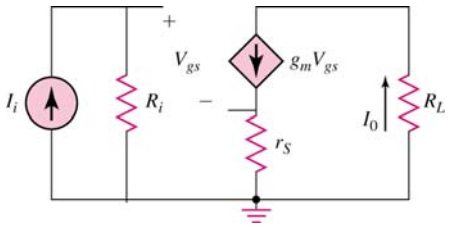
$$g_m = 2\sqrt{(1.104)(0.3974)} = 1.325 \text{ mA/V}$$

$$g'_m = \frac{1.325}{1 + (1.325)(0.04355)} = 1.253 \text{ mA/V}$$

$$\frac{g'_m}{g_m} \Rightarrow 94.5\%$$

7.62

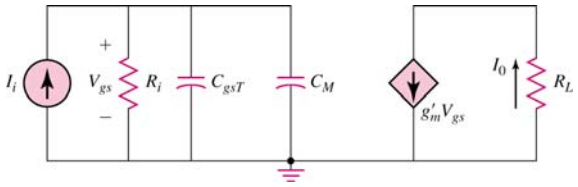
a.



$I_0 = g_m V_{gs}$ and $V_{gs} = I_i R_i - g_m V_{gs} r_S$ so $V_{gs} = \dfrac{I_i R_i}{1 + g_m r_S}$

Then $A_i = \dfrac{I_0}{I_i} = \dfrac{g_m R_i}{1 + g_m r_S}$

b. As an approximation, consider



In this case

$$A_i = \frac{I_0}{I_i} = g'_m R_i \cdot \frac{1}{1 + sR_i\left(C_{gsT} + C_M\right)}$$ where $C_M = C_{gdT}\left(1 + g'_m R_L\right)$ and $g'_m = \dfrac{g_m}{1 + g_m r_s}$

c. As r_S increases, C_M decreases, so the bandwidth increases, but the current gain magnitude decreases.

7.63

(b) $V_{GS} = \left(\dfrac{225}{225 + 500}\right)(10) = 3.103 \text{ V}$

$I_{DQ} = (1)(3.103 - 2)^2 = 1.218 \text{ mA}$

$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1)(1.218)} = 2.207 \text{ mA/V}$

$C_M = C_{gd}\left(1 + g_m R_D\right) = (8)\left[1 + (2.207)(5)\right] = 96.28 \text{ fF}$

(c) $f_{3-dB} = \dfrac{1}{2\pi\tau}$, $\tau = \left(R_i \| R_1 \| R_2\right)\left(C_{gs} + C_M\right)$

Now $R_i \| R_1 \| R_2 = 1 \| 500 \| 225 = 0.9936 \text{ k}\Omega$

$\tau = \left(0.9936 \times 10^3\right)(50 + 96.28) \times 10^{-15} = 1.453 \times 10^{-10} \text{ s}$

$f_{3-dB} = \dfrac{1}{2\pi\left(1.453 \times 10^{-10}\right)} \Rightarrow f_{3-dB} = 1.095 \text{ GHz}$

$A_\upsilon = -g_m R_D\left(\dfrac{R_1 \| R_2}{R_1 \| R_2 + R_i}\right) = -(2.207)(5)\left(\dfrac{155.2}{155.2 + 1}\right) = -10.96$

7.64

(a) $C_M = C_{gd}\left(1+\left|A_v\right|\right) = (0.04)(1+15) = 0.64 \text{ pF}$

(b) $f_H = \dfrac{1}{2\pi\tau}$, $\Rightarrow \tau = \dfrac{1}{2\pi f} = \dfrac{1}{2\pi\left(5\times10^6\right)} = 3.183\times10^{-8} \text{ s}$

$\tau = R_{eq}\left(C_{gs}+C_M\right)$

or $R_{eq} = \dfrac{\tau}{\left(C_{gs}+C_M\right)} = \dfrac{3.183\times10^{-8}}{(0.2+0.64)\times10^{-12}} \Rightarrow R_{eq} = 37.9 \text{ k}\Omega$

7.65

$R_{TH} = R_1 \| R_2 = 33 \| 22 = 13.2 \text{ k}\Omega$

$V_{TH} = \left(\dfrac{R_2}{R_1+R_2}\right)(5) = \left(\dfrac{22}{22+33}\right)(5) = 2 \text{ V}$

$I_{BQ} = \dfrac{2-0.7}{13.2+(121)(4)} = 0.00261 \text{ mA}$

$I_{CQ} = 0.3138$

$r_\pi = \dfrac{(120)(0.026)}{0.3138} = 9.94 \text{ k}\Omega$

$g_m = \dfrac{0.3138}{0.026} = 12.07 \text{ mA/V}$

$r_0 = \dfrac{100}{0.3138} = 318 \text{ k}\Omega$

a.

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)}$$

$C_\pi + C_\mu = \dfrac{g_m}{2\pi f_T} = \dfrac{12.07\times10^{-3}}{2\pi\left(600\times10^6\right)}$

$C_\pi + C_\mu = 3.20 \text{ pF};\ C_\mu = 1 \text{ pF} \Rightarrow C_\pi = 2.20 \text{ pF}$

$C_M = C_\mu\left[1+g_m\left(r_o \| R_C \| R_L\right)\right]$

$= (1)\left[1+(12.07)\left(318 \| 4 \| 5\right)\right]$

$C_M = 27.6 \text{ pF}$

b.

$$\tau = R_{eq}\left(C_\pi + C_M\right)$$

$$R_{eq} = R_1 \| R_2 \| R_S \| r_\pi = 33 \| 22 \| 2 \| r_\pi$$

$$= 1.74 \| 9.94 \Rightarrow R_{eq} = 1.48\ \text{k}\Omega$$

$$\tau = \left(1.48\times10^3\right)\left(2.20 + 27.6\right)\times10^{-12}$$

$$\tau = 4.41\times10^{-8}\ \text{s}$$

$$f_H = \frac{1}{2\pi\tau} = \frac{1}{2\pi\left(4.41\times10^{-8}\right)} \Rightarrow f_H = 3.61\ \text{MHz}$$

$$V_o = -g_m V_\pi \left(r_o \| R_C \| R_L\right)$$

$$V_\pi = \frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S}\cdot V_i$$

$$R_1 \| R_2 \| r_\pi = 33 \| 22 \| 9.94 = 5.67\ \text{k}\Omega$$

$$V_\pi = \frac{5.67}{5.67 + 2}\cdot V_i = (0.739)V_i$$

$$r_o \| R_C \| R_L = 318 \| 4 \| 5 = 2.18\ \text{k}\Omega$$

$$A_\upsilon = -(12.07)(0.739)(2.18)$$

$$A_\upsilon = -19.7$$

7.66

$$R_{TH} = R_1 \| R_2 = 40 \| 5 = 4.44\ \text{k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{5}{5 + 40}\right)(10) = 1.111\ \text{V}$$

$$I_{BQ} = \frac{1.111 - 0.7}{4.44 + (121)(0.5)} = 0.00633\ \text{mA}$$

$$I_{CQ} = 0.760\ \text{mA}$$

$$r_\pi = \frac{(120)(0.026)}{0.760} = 4.11\ \text{k}\Omega$$

$$g_m = \frac{0.760}{0.026} = 29.23\ \text{mA/V}$$

$$r_0 = \infty$$

$$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)}$$

$$C_\pi + C_\mu = \frac{g_m}{2\pi f_T} = \frac{29.23\times10^{-3}}{2\pi\left(250\times10^6\right)}$$

$$C_\pi + C_\mu = 18.6\ \text{pF};\ C_\mu = 3\ \text{pF} \Rightarrow C_\pi = 15.6\ \text{pF}$$

a.

$$C_M = C_\mu\left[1 + g_m\left(R_C \| R_L\right)\right]$$

$$C_M = 3\left[1 + (29.2)(5 \| 2.5)\right] \Rightarrow C_M = 149\ \text{pF}$$

For upper frequency:

$$\tau_H = R_{eq}\left(C_\pi + C_M\right)$$

$$R_{eq} = r_\pi \| R_1 \| R_2 \| R_S = 4.11 \| 40 \| 5 \| 0.5$$

$$R_{eq} = 0.405\ \text{k}\Omega$$

$$\tau_H = \left(0.405\times10^3\right)\left(15.6+149\right)\times10^{-12}$$

$$= 6.67\times10^{-8}\ \text{s}$$

$$f_H = \frac{1}{2\pi\tau_H} \Rightarrow f_H = 2.39\ \text{MHz}$$

For lower frequency:

$$\tau_L = R_{eq}C_{C1}$$

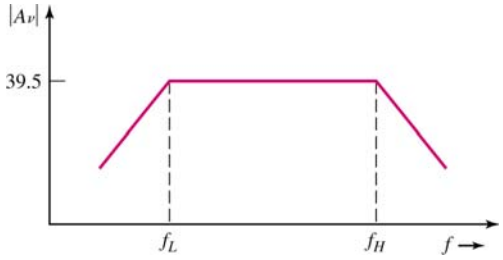
$$R_{eq} = R_S + R_1 \| R_2 \| r_\pi = 0.5 + 40 \| 5 \| 4.11$$

$$R_{eq} = 2.64\ \text{k}\Omega$$

$$\tau_L = \left(2.64\times10^3\right)\left(4.7\times10^{-6}\right) = 1.24\times10^{-2}\ \text{s}$$

$$f_L = \frac{1}{2\pi\tau_L} \Rightarrow f_L = 12.8\ \text{Hz}$$

b.



$$V_0 = -g_m V_\pi \left(R_C \| R_L\right)$$

$$V_\pi = \left(\frac{R_1 \| R_2 \| r_\pi}{R_1 \| R_2 \| r_\pi + R_S}\right)V_i$$

$$V_\pi = \left(\frac{2.135}{2.135+0.5}\right)V_i = 0.8102V_i$$

$$|A_V| = (29.23)(0.8102)\left(5 \| 2.5\right)$$

$$\underline{|A_V| = 39.5}$$

7.67

$$I_D = K_P\left(V_{SG}+V_{TP}\right)^2 = \frac{9-V_{SG}}{R_S}$$

$$(2)(1.2)\left(V_{SG}^2-4V_{SG}+4\right)=9-V_{SG}$$

$$2.4V_{SG}^2-8.6V_{SG}+0.6=0$$

$$V_{SG}=\frac{8.6\pm\sqrt{(8.6)^2-4(2.4)(0.6)}}{2(2.4)}$$

$$V_{SG}=3.512\text{ V}$$

$$g_m = 2K_P\left(V_{SG}+V_{TP}\right)=2(2)(3.512-2)$$

$$g_m = 6.049\ mA/V$$

$$I_D=(2)(3.512-2)^2=4.572\text{ mA}$$

$$r_0=\frac{1}{\lambda I_o}=\frac{1}{(0.01)(4.56)}\Rightarrow r_0=21.9\text{ k}\Omega$$

$$C_M = C_{gdT}\left(1+g_m\left(r_o\|R_D\right)\right)$$

$$C_M=(1)\left[1+(6.04)(21.9\|1)\right]\Rightarrow \underline{C_M=6.785\text{ pF}}$$

a.

b. $$\tau_H=\left(R_i\|R_G\right)\left(C_{gsT}+C_M\right)$$

$$\tau_H=\left(2\|100\right)\times10^3\times(10+6.78)\times10^{-12}$$

$$\tau_H=3.29\times10^{-8}\text{ s}$$

$$f_H=\frac{1}{2\pi\tau_H}\Rightarrow f_H=4.84\text{ MHz}$$

$$V_o=-g_m\left(r_o\|R_D\right)\cdot V_{gs}$$

$$V_{gs}=\left(\frac{R_G}{R_G+R_i}\right)\cdot V_i=\left(\frac{100}{102}\right)\cdot V_i$$

$$A_\upsilon=-(6.04)\left(\frac{100}{102}\right)(21.9\|1)$$

$$A_\upsilon=-5.67$$

7.68

(a) $$I_{DQ}=K_p\left(V_{SGQ}+V_{TP}\right)^2$$

$$0.5=0.5\left(V_{SGQ}-0.5\right)^2\Rightarrow V_{SGQ}=1.5\text{ V}$$

$$R_S=\frac{3-1.5}{0.5}=3\text{ k}\Omega$$

$$V_D=1.5-2=-0.5\text{ V}$$

$$R_D=\frac{-0.5-(-3)}{0.5}=5\text{ k}\Omega$$

(b) $$g_m=2\sqrt{K_pI_{DQ}}=2\sqrt{(0.5)(0.5)}=1\text{ mA/V}$$

$$A_\upsilon=-g_mR_D\left(\frac{R_G}{R_G+R_i}\right)=-(1)(5)\left(\frac{200}{200+4}\right)=-4.90$$

(c) $C_M = C_{gd}\left(1+g_m R_D\right) = (0.08)[1+(1)(5)] = 0.48\text{ pF}$

(d) $f_{3-dB} = \dfrac{1}{2\pi\tau}$

where $\tau = R_{eq}\cdot C_{eq} = \left(R_i \| R_G\right)\left(C_{gs}+C_M\right) = \left(4\|200\right)\times 10^3 \times (0.8+0.48)\times 10^{-12}$

which yields $\tau = 5.02\times 10^{-9}$ s

Then $f_{3-dB} = \dfrac{1}{2\pi\left(5.02\times 10^{-9}\right)} \Rightarrow f_{3-dB} = 31.7\text{ MHz}$

7.69

$$V_G = \left(\frac{R_2}{R_1+R_2}\right)(20)-10 = \left(\frac{22}{22+8}\right)(20)-10$$

$$V_G = 4.67\text{ V}$$

$$I_D = \frac{10-V_{SG}-4.67}{R_S} = K_P\left(V_{SG}+V_{TP}\right)^2$$

$$5.33-V_{SG} = (1)(0.5)\left(V_{SG}^2-4V_{SG}+4\right)$$

$$0.5V_{SG}^2 - V_{SG} - 3.33 = 0$$

$$V_{SG} = \frac{1\pm\sqrt{1+4(0.5)(3.33)}}{2(0.5)} \Rightarrow V_{SG} = 3.77\text{ V}$$

$$g_m = 2K_p\left(V_{SG}+V_{TP}\right) = 2(1)(3.77-2)$$

$$g_m = 3.54\ mA/V$$

b.

$$C_M = C_{gdT}\left(1+g_m\left(R_D\|R_L\right)\right)$$

$$C_M = (3)\left[1+(3.54)\left(2\|5\right)\right] \Rightarrow \underline{C_M = 18.2\text{ pF}}$$

a.

$$\tau = R_{eq}\left(C_{gsT}+C_M\right)$$

$$R_{eq} = R_i\|R_1\|R_2 = 0.5\|8\|22 = 0.461\,\text{k}\Omega$$

$$\tau = \left(0.461\times 10^3\right)(15+18.2)\times 10^{-12}$$

$$= 1.53\times 10^{-8}\text{ s}$$

$$f_H = \frac{1}{2\pi\tau} \Rightarrow f_H = 10.4\text{ MHz}$$

c.

$$V_o = -g_m V_{gs}\left(R_D\|R_L\right)$$

$$V_{gs} = \left(\frac{R_1\|R_2}{R_1\|R_2+R_i}\right)\cdot V_i = \left(\frac{5.87}{5.87+0.5}\right)\cdot V_i = (0.9215)V_i$$

$$A_v = -(3.54)(0.9215)(2\|5) \Rightarrow A_v = -4.66$$

7.70

$$I_E = 0.5\text{ mA} \Rightarrow I_{CQ} = \left(\frac{100}{101}\right)(0.5) = 0.495\text{ mA}$$

$$g_m = \frac{0.495}{0.026} = 19.0\text{ mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{0.495} = 5.25\text{ k}\Omega$$

a. Input: From Eq. (7.114(b))

$$\tau_{P\pi} = \left[\frac{r_\pi}{1+\beta}\Big\|R_E\|R_S\right]C_\pi$$

$$= \left[\frac{5.25}{101}\Big\|0.5\|0.05\right]\times 10^3 \times \left(10\times 10^{-12}\right)$$

$$= 2.43\times 10^{-10}\text{ s}$$

$$f_{H\pi} = \frac{1}{2\pi\tau_{P\pi}} \Rightarrow f_{H\pi} = 656\text{ MHz}$$

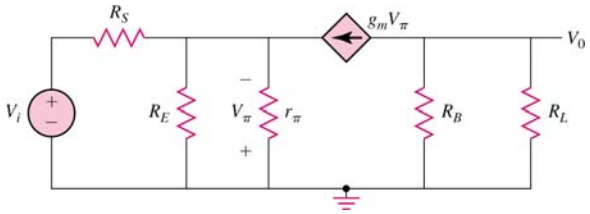
Output: From Eq. (7.115(b))

$$\tau_{P\mu} = \left(R_B\|R_L\right)C_\mu = \left(100\|1\right)\times 10^3 \times \left(10^{-12}\right)$$

$$= 9.90\times 10^{-10}\text{ s}$$

$$f_{H\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{H\mu} = 161\text{ MHz}$$

b.



$$V_o = -g_m V_\pi \left(R_B\|R_L\right)$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - \left(-V_\pi\right)}{R_S} = 0$$

$$V_\pi\left[g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S}\right] = -\frac{V_i}{R_S}$$

$$V_\pi\left[19 + \frac{1}{5.25} + \frac{1}{0.5} + \frac{1}{0.05}\right] = \frac{-V_i}{0.05}$$

$$V_\pi(41.19) = -V_i(20)$$

$$V_\pi = -(0.4856)V_i$$

$$\frac{V_o}{V_i} = -(19)(-0.4856)(100\|1)$$

$$A_\upsilon = 9.14$$

c.

$$\tau = \left(R_B \| R_L\right) C_L = \left(100 \| 1\right) \times 10^3 \times \left(15 \times 10^{-12}\right)$$

$$\tau = 1.485 \times 10^{-8}\ \text{s}$$

$$f = \frac{1}{2\pi\tau} \Rightarrow f = 10.7\ \text{MHz}$$

Since $f < f_{H\mu} \Rightarrow$ 3-dB frequency dominated by C_L

7.71

$$I_{EQ} = \frac{20 - 0.7}{10} = 1.93\ \text{mA}$$

$$I_{CQ} = \left(\frac{100}{101}\right)(1.93) = 1.91\ \text{mA}$$

$$g_m = \frac{1.91}{0.026} = 73.5\ \text{mA/V}$$

$$r_\pi = \frac{(100)(0.026)}{1.91} = 1.36\ \text{k}\Omega$$

a. Input:

$$\tau_{P\pi} = \left[\frac{r_\pi}{1+\beta} \| R_E \| R_S\right] C_\pi$$

$$= \left[\frac{1.36}{101} \| 10 \| 1\right] \times 10^3 \times \left(10 \times 10^{-12}\right)$$

$$\tau_{P\pi} = 1.327 \times 10^{-10}\ \text{s}$$

$$f_{P\pi} = \frac{1}{2\pi\tau_{P\pi}} \Rightarrow f_{P\pi} = 1.20\ \text{GHz}$$

Output:

$$\tau_{P\mu} = \left(R_C \| R_L\right) C_\mu = \left(6.5 \| 5\right) \times 10^3 \times \left(10^{-12}\right)$$

$$\tau_{P\mu} = 2.826 \times 10^{-9}\ \text{s}$$

$$f_{P\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{P\mu} = 56.3\ \text{MHz}$$

b.

$$V_o = -g_m V_\pi \left(R_C \| R_L\right)$$

$$g_m V_\pi + \frac{V_\pi}{r_\pi} + \frac{V_\pi}{R_E} + \frac{V_i - (-V_\pi)}{R_S} = 0$$

$$V_\pi\left(g_m + \frac{1}{r_\pi} + \frac{1}{R_E} + \frac{1}{R_S}\right) = -\frac{V_i}{R_S}$$

$$V_\pi\left(73.5 + \frac{1}{1.36} + \frac{1}{10} + \frac{1}{1}\right) = \frac{-V_i}{1}$$

$$V_\pi(75.34) = -V_i \Rightarrow V_\pi = -(0.01327)V_i$$

$$V_o = -(73.5)(-0.01327)(6.5 \| 5)V_i$$

$$A_v = 2.76$$

c.

$$\tau = \left(R_C \| R_L\right) C_L = \left(6.5 \| 5\right) \times 10^3 \times \left(15 \times 10^{-12}\right)$$

$$\tau = 4.24 \times 10^{-8} \text{ s}$$

$$f = \frac{1}{2\pi\tau} \Rightarrow f = 3.75 \text{ MHz}$$

Since $f < f_{P\mu}$, 3-dB frequency is dominated by C_L

7.72

$$V_{GS} + I_D R_S = 5$$

$$I_D = \frac{5 - V_{GS}}{R_S} = K_n \left(V_{GS} - V_{TN}\right)^2$$

$$5 - V_{GS} = (3)(10)\left(V^2{}_{GS} - 2V_{GS} + 1\right)$$

$$30V^2{}_{GS} - 59V_{GS} + 25 = 0$$

$$V_{GS} = \frac{59 \pm \sqrt{(59)^2 - 4(30)(25)}}{2(30)} \Rightarrow V_{GS} = 1.349 \text{ V}$$

$$g_m = 2K_n \left(V_{GS} - V_{TN}\right) = 2(3)(1.35 - 1)$$

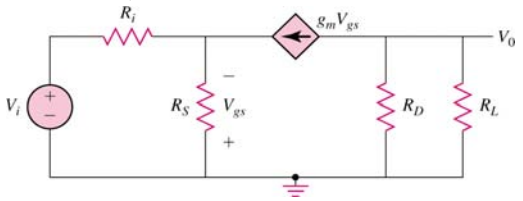
$$g_m = 2.093 \; mA/V$$

On the output:

$$\tau_{P\mu} = \left(R_D \| R_L\right) C_{gdT} = \left(5 \| 4\right) \times 10^3 \times \left(4 \times 10^{-12}\right)$$

$$\tau_{P\mu} = 8.89 \times 10^{-9} \text{ s}$$

$$f_{P\mu} = \frac{1}{2\pi\tau_{P\mu}} \Rightarrow f_{P\mu} = 17.9 \text{ MHz}$$



$$V_0 = -g_m V_{gs} \left(R_D \| R_L\right)$$

$$g_m V_{gs} + \frac{V_{gs}}{R_S} + \frac{V_i - \left(-V_{gs}\right)}{R_S} = 0$$

$$V_{gs}\left(g_m + \frac{1}{R_S} + \frac{1}{R_i}\right) = -\frac{V_i}{R_i}$$

$$V_{gs}\left(2.093 + \frac{1}{10} + \frac{1}{2}\right) = -\frac{V_i}{2}$$

$$V_{gs} = (0.1857) V_i$$

$$A_v = \frac{V_0}{V_i} = (2.093)(0.1857)\left(5 \| 4\right)$$

$$\underline{A_v = 0.864}$$

7.73

dc analysis

$$I_D = \frac{V^+ - V_{SG}}{R_S} = K_P\left(V_{SG} + V_{TP}\right)^2$$

$$5 - V_{SG} = (1)(4)(V_{SG} - 0.8)^2$$

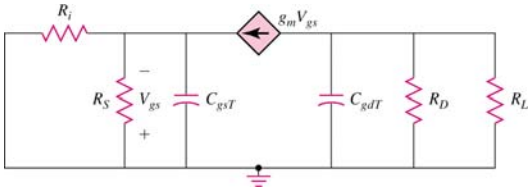
$$= 4\left(V_{SG}^2 - 1.6V_{SG} + 0.64\right)$$

$$4V_{SG}^2 - 5.4V_{SG} - 2.44 = 0$$

$$V_{SG} = \frac{5.4 \pm \sqrt{(5.4)^2 + 4(4)(2.44)}}{2(4)} = 1.707$$

$$g_m = 2K_P\left(V_{SG} + V_{TP}\right) = 2(1)(1.707 - 0.8)$$

$$g_m = 1.81\ mA/V$$



$3 \cdot dB$ frequency due to C_{gsT} : $R_{eq} = \frac{1}{g_m} \| R_S \| R_i$

$$f_A = \frac{1}{2\pi R_{eq} \cdot C_{gsT}}$$

$$R_{eq} = \frac{1}{1.81} \| 4 \| 0.5 = 0.246\ \text{k}\Omega$$

$$f_A = \frac{1}{2\pi(246)\left(4 \times 10^{-12}\right)} = 162\ \text{MHz}$$

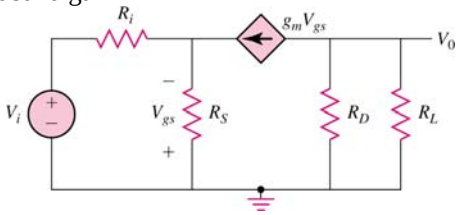
$3 - dB$ frequency due to C_{gdT}

$$f_B = \frac{1}{2\pi\left(R_D \| R_L\right)C_{gdT}}$$

$$= \frac{1}{2\pi\left(2 \| 4\right) \times 10^3 \times 10^{-12}}$$

$$\underline{f = 119\ \text{MHz}}$$

Midband gain



$$V_{gs} = \frac{-\frac{1}{g_m}\|R_S}{\frac{1}{g_m}\|R_S + R_i}\cdot V_i = \frac{-\frac{1}{1.81}\|4}{\frac{1}{1.81}\|4 + 0.5}\cdot V_i$$

$$= -0.492V_i$$

$$V_0 = -g_m V_{gs}\left(R_D \| R_L\right)$$

$$A_v = (0.492)(1.81)(4\|2) \Rightarrow \underline{A_v = 1.19}$$

7.74

$$r_\pi = \frac{(120)(0.026)}{1.02} = 3.059\ \text{k}\Omega$$

$$g_m = 39.23\ \text{mA/V}$$

a.

Input: $f_{H\pi} = \dfrac{1}{2\pi\tau_\pi}$

$$\tau_\pi = \left(R_S \| R_2 \| R_3 \| r_\pi\right)\left(C_\pi + 2C_\mu\right)$$

$$R_{eq} = 0.1\|20.5\|28.3\|3.06 = 0.096\,\text{k}\Omega$$

$$\tau_\pi = (96)[12 + 2(2)]\times 10^{-12} = 1.537\times 10^{-9}\ \text{s}$$

$$f_{H\pi} = \frac{1}{2\pi\left(1.537\times 10^{-9}\right)} \Rightarrow f_{H\pi} = 103.6\ \text{MHz}$$

Output: $f_{H\mu} = \dfrac{1}{2\pi\tau_\mu}$

$$\tau_\mu = \left(R_C \| R_L\right)C_\mu$$

$$= \left(15\|10\right)\times 10^3 \times \left(2\times 10^{-12}\right)$$

$$\tau_\mu = 6.67\times 10^{-9}\ \text{s}$$

$$f_{H\mu} = \frac{1}{2\pi\left(6.67\times 10^{-9}\right)} \Rightarrow f_{H\mu} = 23.9\ \text{MHz}$$

b.

$$A_\upsilon = g_m\left(R_C \| R_L\right)\left(\frac{R_2 \| R_3 \| r_\pi}{R_2 \| R_3 \| r_\pi + R_S}\right)$$

$$R_2 \| R_3 \| r_\pi = 20.5\|28.3\|3.059 = 2.433\,\text{k}\Omega$$

$$A_\upsilon = (39.23)(5\|10)\left(\frac{2.433}{2.433 + 0.1}\right) \Rightarrow A_\upsilon = 125.6$$

c. $C_L = 15\ \text{pF} > C_\mu \Rightarrow C_L$ dominates frequency response.

Chapter 8

8.1

(b) (i) $R_D = \dfrac{24}{4} = 6\,\Omega$

$P_{D,\max} = (12)(2) = 24$ W

(ii) $P_{D,\max} = 30 = (20)I_{DQ} \Rightarrow I_{DQ} = 1.5$ A

$I_{D,\max} = 2(1.5) = 3$ A

$R_D = \dfrac{40}{3} = 13.3\,\Omega$

(c) (i) $I_{D,\max} = 4$ A

(ii) $I_{D,\max} = 3$ A

8.2

(a) $P_{Q,\max} = V_{CEQ} \cdot I_{CQ}$

$25 = \left(\dfrac{24}{2}\right) \cdot I_{CQ} \Rightarrow I_{CQ} = 2.083$ A

$R_L = \dfrac{24 - 12}{2.083} = 5.76\,\Omega$

$I_{BQ} = \dfrac{2.083}{60} = 0.03472$ A

$R_B = \dfrac{24 - 0.7}{0.03472} = 671\,\Omega$

(b) $r_\pi = \dfrac{\beta V_T}{I_{CQ}} = \dfrac{(60)(0.026)}{2.083} = 0.7489\,\Omega$

$I_b = \dfrac{V_p}{r_\pi} = \dfrac{12\,mV}{0.7489} = 16.02$ mA

$I_c = \beta I_b = (60)(0.01602) = 0.9614$ A

$\overline{P}_{avg} = \dfrac{1}{2} I_c^2 R_C = \dfrac{1}{2}(0.9614)^2(5.76) = 2.66$ W

For the transistor,

$P_Q = 25 - 2.66 = 22.34$ W

8.3

(a) $R_L = \dfrac{30}{1.2} = 25\,\Omega$

$I_{BQ} = \dfrac{I_{CQ}}{\beta} = \dfrac{0.6}{80} = 0.0075$ A

$R_B = \dfrac{V_{CC} - V_{BE}(on)}{I_{BQ}} = \dfrac{30 - 0.7}{0.0075} \Rightarrow R_B = 3.91\,\text{k}\Omega$

$P_{Q,\max} = I_{CQ} V_{CEQ} = (0.6)(15) = 9$ W

(b) $P_{Q,\max} = I_{CQ}V_{CEQ} = \left(\frac{V_{CC} - V_{CEQ}}{R_L}\right)\cdot V_{CEQ}$

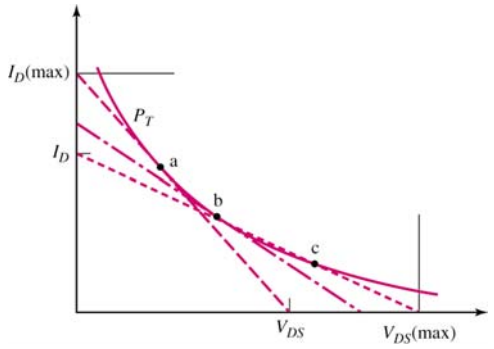
$V_{CEQ} = \frac{1}{2}V_{CC}$

$5 = \frac{1}{25}\left(\frac{V_{CC}}{2}\right)\left(\frac{V_{CC}}{2}\right) \Rightarrow V_{CC} = 22.36 \text{ V};\quad I_{C,\max} = \frac{22.36}{25} = 0.8944 \text{ A}$

(c) For (a): $\Delta I_C = 0.6 \text{ A};\quad \overline{P}_L = \frac{1}{2}(\Delta I_C)^2 R_L = \frac{1}{2}(0.6)^2(25) = 4.5 \text{ W}$

For (b): $\Delta I_C = I_{CQ} = \frac{0.8944}{2} = 0.4472 \text{ A};\quad \overline{P}_L = \frac{1}{2}(0.4472)^2(25) = 2.5 \text{ W}$

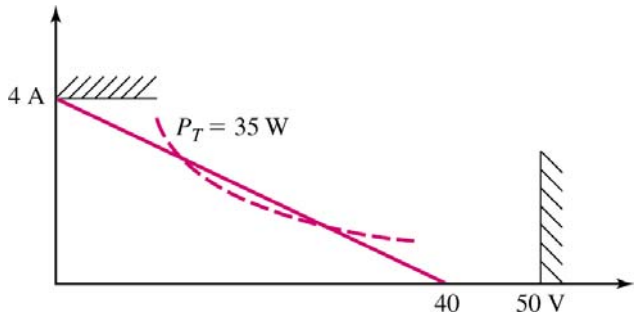
8.4



Point (b): Maximum power delivered to load.
Point (a): Will obtain maximum signal current output.
Point (c): Will obtain maximum signal voltage output.

8.5

a.



b.

$V_{GG} = 5$ V, $I_D = 0.25(5-4)^2 = 0.25$ A, $V_{DS} = 37.5$ V, $P = 9.375$ W

$V_{GG} = 6$ V, $I_D = 0.25(6-4)^2 = 1.0$ A, $V_{DS} = 30$ V, $P = 30$ W

$V_{GG} = 7$ V, $I_D = 0.25(7-4)^2 = 2.25$ A, $V_{DS} = 17.5$ V, $P = 39.375$ W

$$V_{GG} = 8 \text{ V}, \; I_D = 0.25\left[2(8-4)V_{DS} - V_{DS}^2\right]$$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 2.92$$

$I_D = 3.71$ A, $\underline{P = 10.8 \text{ W}}$

$$V_{GG} = 9 \text{ V}, \; I_D = 0.25\left[2(9-4)V_{DS} - V_{DS}^2\right]$$

$$= \frac{40 - V_{DS}}{10} \Rightarrow V_{DS} = 1.88 \text{ V}$$

$I_D = 3.81$ A, $\underline{P = 7.16 \text{ W}}$

c. Yes, at $V_{GG} = 7$ V, $P = 39.375$ W $> P_{D,\max} = 35$ W

8.6

a.

Set $V_{DSQ} = \frac{V_{DD}}{2} = 25$ V

$$I_{DQ} = \frac{50 - 25}{20} = 1.25 \text{ A}$$

$$I_{DQ} = K_n\left(V_{GS} - V_{TN}\right)^2$$

$$\sqrt{\frac{1.25}{0.2}} + 4 = V_{GS} = 6.5 \text{ V}$$

$$V_{GS} = \left(\frac{R_2}{R_1 + R_2}\right)V_{DD}$$

Let $R_1 + R_2 = 100$ kΩ

$$6.5 = \left(\frac{R_2}{100}\right)(50) \Rightarrow \underline{R_2 = 13 \text{ k}\Omega}$$

$$\underline{R_1 = 87 \text{ k}\Omega}$$

b. $P_D = I_{DQ}V_{DSQ} = (1.25)(25) \Rightarrow \underline{P_D = 31.25 \text{ W}}$

c.

$$I_{D,\max} = 2I_{DQ} \Rightarrow \underline{I_{D,\max} = 2.5 \text{ A}}$$

$$V_{DS,\max} = V_{DD} \Rightarrow \underline{V_{DS,\max} = 50 \text{ V}}$$

$$\underline{P_{D,\max} = 31.25 \text{ W}}$$

d.

$$\left|\frac{V_0}{V_i}\right| = g_m R_L$$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(1.25)} = 1\ A/V$$

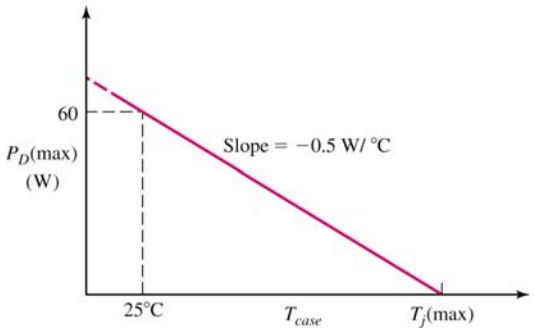
$$|V_0| = (1)(20)(0.5) = 10\ \text{V}$$

$$\overline{P_L} = \frac{1}{2}\cdot\frac{V_0^2}{R_L} = \frac{1}{2}\cdot\frac{(10)^2}{20} \Rightarrow \underline{\overline{P_L} = 2.5\ \text{W}}$$

$$\overline{P_Q} = 31.25 - 2.5 \Rightarrow \underline{\overline{P_Q} = 28.75\ \text{W}}$$

8.7

(a)



(b) $$P_D = P_{D,\max} - (Slope)(T_j - 25)$$

At $P_D = 0$, $T_{j,\max} = \frac{60}{0.5} + 25 \Rightarrow \underline{T_{j,\max} = 145°\text{C}}$

(c) $$P_{D,\max} = \frac{T_{j,\max} - T_{case}}{\theta_{dev-amb}} \quad \text{or} \quad \theta_{dev-amb} = \frac{145-25}{60} \Rightarrow \underline{\theta_{dev-amb} = 2°C/W}$$

8.8

$$P_{D,\text{rated}} = \frac{T_{j,\max} - T_{amb}}{\theta_{\text{dev-case}}}$$

$$\text{or } \theta_{\text{dev-case}} = \frac{T_{j,\max} - T_{\text{amb}}}{P_{D,\text{rated}}}$$

$$= \frac{150-25}{50} = 2.5°\text{C/W}$$

Then $T_{\text{dev}} - T_{\text{amb}} = P_D(\theta_{\text{dev-case}} + \theta_{\text{case-amb}})$

$150 - 25 = P_D(2.5 + \theta_{\text{case-amb}}) \Rightarrow 125 = \underline{P_D(2.5 + \theta_{\text{case-amb}})}$

8.9

(a) $$T_{j,\max} - T_{amb} = P_T(\theta_{dev-case} + \theta_{snk-amb} + \theta_{case-snk})$$

$$120 - 25 = P_T(1.5 + 2.8 + 0.6) \Rightarrow P_T = 19.39\ \text{W}$$

(b) $T_{case} = 25 + (19.39)(0.6 + 2.8) = 90.9°C$

(c) $T_{snk} = 25 + (19.39)(2.8) = 79.3°C$

8.10

(a) $T_{j,\max} - T_{amb} = P(\theta_{dev-case} + \theta_{case-amb})$

$150 - 25 = 30(2.8 + \theta_{case-amb}) \Rightarrow \theta_{case-amb} = 1.37°C/W$

(b) $T_{j,\max} = 25 + 20(2.8 + 1.37) = 108°C$

8.11

(a) $150 - 25 = P_T(3.8 + 1.5 + 4) \Rightarrow P_T = 13.4\text{ W}$

(b) $P = I_{CQ} V_{CEQ}$

$13.4 = (3) \cdot V_{CEQ} \Rightarrow V_{CEQ} = 4.48\text{ V}$

$V_{CC} = V_{CE,\max} = 2V_{CEQ} = 8.96\text{ V}$

8.12

$$\eta = \frac{\overline{P_L}}{\overline{P_S}}$$

$$\overline{P_S} = V_{CC} \cdot I_Q$$

$$\overline{P_L} = V_P \cdot I_P = \left(\frac{V_{CC}}{2}\right)(I_Q)$$

$$\eta = \frac{\frac{1}{2} \cdot V_{CC} \cdot I_Q}{V_{CC} \cdot I_Q} \Rightarrow \underline{\eta = 50\%}$$

8.13

(a) $$A_v = \frac{(1+\beta)R_L}{r_\pi + (1+\beta)R_L} = \frac{(1+\beta)R_L}{\frac{\beta V_T}{I_C} + (1+\beta)R_L}$$

We have $(1+\beta) \cong \beta$

$$A_v = \frac{I_C R_L}{I_C R_L + V_T} = \frac{R_L}{R_L + \frac{V_T}{I_C}} = \frac{R_L}{R_L + \frac{1}{g_m}}$$

(b) (i) $0.9 = \dfrac{8}{8 + \frac{1}{g_m}} \Rightarrow g_m = 1.125\text{ A/V},\quad I_C = 29.25\text{ mA}$

(ii) $0.95 = \dfrac{8}{8 + \frac{1}{g_m}} \Rightarrow g_m = 2.375\text{ A/V},\quad I_C = 61.75\text{ mA}$

(iii) $0.997 = \dfrac{8}{8 + \frac{1}{g_m}} \Rightarrow g_m = 41.54\text{ A/V},\quad I_C = 108\text{ mA}$

8.14

(a) $\overline{P}_L = \frac{1}{2}\cdot\frac{V_p^2}{R_L} \Rightarrow 0.5 = \frac{1}{2}\cdot\frac{V_p^2}{8} \Rightarrow V_p = 2.828\text{ V}$

$$I_p = \frac{V_p}{R_L} = \frac{2.828}{8} = 0.3536\text{ A}$$

(b) For $V_o = -V_p = -2.828$ V

$$\left|I_L\right| = 0.3536 = (0.9)I_O \Rightarrow I_O = 0.393\text{ A}$$

8.15

(a) $V_O = 1.6\text{ V},\ \ I_L = \frac{1.6}{8} = 0.20\text{ A}$

$$I_C \cong 0.2 + 0.25 = 0.45\text{ A}$$

$$g_m = \frac{0.45}{0.026} = 17.31\text{ A/V}$$

$$A_v = \frac{8}{8 + \frac{1}{17.31}} = 0.9928$$

(b) $V_O = 0,\ \ I_L = 0,$ so $I_C = 0.25\text{ A}$

$$g_m = \frac{0.25}{0.026} = 9.615\text{ A/V}$$

$$A_v = \frac{8}{8 + \frac{1}{9.615}} = 0.9872$$

(c) $V_O = -1.6\text{ V},\ \ I_L = -0.2\text{ A},\ \ I_C \cong 0.25 - 0.2 = 0.05\text{ A}$

$$g_m = \frac{0.05}{0.026} = 1.923\text{ A/V}$$

$$A_v = \frac{8}{8 + \frac{1}{1.923}} = 0.939$$

8.16

$v_o(\max) = 4.8$ V

$$i_{C3} = i_{C2} = \frac{-0.7 - (-5)}{1} = 4.3\text{ mA}$$

$v_I = v_o + 0.7$ $\qquad i_L(\max) = -4.3\text{ mA} = \frac{v_S(\min)}{1}$

so $-3.6 \le v_I \le 5.5$ V $\qquad \underline{v_o(\min) = -4.3\text{ V}}$

8.17

$$I_{D3} = \mathrm{K}\left(V_{GS3} - V_{TN}\right)^2 = \frac{0 - V_{GS3} - (-5)}{R}$$

$$12\left(V_{GS3} - 0.5\right)^2 = 5 - V_{GS3}$$

$$2V_{GS3}^2 - 11V_{GS} - 2 = 0$$

$$V_{GS3} = \frac{11 \pm \sqrt{(11)^2 + 4(12)(2)}}{2(12)}$$

$$V_{GS3} = V_{GS2} = 1.072 \text{ V}$$

$$I_{D3} = I_{D2} = 12(1.072 - 0.5)^2 = 3.93 \text{ mA}$$

$$V_{DS2}(\text{sat}) = V_{GS2} - V_{TN} = 1.072 - 0.5 = 0.572 \text{ V}$$

$$v_o(\min):\ i_2(\max) = -3.93 = \frac{V_0(\min)}{1} \Rightarrow \underline{V_0(\min) = -3.93 \text{ V}}$$

$$v_I(\min) = v_o(\min) + V_{TN} = -3.93 + 0.5$$

$$\underline{v_I(\min) = -3.43 \text{ V}}$$

$$v_o(\max) = 5 - V_{DS}(\text{sat}) = 5 - 0.572$$

$$v_o(\max) = 4.43 \text{ V}$$

$$I_{D1}(\max) = 3.93 + \frac{4.43}{1} = 8.36 \text{ mA}$$

$$I_{D1} = 8.36 = 12\left(V_{GS1} - 0.5\right)^2 \Rightarrow V_{GS1} = 1.33 \text{ V}$$

$$v_I(\max) = v_o + V_{GS1} = 4.43 + 1.33 \Rightarrow \underline{v_I(\max) = 5.76 \text{ V}}$$

8.18

(a) For $\upsilon_O = -12 + 0.7 = -11.7$ V, $I_Q = \dfrac{11.3}{0.02} + 50 = 615$ mA

$$I_{REF} = \left(1 + \frac{2}{\beta}\right) \cdot I_Q = \left(1 + \frac{2}{40}\right)(615) = 645.75 \text{ mA}$$

$$R = \frac{0 - 0.7 - (-12)}{0.6475} \Rightarrow R = 17.5\,\Omega$$

For $\upsilon_O = 12 - 0.7 = +11.3$ V, $i_L = \dfrac{11.3}{0.02} = 565$ mA

$$i_{E1}(\max) = I_Q + i_L = 615 + 565 \Rightarrow i_{E1}(\max) = 1.18 \text{ A}$$

(b) $$\overline{P}_L = \frac{1}{2} \cdot \frac{V_O^2}{R_L} = \frac{1}{2} \cdot \frac{(11.3)^2}{20} = 3.19 \text{ W}$$

$$P_S = I_Q(24) = (0.615)(24) = 14.76 \text{ W}$$

Define $\eta = \dfrac{\overline{P}_L}{P_S} = \dfrac{3.19}{14.76} \times 100\% = 21.6\%$

8.19

(a) $V_O = -20\text{ V},\ i_L = \dfrac{-20}{200} = -0.10\text{ A}$

$I_Q = 0.10 + i_{E1}(\min) = 0.10 + 0.02 = 0.12\text{ A}$

$I_{REF} = \left(1 + \dfrac{2}{\beta}\right)\cdot I_Q = \left(1 + \dfrac{2}{50}\right)(0.12) = 0.1248\text{ A}$

$R = \dfrac{0 - 0.7 - (-24)}{0.1248} = 187\,\Omega$

(b) $P_{Q1} = I_Q V_{CE1} = (0.12)(24) = 2.88\text{ W}$

$P = I_Q V_{CE2} + I_{REF}(24) = (0.12)(24) + (0.1248)(24) = 5.88\text{ W}$

(c) $\overline{P_L} = \dfrac{1}{2}\cdot\dfrac{V_O^2}{R_L} = \dfrac{1}{2}\cdot\dfrac{(20^2)}{200} = 1\text{ W}$

$\eta = \dfrac{1}{2.88 + 5.88}\times 100\% = 11.4\%$

8.20

$I_{D1} = K_n(V_{GS} - V_{TN})^2 = 12(0 - (-1.8))^2$

$I_{D1} = 38.9\text{ mA}$

(a)

For $R_L = \infty$

$v_o(\max) = 4.8\text{ V}$

$V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 1.8\text{ V}$

$v_o(\min) = -5 + 1.8 = -3.2\text{ V}$

$v_I = v_o + 0.7 \Rightarrow -2.5 \le v_I \le 5.5\text{ V}$

(b) For $R_L = 500\,\Omega$ $\quad v_o(\max) = 4.8\text{ V}$

$v_o < 0,\ v_o(\min) = -3.2\text{ V} \qquad I_2' = \dfrac{v_o}{R_L} = \dfrac{-3.2}{0.5} = -6.4\text{ mA}$

For

$-2.5 \le v_I \le 5.5\text{ V}$

(c)

For $v_o = -2V,\ I_2'(\max) = -38.9\text{ mA}$

$R_2(\min) = \dfrac{-2}{-38.9} \Rightarrow R_L(\min) = 51.4\,\Omega$

$\overline{P_L} = \dfrac{1}{2}\cdot\dfrac{v_o^2}{R_L} = \dfrac{1}{2}\cdot\dfrac{(2)^2}{51.4} \Rightarrow \overline{P_L} = 38.9\text{ mW}$

$P_L = 10(38.9) = 389\text{ mW}\ \%\ = \dfrac{38.9}{389} = 10\%$

8.21

$$\overline{P_L} = \frac{V_P^2}{R_L} = \frac{\left(V^+\right)^2}{R_L}$$

$$\overline{P_S} = \frac{1}{2}\cdot\frac{\left(V^+\right)^2}{R_L} + \frac{1}{2}\cdot\frac{\left(V^-\right)^2}{R_L},\ V^- = -V^+$$

So $\overline{P_S} = \dfrac{\left(V^+\right)^2}{R_L}$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} \Rightarrow \underline{\eta = 100\%}$$

8.22

(a)

As maximum conversion efficiency

$$\eta = \frac{\pi}{4},\ \frac{V_P}{V_{CC}} = 0.785$$

So $V_p(\max) = (0.785)(5)\left(\dfrac{4}{\pi}\right)$

$$\underline{V_p(\max) = 5\text{ V}}$$

(b) Maximum power dissipation occurs when $V_p = \dfrac{2V_{CC}}{\pi} = \dfrac{2(5)}{\pi} = 3.183\text{ V}$

$$\overline{P_\theta}(\max) = \frac{V_{CC}^2}{\pi^2 R_L}$$

(c)
$$2 = \frac{(5)^2}{\pi^2 R_L} \Rightarrow \underline{R_L = 1.27\ \Omega}$$

8.23

$$\overline{P} = \frac{1}{2}\cdot\frac{V_p^2}{R_L}$$

(a)
$$50 = \frac{1}{2}\cdot\frac{V_p^2}{24} \Rightarrow V_p = 49\text{ V} \Rightarrow \underline{V^+ = 52\text{ V},\ V^- = -52\text{ V}}$$

(b)
$$I_P = \frac{V_P}{R_L} = \frac{49}{24} = \underline{2.04\text{ A}}$$

(c)
$$\eta = \frac{\pi}{4}\cdot\frac{V_P}{V_{CC}} = \frac{\pi}{4}\left(\frac{49}{52}\right)$$

$$\underline{\eta = 74.0\%}$$

8.24

(a)

$$V_{DS} \geq V_{DS}(sat) = V_{GS} - V_{TN} = V_{GS}$$

$$V_{DS} = 10 - V_o(\max) \text{ and } I_D = I_L = K_n (V_{GS})^2$$

$$\frac{V_o(\max)}{R_L} = K_n (V_{GS})^2$$

$$V_{GS} = \sqrt{\frac{V_o(\max)}{R_L \cdot K_n}}$$

$$\text{So } 10 - V_o(\max) = \sqrt{\frac{V_o(\max)}{R_L \cdot K_n}} = \sqrt{\frac{V_o(\max)}{(5)(0.4)}}$$

$$\left[10 - V_0(\max)\right]^2 = \frac{V_0(\max)}{2}$$

$$100 - 20V_0(\max) + V_0^2(\max) = \frac{V_0(\max)}{2}$$

$$V_0^2(\max) - 20.5V_0(\max) + 100 = 0$$

$$V_0(\max) = \frac{20.5 \pm \sqrt{(20.5)^2 - 4(100)}}{2} \Rightarrow \underline{V_0(\max) = 8 \text{ V}}$$

$$i_L = \frac{8}{5} \Rightarrow \underline{i_L = 1.6 \text{ mA}}$$

$$V_{GS} = \sqrt{\frac{i_L}{K_n}} = \sqrt{\frac{1.6}{0.4}} = 2V \Rightarrow \underline{V_I = 10 \text{ V}}$$

b.

$$\overline{P_L} = \frac{1}{2} \cdot \frac{(8)^2}{5} = 6.4 \text{ mW}$$

$$\overline{P_S} = \frac{20(1.6)}{\pi} = 10.2 \text{ mW}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{6.4}{10.2} \Rightarrow \underline{\eta = 62.7\%}$$

8.25

(b) $\upsilon_I = \upsilon_{GSn} + \upsilon_o$

Also $\upsilon_o = i_L R_L = i_{dn} R_L = K R_L (\upsilon_{GSn})^2$, but $\upsilon_{GSn} = \upsilon_I - \upsilon_o$

So $\upsilon_o = K R_L (\upsilon_I - \upsilon_o)^2$

$$\frac{d\upsilon_o}{d\upsilon_I} = 2KR_L(\upsilon_I - \upsilon_o)\left(1 - \frac{d\upsilon_o}{d\upsilon_I}\right)$$

$$\frac{d\upsilon_o}{d\upsilon_I} = \frac{2KR_L(\upsilon_I - \upsilon_o)}{1 + 2KR_L(\upsilon_I - \upsilon_o)}, \text{ also } \upsilon_I - \upsilon_o = \sqrt{\frac{\upsilon_o}{KR_L}}$$

Then $$\frac{d\upsilon_o}{d\upsilon_I} = A_\upsilon = \frac{2\sqrt{KR_L} \cdot \sqrt{\upsilon_o}}{1 + 2\sqrt{KR_L} \cdot \sqrt{\upsilon_o}}$$

We find $\sqrt{KR_L} = \sqrt{(4)(0.05)} = 0.4472$

(i) For $\upsilon_o = 0$, $A_\upsilon = 0$

(ii) For $\upsilon_o = 1\,\text{V}$, $A_\upsilon = \dfrac{2(0.4472)(1)}{1+2(0.4472)(1)} = 0.472$

(iii) For $\upsilon_o = 10\,\text{V}$, $A_\upsilon = \dfrac{2(0.4472)\sqrt{10}}{1+2(0.4472)\sqrt{10}} = 0.739$

8.26

(a) $i_{Cn} = I_S \exp\left(\dfrac{V_{BE}}{V_T}\right) \Rightarrow V_{BE} = V_T \ln\left(\dfrac{i_{Cn}}{I_S}\right) = (0.026)\ln\left(\dfrac{10^{-3}}{2\times10^{-15}}\right) = 0.7004\,\text{V}$

$V_{BB} = 2V_{BE} = 1.40077\,\text{V}$

$P_Q = i_C \cdot \upsilon_{CE} = (1)(5) = 5\,\text{mW}$

(b) For $\upsilon_o = -3.5\,\text{V}$, $i_{Cp} \cong \dfrac{3.5}{1} = 3.5\,\text{mA} \cong i_L$

$\upsilon_{EB} = (0.026)\ln\left(\dfrac{3.5\times10^{-3}}{2\times10^{-15}}\right) = 0.732957\,\text{V}$

$\upsilon_{BE} = 1.40077 - 0.732957 = 0.66781\,\text{V}$

$i_{Cn} = \left(2\times10^{-15}\right)\exp\left(\dfrac{0.66781}{0.026}\right) \Rightarrow i_{Cn} = 0.2857\,\text{mA}$

Then $i_{Cp} \cong 0.2857 + 3.5 = 3.7857\,\text{mA}$

$\upsilon_{EB} = (0.026)\ln\left(\dfrac{3.7857\times10^{-3}}{2\times10^{-15}}\right) = 0.734997\,\text{V}$

$\upsilon_{BE} = 1.40077 - 0.734997 = 0.66577\,\text{V}$

$i_{Cn} = \left(2\times10^{-15}\right)\exp\left(\dfrac{0.66577}{0.026}\right) \Rightarrow i_{Cn} = 0.2642\,\text{mA}$

$i_{Cp} = 3.5 + 0.2642 = 3.764\,\text{mA}$

$\upsilon_I = \upsilon_o - \upsilon_{EB} + \dfrac{V_{BB}}{2} = -3.5 - 0.735 + 0.7004 = -3.535\,\text{V}$

For R_L: $P_{RL} = i_L^2 R_L = (3.5)^2(1) = 12.25\,\text{mW}$

For Q_n: $P_{Qn} = i_{Cn}\upsilon_{CEn} = (0.2642)[5-(-3.5)] = 2.25\,\text{mW}$

For Q_p: $P_{Qp} = i_{Cp}\upsilon_{ECp} = (3.764)[-3.5-(-5)] = 5.65\,\text{mW}$

8.27

(a) (i) $i_{Dn} = K_n(\upsilon_{GSn} - V_{TN})^2$

$1 = 4(\upsilon_{GSn} - 1.5)^2 \Rightarrow \upsilon_{GSn} = \dfrac{V_{BB}}{2} = 2\,\text{V}, \Rightarrow V_{BB} = 4\,\text{V}$

(ii) $P = i_{Dn} \cdot \upsilon_{DSn} = (1)(12) = 12\,\text{mW}$

(b) (i) $i_D = K[\upsilon_{DS}(sat)]^2 = \dfrac{\upsilon_o}{R_L} = \dfrac{V_{DD} - \upsilon_{DS}(sat)}{R_L}$

$$KR_L[\upsilon_{DS}(sat)]^2 + \upsilon_{DS}(sat) - V_{DD} = 0$$

Now $KR_L = (4)(1) = 4$

We have $4[\upsilon_{DS}(sat)]^2 + \upsilon_{DS}(sat) - 12 = 0 \Rightarrow \upsilon_{DS}(sat) = 1.612$ V

$\upsilon_o(\max) = 12 - 1.612 = 10.39$ V

(ii) $i_{Dn} = i_L = 10.39$ mA

$i_{Dp} = 0$

$\upsilon_{GSn} = 3.112$ V

$$\upsilon_I = \frac{-V_{BB}}{2} + \upsilon_{GSn} + \upsilon_o = -2 + 3.112 + 10.39 = 11.5 \text{ V}$$

(iii) For R_L: $P_{RL} = i_L^2 R_L = (10.39)^2(1) = 108$ mW

For M_n: $P_{Mn} = i_{Dn}\upsilon_{DSn} = (10.39)[12 - 10.39] = 16.7$ mW

For M_p: $P_{Mp} = i_{Dp}\upsilon_{SDp} = 0$

8.28

a.

$$v_0 = 24 \text{ V} \Rightarrow i_L = \frac{24}{8} \Rightarrow \underline{i_L \approx i_N = 3 \text{ A}}$$

$$i_{Bn} = \frac{3}{41} \Rightarrow i_{Bn} = 73.2 \text{ mA}$$

For $i_D = 25$ mA $\Rightarrow i_{R1} = 25 + 73.2 = 98.2$ mA

$$V_{BE} = V_T \ln\left(\frac{i_N}{I_S}\right) = (0.026)\ln\left(\frac{3}{6\times10^{-12}}\right)$$

$$= 0.7004 \text{ V}$$

Then $98.2 = \dfrac{30 - (24 + 0.7)}{R_1} \Rightarrow R_1 = \dfrac{5.3}{98.2} \Rightarrow \underline{R_1 = 53.97\ \Omega}$

$$V_D = (0.026)\ln\left(\frac{25\times10^{-3}}{6\times10^{-12}}\right) = 0.5759 \text{ V}$$

$$V_{EB} = 2V_D - V_{BE} = 2(0.5759) - 0.7004$$

$$= 0.4514 \text{ V}$$

$$i_P = I_S \exp\left(\frac{V_{EB}}{V_T}\right) = (6\times10^{-12})\exp\left(\frac{0.4514}{0.026}\right) \Rightarrow \underline{i_P = 0.208 \text{ mA}}$$

b. Neglecting base current

$$i_D \approx \frac{30 - 0.6}{R_1} = \frac{30 - 0.6}{53.97} \Rightarrow \underline{i_D \approx 545 \text{ mA}}$$

$$V_D = (0.026)\ln\left(\frac{0.545}{6\times10^{-12}}\right) = 0.656 \text{ V}$$

Approximation for i_D is okay.

Diodes and transistors matched $\Rightarrow \underline{i_N = i_P = 545 \text{ mA}}$

8.29

(a)

$$I_{D1} = K_1\left(V_{GS1} - V_{TN}\right)^2$$

$$V_{GS1} = \sqrt{\frac{5}{5}} + 1 = 2\ V$$

$$I_{D3} = K_3\left(V_{GS3} - V_{TN}\right)^2$$

$$200 = K_3\left(2-1\right)^2 \Rightarrow \underline{K_{n3} = K_{p4} = 200\ \mu A/V^2}$$

(b)

$$v_I + V_{SG4} + V_{GS3} - V_{GS1} = v_O$$

For v_o large, $i_L = i_1 = K_{n1}\left(V_{GS1} - V_{TN}\right)^2$

$$V_{GS1} = \sqrt{\frac{i_L}{K_{n1}}} + V_{TN} = \sqrt{\frac{v_o}{R_L K_{n1}}} + V_{TN}$$

$$\text{So } v_I + 2 + 2 - \left(\sqrt{\frac{v_o}{(0.5)(5)}} + 1\right) = v_0$$

$$v_I = v_0 + \sqrt{\frac{v_0}{2.5}} - 3$$

$$\frac{dv_I}{dv_I} = 1 = \frac{dv_0}{dv_I} + \frac{1}{2}\cdot\frac{1}{\sqrt{2.5v_0}}\cdot\frac{dv_0}{dv_I}$$

$$1 = \frac{dv_0}{dv_I}\left[1 + \frac{1}{2\sqrt{2.5v_0}}\right]$$

For $v_O = 5\ V$:

$$1 = \frac{dv_0}{dv_I}\left[1 + \frac{1}{2\sqrt{2.5(5)}}\right] = \frac{dv_0}{dv_I}(1.1414) \Rightarrow \underline{\frac{dv_0}{dv_I} = 0.876}$$

8.30

$$v_O = v_I + \frac{V_{BB}}{2} - V_{GS} \text{ and } V_{GS} = \sqrt{\frac{I_{Dn}}{K_n}} + V_{TN}$$

For $v_O \approx 0,\ I_{Dn} = I_{DQ} + i_L = I_{DQ} + \dfrac{v_O}{R_L}$

Then $$v_O = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ} + \left(v_O / R_L\right)}{K_n}}$$ or $$v_O = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}}\cdot\sqrt{1 + \frac{v_O}{I_{DQ}R_L}}$$

For v_O small,

$$v_O \cong v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}}\cdot\sqrt{1 + \frac{1}{2}\cdot\frac{v_O}{I_{DQ}R_L}}$$

$$v_O\left[1 + \frac{1}{2}\cdot\sqrt{\frac{I_{DQ}}{K_n}}\cdot\frac{1}{I_{DQ}R_L}\right] = v_I + \frac{V_{BB}}{2} - V_{TN} - \sqrt{\frac{I_{DQ}}{K_n}}$$

Now

$$\frac{dv_O}{dv_I} = \frac{1}{\left[1 + \frac{1}{2}\cdot\sqrt{\frac{I_{DQ}}{K_n}}\cdot\frac{1}{I_{DQ}R_L}\right]} = 0.95$$

So $\frac{1}{2}\cdot\sqrt{\frac{I_{DQ}}{K_n}}\cdot\frac{1}{I_{DQ}R_L} = \frac{1}{0.95} - 1 = 0.0526$

For $R_L = 0.1\ k\Omega,$ then $\frac{1}{\sqrt{K_n I_{DQ}}} = 0.01052$

Or $\sqrt{K_n I_{DQ}} = 95.1$

We can write $g_m = 2\sqrt{K_n I_{DQ}} = 190\ mA/V$

This is the required transconductance for the output transistor. This implies a very large transistor.

8.31

(a) $R_{TH} = R_1 \| R_2 = 14 \| 10 = 5.833\ \text{k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)\cdot V_{CC} = \left(\frac{10}{10+14}\right)(12) = 5\ \text{V}$$

$$I_{BQ} = \frac{5 - 0.7}{5.833 + (91)(0.02)} = 0.5619\ \text{mA}, \quad I_{CQ} = 50.57\ \text{mA}$$

(b) $R_L = \frac{V_{CC}}{I_{CQ}} = \frac{12}{50.57} \Rightarrow R_L = 237\ \Omega$

(c) $\overline{P}_L(\text{max}) = \frac{1}{2}\cdot\frac{(11)^2}{0.237} = 255\ \text{mW}$

(d) $\overline{P}_S = I_{CQ}V_{CC} = (50.57)(12) = 607\ \text{mW}$

$$\eta = \frac{255}{607}\times 100\% = 42\%$$

8.32

$$I_{CQ} = \frac{V_{CC}}{R_L} = \frac{15}{1} = 15 \text{ mA}$$

$$I_{BQ} = \frac{15}{100} = 0.15 \text{ mA}$$

$$\overline{P_L}(\max) = \frac{1}{2} \cdot \frac{V_{CC}^2}{R_L} = \frac{(15)^2}{2(1)} \Rightarrow \underline{\overline{P_L}(\max) = 112.5 \text{ mW}}$$

Let $R_{TH} = 10 \text{ k}\Omega$

$$V_{TH} = I_{BQ}R_{TH} + V_{BE} + (1+\beta)I_{BQ}R_E$$
$$= (0.15)(10) + 0.7 + (101)(0.15)(0.1)$$

$$V_{TH} = 3.715 = \frac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = \frac{1}{R_1} \cdot (10)(15)$$

$$\underline{R_1 = 40.4 \text{ k}\Omega}$$

$$\underline{R_2 = 13.3 \text{ k}\Omega}$$

8.33

(a) $R_{TH} = R_1 \| R_2 = 2.3 \| 1.75 = 0.9938 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{1.75}{1.75 + 2.3}\right)(12) = 5.185 \text{ V}$$

$$I_{BQ} = \frac{5.185 - 0.7}{0.9938 + (41)(0.02)} = 2.473 \text{ mA}, \quad I_{CQ} = 98.91 \text{ mA}$$

(b) Want $R_L' = \frac{V_{CC}}{I_{CQ}} = \frac{12}{0.09891} = 121.3\,\Omega = a^2 R_L = a^2(8) \Rightarrow a = 3.89$

(c) $\overline{P}_L = \frac{1}{2} \cdot \frac{(9)^2}{(0.1213)} = 333.9 \text{ mW}$

$\overline{P}_S = I_{CQ}V_{CC} = (98.91)(12) \Rightarrow 1.187 \text{ W}$

(d) $\eta = \frac{0.3339}{1.187} \times 100\% = 28.1\%$

8.34

a. Assuming the maximum power is being delivered, then

$$V_o'(\text{peak}) = 36 \text{ V} \Rightarrow V_o = \frac{36}{4} = 9 \text{ V} \Rightarrow V_{\text{rms}} = \frac{9}{\sqrt{2}} \Rightarrow \underline{V_{\text{rms}} = 6.36 \text{ V}}$$

b. $$V_o = \frac{36}{\sqrt{2}} \Rightarrow \underline{V_o = 25.5 \text{ V}}$$

c. Secondary $$I_{\text{rms}} = \frac{\overline{P_L}}{V_{\text{rms}}} = \frac{2}{6.36} \Rightarrow \underline{I_{\text{rms}} = 0.314 \text{ A}}$$

Primary $$I_P = \frac{0.314}{4} \Rightarrow \underline{I_P = 78.6 \text{ mA}}$$

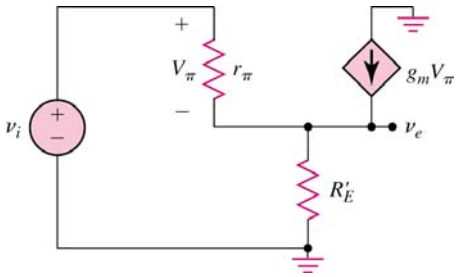
d.

$$\overline{P_S} = I_{CQ}.V_{CC} = (0.15)(36) = 5.4 \text{ W}$$

$$\eta = \frac{2}{5.4} \Rightarrow \underline{\eta = 37\%}$$

8.35

a.



$$v_e = \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right) R'_E = V_\pi \left(\frac{1}{r_\pi} + g_m\right) R'_E$$

$$= V_\pi \left(\frac{1+\beta}{r_\pi}\right) R'_E$$

$$v_i = V_\pi + v_e \Rightarrow V_\pi = v_i - v_e$$

$$v_e = (v_i - v_e)\left(\frac{1+\beta}{r_\pi}\right) R'_E$$

$$\frac{v_e}{v_i} = \frac{\dfrac{1+\beta}{r_\pi} \cdot R'_E}{1 + \dfrac{1+\beta}{r_\pi} \cdot R'_E} = \underline{\frac{(1+\beta)R'_E}{r_\pi + (1+\beta)R'_E} = \frac{v_e}{v_i}} \text{ where } \underline{R'_E = \left(\frac{n_1}{n_2}\right)^2 R_L}$$

$$v_0 = \frac{v_e}{\left(\dfrac{n_1}{n_2}\right)} \text{ so } v_e - v_0\left(\frac{n_1}{n_2}\right)$$

$$\text{so } \frac{v_0}{v_i} = \frac{1}{\left(\dfrac{n_1}{n_2}\right)} \cdot \frac{(1+\beta)R'_E}{r_\pi + (1+\beta)R'_E}$$

b.

$$\overline{P_L} = \frac{1}{2} \cdot I_P^2 R_L,\ a = \frac{n1}{n2},\ I_{CQ} = \frac{I_P}{a} \text{ so } \overline{P_L} = \frac{1}{2}.a^2 I_{CQ}^2 R_L$$

$$\overline{P_S} = I_{CQ}.V_{CC}$$

For $\eta = 50\%$:

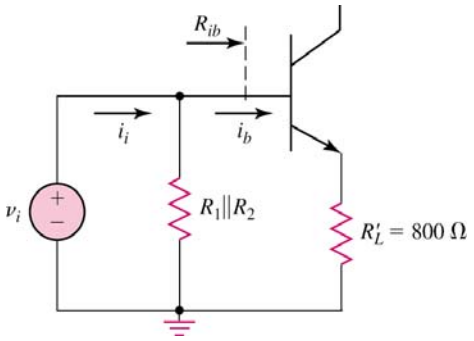
$$\frac{\overline{P_L}}{\overline{P_S}} = 0.5 = \frac{\dfrac{1}{2} \cdot a^2 I_{CQ}^2 R_L}{I_{CQ} \cdot V_{CC}} = \frac{a^2 I_{CQ} R_L}{2V_{CC}} \text{ so } a^2 = \frac{V_{CC}}{I_{CQ} \cdot R_L} = \frac{V_{CC}}{(0.1)(50)} \Rightarrow \underline{a^2 = \frac{V_{CC}}{5}}$$

c.

$$R_0 = \frac{r_\pi}{1+\beta} = \frac{\beta V_T}{(1+\beta) I_{CQ}} = \frac{49(0.026)}{(50)(0.1)} \Rightarrow \underline{R_0 = 0.255\ \Omega}$$

8.36

a. With a 10:1 transformer ratio, we need a current gain of 8 through the transistor.



$i_e = (1+\beta) i_b$ and $i_b = \left(\dfrac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right) i_i$ so we need $\dfrac{i_e}{i_i} = 8 = (1+\beta)\left(\dfrac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}} \right)$ where

$R_{ib} = r_\pi + (1+\beta) R'_L \approx (1+\beta) R'_L = (101)(0.8) = 80.8$

Then $8 = (101)\left(\dfrac{R_1 \| R_2}{R_1 \| R_2 + 80.8} \right)$

$\dfrac{R_1 \| R_2}{R_1 \| R_2 + 80.8} = 0.0792$ or $R_1 \| R_2 = 6.95\ \text{k}\Omega$

Set $\dfrac{2V_{CC}}{2I_{CQ}} = R'_L \Rightarrow I_{CQ} = \dfrac{V_{CC}}{R'_L} = \dfrac{12}{0.8} = 15\ \text{mA}$

$I_{BQ} = \dfrac{15}{100} = 0.15\ \text{mA}$

$V_{TH} = I_{BQ} R_{TH} + V_{BE}$

$\dfrac{1}{R_1} \cdot R_{TH} \cdot V_{CC} = I_{BQ} R_{TH} + V_{BE}$

$\dfrac{1}{R_1}(6.95)(12) = (0.15)(6.95) + 0.7 \Rightarrow \underline{R_1 = 47.9\ \text{k}\Omega}$ then $\underline{R_2 = 8.13\ \text{k}\Omega}$

b.

$I_e = 0.9 I_{CQ} = 13.5\ \text{mA} = \dfrac{I_L}{a} \Rightarrow I_L = 135\ \text{mA}$

$\overline{P_L} = \dfrac{1}{2}(0.135)^2 (8) \Rightarrow \underline{\overline{P_L} = 72.9\ \text{mW}}$

$\overline{P_S} = V_{CC} I_{CQ} = (12)(15) \Rightarrow \underline{\overline{P_S} = 180\ \text{mW}}$

$\eta = \dfrac{\overline{P_L}}{P_S} \Rightarrow \underline{\eta = 40.5\%}$

8.37

a.

$V_P = \sqrt{2R_L \overline{P_L}}$

$V_P = \sqrt{2(8)(2)} = 5.66$ V = peak output voltage

$I_P = \dfrac{V_P}{R_L} = \dfrac{5.66}{8} = 0.708$ A = peak output current

Set $V_e = 0.9V_{CC} = aV_P$ to minimize distortion

Then $a = \dfrac{(0.9)(18)}{5.66} \Rightarrow \underline{a = 2.86}$

b.

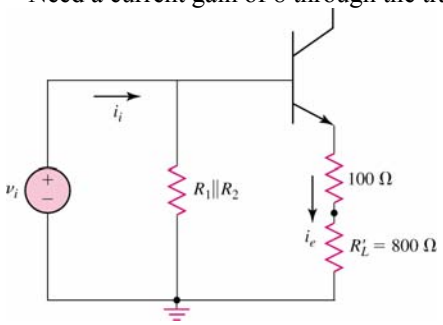
Now $I_{CQ} = \dfrac{1}{0.9}\left(\dfrac{I_P}{a}\right) = \dfrac{1}{0.9}\left(\dfrac{0.708}{2.86}\right) \Rightarrow I_{CQ} = 0.275$ A

Then $P_Q = V_{CC}I_Q = (18)(0.275) \Rightarrow \underline{P_Q = 4.95 \text{ W}}$ Power rating of transistor

---

8.38

a. Need a current gain of 8 through the transistor.



$\dfrac{i_b}{i_i} = 8 = (1+\beta)\left(\dfrac{R_1 \| R_2}{R_1 \| R_2 + R_{ib}}\right)$ where $R_{ib} \approx (1+\beta)(0.9) = 90.9$ kΩ

$\dfrac{8}{101} = \left(\dfrac{R_1 \| R_2}{R_1 \| R_2 + 90.9}\right) = 0.0792$ or $R_1 \| R_2 = 7.82$ kΩ

Set $\dfrac{2V_{CC}}{2I_{CQ}} = 0.9$ kΩ $\Rightarrow I_{CQ} = \dfrac{12}{0.9} = 13.3$ mA

$I_{BQ} = \dfrac{13.3}{100} = 0.133$ mA

Then $\dfrac{1}{R_1}(7.82)(12) = (0.133)(7.82) + 0.7 \Rightarrow \underline{R_1 = 53.9 \text{ kΩ}}$ and $\underline{R_2 = 9.15 \text{ kΩ}}$

b.

$$I_e = (0.9)I_{CQ} = 12 \text{ mA} = \frac{I_L}{a} \Rightarrow I_L = 120 \text{ mA}$$

$$\overline{P_L} = \frac{1}{2}(0.12)^2(8) \Rightarrow \underline{\overline{P_L} = 57.6 \text{ mW}}$$

$$\overline{P_S} = V_{CC}I_{CQ} = (12)(13.3) \Rightarrow \underline{\overline{P_S} = 159.6 \text{ mW}}$$

$$\eta = \frac{\overline{P_L}}{\overline{P_S}} = \frac{57.6}{159.6} \Rightarrow \underline{\eta = 36.1\%}$$

8.39

(a) $$V_{BB} = 2V_T \ln\left(\frac{I_{Bias}}{I_{SD}}\right) = 2(0.026)\ln\left(\frac{10^{-3}}{5\times10^{-16}}\right) = 1.473 \text{ V}$$

(b) $$I_{CQ} = I_{SQ} \exp\left(\frac{1.473/2}{0.026}\right) \Rightarrow I_{CQ} = 14 \text{ mA}$$

8.40

(a) $$V_{BE} = V_T \ln\left(\frac{I_{CQ}}{I_{SQ}}\right) = (0.026)\ln\left(\frac{4\times10^{-3}}{2\times10^{-15}}\right) = 0.73643 = V_{BEn} = V_{EBp} = V_D$$

$$I_{Bias} = I_{SD} \exp\left(\frac{V_D}{V_T}\right) = (4\times10^{-16})\exp\left(\frac{0.73643}{0.026}\right) \Rightarrow I_{Bias} = 0.8 \text{ mA}$$

(b) $V_{BB} = 2V_D = 1.473$ V

(c) $\upsilon_I = -V_{EBp} = -0.7364$ V

8.41

(a) $$V_{D1} = (0.026)\ln\left(\frac{0.5\times10^{-3}}{10^{-16}}\right) = 0.76025 \text{ V}$$

$$V_{D2} = (0.026)\ln\left(\frac{0.5\times10^{-3}}{4\times10^{-16}}\right) = 0.72421 \text{ V}$$

$$V_{BB} = V_{D1} + V_{D2} = 1.48446 \text{ V}$$

(b) $$i_{Cn} = i_{Cp} = I_{SQn} \exp\left(\frac{\upsilon_{BEn}}{V_T}\right) = I_{SQp} \exp\left(\frac{\upsilon_{EBp}}{V_T}\right)$$

$$\frac{I_{SQp}}{I_{SQn}} = \frac{\exp\left(\frac{\upsilon_{BEn}}{V_T}\right)}{\exp\left(\frac{\upsilon_{EBp}}{V_T}\right)} = \frac{\exp\left(\frac{\upsilon_{BEn}}{V_T}\right)}{\exp\left(\frac{V_{BB}-\upsilon_{BEn}}{V_T}\right)} = \exp\left(\frac{2\upsilon_{BEn}-V_{BB}}{V_T}\right)$$

$$\frac{2\upsilon_{BEn}-V_{BB}}{V_T} = \ln\left(\frac{I_{SQp}}{I_{SQn}}\right)$$

$$\upsilon_{BEn} = \frac{1}{2}\left[V_{BB} + V_T \ln\left(\frac{I_{SQp}}{I_{SQn}}\right)\right] = \frac{1}{2}\left[1.48446 + (0.026)\ln\left(\frac{1.6\times10^{-15}}{8\times10^{-16}}\right)\right] = 0.75124 \text{ V}$$

$$\upsilon_{EBp} = V_{BB} - \upsilon_{BEn} = 0.73322 \text{ V}$$

(c) $$I_{CQ} = I_{SQn} \exp\left(\frac{\upsilon_{BEn}}{V_T}\right) = \left(8\times10^{-16}\right)\exp\left(\frac{0.75124}{0.026}\right)$$

$$\Rightarrow i_{Cn} = i_{Cp} = 2.828 \text{ mA}$$

(d) $$\upsilon_I = -\upsilon_{EBp} = -0.73322 \text{ V}$$

8.42

a. All transistors are matched.

$$3 \text{ mA} = i_{E1} + i_{B3} = \left(\frac{1+\beta}{\beta}\right) i_C + \frac{i_C}{\beta}$$

$$3 = \left(\frac{61}{60} + \frac{1}{60}\right) i_C \Rightarrow \underline{i_C = 2.90 \text{ mA}}$$

b.

For $v_o = 6$ V, let $\underline{R_L = 200\ \Omega}$.

$$i_o = \frac{6}{200} = 0.03 \text{ A} = 30 \text{ mA} \cong i_{E3}$$

$$i_{B3} = \frac{30}{61} = 0.492 \text{ mA}$$

$$i_{E1} = 3 - 0.492 = 2.508 \text{ mA}$$

$$i_{B1} = \frac{2.508}{61} \Rightarrow \underline{i_{B1} = 41.11\ \mu\text{A}}$$

$$i_{E2} \cong 3 \text{ mA} \Rightarrow i_{B2} = \frac{3}{61} \Rightarrow 49.18\ \mu\text{A}$$

$$i_I = i_{B2} - i_{B1} = 49.18 - 41.11 \Rightarrow i_I = 8.07\ \mu\text{A}$$

Current gain

$$A_i = \frac{30\times10^{-3}}{8.07\times10^{-6}} \Rightarrow \underline{A_i = 3.72\times10^3}$$

$$V_{BE3} = V_T \ln\left(\frac{i_{E3}}{I_S}\right) = \left(0.026\right)\ln\left(\frac{30\times10^{-3}}{5\times10^{-13}}\right)$$

$$V_{BE3} = 0.6453 \text{ V}$$

$$V_{EB1} = V_T \ln\left(\frac{i_{E1}}{I_S}\right) = \left(0.026\right)\ln\left(\frac{2.508\times10^{-3}}{5\times10^{-13}}\right)$$

$$V_{EB1} = 0.5807 \text{ V}$$

$$v_I = v_0 + V_{BE3} - V_{EB1} = 6 + 0.6453 - 0.5807$$

$$v_I = 6.0646 \text{ V}$$

Voltage gain

$$A_v = \frac{v_0}{v_I} = \frac{6}{6.0646} \Rightarrow \underline{A_v = 0.989}$$

8.43

a. For $i_0 = 1$ A, $I_{B3} \cong \frac{1}{50} \Rightarrow 20$ mA

We can then write $\frac{10 - V_{EB1}}{R_1} = 2\left[\frac{10 - \left(v_{0,\max} + V_{BE3}\right)}{R_1} - 20\right]$

If, for simplicity, we assume $V_{EB1} = V_{BE3} = 0.7$ V, then $\frac{10 - V_{BE}}{R_1} = \frac{2v_{o,\max}}{R_1} + 40$

If we assume $v_{0,\max} = 4$ V, then $\frac{9.3}{R_1} = \frac{2(4)}{R_1} + 40$ which yields $\underline{R_1 = R_2 = 32.5\ \Omega}$

b. For $v_I = 0$, $I_{E1} = \frac{9.3}{32.5} \Rightarrow \underline{I_{E1} = 0.286\text{ A} = I_{E2}}$

Since $I_{S3,4} = 10I_{S1,2}$, then $\underline{I_{E3} = I_{E4} = 2.86\text{ A}}$

c.

We can write

$$R_0 = \frac{1}{2}\left\{\frac{r_{\pi 3} + R_1 \left\| \frac{r_{\pi 1}}{1+\beta_1}\right.}{1+\beta_3}\right\}$$

Now $r_{\pi 3} = \frac{\beta_3 V_T}{I_{C3}} = \frac{(50)(0.026)}{2.86} = 0.4545\ \Omega$

$$r_{\pi 1} = \frac{\beta_1 V_T}{I_{C1}} = \frac{(120)(0.026)}{0.286} = 10.91\ \Omega$$

So

$$R_0 = \frac{1}{2}\left\{\frac{0.4545 + 32.5 \left\| \frac{10.91}{121}\right.}{51}\right\}$$

$$32.5 \left\| \frac{10.91}{121} = 32.5 \right\| 0.0902 = 0.0900$$

Then $R_0 = \frac{1}{2}\left\{\frac{0.4545 + 0.0900}{51}\right\}$ or $\underline{R_0 = 0.00534\ \Omega}$

8.44

$$R_i = \frac{1}{2}\left\{r_{\pi 1} + (1+\beta)\left[R_1 \| \left(r_{\pi 3} + (1+\beta)2R_L\right)\right]\right\}$$

$i_{C1} \approx 7.2$ mA and $i_{C3} \approx 7.2$ mA

Then $r_\pi = \dfrac{(60)(0.026)}{7.2} = 0.217$ kΩ

So $R_i = \dfrac{1}{2}\left\{0.217 + (61)\left[2 \| \left(0.217 + (61)(0.2)\right)\right]\right\}$

$$= \frac{1}{2}\left\{0.217 + 61\left[2 \| 12.4\right]\right\} \text{ or } R_i = 52.6 \text{ kΩ}$$

8.45

(b) For M_{n3}; $\upsilon_{GSn3} = \sqrt{\dfrac{I_{DQ}}{K_{n3}}} + V_{TN} = \sqrt{\dfrac{5}{5}} + 1 = 2$ V

$\upsilon_{SGp1} = \upsilon_{GSn3} = 2$ V

$I_{DQ1} = 2(2-1)^2 = 2$ mA

$R_1 = R_2 = \dfrac{10-2}{2} = 4$ kΩ

(c) $I_{DQ1} = I_{DQ2} = 2$ mA

(d) For $\upsilon_o = 3.5$ V, Assume M_{p4} cutoff, so $I_{DQ4} = 0$

$I_{Dn3} = i_o = \dfrac{\upsilon_o}{R_L} = \dfrac{3.5}{0.15} = 23.33$ mA

$\upsilon_{GSn3} = \sqrt{\dfrac{23.33}{5}} + 1 = 3.160$ V

$I_{Dp1} = I_{R1} = \dfrac{10 - (\upsilon_o + \upsilon_{GSn3})}{R_1} = \dfrac{10 - (3.16 + 3.5)}{4} = 0.835$ mA

$\upsilon_{SGp1} = \sqrt{\dfrac{0.835}{2}} + 1 = 1.646$ V

$\upsilon_I = \upsilon_o + \upsilon_{GSn3} - \upsilon_{SGp1} = 3.5 + 3.160 - 1.646 = 5.014$ V

$\upsilon_I = \upsilon_{GSn2} + K_{n2}R_2(\upsilon_{GSn2} - V_{TN})^2 + V^-$

$15.014 = \upsilon_{GSn2} + 8(\upsilon_{GSn2}^2 - 2\upsilon_{GSn2} + 1)$

or $8\upsilon_{GSn2}^2 - 15\upsilon_{GSn2} - 7.014 = 0 \Rightarrow \upsilon_{GSn2} = 2.2625$ V

$I_{Dn2} = 2(2.2625-1)^2 = 3.188$ mA

$\upsilon_{G4} = \upsilon_I - \upsilon_{GSn2} = 5.014 - 2.2625 = 2.75$ V

$\upsilon_{SGp4} = \upsilon_o - \upsilon_{G4} = 3.5 - 2.75 = 0.75$ V, $\Rightarrow M_{p4}$ cutoff

$\overline{P}_L = \dfrac{\upsilon_o^2}{R_L} = \dfrac{(3.5)^2}{0.15} = 81.7$ mW

8.46

For $\upsilon_I = -1.5\,\text{V}$ and $\upsilon_O = 0$, $\Rightarrow \upsilon_{SG2} = 1.5\,\text{V} = \upsilon_{GS1}$

$$i_{D1} = i_{D2} = 0.5 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_1 (1.5-0.8)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 20.4$$

$$0.5 = \left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)_2 (1.5-0.8)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 51.0$$

$$\upsilon_{SG4} = 1.5\,\text{V},\quad 0.2 = \left(\frac{0.04}{2}\right)\left(\frac{W}{L}\right)_4 (1.5-0.8)^2 \Rightarrow \left(\frac{W}{L}\right)_4 = 20.4$$

$$0.2 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_3 (1.5-0.8)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 8.16$$

8.47

For $v_0 = 0$

$$I_Q = I_{C3} + I_{C2} + I_{E1}$$

$$I_{B3} = I_{E2} = \left(\frac{1+\beta_n}{\beta_n}\right) I_{C2} = \frac{I_{C3}}{\beta_n}$$

$$I_{C3} = (1+\beta_n) I_{C2}$$

$$I_{B2} = I_{C1} = \left(\frac{\beta_P}{1+\beta_P}\right) I_{E1} = \frac{I_{C2}}{\beta_n}$$

$$I_{C2} = \beta_n \left(\frac{\beta_P}{1+\beta_P}\right) I_{E1}$$

$$I_{C3} = (1+\beta_n)\beta_n \left(\frac{\beta_P}{1+\beta_p}\right) I_{E1}$$

$$I_Q = (1+\beta_n)\beta_n \left(\frac{\beta_P}{1+\beta_P}\right) I_{E1} + \beta_n \left(\frac{\beta_P}{1+\beta_P}\right) I_{E1} + I_{E1}$$

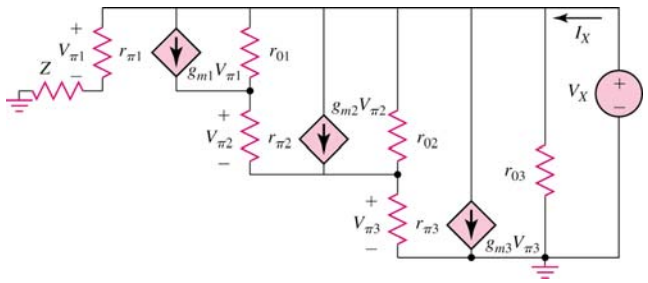
$$= (51)(50)\left(\frac{10}{11}\right) I_{E1} + (50)\left(\frac{10}{11}\right) I_{E1} + I_{E1}$$

$$I_Q = 2318.18 I_{E1} + 45.45 I_{E1} + I_{E1}$$

$$I_{E1} = 1.692\ \mu\text{A} \Rightarrow \underline{I_{C1} = 1.534\ \mu\text{A}}$$

$$I_{C2} = (50)\left(\frac{10}{11}\right)(1.692) \Rightarrow I_{C2} = 76.9\ \mu\text{A}$$

$$I_{C3} = (51)(50)\left(\frac{10}{11}\right)(1.692) \Rightarrow \underline{I_{C3} = 3.92\ \text{mA}}$$



Because of $r_{\pi 1}$ and Z, neglect effect of r_0. Then neglecting r_{01}, r_{02} and r_{03}, we find

$$I_X = g_{m3}V_{\pi 3} + g_{m2}V_{\pi 2} + g_{m1}V_{\pi 1} + \frac{V_X}{r_{\pi 1} + Z}$$

Now

$$V_{\pi 1} = \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z}\right)V_X, \; V_{\pi 2} \cong g_{m1}V_{\pi 1}r_{\pi 2}$$

and

$$V_{\pi 3} = \left(g_{m1}V_{\pi 1} + g_{m2}V_{\pi 2}\right)r_{\pi 3}$$
$$= \left[g_{m1}V_{\pi 1} + g_{m2}\left(g_{m1}V_{\pi 1}r_{\pi 2}\right)\right]r_{\pi 3}$$

$$V_{\pi 3} = \left(\frac{r_{\pi 1}}{r_{\pi 1} + Z}\right)\left[g_{m1} + g_{m1}g_{m2}r_{\pi 2}\right]r_{\pi 3} \cdot V_X$$

$$V_{\pi 3} = \frac{\left(\beta_1 + \beta_1\beta_2\right)r_{\pi 3}}{r_{\pi 1} + Z} \cdot V_X$$

and $V_{\pi 2} = g_{m1}\left(\frac{r_{\pi 1}}{r_{\pi 1} + Z}\right)r_{\pi 2}V_X = \left(\frac{\beta_1 r_{\pi 2}}{r_{\pi 1} + Z}\right)V_X$

Then $I_X = \frac{\left(\beta_1 + \beta_1\beta_2\right)\beta_3}{r_{\pi 1} + Z} \cdot V_X + \frac{\beta_1\beta_2}{r_{\pi 1} + Z} \cdot V_X + \frac{\beta_1}{r_{\pi 1} + Z} \cdot V_X + \frac{V_X}{r_{\pi 1} + Z}$

Then

$$R_0 = \frac{V_X}{I_X} = \frac{r_{\pi 1} + Z}{1 + \beta_1 + \beta_1\beta_2 + \left(\beta_1 + \beta_1\beta_2\right)\beta_3}$$

$$r_{\pi 1} = \frac{(10)(0.026)}{1.534} = 0.169 \text{ M}\Omega$$

$Z = 25 \text{ k}\Omega$

Then

$$R_0 = \frac{169 + 25}{1 + (10) + (10)(50) + \left[10 + (10)(50)\right](50)}$$

$$R_0 = \frac{194}{26,011} = 0.00746 \text{ k}\Omega \text{ or } \underline{R_o = 7.46 \; \Omega}$$

8.48

a Neglect base currents.

$$V_{BB} = 2V_D = 2V_T \ln\left(\frac{I_{\text{Bias}}}{I_S}\right)$$

$$= 2(0.026)\ln\left(\frac{5 \times 10^{-3}}{10^{-13}}\right) \Rightarrow \underline{V_{BB} = 1.281 \text{ V}}$$

$$V_{BE1} + V_{EB3} = V_{BB}$$

$$I_{E1} = I_{E3} + I_{C2}$$

$$I_{B2} = I_{C3} = \left(\frac{\beta_P}{1+\beta_P}\right) I_{E3}$$

$$I_{C2} = \beta_n I_{B2} = \beta_n \left(\frac{\beta_P}{1+\beta_P}\right) I_{E3}$$

$$I_{E1} = I_{E3} + \beta_n \left(\frac{\beta_P}{1+\beta_P}\right) I_{E3}$$

$$I_{E1} = I_{E3}\left[1 + \beta_n \left(\frac{\beta_P}{1+\beta_P}\right)\right]$$

$$\left(\frac{1+\beta_n}{\beta_n}\right) I_{C1} = \left(\frac{1+\beta_P}{\beta_P}\right) I_{C3}\left[1 + \beta_n \left(\frac{\beta_P}{1+\beta_P}\right)\right]$$

$$V_{BE1} = V_T \ln\left[\frac{I_{C1}}{I_S}\right],\ V_{EB3} = V_T \ln\left[\frac{I_{C3}}{I_S}\right]$$

$$(1.01) I_{C1} = \left(\frac{21}{20}\right) I_{C3}\left[1 + (100)\left(\frac{20}{21}\right)\right]$$

$$= I_{C3}\left[\frac{21}{20} + 100\right] = 101.05 I_{C3}$$

$$I_{C1} = 100.05 I_{C3}$$

$$V_T \ln\left(\frac{100.05 I_{C3}}{I_S}\right) + V_T \ln\left(\frac{I_{C3}}{I_S}\right) = V_{BB}$$

$$V_T \ln\left(\frac{100.05 I_{C3}^2}{I_S^2}\right) = V_{BB}$$

$$\frac{100.05 I_{C3}^2}{I_S^2} = \exp\left(\frac{V_{BB}}{V_T}\right)$$

$$I_{C3} = \frac{I_S}{\sqrt{100.05}} \sqrt{\exp\left(\frac{V_{BB}}{V_T}\right)} = \underline{0.4995 \text{ mA} = I_{C3}}$$

Then $I_{E3} = 0.5245$ mA

Now $I_{C1} = 100.05 I_{C3} = \underline{49.97 \text{ mA} = I_{C1}}$

$$I_{C2} = (100)\left(\frac{20}{21}\right)(0.5245) = \underline{49.95 \text{ mA} = I_{C2}}$$

$$V_{BE1} = V_T \ln\left(\frac{I_{C1}}{I_S}\right) = 0.026 \ln\left(\frac{49.97 \times 10^{-3}}{10^{-13}}\right)$$

$$= 0.70037$$

$$V_{EB3} = V_T \ln\left(\frac{I_{C3}}{I_S}\right) = 0.026 \ln\left(\frac{0.4995 \times 10^{-3}}{10^{-13}}\right)$$

$$= 0.58062$$

Note: $V_{BE1} + V_{EB3} = 0.70037 + 0.58062 = 1.28099$

$$= V_{BB}$$

b.

$$v_0 = 10 \text{ V} \Rightarrow i_{E1} \approx \frac{10}{100} = 0.10 \text{ A} = i_{C1}$$

$$i_{B1} = \frac{100}{100} = 1 \text{ mA}$$

$$V_{BB} = 2(0.026)\ln\left(\frac{4\times10^{-3}}{10^{-13}}\right) = 1.2694 \text{ V}$$

$$V_{BE1} = (0.026)\ln\left(\frac{0.1}{10^{-13}}\right) = 0.7184$$

$$V_{EB3} = 1.2694 - 0.7184 = 0.55099 \text{ V}$$

$$I_{C3} = 10^{-13}\exp\left(\frac{0.55099}{0.026}\right) = 0.1598 \text{ mA}$$

$$\overline{P_L} = \frac{V_0^2}{R_L} = \frac{(10)^2}{100} \Rightarrow \underline{\overline{P_L} = 1 \text{ W}}$$

$$P_{Q1} = i_{C1}\cdot v_{CE1} = (0.1)(12-10) \Rightarrow \underline{P_{Q1} = 0.2 \text{ W}}$$

$$P_{Q3} = i_{C3}\cdot v_{EC3} = (0.1598)(10-[0.7-12]) \Rightarrow \underline{P_{Q3} = 3.40 \text{ mW}}$$

$$i_{C2} = (100)(i_{C3}) = (100)(0.1598) = 15.98 \text{ mA}$$

$$P_{Q2} = i_{C2}\cdot v_{CE2} = (15.98)(10-[-12]) \Rightarrow \underline{P_{Q2} = 0.352} \text{ W}$$

8.49

a.

$$V_{BB} = 3(0.026)\ln\left(\frac{10\times10^{-3}}{2\times10^{-12}}\right) \Rightarrow \underline{V_{BB} = 1.74195 \text{ V}}$$

$$V_{BE1} + V_{BE2} + V_{EB3} = V_{BB}$$

$$I_{C1} \approx \frac{I_{C2}}{\beta_n},\ I_{C3} \approx \frac{I_{C2}}{\beta_n^2}$$

$$V_T \ln\left(\frac{I_{C1}}{I_S}\right) + V_T \ln\left(\frac{I_{C2}}{I_S}\right) + V_T \ln\left(\frac{I_{C3}}{I_S}\right) = V_{BB}$$

$$V_T \ln\left[\frac{I_{C2}^3}{\beta_n^3 I_S^3}\right] = V_{BB}$$

$$I_{C2} = \beta_n I_S \sqrt[3]{\exp\left(\frac{V_{BB}}{V_T}\right)}$$

$$= (20)\left(20\times10^{-12}\right)\sqrt[3]{\exp\left(\frac{1.74195}{0.026}\right)}$$

$$I_{C2} = 0.20 \text{ A},\ I_{C1} \approx 10 \text{ mA},\ I_{C3} \approx 0.5 \text{ mA}$$

$$V_{BE1} = (0.026)\ln\left(\frac{10\times10^{-3}}{2\times10^{-12}}\right) \Rightarrow \underline{V_{BE1} = 0.58065 \text{ V}}$$

$$V_{BE2} = (0.026)\ln\left(\frac{0.2}{2\times10^{-12}}\right) \Rightarrow \underline{V_{BE2} = 0.6585 \text{ V}}$$

$$V_{EB3} = (0.026)\ln\left(\frac{0.5\times10^{-3}}{2\times10^{-12}}\right) \Rightarrow \underline{V_{EB3} = 0.50276 \text{ V}}$$

b.

$$\overline{P_L} = 10 \text{ W} = \frac{1}{2}\cdot\frac{V_0^2}{R_L} = \frac{1}{2}\cdot\frac{V_0^2}{20} \Rightarrow V_0(\max) = 20 \text{ V}$$

For $v_0(\max)$:

$$P_L = \frac{v_0^2}{R_L} = \frac{(20)^2}{20} \Rightarrow \underline{P_L = 20 \text{ W}}$$

$$i_0(\max) = -\frac{20}{20} = -1 \text{ A}$$

$$i_{C5} + i_{C4} + i_{E3} = -i_o(\max) = 1 \text{ A}$$

$$i_{C5} + \frac{i_{C5}}{\beta_n}\cdot\left(\frac{\beta_n}{1+\beta_n}\right) + \frac{i_{C4}}{\beta_n}\left(\frac{1+\beta_p}{\beta_p}\right) = 1$$

$$i_{C5} + \frac{i_{C5}}{\beta_n}\left(\frac{\beta_n}{1+\beta_n}\right) + \frac{i_{C5}}{\beta_n}\left(\frac{\beta_n}{1+\beta_n}\right)\left[\frac{1}{\beta_n}\left(\frac{1+\beta_p}{\beta_p}\right)\right] = 1$$

$$i_{C5}\left[1 + \frac{1}{20}\left(\frac{20}{21}\right)\right] + \left(\frac{1}{21}\right)\left[\frac{1}{20}\left(\frac{6}{5}\right)\right] = 1$$

$i_{C5}\left(1.05048\right)=1 \quad \underline{i_{C5}=0.952\text{ A}}$

$\underline{i_{C4}=0.0453\text{ A}}$

$i_{E3}=0.00272\text{ A}$

$$i_{C3}=0.00272\left(\frac{5}{6}\right)$$

$$\underline{=0.002267\text{ A}}$$

$$V_{EB3}=\left(0.026\right)\ln\left(\frac{2.267\times10^{-3}}{2\times10^{-12}}\right)=0.54206\text{ V}$$

$$V_{BE1}+V_{BE2}=1.74195-0.54206=1.19989$$

$$V_T\ln\left(\frac{I_{C2}}{\beta_n I_S}\right)+V_T\ln\left(\frac{I_{C2}}{I_S}\right)=1.19989$$

$$i_{C2}=\sqrt{\beta_n}\cdot I_S\sqrt{\exp\left(\frac{1.19989}{0.026}\right)}$$

$$=\sqrt{20}\left(18.83\right)\text{ mA}$$

$$\underline{i_{C2}=93.9\text{ mA}}$$

$$i_{C1}=\frac{i_{C2}}{\beta_n}\left(\frac{\beta_n}{1+\beta_n}\right)=\frac{93.9}{21}=\underline{4.47\text{ mA}}$$

$$P_{Q2}=I_{C2}\left(24-\left(-20\right)\right)=\left(0.0939\right)\left(44\right)=\underline{4.13\text{ W}}$$

$$P_{Q5}=\left(0.952\right)\left(-10-\left(-24\right)\right)=\underline{13.3\text{ W}}$$

Chapter 9

9.1

(a)

$$v_O = A_d\left(v_2 - v_1\right)$$

$$1 = A_d\left(10^{-3} - \left(-10^{-3}\right)\right) \Rightarrow A_d = 500$$

(b)

$$1 = 500\left(v_2 - 10^{-3}\right) = 1 + 0.5 = 500v_2$$

$$\underline{v_2 = 3 \text{ mV}}$$

(c)

$$5 = 500\left(1 - v_1\right) \Rightarrow 500v_1 = 495$$

$$\underline{v_1 = 0.990 \text{ V}}$$

(d) $v_O = 0$

(e)

$$-3 = 500\left(v_2 - \left(-0.5\right)\right)$$

$$-250 - 3 = 500v_2$$

$$\underline{v_2 = -0.506 \text{ V}}$$

9.2

(a) $$\upsilon_2 = \frac{\upsilon_O}{A_{od}} = \frac{-2}{10^4} = -2\times10^{-4} \text{ V}$$

$$\upsilon_2 = \left(\frac{1}{1+2000}\right)\cdot\upsilon_I$$

$$-2\times10^{-4} = \left(\frac{1}{2001}\right)\cdot\upsilon_I \Rightarrow \upsilon_I = -0.4002 \text{ V}$$

(b) $$\upsilon_2 = \left(\frac{1}{1+2000}\right)\cdot\upsilon_I$$

$$\upsilon_2 = \left(\frac{1}{2001}\right)(2) = 0.9995\times10^{-3} \text{ V}$$

$$\upsilon_O = 1 = A_{od}\upsilon_2 = A_{od}\left(0.9995\times10^{-3}\right) \Rightarrow A_{od} = 1000.5$$

9.3

(a) $$\upsilon_O = A_{od}\left(\upsilon_2 - \upsilon_1\right) = \left(5\times10^3\right)\left(2.0000 - 2.0010\right) = -5 \text{ V}$$

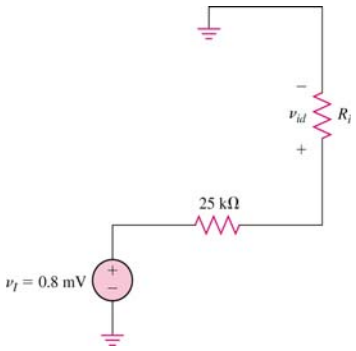
(b) $$\upsilon_O = A_{od}\left(\upsilon_2 - \upsilon_1\right)$$

$$-3.000 = \left(2\times10^4\right)\left(3.0025 - \upsilon_1\right) \Rightarrow \upsilon_1 = 3.00265 \text{ V}$$

(c) $$\upsilon_O = A_{od}\left(\upsilon_2 - \upsilon_1\right)$$

$$1.80 = A_{od}\left(0.01 - \left(-0.01\right)\right)\times10^{-3} \Rightarrow A_{od} = 9\times10^4$$

9.4



$$v_{id} = \left(\frac{R_i}{R_i + 25}\right) v_I$$

$$0.790 = \left(\frac{R_i}{R_i + 25}\right)(0.80)$$

$$0.9875\left(R_i + 25\right) = R_i$$

$$24.6875 = 0.0125 R_i$$

$$\underline{R_i = 1975 \text{ K}}$$

9.5

(a) $A_v = \dfrac{-R_2}{R_1} = \dfrac{-200}{20} = -10$

(b) $A_v = \dfrac{-120}{40} = -3$

(c) $A_v = \dfrac{-40}{40} = -1$

9.6

$$\left.\begin{array}{l} A_v = -\dfrac{200}{20} = -10 \\ \text{and} \\ R_i = 20 \text{ k}\Omega \end{array}\right\} \text{for each case}$$

9.7

a.

$$A_v = -\frac{100}{10} = -10$$

$$R_i = R_1 = 10 \text{ k}\Omega$$

b.

$$A_v = -\frac{100 \| 100}{10} = -5$$

$$R_i = R_1 = 10 \text{ k}\,\Omega$$

c.

$$A_v = -\frac{100}{10+10} = -5$$

$$R_i = 10+10 = 20 \text{ K}$$

9.8

(a) $|i_2| = \left|\frac{0-\upsilon_O}{R_2}\right| \Rightarrow R_2 = \frac{4}{20\times10^{-6}} \Rightarrow R_2 = 200\,\text{k}\Omega$

$$A_\upsilon = \frac{-R_2}{R_1} \Rightarrow -12 = \frac{-200}{R_1} \Rightarrow R_1 = 16.67\,\text{k}\Omega$$

(b) $i_2 = \frac{0-\upsilon_O}{R_2} = \frac{0-1.5}{200\times10^3} \Rightarrow i_1 = i_2 = -7.5\,\mu\text{A}$

$$\upsilon_I = \frac{\upsilon_O}{A_\upsilon} = \frac{+1.5}{-12} \Rightarrow \upsilon_I = -0.125\,\text{V}$$

9.9

$$A_v = -\frac{R_2}{R_1}$$

(a) $A_v = -10$

(b) $A_v = -1$

(c) $A_v = -0.20$

(d) $A_v = -10$

(e) $A_v = -2$

(f) $A_v = -1$

9.10

(a) $-3 = \frac{-R_2}{R_1} = \frac{-200}{R_1} \Rightarrow R_2 = 200\,\text{k}\Omega$, $R_1 = 66.67\,\text{k}\Omega$

(b) $-8 = \frac{-R_2}{R_1} = \frac{-200}{R_1} \Rightarrow R_2 = 200\,\text{k}\Omega$, $R_1 = 25\ \text{k}\Omega$

(c) $-20 = \frac{-R_2}{R_1} = \frac{-200}{R_1} \Rightarrow R_2 = 200\,\text{k}\Omega$, $R_1 = 10\,\text{k}\Omega$

(d) $-0.5 = \frac{-R_2}{R_1} = \frac{-R_2}{200} \Rightarrow R_2 = 100\,\text{k}\Omega$, $R_1 = 200\,\text{k}\Omega$

9.11

(a) $|i_1| = \frac{\upsilon_I}{R_1} \Rightarrow R_1 = \frac{0.25}{50\times10^{-6}} \Rightarrow R_1 = 5\,\text{k}\Omega$

$$A_\upsilon = \frac{-R_2}{R_1} \Rightarrow -6.5 = \frac{-R_2}{5} \Rightarrow R_2 = 32.5\,\text{k}\Omega$$

(b) $\upsilon_I = \dfrac{\upsilon_O}{A_\upsilon} = \dfrac{-4}{-6.5} = 0.6154\ \text{V}$

$i_1 = i_2 = \dfrac{4}{32.5} = 0.123\,\text{mA}$

9.12

(a) $A_\upsilon = \dfrac{-R_2}{R_1}$

$-20 = \dfrac{-R_2}{25} \Rightarrow R_1 = 25\,\text{k}\Omega\,,\quad R_2 = 500\,\text{k}\Omega$

(b) $-20 = \dfrac{-1000}{R_1} \Rightarrow R_1 = 50\,\text{k}\Omega\,,\quad R_2 = 1\,\text{M}\Omega$

(c) For (a), $i_1 = \dfrac{\upsilon_I}{R_1} = \dfrac{-0.2}{25} \Rightarrow i_1 = -8\,\mu\,\text{A}$

For (b), $i_1 = \dfrac{\upsilon_I}{R_1} = \dfrac{-0.2}{50} \Rightarrow i_1 = -4\,\mu\,\text{A}$

9.13

a.

$A_v = \dfrac{R_2}{R_1} \Rightarrow \dfrac{1.05R_2}{0.95R_1} = 1.105\left(\dfrac{R_2}{R_1}\right)$

$\dfrac{0.95R_2}{1.05R_1} = 0.905\left(\dfrac{R_2}{R_1}\right)$

Deviation in gain is +10.5% and -9.5%

b.

$A_v \Rightarrow \dfrac{1.01R_2}{0.99R_1} = 1.02\left(\dfrac{R_2}{R_1}\right) \Rightarrow \dfrac{0.99R_2}{1.01R_1} = 0.98\left(\dfrac{R_2}{R_1}\right)$

Deviation in gain = ±2%

9.14

(a) (i) $\upsilon_O = \dfrac{-R_2}{R_1}\cdot\upsilon_I = \dfrac{-15}{1}\left(-0.20\right) = 3\,\text{V}$

(ii) $i_2 = \dfrac{0-\upsilon_O}{R_2} = \dfrac{-3}{15} = -0.20\ \text{mA}$

$i_L = \dfrac{\upsilon_O}{R_L} = \dfrac{3}{4} = 0.75\ \text{mA}$

$i_O + i_2 = i_L \Rightarrow i_O = 0.75 - \left(-0.20\right) = 0.95\ \text{mA}$

(b) (i) $\upsilon_O = \dfrac{-15}{1}(0.05) = -0.75\ \text{V}$

(ii) $i_2 = \dfrac{0-(-0.75)}{15} = 0.05\ \text{mA}$

$i_L = \dfrac{-0.75}{4} = -0.1875\ \text{mA}$

$i_O = i_L - i_2 = -0.1875 - 0.05 = -0.2375\ \text{mA}$

(c) (i) $\upsilon_O = \dfrac{-15}{1}(8\sin\omega t)\ (\text{mV}) \Rightarrow \upsilon_O = -0.12\sin\omega t\ (\text{V})$

(ii) $i_2 = \dfrac{0.12\sin\omega t}{15} \Rightarrow i_2 = 8\sin\omega t\ \ (\mu\ \text{A})$

$i_L = \dfrac{-0.12\sin\omega t}{4} \Rightarrow i_L = -30\sin\omega t\ \ (\mu\ \text{A})$

$i_O = i_L - i_2 = -38\sin\omega t\ \ (\mu\ \text{A})$

9.15

$$A_v = -\frac{R_2}{R_1 + R_S}$$

$A_v = -30 \pm 2.5\% \Rightarrow 29.25 \le |A_v| \le 30.75$

So $\dfrac{R_2}{R_1 + 2} = 29.25$ and $\dfrac{R_2}{R_1 + 1} = 30.75$

We have $29.25(R_1 + 2) = 30.75(R_1 + 1)$

Which yields $\underline{R_1 = 18.5\ k\Omega}$ and $\underline{R_2 = 599.6\ k\Omega}$

For $v_I = 25\ mV$, then $\underline{0.731 \le |v_O| \le 0.769\ V}$

9.16

$\upsilon_{O1} = \dfrac{-R_2}{R_1}\cdot\upsilon_I = \dfrac{-80}{10}(-0.15) = 1.2\ \text{V}$

$\upsilon_O = \dfrac{-R_4}{R_3}\cdot\upsilon_{O1} = \dfrac{-100}{20}(1.2) = -6\ \text{V}$

$i_1 = i_2 = \dfrac{\upsilon_I}{R_1} = \dfrac{-(0.15)}{10} \Rightarrow i_1 = i_2 = -15\ \mu\ \text{A}$

$i_3 = i_4 = \dfrac{\upsilon_{O1}}{R_3} = \dfrac{1.2}{20} \Rightarrow i_3 = i_4 = 60\ \mu\ \text{A}$

At υ_{O1}: $i_2 + i_{O1} = i_3 \Rightarrow i_{O1} = 60 - (-15) = 75\ \mu\ \text{A}$; Out of Op-Amp

At υ_O: $i_{O2} = i_4 = 60\ \mu\ \text{A}$; Into Op-Amp

9.17

$$\frac{\upsilon_O}{\upsilon_I} = \left(\frac{-R_2}{R_1}\right)\left(\frac{-R_4}{R_3}\right) = 100$$

For $\upsilon_I = 50\ \text{mV}$, $\upsilon_O = (100)(0.05) = 5\ \text{V}$

If $|i_4| = 50\,\mu$A, $R_4 = \dfrac{5}{50\times10^{-6}} \Rightarrow R_4 = 100\,\text{k}\Omega$

Set $R_1 = 10\,\text{k}\Omega$

Then $\dfrac{\upsilon_O}{\upsilon_I} = 100 = \left(\dfrac{R_2}{10}\right)\left(\dfrac{100}{R_3}\right) \Rightarrow \dfrac{R_2}{R_3} = 10$

Set $R_2 = 100\,\text{k}\Omega$, $R_3 = 10\,\text{k}\Omega$

9.18

$$A_\upsilon = \left(\frac{-R_2}{R_1}\right)\left(\frac{-R_4}{R_3}\right)\left(\frac{-R_6}{R_5}\right) = -300$$

For $\upsilon_O = 6$ V, set $|i_6| = \dfrac{\upsilon_O}{R_6} \Rightarrow 60\,\mu\text{A} = \dfrac{6}{R_6} \Rightarrow R_6 = 100\,\text{k}\Omega$

Set $R_6 = 200\,\text{k}\Omega$ so that $|i_6| = 30\ \mu$A

Set $R_1 = 20\,\text{k}\Omega$

Now $300 = \left(\dfrac{R_2}{20}\right)\left(\dfrac{R_4}{R_3}\right)\left(\dfrac{200}{R_5}\right)$

For example, set $R_2 = 100\,\text{k}\Omega$ and $R_5 = 20\,\text{k}\Omega$

Then $300 = \left(\dfrac{100}{20}\right)\left(\dfrac{R_4}{R_3}\right)\left(\dfrac{200}{20}\right) = 50\left(\dfrac{R_4}{R_3}\right)$

Or $\dfrac{R_4}{R_3} = 6$, set $R_3 = 20\,\text{k}\Omega$ and $R_4 = 120\,\text{k}\Omega$

9.19

(a) $\upsilon_O = \dfrac{-R_2}{R_1}\cdot\upsilon_I = -\left(\dfrac{22}{1}\right)(-0.40) = 8.8$ V

(b) $A_\upsilon = \dfrac{-R_2}{R_1}\cdot\dfrac{1}{\left[1+\dfrac{1}{A_{od}}\left(1+\dfrac{R_2}{R_1}\right)\right]} = -(22)\cdot\dfrac{1}{\left[1+\dfrac{1}{5\times10^3}(23)\right]} = -21.8993$

$\upsilon_O = (-21.8993)(-0.40) = 8.7597$ V

(c) $A_\upsilon = -(0.998)(22) = -21.956$

$$-21.956 = -(22)\cdot\frac{1}{\left[1+\dfrac{1}{A_{od}}(23)\right]} \Rightarrow A_{od} = 1.1477\times10^4$$

9.20

(a)

$$A_v = -\frac{R_2}{R_1}\cdot\frac{1}{\left[1+\frac{1}{A_{od}}\left(1+\frac{R_2}{R_1}\right)\right]}$$

$$= -\frac{100}{25}\cdot\frac{1}{\left[1+\frac{1}{5\times10^3}(5)\right]}$$

$$A_v = -3.9960$$

(b) $v_O = -3.9960(1.00) \Rightarrow \underline{v_O = -3.9960\text{ V}}$

(c) $\frac{4-3.9960}{4}\times100\% = \underline{0.10\%}$

(d)

$$v_O = A_{od}\left(v_2 - v_1\right) = -A_{od}v_1$$

$$v_1 = -\frac{v_O}{A_{od}} = \frac{-(-3.9960)}{5\times10^{+3}}$$

$$\underline{v_1 = 0.7992\text{ mV}}$$

9.21

(a) $$A_\upsilon = \frac{-100}{10}\cdot\frac{1}{\left[1+\frac{1}{7\times10^3}\left(1+\frac{100}{10}\right)\right]} = -9.98431$$

$$\upsilon_I = \frac{\upsilon_O}{A_\upsilon} = \frac{7}{-9.9843} = -0.7011\,\text{V}$$

$$\upsilon_1 = \frac{-\upsilon_O}{A_\upsilon} = \frac{-7}{7\times10^3} \Rightarrow \upsilon_1 = -1\,\text{mV}$$

(b) $$A_{od} = \frac{-\upsilon_O}{\upsilon_1} = \frac{-(-5)}{0.2\times10^{-3}} = 2.5\times10^4$$

$$A_\upsilon = (-10)\cdot\frac{1}{\left[1+\frac{1}{2.5\times10^4}(11)\right]} = -9.9956$$

$$\upsilon_I = \frac{\upsilon_O}{A_\upsilon} = \frac{-5}{-9.9956} = 0.50022\text{ V}$$

9.22

(a) $$A_\upsilon = \frac{-R_2}{R_1}\left(1+\frac{R_3}{R_4}+\frac{R_3}{R_2}\right) = \frac{-50}{10}\left(1+\frac{50}{5}+\frac{50}{50}\right) = -60$$

(b) (i) $$-100 = -5\left(1+\frac{50}{R_4}+\frac{50}{50}\right) \Rightarrow R_4 = 2.78\,\text{k}\Omega$$

(ii) $$-150 = -5\left(1+\frac{50}{R_4}+\frac{50}{50}\right) \Rightarrow R_4 = 1.79\,\text{k}\Omega$$

9.23

a.

$$A_v = -\frac{R_2}{R_1}\left(1+\frac{R_3}{R_4}+\frac{R_3}{R_2}\right)$$

$$R_1 = 500\ \text{k}\Omega$$

$$80 = \frac{R_2}{500}\left(1+\frac{R_3}{R_4}+\frac{R_3}{R_2}\right)$$

Set $\underline{R_2 = R_3 = 500\ \text{k}\Omega}$

$$80 = 1\left(1+\frac{500}{R_4}+1\right) = 2+\frac{500}{R_4} \Rightarrow \underline{R_4 = 6.41\ \text{k}\Omega}$$

b.

For $v_I = -0.05$ V

$$i_1 = i_2 = \frac{-0.05}{500\ \text{k}\Omega} \Rightarrow \underline{i_1 = i_2 = -0.1\ \mu\text{A}}$$

$$v_X = -i_2 R_2 = -\left(-0.1\times10^{-6}\right)\left(500\times10^{3}\right) = 0.05$$

$$i_4 = -\frac{v_X}{R_4} = -\frac{0.05}{6.41} \Rightarrow \underline{i_4 = -7.80\ \mu\text{A}}$$

$$i_3 = i_2 + i_4 = -0.1 - 7.80 \Rightarrow \underline{i_3 = -7.90\ \mu\text{A}}$$

9.24

(a)

$$A_v = -1000 = \frac{-R_2}{R_1} = \frac{-500}{R_1}$$

$$\underline{R_1 = 0.5\ \text{K}}$$

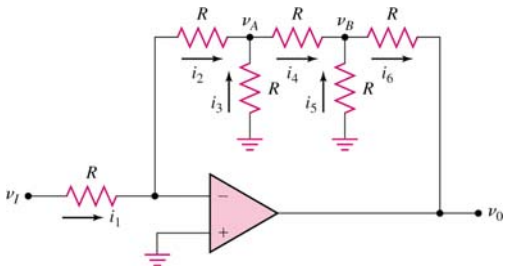
(b)

$$A_v = \frac{-R_2}{R_1}\left(1+\frac{R_3}{R_4}+\frac{R_3}{R_2}\right)$$

$$-1000 = \frac{-250}{R_1}\left(1+\frac{500}{250}+\frac{500}{250}\right) = \frac{-1250}{R_1}$$

$$\underline{R_1 = 1.25\ \text{K}}$$

9.25



$$i_1 = \frac{v_I}{R} = i_2$$

$$v_A = -i_2 R = -\left(\frac{v_I}{R}\right)R = -v_I$$

$$i_3 = -\frac{v_A}{R} = \frac{v_I}{R}$$

$$i_4 = i_2 + i_3 = -\frac{v_A}{R} - \frac{v_A}{R} = -\frac{2v_A}{R} = \frac{2v_I}{R}$$

$$v_B = v_A - i_4 R = -v_I - \left(\frac{2v_I}{R}\right)(R) = -3v_I$$

$$i_5 = -\frac{v_B}{R} = -\frac{(-3v_I)}{R} = \frac{3v_I}{R}$$

$$i_6 = i_4 + i_5 = \frac{2v_I}{R} + \frac{3v_I}{R} = \frac{5v_I}{R}$$

$$v_0 = v_B - i_6 R = -3v_I - \left(\frac{5v_I}{R}\right)R \Rightarrow \underline{\frac{v_0}{v_I} = -8}$$

From Figure 9.12 $\Rightarrow \underline{A_v = -3}$

9.26

(a) $$A_\upsilon = \frac{-R_2}{R_1}\cdot\frac{1}{\left[1+\frac{1}{A_{od}}\left(1+\frac{R_2}{R_1}\right)\right]} = \frac{-200}{20}\cdot\frac{1}{\left[1+\frac{1}{5\times10^4}\left(1+\frac{200}{20}\right)\right]} = -9.9978$$

(b) $$\upsilon_I = \frac{\upsilon_O}{A_\upsilon} = \frac{-4.80}{-9.9978} = 0.4801056\ \text{V}$$

(c) $$\upsilon_1 = \frac{-\upsilon_O}{A_{od}} = \frac{-(-4.80)}{5\times10^4} \Rightarrow \upsilon_1 = 96\,\mu\,\text{V}$$

(d) $$\frac{4.801056 - 4.8}{4.8}\times100\% = 0.022\%$$

9.27

(a) $$A_\upsilon = \frac{-R_2}{R_1}\cdot\frac{1}{\left[1+\frac{1}{A_{od}}\left(1+\frac{R_2}{R_1}\right)\right]} = (-1)\cdot\frac{1}{\left[1+\frac{1}{2.5\times10^3}(2)\right]} = -0.9992$$

(i) $$\upsilon_O = (-0.9992)(-0.8) = 0.7993605\ \text{V}$$

(ii) $$\frac{0.8 - 0.7993605}{0.8}\times100\% \cong 0.08\%$$

(b) $$A_\upsilon = (-1)\cdot\frac{1}{\left[1+\frac{1}{2\times10^2}(2)\right]} = -0.990099$$

(i) $$\upsilon_O = (-0.990099)(-0.8) = 0.79208\ \text{V}$$

(ii) $$\frac{0.8 - 0.79208}{0.9}\times100\% = 0.99\%$$

9.28

$$i_i = \frac{v_I}{R_1} = i_2 = -\frac{v_O}{R_2} \Rightarrow \frac{v_O}{v_I} = -\frac{R_2}{R_1}$$

(a)

(b)

$$i_2 = i_1 = \frac{v_I}{R_1} = i_3 + \frac{v_O}{R_L} = i_3 + \frac{1}{R_L}\left(-\frac{R_2}{R_1}\cdot v_I\right)$$

Then $\underline{i_3 = \frac{v_I}{R_1}\left(1+\frac{R_2}{R_L}\right)}$

9.29

$$V_{X\,\max} = \left(\frac{R_3 \| R_1}{R_3 \| R_1 + R_4}\right)\cdot V^+ = \left(\frac{0.1 \| 1}{0.1 \| 1 + 10}\right)(10) \Rightarrow V_{X\,\max} = 0.09008\ V$$

$$|v_O| = \frac{R_2}{R_1}\cdot V_{X\,\max}$$

$$10 = \frac{R_2}{R_1}(0.09008) \Rightarrow \frac{R_2}{R_1} = 111$$

So $\underline{R_2 = 111\ k\Omega}$

9.30

$$\upsilon_O = -\frac{R_F}{R_1}\cdot\upsilon_{I1} - \frac{R_F}{R_2}\cdot\upsilon_{I2} - \frac{R_F}{R_3}\cdot\upsilon_{I3} = -\frac{120}{40}\cdot\upsilon_{I1} - \frac{120}{20}\cdot\upsilon_{I2} - \frac{120}{60}\cdot\upsilon_{I3}$$

$$\upsilon_O = -3\upsilon_{I1} - 6\upsilon_{I2} - 2\upsilon_{I3}$$

(a) $\upsilon_O = -3(-0.25) - 6(0.10) - 2(1.5) = -2.85\ \text{V}$

(b) $0.5 = -3(\upsilon_{I1}) - 6(0.25) - 2(-1.2) \Rightarrow \upsilon_{I1} = 0.133\ \text{V}$

9.31

(a) $\upsilon_O = -2.5(1.2\upsilon_{I1} + 2.5\upsilon_{I2} + 0.25\upsilon_{I3}) = -3\upsilon_{I1} - 6.25\upsilon_{I2} - 0.625\upsilon_{I3}$

Then $\frac{R_F}{R_1} = 3$, $\frac{R_F}{R_2} = 6.25$, $\frac{R_F}{R_3} = 0.625$

R_3 is the largest resistor, so set $R_3 = 400\ \text{k}\Omega$

Then $R_F = 250\ \text{k}\Omega$, $R_1 = 83.3\ \text{k}\Omega$, $R_2 = 40\ \text{k}\Omega$

(b) $\upsilon_O = -3(-1) - 6.25(0.25) - 0.625(2) = 0.1875\ \text{V}$

$$|i_F| = \frac{\upsilon_O}{R_F} = \frac{0.1875}{250} \Rightarrow |i_F| = 0.75\ \mu\text{A}$$

9.32

$$\upsilon_O = -2(\upsilon_{I1} + 3\upsilon_{I2}) = -2\upsilon_{I1} - 6\upsilon_{I2}$$

Then $\frac{R_F}{R_1} = 2$, $\frac{R_F}{R_2} = 6$

For $\upsilon_{I1} = -1\ \text{V}$, $\upsilon_{I2} = -0.5\ \text{V}$

Then $\upsilon_O = -2(-1) - 6(-0.5) = 5$ V

Set $|i_F| = 80\,\mu\text{A} = \dfrac{\upsilon_O}{R_F} = \dfrac{5}{R_F} \Rightarrow R_F = 62.5\,\text{k}\Omega$

Then $R_1 = 31.25\,\text{k}\Omega$, $R_2 = 10.42\,\text{k}\Omega$

9.33

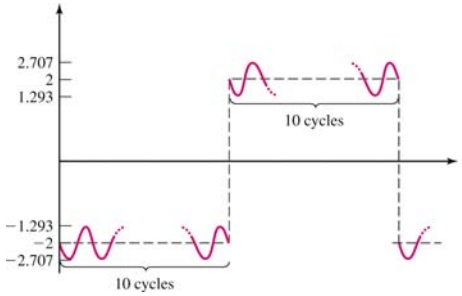
$$v_{I1} = (0.05)\sqrt{2}\sin(2\pi ft) = 0.0707\sin(2\pi ft)$$

$$f = 1\,kHz \Rightarrow T = \frac{1}{10^3} \Rightarrow 1\,ms \quad v_{I2} \Rightarrow T_2 = \frac{1}{100} \Rightarrow 10\,ms$$

$$v_O = -\frac{R_F}{R_1}\cdot v_{I1} - \frac{R_F}{R_2}\cdot v_{I2} = -\frac{10}{1}\cdot v_{I1} - \frac{10}{5}\cdot v_{I2}$$

$$v_O = -(10)\left(0.0707\sin(2\pi ft)\right) - (2)(\pm 1\,V)$$

$$v_O = -0.707\sin(2\pi ft) - (\pm 2\,V)$$



9.34

$$\upsilon_O = -\frac{R_F}{R_1}\cdot\upsilon_{I1} - \frac{R_F}{R_2}\cdot\upsilon_{I2}$$

$$-0.5\sin\omega t = -\frac{100}{R_1}(0.004 + 0.125\sin\omega t) - \frac{100}{R_2}(-0.006)$$

Set $-0.5\sin\omega t = -\dfrac{100}{R_1}(0.125\sin\omega t) \Rightarrow R_1 = 25\,\text{k}\Omega$

We have $0 = -\dfrac{100}{R_1}(0.004) - \dfrac{100}{R_2}(-0.006)$

$$0 = -\frac{0.4}{25} + \frac{0.6}{R_2} \Rightarrow R_2 = 37.5\ \text{k}\Omega$$

9.35

(a) $\upsilon_O = -2\left[\dfrac{\upsilon_{I1}}{4} + 2\upsilon_{I2} + \upsilon_{I3}\right] = -\dfrac{1}{2}\upsilon_{I1} - 4\upsilon_{I2} - 2\upsilon_{I3}$

Then $\dfrac{R_F}{R_1} = \dfrac{1}{2}$, $\dfrac{R_F}{R_2} = 4$, $\dfrac{R_F}{R_3} = 2$

Set $R_1 = 250\,\text{k}\Omega$, Then $R_F = 125\,\text{k}\Omega$, $R_2 = 31.25\,\text{k}\Omega$, $R_3 = 62.5\,\text{k}\Omega$

(b) For $\upsilon_{I1} = -2\,\text{V},\ \upsilon_{I2} = 0,\ \upsilon_{I3} = -1\,\text{V}$

$$\upsilon_O = -\frac{1}{2}(-2) - 4(0) - 2(-1) = 3\,\text{V}$$

For $\upsilon_{I1} = 2\,\text{V},\ \upsilon_{I2} = 0.5\,\text{V},\ \upsilon_{I3} = 0$

$$\upsilon_O = -\frac{1}{2}(2) - 4(0.5) - 2(0) = -3\,\text{V}$$

Then $-3 \le \upsilon_O \le +3\,\text{V}$

$$|i_F|_{\max} = \frac{|\upsilon_O|_{\max}}{R_F} = \frac{3}{125} \Rightarrow |i_F|_{\max} = 24\,\mu\text{A}$$

9.36

$$\upsilon_O = -\frac{R_F}{R_1}\cdot\upsilon_{I1} - \frac{R_F}{R_2}\cdot\upsilon_{I2} - \frac{R_F}{R_3}\cdot\upsilon_{I3}$$

$$-6\sin\omega t = -\frac{R_F}{R_1}(2 + 2\sin\omega t) - \frac{R_F}{R_2}(0.5\sin\omega t) - \frac{R_F}{R_3}(-4)$$

We have $-\frac{R_F}{R_1}(2) + \frac{R_F}{R_3}(4) = 0 \Rightarrow R_3 = 2R_1$

Also $-6 = -\frac{R_F}{R_1}(2) - \frac{R_F}{R_2}(0.5)$

For $\upsilon_O = 6\,\text{V},\ |i_F|_{\max} = 120\,\mu\text{A} = \frac{6}{R_F} \Rightarrow R_F = 50\,\text{k}\Omega$

For $|\upsilon_{I1}|_{\max} = 4\,\text{V},\ |i_1|_{\max} = 120\,\mu\text{A} = \frac{4}{R_1} \Rightarrow R_1 = 33.33\,\text{k}\Omega$ and $R_3 = 2R_1 = 66.66\,\text{k}\Omega$

Now $6 = \frac{(50)(2)}{R_1} + \frac{(50)(0.5)}{R_2} = \frac{100}{R_1} + \frac{25}{R_2} = \frac{100}{33.33} + \frac{25}{R_2} \Rightarrow R_2 = 8.33\,\text{k}\Omega$

9.37

a.

$$v_0 = -\frac{R_F}{R_3}\cdot a_3(-5) - \frac{R_F}{R_2}\cdot a_2(-5) - \frac{R_F}{R_1}\cdot a_1(-5) - \frac{R_F}{R_0}\cdot a_0(-5)$$

So $v_0 = \frac{R_F}{10}\left[\frac{a_3}{2} + \frac{a_2}{4} + \frac{a_1}{8} + \frac{a_0}{16}\right](5)$

b. $v_0 = 2.5 = \frac{R_F}{10}\cdot\frac{1}{2}\cdot 5 \Rightarrow \underline{R_F = 10\ \text{k}\Omega}$

c.

i. $v_0 = \frac{10}{10}\cdot\frac{1}{16}\cdot 5 \Rightarrow \underline{v_0 = 0.3125\ \text{V}}$

ii. $v_0 = \frac{10}{10}\left[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right](5) \Rightarrow \underline{v_0 = 4.6875\ \text{V}}$

9.38

(a) $\upsilon_O = \left(\frac{-10}{1}\right)\left(\frac{-20}{1}\right)\cdot\upsilon_{I1} - \left(\frac{20}{1}\right)\cdot\upsilon_{I2} = 200\upsilon_{I1} - 20\upsilon_{I2}$

(b) $\upsilon_O = (200)(5) - (20)(-25 - 50\sin\omega t) = 1000 + 500 + 1000\sin\omega t$ (mV)

$\upsilon_O = 1.5 + 1.0\sin\omega t$ (V)

(c) For the 20 kΩ resistor:

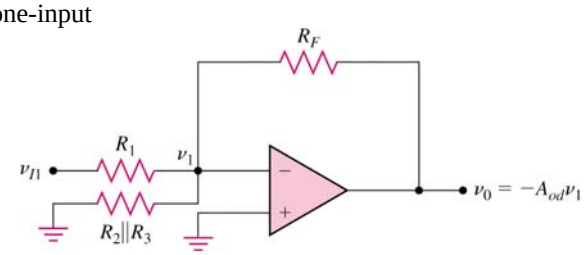
$$|i|_{\max} = \frac{2.5}{20} \Rightarrow |i|_{\max} = 0.125 \text{ mA}$$

For the 10 kΩ resistor:

$$|\upsilon_{O1}| = \left(\frac{10}{1}\right)(5) = 50 \text{ mV}, \quad |i|_{\max} = \frac{50\,mV}{10k\Omega} \Rightarrow |i_{\max}| = 5\,\mu\text{A}$$

9.39

For one-input



$$\upsilon_1 = -\frac{\upsilon_O}{A_{od}}$$

$$\frac{\upsilon_{I1} - \upsilon_1}{R_1} = \frac{\upsilon_1}{R_2 \| R_3} + \frac{\upsilon_1 - \upsilon_O}{R_F}$$

$$\frac{\upsilon_{I1}}{R_1} = \upsilon_1\left[\frac{1}{R_1} + \frac{1}{R_2 \| R_3} + \frac{1}{R_F}\right] - \frac{\upsilon_O}{R_F}$$

$$= -\frac{\upsilon_O}{A_{od}}\left[\frac{1}{R_1} + \frac{1}{R_2 \| R_3} + \frac{1}{R_F}\right] - \frac{\upsilon_O}{R_F}$$

$$= -\upsilon_O\left[\frac{1}{A_{od}R_F} + \frac{1}{R_F} + \frac{1}{A_{od}}\left(\frac{1}{R_1} + \frac{1}{R_2 \| R_3}\right)\right]$$

$$= -\frac{\upsilon_O}{R_F}\left[\frac{1}{A_{od}} + 1 + \frac{1}{A_{od}}\cdot\frac{R_F}{R_1 \| R_2 \| R_3}\right]$$

$$\upsilon_O = -\frac{R_F}{R_1}\cdot\upsilon_{I1}\cdot\frac{1}{\left[1 + \frac{1}{A_{od}}\left(1 + \frac{R_F}{R_P}\right)\right]} \quad \text{where} \quad R_P = R_1 \| R_2 \| R_3$$

Therefore, for three-inputs $\upsilon_O = \dfrac{-1}{1 + \frac{1}{A_{od}}\left(1 + \frac{R_F}{R_P}\right)}\cdot\left(\frac{R_F}{R_1}\cdot\upsilon_{I1} + \frac{R_F}{R_2}\cdot\upsilon_{I2} + \frac{R_F}{R_3}\cdot\upsilon_{I3}\right)$

9.40

(a) $A_\upsilon = \left(1+\dfrac{R_2}{R_1}\right) = \left(1+\dfrac{150}{15}\right) = 11$

(b) $A_\upsilon = \left(1+\dfrac{150}{50}\right) = 4$

(c) $A_\upsilon = \left(1+\dfrac{20}{50}\right) = 1.4$

(d) $A_\upsilon = \left(1+\dfrac{20}{20}\right) = 2$

9.41

(a) $A_\upsilon = 15 = \left(1+\dfrac{R_2}{R_1}\right) \Rightarrow \dfrac{R_2}{R_1} = 14$

For $\upsilon_O = -7.5\text{ V} \Rightarrow \upsilon_1 = -0.5\text{ V}$

$|i| = 120\,\mu\text{A} = \dfrac{7.5-0.5}{R_2} \Rightarrow R_2 = 58.33\,\text{k}\Omega\text{ ,}\quad R_1 = 4.17\,\text{k}\Omega$

(b) $\upsilon_O = (15)(0.25) = 3.75\text{ V}$

$i_1 = i_2 = \dfrac{0.25}{4.17} \Rightarrow i_1 = i_2 = 60\,\mu\text{A}$

9.42

(a) $A_\upsilon = \left(1+\dfrac{R_2}{R_1}\right)$

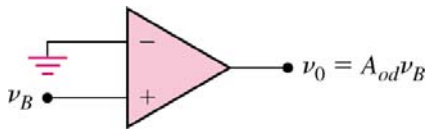
$3 = \left(1+\dfrac{R_2}{R_1}\right) \Rightarrow \dfrac{R_2}{R_1} = 2\text{ ,}\quad \text{Set } R_2 = 290\,\text{k}\Omega\text{ ,}\quad R_1 = 145\,\text{k}\Omega$

(b) $9 = \left(1+\dfrac{R_2}{R_1}\right) \Rightarrow \dfrac{R_2}{R_1} = 8\text{ ,}\quad \text{Set } R_2 = 290\,\text{k}\Omega\text{ ,}\quad R_1 = 36.25\,\text{k}\Omega$

(c) $30 = \left(1+\dfrac{R_2}{R_1}\right) \Rightarrow \dfrac{R_2}{R_1} = 29\text{ , Set } R_2 = 290\,\text{k}\Omega\text{ ,}\quad R_1 = 10\,\text{k}\Omega$

(d) $1 = \left(1+\dfrac{R_2}{R_1}\right) \Rightarrow \dfrac{R_2}{R_1} = 0\text{ , Set } R_2 = 0\text{ ,}\quad R_1 = 290\,\text{k}\Omega$

9.43



$v_B = \left(\dfrac{1}{1+500}\right)v_I \quad v_0 = A_{od}\left(\dfrac{1}{501}\right)v_i$

---

a. $2.5 = A_{od}\left(\frac{1}{501}\right)(5) \Rightarrow \underline{A_{od} = 250.5}$

b. $v_0 = 5000\left(\frac{1}{501}\right)(5) \Rightarrow \underline{v_0 = 49.9\text{ V}}$

9.44

$$v_0 = \left(1+\frac{50}{50}\right)\left[\left(\frac{20}{20+40}\right)v_{I2} + \left(\frac{40}{20+40}\right)v_{I1}\right]$$

$$\underline{v_0 = 1.33v_{I1} + 0.667v_{I2}}$$

9.45

(a)

$$\frac{v_{I1}-v_2}{20} + \frac{v_{I2}-v_2}{40} = \frac{v_2}{10}$$

$$v_O = \left(1+\frac{100}{50}\right)v_2 = 3v_2$$

Now $2v_{I1} - 2v_2 + v_{I2} - v_2 = 4v_2$

$$2v_{I1} + v_{I2} = 7v_2 = 7\left(\frac{v_o}{3}\right)$$

So $\underline{v_O = \frac{6}{7}\cdot v_{I1} + \frac{3}{7}\cdot v_{I2}}$

(b) $v_O = \frac{6}{7}(0.2) + \left(\frac{3}{7}\right)(0.3) \Rightarrow \underline{v_O = 0.3\text{ V}}$

(c) $v_O = \left(\frac{6}{7}\right)(0.25) + \left(\frac{3}{7}\right)(-0.4) \Rightarrow \underline{v_O = 42.86\text{ mV}}$

9.46

(a) $\upsilon_2 = \left(\frac{R_4}{R_3+R_4}\right)\cdot\upsilon_I$

$$\upsilon_O = \left(1+\frac{R_2}{R_1}\right)\cdot\upsilon_2 = \left(1+\frac{R_2}{R_1}\right)\cdot\frac{1}{\left(1+\frac{R_3}{R_4}\right)}\cdot\upsilon_I$$

(b) $6 = \left(1+\frac{R_2}{R_1}\right)\cdot\frac{1}{\left(1+\frac{25}{50}\right)} \Rightarrow \left(1+\frac{R_2}{R_1}\right) = 9 \Rightarrow \frac{R_2}{R_1} = 8$

Set $R_2 = 200\text{ k}\Omega$, $R_1 = 25\text{ k}\Omega$

9.47

(a)

$$\frac{v_O}{v_I} = \left(1 + \frac{50x}{(1-x)50}\right)$$

$$\frac{v_O}{v_I} = \left(1 + \frac{x}{1-x}\right) = \frac{1-x+x}{1-x}$$

$$A_v = \frac{v_O}{v_I} = \frac{1}{1-x}$$

(b) $1 \le A_v \le \infty$

(c) If $x = 1$, gain goes to infinity.

9.48

(a) $\upsilon_X = \left(\frac{\upsilon_I}{R}\right)(2R) + \upsilon_I = 3\upsilon_I$

$$\frac{\upsilon_X - \upsilon_I}{2R} + \frac{\upsilon_X}{R} + \frac{\upsilon_X - \upsilon_O}{2R} = 0$$

$$\upsilon_X\left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{2R}\right) - \frac{\upsilon_I}{2R} = \frac{\upsilon_O}{2R}$$

$$3\upsilon_I\left(\frac{2}{R}\right) - \frac{\upsilon_I}{2R} = \frac{\upsilon_O}{2R}$$

so $\frac{\upsilon_O}{\upsilon_I} = 11$

(b) For $\upsilon_I = 0.25$ V, $\Rightarrow \upsilon_O = 2.75$ V

(c) $R = 30\ \text{k}\Omega$, $\upsilon_I = -0.15$ V

For R_1: $|i| = \frac{0.15}{30} \Rightarrow 5\ \mu$ A

For R_2: $|i| = 5\ \mu$ A

$\upsilon_X = 3\upsilon_I = -0.45$ V

For R_4: $|i| = \frac{0.45}{30} \Rightarrow 15\ \mu$ A

$\upsilon_O = (11)(-0.15) = -1.65$ V

For R_3: $|i| = \frac{1.65 - 0.45}{60} \Rightarrow 20\ \mu$ A

9.49

(a) $\dfrac{v_O}{v_I} = 1$

(b) From Exercise TYU9.7

$$\frac{v_O}{v_I} = \frac{\left(1+\dfrac{R_2}{R_1}\right)}{\left[1+\dfrac{1}{A_{od}}\left(1+\dfrac{R_2}{R_1}\right)\right]}$$

But $R_2 = 0,\ R_1 = \infty$

$$\frac{v_O}{v_I} = \frac{1}{1+\dfrac{1}{A_{od}}} = \frac{1}{1+\dfrac{1}{1.5\times 10^5}} \Rightarrow \underline{\frac{v_O}{v_I} = 0.999993}$$

Want $\dfrac{v_O}{v_I} = 0.990 = \dfrac{1}{1+\dfrac{1}{A_{od}}} \Rightarrow \underline{A_{od} = 99}$

(b)

9.50

(a) $\upsilon_O = A_{od}(\upsilon_2 - \upsilon_1) = A_{od}(\upsilon_I - \upsilon_O)$

$\upsilon_O(1 + A_{od}) = A_{od}\upsilon_I$

$$A_\upsilon = \frac{\upsilon_O}{\upsilon_I} = \frac{1}{1+\dfrac{1}{A_{od}}} = \frac{1}{1+\dfrac{1}{20}} = 0.9524$$

(b) $$A_\upsilon = \frac{1}{1+\dfrac{1}{200}} = 0.995$$

(c) $$A_\upsilon = \frac{1}{1+\dfrac{1}{2000}} = 0.9995$$

(d) $$A_\upsilon = \frac{1}{1+\dfrac{1}{20000}} = 0.99995$$

9.51

(a) $$A_{\upsilon 1} = \frac{\upsilon_{O1}}{\upsilon_I} = \left(1+\frac{R_2}{R_1}\right)$$

$$A_{\upsilon 2} = \frac{\upsilon_{O2}}{\upsilon_I} = -\left(1+\frac{R_2}{R_1}\right)$$

(b) $$\upsilon_{O1} = \left(1+\frac{60}{20}\right)(-0.5) = -2\ \text{V}$$

$$\upsilon_{O2} = -\left(1+\frac{60}{20}\right)(-0.5) = 2\ \text{V}$$

(c) $\upsilon_{O1} - \upsilon_{O2} = 2\left(1 + \frac{R_2}{R_1}\right) \cdot \upsilon_I = 2\left(1 + \frac{60}{20}\right)(0.8) = 6.4 \text{ V}$

9.52

(a) $i_L = \frac{v_I}{R_1}$

(b)

$v_{O1} = i_L R_L + v_I = i_L R_L + i_L R_1$

$v_{OI}(\max) \cong 10 \text{ V} = i_L(1+9) = 10 i_L$

So $\underline{i_L(\max) \cong 1 \text{ mA}}$

Then $v_I(\max) \cong i_L R_1 = (1)(9) \Rightarrow \underline{v_I(\max) \cong 9 \text{ V}}$

9.53

(a) $\upsilon_O = \left(\frac{20}{20+40}\right) \cdot \upsilon_I = (0.3333)\upsilon_I$

(i) $\upsilon_O = 1 \text{ V}$

(ii) $\upsilon_O = -1.67 \text{ V}$

(b) $\upsilon_O = \left(\frac{20}{20+40}\right) \cdot \upsilon_I = (0.3333)\upsilon_I$

(i) $\upsilon_O = 1 \text{ V}$

(ii) $\upsilon_O = -1.67 \text{ V}$

(c) $\upsilon_O = \left(1 + \frac{10}{10}\right)\left(\frac{6}{6+48}\right) \cdot \upsilon_I = (0.2222)\upsilon_I$

(i) $\upsilon_O = 0.667 \text{ V}$

(ii) $\upsilon_O = -1.111 \text{ V}$

9.54

a.

$R_{in} = \frac{v_1}{i_1}$ and $\frac{v_1 - v_0}{R_F} = i_1$ and $v_0 = -A_{od} v_1$

So $i_1 = \frac{v_1 - (-A_{od} v_1)}{R_F} = \frac{v_1(1 + A_{od})}{R_F}$

Then $\underline{R_{in} = \frac{v_1}{i_1} = \frac{R_F}{1 + A_{od}}}$

b.

$$i_1 = \left(\frac{R_S}{R_S + R_{in}}\right) i_S \text{ and } v_0 = -A_{od} \cdot \frac{R_F}{1 + A_{od}} \cdot i_1$$

$$\text{So } v_0 = -R_F \left(\frac{A_{od}}{1 + A_{od}}\right)\left(\frac{R_S}{R_S + R_{in}}\right) i_S$$

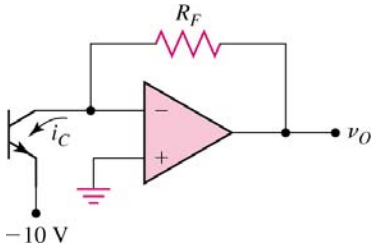
$$R_{in} = \frac{R_F}{1 + A_{od}} = \frac{10}{1001} = 0.009990$$

$$v_0 = -R_F \left(\frac{1000}{1001}\right)\left(\frac{R_S}{R_S + 0.009990}\right) i_S$$

$$\text{Want } \left(\frac{1000}{1001}\right)\left(\frac{R_S}{R_S + 0.009990}\right) \le 0.990$$

which yields $\underline{R_S \ge 1.099 \text{ k}\Omega}$

9.55



$$v_O = i_C R_F, \quad 0 \le i_C \le 8 \text{ mA}$$

For $v_O(\max) = 8$ V, Then $\underline{R_F = 1\ k\Omega}$

9.56

$$i = \frac{v_I}{R} \text{ so } 1 = \frac{10}{R} \Rightarrow \underline{R = 10 \text{ k}\Omega}$$

In the ideal op-amp, R_1 has no influence.

Output voltage: $$v_0 = \left(1 + \frac{R_2}{R}\right) v_I$$

v_0 must remain within the bias voltages of the op-amp; the larger the R_2, the smaller the range of input voltage v_I in which the output is valid.

9.57

(a) $i_L = \dfrac{-\upsilon_I}{R_2}$; $R_2 = \dfrac{-(-5)}{5} = 1\,\text{k}\Omega$

Set $\dfrac{R_F}{R_1 R_3} = \dfrac{1}{R_2}$; For example, set $R_1 = R_F = 10\,\text{k}\Omega$, $R_3 = 1\,\text{k}\Omega$

(b) $\upsilon_L = (5)(0.2) = 1\,\text{V} = \upsilon_1$

$$i_1 = i_2 = \frac{\upsilon_I - \upsilon_1}{R_1} = \frac{-5-1}{10} = -0.6 \text{ mA}$$

$$\upsilon_O = \upsilon_1 - i_2 R_F = 1 - (-0.6)(10) = 7 \text{ V}$$

$$i_3 = \frac{\upsilon_O - \upsilon_L}{R_3} = \frac{7-1}{1} = 6 \text{ mA}$$

$$i_4 = \frac{\upsilon_L}{R_2} = \frac{1}{1} = 1 \text{mA}$$

For the op-amp: $i_O + i_2 = i_3 \Rightarrow i_O = i_3 - i_2 = 6 - (-0.6) = 6.6$ mA

9.58

(a)

$i_1 = i_2$ and $i_2 = \frac{v_x}{R_2} + i_D, \quad v_x = -i_2 R_F$

Then $i_1 = -i_1\left(\frac{R_F}{R_2}\right) + i_D$

Or $\underline{i_D = i_1\left(1 + \frac{R_F}{R_2}\right)}$

(b)

$R_1 = \frac{v_I}{i_1} = \frac{5}{1} \Rightarrow \underline{R_1 = 5 \text{ k}\Omega}$

$12 = (1)\left(1 + \frac{R_F}{R_2}\right) \Rightarrow \underline{\frac{R_F}{R_2} = 11}$

For example, $\underline{R_2 = 5 \text{ k}\Omega, \quad R_F = 55 \text{ k}\Omega}$

9.59

(1) $I_X = \frac{V_X}{R_2} + \frac{V_X - v_O}{R_3}$

(2) $\frac{V_X}{R_1} + \frac{V_X - v_O}{R_F} = 0$

From (2) $v_O = V_X\left(1 + \frac{R_F}{R_1}\right)$

Then (1) $I_X = V_X\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{1}{R_3} \cdot V_X\left(1 + \frac{R_F}{R_1}\right)$

$$\frac{I_X}{V_X} = \frac{1}{R_0} = \frac{1}{R_2} + \frac{1}{R_3} - \frac{1}{R_3} - \frac{R_F}{R_1 R_3} = \frac{1}{R_2} - \frac{R_F}{R_1 R_3}$$

$$= \frac{R_1 R_3 - R_2 R_F}{R_1 R_2 R_3}$$

or $\underline{R_o = \frac{R_1 R_2 R_3}{R_1 R_3 - R_2 R_F}}$

Note: If $\frac{R_F}{R_1 R_3} = \frac{1}{R_2} \Rightarrow R_2 R_F = R_1 R_3$ then $R_o = \infty,$ which corresponds to an ideal current source.

9.60

(a) $R_{id} = R_1 + R_3 = 30\,\text{k}\Omega$; $R_1 = R_3 = 15\,\text{k}\Omega$

$$\frac{R_2}{R_1} = \frac{R_4}{R_3} = 15 \Rightarrow R_2 = R_4 = 225\,\text{k}\Omega$$

(b) $\upsilon_O = i_L R_L = (0.25)(10) = 2.5\,\text{V}$

$$\upsilon_{I2} - \upsilon_{I1} = \frac{\upsilon_O}{A_d} = \frac{2.5}{15} = 0.1667\,\text{V}$$

(c) $\upsilon_O = A_d(\upsilon_{I2} - \upsilon_{I1}) = 15(1.2 - 1.5) = -4.5\,\text{V}$

$$i_L = \frac{\upsilon_O}{R_L} = \frac{-4.5}{10} = -0.45\,\text{mA}$$

(d) $\upsilon_O = (0.5)(10) = 5\,\text{V}$

$$\upsilon_{I2} - \upsilon_{I1} = \frac{\upsilon_O}{A_d} = \frac{5}{15} = 0.333\,\text{V}$$

$$\upsilon_{I1} = 2 - 0.333 = 1.667\,\text{V}$$

9.61

(a) $\dfrac{R_2}{R_1} = \dfrac{R_4}{R_3} = 40$; Set $R_2 = R_4 = 250\,\text{k}\Omega$, $R_1 = R_3 = 6.25\,\text{k}\Omega$

(b) $\dfrac{R_2}{R_1} = \dfrac{R_4}{R_3} = 25$; Set $R_2 = R_4 = 250\,\text{k}\Omega$, $R_1 = R_3 = 10\,\text{k}\Omega$

(c) $\dfrac{R_2}{R_1} = \dfrac{R_4}{R_3} = 5$; Set $R_2 = R_4 = 250\,\text{k}\Omega$, $R_1 = R_3 = 50\,\text{k}\Omega$

(d) $\dfrac{R_2}{R_1} = \dfrac{R_4}{R_3} = 0.5$; Set $R_2 = R_4 = 125\,\text{k}\Omega$, $R_1 = R_3 = 250\,\text{k}\Omega$

9.62

We have

$$v_O = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4/R_3}{1 + R_4/R_3}\right)v_{I2} - \left(\frac{R_2}{R_1}\right)v_{I1} \text{ or } v_O = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{1 + R_3/R_4}\right)v_{I2} - \left(\frac{R_2}{R_1}\right)v_{I1}$$

Set $R_2 = 50(1+x)$, $R_1 = 50(1-x)$

$R_3 = 50(1-x)$, $R_4 = 50(1+x)$

$$v_O = \left[1 + \left(\frac{1+x}{1-x}\right)\right]\left[\frac{1}{1 + \left(\frac{1-x}{1+x}\right)}\right]v_{I2} - \left(\frac{1+x}{1-x}\right)v_{I1}$$

$$v_O = \left[\frac{1-x+(1+x)}{1-x}\right]\cdot\left[\frac{1+x}{1+x+(1-x)}\right]v_{I2} - \left(\frac{1+x}{1-x}\right)v_{I1}$$

$$= \left(\frac{1+x}{1-x}\right)v_{I2} - \left(\frac{1+x}{1-x}\right)v_{I1}$$

For $v_{I1} = v_{I2} \Rightarrow v_O = 0$

Set $R_2 = 50(1+x)$ $R_1 = 50(1-x)$

$R_3 = 50(1+x)$ $R_4 = 50(1-x)$

$$v_O = \left(1+\frac{1+x}{1-x}\right)\left(\frac{1}{1+\frac{1+x}{1-x}}\right)v_{I2} - \left(\frac{1+x}{1-x}\right)v_{I1}$$

$$= v_{I2} - \left(\frac{1+x}{1-x}\right)v_{I1}$$

$v_{I1} = v_{I2} = v_{cm}$

$$\frac{v_O}{v_{cm}} = 1 - \frac{1+x}{1-x} = \frac{1-x-(1+x)}{1-x} = \frac{-2x}{1-x}$$

Set $R_2 = 50(1-x)$ $R_1 = 50(1+x)$

$R_3 = 50(1-x)$ $R_4 = 50(1+x)$

$$v_O = \left(1+\frac{1-x}{1+x}\right)\left(\frac{1}{1+\frac{1-x}{1+x}}\right)v_{I2} - \left(\frac{1-x}{1+x}\right)v_{I1}$$

$$= \left(1-\frac{1-x}{1+x}\right)v_{cm}$$

$$A_{cm} = \frac{1+x-(1-x)}{1+x} = \frac{2x}{1+x}$$

Worst common-mode gain

$$A_{cm} = \frac{-2x}{1-x}$$

(b)

For $x = 0.01$, $A_{cm} = \dfrac{-2x}{1-x} = \dfrac{-2(0.01)}{1-0.01} = -0.0202$

For $x = 0.02$, $A_{cm} = \dfrac{-2(0.02)}{1-0.02} = -0.04082$

For $x = 0.05$, $A_{cm} = \dfrac{-2(0.05)}{1-0.05} = -0.1053$

For this condition, set $v_{I2} = +\dfrac{1}{2}$, $v_{I1} = -\dfrac{1}{2} \Rightarrow v_d = 1$ V

$$A_d = \frac{1}{2}\left[1+\left(\frac{1+x}{1-x}\right)\right] = \frac{1}{2}\left[\frac{1-x+(1+x)}{1-x}\right] = \frac{1}{2}\cdot\frac{2}{1-x} = \frac{1}{1-x}$$

For $x = 0.01$ $A_d = 1.010$ $\quad CMRR_{dB} = 20\log_{10}\left|\dfrac{1.010}{0.0202}\right| = 33.98$ dB

For $x = 0.02$, $A_d = \dfrac{1}{0.98} = 1.020$ $\quad CMRR_{dB} = 20\log_{10}\left|\dfrac{1.020}{0.04082}\right| = 27.96$ dB

For $x = 0.05$ $A_d = \dfrac{1}{0.95} = 1.0526$ $\quad CMRR_{dB} = 20\log_{10}\left|\dfrac{1.0526}{0.1053}\right| \cong 20$ dB

9.63

(a) $\upsilon_O = 10(\upsilon_2 - \upsilon_1) = 10(1.4 - 1.8) = -4\text{ V}$

$$i_3 = i_4 = \frac{\upsilon_2}{R + 10R} = \frac{1.4}{(11)(10)} = 0.0127\text{ mA}$$

$$\upsilon_X = \upsilon_Y = \left(\frac{10}{11}\right)\cdot\upsilon_2 = \left(\frac{10}{11}\right)(1.4) = 1.273\text{ V}$$

$$i_1 = i_2 = \frac{\upsilon_1 - \upsilon_X}{R} = \frac{1.8 - 1.273}{10} = 0.0527\text{ mA}$$

(b) $\upsilon_O = 10(\upsilon_2 - \upsilon_1) = 10(3.6 - 3.2) = 4\text{ V}$

$$\upsilon_X = \upsilon_Y = \left(\frac{10}{11}\right)(3.6) = 3.273\text{ V}$$

$$i_3 = i_4 = \frac{\upsilon_2}{(11)(10)} = \frac{3.6}{110} = 0.0327\text{ mA}$$

$$i_1 = i_2 = \frac{3.2 - 3.273}{10} = -0.00727\text{ mA}$$

(c) $\upsilon_O = 10(-1.35 - (-1.20)) = -1.5\text{ V}$

$$i_3 = i_4 = \frac{-1.35}{(11)(10)} = -0.0123\,\text{mA}$$

$$\upsilon_X = \upsilon_Y = \left(\frac{10}{11}\right)(-1.35) = -1.227\text{ V}$$

$$i_1 = i_2 = \frac{-1.2 - (-1.227)}{10} = 0.00273\,\text{mA}$$

9.64

(a) $I_E = (1+\beta)\cdot I_B = (76)(1.2) = 91.2\text{ mA}$

$$R = \frac{10}{91.2} \Rightarrow R = 109.6\,\Omega$$

(b) $I_E = (101)(0.2) = 20.2\text{ mA}$

$$R = \frac{10}{20.2} = 0.495\,\text{k}\Omega$$

(c) $I_E = \dfrac{6}{0.1096} = 54.74\text{ mA}$

$$I_O = \frac{54.74}{76} = 0.72\text{ mA}$$

(d) $I_E = \dfrac{4}{0.495} = 8.08\text{ mA}$

$$I_O = \frac{8.08}{101} = 0.080\text{ mA}$$

9.65

(a) $\upsilon_{O1} = \dfrac{-R_2}{R_1}\cdot\upsilon_{CM}$

$$\upsilon_{O2} = \left(\frac{R_4}{R_3+R_4}\right)\cdot\upsilon_{CM}\cdot\left(1+\frac{R_2}{R_1}\right)$$

$$\upsilon_O = \upsilon_{O1}+\upsilon_{O2} = \left[\frac{\dfrac{R_4}{R_3}}{1+\dfrac{R_4}{R_3}}\cdot\left(1+\frac{R_2}{R_1}\right)-\frac{R_2}{R_1}\right]\cdot\upsilon_{CM}$$

$$A_{CM} = \frac{\upsilon_O}{\upsilon_{CM}} = \frac{\left(\dfrac{R_4}{R_3}\right)\left(1+\dfrac{R_2}{R_1}\right)-\left(\dfrac{R_2}{R_1}\right)\left(1+\dfrac{R_4}{R_3}\right)}{\left(1+\dfrac{R_4}{R_3}\right)} = \frac{\dfrac{R_4}{R_3}-\dfrac{R_2}{R_1}}{1+\dfrac{R_4}{R_3}}$$

(b) $$A_{CM} = \frac{\dfrac{86.4}{9.6}-\dfrac{62.4}{10.4}}{1+\dfrac{86.4}{9.6}} = \frac{9-6}{10} = 0.3$$

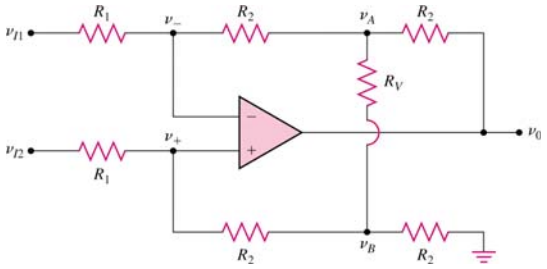
(c) $$A_{CM} = \frac{\dfrac{80.8}{19.8}-\dfrac{79.2}{20.2}}{1+\dfrac{80.8}{19.8}} = 0.03149$$

or

$$A_{CM} = \frac{\dfrac{79.2}{20.2}-\dfrac{80.8}{19.8}}{1+\dfrac{79.2}{20.2}} = -0.0325$$

$$\Rightarrow \left|A_{CM}\right|_{\max} = 0.0325$$

9.66



$$\frac{v_{I1}-v_A}{R_1+R_2}=\frac{v_A-v_B}{R_v}+\frac{v_A-v_0}{R_2} \qquad (1)$$

$$\frac{v_{I2}-v_B}{R_1+R_2}=\frac{v_B-v_A}{R_v}+\frac{v_B}{R_2} \qquad (2)$$

$$v_-=\left(\frac{R_1}{R_1+R_2}\right)v_A+\left(\frac{R_2}{R_1+R_2}\right)v_{I1} \qquad (3)$$

$$v_+=\left(\frac{R_1}{R_1+R_2}\right)v_B+\left(\frac{R_2}{R_1+R_2}\right)v_{I2} \qquad (4)$$

Now $v_-=v_+ \Rightarrow R_1v_A+R_2v_{I1}=R_1v_B+R_2v_{I2}$

So that $v_A=v_B+\frac{R_2}{R_1}\left(v_{I2}-v_{I1}\right)$

$$\frac{v_{I1}}{R_1+R_2}=v_A\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)-\frac{v_B}{R_V}-\frac{v_0}{R_2} \qquad (1)$$

$$\frac{v_{I2}}{R_1+R_2}=v_B\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)-\frac{v_A}{R_V} \qquad (2)$$

Then

$$\frac{v_{I1}}{R_1+R_2}=v_B\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)-\frac{v_B}{R_V}-\frac{v_0}{R_2}+\left(\frac{R_2}{R_1}\right)\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)\left(v_{I2}-v_{I1}\right) \qquad (1)$$

$$\frac{v_{I2}}{R_1+R_2}=v_B\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)-\frac{1}{R_V}\left[v_B+\frac{R_2}{R_1}\left(v_{I2}-v_{I1}\right)\right] \qquad (2)$$

Subtract (2) from (1)

$$\frac{1}{R_1+R_2}\left(v_{I1}-v_{I2}\right)=\left(\frac{R_2}{R_1}\right)\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)\left(v_{I2}-v_{I1}\right)-\frac{v_0}{R_2}+\frac{1}{R_V}\cdot\frac{R_2}{R_1}\left(v_{I2}-v_{I1}\right)$$

$$\frac{v_0}{R_2}=\left(v_{I2}-v_{I1}\right)\left\{\left(\frac{R_2}{R_1}\right)\left(\frac{1}{R_1+R_2}+\frac{1}{R_V}+\frac{1}{R_2}\right)+\frac{1}{R_1+R_2}+\frac{1}{R_V}\cdot\frac{R_2}{R_1}\right\}$$

$$v_0=\left(v_{I2}-v_{I1}\right)\left(\frac{R_2}{R_1}\right)\left\{\frac{R_2}{R_1+R_2}+\frac{R_2}{R_V}+1+\frac{R_1}{R_1+R_2}+\frac{R_2}{R_V}\right\}$$

$$\underline{v_0=\frac{2R_2}{R_1}\left(1+\frac{R_2}{R_V}\right)\left(v_{I2}-v_{I1}\right)}$$

9.67

(a) $i_1=\dfrac{\upsilon_{I1}-\upsilon_{I2}}{R_1}=\dfrac{\left(1.2-0.08\sin\omega t\right)-\left(1.2+0.08\sin\omega t\right)}{10}\Rightarrow i_1=-16\sin\omega t\ \ (\mu\text{ A})$

$\upsilon_{O1}=\left(1.2-0.08\sin\omega t\right)-\left(0.016\sin\omega t\right)(40)=1.2-0.72\sin\omega t\ \ (\text{V})$

$\upsilon_{O2}=\left(1.2+0.08\sin\omega t\right)-\left(-0.016\sin\omega t\right)(40)=1.2+0.72\sin\omega t\ \ (\text{V})$

$\upsilon_O=\dfrac{R_4}{R_3}\left(\upsilon_{O2}-\upsilon_{O1}\right)=\left(\dfrac{120}{40}\right)(2)(0.72\sin\omega t)=4.32\sin\omega t\ \ (\text{V})$

(b) $i_i = \dfrac{\left(-0.65+0.05\sin\omega t\right)-\left(-0.60-0.05\sin\omega t\right)}{10}$

$i_1 = -5+10\sin\omega t \ \ (\mu\text{ A})$

$\upsilon_{O1} = -0.65+0.05\sin\omega t+(40)\left(-0.005+0.010\sin\omega t\right) = -0.85+0.45\sin\omega t \ \text{(V)}$

$\upsilon_{O2} = -0.60-0.05\sin\omega t-(40)\left(-0.005+0.01\sin\omega t\right) = -0.40-0.45\sin\omega t \ \text{(V)}$

$\upsilon_O = \left(\dfrac{120}{40}\right)\left[\left(-0.40-0.45\sin\omega t\right)-\left(-0.85+0.45\sin\omega t\right)\right] = 1.35-2.7\sin\omega t \ \text{(V)}$

9.68

(a) $v_{OB} = \left(1+\dfrac{40}{12}\right)v_I = 2.1667\sin\omega t$

(b) $v_{OC} = -\dfrac{30}{12}v_I = -1.25\sin\omega t$

(c)

$v_O = v_{OB}-v_{OC} = 2.1667\sin\omega t-\left(-1.25\sin\omega t\right)$

$v_O = 3.417\sin\omega t$

(d) $\dfrac{v_O}{v_I} = \dfrac{3.417}{0.5} = 6.83$

9.69

(a) $i_O = \dfrac{\upsilon_{I1}-\upsilon_{I2}}{R}$

(b) $R = \dfrac{0.25-\left(-0.25\right)}{5} \Rightarrow R = 100\,\Omega$

(c) $\upsilon_{O1} = \upsilon_{I1}+i_O R_L = 0.25+(5)(1) = 5.25\text{ V}$

$\upsilon_{O2} = \upsilon_{I2} = -0.25\text{ V}$

(d) $i_O = \dfrac{\upsilon_{I1}-\upsilon_{I2}}{R} = \dfrac{1.25-1.75}{0.5} = -1\text{ mA}$

$\upsilon_{O1} = \upsilon_{I1}+i_O R_L = 1.25-(1)(3) = -1.75\text{ V}$

$\upsilon_{O2} = \upsilon_{I2} = 1.75\text{ V}$

9.70

$A_d = \dfrac{v_O}{v_{I2}-v_{I1}} = \dfrac{R_4}{R_3}\left(1+\dfrac{2R_2}{R_1}\right)$

$\left|v_O\right| = \dfrac{200}{50}\left(1+\dfrac{2(115)}{R_1}\right)\left(0.06\sin\omega t\right)$

For $\left|v_O\right| = 0.5 \quad \dfrac{230}{R_1} = 1.0833 \Rightarrow R_1 = 212.3\text{ K}$

$\left|v_O\right| = 8\text{ V} \qquad \dfrac{230}{R_1} = 32.33 \Rightarrow R_1 = 7.11\text{ K} \Rightarrow R_{1f} = 7.11\text{ K}$, R_1(potentiometer) = 205.2 K

9.71

For $\upsilon_O = 10$ V, $A_d = 200 \Rightarrow \upsilon_{I2} - \upsilon_{I1} = \dfrac{10}{200} = 0.05$ V

$$R_1(\textit{fixed}) = \frac{0.05}{50\times10^{-6}} \Rightarrow R_1(\textit{fixed}) = 1\,\text{k}\Omega$$

$$A_d = \frac{R_4}{R_3}\left(1+\frac{2R_2}{R_1}\right)$$

$$200 = (2.5)\left(1+\frac{2R_2}{1}\right) \Rightarrow R_2 = 39.5\,\text{k}\Omega$$

For $A_d = 5$

$$5 = (2.5)\left[1+\frac{2(39.5)}{R_1}\right] \Rightarrow R_1 = 79 = R_1(\text{var}) + R_1(\textit{fixed}) = R_1(\text{var}) + 1$$

$$R_1(\text{var}) = 78\,\text{k}\Omega$$

9.72

$$\upsilon_O(\upsilon_{O1}) = -\frac{R_4}{R_3'}\cdot\upsilon_{O1}$$

$$\upsilon_O(\upsilon_{O2}) = \left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_4}{R_3'}\right)\cdot\upsilon_{O2}$$

$\upsilon_O = \upsilon_O(\upsilon_{O1}) + \upsilon_O(\upsilon_{O2})$ and $\upsilon_{O1} = \upsilon_{O2} \equiv \upsilon_{CM}$

Then

$$A_{CM} = \frac{\upsilon_O}{\upsilon_{CM}} = \left(\frac{R_4}{R_3+R_4}\right)\left(1+\frac{R_4}{R_3'}\right) - \left(\frac{R_4}{R_3'}\right)$$

$R_4 = 2R_3 = 60\,\text{k}\Omega$, $R_3 = 30\,\text{k}\Omega$, $R_3' = 30\,\text{k}\Omega \pm 5\%$

For $R_3' = 30\,\text{k}\Omega - 5\% = 28.5\,\text{k}\Omega$

$$A_{CM} = \left(\frac{60}{60+30}\right)\left(1+\frac{60}{28.5}\right) - \left(\frac{60}{28.5}\right) = -0.03509$$

For $R_3' = 30\,\text{k}\Omega + 5\% = 31.5\,\text{k}\Omega$

$$A_{CM} = \left(\frac{60}{60+30}\right)\left(1+\frac{60}{31.5}\right) - \left(\frac{60}{31.5}\right) = +0.03175$$

Then $-0.03509 \le A_{CM} \le +0.03175$

9.73

(a) $R_1C_2 = (20\times10^3)(0.02\times10^{-6}) = 4\times10^{-4}$ s

$$\upsilon_O = \frac{-1}{4\times10^{-4}}\int(0.25)\cos\omega t dt = \frac{-0.25}{(4\times10^{-4})\omega}\cdot\sin\omega t$$

For $|\upsilon_O| = 0.25 \Rightarrow (4\times10^{-4})(2\pi f) = 1 \Rightarrow f = 398$ Hz

Phase $= 90°$

(b) (i) $|\upsilon_O| = 1.5 = \dfrac{0.25}{2\pi f\left(4\times10^{-4}\right)} \Rightarrow f = 66.3\text{ Hz}$

(ii) $|\upsilon_O| = 0.15 = \dfrac{0.25}{2\pi f\left(4\times10^{-4}\right)} \Rightarrow f = 663\text{ Hz}$

9.74

(a) $\upsilon_O = \dfrac{-1}{R_1C_2}\displaystyle\int_0^t \upsilon_I(t')dt' = \dfrac{-1}{R_1C_2}(0.25)t'\Big|_0^{1.2}$

$-5 = \dfrac{-(0.25)(1.2)}{R_1C_2} \Rightarrow R_1C_2 = 60\text{ ms}$

(b) (i) $0 = -5 + \dfrac{(0.10)}{0.06}\cdot t'' \Rightarrow t'' = 3\text{ s},\quad t = 4.2\text{ s}$

(ii) $5 = -5 + \dfrac{(0.10)}{0.06}\cdot t'' \Rightarrow t'' = 6\text{ s},\quad t = 7.2\text{ s}$

9.75

(a) $A_\upsilon = \dfrac{-Z_2}{R_1}$, where $Z_2 = R_2 \left\| \dfrac{1}{j\omega C_2} = \dfrac{R_2\left(\dfrac{1}{j\omega C_2}\right)}{R_2 + \dfrac{1}{j\omega C_2}} = \dfrac{R_2}{1 + j\omega R_2C_2}\right.$

$A_\upsilon = \dfrac{-R_2}{R_1}\cdot\dfrac{1}{1 + j\omega R_2C_2}$

(b) At $\omega = 0$, $A_\upsilon(0) = \dfrac{-R_2}{R_1}$

(c) $|A_\upsilon| = \dfrac{R_2}{R_1}\cdot\dfrac{1}{\sqrt{1+(\omega R_2C_2)^2}}$

Set $1+(\omega R_2C_2)^2 = 2 \Rightarrow \omega = \dfrac{1}{R_2C_2} \Rightarrow f = \dfrac{1}{2\pi R_2C_2}$

9.76

(a) $R_1 = 20\text{ k}\Omega$

$\dfrac{R_2}{R_1} = 15 \Rightarrow R_2 = 300\text{ k}\Omega$

$\omega = \dfrac{1}{R_2C_2} = 2\pi f$

$C_2 = \dfrac{1}{2\pi fR_2} = \dfrac{1}{2\pi\left(5\times10^3\right)\left(300\times10^3\right)} \Rightarrow C_2 = 106\text{ pF}$

(b) $R_1 = 15\,\text{k}\Omega$

$$\frac{R_2}{R_1} = 25 \Rightarrow R_2 = 375\,\text{k}\Omega$$

$$C_2 = \frac{1}{2\pi\left(15\times10^3\right)\left(375\times10^3\right)} \Rightarrow C_2 = 28.3\,\text{pF}$$

9.77

(a) $A_v = \dfrac{-R_2}{Z_1}$, where $Z_1 = R_1 + \dfrac{1}{j\omega C_1} = \dfrac{1+ j\omega R_1 C_1}{j\omega C_1}$

$$A_v = \frac{-j\omega R_2 C_1}{1+ j\omega R_1 C_1} = \frac{-R_2}{R_1}\cdot\frac{j\omega R_1 C_1}{1+ j\omega R_1 C_1}$$

(b) As $\omega \Rightarrow \infty$, $A_v = \dfrac{-R_2}{R_1}$

(c) $\left|A_v\right| = \dfrac{R_2}{R_1}\cdot\dfrac{\omega R_1 C_1}{\sqrt{1+\left(\omega R_1 C_1\right)^2}}$

Set $\dfrac{\omega R_1 C_1}{\sqrt{1+\left(\omega R_1 C_1\right)^2}} = \dfrac{1}{\sqrt{2}} \Rightarrow \omega = \dfrac{1}{R_1 C_1} \Rightarrow f = \dfrac{1}{2\pi R_1 C_1}$

9.78

(a) Set $R_2 = 350\,\text{k}\Omega$

$$\frac{R_2}{R_1} = 15 \Rightarrow R_1 = 23.33\,\text{k}\Omega$$

$$2\pi f = \frac{1}{R_1 C_1} \Rightarrow C_1 = \frac{1}{2\pi R_1 f} = \frac{1}{2\pi\left(23.33\times10^3\right)\left(20\times10^3\right)} \Rightarrow C_1 = 341\,\text{pF}$$

(b) Set $R_1 = 20\,\text{k}\Omega$

$$\frac{R_2}{R_1} = 25 \Rightarrow R_2 = 500\,\text{k}\Omega$$

$$C_1 = \frac{1}{2\pi\left(20\times10^3\right)\left(35\times10^3\right)} \Rightarrow C_1 = 227\,\text{pF}$$

9.79

Assuming the Zener diode is in breakdown,

$$v_O = -\frac{R_2}{R_1}\cdot V_z = -\frac{1}{1}(6.8) \Rightarrow v_O = -6.8\ V$$

$$i_2 = \frac{0 - v_O}{R_2} = \frac{0-(-6.8)}{1} \Rightarrow i_2 = 6.8\ mA$$

$$i_z = \frac{10 - V_z}{R_s} - i_2 = \frac{10-6.8}{5.6} - 6.8 \Rightarrow i_z = -6.2\ mA\text{!!!}$$

Circuit is not in breakdown. Now

$$\frac{10-0}{R_s+R_1}=i_2=\frac{10}{5.6+1}\Rightarrow \underline{i_2=1.52\ mA}$$

$$v_O=-i_2R_2=-(1.52)(1)\Rightarrow \underline{v_O=-1.52\ V}$$

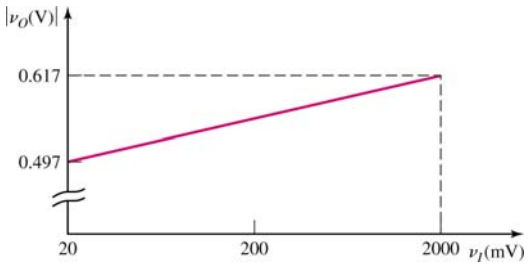
$$\underline{i_z=0}$$

9.80

$$v_O=-V_T\ln\left(\frac{v_I}{I_sR_1}\right)=-(0.026)\ln\left[\frac{v_I}{(10^{-14})(10^4)}\right]\Rightarrow v_O=-0.026\ln\left(\frac{v_I}{10^{-10}}\right)$$

For $v_I=20\ mV$, $|v_O|=0.497\ V$

For $v_I=2\ V$, $|v_O|=0.617\ V$



9.81

$$v_0 = \left(\frac{333}{20}\right)(v_{01} - v_{02}) = 16.65(v_{01} - v_{02})$$

$$v_{01} = -v_{BE1} = -V_T \ln\left(\frac{i_{C1}}{I_S}\right)$$

$$v_{02} = -v_{BE2} = -V_T \ln\left(\frac{i_{C2}}{I_S}\right)$$

$$v_{01} - v_{02} = -V_T \ln\left(\frac{i_{C1}}{i_{C2}}\right) = V_T \ln\left(\frac{i_{C2}}{i_{C1}}\right)$$

$$i_{C2} = \frac{v_2}{R_2}, \quad i_{C1} = \frac{v_1}{R_1}$$

So $v_{01} - v_{02} = V_T \ln\left(\frac{v_2}{R_2} \cdot \frac{R_1}{v_1}\right)$

Then

$$v_0 = (16.65)(0.026)\ln\left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2}\right)$$

$$v_0 = 0.4329 \ln\left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2}\right)$$

$$\ln(x) = \log_e(x) = \left[\log_{10}(x)\right] \cdot \left[\log_e(10)\right]$$

$$= 2.3026 \log_{10}(x)$$

Then $\underline{v_0 \cong (1.0)\log_{10}\left(\frac{v_2}{v_1} \cdot \frac{R_1}{R_2}\right)}$

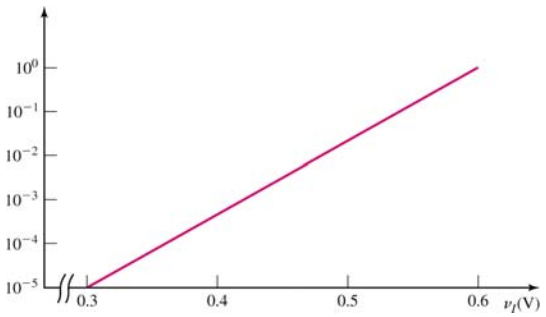
9.82

$$v_O = -I_s R\left(e^{v_I/V_T}\right) = -\left(10^{-14}\right)\left(10^4\right)e^{v_I/V_T}$$

$$|v_O| = \left(10^{-10}\right)e^{v_I/0.026}$$

For $v_I = 0.30\ V$, $|v_o| = 1.03 \times 10^{-5}\ V$

For $v_I = 0.60\ V$, $|v_o| = 1.05\ V$



9.83

From Figure 9.40

$$\upsilon_O = -\frac{R_F}{R_1}\cdot\upsilon_{I2} - \frac{R_F}{R_2}\cdot\upsilon_{I4} + \left(1+\frac{R_F}{R_N}\right)\left[\frac{R_P}{R_A}\cdot\upsilon_{I1} + \frac{R_P}{R_B}\cdot\upsilon_{I3}\right]$$

$$= -10\upsilon_{I2} - \upsilon_{I4} + 2\upsilon_{I1} + 3\upsilon_{I3}$$

Then $\frac{R_F}{R_1} = 10$, $\frac{R_F}{R_2} = 1$, $\left(1+\frac{R_F}{R_N}\right)\left(\frac{R_P}{R_A}\right) = 2$, $\left(1+\frac{R_F}{R_N}\right)\left(\frac{R_P}{R_B}\right) = 3$

Set $R_F = 500\,\text{k}\Omega$, $R_1 = 50\,\text{k}\Omega$, $R_2 = 500\,\text{k}\Omega$

Now $R_N = R_1 \| R_2 = 50 \| 500 = 45.45\,\text{k}\Omega$

Then $\left(1+\frac{500}{45.45}\right)\left(\frac{R_P}{R_A}\right) = 12\left(\frac{R_P}{R_A}\right) = 2$, Also $\left(1+\frac{500}{45.45}\right)\left(\frac{R_P}{R_B}\right) = 12\left(\frac{R_P}{R_B}\right) = 3$

Let $R_A = 500\,\text{k}\Omega$, then $R_B = \frac{2}{3}R_A = 333.3\,\text{k}\Omega$

Then $R_P = 83.33\,\text{k}\Omega = R_A \| R_B \| R_C$

We find $R_A \| R_B = 500 \| 333.3 = 200\,\text{k}\Omega$

So $200 \| R_C = 83.33 \Rightarrow R_C = 142.8\,\text{k}\Omega$

9.84

$$\upsilon_O = \left(1+\frac{R_F}{R_N}\right)\left[\frac{R_P}{R_A}\cdot\upsilon_{I1} + \frac{R_P}{R_B}\cdot\upsilon_{I2} + \frac{R_P}{R_C}\cdot\upsilon_{I3}\right] - \frac{R_R}{R_1}\cdot\upsilon_{I4} - \frac{R_F}{R_2}\cdot\upsilon_{I5}$$

$$= 3\upsilon_{I1} + 1.5\upsilon_{I2} + 2\upsilon_{I3} - 4\upsilon_{I4} - 6\upsilon_{I5}$$

We have $\frac{R_F}{R_1} = 4$, $\frac{R_F}{R_2} = 6$; Set $R_F = 250\,\text{k}\Omega$, $R_1 = 62.5\,\text{k}\Omega$, $R_2 = 41.67\,\text{k}\Omega$

Now $R_N = R_1 \| R_2 = 62.5 \| 41.67 = 25\,\text{k}\Omega$

Also $\left(1+\frac{R_F}{R_N}\right) = \left(1+\frac{250}{25}\right) = 11$

Now $\frac{(11)R_P}{R_A} = 3$, $\frac{(11)R_P}{R_B} = 1.5$, $\frac{(11)R_P}{R_C} = 2 \Rightarrow \frac{R_A}{R_B} = \frac{1}{2}$, $\frac{R_A}{R_C} = \frac{2}{3}$

Set $R_B = 250\,\text{k}\Omega$, $R_A = 125\,\text{k}\Omega$, $R_C = 187.5\,\text{k}\Omega$

This yields $R_P = 34.09\,\text{k}\Omega$, We have $R_p = R_A \| R_B \| R_C \| R_D$

We find $R_A \| R_B \| R_C = 125 \| 250 \| 187.5 = 57.69\,\text{k}\Omega$

Then $57.69 \| R_D = 34.09 \Rightarrow R_D = 83.3\ \text{k}\Omega$

9.85

$$\frac{V_O}{V_Z} = \left(1+\frac{R_2}{R_1}\right) = \frac{12}{5.6} \Rightarrow \frac{R_2}{R_1} = 1.143$$

$$I_F = \frac{V_O - V_Z}{R_F}\ ;\ \text{Set } I_F = I_Z(\min) = 1.2\ \text{mA}$$

Then $R_F = \frac{12-5.6}{1.2} = 5.33\,\text{k}\Omega$

Set $I_{D1} = 0.15\,\text{mA}$

$V_Z' = V_Z + V_\gamma = 5.6 + 0.7 = 6.3\,\text{V}$

Let $I_4 = 0.2\,\text{mA}$, $\Rightarrow R_4 = \dfrac{6.3}{0.2} = 31.5\,\text{k}\Omega$

Then $I_3 = 0.2 + 0.15 = 0.35\,\text{mA}$

So $R_3 = \dfrac{V_S - V_Z'}{I_3} = \dfrac{10 - 6.3}{0.35} = 10.6\,\text{k}\Omega$

9.86

$$R_1 = \frac{V_O - V_Z}{I_Z} = \frac{12 - 5.6}{2} = 3.2\,\text{k}\Omega$$

$$\frac{V_O}{V_Z} = \left(1 + \frac{R_2}{R_3}\right) = \frac{12}{5.6} \Rightarrow \frac{R_2}{R_3} = 1.143$$

Let $I_R = 2\,\text{mA}$, $\Rightarrow R_2 + R_3 = \dfrac{V_O}{I_R} = \dfrac{12}{2} = 6\,\text{k}\Omega$

Then $1.143R_3 + R_3 = 6$, $\Rightarrow R_3 = 2.8\,\text{k}\Omega$ and $R_2 = 3.2\,\text{k}\Omega$

Let $I_{R4} = 4\,\text{mA}$, $R_4 = \dfrac{V_{IN} - V_O}{I_{R4}} = \dfrac{15 - 12}{4} = 0.75\,\text{k}\Omega$

9.87

Let $R_1 = R_2 = R_3 = 20\,\text{k}\Omega$

Let $R_T = 20(1+\delta)\ \text{k}\Omega$

Now $\upsilon_{O1} = \upsilon_A = \left(\dfrac{R_3}{R_3 + R_1}\right)\cdot V^+ = \dfrac{1}{2}(10) = 5\,\text{V}$

$$\upsilon_{O2} = \upsilon_B = \left(\frac{R_T}{R_T + R_2}\right)\cdot V^+ = \left[\frac{20(1+\delta)}{20(1+\delta)+20}\right](10) = \frac{10(1+\delta)}{2+\delta}$$

So $\upsilon_{OA} = \upsilon_A - \upsilon_B = 5 - \dfrac{10(1+\delta)}{2+\delta} = \dfrac{5(2+\delta) - 10(1+\delta)}{2+\delta} \cong -\dfrac{5\delta}{2} = -2.5\delta$

We have $\delta = y\left(\dfrac{T - 300}{300}\right)$; At $T = 350$, $R_T = 21\,\text{k}\Omega$, $\Rightarrow 21 = 20(1+\delta) \Rightarrow \delta = 0.05$

Then $0.05 = y\left(\dfrac{350 - 300}{300}\right) \Rightarrow y = 0.30$

For $\delta = 0.05$, $|\upsilon_{OA}| = (2.5)(0.05) = 0.125\ \text{V}$

For the instrumentation amplifier, $\upsilon_O = 5 = \dfrac{R_4}{R_3}\left(1 + \dfrac{2R_2}{R_1}\right)(0.125)$

For example, set $\dfrac{R_4}{R_3} = 4$ and $\dfrac{R_2}{R_1} = 4.5$

9.88

(a) $$\upsilon_A = \left(\frac{R-\Delta R}{R-\Delta R+R+\Delta R}\right)\cdot V^+ = \left(\frac{R-\Delta R}{2R}\right)\cdot V^+$$

$$\upsilon_B = \left(\frac{R+\Delta R}{R+\Delta R+R-\Delta R}\right)\cdot V^+ = \left(\frac{R+\Delta R}{2R}\right)\cdot V^+$$

$$\upsilon_{O1} = \upsilon_A - \upsilon_B = \left[\frac{R-\Delta R}{2R} - \frac{R+\Delta R}{2R}\right]\cdot V^+ = -\frac{\Delta R}{R}\cdot V^+ = -\Delta R\left(\frac{9}{20\times 10^3}\right)$$

or $$\upsilon_{O1} = -\left(4.5\times 10^{-4}\right)\left(\Delta R\right)$$

(b) For an instrumentation amplifier,

$$\upsilon_O = \frac{R_4}{R_3}\left(1+\frac{2R_2}{R_1}\right)\cdot \upsilon_{O1}$$

For $\Delta R = 200\,\Omega$, $\upsilon_O = -5$ V

$$-5 = \frac{R_4}{R_3}\left(1+\frac{2R_2}{R_1}\right)\left(-4.5\times 10^{-4}\right)\left(200\right)$$

or $$55.55 = \frac{R_4}{R_3}\left(1+\frac{2R_2}{R_1}\right)$$

For example, set $\dfrac{R_4}{R_3} = 6$ and $\dfrac{R_2}{R_1} = 4.13$

Chapter 10

10.1

a.
$$I_1 = I_2 = \frac{0 - 2V_\gamma - V^-}{R_1 + R_2}$$
$$2V_\gamma + I_2 R_2 = V_{BE} + I_C R_3$$
$$2V_\gamma + \frac{R_2}{R_1 + R_2}\left(-2V_\gamma - V^-\right) = V_{BE} + I_C R_3$$
$$\underline{I_C = \frac{1}{R_3}\left\{2V_\gamma - \left(2V_\gamma + V^-\right)\left(\frac{R_2}{R_1 + R_2}\right) - V_{BE}\right\}}$$

b.
$V_\gamma = V_{BE}$ and $R_1 = R_2$
$$I_C = \frac{1}{R_3}\left\{2V_\gamma - \frac{1}{2}\left(2V_\gamma + V^-\right) - V_{BE}\right\}$$
or $$\underline{I_C = \frac{-V^-}{2R_3}}$$

c.
$$I_C = 2\text{ mA} = \frac{-(-10)}{2R_3} \Rightarrow \underline{R_3 = 2.5\text{ k}\Omega}$$
$$I_1 = I_2 = 2\text{ mA} = \frac{-2(0.7) - (-10)}{R_1 + R_2} \Rightarrow R_1 + R_2 = 4.3\text{ k}\Omega \Rightarrow \underline{R_1 = R_2 = 2.15\text{ k}\Omega}$$

10.2

(a) (i) $I_O = 50\,\mu\text{A},\ V_{BE1} = (0.026)\ln\left(\frac{50\times10^{-6}}{10^{-16}}\right) = 0.7004\text{ V}$

(ii) $I_O = 150\,\mu\text{A},\ V_{BE1} = (0.026)\ln\left(\frac{150\times10^{-6}}{10^{-16}}\right) = 0.7289\text{ V}$

(iii) $I_O = 1.5\,\text{mA},\ V_{BE1} = (0.026)\ln\left(\frac{1.5\times10^{-3}}{10^{-16}}\right) = 0.7888\text{ V}$

(b) (i) $I_O = \frac{50}{1+\frac{2}{50}} = 48.08\,\mu\text{A},\ V_{BE1} = (0.026)\ln\left(\frac{48.0769\times10^{-6}}{10^{-16}}\right) = 0.6994\text{ V}$

(ii) $I_O = \frac{150}{1+\frac{2}{50}} = 144.23\,\mu\text{A},\ V_{BE1} = (0.026)\ln\left(\frac{144.23\times10^{-6}}{10^{-16}}\right) = 0.7279\text{ V}$

(iii) $I_O = \frac{1.5}{1+\frac{2}{50}} = 1.4423\,\text{mA},\ V_{BE1} = (0.026)\ln\left(\frac{1.4423\times10^{-3}}{10^{-16}}\right) = 0.7878\text{ V}$

10.3

$$I_{C1} \cong \frac{I_{REF}}{1+\frac{2}{\beta}} = \frac{200}{1+\frac{2}{80}} = 195.12\ \mu\text{A}$$

$$V_{BE1} = V_{BE2} = (0.026)\ln\left(\frac{195.12\times10^{-6}}{5\times10^{-15}}\right) = 0.6341\text{ V}$$

$$I_O = I_{S2}\exp\left(\frac{V_{BE2}}{V_T}\right) = \left(2\times10^{-15}\right)\exp\left(\frac{0.6341}{0.026}\right) \Rightarrow I_O = 78.05\ \mu\text{A}$$

10.4

$$I_{C1} = \frac{I_{REF}}{1+\frac{2}{\beta}} = \frac{150}{1+\frac{2}{120}} = 147.54\ \mu\text{A}$$

$$V_{BE1} = V_{BE2} = (0.026)\ln\left(\frac{147.54\times10^{-6}}{10^{-16}}\right) = 0.7285\text{ V}$$

$$I_O = I_{S2}\exp\left(\frac{V_{BE2}}{V_T}\right) = \left(3\times10^{-16}\right)\exp\left(\frac{0.7285}{0.026}\right) \Rightarrow I_O = 0.4426\text{ mA}$$

10.5

Approximation: $I_{C1} \cong \dfrac{I_{REF}}{1+\frac{2}{\beta}} = \dfrac{200}{1+\frac{2}{40}} = 190.48\ \mu\text{A}$

$$V_{BE1} = (0.026)\ln\left(\frac{190.48\times10^{-6}}{5\times10^{-15}}\right) = 0.63345\text{ V}$$

$$I_{B2} \cong \frac{I_{C1}}{\beta} = \frac{190.48}{40} = 4.762\ \mu\text{A}$$

$$V_{BE2} = V_{BE1} - I_{B2}R = 0.63345 - (0.004762)(2) = 0.62393\text{ V}$$

$$I_O = \left(5\times10^{-15}\right)\exp\left(\frac{0.62393}{0.026}\right) \Rightarrow I_O = 132.07\ \mu\text{A}$$

10.6

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1} \Rightarrow 0.250 = \frac{3-0.7-(-3)}{R_1}$$

$$\underline{R_1 = 21.2\text{ K}}$$

$$I_{C1} = I_{C2} = \frac{I_{REF}}{1+\frac{2}{\beta}} = \frac{0.250}{1+\frac{2}{60}} \Rightarrow I_{C1} = I_{C2} = 0.2419\text{ mA}$$

$$I_{B1} = I_{B2} = 4.03\ \mu\text{A}$$

10.7

$$I_{REF} = \frac{V^+ - V_{BE}(\text{on}) - V^-}{R_1} = \frac{5 - 0.7 - (-5)}{18.3}$$

$$I_{REF} = 0.5082 \text{ mA}$$

$$I_{C1} = I_{C2} = \frac{I_{REF}}{1 + \frac{2}{\beta}} = \frac{0.5082}{1 + \frac{2}{80}} \Rightarrow I_{C1} = I_{C2} = 0.4958 \text{ mA}$$

$$I_{B1} = I_{B2} = (6.198\ \mu\text{A})$$

10.8

(a) $P = (I_O + I_{REF})(V^+ - V^-)$

$$1.8 = (0.25 + I_{REF})(5) \Rightarrow I_{REF} = 0.11 \text{ mA}$$

$$V_{BE1} = V_{BE2} = (0.026)\ln\left(\frac{0.25 \times 10^{-3}}{10^{-15}}\right) = 0.68236 \text{ V}$$

(b) $0.11 \times 10^{-3} = I_{S1} \exp\left(\frac{0.68236}{0.026}\right) \Rightarrow I_{S1} = 4.4 \times 10^{-16} \text{ A}$

(c) $R_1 = \frac{5 - 0.68236}{0.11} = 39.25 \text{ k}\Omega$

10.9

(a) $I_{REF} = \left(1 + \frac{2}{\beta}\right) \cdot I_O = \left(1 + \frac{2}{120}\right)(0.5) = 0.5083 \text{ mA}$

$$R_1 = \frac{5 - 0.7 - (-5)}{0.5083} = 18.3 \text{ k}\Omega$$

(b) $R_o = \frac{V_A}{I_O} = \frac{100}{0.5} = 200 \text{ k}\Omega$

$$\Delta I_O = \frac{\Delta V_{CE2}}{R_o} = \frac{7 - 0.7}{200} = 0.0315 \text{ mA}$$

$$\frac{\Delta I_O}{I_O} = \frac{0.0315}{0.5} \Rightarrow 6.3\%$$

10.10

$$I_0 = nI_{C1}$$

$$I_{REF} = I_{C1} + I_{B1} + I_{B2} = I_{C1} + \frac{I_{C1}}{\beta} + \frac{I_0}{\beta}$$

$$I_{REF} = I_{C1}\left(1 + \frac{1}{\beta} + \frac{n}{\beta}\right) = I_{C1}\left(1 + \frac{1+n}{\beta}\right)$$

$$= \frac{I_0}{n}\left(1 + \frac{1+n}{\beta}\right) \text{ or } I_0 = \frac{nI_{REF}}{\left(1 + \frac{1+n}{\beta}\right)}$$

10.11

$$I_O = \frac{I_{REF}}{1+\frac{2}{\beta}} \Rightarrow I_{REF} = (0.20)\left(1+\frac{2}{40}\right) = 0.210 \text{ mA}$$

$$R_1 = \frac{5-0.7}{I_{REF}} = \frac{4.3}{0.21} \Rightarrow \underline{R_1 = 20.5 \text{ K}}$$

10.12

(a) $I_{REF} = \left(1+\frac{2}{\beta}\right)\cdot I_O = \left(1+\frac{2}{80}\right)(0.12) = 0.123 \text{ mA}$

$$R_1 = \frac{5-0.7}{0.123} = 34.96 \text{ k}\Omega$$

(b) $r_o = \frac{V_A}{I_O} = \frac{80}{0.12} = 666.7 \text{ k}\Omega$

(i) $\Delta I_O = \frac{\Delta V_{EC}}{r_o} = \frac{2-0.7}{666.7} \Rightarrow \Delta I_O = 1.95\,\mu\text{A}$

(ii) $\Delta I_O = \frac{4-0.7}{666.7} \Rightarrow \Delta I_O = 4.95\,\mu\text{ A}$

10.13

a. $I_{REF} = 1 = \frac{5-0.7-(-5)}{R_1} \Rightarrow \underline{R_1 = 9.3 \text{ k}\Omega}$

b. $I_0 = 2I_{REF} \Rightarrow \underline{I_0 = 2 \text{ mA}}$

c. For $V_{EC2}(\min) = 0.7 \Rightarrow R_{C2} = \frac{5-0.7}{2} \Rightarrow \underline{R_{C2} = 2.15 \text{ k}\Omega}$

10.14

$$I_O = 0.50 \text{ mA} \Rightarrow I_{OA} = I_{OB} = 0.25 \text{ mA}$$

$$I_{REF} = I_{OA}\left(1+\frac{3}{\beta}\right) = 0.25\left(1+\frac{3}{60}\right)$$

$$I_{REF} = 0.2625 \text{ mA}$$

$$R_1 = \frac{2.5-0.7}{0.2625} \Rightarrow \underline{R_1 = 6.86 \text{ K}}$$

10.15

Similar to Figure P10.14 biased at V^+ and V^-.

$$R_1 = \frac{2.5-0.7-(-2.5)}{0.2} = 21.5 \text{ k}\Omega$$

10.16

$I_2 = 2I_1$ and $I_3 = 3I_1$

(a) $I_2 = 1.0\ mA,\ I_3 = 1.5\ mA$

(b) $I_1 = 0.25\ mA,\ I_3 = 0.75\ mA$

(c) $I_1 = 0.167\ mA,\ I_2 = 0.333\ mA$

10.17

a.

$$I_0 = I_{C1} \text{ and } I_{REF} = I_{C1} + I_{B3} = I_{C1} + \frac{I_{E3}}{1+\beta}$$

$$I_{E3} = I_{B1} + I_{B2} + \frac{V_{BE}}{R_2} = \frac{2I_{C1}}{\beta} + \frac{V_{BE}}{R_2}$$

$$I_{REF} = I_{C1} + \frac{2I_{C1}}{\beta(1+\beta)} + \frac{V_{BE}}{(1+\beta)R_2}$$

$$I_{REF} - \frac{V_{BE}}{(1+\beta)R_2} = I_0\left(1 + \frac{2}{\beta(1+\beta)}\right)$$

$$\underline{I_0 = \frac{I_{REF} - \dfrac{V_{BE}}{(1+\beta)R_2}}{\left(1 + \dfrac{2}{\beta(1+\beta)}\right)}}$$

$$I_{REF} = (0.70)\left(1 + \frac{2}{(80)(81)}\right) + \frac{0.7}{(81)(10)}$$

$$I_{REF} = 0.700216 + 0.000864$$

$$\underline{I_{REF} = 0.7011\ \text{mA}} = \frac{10 - 2(0.7)}{R_1} \Rightarrow \underline{R_1 = 12.27\ \text{k}\Omega}$$

b.

10.18

a.

$$I_{0i} = I_{CR} \text{ and } I_{REF} = I_{CR} + I_{BS} = I_{CR} + \frac{I_{ES}}{1+\beta}$$

$$I_{ES} = I_{BR} + I_{B1} + I_{B2} + \ldots + I_{BN} = (1+N)I_{BR}$$

$$= \frac{(1+N)I_{CR}}{\beta}$$

Then $I_{REF} = I_{CR} + \dfrac{(1+N)I_{CR}}{\beta(1+\beta)}$

or $\underline{I_{0i} = \dfrac{I_{REF}}{\left(1 + \dfrac{(1+N)}{\beta(1+\beta)}\right)}}$

$$I_{REF} = (0.5)\left[1 + \frac{6}{(50)(51)}\right] = 0.5012 \text{ mA}$$

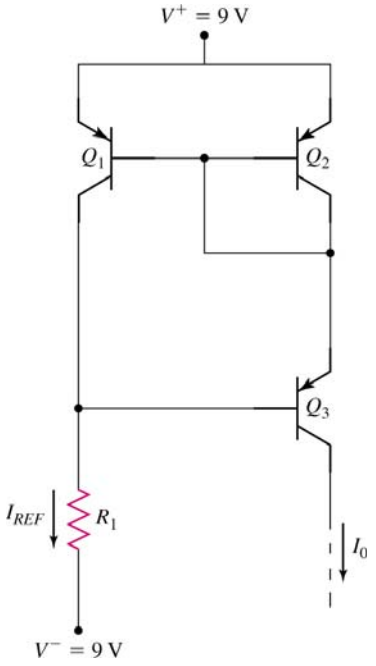
b. $$R_1 = \frac{5 - 2(0.7) - (-5)}{0.5012} \Rightarrow \underline{R_1 = 17.16 \text{ k}\Omega}$$

10.19

$$I_{REF} = I_O\left[1 + \frac{2}{\beta(1+\beta)}\right] = (0.15)\left[1 + \frac{2}{(40)(41)}\right] = 0.15018 \text{ mA}$$

$$R_1 = \frac{3 - 0.7 - 0.7 - (-3)}{0.15018} = 30.63 \text{ k}\Omega$$

10.20



$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \frac{2}{\beta(2+\beta)}\right)}$$

For $I_0 = 0.8$ mA

$$I_{REF} = (0.8)\left(1 + \frac{2}{25(27)}\right) \Rightarrow \underline{I_{REF} = 0.8024 \text{ mA}}$$

$$R_1 = \frac{18 - 2(0.7)}{0.8024} \Rightarrow \underline{R_1 = 20.69 \text{ k}\Omega}$$

10.21

The analysis is exactly the same as in the text. We have

$$I_0 = I_{REF} \cdot \frac{1}{\left(1 + \dfrac{2}{\beta(2+\beta)}\right)}$$

10.22

(a) $I_O = 0.5\,\text{mA}, \quad I_{B2} = \dfrac{0.5}{90} = 0.005556\,\text{mA}$

$$I_{B1} = \frac{I_{B2}}{2} = 0.002778\,\text{mA}$$

$$I_{E3} = I_{B1} + I_{B2} = 0.008334\,\text{mA}$$

$$I_{B3} = \frac{0.008334}{61} = 0.0001366\,\text{mA}$$

$$I_{C1} = \frac{I_O}{2} = 0.25\,\text{mA}$$

$$I_{REF} = I_{C1} + I_{B3} = 0.2501366\,\text{mA}$$

$$R_1 = \frac{5 - 0.6 - 0.7 - (-5)}{0.2501366} = 34.78\,\text{k}\Omega$$

(b) $I_{B1} = 0.002778\,\text{mA}$

$I_{B2} = 0.005556\,\text{mA}$

$I_{E3} = 0.008334\,\text{mA}$

$I_{B3} = 0.0001366\,\text{mA}$

10.23

(a)

Assuming $R_O \approx \dfrac{\beta r_{o3}}{2}$

$$r_{O3} = \frac{V_A}{I_O} = \frac{V_A}{I_{REF}} = \frac{100}{0.25} = 400\ \text{K}$$

$$R_O = \frac{(100)(400)}{2} \Rightarrow \underline{R_O = 20\ \text{M}\Omega}$$

(b)

$$R_O = \frac{\Delta V}{\Delta I_O} \Rightarrow \Delta I_O = \frac{\Delta V}{20\ \text{M}\Omega} = \frac{5}{20\ \text{M}\Omega}$$

$$\underline{\Delta I_O = 0.25\ \mu\text{A}}$$

10.24

$$I_{REF} = \frac{V^+ - V_{BE1} - V^-}{R_1} = \frac{5 - 0.7}{9.3}$$

$$\underline{I_{REF} = 0.4624 \text{ mA}}$$

$$I_O = \frac{V_T}{R_E} \ln\left(\frac{I_{REF}}{I_O}\right) = \frac{0.026}{1.5} \ln\left(\frac{0.4624}{I_O}\right)$$

$$I_O = 0.01733 \ln\left(\frac{0.4624}{I_O}\right)$$

By trial and error

$$I_O = 41.7\,\mu\text{A}$$

$$V_{BE2} = 0.7 - I_O R_E = 0.7 - (0.0417)(1.5)$$

$$V_{BE2} = 0.6375 \text{ V}$$

10.25

(a) $$V_{BE1} = (0.026)\ln\left(\frac{200\times10^{-6}}{5\times10^{-15}}\right) = 0.6347 \text{ V}$$

$$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right)$$

$$I_O(0.5) = (0.026)\ln\left(\frac{0.2}{I_O}\right)$$

$$I_O \cong 61.4\,\mu\text{A}$$

$$V_{BE2} = (0.026)\ln\left(\frac{61.4\times10^{-6}}{5\times10^{-15}}\right) = 0.6040 \text{ V}$$

(b) $$V_{BE1} = (0.026)\ln\left(\frac{200\times10^{-6}}{5\times10^{-15}}\right) = 0.6347 \text{ V}$$

$$V_{BE1} - V_{BE2} = I_O R_E$$

$$V_T\left[\ln\left(\frac{I_{REF}}{I_{S1}}\right) - \ln\left(\frac{I_O}{I_{S2}}\right)\right] = I_O R_E$$

$$V_T \ln\left(\frac{I_{REF}}{I_O} \cdot \frac{I_{S2}}{I_{S1}}\right) = I_O R_E$$

$$(0.026)\ln\left[\left(\frac{0.2}{I_O}\right)\left(\frac{7}{5}\right)\right] = I_O(0.5)$$

$$I_O \cong 71.2\,\mu\text{A}$$

$$V_{BE2} = (0.026)\ln\left(\frac{71.2\times10^{-6}}{7\times10^{-15}}\right) = 0.5991 \text{ V}$$

10.26

(a) $V_{BE1} = (0.026)\ln\left(\dfrac{100\times10^{-6}}{5\times10^{-15}}\right) = 0.61669\text{ V}$

$V_{BE2} = V_{BE1} + I_{REF}R_E = 0.61669 + (0.1)(0.7) = 0.68669\text{ V}$

$I_O = (5\times10^{-15})\exp\left(\dfrac{0.68669}{0.026}\right) \Rightarrow I_O = 1.477\text{ mA}$

(b) $V_{BE1} = 0.61669\text{ V}$

$V_{BE2} = 0.68669\text{ V}$

$I_O = (2\times10^{-15})\exp\left(\dfrac{0.68669}{0.026}\right) \Rightarrow I_O = 0.5906\text{ mA}$

10.27

(a) $I_{REF} = \dfrac{5 - 0.7 - (-5)}{50} = 0.186\text{ mA}$

$I_O R_E = V_T\ln\left(\dfrac{I_{REF}}{I_O}\right)$

$I_O(3) = (0.026)\ln\left(\dfrac{0.186}{I_O}\right)$

$I_O \cong 19.53\,\mu\text{A}$

$V_{BE2} = V_{BE1} - I_O R_E = 0.7 - (0.01953)(3) = 0.6414\text{ V}$

(b) $r_{o2} = \dfrac{V_A}{I_O} = \dfrac{80}{0.01953} \Rightarrow r_{o2} = 4.096\text{ M}\Omega$

$g_{m2} = \dfrac{0.01953}{0.026} = 0.7512\text{ mA/V}$

$r_{\pi2} = \dfrac{(120)(0.026)}{0.01953} = 159.8\text{ k}\Omega$

$R_E \| r_{\pi2} = 3\|159.8 = 2.945\text{ k}\Omega$

$R_o = (4.096)[1 + (0.7512)(2.945)] = 13.16\text{ M}\Omega$

10.28

$R_o = r_{o2}\left[1 + g_{m2}\left(R_E \| r_{\pi2}\right)\right]$

From 10.27, $I_O = 19.53\,\mu\text{A}$

$g_{m2} = \dfrac{I_O}{V_T} = \dfrac{0.01953}{0.026} = 0.7512\text{ mA/V}$

$r_{\pi2} = \dfrac{\beta V_T}{I_O} = \dfrac{(80)(0.026)}{0.01953} = 106.5\text{ k}\Omega$

$r_{o2} = \dfrac{V_A}{I_O} = \dfrac{80}{0.01953} \Rightarrow 4.096\text{ M}\Omega$

Then $R_o = (4.096)[1 + (0.7512)(3\|106.5)] = 13.07\text{ M}\Omega$

$\Delta I_O = \dfrac{\Delta V_O}{R_o} = \dfrac{5}{13.07} = 0.382\,\mu\text{A}$

10.29

$$I_{REF} = \frac{5 - 0.7 - (-5)}{R_1} = 0.50$$

$$\underline{R_1 = 18.6 \text{ K}}$$

$$I_O R_E = V_T \ln\left(\frac{I_{REF}}{I_O}\right)$$

$$R_E = \frac{0.026}{0.050}\ln\left(\frac{0.50}{0.050}\right)$$

(a) $R_E = 1.20$ K

$$R_O = r_{c2}\left[1 + R_E' g_{m2}\right]$$

$$R_E' = R_E \| r_{\pi 2}$$

$$r_{\pi 2} = \frac{(75)(0.026)}{0.050} = 39 \text{ K} \qquad g_{m2} = \frac{0.050}{0.026} = 1.923 \text{ mA/V}$$

$$r_{o2} = \frac{V_A}{I_O} = \frac{100}{0.05} \Rightarrow 2 \text{ M}\Omega \qquad R_E' = 1.20 \| 39 = 1.164 \text{ K}$$

(b) $R_O = 2\left[1 + (1.164)(1.923)\right] \Rightarrow R_O = (6.477) \text{ M}\Omega$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{5}{6.477} = 0.772 \ \mu\text{A}$$

(c) $$\frac{\Delta I_O}{I_O} \times 100\% = \frac{0.772}{50} \times 100 = \underline{1.54\%}$$

10.30

Let $R_1 = 10 \text{ k}\Omega$

Then $I_{REF} = \dfrac{3 - 0.7 - (-3)}{10} = 0.53 \text{ mA}$

$$R_E = \frac{V_T}{I_O}\ln\left(\frac{I_{REF}}{I_O}\right) = \frac{(0.026)}{(0.05)}\ln\left(\frac{0.53}{0.05}\right) \Rightarrow R_E = 1.228 \text{ k}\Omega$$

10.31

$$V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$0.7 = (0.026)\ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03 \times 10^{-15} \text{ A}$$

At 2 mA, $V_{BE} = (0.026)\ln\left(\dfrac{2 \times 10^{-3}}{2.03 \times 10^{-15}}\right)$

$= 0.718$ V

$$R_1 = \frac{15 - 0.718}{2} \Rightarrow \underline{R_1 = 7.14 \text{ k}\Omega}$$

$$R_E = \frac{V_T}{I_0}\ln\left(\frac{I_{REF}}{I_0}\right) = \frac{0.026}{0.050} \cdot \ln\left(\frac{2}{0.050}\right) \Rightarrow \underline{R_E = 1.92 \text{ k}\Omega}$$

10.32

(a) $I_{REF} = \dfrac{3-0.7-(-3)}{20} = 0.265\text{ mA}$

(b) $R_E = \dfrac{V_T}{I_O}\ln\left(\dfrac{I_{REF}}{I_O}\right) = \dfrac{(0.026)}{(0.1)}\ln\left(\dfrac{0.265}{0.1}\right) \Rightarrow R_E = 253\,\Omega$

10.33

$$I_{REF} \approx \frac{10-0.7-(-10)}{40} = 0.4825\text{ mA}$$

$$V_{BE} \cong V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$0.7 = (0.026)\ln\left(\frac{10^{-3}}{I_S}\right) \Rightarrow I_S = 2.03\times10^{-15}\text{ A}$$

Now

$$V_{BE} = (0.026)\ln\left(\frac{0.4825\times10^{-3}}{2.03\times10^{-15}}\right) = 0.681\text{ V}$$

$\underline{V_{BE1} = 0.681\text{ V}}$

So

$$I_{REF} \cong \frac{10-0.681-(-10)}{40} \Rightarrow \underline{I_{REF} = 0.483\text{ mA}}$$

$$I_0 R_E = V_T \ln\left(\frac{I_{REF}}{I_0}\right)$$

$$I_0(12) = (0.026)\ln\left(\frac{0.483}{I_0}\right)$$

By trial and error.

$\Rightarrow \underline{I_0 \cong 8.7\ \mu\text{A}}$

$V_{BE2} = V_{BE1} - I_0 R_E = 0.681 - (0.0087)(12) \Rightarrow \underline{V_{BE2} = 0.5766\text{ V}}$

10.34

$$V_{BE1} + I_{REF}R_{E1} = V_{BE2} + I_0 R_{E2}$$

$$V_{BE1} - V_{BE2} = I_0 R_{E2} - I_{REF} R_{E1}$$

For matched transistors

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$$

$$V_{BE2} = V_T \ln\left(\frac{I_0}{I_S}\right)$$

Then $\underline{V_T \ln\left(\dfrac{I_{REF}}{I_0}\right) = I_0 R_{E2} - I_{REF} R_{E1}}$

Output resistance looking into the collector of Q_2 is increased.

10.35

(a) $V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026)\ln\left(\frac{0.5\times10^{-3}}{10^{-15}}\right) = 0.70038 \text{ V}$

$V^+ = I_{REF}R_1 + V_{BE1} + I_{REF}R_{E1} + V^-$

Then $R_1 = \frac{3 - 0.70038 - (0.5)(0.5) - (-3)}{0.5} = 10.1 \text{ k}\Omega$

$V_{BE2} = (0.026)\ln\left(\frac{0.2\times10^{-3}}{10^{-15}}\right) = 0.67656 \text{ V}$

$V_{BE1} + I_{REF}R_{E1} = V_{BE2} + I_O R_{E2}$

Then $R_{E2} = \frac{0.70038 + (0.5)(0.5) - 0.67656}{0.2} = 1.37 \text{ k}\Omega$

(b) $R_1 = 10.1 \text{ k}\Omega$

$V_{BE2} = (0.026)\ln\left(\frac{0.2\times10^{-3}}{2\times10^{-15}}\right) = 0.65854 \text{ V}$

$R_{E2} = \frac{0.70038 + (0.5)(0.5) - 0.65854}{0.2} = 1.46 \text{ k}\Omega$

10.36

Assume all transistors are matched.

a.

$2V_{BE1} = V_{BE3} + I_0 R_E$

$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right)$

$V_{BE3} = V_T \ln\left(\frac{I_0}{I_S}\right)$

$2V_T \ln\left(\frac{I_{REF}}{I_S}\right) - V_T \ln\left(\frac{I_0}{I_S}\right) = I_0 R_E$

$V_T\left[\ln\left(\frac{I_{REF}}{I_S}\right)^2 - \ln\left(\frac{I_0}{I_S}\right)\right] = I_0 R_E$

$\underline{V_T \ln\left(\frac{I^2_{REF}}{I_0 I_S}\right) = I_0 R_E}$

b.

$V_{BE} = 0.7 \text{ V at } 1 \text{ mA} \Rightarrow 10^{-3} = I_S \exp\left(\frac{0.7}{0.026}\right) \text{ or } I_S = 2.03\times10^{-15} \text{ A}$

$V_{BE} \text{ at } 0.1 \text{ mA} \Rightarrow V_{BE} = (0.026)\ln\left(\frac{0.1\times10^{-3}}{2.03\times10^{-15}}\right) = 0.640 \text{ V}$

Since $I_0 = I_{REF}$, then $V_{BE} = I_0 R_E \Rightarrow R_E = \frac{0.640}{0.1}$ or $\underline{R_E = 6.4 \text{ k}\Omega}$

10.37

(a) $I_{REF} = \dfrac{5 - 0.7 - (-5)}{10} = 0.93\ \text{mA}$

$$I_{O2}R_{E2} = V_T \ln\left(\frac{I_{REF}}{I_{O2}}\right)$$

$$I_{O2}(1) = (0.026)\ln\left(\frac{0.93}{I_{O2}}\right)$$

$$I_{O2} \cong 68\,\mu\text{A}$$

$$I_{O3}(2) = (0.026)\ln\left(\frac{0.93}{I_{O3}}\right)$$

$$I_{O3} \cong 40.7\,\mu\text{A}$$

(b) $R_{E2} = \dfrac{V_T}{I_{O2}}\ln\left(\dfrac{I_{REF}}{I_{O2}}\right) = \dfrac{(0.026)}{(0.02)}\ln\left(\dfrac{0.93}{0.02}\right) \Rightarrow R_{E2} = 4.99\ \text{k}\Omega$

$$R_{E3} = \frac{(0.026)}{(0.08)}\ln\left(\frac{0.93}{0.08}\right) \Rightarrow R_{E3} = 0.797\ \text{k}\Omega$$

10.38

(a)

$$V_{BE1} = V_{BE2}$$

$$I_{REF} = \frac{V^+ - 2V_{BE1} - V^-}{R_1 + R_2}$$

Now

$$2V_{BE1} + I_{REF}R_2 = V_{BE3} + I_O R_E$$

or

$$I_O R_E = 2V_{BE1} - V_{BE3} + I_{REF}R_2$$

We have

$$V_{BE1} = V_T \ln\left(\frac{I_{REF}}{I_S}\right) \text{ and } V_{BE3} = V_T \ln\left(\frac{I_O}{I_S}\right)$$

(b)

Let $R_1 = R_2$ and $I_O = I_{REF} \Rightarrow V_{BE1} = V_{BE3} \equiv V_{BE}$

Then

$$V_{BE} = I_O R_E - I_{REF}R_2 = I_O(R_E - R_2)$$

so

$$I_{REF} = I_O = \frac{V^+ - V^- - 2I_O(R_E - R_2)}{2R_2}$$

$$= \frac{V^+ - V^-}{2R_2} - I_O\left(\frac{R_E}{R_2}\right) + I_O$$

Then

$$I_O = \frac{V^+ - V^-}{2R_\varepsilon}$$

(c)

Want $I_O = 0.5\ mA$

So $R_E = \dfrac{5-(-5)}{2(0.5)} \Rightarrow \underline{R_E = 10\ k\Omega}$

$$2R_2 = \frac{5-2(0.7)-(-5)}{0.5} = 17.2\ k\Omega$$

Then $\underline{R_1 = R_2 = 8.6\ k\Omega}$

10.39

a.

$$I_{REF} = \frac{20-0.7-0.7}{12} = 1.55\ \text{mA}$$

$$I_{01} = 2I_{REF} = 3.1\ \text{mA}$$

$$I_{02} = I_{REF} = 1.55\ \text{mA}$$

$$I_{03} = 3I_{REF} = 4.65\ \text{mA}$$

b.

$$V_{CE1} = -I_{01}R_{C1} - (-10) = -(3.1)(2) + 10 \Rightarrow \underline{V_{CE1} = 3.8\ \text{V}}$$

$$V_{EC2} = 10 - I_{02}R_{C2} = 10 - (1.55)(3) \Rightarrow \underline{V_{EC2} = 5.35\ \text{V}}$$

$$V_{EC3} = 10 - I_{03}R_{C3} = 10 - (4.65)(1) \Rightarrow \underline{V_{EC3} = 5.35\ \text{V}}$$

10.40

(a) $$I_{REF} = \frac{10-0.7-0.7-(-10)}{24} = 0.775\ \text{mA}$$

$$I_{O1} = 2I_{REF} = 1.55\ \text{mA}$$

$$I_{O2} = I_{REF} = 0.775\ \text{mA}$$

$$I_{O3} = 3I_{REF} = 2.325\ \text{mA}$$

(b) $$R_{C1} = \frac{0-0.7-(-10)}{1.55} = 6\ \text{k}\Omega$$

$$R_{C2} = \frac{10-0.7}{0.775} = 12\ \text{k}\Omega$$

$$R_{C3} = \frac{10-0.7}{2.325} = 4\ \text{k}\Omega$$

10.41

$$I_{C1} = I_{C2} = \frac{10-0.7-0.7-(-10)}{10} = 1.86\ \text{mA}$$

$$I_{C3} = I_{C4} = 1.86\ \text{mA}$$

$$I_{C5}(0.5) = 0.026 \ln\left(\frac{1.86}{I_{C5}}\right)$$

By Trial and error.

$$\Rightarrow I_{C5} = 0.136\ \text{mA} = I_{C6} = I_{C7}$$

$$2I_{C3}(0.8)+V_{CE3}=10 \Rightarrow \quad V_{CE3}=10-2(1.86)(0.8)$$
$$V_{CE3}=7.02 \text{ V}$$
$$5=V_{EB6}+V_{CE5}+I_{C5}(0.5)-10$$
$$V_{CE5}=5+10-0.7-(0.136)(0.5)$$
$$\underline{V_{CE5}=14.2 \text{ V}}$$
$$5=V_{EC7}+I_{C7}(0.8)$$
$$V_{EC7}=5-(0.136)(0.8)$$
$$\underline{V_{EC7}=4.89 \text{ V}}$$

10.42

$$I_{C1}=I_{C2}=\frac{10-0.7-0.7-(-10)}{10} \Rightarrow I_{C1}=I_{C2}=1.86 \text{ mA}$$
$$I_{C4}=I_{C5}=1.86 \text{ mA}$$
$$I_{C3}R_{E1}=V_T \ln\left(\frac{I_{C1}}{I_{C3}}\right) \Rightarrow I_{C3}(0.3)=0.026\ln\left(\frac{1.86}{I_{C3}}\right)$$

By trial and error $I_{C3}=0.195$ mA

$$I_{C6}R_{E2}=V_T \ln\left(\frac{I_{C5}}{I_{C6}}\right) \Rightarrow I_{C6}(0.5)=0.026\ln\left(\frac{1.86}{I_{C6}}\right)$$

By trial and error $I_{C6}=0.136$ mA

10.43

$$I_{REF}=\frac{10-0.7}{6.3+3}=1 \text{ mA}$$
$V_{BE}(Q_R)=0.7$ V as assumed
$$V_{RER}=I_{REF}\cdot R_{ER}=(1)(3)=3 \text{ V}$$
$$V_{RE1}=3 \text{ V} \Rightarrow R_{E1}=\frac{V_{RE1}}{I_{01}}=\frac{3}{1} \Rightarrow \underline{R_{E1}=3 \text{ k}\Omega}$$
$$V_{RE2}=3 \text{ V} \Rightarrow R_{E2}=\frac{V_{RE2}}{I_{02}}=\frac{3}{2} \Rightarrow \underline{R_{E2}=1.5 \text{ k}\Omega}$$
$$V_{RE3}=3 \text{ V} \Rightarrow R_{E3}=\frac{V_{RE3}}{I_{03}}=\frac{3}{4} \Rightarrow \underline{R_{E3}=0.75 \text{ k}\Omega}$$
$I_{01}=1$ mA
$I_{02}=2$ mA
$I_{03}=4$ mA

10.44

$$I_{REF} = \frac{2.5 - V_{GS}}{15} = \left(\frac{0.08}{2}\right)(6)(V_{GS} - 0.5)^2$$

$$2.5 - V_{GS} = 3.6\left(V_{GS}^2 - V_{GS} + 0.25\right)$$

$$3.6V_{GS}^2 - 2.6V_{GS} - 1.6 = 0$$

$$V_{GS} = \frac{2.6 \pm \sqrt{6.76 + 23.04}}{2(3.6)}$$

$$V_{GS} = 1.12 \text{ V} \left(1.1193\right)$$

$$I_{REF} = \frac{2.5 - 1.1193}{15} \Rightarrow I_{REF} = 92.0\ \mu\text{A}\ \left(92.05\right)$$

$$I_o = 92.0\ \mu\text{A}$$

$$V_{DS2}\left(\text{sat}\right) = V_{GS} - V_{TN} = 1.1193 - 0.5$$

$$V_{DS2}\left(\text{sat}\right) = 0.619 \text{ V}$$

10.45

(a)

$$I_{REF} = 50 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_1 (V_{GS} - 0.5)^2$$

$$I_{REF} = 0.050 = \frac{2.0 - V_{GS}}{R}$$

Design such that $V_{DS2}\left(\text{sat}\right) = 0.25 = V_{GS} - 0.5$

$$V_{GS} = 0.75 \text{ V}$$

So $$0.050 = \frac{2 - 0.75}{R} \Rightarrow \underline{R = 25 \text{ K}}$$

$$50 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_1 (0.75 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 20}$$

$$\frac{\left(\frac{W}{L}\right)_1}{\left(\frac{W}{L}\right)_2} = \frac{I_{REF}}{I_O} \Rightarrow \frac{20}{\left(\frac{W}{L}\right)_2} = \frac{50}{100} \Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 40}$$

(b)

$$R_O = \frac{1}{\lambda I_O} = \frac{1}{(0.015)(0.1)} \Rightarrow R_O = 667 \text{ K}$$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{1}{666} \Rightarrow 1.5\ \mu\text{A}$$

(c)

$$\frac{\Delta I_O}{I_O} \times 100\% = \left(\frac{1.5}{100}\right) \times 100\% \Rightarrow \underline{1.5\%}$$

10.46

$$I_{REF} = 250 = \left(\frac{80}{2}\right)(3)(V_{GS} - 1)^2$$

$$V_{GS} = 2.44 \text{ V}$$

$$I_O = 250\mu\text{A at } V_{DS2} = V_{GS} = 2.44 \text{ V}$$

(a)

$$R_O = \frac{1}{\lambda I_O} = \frac{1}{(0.02)(0.25)} = 200 \text{ K}$$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{3 - 2.44}{200} \Rightarrow 2.8 \ \mu\text{A}$$

(i) $\underline{I_O = 252.8 \ \mu\text{A}}$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{4.5 - 2.44}{200} \Rightarrow 10.3 \ \mu\text{A}$$

(ii) $\underline{I_O = 260.3 \ \mu\text{A}}$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{6 - 2.44}{200} \Rightarrow 17.8 \ \mu\text{A}$$

(iii) $\underline{I_O = 267.8 \ \mu\text{A}}$

$$I_O = \frac{4.5}{3}(250) = 375 \ \mu\text{A at } V_{DS} = 2.44 \text{ V}$$

(b)

$$R_O = \frac{1}{\lambda I_O} = \frac{1}{(0.02)(0.375)} = 133.3 \text{ K}$$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{3 - 2.44}{133.3} \Rightarrow 4.20 \ \mu\text{A}$$

(i) $\underline{I_O = 379.2 \ \mu\text{A}}$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{4.5 - 2.44}{133.3} \Rightarrow 15.5 \ \mu\text{A}$$

(ii) $\underline{I_O = 390.5 \ \mu\text{A}}$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{6 - 2.44}{133.3} \Rightarrow 26.7 \ \mu\text{A}$$

(iii) $\underline{I_O = 401.7 \ \mu\text{A}}$

10.47

(a) $I_{REF} = K_{n1}(V_{GS1} - V_{TN1})^2$

$$0.2 = 0.2(V_{GS1} - 0.4)^2 \Rightarrow V_{GS1} = V_{GS2} = 1.4 \text{ V}$$

$$I_O = K_{n2}(V_{GS2} - V_{TN2})^2$$

Now $I_O = (0.2 - 0.01)(1.4 - 0.4)^2 = 0.19 \text{ mA}$

$$I_O = (0.2 + 0.01)(1.4 - 0.4)^2 = 0.21 \text{ mA}$$

So $0.19 \le I_O \le 0.21 \text{ mA}$

(b) $I_O = (0.2)[1.4-(0.4-0.02)]^2 = 0.2081$ mA

$I_O = (0.2)[1.4-(0.4+0.02)]^2 = 0.1921$ mA

So $0.1921 \le I_O \le 0.2081$ mA

10.48

(a) $I_{REF} = K_{n1}(V_{GS1}-V_{TN})^2$

$0.2 = 0.2(V_{GS1}-0.5)^2 \Rightarrow V_{GS1} = 1.5$ V

$V_{GS1} = V_{GS2} + I_O R_S = V_{GS2} + K_{n2} R_S (V_{GS2}-V_{TN})^2$

$1.5 = V_{GS2} + (0.2)(10)(V_{GS2}^2 - V_{GS2} + 0.25)$

or $2V_{GS2}^2 - V_{GS2} - 1 = 0 \Rightarrow V_{GS2} = 1.0$ V

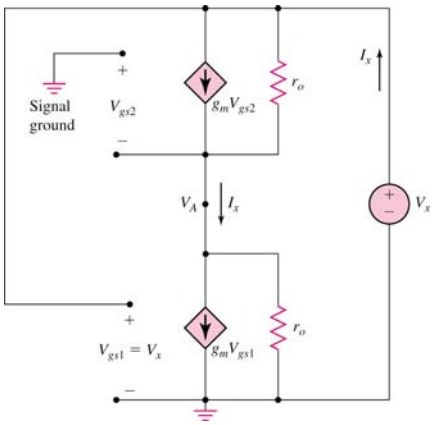
$$I_O = \frac{V_{GS1}-V_{GS2}}{R_S} = \frac{1.5-1.0}{10} \Rightarrow I_O = 50\,\mu\text{A}$$

(b) $I_O = 0.5 I_{REF} = 0.1$ mA

$0.1 = 0.2(V_{GS2}-0.5)^2 \Rightarrow V_{GS2} = 1.207$ V

$$R_S = \frac{V_{GS1}-V_{GS2}}{I_O} = \frac{1.5-1.207}{0.1} = 2.93\,\text{k}\Omega$$

10.49



(1) $$I_x = \frac{V_x - V_A}{r_o} + g_m V_{gs2}$$

$$I_x = \frac{V_A}{r_o} + g_m V_{gs1}$$

(2) $V_{gs1} = V_x$, $V_{gs2} = -V_A$

So

(1) $$I_x = \frac{V_x}{r_o} - V_A\left(\frac{1}{r_o} + g_m\right)$$

(2) $$I_x = \frac{V_A}{r_o} + g_m V_x \Rightarrow V_A = r_o\left[I_x - g_m V_x\right]$$

Then

$$I_x = \frac{V_x}{r_o} - r_o\left(I_x - g_m V_x\right)\left(\frac{1}{r_o} + g_m\right)$$

$$I_x = \frac{V_x}{r_o} - r_o\left[\frac{I_x}{r_o} + g_m I_x - \frac{g_m}{r_o}\cdot V_x - g_m^2 V_x\right]$$

$$I_x = \frac{V_x}{r_o} - I_x - g_m r_o I_x + g_m V_x + g_m^2 r_o V_x$$

$$I_x\left[2 + g_m r_o\right] = V_x\left[\frac{1}{r_o} + g_m + g_m^2 r_o\right]$$

Since $g_m >> \frac{1}{r_o}$

$$I_x\left[2 + g_m r_o\right] \cong V_x\left(g_m\right)\left(1 + g_m r_o\right)$$

Then $\frac{V_x}{I_x} = R_o = \frac{2 + g_m r_o}{g_m\left(1 + g_m r_o\right)}$

Usually, $g_m r_o >> 2,$ so that $R_o \cong \frac{1}{g_m}$

10.50

$$V_{GS2} = V_{DS2}(sat) + V_{TN} = 1 + 0.5 = 1.5 \text{ V}$$

$$I_O = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_2 \left(V_{GS2} - V_{TN}\right)^2$$

$$0.15 = \left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)_2 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 3.75$$

$$I_{REF} = 0.5 = \left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)_1 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 12.5$$

$$V_{GS3} = \left(V^+ - V^-\right) - V_{GS1} = 1.8 - (-1.8) - 1.5 = 2.1 \text{ V}$$

$$I_{REF} = 0.5 = \left(\frac{0.08}{2}\right)\left(\frac{W}{L_3}\right)(2.1 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 4.88$$

10.51

(a)

$$I_{REF} = \left(\frac{60}{2}\right)(20)(V_{GS1} - 0.7)^2 = \left(\frac{60}{2}\right)(3)(V_{GS3} - 0.7)^2$$

$$V_{GS1} + V_{GS3} = 5$$

$$\sqrt{\frac{20}{3}}(V_{GS1} - 0.7) = 5 - V_{GS1} - 0.7$$

$$3.582V_{GS1} = 6.107 \Rightarrow V_{GS1} = V_{GS2} = 1.705 \text{ V}$$

$$I_O = \left(\frac{60}{2}\right)(12)(1.705 - 0.7)^2 = 363.6 \ \mu\text{A at } V_{DS2} = 1.705 \text{ V}$$

$$I_{REF} = \left(\frac{60}{2}\right)(20)(1.705 - 0.7)^2 = 606 \ \mu\text{A}$$

$$R_O = \frac{1}{\lambda I_O} = \frac{1}{(0.015)(0.3636)} = 183.4 \text{ K}$$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{1.5 - 1.705}{183.4} \Rightarrow -1.12 \ \mu\text{A}$$

(b) $I_O = 362.5 \ \mu\text{A}$

$$\Delta I_O = \frac{\Delta V}{R_O} = \frac{3 - 1.705}{183.4} \Rightarrow 7.06 \ \mu\text{A}$$

(c) $I_O = 370.7 \ \mu\text{A}$

10.52

$$I_{REF} = \left(\frac{50}{2}\right)(15)(V_{SG1} - 0.5)^2 = \left(\frac{50}{2}\right)(3)(V_{SG3} - 0.5)^2$$

$$V_{SG1} + V_{SG3} = 10 \Rightarrow V_{SG3} = 10 - V_{SG1}$$

$$\sqrt{\frac{15}{3}}(V_{SG1} - 0.5) = 10 - V_{SG1} - 0.5$$

$$3.236V_{SG1} = 10.618 \Rightarrow V_{SG1} = 3.28 \text{ V}$$

$$I_{REF} = \left(\frac{50}{2}\right)(15)(3.28 - 0.5)^2 \Rightarrow I_{REF} = 2.90 \text{ mA}$$

$$I_O = I_{REF} = 2.90 \text{ mA}$$

$$V_{SD2}(\text{sat}) = V_{SG2} + V_{TP} = 3.28 - 0.5 \Rightarrow \underline{V_{SD2}(\text{sat}) = 2.78 \text{ V}}$$

10.53

$$V_{SD2}(\text{sat}) = 1.2 = V_{SG2} - 0.35 \Rightarrow V_{SG2} = 1.55 \text{ V}$$

$$I_O = 100 = \left(\frac{50}{2}\right)\left(\frac{W}{L}\right)_2 (1.55 - 0.35)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 2.78$$

$$\frac{I_{REF}}{I_O} = \frac{\left(W/L\right)_1}{\left(W/L\right)_2} \Rightarrow \frac{200}{100} = \frac{\left(W/L\right)_1}{2.78} \Rightarrow \left(\frac{W}{L}\right)_1 = 5.56$$

$$V_{SG1} + V_{SG3} = 4 \Rightarrow V_{SG3} = 2.45 \text{ V}$$

$$I_{REF} = 200 = \left(\frac{50}{2}\right)\left(\frac{W}{L}\right)_3 (2.45 - 0.35)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 1.81$$

10.54

$$I_{REF} = \left(\frac{80}{2}\right)(25)(V_{SG1} - 1.2)^2 = \left(\frac{80}{2}\right)(4)(V_{SG3} - 1.2)^2$$

$$V_{SG1} + 2V_{SG3} = 10 \Rightarrow V_{SG3} = \frac{10 - V_{SG1}}{2}$$

Then $$\sqrt{\frac{25}{4}}(V_{SG1} - 1.2) = \frac{10 - V_{SG1}}{2} - 1.2$$

$$3V_{SG1} = 6.8 \Rightarrow V_{SG1} = 2.27 \text{ V}$$

$$I_{REF} = \left(\frac{80}{2}\right)(25)(2.267 - 1.2)^2 \Rightarrow \underline{I_{REF} = I_O = 1.14 \text{ mA}}$$

$$V_{SD2}(\text{sat}) = V_{SG2} + V_{TP} = 2.27 - 1.2 \Rightarrow \underline{V_{SD2}(\text{sat}) = 1.07 \text{ V}}$$

10.55

$$V_{SG2} = V_{SD2}(sat) - V_{TP} = 1 + 0.5 = 1.5 \text{ V}$$

$$I_O = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_2 (V_{SG2} + V_{TP})^2$$

$$80 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_2 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 2.67$$

$$I_{REF} = 250 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_1 (1.5 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 8.33$$

$$V_{SG3} = V_{SG4}$$

Then $2V_{SG3} = V^+ - V^- - V_{SG1} = 3 - (-3) - 1.5 = 4.5 \text{ V}$

So $V_{SG3} = 2.25 \text{ V}$

$$250 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_{3,4} (2.25 - 0.5)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 2.72$$

10.56

(a) $I_{REF} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_1 (V_{SG1} + V_{TP})^2 = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$

$V_{SG3} = 3 - V_{SG1}$

$\sqrt{25}(V_{SG1} - 0.4) = \sqrt{5}(3 - V_{SG1} - 0.4)$

$3.236V_{SG1} = 3.4944 \Rightarrow V_{SG1} = 1.08\text{ V}$ and $V_{SG3} = 1.92\text{ V}$

$I_{REF} = \left(\frac{60}{2}\right)(25)(1.08 - 0.4)^2 \Rightarrow I_{REF} = 0.347\text{ mA}$

$I_O = \left(\frac{60}{2}\right)(15)(1.08 - 0.4)^2 \Rightarrow I_O = 0.208\text{ mA}$

(b) $V_{SD2}(sat) = V_{SG2} + V_{TP} = 1.08 - 0.4 = 0.68\text{ V}$

$R = \frac{3 - 0.68}{0.208} = 11.15\text{ k}\Omega$

10.57

$V_{SD2}(sat) = 0.35 = V_{SG2} + V_{TP} = V_{SG2} - 0.4 \Rightarrow V_{SG2} = 0.75\text{ V}$

$I_{REF} = 220 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_1 (0.75 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 59.9$

$I_O = 80 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_2 (0.75 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 21.8$

$V_{SG3} = 3 - 0.75 = 2.25\text{ V}$

$220 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_3 (2.25 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 2.14$

10.58

(a) $I_{REF} = 100 = 100(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 1.5\text{ V}$

For $V_{D4} = -2\text{ V}$, $V_{DS4} + V_{DS2} = 3\text{ V} = V_{DS3} + V_{DS1}$

Then $I_O = 100\ \mu\text{A}$

(b) $R_o = r_{o4} + r_{o2}(1 + g_m r_{o4})$

$g_m = 2\sqrt{K_n I_O} = 2\sqrt{(0.1)(0.1)} = 0.2\text{ mA/V}$

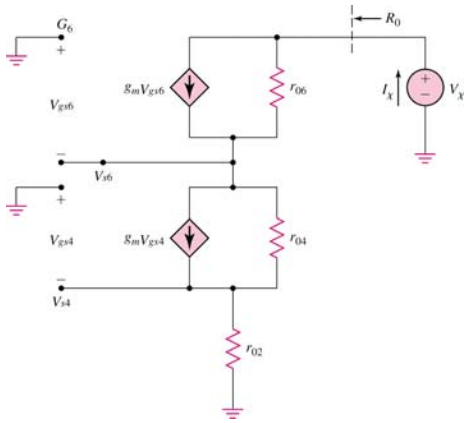
$r_{o2} = r_{o4} = \frac{1}{\lambda I_O} = \frac{1}{(0.02)(0.1)} = 500\text{ k}\Omega$

$R_o = 500 + 500[1 + (0.2)(500)] \Rightarrow R_o = 51\text{ M}\Omega$

$\Delta I_O = \frac{\Delta V_{D4}}{R_o} = \frac{4}{51} = 0.07843\ \mu\text{A}$

$\frac{\Delta I_O}{I_O} \times 100\% = \left(\frac{0.07843}{100}\right) \times 100\% = 0.07843\%$

10.59



$$V_{gs4} = -I_X r_{02}$$

$$V_{S6} = \left(I_X - g_m V_{gs4}\right) r_{04} + I_X r_{02}$$

$$= \left(I_X + g_m I_X r_{02}\right) r_{04} + I_X r_{02}$$

$$V_{S6} = I_X\left[r_{02} + \left(1 + g_m r_{02}\right) r_{04}\right] = -V_{gs6}$$

$$I_X = g_m V_{gs6} + \frac{V_X - V_{S6}}{r_{06}} = \frac{V_X}{r_{06}} - V_{S6}\left(g_m + \frac{1}{r_{06}}\right)$$

$$I_X = \frac{V_X}{r_{06}} - I_X\left(g_m + \frac{1}{r_{06}}\right)\left[r_{02} + \left(1 + g_m r_{02}\right) r_{04}\right]$$

$$I_X\left\{1 + \left(g_m + \frac{1}{r_{06}}\right)\left[r_{02} + \left(1 + g_m r_{02}\right) r_{04}\right]\right\} = \frac{V_X}{r_{06}}$$

$$\frac{V_X}{I_X} = R_0 = r_{06} + \left(1 + g_m r_{06}\right)\left[r_{02} + \left(1 + g_m r_{02}\right) r_{04}\right]$$

$$I_0 \approx I_{REF} = 0.2 \text{ mA} = 0.2\left(V_{GS} - 1\right)^2$$

$$V_{GS} = 2 \text{ V}$$

$$g_m = 2K_n\left(V_{GS} - V_{TN}\right) = 2(0.2)(2-1) = 0.4 \; mA/V$$

$$r_{02} = r_{04} = r_{06} = \frac{1}{\lambda I_0} = \frac{1}{(0.02)(0.2)} = 250 \text{ k}\Omega$$

$$R_0 = 250 + \left[1 + (0.4)(250)\right] \times \left\{250 + \left[1 + (0.4)(250)\right](250)\right\}$$

$$R_0 = 2575750 \text{ k}\Omega \Rightarrow \underline{R_0 = 2.58 \times 10^9 \, \Omega}$$

10.60

$$I_{REF} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_3 \left(V_{GS3} - V_{TN}\right)^2 = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_4 \left(V_{SG4} + V_{TP}\right)^2$$

$$\left(\frac{100}{2}\right)(5)\left(V_{GS3} - 0.4\right)^2 = \left(\frac{60}{2}\right)(10)\left(V_{SG4} - 0.4\right)^2$$

We find $V_{SG4} = 0.91287 V_{GS3} + 0.03485$

$$I_{REF} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 \left(V_{GS1} - V_{TN}\right)^2 = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_3 \left(V_{GS3} - V_{TN}\right)^2$$

$$\sqrt{20}\left(V_{GS1}-0.4\right)=\sqrt{5}\left(V_{GS3}-0.4\right)$$

Now $V_{SG4}+V_{GS3}+V_{GS1}=6$

$$0.91287V_{GS3}+0.03485+V_{GS3}+V_{GS1}=6$$

Then $V_{GS3}=3.1184-0.52277V_{GS1}$

And $2\left(V_{GS1}-0.4\right)=3.1184-0.52277V_{GS1}-0.4$

So $V_{GS1}=1.395$ V

$V_{GS3}=2.389$ V

$V_{SG4}=2.216$ V

$$I_{REF}=\left(\frac{0.1}{2}\right)(20)(1.395-0.4)^2=0.99\text{ mA}$$

$$I_O=\left(\frac{0.1}{2}\right)(20)(1.395-0.4)^2=0.99\text{ mA}$$

$$V_{DS2}(sat)=1.395-0.4=0.995\text{ V}$$

10.61

$$V_{DS2}(sat)=0.5\text{ V}=V_{GS2}-V_{TN}=V_{GS2}-0.4\Rightarrow V_{GS2}=0.9\text{ V}$$

$$I_O=\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_2\left(V_{GS2}-V_{TN}\right)^2$$

$$50=\left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_2(0.9-0.4)^2\Rightarrow\left(\frac{W}{L}\right)_2=4$$

$$I_{REF}=500=\left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_1(0.9-0.4)^2\Rightarrow\left(\frac{W}{L}\right)_1=40$$

$$V_{GS3}=V_{SG4}$$

$$6=V_{GS1}+V_{GS3}+V_{SG4}=0.9+2V_{GS3}\Rightarrow V_{GS3}=V_{SG4}=2.55\text{ V}$$

$$500=\left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_3(2.55-0.4)^2\Rightarrow\left(\frac{W}{L}\right)_3=2.16$$

$$500=\left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_4(2.55-0.4)^2\Rightarrow\left(\frac{W}{L}\right)_4=3.61$$

10.62

a. As a first approximation

$$I_{REF}=80=80\left(V_{GS1}-1\right)^2\Rightarrow V_{GS1}=2\text{ V}$$

Then $V_{DS1}\cong 2(2)=4$ V

The second approximation

$$80=80\left(V_{GS1}-1\right)^2\left[1+(0.02)(4)\right]$$

Or $$\frac{80}{86.4}=\left(V_{GS1}-1\right)^2\Rightarrow V_{GS1}=1.962$$

Then

$$I_O = K_n\left(V_{GS1} - V_{TN}\right)^2\left(1+\lambda_n V_{GS1}\right)$$
$$= 80\left(1.962-1\right)^2\left[1+\left(0.02\right)\left(1.962\right)\right]$$

Or $\underline{I_0 = 76.94\ \mu\text{A}}$

b. From a PSpice analysis, $I_0 = 77.09\ \mu\text{A}$ for $V_{D3} = -1\ \text{V}$ and $I_0 = 77.14\ \mu\text{A}$ for $V_{D3} = 3\ \text{V}$.
The change is $\Delta I_0 \approx 0.05\ \mu\text{A}$ or 0.065%.

10.63

a. For a first approximation,

$$I_{REF} = 80 = 80\left(V_{GS4} - 1\right)^2 \Rightarrow V_{GS4} = 2\ \text{V}$$

As a second approximation

$$I_{REF} = 80 = 80\left(V_{GS4} - 1\right)^2\left[1+\left(0.02\right)\left(2\right)\right]$$

Or $V_{GS4} = 1.98\ \text{V} = V_{GS1}$

$$I_O = K_n\left(V_{GS2} - V_{TN}\right)^2\left(1+\lambda V_{GS2}\right)$$

To a very good approximation $I_0 = 80\ \mu\text{A}$

b. From a PSpice analysis, $I_0 = 80.00\ \mu\text{A}$ for $V_{D3} = -1\ \text{V}$ and the output resistance is $R_0 = 76.9\ \text{M}\Omega$.

Then

For $V_D = +3\ \text{V}$

$$\Delta I_0 = \frac{1}{R_0}\cdot V_{D3} = \frac{4}{76.9} = 0.052\ \mu\text{A}$$

$$I_0 = 80.05\ \mu\text{A}$$

10.64

$$V_{DS3}\left(sat\right) = V_{GS3} - V_{TN} \text{ or } V_{GS3} = V_{DS3}\left(sat\right) + V_{TN} = 0.2 + 0.8 = 1.0$$

$$I_D = \frac{k_n'}{2}\left(\frac{W}{L}\right)\left(V_{GS3} - V_{TN}\right)^2$$

(a)
$$50 = 48\left(\frac{W}{L}\right)\left(0.2\right)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 26}$$

$$V_{GS5} - V_{TN} = 2\left(V_{GS3} - V_{TN}\right)$$

(b)
$$V_{GS5} = 0.8 + 2\left(0.2\right) \Rightarrow \underline{V_{GS5} = 1.2\ V}$$

(c)
$$V_{D1}\left(\min\right) = 2V_{DS}\left(sat\right) = 2\left(0.2\right) \Rightarrow \underline{V_{D1}\left(\min\right) = 0.4\ V}$$

10.65

(a)

$$K_{n1} = \frac{k_n'}{2}\left(\frac{W}{L}\right)_1 = 50(5) = 250\ \mu A/V^2$$

$$R = \frac{1}{\sqrt{K_{n1} I_{D1}}}\left(1 - \sqrt{\frac{(W/L)_1}{(W/L)_2}}\right)$$

$$= \frac{1}{\sqrt{(0.25)(0.05)}}\left(1 - \sqrt{\frac{5}{50}}\right) = (8.944)(0.6838)$$

$$R = 6.12\ k\Omega$$

(b)

$$V^+ - V^- = V_{SD3}(sat) + V_{GS1}$$

$$V_{SD3}(sat) = V_{SG3} + V_{TP}$$

$$I_{D1} = 50 = 20(5)(V_{SG3} - 0.5)^2 \Rightarrow V_{SG3} = 1.207\ V$$

Then $V_{SD3}(sat) = 1.21 - 0.5 = 0.707\ V$

Also $I_{D1} = 50 = 50(5)(V_{GS1} - 0.5)^2 \Rightarrow V_{GS1} = 0.9472\ V$

Then $\underline{\left(V^+ - V^-\right)_{\min} = 0.71 + 0.947 = 1.66\ V}$

(c)

$$I_{O1} = 25 = 50\left(\frac{W}{L}\right)_5 (0.947 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_5 = 2.5}$$

$$I_{O2} = 75 = 20\left(\frac{W}{L}\right)_6 (1.207 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_6 = 7.5}$$

10.66

$$V_{GS3} = V_{GS4} = V_{GS5} \Rightarrow V_{GS3} = \frac{5}{3}\ \text{V}$$

$$I_{REF} = 0.1 = \left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)_3 \left(\frac{5}{3} - 0.7\right)^2 \Rightarrow \left(\frac{W}{L}\right)_{3,4,5} = 2.68$$

$$I_{O1} = 0.2 = \left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)_1 \left(\frac{5}{3} - 0.7\right)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 5.35$$

$$I_{O2} = 0.3 = \left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)_2 \left(\frac{5}{3} - 0.7\right)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 8.03$$

10.67

$$P = \left(I_{REF} + I_{O1} + I_{O2} + I_{O3}\right)\left(V^+ - V^-\right)$$

$$5 = \left(I_{REF} + 0.1 + 0.2 + 0.4\right)\left[1.8 - (-1.8)\right] \Rightarrow I_{REF} = 0.689\ \text{mA}$$

$$V_{DS2}(sat) = 0.4 = V_{GS2} - 0.4 \Rightarrow V_{GS2} = 0.8\ \text{V} = V_{GS}$$

$$I_{REF} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 (V_{GS1} - V_{TN})^2$$

$$0.689 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_1 (0.8-0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_1 = 86.1$$

$$I_{O1} = 0.1 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_2 (0.8-0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_2 = 12.5$$

$$I_{O2} = 0.2 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_3 (0.8-0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_3 = 25$$

$$I_{O3} = 0.4 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_4 (0.8-0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_4 = 50$$

10.68

$$24 = I_{REF}R + V_{SG} + V_{GS}$$

$$V_{SG} = \sqrt{\frac{I_{REF}}{\left(\frac{k_p'}{2}\right)(1)}} - V_{TP} = 5.7735\sqrt{I_{REF}} + 0.8$$

$$V_{GS} = \sqrt{\frac{I_{REF}}{\left(\frac{k_n'}{2}\right)(1)}} + V_{TN} = 4.472\sqrt{I_{REF}} + 0.8$$

So $24 = I_{REF}(100) + 5.7735\sqrt{I_{REF}} + 0.8 + 4.4721\sqrt{I_{REF}} + 0.8$

Let $\sqrt{I_{REF}} = x$

Then $100x^2 + 10.2456x - 22.4 = 0 \Rightarrow x = 0.4248 \Rightarrow x^2 = I_{REF} = 0.1805$ mA

$I_1 = (0.2)I_{REF} = 0.0361$ mA

$I_2 = (1.25)I_{REF} = 0.2256$ mA

$I_3 = (0.8)I_{REF} = 0.1444$ mA

$I_4 = (4)I_{REF} = 0.722$ mA

10.69

$$10 = I_{REF}R + V_{SG} + V_{GS}$$

$$V_{SG} = \sqrt{\frac{I_{REF}}{\left(\frac{0.06}{2}\right)(1)}} + 0.8 = 5.7735\sqrt{I_{REF}} + 0.8$$

$$V_{GS} = \sqrt{\frac{I_{REF}}{\left(\frac{0.1}{2}\right)(1)}} + 0.8 = 4.472\sqrt{I_{REF}} + 0.8$$

$$8.4 = 100I_{REF} + 10.2456\sqrt{I_{REF}}$$

Let $\sqrt{I_{REF}} = x$

Then $100x^2 + 10.2456x - 8.4 = 0 \Rightarrow x = 0.2431 \Rightarrow x^2 = I_{REF} = 59.09\,\mu$A

$I_1 = (0.2)I_{REF} = 11.82\,\mu$A, $I_2 = (1.25)I_{REF} = 73.87\,\mu$A

$I_3 = (0.8)I_{REF} = 47.27\,\mu$A, $I_4 = (4)I_{REF} = 236.4\,\mu$A

10.70

$$I_{D2} = \frac{\left(W/L\right)_2}{\left(W/L\right)_1} \cdot I_{REF} = \frac{9}{15}(200) \Rightarrow \underline{I_{D2} = 120\ \mu\text{A}}$$

$$I_O = \frac{\left(W/L\right)_4}{\left(W/L\right)_3} \cdot I_{D2} = \left(\frac{20}{9}\right)(120) \Rightarrow \underline{I_O = 267\ \mu\text{A}}$$

$$I_O = 266.7 = \left(\frac{40}{2}\right)(20)\left(V_{SG4} - 0.6\right)^2$$

$$V_{SG4} = 1.416\ \text{V}$$

$$V_{SD4}(\text{sat}) = 1.416 - 0.6 \Rightarrow \underline{V_{SD4}(\text{sat}) = 0.816\ \text{V}}$$

10.71

For M_1: $I_{REF} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 \left(V_{GS1} - V_{TN}\right)^2$

$$100 = \left(\frac{100}{2}\right)(4)\left(V_{GS1} - 0.4\right)^2 \Rightarrow V_{GS1} = V_{GS2} = 1.107\ \text{V}$$

$$I_{D2} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_2 \left(V_{GS2} - V_{TN}\right)^2 = \left(\frac{100}{2}\right)(2.5)(1.107 - 0.4)^2 \Rightarrow I_{D2} = 62.5\ \mu\text{A}$$

Also $I_{D2} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_3 \left(V_{SG3} + V_{TP}\right)^2$

$$62.5 = \left(\frac{60}{2}\right)(6)\left(V_{SG3} - 0.4\right)^2 \Rightarrow V_{SG3} = V_{SG4} = 0.9893\ \text{V}$$

$$I_O = \left(\frac{60}{2}\right)(4)(0.9893 - 0.4)^2 = 41.67\ \mu\text{A}$$

10.72

$$I_{REF} = 50 = \left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_1 \left(V_{SG1} - 0.6\right)^2$$

$$I_{REF} = \frac{1.75 - V_{SG1}}{R} = 50$$

$$V_{SD2}(\text{sat}) = 0.35 = V_{SG2} - 0.6 \Rightarrow V_{SG2} = 0.95\ \text{V}$$

$$R = \frac{1.75 - 0.95}{0.05} \Rightarrow \underline{R = 16\ \text{K}}$$

$$50 = \left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_1 (0.95 - 0.6)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_1 = 20.4}$$

$$\frac{I_{O1}}{I_{REF}} = \frac{120}{50} = \frac{\left(W/L\right)_2}{(20.4)} \Rightarrow \underline{\left(\frac{W}{L}\right)_2 = 49}$$

$$\frac{I_{D3}}{I_{REF}} = \frac{25}{50} = \frac{\left(W/L\right)_3}{(20.4)} \Rightarrow \underline{\left(\frac{W}{L}\right)_3 = 10.2}$$

$$V_{DS5}(\text{sat}) = 0.35 = V_{GS5} - 0.4 \Rightarrow V_{GS5} = 0.75 \text{ V}$$

$$I_{O2} = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_5 (0.75 - 0.4)^2 = 150 \Rightarrow \left(\frac{W}{L}\right)_5 = 24.5$$

$$\frac{I_{D4}}{I_{O2}} = \frac{I_{D3}}{I_{O2}} = \frac{25}{150} = \frac{\left(W/L\right)_4}{24.5} \Rightarrow \left(\frac{W}{L}\right)_4 = 4.08$$

10.73

For $v_{GS} = 0,\ i_D = I_{DSS}\left(1 + \lambda v_{DS}\right)$

a. $V_D = -5 \text{ V}, \quad v_{DS} = 5$

$$i_D = (2)\left[1 + (0.05)(5)\right] \Rightarrow \underline{i_D = 2.5 \text{ mA}}$$

b. $V_D = 0, \quad v_{DS} = 10$

$$i_D = (2)\left[1 + (0.05)(10)\right] \Rightarrow \underline{i_D = 3 \text{ mA}}$$

c. $V_D = 5 \text{ V}, \quad v_{DS} = 15 \text{ V}$

$$i_D = (2)\left[1 + (0.05)(15)\right] \Rightarrow \underline{i_D = 3.5 \text{ mA}}$$

10.74

$$I_0 = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$2 = 4\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\frac{V_{GS}}{V_P} = 1 - \sqrt{\frac{2}{4}} = 0.293$$

So $V_{GS} = (0.293)(-4) = -1.17 \text{ V}$

Then $I_0 = \dfrac{V_S}{R}$ and $V_S = -V_{GS}$

$$R = \frac{-V_{GS}}{I_0} = -\frac{(-1.17)}{2} \Rightarrow \underline{R = 0.586 \text{ k}\Omega}$$

Need $v_{DS} \geq v_{DS}(\text{sat}) = v_{GS} - V_P$

$= -1.17 - (-4)$

$v_{DS} \geq 2.83 \text{ V}$

So $V_D \geq v_{DS}(\text{sat}) + V_S = 2.83 + 1.17 \Rightarrow \underline{V_D \geq 4 \text{ V}}$

10.75

(a) $$A_\upsilon = \frac{-\frac{1}{V_T}}{\frac{1}{V_{AN}}+\frac{1}{V_{AP}}} = \frac{-\frac{1}{0.026}}{\frac{1}{120}+\frac{1}{80}} = \frac{-38.46}{0.00833+0.0125} \Rightarrow A_\upsilon = -1846$$

(b) $$I_{CQ} = I_{S1}\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1+\frac{V_{CE1}}{V_{AN}}\right)$$

$$200\times10^{-6} = \left(5\times10^{-16}\right)\left[\exp\left(\frac{V_I}{V_T}\right)\right]\left(1+\frac{1.25}{120}\right)$$

or $$V_I = (0.026)\ln\left[\frac{200\times10^{-6}}{\left(5\times10^{-16}\right)(1+1.25/120)}\right] = 0.6943\text{ V}$$

(c) $$200\times10^{-6} = \left(10^{-15}\right)\left[\exp\left(\frac{V_{EB}}{V_T}\right)\right]\left(1+\frac{1.25}{80}\right)$$

$$V_{EB} = (0.026)\ln\left[\frac{200\times10^{-6}}{\left(10^{-15}\right)(1+1.25/80)}\right] = 0.6762\text{ V}$$

$$V_{EB} = 0.6762 = V^+ - V_B \Rightarrow V_B = 1.824\text{ V}$$

10.76

(a) $$A_\upsilon = -g_{m1}\left(r_{o1}\|r_{o2}\right)$$

$$r_{o1} = \frac{1}{\lambda_1 I_{DQ}} = \frac{1}{(0.02)(0.2)} = 250\text{ k}\Omega,\quad r_{o2} = \frac{1}{\lambda_2 I_{DQ}} = \frac{1}{(0.03)(0.2)} = 166.7\text{ k}\Omega$$

$$-100 = -g_{m1}\left(250\|166.7\right) \Rightarrow g_{m1} = 1\text{ mA/V}$$

$$g_{m1} = 1 = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 I_{DQ}} = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_1 (0.2)} \Rightarrow \left(\frac{W}{L}\right)_1 = 25$$

(b) $$I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 (V_I - V_{TN})^2 (1+\lambda_1 V_{DS1})$$

$$0.2 = \left(\frac{0.1}{2}\right)(25)(V_I - 0.5)^2\left[1+(0.02)(1.25)\right] \Rightarrow V_I = 0.895\text{ V}$$

(c) $$K_{n1} = K_{p2} \Rightarrow \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_2$$

Then $$\left(\frac{W}{L}\right)_2 = \left(\frac{100}{60}\right)(25) = 41.67$$

$$I_{DQ} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_2 (V_{SG2} + V_{TP})^2 (1+\lambda_2 V_{SD2})$$

$$0.2 = \left(\frac{0.06}{2}\right)(41.67)(V_{SG2} - 0.5)^2\left[1+(0.03)(1.25)\right] \Rightarrow V_{SG2} = 0.8927\text{ V}$$

$$V_{SG2} = 0.8927 = V^+ - V_G \Rightarrow V_G = 1.607\text{ V}$$

10.77

a.
$$I_{REF} = I_{S1}\exp\left(\frac{V_{EB1}}{V_T}\right)$$
$$\text{or } V_{EB1} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026)\ln\left(\frac{1\times10^{-3}}{5\times10^{-13}}\right) \Rightarrow \underline{V_{EB1} = 0.5568}$$

b.
$$R_1 = \frac{5 - 0.5568}{1} \Rightarrow \underline{R_1 = 4.44\ \text{k}\Omega}$$

c. From Equations (10.79) and (10.80) and letting $V_{CE0} = V_{EC2} = 2.5$ V
$$10^{-12}\exp\left(\frac{V_I}{V_T}\right)\left[1+\frac{2.5}{120}\right] = 10^{-3}\left(\frac{1+\frac{2.5}{80}}{1+\frac{0.5568}{80}}\right)$$
$$1.0208333\times10^{-12}\exp\left(\frac{V_I}{V_T}\right) = \left(10^{-3}\right)\left(\frac{1.03125}{1.00696}\right)$$
Then $V_I = 0.026\ln\left(1.003222\times10^{9}\right)$
So $V_I = 0.5389\ V$

d.
$$A_v = \frac{-(1/V_T)}{(1/V_{AN})+(1/V_{AP})}$$
$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120}+\frac{1}{80}} = \frac{-38.46}{0.00833+0.0125}$$
$$\underline{A_v = -1846}$$

10.78

a. $$V_{BE} = V_T \ln\left(\frac{I_{REF}}{I_{S1}}\right) = (0.026)\ln\left(\frac{0.5\times10^{-3}}{10^{-12}}\right) \Rightarrow \underline{V_{BE} = 0.5208}$$

b. $$R_1 = \frac{5 - 0.5208}{0.5} \Rightarrow \underline{R_1 = 8.96\ \text{k}\Omega}$$

c. Modify Eqs. 10.79 and 10.80 to apply to pnp and npn, and set the two equation equal to each other.

$$I_{CO} = I_{SO}\exp\left(\frac{V_{EBO}}{V_T}\right)\left(1+\frac{V_{ECO}}{V_{AP}}\right) = I_{C2} = I_{S2}\exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{V_{CE2}}{V_{AN}}\right)$$

$$5\times10^{-13}\exp\left(\frac{V_{EBO}}{V_T}\right)\left(1+\frac{2.5}{80}\right) = 10^{-12}\exp\left(\frac{V_{BE}}{V_T}\right)\left(1+\frac{2.5}{120}\right)$$

$$5.15625\times10^{-13}\exp\left(\frac{V_{EBO}}{V_T}\right) = 1.020833\times10^{-12}\exp\left(\frac{V_{BE}}{V_T}\right)$$

$$\frac{\exp\left(\frac{V_{EBO}}{V_T}\right)}{\exp\left(\frac{V_{BE}}{V_T}\right)} = 1.9798 = \exp\left(\frac{V_{EBO} - V_{BE}}{V_T}\right)$$

$$V_{EBO} = V_{BE} + V_T \ln(1.9798) = 0.5208 + (0.026)\ln(1.9798)$$

$$V_{EBO} = 0.5386 \Rightarrow V_I = 5 - 0.5386 \Rightarrow \underline{V_I = 4.461\ \text{V}}$$

$$A_v = \frac{-(1/V_T)}{(1/V_{AN}) + (1/V_{AP})}$$

$$A_v = \frac{-\frac{1}{0.026}}{\frac{1}{120}+\frac{1}{80}} = \frac{-38.46}{0.00833 + 0.0125}$$

d. $$\underline{A_v = -1846}$$

10.79

(a) For M_O: $$I_{DQ} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_O (V_I - V_{TN})^2(1+\lambda_n V_{DSO})$$

$$100 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_O (1.2-0.5)^2[1+(0.02)(1.5)] \Rightarrow \left(\frac{W}{L}\right)_O = 3.96$$

For M_1, M_2: For $I_{REF} = I_O \Rightarrow V_{SD2} = V_{SD1} = V_{SG} = 1.5$ V

$$100 = \left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_2 (1.5-0.5)^2[1+(0.02)(1.5)] \Rightarrow \left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1 = 3.24$$

For M_3: $$V_{SG3} = 3 - 1.5 = 1.5\ \text{V} = V_{SD3} \Rightarrow \left(\frac{W}{L}\right)_3 = 3.24$$

(b) $A_\upsilon = -g_{mO}\left(r_{oO} \| r_{o2}\right)$

$$g_{mO} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_O I_{DQ}} = 2\sqrt{\left(\frac{0.1}{2}\right)(3.96)(0.1)} = 0.2814 \text{ mA/V}$$

$$r_{oO} = r_{o2} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$A_\upsilon = -(0.2814)(500 \| 500) = -70.35$$

10.80

(a) $I_{REF} = K_p\left(V_{SG} + V_{TP}\right)^2\left(1 + \lambda_p V_{SD1}\right)$

$$0.080 = 0.1(V_{SG} - 0.5)^2\left[1 + (0.02)V_{SG}\right]$$

Approximation, for $\lambda = 0 \Rightarrow V_{SG} = 1.394$ V

(b) $I_O = I_2$

$$K_n\left(V_I - V_{TN}\right)^2\left(1 + \lambda_n V_{DSO}\right) = K_p\left(V_{SG} + V_{TP}\right)^2\left(1 + \lambda_p V_{SD2}\right)$$

$K_n = K_p$, $V_{DSO} = V_{SD2}$, $\lambda_n = \lambda_p$

So $V_I = 1.394$ V

(c) $I_O \cong I_{REF} = 80\,\mu$ A

$A_\upsilon = -g_{mo}\left(r_{oo} \| r_{o2}\right)$

$$g_{mo} = 2\sqrt{K_n I_O} = 2\sqrt{(0.1)(0.08)} = 0.1789 \text{ mA/V}$$

$$r_{oo} = r_{o2} = \frac{1}{\lambda I_O} = \frac{1}{(0.02)(0.08)} = 625 \text{ k}\Omega$$

$$A_\upsilon = -(0.1789)(625 \| 625) = -55.9$$

10.81

(a) $I_{REF} = I_{CO} = \dfrac{3 - 0.6}{47} \Rightarrow I_{CO} = 51.06\,\mu$ A

$$r_{oo} = \frac{V_{AN}}{I_{CO}} = \frac{120}{0.05106} \Rightarrow r_{oo} = 2.35 \text{ M}\Omega$$

$$r_{o2} = \frac{V_{AP}}{I_{CO}} = \frac{90}{0.05106} \Rightarrow r_{o2} = 1.763 \text{ M}\Omega$$

$$g_m = \frac{I_{CO}}{V_T} = \frac{0.05106}{0.026} = 1.964 \text{ mA/V}$$

$$A_\upsilon = -g_m\left(r_{oo} \| r_{o2}\right) = -(1.964)(2350 \| 1763) = -1978$$

(b) $A_\upsilon = -g_m\left(r_{oo} \| r_{o2} \| R_L\right) = -(1.964)(2350 \| 1763 \| 300) = -454$

(c) $A_\upsilon = -(1.964)(2350 \| 1763 \| 150) = -256$

10.82

$$I_{REF} = \frac{5 - 0.6}{35} = 0.1257 \text{ mA}$$

Then

$$I_{CO} = 2I_{REF} = 0.2514 \text{ mA}$$

From Eq. 10.96

$$A_v = \frac{\dfrac{-0.2514}{0.026}}{\dfrac{0.2514}{120} + \dfrac{0.2514}{80} + \dfrac{1}{R_L}} = \frac{-9.6692}{0.002095 + 0.0031425 + \dfrac{1}{R_L}}$$

$$A_v = \frac{-9.6692}{0.0052375 + \dfrac{1}{R_L}}$$

(a) $R_L = \infty \; A_v = -1846$

(b) $R_L = 250 \text{ K}, \; A_v = -1047$

10.83

(a) To a good approximation, output resistance is the same as the widlar current source.

$$R_0 = r_{02}\left[1 + g_{m2}\left(r_{\pi 2} \,\|\, R_E\right)\right]$$

(b) $$A_v = -g_{m0}\left(r_0 \,\|\, R_L \,\|\, R_0\right)$$

10.84

Output resistance of Wilson source

$$R_0 \cong \frac{\beta r_{03}}{2}$$

Then

$$A_v = -g_m\left(r_0 \,\|\, R_0\right)$$

$$r_{03} = \frac{V_{AP}}{I_{REF}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

$$r_0 = \frac{V_{AN}}{I_{REF}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$g_m = \frac{I_{REF}}{V_T} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$A_v = -7.69\left[600 \left\| \frac{(80)(400)}{2} \right.\right] = -7.69\left[600 \,\|\, 16{,}000\right] \Rightarrow \underline{A_v = -4448}$$

10.85

(a) $I_{D2} = I_{D0} = I_{REF} = 200\ \mu\text{A}$

For M_2; $r_{o2} = \dfrac{1}{\lambda_P I_{D2}} = \dfrac{1}{(0.02)(0.2)} = 250\ \text{K}$

$$g_{m2} = 2\sqrt{K_P I_{D2}} = 2\sqrt{\left(\frac{0.04}{2}\right)(35)(0.2)}$$

$$g_{m2} = 0.748\ \text{mA/V}$$

For M_0; $r_o = \dfrac{1}{\lambda_n I_{Do}} = \dfrac{1}{(0.015)(0.2)} = 333\ \text{K}$

$$g_{mo} = 2\sqrt{\left(\frac{0.08}{2}\right)(20)(0.2)} \Rightarrow g_{mo} = 0.80\ \text{mA/V}$$

$$A_v = -g_{mo}\left(r_{o2} \,||\, r_{oo}\right) = -(0.80)(250 \,||\, 333)$$

(b) $A_v = -114.3$

Want $A_v = -57.15 = -0.80(142.8 \,||\, R_L)$

(c) $142.8 \,||\, R_L = 71.375 = \dfrac{142.8 R_L}{142.8 + R_L} \Rightarrow R_L = 143\ \text{K}$

10.86

Assume M_1, M_2 matched

$$I_{REF} = I_{D2} = I_{Do} = 200\ \mu\text{A}$$

$$r_{o2} = \frac{1}{\lambda_p I_{D2}} = \frac{1}{(0.02)(0.2)} = 250\ \text{K}$$

$$r_{oo} = \frac{1}{\lambda_n I_{D0}} = \frac{1}{(0.015)(0.2)} = 333\ \text{K}$$

$$A_v = -g_{mo}\left(r_{o2} \,\|\, r_{oo}\right)$$

$$-100 = -g_{mo}(250 \,\|\, 333) \Rightarrow g_{mo} = 0.70\ \text{mA/V}$$

$$g_{mo} = 2\sqrt{\left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)_0 (0.2)} = 0.70$$

$$\left(\frac{W}{L}\right)_0 = 15.3$$

Now $\left(\dfrac{k_n'}{2}\right)\left(\dfrac{W}{L}\right)_0 = \left(\dfrac{k_p'}{2}\right)\left(\dfrac{W}{L}\right)_2$

$$\left(\frac{80}{2}\right)(15.3) = \left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_2$$

$$\left(\frac{W}{L}\right)_2 = \left(\frac{W}{L}\right)_1 = 30.6$$

10.87

$$g_{m1} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_1 I_{D1}} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o1} = \frac{1}{\lambda_n I_{D1}} = \frac{1}{(0.02)(0.1)} = 500\,\text{k}\Omega$$

$$R_{o2} = r_{o2} + r_{o3}\left(1 + g_{m2} r_{o2}\right)$$

$$g_{m2} = 2\sqrt{\left(\frac{0.06}{2}\right)(80)(0.1)} = 0.9798 \text{ mA/V}$$

$$r_{o2} = r_{o3} = \frac{1}{(0.02)(0.1)} = 500\,\text{k}\Omega$$

$$R_{o2} = 500 + 500\left[1 + (0.9798)(500)\right] = 245{,}949\,\text{k}\Omega$$

$$A_v = -g_{m1}\left(r_{o1} \| R_{o2}\right) = -(0.6325)(500 \| 245{,}949) = -316$$

10.88

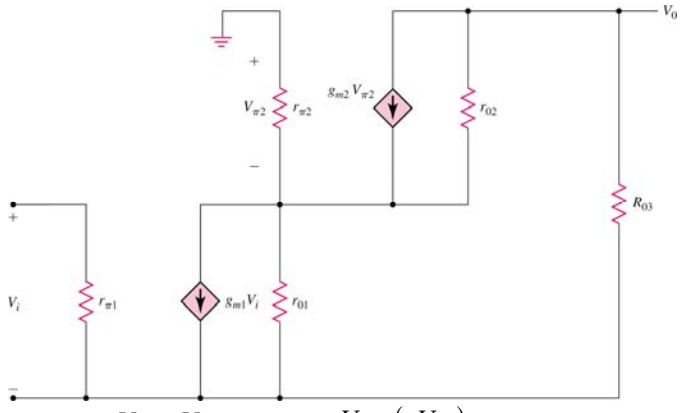
$$A_v = \frac{-g_m^2}{\dfrac{1}{r_{o3} r_{o4}} + \dfrac{1}{r_{o1} r_{o2}}}$$

$$g_m = 2\sqrt{\left(\frac{0.1}{2}\right)(25)(0.08)} = 0.6325 \text{ mA/V}$$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.04)(0.08)} = 312.5\,\text{k}\Omega$$

$$A_v = \frac{-(0.6325)^2}{\dfrac{1}{(312.5)^2} + \dfrac{1}{(312.5)^2}} = \frac{-0.40}{2(0.00001024)} = -19{,}531$$

10.89



(1) $$g_{m1} V_i = \frac{V_{\pi 2}}{r_{\pi 2}} + \frac{V_{\pi 2}}{r_{o1}} + g_{m2} V_{\pi 2} + \frac{V_O - (-V_{\pi 2})}{r_{o2}}$$

(2) $$\frac{V_O}{R_{O3}} + \frac{V_O - (-V_{\pi 2})}{r_{o2}} + g_{m2} V_{\pi 2} = 0$$

(1) $$g_{m1}V_i = V_{\pi 2}\left(\frac{1}{r_{\pi 2}}+\frac{1}{r_{o1}}+g_{m2}+\frac{1}{r_{o2}}\right)+\frac{V_O}{r_{o2}}$$

(2) $$V_O\left(\frac{1}{R_{O3}}+\frac{1}{r_{o2}}\right)+V_{\pi 2}\left(\frac{1}{r_{o2}}+g_{m2}\right)=0$$

$$g_m >> \frac{1}{r_o}$$

(1) $$g_{m1}V_i = V_{\pi 2}\left(\frac{1+\beta}{r_{\pi 2}}\right)+\frac{V_O}{r_{o2}}$$

(2) $$V_O\left(\frac{1}{R_{O3}}+\frac{1}{r_{o2}}\right)+V_{\pi 2}\cdot g_{m2}=0$$

$$V_{\pi 2} = -\frac{V_O}{g_{m2}}\left(\frac{1}{R_{O3}}+\frac{1}{r_{o2}}\right)$$

Then

(1) $$g_{m1}V_i = -\frac{V_O}{g_{m2}}\left(\frac{1}{R_{O3}}+\frac{1}{r_{o2}}\right)\left(\frac{1+\beta}{r_{\pi 2}}\right)+\frac{V_O}{r_{o2}}$$

$$= -V_O\left(\frac{1}{R_{O3}}+\frac{1}{r_{o2}}\right)\left(\frac{1+\beta}{\beta}\right)+\frac{V_O}{r_{o2}}$$

$$\approx -\frac{V_O}{R_{O3}}\left(\frac{1+\beta}{\beta}\right)$$

$$\frac{V_O}{V_i} = -g_{m1}R_{O3}\left(\frac{\beta}{1+\beta}\right)$$

From Equation (10.20) $R_{O3} \approx \beta r_{O3}$

So

$$A_v = \frac{V_O}{V_i} = \frac{-g_{m1}r_{o3}\beta^2}{1+\beta} \qquad g_m = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_{o3} = \frac{80}{0.25} = 320 \text{ K}$$

$$A_v = \frac{-(9.615)(320)(120)^2}{121} = -366{,}165$$

10.90 Design Problem

10.91

Let $(W/L)_{1,2} = 50$ and $(W/L) = 25$ for all other transistors

Let $I_{REF} = 80\,\mu$ A

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.04)(0.08)} = 312.5\ \text{k}\Omega$$

$$g_m = 2\sqrt{\left(\frac{0.06}{2}\right)(50)(0.08)} = 0.6928\ \text{mA/V}$$

$$A_v = \frac{-g_m^2}{\dfrac{1}{r_{o3}r_{o4}} + \dfrac{1}{r_{o1}r_{o2}}} = \frac{-(0.6928)^2}{\dfrac{1}{(312.5)^2} + \dfrac{1}{(312.5)^2}} = \frac{-0.480}{2(0.00001024)}$$

$$A_v = -23{,}438$$

Chapter 11

11.1

(a) $CMRR_{dB} = \infty \Rightarrow \upsilon_o = A_d \upsilon_d = (250)(1.5 \sin \omega t)$ (mV)

$\upsilon_o = 0.375 \sin \omega t$ (V)

(b) $CMRR_{dB} = 80 \text{ dB} \Rightarrow CMRR = 10^4 = \dfrac{250}{A_{cm}} \Rightarrow A_{cm} = 0.025$

$\upsilon_o = (250)(0.0015 \sin \omega t) + (0.025)(3 \sin \omega t)$

$\upsilon_o = 0.45 \sin \omega t$ (V)

(c) $CMRR_{dB} = 50 \text{ dB} \Rightarrow CMRR = 316.2 = \dfrac{250}{A_{cm}} \Rightarrow A_{cm} = 0.791$

$\upsilon_o = (250)(0.0015 \sin \omega t) + (0.791)(3 \sin \omega t)$

$\upsilon_o = 2.75 \sin \omega t$ (V)

11.2

(a) (i) $\upsilon_{o1} = -g_m R \upsilon_1 = -(1)(5)(0.7 + 0.1 \sin \omega t)$

$\upsilon_{o1} = -3.5 - 0.5 \sin \omega t$ (V)

(ii) $\upsilon_{o2} = -g_m R \upsilon_2 = -(1)(5)(0.7 - 0.1 \sin \omega t)$

$\upsilon_{o2} = -3.5 + 0.5 \sin \omega t$ (V)

(iii) $\upsilon_{o1} - \upsilon_{o2} = -1.0 \sin \omega t$ (V)

(b) $\Delta(\upsilon_1 - \upsilon_2) = (0.7 + 0.1 \sin \omega t) - (0.7 - 0.1 \sin \omega t) = 0.2 \sin \omega t$

(c) (i) $A_{d1} = \dfrac{-0.5}{0.2} = -2.5$

(ii) $A_{d2} = \dfrac{+0.5}{0.2} = +2.5$

(iii) $A_{d3} = \dfrac{-1}{0.2} = -5$

11.3

(a) Neglect dc base currents

$I_E = I_{C1} + I_{C2} = 0.2 \text{ mA} = \dfrac{-0.7 - (-3)}{R_E} \Rightarrow R_E = 11.5 \text{ k}\Omega$

$\upsilon_{o1} = 1.2 \text{ V}, \Rightarrow R_C = \dfrac{3 - 1.2}{0.1} = 18 \text{ k}\Omega$

(c) For $V_{CB} = 0 \Rightarrow V_{CE} = 0.7 \text{ V}$

$3 = I_C(18) + 0.7 + 2I_C(11.5) - 3$

So $I_C = 0.1293 \text{ mA}$

$\upsilon_{o1} = \upsilon_{cm+} = 3 - (0.1293)(18) = 0.673 \text{ V}$

$\upsilon_{cm-} = -2.3 \text{ V}$

So $-2.3 \le \upsilon_{cm} \le 0.673 \text{ V}$

11.4

a.

$$I_1 = \frac{10-2(0.7)}{8.5} \Rightarrow \underline{I_1 = 1.01 \text{ mA}}$$

$$I_{C2} = \frac{I_1}{1+\dfrac{2}{\beta(1+\beta)}} = \frac{1.01}{1+\dfrac{2}{(100)(101)}} \Rightarrow \underline{I_{C2} \cong 1.01 \text{ mA}}$$

$$I_{C4} = \left(\frac{100}{101}\right)\left(\frac{1.01}{2}\right) \Rightarrow \underline{I_{C4} \cong 0.50 \text{ mA}}$$

$$V_{CE2} = (0-0.7)-(-5) \Rightarrow \underline{V_{CE2} = 4.3 \text{ V}}$$

$$V_{CE4} = \left[5-(0.5)(2)\right]-(-0.7) \Rightarrow \underline{V_{CE4} = 4.7 \text{ V}}$$

b.

For $V_{CE4} = 2.5$ V $\Rightarrow V_{C4} = -0.7 + 2.5 = 1.8$ V

$$I_{C4} = \frac{5-1.8}{2} \Rightarrow \underline{I_{C4} = 1.6 \text{ mA}}$$

$$I_{C2} + \left(\frac{1+\beta}{\beta}\right)(2I_{C4}) = \left(\frac{101}{100}\right)(2)(1.6) \Rightarrow \underline{I_{C2} = 3.23 \text{ mA}}$$

$$\underline{I_1 \approx I_{C2} = 3.23 \text{ mA}}$$

$$R_1 = \frac{10-2(0.7)}{3.23} \Rightarrow \underline{R_1 = 2.66 \text{ k}\Omega}$$

11.5

a. Neglecting base currents

$$I_1 = I_3 = 400 \ \mu\text{A} \Rightarrow \text{R}_1 = \frac{30-0.7}{0.4} \Rightarrow \underline{R_1 = 73.25 \text{ k}\Omega}$$

$$V_{CE1} = 10 \text{ V} \Rightarrow \text{V}_{C1} = 9.3 \text{ V}$$

$$R_C = \frac{15-9.3}{0.2} \Rightarrow \underline{R_C = 28.5 \text{ k}\Omega}$$

b.

$$r_\pi = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

$$r_0(Q_3) = \frac{50}{0.4} = 125 \text{ k}\Omega$$

We have

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)} = \frac{(100)(28.5)}{2(13+10)} \Rightarrow \underline{A_d = 62}$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B}\left\{\frac{1}{1+\dfrac{2r_0(1+\beta)}{r_\pi + R_B}}\right\}$$

$$= -\frac{(100)(28.5)}{13+10}\left\{\frac{1}{1+\dfrac{2(125)(101)}{13+10}}\right\} \Rightarrow \underline{A_{cm} = -0.113}$$

$$CMRR_{dB} = 20\log_{10}\left(\frac{62}{0.113}\right) \Rightarrow \underline{CMRR_{dB} = 54.8 \text{ dB}}$$

c.

$$R_{id} = 2\left(r_\pi + R_B\right) = 2\left(13+10\right) \Rightarrow \underline{R_{id} = 46\ \text{k}\Omega}$$

$$R_{icm} = \frac{1}{2}\left[r_\pi + R_B + 2\left(1+\beta\right)r_0\right]$$

$$= \frac{1}{2}\left[13+10+2\left(101\right)\left(125\right)\right] \Rightarrow \underline{R_{icm} = 12.6\ \text{M}\Omega}$$

11.6

(a) $v_{CM}\left(\max\right) \Rightarrow V_{CB} = 0$ so that $v_{CM}\left(\max\right) = 5 - \frac{I_Q}{2}\left(R_C\right) = 5 - \frac{\left(0.5\right)}{2}\left(8\right)$

$\underline{v_{CM}\left(\max\right) = 3\ \text{V}}$

(b)

$$\Delta I = g_m \cdot \frac{V_d}{2} = \left(\frac{I_{CQ}}{V_T}\right)\cdot\frac{V_d}{2} = \left(\frac{0.25}{0.026}\right)\left(\frac{0.018}{2}\right) = 0.08654\ \text{mA}$$

$$\Delta V_{C2} = \Delta I \cdot R_C = \left(0.08654\right)\left(8\right) = 0.692\ \text{V}$$

(c)

$$\Delta I = \left(\frac{0.25}{0.026}\right)\left(\frac{0.010}{2}\right) = 0.04808\ \text{mA}$$

$$\Delta V_{C2} = \left(0.04808\right)\left(8\right) = 0.385\ \text{V}$$

11.7

$$P = \left(I_1 + I_{C4}\right)\left(V^+ - V^-\right)$$

$$I_1 \cong I_{C4} \text{ so } 1.2 = 2I_1\left(6\right) \Rightarrow \underline{I_1 = I_{C4} = 0.1\ mA}$$

$$R_1 = \frac{3-0.7-\left(-3\right)}{0.1} \Rightarrow \underline{R_1 = 53\ k\Omega}$$

$$\text{For } v_{CM} = +1V \Rightarrow V_{C1} = V_{C2} = 1\ V \Rightarrow R_C = \frac{3-1}{0.05} \Rightarrow \underline{R_C = 40\ k\Omega}$$

One-sided output

$$A_d = \frac{1}{2}g_m R_C \text{ where } g_m = \frac{0.05}{0.026} = 1.923\ mA/V$$

Then

$$A_d = \frac{1}{2}\left(1.923\right)\left(40\right) \Rightarrow \underline{A_d = 38.5}$$

11.8

a.

$$0 = 0.7 + \frac{I_E}{2}\left(2\right) + I_E\left(85\right) - 5$$

$$I_E = \frac{5-0.7}{85+1} \Rightarrow \underline{I_E = 0.050\ \text{mA}}$$

$$I_{C1} = I_{C2} = \left(\frac{\beta}{1+\beta}\right)\left(\frac{I_E}{2}\right) = \left(\frac{100}{101}\right)\left(\frac{0.050}{2}\right)$$

Or $\underline{I_{C1} = I_{C2} = 0.0248\ \text{mA}}$

$$V_{CE1} = V_{CE2} = \left[5 - I_{C1}\left(100\right)\right] - \left(-0.7\right)$$

So $\underline{V_{CE1} = V_{CE2} = 3.22\ \text{V}}$

b. $v_{cm}(\max)$ for $V_{CB} = 0$ and $V_C = 5 - I_{C1}(100) = 2.52$ V

So $\underline{v_{cm}(\max) = 2.52 \text{ V}}$

$v_{cm}(\min)$ for Q_1 and Q_2 at the edge of cutoff $\Rightarrow \underline{v_{cm}(\min) = -4.3 \text{ V}}$

(c) Differential-mode half circuits

$$-\frac{v_d}{2} = V_\pi + \left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right).R'_E$$

$$= V_\pi\left[1 + \frac{(1+\beta)}{r_\pi}R'_E\right]$$

Then

$$V_\pi = \frac{-(v_d/2)}{\left[1 + \frac{(1+\beta)}{r_\pi}R'_E\right]}$$

$$v_o = -g_m V_\pi R_C \Rightarrow A_d = \frac{1}{2}\cdot\frac{\beta R_C}{r_\pi + (1+\beta)R'_E}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.0248} = 105\ k\Omega \quad R'_E = 2\ k\Omega$$

Then

$$A_d = \frac{1}{2}\cdot\frac{(100)(100)}{105 + (101)(2)} \Rightarrow \underline{A_d = 16.3}$$

11.9

a. For $v_1 = v_2 = 0$ and neglecting base currents

$$R_E = \frac{-0.7 - (-10)}{0.15} \Rightarrow \underline{R_E = 62 \text{ k}\Omega}$$

b.

$$A_d = \frac{v_{02}}{v_d} = \frac{\beta R_C}{2(r_\pi + R_B)}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.075} = 34.7 \text{ k}\Omega$$

$$A_d = \frac{(100)(50)}{2(34.7 + 0.5)} \Rightarrow \underline{A_d = 71.0}$$

$$A_{cm} = -\frac{\beta R_C}{r_\pi + R_B}\left[\frac{1}{1 + \frac{2R_E(1+\beta)}{r_\pi + R_B}}\right]$$

$$= -\frac{(100)(50)}{34.7 + 0.5}\left[\frac{1}{1 + \frac{2(62)(101)}{34.7 + 0.5}}\right] \Rightarrow \underline{A_{cm} = -0.398}$$

$$CMRR_{dB} = 20\log_{10}\left|\frac{71.0}{0.398}\right| \Rightarrow \underline{CMRR_{dB} = 45.0 \text{ dB}}$$

c.

$$R_{id} = 2\left(r_\pi + R_B\right)$$
$$R_{id} = 2\left(34.7 + 0.5\right) \Rightarrow \underline{R_{id} = 70.4\ \text{k}\Omega}$$

Common-mode input resistance

$$R_{icm} = \frac{1}{2}\left[r_\pi + R_B + 2\left(1+\beta\right)R_E\right]$$
$$= \frac{1}{2}\left[34.7 + 0.5 + 2\left(101\right)\left(62\right)\right] \Rightarrow \underline{R_{icm} = 6.28\ \text{M}\Omega}$$

11.10

(a) $I_{E1} = I_{E2} = \left(\frac{81}{80}\right)(50) = 50.625 \Rightarrow I_E = 101.25\ \mu\text{A}$

$$R_E = \frac{3 - 0.6}{0.10125} = 23.7\ \text{k}\Omega$$
$$R_C = \frac{-1.5 - (-3)}{0.05} = 30\ \text{k}\Omega$$

(b) (i) $I_E = \frac{3 - (1 + 0.6)}{23.7} \Rightarrow I_E = 59.07\ \mu\text{A}$

$$I_{C1} = I_{C2} = \frac{1}{2}\left(\frac{80}{81}\right)(59.07) = 29.17\ \mu\text{A}$$
$$\upsilon_{C1} = \upsilon_{C2} = I_C R_C - 3 = (0.02917)(30) - 3 = -2.125\ \text{V}$$

(ii) $g_m = \frac{0.02917}{0.026} = 1.122\ \text{mA/V}$

$$\upsilon_d = 12\ \text{mV}, \quad \frac{\upsilon_d}{2} = 6\ \text{mV}$$
$$\upsilon_{C2} = -2.125 - (1.122)(30)(0.006) = -2.327\ \text{V}$$
$$\upsilon_{C1} = -2.125 + (1.122)(30)(0.006) = -1.923\ \text{V}$$

11.11

(a)

$$v_1 = v_2 = 0$$
$$I_{E1} = I_{E2} \cong 6\ \mu\text{A}$$
$$\beta = 60$$
$$I_{C1} = I_{C2} = 5.90\ \mu\text{A}$$
$$v_{C1} = v_{C2} = (5.90)(0.360) - 3$$
$$= -0.875\ \text{V}$$
$$V_{EC1} = V_{EC2} = +0.6 - (-0.875)$$
$$= 1.475\ \text{V}$$

(b)

(i)

$$g_m = \frac{5.90}{0.026} \Rightarrow 0.227\ \text{mA/V}$$
$$A_d = g_m R_C = (0.227)(360) = 81.7$$
$$A_{cm} = 0$$

(ii)

$$A_d = \frac{g_m R_C}{2} = 40.8 \quad r_\pi = \frac{(60)(0.026)}{0.0059}$$
$$= 264 \text{ K}$$
$$A_{cm} = \frac{-(0.227)(360)}{1 + \dfrac{2(61)(4000)}{264}} = -0.0442$$

11.12

$$I_Q = 0.2 \text{ mA}, \Rightarrow I_{C1} = I_{C2} = \frac{1}{2}\left(\frac{80}{81}\right)(0.2) = 0.09877 \text{ mA}$$

(a) For $V_{CB} = 0$, $\upsilon_{C1} = \upsilon_{C2} = \upsilon_{CM} = -2.5 = I_C R_C - 5$

So $R_C = \dfrac{5 - 2.5}{0.09877} = 25.3 \text{ k}\Omega$

(b) $A_d = g_m R_C$, $g_m = \dfrac{0.09877}{0.026} = 3.799 \text{ mA/V}$

$A_d = (3.799)(25.3) = 96.1$

(c) $\upsilon_d = 14 \text{ mV}$, $\dfrac{\upsilon_d}{2} = 7 \text{ mV}$

$$\upsilon_{C1} = -2.5 - g_m R_C \cdot \frac{\upsilon_d}{2} = -2.5 - (3.799)(25.3)(0.007) = -3.173 \text{ V}$$
$$\upsilon_{C2} = -2.5 + (3.799)(25.3)(0.007) = -1.827 \text{ V}$$

(d) $CMRR_{dB} = 60 \text{ dB} \Rightarrow CMRR = 1000$

$$1000 = \frac{1}{2}\left[1 + \frac{(81)(0.2)R_o}{(0.026)(80)}\right] \Rightarrow R_o = 257 \text{ k}\Omega$$

11.13

(a) Neglect dc base currents

$$I_E = I_{C1} + I_{C2} = 240\,\mu\text{A}$$
$$R_E = \frac{-0.7 - (-5)}{0.24} = 17.9 \text{ k}\Omega$$
$$R_C = \frac{5 - 3}{0.12} = 16.7 \text{ k}\Omega$$

(b) $g_m = \dfrac{0.12}{0.026} = 4.615 \text{ mA/V}$

$A_d = g_m R_C = (4.615)(16.667) = 76.9$

(c) $A_{cm} \cong \dfrac{\Delta R_C}{R_E} = \dfrac{0.5}{17.9} = 0.0279$

$$CMRR_{dB} = 20 \log_{10}\left(\frac{76.9}{0.0279}\right) = 68.8 \text{ dB}$$

11.14

$$v_1 = v_2 = 0$$

$$I_E = \frac{-0.7-(-10)}{75}$$

$$= 0.124 \text{ mA}$$

$$I_{C1} = I_{C2} = 0.0615 \text{ mA}$$

$$g_m = \frac{0.0615}{0.026} = 2.365 \text{ mA/V}$$

$$\frac{\Delta g_m}{g_m} = 0.01$$

$$g_{m1} = 2.377 \text{ mA/V}$$

$$g_{m2} = 2.353 \text{ mA/V}$$

$$r_\pi = \frac{(120)(0.026)}{0.0615} = 50.7 \text{ K}$$

$$\Delta I = g_m \frac{V_d}{2}$$

$$\Delta v_{C1} = -g_{m1} \frac{V_d}{2} R_c$$

$$\Delta v_{C2} = +g_{m2} \frac{V_d}{2} R_c$$

$$v_o = \Delta v_{C1} - \Delta v_{C2} = -g_{m1} \frac{V_d}{2} R_C - g_{m2} \frac{V_d}{2} R_C$$

$$= -\frac{V_d}{2} R_C \left(g_{m1} + g_{m2}\right)$$

$$A_d = -\frac{R_C}{2}\left(g_{m1} + g_{m2}\right) = \frac{-50}{2}\left(2.377 + 2.353\right) \Rightarrow A_d = -118.25$$

Common-Mode

$$\Delta v_{C1} = \frac{-g_{m1} R_C v_{cm}}{1 + \left(\frac{1+\beta}{r_\pi}\right)(2R_E)} \quad \Delta v_{C2} = \frac{-g_{m2} R_C v_{cm}}{1 + \left(\frac{1+\beta}{r_\pi}\right)(2R_E)}$$

$$\frac{v_o}{v_{cm}} = A_{cm} = \frac{-\left(g_{m1} - g_{m2}\right) R_C}{1 + \left(\frac{1+\beta}{r_\pi}\right)(2R_E)} = \frac{-(2.377 - 2.353)(50)}{1 + \left(\frac{121}{50.7}\right)(2)(75)}$$

$$= \frac{-1.2}{358.99} \Rightarrow A_{cm} = -0.003343$$

$$CMRR\Big|_{dB} = \underline{91 \text{ dB}}$$

11.15

(a)

$$v_1 = v_2 = 0$$

$$v_E = +0.7 \text{ V}$$

$$I_E = \frac{5 - 0.7}{1} = 4.3 \text{ mA}$$

$$I_{C1} = I_{C2} = 2.132 \text{ mA}$$

$$v_{C1} = v_{C2} = (2.132)(1) - 5$$

$$= -2.87 \text{ V}$$

(b)

$v_1 = 0.5, \ v_2 = 0 \quad Q_2$ on

Q_1 off

$$I_{C1} = 0, \ I_{C2} = 4.3\left(\frac{120}{121}\right) \text{ mA} = 4.264 \text{ mA}$$

$$\underline{v_{C1} = -5 \text{ V}} \quad v_{C2} = (4.264)(1) - 5$$

$$\underline{v_{C2} = -0.736 \text{ V}}$$

(c)

$$\underline{v_E \approx 0.7 \text{ V}} \quad g_m = \frac{2.132}{0.026} = 82.0 \text{ mA/V}$$

$$\Delta I = g_m \frac{v_d}{2} \quad \Delta v_C = \Delta I \cdot R_C = g_m \frac{V_d}{2} \cdot R_C = \frac{(82.0)}{2} \cdot V_d (1) = 41.0 V_d$$

$$V_d = 0.015 \Rightarrow \Delta v_c = 0.615 \text{ V}$$

$$v_{C2} \downarrow \ v_{C1} \uparrow$$

$$v_{C1} = -2.87 + 0.615 = -2.255 \text{ V}$$

$$v_{C2} = -2.87 - 0.615 = -3.485 \text{ V}$$

11.16

(a)

$$g_m = \frac{I_C}{V_T} = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

$$A_d = \frac{v_o}{v_d} = \frac{1}{0.01} = 100$$

$$A_d = g_m R_C$$

$$100 = 38.46 \ R_C$$

$$\underline{R_c = 2.6 \text{ K}}$$

(b)

With $v_1 = v_2 = 0$

$$v_{C1} = v_{C2} = 10 - (1)(2.6) = 7.4 \ V \Rightarrow v_{cm}(\max) = 7.4 \ V$$

11.17

(a) $I_O = I_{E1} + I_{E2} \Rightarrow I_{E1} = I_{E2} = 0.4\,\text{mA}$

(i) $\upsilon_{O1} - \upsilon_{O2} = \left[V^+ - I_{E1}R_{C1}\right] - \left[V^+ - I_{E2}R_{C2}\right] = I_{E2}R_{C2} - I_{E1}R_{C1}$

$\upsilon_{O1} - \upsilon_{O2} = 0$

(ii) $\upsilon_{O1} - \upsilon_{O2} = (0.4)(7.6 - 7.4) = 0.08\,\text{V}$

(b) $V_{BE1} = V_{BE2}$

$$0.8 = \left(2.9\times10^{-15} + 3.1\times10^{-15}\right)\cdot\exp\left(\frac{V_{BE}}{V_T}\right) \Rightarrow \exp\left(\frac{V_{BE}}{V_T}\right) = 1.333\times10^{14}$$

Then $I_{E1} = \left(2.9\times10^{-15}\right)\left(1.333\times10^{14}\right) = 0.3867\ \text{mA}$

$I_{E2} = \left(3.1\times10^{-15}\right)\left(1.333\times10^{14}\right) = 0.4133\ \text{mA}$

(i) $\upsilon_{O1} - \upsilon_{O2} = (0.4133 - 0.3867)(7.5) = 0.1995\,\text{V}$

(ii) $\upsilon_{O1} - \upsilon_{O2} = (0.4133)(7.6) - (0.3867)(7.4) = 0.2795\,\text{V}$

11.18

(a) $$\frac{i_{C1}}{I_Q} = 0.20 = \frac{1}{1+\exp\left(\dfrac{-\upsilon_d}{V_T}\right)} \Rightarrow \exp\left(\frac{-\upsilon_d}{V_T}\right) = 4$$

so that $-\upsilon_d = (0.026)\ln(4) \Rightarrow \upsilon_d = -36.0\,\text{mV}$

(b) $$\frac{i_{C2}}{I_Q} = 0.90 = \frac{1}{1+\exp\left(\dfrac{+\upsilon_d}{V_T}\right)} \Rightarrow \exp\left(\frac{+\upsilon_d}{V_T}\right) = 0.1111$$

so that $\upsilon_d = (0.026)\ln(0.1111) = -57.1\,\text{mV}$

11.19

(a) $$\frac{\left[0.5I_Q + \left(\dfrac{I_Q}{4V_T}\right)\cdot\upsilon_d(\max)\right] - \dfrac{I_Q}{1+\exp\left(-\upsilon_d(\max)/V_T\right)}}{\left[0.5I_Q + \left(\dfrac{I_Q}{4V_T}\right)\cdot\upsilon_d(\max)\right]} = 0.005$$

$$(0.995)\left[0.5 + \left(\frac{1}{4V_T}\right)\cdot\upsilon_d(\max)\right] = \frac{1}{1+\exp\left(-\upsilon_d(\max)/V_T\right)}$$

By trial and error, $\upsilon_d(\max) \cong 14\,\text{mV}$

(b) $$(0.985)\left[0.5 + \left(\frac{1}{4V_T}\right)\cdot\upsilon_d(\max)\right] = \frac{1}{1+\exp\left(-\upsilon_d(\max)/V_T\right)}$$

By trial and error, $\upsilon_d(\max) \cong 21.2\,\text{mV}$

11.20

For $v_{CM} = 3.5\ V$ and a maximum peak-to-peak swing in the output voltage of 2 *V*, we need the quiescent collector voltage to be

$$V_C = 3.5 + 1 = 4.5\ V$$

Assume the bias is $\pm 10\ V$, and $I_Q = 0.5\ mA$.

Then $I_C = 0.25\ mA$

Now $R_C = \dfrac{10 - 4.5}{0.25} \Rightarrow \underline{R_C = 22\ k\Omega}$

In this case, $r_\pi = \dfrac{(100)(0.026)}{0.25} = 10.4\ k\Omega$

Then

$$A_d = \frac{(100)(22)}{2(10.4 + 0.5)} = 101$$

So gain specification is met.

For $CMRR_{dB} = 80\ dB \Rightarrow$

$$CMRR = 10^4 = \frac{1}{2}\left[1 + \frac{(1+\beta) I_Q R_o}{V_T \beta}\right] = \frac{1}{2}\left[1 + \frac{(101)(0.5) R_o}{(0.026)(100)}\right] \Rightarrow \underline{R_o = 1.03\ M\Omega}$$

Need to use a Modified Widlar current source.

$$R_o = r_o\left[1 + g_m\left(R_{E1} \| r_\pi\right)\right]$$

If $V_A = 100\ \text{V}$, then $r_o = \dfrac{100}{0.5} = 200\ \text{k}\,\Omega$

$$r_\pi = \frac{(100)(0.026)}{0.5} = 5.2\ \text{k}\,\Omega$$

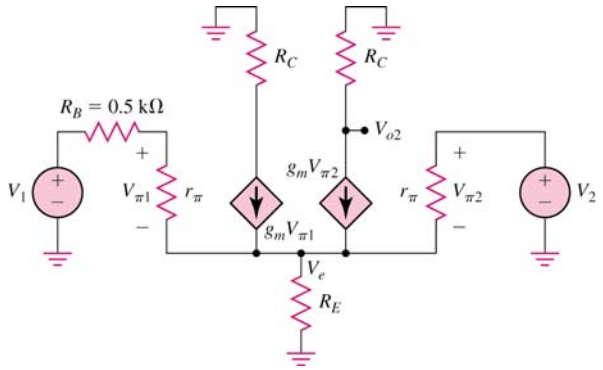
$$g_m = \frac{0.5}{0.026} = 19.23\ \text{mA/V}$$

Then $1030 = 200\left[1 + (19.23)\left(R_{E1} \| r_\pi\right)\right] \Rightarrow R_{E1} \| r_\pi = 0.216\ \text{k}\,\Omega \Rightarrow R_{E1} \| 5.2 = 0.216$

So, $R_{E1} = 225\,\Omega$; also $I_{REF} \cong 0.5\ \text{mA}$

11.21

(a) $R_E = \dfrac{-0.7 - (-10)}{0.25} \Rightarrow \underline{R_E = 37.2\ k\Omega}$



(b)

$$\frac{V_{\pi 1}}{r_\pi}+g_m V_{\pi 1}+\frac{V_{\pi 2}}{r_\pi}+g_m V_{\pi 2}=\frac{V_e}{R_E} \text{ or } (1)\left(\frac{1+\beta}{r_\pi}\right)\left(V_{\pi 1}+V_{\pi 2}\right)=\frac{V_e}{R_E}$$

$$\frac{V_{\pi 1}}{r_\pi}=\frac{V_1-V_e}{R_B+r_\pi}\Rightarrow V_{\pi 1}=\left(\frac{r_\pi}{r_\pi+R_B}\right)\left(V_1-V_e\right)$$

$$V_{\pi 2}=V_2-V_e$$

Then

$$(1)\left(\frac{1+\beta}{r_\pi}\right)\left[\frac{r_\pi}{r_\pi+R_B}\left(V_1-V_e\right)+\left(V_2-V_e\right)\right]=\frac{V_e}{R_E}$$

From this, we find

$$V_e=\frac{V_1+\dfrac{r_\pi+R_B}{r_\pi}\cdot V_2}{\left[\dfrac{r_\pi+R_B}{R_E(1+\beta)}+1+\dfrac{r_\pi+R_B}{r_\pi}\right]}$$

Now

$$V_o=-g_m V_{\pi 2}R_C=-g_m R_C\left(V_2-V_e\right)$$

We have

$$r_\pi=\frac{(120)(0.026)}{0.125}\cong 25\ k\Omega,\quad g_m=\frac{0.125}{0.026}=4.81\ mA/V$$

(i)

Set $V_1=\dfrac{V_d}{2}$ and $V_2=-\dfrac{V_d}{2}$

Then

$$V_e=\frac{\dfrac{V_d}{2}\left(1-\left(\dfrac{25+0.5}{25}\right)\right)}{\left[\dfrac{25+0.5}{(37.2)(121)}+1+\dfrac{25+0.5}{25}\right]}=\frac{\dfrac{V_d}{2}(-0.02)}{2.026}$$

So

$$V_e=-0.00494V_d$$

Now

$$V_o=-(4.81)(50)\left(-\frac{V_d}{2}-(-0.00494)V_d\right)\Rightarrow \underline{A_d=\frac{V_o}{V_d}=119}$$

(ii)

Set $V_1=V_2=V_{cm}$

Then

$$V_e=\frac{V_{cm}\left(1+\dfrac{25+0.5}{25}\right)}{\left[\dfrac{25+0.5}{(37.2)(121)}+1+\dfrac{25+0.5}{25}\right]}=\frac{V_{cm}(-2.02)}{2.02567}$$

$$V_e=V_{cm}(0.9972)$$

Then

$$V_o=-(4.81)(50)\left[V_{cm}-V_{cm}(0.9972)\right]$$

or $\underline{A_{cm}=\dfrac{V_o}{V_{cm}}=-0.673}$

11.22

(a) $A_d = g_m R_C$

$$A_d = \frac{1.2}{0.016} = 75$$

$$g_m = \frac{0.125}{0.026} = 4.808 \text{ mA/V}$$

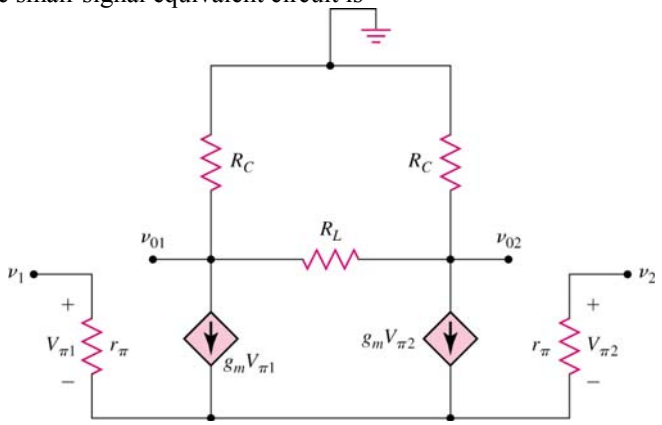
Then $R_C = \dfrac{75}{4.808} = 15.6 \text{ k}\Omega$

(b) For $V_{CB} = 0 \Rightarrow \upsilon_{C1} = \upsilon_{C2} = \upsilon_{CM} = 3 - (0.125)(15.6) = 1.05$ V

(c) $CMRR = \dfrac{1}{2}\left[1 + \dfrac{(0.25)(4000)}{(0.026)}\right] = 19{,}231 \Rightarrow CMRR_{dB} = 85.7$ dB

11.23

The small-signal equivalent circuit is



A KVL equation: $v_1 = V_{\pi 1} - V_{\pi 2} + v_2$

$$v_1 - v_2 = V_{\pi 1} - V_{\pi 2}$$

A KCL equation

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + \frac{V_{\pi 2}}{r_\pi} + g_m V_{\pi 2} = 0$$

$$(V_{\pi 1} + V_{\pi 2})\left(\frac{1}{r_\pi} + g_m\right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

Then $v_1 - v_2 = 2V_{\pi 1} \Rightarrow V_{\pi 1} = \dfrac{1}{2}(v_1 - v_2)$ and $V_{\pi 2} = -\dfrac{1}{2}(v_1 - v_2)$

At the v_{01} node:

$$\frac{v_{01}}{R_C} + \frac{v_{01} - v_{02}}{R_L} + g_m V_{\pi 1} = 0$$

$$v_{01}\left(\frac{1}{R_C} + \frac{1}{R_L}\right) - v_{02}\left(\frac{1}{R_L}\right) = \frac{1}{2} g_m (v_2 - v_1) \qquad (1)$$

At the v_{02} node:

$$\frac{v_{02}}{R_C} + \frac{v_{02} - v_{01}}{R_L} + g_m V_{\pi 2} = 0$$

$$v_{02}\left(\frac{1}{R_C}+\frac{1}{R_L}\right)-v_{01}\left(\frac{1}{R_L}\right)=\frac{1}{2}g_m\left(v_1-v_2\right) \quad (2)$$

From (1):

$$v_{02}=v_{01}\left(1+\frac{R_L}{R_C}\right)-\frac{1}{2}g_mR_L\left(v_2-v_1\right)$$

Substituting into (2)

$$v_{01}\left(1+\frac{R_L}{R_C}\right)\left(\frac{1}{R_C}+\frac{1}{R_L}\right)-\frac{1}{2}g_mR_L\left(v_2-v_{1/}\right)\left(\frac{1}{R_C}+\frac{1}{R_L}\right)-v_{01}\left(\frac{1}{R_L}\right)=\frac{1}{2}g_m\left(v_1-v_2\right)$$

$$v_{01}\left(\frac{1}{R_C}+\frac{R_L}{R_C^2}+\frac{1}{R_C}\right)=\frac{1}{2}g_m\left(v_1-v_2\right)\left[1-\left(\frac{R_L}{R_C}+1\right)\right]$$

$$\frac{v_{01}}{R_C}\left(2+\frac{R_L}{R_C}\right)=-\frac{1}{2}g_m\left(\frac{R_L}{R_C}\right)\left(v_1-v_2\right)$$

For $v_1-v_2=v_d$

$$A_{v1}=\frac{v_{01}}{v_d}=\frac{-\frac{1}{2}g_mR_L}{\left(2+\frac{R_L}{R_C}\right)}$$

From symmetry:

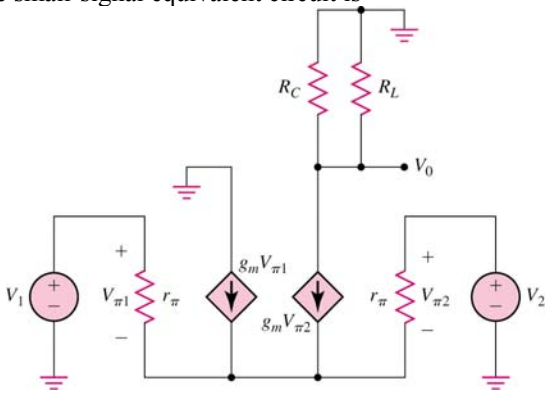
$$A_{v2}=\frac{v_{02}}{v_d}=\frac{\frac{1}{2}g_mR_L}{\left(2+\frac{R_L}{R_C}\right)}$$

Then

$$A_v=\frac{v_{02}-v_{01}}{v_d}=\frac{g_mR_L}{\left(2+\frac{R_L}{R_C}\right)}$$

11.24

The small-signal equivalent circuit is



KVL equation: $v_1=V_{\pi 1}-V_{\pi 2}+v_2$ or $v_1-v_2=V_{\pi 1}-V_{\pi 2}$

KCL equation:

$$\frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 1} + g_m V_{\pi 2} + \frac{V_{\pi 2}}{r_\pi} = 0$$

$$\left(V_{\pi 1} + V_{\pi 2}\right)\left(\frac{1}{r_\pi} + g_m\right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}$$

Then $\upsilon_1 - \upsilon_2 = -2V_{\pi 2}$, or $V_{\pi 2} = -\frac{1}{2}\left(\upsilon_1 - \upsilon_2\right)$

Now $\upsilon_o = -g_m V_{\pi 2}\left(R_C \| R_L\right)$

$$= \frac{1}{2} g_m \left(R_C \| R_L\right)\left(\upsilon_1 - \upsilon_2\right)$$

For $\upsilon_1 - \upsilon_2 \equiv \upsilon_d \Rightarrow A_d = \frac{\upsilon_o}{\upsilon_d} = \frac{1}{2} g_m \left(R_C \| R_L\right)$

11.25

We have $V_{C2} = -g_m V_{\pi 2} R_C = -g_m \left(V_{b2} - V_e\right) R_C$

and

$V_{C1} = -g_m V_{\pi 1} R_C = -g_m \left(V_{b1} - V_e\right) R_C$

Then

$V_0 = V_{C2} - V_{C1}$

$= -g_m \left(V_{b2} - V_e\right) R_C - \left[-g_m \left(V_{b1} - V_e\right) R_C\right]$

$= g_m R_C \left(V_{b1} - V_{b2}\right)$

Differential gain $\underline{A_d = \frac{V_0}{V_{b1} - V_{b2}} = g_m R_C}$

Common-mode gain $\underline{A_{cm} = 0}$

11.26

(a)

$v_{cm} = 3\ V \Rightarrow V_{C1} = V_{C2} = 3\ V$

Then $R_C = \frac{10 - 3}{0.1} \Rightarrow \underline{R_C = 70\ k\Omega}$

(b)

$CMRR_{dB} = 75\ dB \Rightarrow CMRR = 5623$

Now

$$CMRR = \frac{1}{2}\left[1 + \frac{\left(1 + \beta\right) I_Q R_o}{\beta V_T}\right]$$

$$5623 = \frac{1}{2}\left[1 + \frac{(151)(0.2) R_o}{(150)(0.026)}\right] \Rightarrow R_o = 1.45\ M\Omega$$

Use a Widlar current source.

$R_o = r_o\left[1 + g_m R_E'\right]$

Let V_A of current source transistor be 100 V.

Now $r_o = \dfrac{100}{0.2} = 500\ \text{k}\Omega$, $g_m = \dfrac{0.2}{0.026} = 7.69\ \text{mA/V}$

$$r_\pi = \frac{(150)(0.026)}{0.2} = 19.5\ \text{k}\Omega$$

So $1450 = 500\left[1 + (7.69)R_E'\right] \Rightarrow R_E' = 0.247\ \text{k}\Omega$

Now $R_E' = R_E \| r_\pi \Rightarrow 0.247 = R_E \| 19.5 \Rightarrow R_E = 250\,\Omega$

We have $I_Q R_E = V_T \ln\left(\dfrac{I_{REF}}{I_Q}\right)$

$$(0.2)(0.250) = (0.026)\ln\left[\frac{I_{REF}}{(0.2)}\right] \Rightarrow I_{REF} = 1.37\ \text{mA}$$

Then $R_1 = \dfrac{10 - 0.7 - (-10)}{1.37} \Rightarrow R_1 = 14.1\,\text{k}\Omega$

11.27

$$\upsilon_A = \left(\frac{R_A}{R_A + R}\right)\cdot V^+ = \left[\frac{R(1+\delta)}{R(1+\delta)+R}\right]\cdot V^+ = \left(\frac{1+\delta}{2+\delta}\right)\cdot V^+$$

$$\upsilon_B = \frac{1}{2}V^+ = 2.5\ \text{V}$$

For $\delta = +0.01$, $\upsilon_A = \left(\dfrac{1.01}{2.01}\right)(5) = 2.5124\ \text{V}$

For $\delta = -0.01$, $\upsilon_A = \left(\dfrac{0.99}{1.99}\right)(5) = 2.4874\ \text{V}$

$\upsilon_d \cong 12.5\ \text{mV}$

$R_B \cong R \| R = 20\ \text{k}\Omega$

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(120)(0.026)}{0.1} = 31.2\ \text{k}\Omega$$

$$A_d = \frac{\beta R_C}{2(r_\pi + R_B)} = \frac{(120)(15)}{2(31.2+20)} = 17.58$$

$$\upsilon_o = A_d \cdot \upsilon_d = (17.58)(0.0125) = 0.22\ \text{V}$$

$$-0.22 \le \upsilon_{O2} \le +0.22\ \text{V}$$

11.28

(a) $$R_{id} = 2r_\pi = \frac{2\beta V_T}{I_{CQ}} = \frac{2(120)(0.026)}{0.125} = 49.9\ \text{k}\Omega$$

(b) $$R_{icm} = \frac{1}{2}\left[r_\pi + (1+\beta)(2R_o)\right] = \frac{1}{2}\left[\frac{(120)(0.026)}{0.125} + (121)(2)(8000)\right]$$

$$R_{icm} = 968\ \text{M}\Omega$$

11.29

(a)

$$I_1 = \frac{10 - 0.7 - (-10)}{R_1} = 0.5 \Rightarrow \underline{R_1 = 38.6 \text{ K}}$$

$$R_2 = \frac{0.026}{0.14} \ln\left(\frac{0.5}{0.14}\right) \Rightarrow \underline{R_2 = 236\ \Omega}$$

(b)

$$R_{icm} \approx (1+\beta) R_o$$

$$R_o = r_{o4}\left(1 + g_{m4} R'_E\right) \quad g_{m4} = \frac{0.14}{0.026} = 5.385 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(180)(0.026)}{0.14} = 33.4 \text{ K}$$

$$R'_E = 33.4 \| 0.236 = 0.234 \text{ K}$$

$$r_{o4} = \frac{100}{0.14} = 714 \text{ K}$$

$$R_o = 714\left[1 + (5.385)(0.234)\right]$$

$$= 1614 \text{ K}$$

$$R_{icm} = (181)(1614) \approx 292 \text{ M}\Omega$$

(c)

$$A_{cm} = \frac{-g_{m1} R_C}{1 + \dfrac{2(1+\beta) R_o}{r_{\pi 1}}} \quad g_{m1} = \frac{0.07}{0.026} = 2.692 \text{ mA/V}$$

$$r_{\pi 1} = \frac{(180)(0.026)}{0.07} = 66.86 \text{ K}$$

$$A_{cm} = \frac{-(2.692)(40)}{1 + \dfrac{2(181)(1614)}{66.86}}$$

$$\underline{A_{cm} = -0.0123}$$

11.30

$$A_{d1} = g_{m1}\left(R_1 \| r_{\pi 3}\right)$$

$$g_{m1} = \frac{I_{Q1}/2}{V_T} = 19.23 I_{Q1}$$

$$r_{\pi 3} = \frac{\beta V_T}{I_{Q2}/2} = \frac{2(100)(0.026)}{I_{Q2}} = \frac{5.2}{I_{Q2}}$$

$$A_{d2} = \frac{g_{m3} R_2}{2}, \quad g_{m3} = \frac{I_{Q2}/2}{V_T} = 19.23 I_{Q2}$$

$$\text{Then } 30 = \frac{(19.23) I_{Q2}}{2} \cdot R_2 \Rightarrow I_{Q2} R_2 = 3.12\ V$$

Maximum $v_{o2} - v_{o1} = \pm 18\ mV$ for linearity

$v_{o3}(\max) = (\pm 18)(30)\ mV \Rightarrow \pm 0.54\ V$

so $I_{Q2} R_2 = 3.12\ V$ is OK.

From A_{v1}:

$$20 = 19.23 I_{Q1}\left(R_1 \| r_{\pi 3}\right) = 19.23 I_{Q1}\left[\frac{R_1\left(\frac{5.2}{I_{Q2}}\right)}{R_1 + \left(\frac{5.2}{I_{Q2}}\right)}\right]$$

$$20 = \frac{19.23 I_{Q1} R_1 (5.2)}{I_{Q2} R_1 + 5.2}$$

Let $\frac{I_{Q1}}{2} \cdot R_1 = 5\text{ V}, \Rightarrow I_{Q1} R_1 = 10\text{ V}$

Then $20 = \frac{19.23(10)(5.2)}{I_{Q2} R_1 + 5.2} \Rightarrow I_{Q2} R_1 = 44.8\text{ V}$

Now $I_{Q1} R_1 = 10 \Rightarrow R_1 = \frac{10}{I_{Q1}}$

So $I_{Q2}\left(\frac{10}{I_{Q1}}\right) = 44.8 \Rightarrow \frac{I_{Q2}}{I_{Q1}} = 4.48$

Let $I_{Q1} = 100\,\mu\text{A}$, then $I_{Q2} = 448\,\mu\text{A}$

Then

$I_{Q2} R_2 = 3.12 \Rightarrow R_2 = 6.96\,\text{k}\Omega$

$I_{Q1} R_1 = 10 \Rightarrow R_1 = 100\,\text{k}\Omega$

11.31

a.

$$I_1 = \frac{20 - V_{GS3}}{50} = 0.25\left(V_{GS3} - 2\right)^2$$

$$20 - V_{GS3} = 12.5\left(V_{GS3}^2 - 4V_{GS3} + 4\right)$$

$$12.5 V_{GS3}^2 - 49 V_{GS3} + 30 = 0$$

$$V_{GS3} = \frac{49 \pm \sqrt{(49)^2 - 4(12.5)(30)}}{2(12.5)} \Rightarrow \underline{V_{GS3} = 3.16\text{ V}}$$

$$I_1 = \frac{20 - 3.16}{50} \Rightarrow \underline{I_1 = I_Q = 0.337\text{ mA}}$$

$$I_{D1} = \frac{I_Q}{2} \Rightarrow \underline{I_{D1} = 0.168\text{ mA}}$$

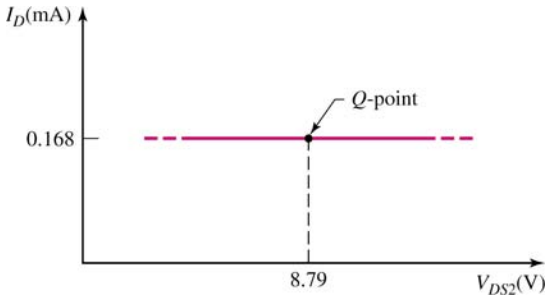
$$0.168 = 0.25\left(V_{GS1} - 2\right)^2 \Rightarrow V_{GS1} = 2.82\text{ V}$$

$$V_{DS4} = -2.82 - (-10) \Rightarrow \underline{V_{DS4} = 7.18\text{ V}}$$

$$V_{D1} = 10 - (0.168)(24) = 5.97\text{ V}$$

$$V_{DS1} = 5.97 - (-2.82) \Rightarrow \underline{V_{DS1} = 8.79\text{ V}}$$

(b)



(c)

$$\text{Max } v_{CM} \Rightarrow V_{DS1} = V_{DS2} = V_{DS}(sat) = V_{GS1} - V_{TN}$$
$$2.82 - 2 = 0.82\ V$$
$$\text{Now } V_{D1} = 10 - (0.168)(24) = 5.97\ V$$
$$V_S(\max) = 5.97 - V_{DS1}(sat) = 5.97 - 0.82$$
$$V_S(\max) = 5.15\ V$$
$$v_{CM}(\max) = V_S(\max) + V_{GS1} = 5.15 + 2.82$$
$$\underline{v_{CM}(\max) = 7.97V}$$
$$v_{CM}(\min) = V^- + V_{DS4}(sat) + V_{GS1}$$
$$V_{DS4}(sat) = V_{GS4} - V_{TN} = 3.16 - 2 = 1.16\ V$$
$$\text{Then } v_{CM}(\min) = -10 + 1.16 + 2.82 \Rightarrow \underline{v_{CM}(\min) = -6.02\ V}$$

11.32

(a) $I_{D1} = K_{n1}(V_{GS1} - V_{TN})^2$

$$60 = 100(V_{GS1} - 0.3)^2 \Rightarrow V_{GS1} = 1.075\ \text{V}$$
$$V_{D1} = V_{DS1} - V_{GS1} + V_1 = 4 - 1.075 - 1.15 = 1.775\ \text{V}$$
$$R_D = \frac{3 - 1.775}{0.06} = 20.4\ \text{k}\Omega$$

(i) $I_Q = I_1 = I_{D1} + I_{D2} = 120\ \mu\text{A}$

$$I_1 = K_3(V_{GS3} - V_{TN})^2$$
$$120 = 200(V_{GS3} - 0.3)^2 \Rightarrow V_{GS3} = 1.075\ \text{V}$$

(ii) $R_1 = \dfrac{3 - 1.075 - (-3)}{0.12} = 41\ \text{k}\Omega$

(iii) $V_{GS1} = V_{GS4} = 1.075\ \text{V}$

(b) $r_o = \dfrac{1}{\lambda_4 I_Q} = \dfrac{1}{(0.01)(0.12)} = 833.3\ \text{k}\Omega$

$$\Delta I_Q = \frac{\Delta V_{DS4}}{r_o} = \frac{2.3}{833.3} \Rightarrow \Delta I_Q = 2.76\ \mu\text{A}$$

11.33

(a)

$$I_Q = 160\ \mu\text{A}$$

$$I_D = \frac{k_n'}{2}\left(\frac{W}{L}\right)\left(V_{GS} - V_{TN}\right)^2$$

$$80 = \frac{80}{2}(4)\left(V_{OS} - 0.5\right)^2$$

$$80 = 160\left(V_{o5} - 0.5\right)^2$$

$$V_{GS} = \sqrt{\frac{80}{160}} + 0.5 = 1.207\ \text{V}$$

$$R_D = \frac{5-2}{0.08} = 37.5\ \text{K} \quad V_{DS} = 2 - (-1.207) = 3.21\ \text{V}$$

(c)

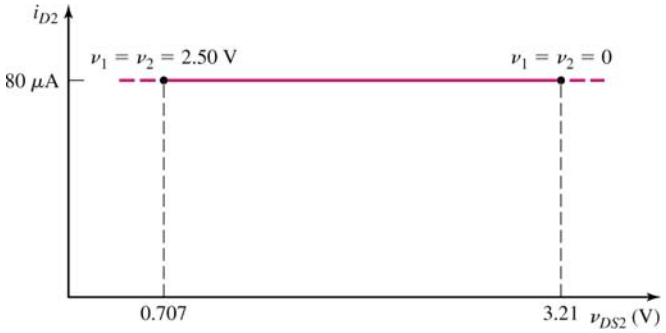
$$V_{DS}\left(sat\right) = V_{GS} - V_{TN} = 1.207 - 0.5 = 0.707\ \text{V}$$

Then $V_S = V_{O2} - V_{DS}\left(sat\right) = 2 - 0.707 = +1.29\ \text{V}$

And $v_1 = v_2 = v_{cm} = V_{GS} + V_S = 1.207 + 1.29$

$$\underline{v_{cm} = 2.50\ \text{V}}$$

(b)



11.34

(a) $\upsilon_{CM}\left(\max\right) = \upsilon_{D2} - \upsilon_{DS2}\left(sat\right) + \upsilon_{GS2}$

$$i_{D2} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left(\upsilon_{GS2} - V_{TN}\right)^2$$

$$0.09 = \left(\frac{0.1}{2}\right)(4)\left(\upsilon_{GS2} - 0.35\right)^2 \Rightarrow \upsilon_{GS2} = 1.021\ \text{V}$$

$$\upsilon_{DS2}\left(sat\right) = 1.0208 - 0.35 = 0.6708\ \text{V}$$

Then $2.25 = \upsilon_{D2} - 0.6708 + 1.021 \Rightarrow \upsilon_{D2} = 1.90\ \text{V}$

$$R_D = \frac{3 - 1.90}{0.09} = 12.2\ \text{k}\Omega$$

(b) (i) $\upsilon_{D2} = 1.90\ \text{V}$

(ii) $g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)I_{DQ}} = 2\sqrt{\left(\frac{0.1}{2}\right)(4)(0.09)} = 0.2683\ \text{mA/V}$

$$\upsilon_{D2} = 1.90 + g_m \cdot \frac{\upsilon_d}{2} \cdot R_D = 1.90 + (0.2683)\left(\frac{0.12}{2}\right)(12.2) = 2.096\ \text{V}$$

(iii) $\upsilon_{D2} = 1.90 - g_m \cdot \frac{\upsilon_d}{2} \cdot R_D = 1.90 - (0.2683)\left(\frac{0.05}{2}\right)(12.2) = 1.818 \text{ V}$

11.35

(a) $I_Q = I_1 = K_{n3}(V_{GS3} - V_{TN})^2$

$0.25 = 0.20(V_{GS3} - 0.4)^2 \Rightarrow V_{GS3} = 1.518 \text{ V}$

$R_1 = \frac{5 - 1.518 - (-5)}{0.25} = 33.9 \text{ k}\Omega$

(b) $I_{D1} = I_{D2} = \frac{0.25}{2} = 0.125 \text{ mA}$

$\upsilon_{O1} - \upsilon_{O2} = (V^+ - I_{D1}R_{D1}) - (V^+ - I_{D2}R_{D2}) = I_{D2}R_{D2} - I_{D1}R_{D1}$

(i) $\upsilon_{O1} - \upsilon_{O2} = 0$

(ii) $\upsilon_{O1} - \upsilon_{O2} = (0.125)(15.5 - 14.5) = 0.125 \text{ V}$

(c) $I_Q = I_{D1} + I_{D2} = (K_{n1} + K_{n2})(V_{GS} - V_{TN})^2$

$0.25 = (0.125 + 0.115)(V_{GS} - V_{TN})^2 \Rightarrow (V_{GS} - V_{TN})^2 = 1.04166$

$I_{D1} = (0.125)(1.04166) = 0.1302 \text{ mA}$

$I_{D2} = (0.115)(1.04166) = 0.1198 \text{ mA}$

(i) $\upsilon_{O1} - \upsilon_{O2} = (15)(0.1198 - 0.1302) = -0.156 \text{ V}$

(ii) $\upsilon_{O1} - \upsilon_{O2} = (0.1198)(15.5) - (0.1302)(14.5) = -0.031 \text{ V}$

11.36

(a) $\frac{i_{D1}}{I_Q} = \frac{1}{2} + \sqrt{\frac{K_n}{2I_Q}} \cdot \upsilon_d \sqrt{1 - \left(\frac{K_n}{2I_Q}\right) \cdot \upsilon_d^2}$

$0.20 = 0.50 + \sqrt{\frac{0.20}{2(0.15)}} \cdot \upsilon_d \sqrt{1 - \left(\frac{0.20}{2(0.15)}\right) \cdot \upsilon_d^2}$

$-0.3674 = \upsilon_d \sqrt{1 - (0.6667)\upsilon_d^2}$

$0.135 = \upsilon_d^2 \left[1 - (0.6667)\upsilon_d^2\right]$

$0.6667\upsilon_d^4 - \upsilon_d^2 + 0.135 = 0 \Rightarrow \upsilon_d^2 = 0.15 \Rightarrow \upsilon_d = -0.3873 \text{ V}$

(b) $0.80 = 0.50 - \sqrt{\frac{0.20}{2(0.15)}} \cdot \upsilon_d \sqrt{1 - \left(\frac{0.20}{2(0.15)}\right)\upsilon_d^2}$

$0.3674 = -\upsilon_d \sqrt{1 - (0.6667)\upsilon_d^2} \Rightarrow \upsilon_d = -0.3873 \text{ V}$

(c) $\upsilon_d = \upsilon_{d,\max} = \sqrt{\frac{I_Q}{K_n}} = \sqrt{\frac{0.15}{0.20}} = +0.866 \text{ V}$

11.37

$$\frac{i_{D1}}{I_Q}\Big|_{Lin} = 0.5 + \frac{g_f \upsilon_d}{I_Q}, \text{ Now } g_f = \sqrt{\frac{K_n I_Q}{2}} = \sqrt{\frac{(0.2)(0.10)}{2}} = 0.10 \text{ mA/V}$$

$$= 0.5 + \frac{(0.1)}{(0.1)} \cdot \upsilon_d = 0.5 + \upsilon_d$$

We have $\dfrac{K_n}{2I_Q} = \dfrac{(0.2)}{2(0.1)} = 1$

(a) $$\frac{0.5 + \upsilon_d - \left[0.5 + \upsilon_d \sqrt{1 - \upsilon_d^2}\right]}{0.5 + \upsilon_d} = 0.005$$

$$0.0025 = \upsilon_d \left[0.995 - \sqrt{1 - \upsilon_d^2}\right], \Rightarrow \upsilon_d \cong 0.19 \text{ V}$$

(b) $$\frac{0.5 + \upsilon_d - \left[0.5 + \upsilon_d \sqrt{1 - \upsilon_d^2}\right]}{0.5 + \upsilon_d} = 0.015$$

$$0.0075 = \upsilon_d \left[0.985 - \sqrt{1 - \upsilon_d^2}\right], \Rightarrow \upsilon_d \cong 0.285 \text{ V}$$

11.38

(b)

$$g_m = 2\sqrt{K_p I_D} = 2\sqrt{(0.05)(0.008696)}$$

$$= 0.0417 \text{ mA/V}$$

$$\Delta I = g_m \frac{V_d}{2} = (0.0417)(0.05) = 0.002085 \text{ mA}$$

$$\Delta v_D = (0.002085)(510) = 1.063$$

$$v_{D2} \uparrow \Rightarrow v_{D2} = 1.063 - 4.565 = -3.502 \text{ V}$$

$$v_{D1} = -1.063 - 4.565 = -5.628 \text{ V}$$

$$9 = I_S R_S + V_{SG} + 1$$

$$\underline{I_S = 2I_D}$$

$$8 = 2K_P R_S \left(V_{SG} + V_{TP}\right)^2 + V_{SG}$$

$$8 = (2)(0.05)(390)\left(V_{SG} - 0.8\right)^2 + V_{SG}$$

$$8 = 39\left(V_{SG}^2 - 1.6V_{SG} + 0.64\right) + V_{SG}$$

$$39V_{SG}^2 - 61.4V_{SG} + 16.96 = 0$$

$$V_{SG} = \frac{61.4 \pm \sqrt{3769.96 - 4(39)(16.96)}}{2(39)}$$

$$= 1.217 \text{ V} \quad V_S = 2.217$$

$$I_S = 0.01739 \text{ mA} \quad I_{D1} = I_{D2} \Rightarrow 8.696 \ \mu\text{A}$$

$$v_{D1} = v_{D2} = (8.696)(0.510) - 9 = -4.565 \text{ V}$$

(b)

$$g_m = 2\sqrt{K_P I_{DQ}} = 2\sqrt{(0.05)(0.008696)} = 0.0417 \text{ mA/V}$$

$$\Delta v_D = \Delta I_D \cdot R_D \quad \Delta I_D = g_m \cdot \frac{V_d}{2} = (0.0417)(0.05) = 0.002085 \text{ mA}$$

$$\Delta v_D = (0.002085)(510) = 1.063 \text{ V}$$

$v_1 \uparrow, I_{D1} \downarrow, v_{D1} \downarrow$

$v_{D1} = -4.565 - 1.063 = -5.628\ \text{V}$

$v_{D2} = -4.565 + 1.063 = -3.502\ \text{V}$

11.39

(a)

$v_1 = v_2 = 0$

$I_D = K_n\left(V_{SG} + V_{TP}\right)^2$

$I_D = 6\ \mu\text{A}$

$\sqrt{\frac{6}{30}} + 0.4 = V_{SG}$

$V_{SG} = 0.847\ \text{V}$

$V_S = +0.847\ \text{V}$

$v_D = I_D R_D - 3$

$= (6)(0.36) - 3 = -0.84\ \text{V}$

$V_{SD} = V_S - v_D = 0.847 - (-0.84)$

$v_{SD} = 1.69\ \text{V}$

(b)

(i)

$A_d = g_m R_D \quad g_m = 2\sqrt{K_n I_D}$

$= 2\sqrt{(30)(6)} = 26.83\ \mu\text{A/V}$

$A_d = (26.83)(0.36) \Rightarrow A_d = 9.66$

$A_{cm} = 0$

(ii)

$A_d = \frac{g_m R_D}{2} = \frac{(26.83)(0.36)}{2} \Rightarrow A_d = 4.83$

$A_{cm} = \frac{-g_m R_D}{1 + 2g_m R_O} = \frac{-(26.83)(0.36)}{1 + 2(26.83)(4)} = \underline{-0.0448}$

11.40

(a) $g_m = 2\sqrt{K_p I_{DQ}} = 2\sqrt{(0.12)(0.075)} = 0.1897\ \text{mA/V}$

(i) $\upsilon_{D1} - \upsilon_{D2} = g_m R_D \upsilon_d = (0.1897)(30)(0.1) = 0.5692\ \text{V}$

(ii) $\upsilon_{D1} - \upsilon_{D2} = g_m R_D \upsilon_d = (0.1897)(30)(0.2) = 1.138\ \text{V}$

(b) (i) $\Delta\upsilon_{D2} = g_m R_D \cdot \frac{\upsilon_d}{2} = (0.1897)(30)(0.1) = 0.5692\ \text{V}$

(ii) $A_{cm} = \frac{-g_m R_D}{1 + 2g_m R_o} = \frac{-(0.1897)(30)}{1 + 2(0.1897)(4000)} = -0.003748$

$A_d = \frac{g_m R_D}{2} = \frac{(0.1897)(30)}{2} = 2.846$

$\Delta\upsilon_{D2} = A_d \upsilon_d + A_{cm}\upsilon_{cm}$, let $\upsilon_{cm} = 1\ \text{V},\ \upsilon_d = 0.2\ \text{V}$

Then $\Delta\upsilon_{D2} = (2.846)(0.2) - (0.003748)(1) = 0.5655\ \text{V}$

11.41

For $v_1 = v_2 = 0$

$$0 = V_{GS} + 2I_D R_S - 10$$

$$10 = V_{GS} + 2K_n R_S \left(V_{GS} - V_{TN}\right)^2$$

$$= V_{GS} + 2(0.15)(75)(V_{GS} - 1)^2$$

$$22.5V_{GS}^2 - 44V_{GS} + 12.5 = 0$$

So $V_{GS} = 1.61\ V$ and $I_D = (0.15)(1.61-1)^2 \Rightarrow 55.9\ \mu$A

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{(0.15)(0.0559)}$$

$$g_m = 0.1831 \text{ mA/V}$$

Use Half-circuits – Differential gain

$$v_{D1} = -g_m\left(\frac{V_d}{2}\right)\left(R_D + \frac{\Delta R}{2}\right)$$

$$v_{o2} = g_m\left(\frac{V_d}{2}\right)\left(R_D - \frac{\Delta R}{2}\right)$$

$$v_o = v_{D1} - v_{D2} = -g_m V_d R_D$$

$$A_d = \frac{v_o}{V_d} = -g_m R_D$$

Now – Common-Mode Gain

$$V_i = V_{gs} + g_m V_{gs}(2R_S) = V_{cm}$$

$$V_{gs} = \frac{V_{cm}}{1 + g_m(2R_S)}$$

$$v_{D1} = \frac{-g_m\left(R_D + \frac{\Delta R_D}{2}\right)V_{cm}}{1 + g_m(2R_S)}$$

$$v_{D2} = \frac{-gm\left(R_D - \frac{\Delta R_D}{2}\right)V_{cm}}{1 + g_m(2R_S)}$$

$$v_O = v_{D1} - v_{D2}$$

So $v_o = \dfrac{-g_m(\Delta R_D)V_{cm}}{1 + g_m(2R_D)}$

$$A_{cm} = \frac{v_o}{V_{cm}} = \frac{-g_m(\Delta R_D)}{1 + g_m(2R_S)}$$

Then

$$A_d = -(0.1831)(50) = -9.16$$

$$A_{cm} = \frac{-(0.1831)(0.5)}{1 + (0.1831)(2)(75)} = -0.003216$$

$$CMRR\Big|_{bB} = 69.1 \text{ dB}$$

11.42

From 11.41

$I_{D1} = I_{D2} = 55.9\ \mu\text{A}$

$g_m = 0.183$ mA/V

$A_d:\quad \Delta v_{D1} = -g_{m1}\dfrac{V_d}{2}\cdot R_D \quad \Delta v_{D2} = +g_{m2}\left(\dfrac{+V_d}{2}\right)R_D$

$$v_O = \Delta v_{D1} - \Delta v_{D2} = -g_{m1}\frac{V_d}{2}R_D - g_{m2}\frac{V_d}{2}R_D$$

$$v_O = \frac{-V_d}{2}\cdot R_D\left(g_{m2}+g_{m1}\right) = \frac{-V_d}{2}\cdot R_D\left(g_m - \frac{\Delta g_m}{2} + \left(g_m - \frac{\Delta g_m}{2}\right)\right)$$

$$A_d = -g_m R_D = -(0.183)(50) = \underline{\underline{-9.15}}$$

$$A_{CM}:\quad v_O = \Delta v_{D1} - \Delta v_{D2} = \frac{-\left(g_m + \dfrac{\Delta g_M}{2}\right)R_D v_{cm}}{1+g_m(2R_S)} + \frac{\left(g_m - \dfrac{\Delta g_M}{2}\right)R_D v_{CM}}{1+g_m(2R_S)}$$

$$A_{cm} = \frac{v_O}{v_{cm}} = \frac{-\Delta g_m R_D}{1+g_m(2R_S)} \quad \Delta g_m = (0.01)(0.183) = 0.00183$$

$$A_{cm} = \frac{-(0.00183)(50)}{1+(0.183)(2)(75)} = -0.003216$$

$$CMRR\Big|_{dB} = 69.1\ \text{dB}$$

11.43

(a) $5 = I_S R_S + V_{SG} = 2K_p\left(V_{SG}+V_{TP}\right)^2 R_S + V_{SG}$

$5 = 2(1.2)(2)\left(V_{SG}^2 - 1.2V_{SG} + 0.36\right) + V_{SG}$

$4.8V_{SG}^2 - 4.76V_{SG} - 3.272 = 0 \Rightarrow V_{SG} = 1.459\ \text{V} = \upsilon_S$

$I_S = 2(1.2)(1.459-0.6)^2 = 1.77\ \text{mA}$

$I_{D1} = I_{D2} = 0.885\ \text{mA}$

$\upsilon_{D1} = \upsilon_{D2} = I_D R_D - 5 = (0.885)(1) - 5 = -4.115\ \text{V}$

(b) $5 = 2(1.2)(2)\left(V_{SG}^2 - 1.2V_{SG} + 0.36\right) + V_{SG} + 1$

$4.8V_{SG}^2 - 4.76V_{SG} - 2.272 = 0 \Rightarrow V_{SG} = 1.344\ \text{V}$

$\upsilon_S = V_{SG} + 1 = 2.344\ \text{V}$

$I_{D1} = I_{D2} = (1.2)(1.344-0.6)^2 = 0.664\ \text{mA}$

$\upsilon_{D1} = \upsilon_{D2} = (0.664)(1) - 5 = -4.336\ \text{V}$

(c) Using part (a), $g_m = 2\sqrt{(1.2)(0.885)} = 2.061\ \text{mA/V}$

$$\upsilon_{D1} = -4.115 + g_m R_D\cdot\frac{\upsilon_d}{2} = -4.115 + (2.061)(1)(0.1) = -3.909\ \text{V}$$

$$\upsilon_{D2} = -4.115 - (2.061)(1)(0.1) = -4.321\ \text{V}$$

(d) Using part (b), $g_m = 2\sqrt{(1.2)(0.664)} = 1.785\ \text{mA/V}$

$$\upsilon_{D1} = -4.336 + g_m R_D\cdot\frac{\upsilon_d}{2} = -4.336 + (1.785)(1)(0.1) = -4.158\ \text{V}$$

$$\upsilon_{D2} = -4.336 - (1.785)(1)(0.1) = -4.515\ \text{V}$$

11.44

(a)

$$g_m = 2\sqrt{K_n I_D}$$
$$= 2\sqrt{(0.4)(1)}$$
$$g_m = 1.265 \text{ mA/V}$$
$$A_d = \frac{v_o}{v_d} = \frac{1}{0.1} = 10$$
$$A_d = g_m R_D$$
$$10 = (1.265) R_D$$
$$\underline{R_D = 7.91 \text{ K}}$$

(b)

Quiescent $v_1 = v_2 = 0$

$$v_{D1} = v_{D2} = 10 - (1)(7.91) = 2.09 \text{ V}$$
$$V_{GS} = \sqrt{\frac{I_D}{K_n}} + V_{TN} = \sqrt{\frac{1}{0.4}} + 0.8 = 2.38 \text{ V}$$
$$V_{DS}(sat) = 2.38 - 0.8 = 1.58$$
$$\text{So } v_{cm} = v_D - V_{DS}(sat) + V_{GS}$$
$$= 2.09 - 1.58 + 2.38$$
$$\underline{v_{cm} = 2.89 \text{ V}}$$

11.45

$$A_d = \frac{g_m R_D}{2}$$

For $v_{CM} = 2.5\ V$

$$I_{D1} = I_{D2} = \frac{I_Q}{2} = 0.25\ mA$$

Let $V_{D1} = V_{D2} = 3\ V$, then $R_D = \frac{10-3}{0.25} \Rightarrow \underline{R_D = 28\ k\Omega}$

Then $100 = \frac{g_m(28)}{2} \Rightarrow g_m = 7.14\ mA/V$

And $g_m = 2\sqrt{\frac{k_n'}{2}\left(\frac{W}{L}\right) I_D}$

$$7.14 = 2\sqrt{\left(\frac{0.080}{2}\right)\left(\frac{W}{L}\right)(0.25)} \Rightarrow$$

$\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 1274$ (Extremely large transistors to meet the gain requirement.)

Need $|A_{CM}| = 0.10$

From Eq. (11.82(b))

$$|A_{CM}| = \frac{g_m R_D}{1 + 2 g_m R_o}$$

$$\text{So } 0.10 = \frac{(7.14)(28)}{1 + 2(7.14) R_o} \Rightarrow R_o = 140\ k\Omega$$

For the basic 2-transistor current source

$$R_o = r_o = \frac{1}{\lambda I_Q} = \frac{1}{(0.01)(0.5)} = 200\ k\Omega$$

This current source is adequate to meet common-mode gain requirement.

11.46

Not in detail, Approximation looks good.

a.

$$I_S = \frac{-V_{GS1}-(-5)}{R_S} \text{ and } I_S = 2I_D = 2K_n\left(V_{GS1}-V_{TN}\right)^2$$

$$\frac{5-V_{GS1}}{20} = 2(0.050)(V_{GS1}-1)^2$$

$$5-V_{GS1} = 2\left(V_{GS1}^2 - 2V_{GS1}+1\right)$$

$$2V_{GS1}^2 - 3V_{GS1} - 3 = 0$$

$$V_{GS1} = \frac{3 \pm \sqrt{(3)^2+4(2)(3)}}{2(2)} \Rightarrow V_{GS1} = 2.186 \text{ V}$$

$$I_S = \frac{5-2.186}{20} \Rightarrow \underline{I_S = 0.141 \text{ mA}}$$

$$I_{D1} = I_{D2} = \frac{I_S}{2} \Rightarrow \underline{I_{D1} = I_{D2} = 0.0704 \text{ mA}}$$

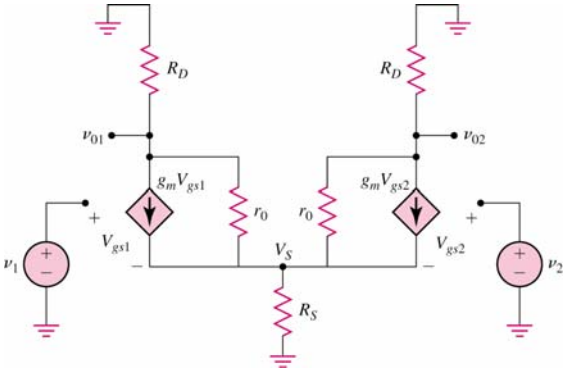
$$v_{02} = 5-(0.0704)(25) \Rightarrow \underline{v_{02} = 3.24 \text{ V}}$$

b.

$$g_m = 2K_n\left(V_{GS}-V_{TN}\right) = 2(0.05)(2.186-1)$$

$$g_m = 0.119 \text{ mA/V}$$

$$r_0 = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.02)(0.0704)} = 710 \text{ k}\Omega$$



$$V_{gs1} = v_1 - V_S, \quad V_{gs2} = v_2 - V_S$$

$$\frac{v_{01}}{R_D} + g_m V_{gs1} + \frac{v_{01}-V_S}{r_0} = 0$$

$$v_{01}\left(\frac{1}{R_D}+\frac{1}{r_0}\right) + g_m\left(v_1 - V_S\right) - \frac{V_S}{r_0} = 0 \qquad (1)$$

$$\frac{v_{02}}{R_D} + g_m V_{gs2} + \frac{v_{02}-V_S}{r_0} = 0$$

$$v_{02}\left(\frac{1}{R_D}+\frac{1}{r_0}\right) + g_m\left(v_2 - V_S\right) - \frac{V_S}{r_0} = 0 \qquad (2)$$

$$g_m V_{gs1} + \frac{v_{01} - V_S}{r_0} + \frac{v_{02} - V_S}{r_0} + g_m V_{gs2} = \frac{V_S}{R_S}$$

$$g_m (v_1 - V_S) + \frac{v_{01}}{r_0} + \frac{v_{02}}{r_0} - \frac{2V_S}{r_0} + g_m (v_2 - V_S) = \frac{V_S}{R_S}$$

$$g_m (v_1 + v_2) + \frac{v_{01}}{r_0} + \frac{v_{02}}{r_0} = V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right\} \qquad (3)$$

From (1)

$$v_{01} = \frac{V_S \left(g_m + \frac{1}{r_0} \right) - g_m v_1}{\left(\frac{1}{R_D} + \frac{1}{r_0} \right)}$$

Then

$$g_m (v_1 + v_2) + \frac{V_S \left(g_m + \frac{1}{r_0} \right) - g_m v_1}{r_0 \left(\frac{1}{R_D} + \frac{1}{r_0} \right)} + \frac{v_{02}}{r_0} = V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right\} \qquad (3)$$

$$g_m (v_1 + v_2) r_0 \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + V_S \left(g_m + \frac{1}{r_0} \right) - g_m v_1 + v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) = V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right\} \cdot r_0 \left(\frac{1}{R_D} + \frac{1}{r_0} \right)$$

$$g_m (v_1 + v_2) \left(1 + \frac{r_0}{R_D} \right) - g_m v_1 + v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) = V_S \left\{ \left(2g_m + \frac{2}{r_0} + \frac{1}{R_S} \right) \left(1 + \frac{r_0}{R_D} \right) - \left(g_m + \frac{1}{r_0} \right) \right\}$$

$$g_m \left(v_1 \cdot \frac{r_0}{R_D} + v_2 + v_2 \cdot \frac{r_0}{R_D} \right) + v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) = V_S \left\{ 2g_m + \frac{2}{r_0} + \frac{1}{R_S} + 2g_m \cdot \frac{r_0}{R_D} + \frac{2}{R_D} + \frac{r_0}{R_S R_D} - g_m - \frac{1}{r_0} \right\}$$

$$g_m \left(v_1 \cdot \frac{r_0}{R_D} + v_2 + v_2 \cdot \frac{r_0}{R_D} \right) + v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) = V_S \left\{ 2g_m + \frac{1}{r_0} + \frac{1}{R_S} \left(1 + \frac{r_0}{R_D} \right) + \frac{2}{R_D} (1 + g_m r_0) \right\} \qquad (4)$$

$$v_{02} \left(\frac{1}{R_D} + \frac{1}{r_0} \right) + g_m v_2 = V_S \left(g_m + \frac{1}{r_0} \right)$$

Then substituting into (2),

$$(0.119) \left[v_1 \frac{710}{25} + v_2 + v_2 \frac{710}{25} \right] + v_{02} \left[\frac{1}{25} + \frac{1}{710} \right]$$

$$= V_S \left\{ 0.119 + \frac{1}{710} + \frac{1}{20} \left(1 + \frac{710}{25} \right) + \frac{2}{25} \left[1 + (0.119)(710) \right] \right\}$$

Substitute numbers: (4)

$$(0.119)[28.4 v_1 + 29.4 v_2] + (0.0414) v_{02} = V_S \{ 0.1204 + 1.470 + 6.8392 \}$$

$$= V_S (8.4296)$$

or $V_S = 0.4010 v_1 + 0.4150 v_2 + 0.00491 v_{02}$

Then $v_{02} \left(\frac{1}{25} + \frac{1}{710} \right) + (0.119) v_2 = V_S \left(0.119 + \frac{1}{710} \right)$ (2)

$$v_{02}(0.0414)+v_2(0.119)=(0.1204)[0.401v_1+0.4150v_2+0.00491v_{02}]$$
$$v_{02}(0.0408)=(0.04828)v_1-(0.0690)v_2$$
$$v_{02}=(1.183)v_1-(1.691)v_2$$

Now $v_1=v_{cm}+\dfrac{v_d}{2}$

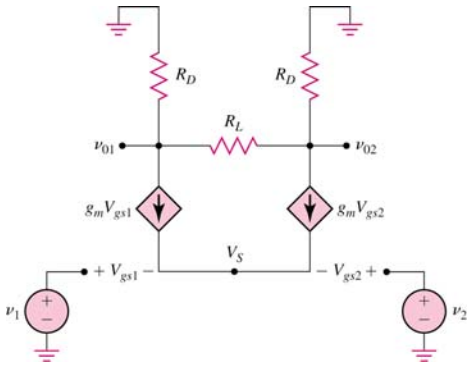
$$v_2=v_{cm}-\frac{v_d}{2}$$

So $v_{02}=(1.183)\left(v_{cm}+\dfrac{v_d}{2}\right)-(1.691)\left(v_{cm}-\dfrac{v_d}{2}\right)$

Or $v_{02}=1.437v_d-0.508v_{cm}\Rightarrow \underline{A_d=1.437},\quad \underline{A_{cm}=-0.508}$

$$CMRR_{dB}=20\log_{10}\left(\frac{1.437}{0.508}\right)\Rightarrow \underline{CMRR_{dB}=9.03\text{ dB}}$$

11.47



KVL:

$$v_1=V_{gs1}-V_{gs2}+v_2$$
$$\text{So } v_1-v_2=V_{gs1}-V_{gs2}$$

KCL:

$$g_mV_{gs1}+g_mV_{gs2}=0\Rightarrow V_{gs1}=-V_{gs2}$$
$$\text{So } V_{gs1}=\frac{1}{2}(v_1-v_2),\quad V_{gs2}=-\frac{1}{2}(v_1-v_2)$$

Now

$$\frac{v_{02}}{R_D}+\frac{v_{02}-v_{01}}{R_L}=-g_mV_{gs2}$$
$$=v_{02}\left(\frac{1}{R_D}+\frac{1}{R_L}\right)-\frac{v_{01}}{R_L}\qquad(1)$$
$$\frac{v_{01}}{R_D}+\frac{v_{01}-v_{02}}{R_L}=-g_mV_{gs1}$$
$$=v_{01}\left(\frac{1}{R_D}+\frac{1}{R_L}\right)-\frac{v_{02}}{R_L}\qquad(2)$$

From (1):

$$v_{01}=v_{02}\left(1+\frac{R_L}{R_D}\right)+g_mR_LV_{gs2}$$

Substitute into (2):

$$-g_m V_{gs1} = v_{02}\left(1+\frac{R_L}{R_D}\right)\left(\frac{1}{R_D}+\frac{1}{R_L}\right)+g_m R_L\left(\frac{1}{R_D}+\frac{1}{R_L}\right)V_{gs2}-\frac{v_{02}}{R_L}$$

$$-g_m\cdot(v_1-v_2)+g_m\left(1+\frac{R_L}{R_D}\right)\left(\frac{1}{2}\right)(v_1-v_2)=v_{02}\left(\frac{1}{R_D}+\frac{R_L}{R_D^2}+\frac{1}{R_D}\right)$$

$$\frac{1}{2}g_m\left(\frac{R_L}{R_D}\right)(v_1-v_2)=\frac{v_{02}}{R_D}\left(2+\frac{R_L}{R_D}\right)\Rightarrow A_{d2}=\frac{v_{02}}{v_1-v_2}=\frac{\frac{1}{2}\cdot g_m R_L}{\left(2+\frac{R_L}{R_D}\right)}$$

$$A_{d1}=\frac{v_{01}}{v_1-v_2}=\frac{-\frac{1}{2}\cdot g_m R_L}{\left(2+\frac{R_L}{R_D}\right)}$$

From symmetry

$$A_v=\frac{v_{02}-v_{01}}{v_1-v_2}=\frac{g_m R_L}{\left(2+\frac{R_L}{R_D}\right)}$$

Then

11.48

(a) $\upsilon_o=-g_m\left(R_D\|R_L\right)\left(\frac{-\upsilon_d}{2}\right)$

$$A_d=\frac{1}{2}g_m\left(R_D\|R_L\right)$$

(b) $\upsilon_o=-g_m\left(R_D\|R_L\right)\upsilon_{gs}$

$$\upsilon_{cm}=\upsilon_{gs}+g_m\upsilon_{gs}\left(2R_o\right)$$

$$\upsilon_{gs}=\frac{\upsilon_{cm}}{1+2g_m R_o}$$

$$A_{cm}=\frac{-g_m\left(R_D\|R_L\right)}{1+2g_m R_o}$$

11.49

(a) $A_d=\frac{\Delta\upsilon_o}{\Delta\upsilon_d}=\frac{0.5}{0.1}=5$

$$A_d=5=\frac{g_m R_D}{2},\quad g_m=2\sqrt{K_n I_{DQ}}=2\sqrt{(0.15)(0.1)}=0.2449\ \text{mA/V}$$

$$R_D=\frac{2(5)}{0.2449}=40.8\,\text{k}\Omega$$

(b) $\upsilon_{CM}(\max)=\upsilon_O-\upsilon_{DS}(sat)+\upsilon_{GS}$

$$i_D=K_n\left(\upsilon_{GS}-V_{TN}\right)^2$$

$$0.1=0.15(\upsilon_{GS}-0.4)^2\Rightarrow\upsilon_{GS}=1.216\ \text{V}$$

$$\upsilon_{DS}(sat)=1.216-0.4=0.8165\,\text{V}$$

$$\upsilon_O=5-(0.1)(40.8)=0.92\,\text{V}$$

$$\upsilon_{CM}(\max)=0.92-0.8165+1.216=1.32\,\text{V}$$

11.50

$V_{d1} = -g_m V_{gs1} R_D = -g_m R_D (V_1 - V_s)$

$V_{d2} = -g_m V_{gs2} R_D = -g_m R_D (V_2 - V_s)$

Now $V_o = V_{d2} - V_{d1} = -g_m R_D (V_2 - V_s) - (-g_m R_D (V_1 - V_s))$

$V_o = g_m R_D (V_1 - V_2)$

Define $V_1 - V_2 \equiv V_d$

Then $\underline{A_d = \frac{V_o}{V_d} = g_m R_D}$ and $\underline{A_{cm} = 0}$

11.51

(a) $\upsilon_{CM} = \upsilon_O - \upsilon_{DS}(sat) + \upsilon_{GS}$

$i_D = K_n (\upsilon_{GS} - V_{TN})^2$

$0.1 = \left(\frac{0.1}{2}\right)(10)(\upsilon_{GS} - 0.4)^2 \Rightarrow \upsilon_{GS} = 0.8472 \text{ V}$

$\upsilon_{DS}(sat) = 0.8472 - 0.4 = 0.4472 \text{ V}$

Then $\upsilon_O = 1.5 + 0.4472 - 0.8472 = 1.1 \text{ V}$

$\upsilon_O = 3 - (0.1) R_D = 1.1 \Rightarrow R_D = 19 \text{ k}\Omega$

Now $A_d = \frac{g_m R_D}{2}$, where $g_m = 2\sqrt{\left(\frac{0.1}{2}\right)(10)(0.1)} = 0.4472 \text{ mA/V}$

$A_d = \frac{(0.4472)(19)}{2} = 4.248$

$CMRR_{dB} = 50 \text{ dB} \Rightarrow CMRR = 316.2$

$CMRR = \frac{1}{2}\left[1 + 2\sqrt{K_n I_Q} \cdot R_o\right] \Rightarrow 316.2 = \frac{1}{2}\left[1 + 2\sqrt{\left(\frac{0.1}{2}\right)(10)(0.2)} \cdot R_o\right] \Rightarrow R_o = 998 \text{ k}\Omega$

(b) Use cascode current source similar to Figure 10.18 with $\upsilon_{DS2}(sat) = 0.3 \text{ V}$.

11.52

(a) From Problem 11.27, $\upsilon_d \cong 12.5 \text{ mV}$.

$A_d = \frac{g_m R_D}{2}$, where $g_m = 2\sqrt{(1)(0.1)} = 0.6325 \text{ mA/V}$

$A_d = \frac{(0.6325)(20)}{2} = 6.325$

$\upsilon_O = (6.325)(0.0125) = 0.0791 \text{ V}$

So $-0.0791 \le \upsilon_O \le 0.0791 \text{ V}$

11.53

From previous results

$$A_{d1} = \frac{v_{o2} - v_{o1}}{v_1 - v_2} = g_{m1} R_1 = \sqrt{2K_{n1} I_{Q1}} \cdot R_1 = 20$$

and $A_{d2} = \dfrac{v_{o3}}{v_{o2} - v_{o1}} = \dfrac{1}{2} g_{m3} R_2 = \dfrac{1}{2}\sqrt{2K_{n3} I_{Q2}} \cdot R_2 = 30$

Set $\dfrac{I_{Q1} R_1}{2} = 5\ V$ and $\dfrac{I_{Q2} R_2}{2} = 2.5\ V$

Let $I_{Q1} = I_{Q2} = 0.1\ mA$

Then $R_1 = 100\ k\Omega, \quad R_2 = 50\ k\Omega$

Then $2\left(\dfrac{0.06}{2}\right)\left(\dfrac{W}{L}\right)_1 (0.1) = \left(\dfrac{20}{100}\right)^2 \Rightarrow \underline{\left(\dfrac{W}{L}\right)_1 = \left(\dfrac{W}{L}\right)_2 = 6.67}$

and $2\left(\dfrac{0.060}{2}\right)\left(\dfrac{W}{L}\right)_3 (0.1) = \left(\dfrac{2(30)}{50}\right)^2 \Rightarrow \underline{\left(\dfrac{W}{L}\right)_3 = \left(\dfrac{W}{L}\right)_4 = 240}$

11.54

a.

$$i_{D1} = I_{DSS}\left(1 - \frac{v_{GS1}}{V_P}\right)^2$$

$$i_{D2} = I_{DSS}\left(1 - \frac{v_{GS2}}{V_P}\right)^2$$

$$\sqrt{i_{D1}} - \sqrt{i_{D2}} = \sqrt{I_{DSS}}\left(1 - \frac{v_{GS1}}{V_P}\right) - \sqrt{I_{DSS}}\left(1 - \frac{v_{GS2}}{V_P}\right)$$

$$= \frac{\sqrt{I_{DSS}}}{V_P}\left(v_{GS2} - v_{GS1}\right)$$

$$= -\frac{\sqrt{I_{DSS}}}{V_P} \cdot v_d = \frac{\sqrt{I_{DSS}}}{(-V_P)} \cdot v_d$$

$$i_{D1} + i_{D2} = I_Q \Rightarrow i_{D2} = I_Q - i_{D1}$$

$$\left(\sqrt{i_{D1}} - \sqrt{I_Q - i_{D1}}\right)^2 = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$

$$i_{D1} - 2\sqrt{i_{D1}\left(I_Q - i_{D1}\right)} + \left(I_Q - i_{D1}\right) = \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2$$

Then $\sqrt{i_{D1}\left(I_Q - i_{D1}\right)} = \dfrac{1}{2}\left[I_Q - \dfrac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right]$

Square both sides

$$i_{D1}^2 - i_{D1} I_Q + \frac{1}{4}\left[I_Q - \frac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right]^2 = 0$$

$$i_{D1} = \frac{I_Q \pm \sqrt{I_Q^2 - 4\left(\dfrac{1}{4}\right)\left[I_Q - \dfrac{I_{DSS}}{(-V_P)^2} \cdot v_d^2\right]^2}}{2}$$

$$i_{D1} = \frac{I_Q}{2} \pm \frac{1}{2}\sqrt{I_Q^2 - \left[I_Q^2 - \frac{2 I_Q I_{DSS} v_d^2}{(-V_P)^2} + \left(\frac{I_{DSS} v_d^2}{(-V_P)^2}\right)^2\right]}$$

Use + sign

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2}\sqrt{\frac{2I_Q I_{DSS}}{(-V_P)^2}\cdot v_d^2 - \left(\frac{I_{DSS}}{(-V_P)^2}\cdot v_d^2\right)^2}$$

$$i_{D1} = \frac{I_Q}{2} + \frac{1}{2}\frac{I_Q}{(-V_P)} v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

Or

$$\frac{i_{D1}}{I_Q} = \frac{1}{2} + \left(\frac{1}{-2V_P}\right)\cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

We had

$$i_{D2} = I_Q - i_{D1}$$

Then

$$\frac{i_{D2}}{I_Q} = \frac{1}{2} - \left(\frac{1}{-2V_P}\right)\cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

b.

If $i_{D1} = I_Q$, then

$$1 = \frac{1}{2} + \left(\frac{1}{-2V_P}\right)\cdot v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

$$|V_P| = v_d \sqrt{\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2}$$

Square both sides

$$|V_P|^2 = v_d^2\left[\frac{2I_{DSS}}{I_Q} - \left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{v_d}{V_P}\right)^2\right]$$

$$\left(\frac{I_{DSS}}{I_Q}\right)^2\left(\frac{1}{V_P}\right)^2\left(v_d^2\right)^2 - \frac{2I_{DSS}}{I_Q}\cdot v_d^2 + |V_P|^2 = 0$$

$$v_d^2 = \frac{\dfrac{2I_{DSS}}{I_Q} \pm \sqrt{\left(\dfrac{2I_{DSS}}{I_Q}\right)^2 - 4\left(\dfrac{I_{DSS}}{I_Q}\right)^2\left(\dfrac{1}{V_P}\right)^2 (V_P)^2}}{2\left(\dfrac{2I_{DSS}}{I_Q}\right)^2\left(\dfrac{1}{V_P}\right)^2}$$

$$v_d^2 = (V_P)^2\left(\frac{I_Q}{I_{DSS}}\right)$$

Or $\underline{|v_d| = |V_P|\left(\dfrac{I_Q}{I_{DSS}}\right)^{1/2}}$

c. For v_d small,

$$i_{D1} \approx \frac{I_Q}{2} + \frac{1}{2}\cdot\frac{I_Q}{(-V_P)}\cdot v_d\sqrt{\frac{2I_{DSS}}{I_Q}}$$

$$g_f = \left.\frac{di_{D1}}{d_{vd}}\right|_{v_d\to 0} = \frac{1}{2}\cdot\frac{I_Q}{(-V_P)}\cdot\sqrt{\frac{2I_{DSS}}{I_Q}}$$

Or $\Rightarrow \underline{g_f(\max) = \left(\dfrac{1}{-V_P}\right)\sqrt{\dfrac{I_Q I_{DSS}}{2}}}$

11.55

a.

$$I_Q = I_{D1} + I_{D2} \Rightarrow \underline{I_Q = 1 \text{ mA}}$$

$$v_0 = 7 = 10 - (0.5)R_D \Rightarrow \underline{R_D = 6 \text{ k}\Omega}$$

b.

$$g_f(\max) = \left(\frac{1}{-V_P}\right)\sqrt{\frac{I_Q \cdot I_{DSS}}{2}}$$

$$g_f(\max) = \left(\frac{1}{4}\right)\sqrt{\frac{(1)(2)}{2}} \Rightarrow \underline{g_f(\max) = 0.25 \text{ mA/V}}$$

c.

$$A_d = \frac{g_m R_D}{2} = g_f(\max)\cdot R_D$$

$$A_d = (0.25)(6) \Rightarrow \underline{A_d = 1.5}$$

11.56

a.

$$I_S = \frac{-V_{GS} - (-5)}{R_S} = (2)I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$5 - V_{GS} = (2)(0.8)(20)\left(1 - \frac{V_{GS}}{(-2)}\right)^2$$

$$5 - V_{GS} = (2)16\left(1 + V_{GS} + \frac{1}{4}V_{GS}^2\right)$$

$$8V_{GS}^2 + 33V_{GS} + 27 = 0$$

$$V_{GS} = \frac{-33 \pm \sqrt{1089 - 4(8)(27)}}{2(8)}$$

$$= -1.125 \text{ V}$$

$$I_S = \frac{5 - (-1.125)}{20}$$

$$= 0.306 \text{ mA}$$

$$I_{D1} = I_{D2} = 0.153 \text{ mA}$$

$$v_{o2} = 1.17 \text{ V}$$

(b)

11.57

Equivalent circuit and analysis is identical to that in problem 11.47.

$$A_{d2} = \frac{\frac{1}{2}\cdot g_m R_L}{\left(2+\frac{R_L}{R_D}\right)}$$

$$A_{d1} = \frac{-\frac{1}{2}\cdot g_m R_L}{\left(2+\frac{R_L}{R_D}\right)}$$

$$A_v = \frac{v_{02}-v_{01}}{v_d} = \frac{g_m R_L}{\left(2+\frac{R_L}{R_D}\right)}$$

11.58

(a) $A_d = g_m\left(r_{o2}\|r_{o4}\right)$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.2}{0.026} = 7.692\ \text{mA/V}$$

$$r_{o2} = \frac{V_{A2}}{I_{CQ}} = \frac{120}{0.2} = 600\ \text{k}\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{CQ}} = \frac{80}{0.2} = 400\ \text{k}\Omega$$

$$A_d = (7.692)(600\|400) = 1846$$

(b) $R_o = r_{o2}\|r_{o4} = 600\|400 = 240\ \text{k}\Omega$

(c) $A_d = g_m\left(r_{o2}\|r_{o4}\|R_L\right)$

$$A_d = (0.75)(1846) = 1384.5 = (7.692)(240\|R_L) \Rightarrow R_L = 720\ \text{k}\Omega$$

11.59

(a)

$$I_Q = 250\ \mu\text{A} \quad I_{REF} = I_Q\left(1+\frac{2}{\beta}\right)$$

$$= 250\left(1+\frac{2}{180}\right) = 252.8\ \mu\text{A}$$

$$R_1 = \frac{5-(0.7)-(-5)}{0.2528} \Rightarrow \underline{R_1 = 36.8\ \text{K}}$$

(b)

$$A_d = g_m\left(r_{o2}\|r_{o4}\right) \qquad g_m = \frac{0.125}{0.026} = 4.808\ \text{mA/V}$$

$$r_{o2} = \frac{150}{0.125} = 1200\ \text{K}$$

$$A_d = (4.808)(1200\|800) \quad r_{o4} = \frac{100}{0.125} = 800\ \text{K}$$

$$\underline{A_d = 2308}$$

(c)

$$R_{id} = 2r_\pi = \frac{2(180)(0.026)}{0.125} \Rightarrow \underline{R_{id} = 74.9 \text{ K}}$$

$$R_o = r_{o2} \| r_{o4} = 1200 \| 800 = \underline{480 \text{ K} = R_o}$$

(d)

$$v_{cm}(\max) = 5 - 0.7 = 4.3 \text{ V}$$

$$v_{cm}(\min) = 0.7 + 0.7 - 5 = -3.6 \text{ V}$$

11.60

a.

$$I_0 = I_{B3} + I_{B4} \approx 2\left(\frac{I_Q}{2}\right)\left(\frac{1}{\beta}\right)$$

$$I_0 = \frac{I_Q}{\beta} = \frac{0.2}{100} \Rightarrow \underline{I_0 = 2\ \mu\text{A}}$$

b.

$$r_{02} = r_{04} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000\ \text{k}\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$A_d = g_m(r_{02} \| r_{04}) = (3.846)(1000 \| 1000) \Rightarrow \underline{A_d = 1923}$$

c.

$$A_d = g_m(r_{02} \| r_{04} \| R_L)$$

$$A_d = (3.846)(1000 \| 1000 \| 250) \Rightarrow \underline{A_d = 641}$$

11.61

(a) $$A_d = \frac{\dfrac{I_Q}{2V_T}}{\dfrac{I_Q}{2V_{A2}} + \dfrac{I_Q}{2V_{A4}} + \dfrac{1}{R_L}}$$

$$1000 = \frac{\dfrac{I_Q}{2(0.026)}}{\dfrac{I_Q}{2(90)} + \dfrac{I_Q}{2(60)} + \dfrac{1}{250}}$$

$$1000(I_Q)(0.005556 + 0.008333) + 4 = I_Q(19.23) \Rightarrow I_Q = 0.749 \text{ mA}$$

(b) $$\upsilon_{CM}(\max) = V^+ - 2V_{EB}(on) = 5 - 2(0.6) = 3.8 \text{ V}$$

11.62

(b) $\upsilon_{o1} = -g_{m1}\left(r_{o1} \| R_{o3}\right)\left(\dfrac{\upsilon_d}{2}\right)$

$$A_{d1} = \frac{\upsilon_{o1}}{\upsilon_d} = -\frac{1}{2} g_{m1}\left(r_{o1} \| R_{o3}\right)$$

$$g_{m1} = \frac{I_{CQ}}{V_T} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$r_{o1} = \frac{V_{A1}}{I_{CQ}} = \frac{120}{0.2} = 600 \text{ k}\Omega$$

$$R_{o3} = r_{o3} \| r_{\pi 3} \left\| \frac{1}{g_{m3}} \right.,\ \text{where } g_{m3} = 7.692 \text{ mA/V},$$

$$r_{\pi 3} = \frac{(120)(0.026)}{0.2} = 15.6 \text{ k}\Omega,\ r_{o3} = 400 \text{ k}\Omega$$

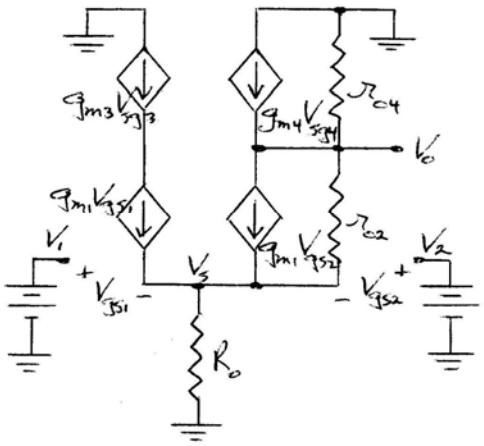
Then $R_{o3} = 400 \left\| 15.6 \right\| \dfrac{1}{7.692} = 15.014 \| 0.130 = 0.1289 \text{ k}\Omega$

$$A_{d1} = -\frac{1}{2}(7.692)(600 \| 0.1289) = -0.4956$$

(c) $A_{d2} = +\dfrac{1}{2} g_{m2}\left(r_{o2} \| R_{o4}\right) = +0.4956$

(d) $A_{d3} = +g_{m1}\left(r_{o1} \| R_{o3}\right) = 2(0.4956) = 0.9912$

11.63



$g_{m4}V_{sg4} = g_{m3}V_{sg3} = g_{m1}V_{gs1}$

$V_{gs1} = V_1 - V_s$, and $V_{gs2} = V_2 - V_s$

$$g_{m1}V_{gs1} + g_{m1}V_{gs2} + \frac{V_o - V_s}{r_{o2}} = \frac{V_s}{R_o}$$

$$g_{m1}(V_1 + V_2 - 2V_s) + \frac{V_o}{r_{o2}} = V_s\left(\frac{1}{R_o} + \frac{1}{r_{o2}}\right)$$

$$g_{m1}(V_1+V_2)+\frac{V_o}{r_{o2}}=V_s\left(2g_{m1}+\frac{1}{R_o}+\frac{1}{r_{o2}}\right)$$

Now $g_{m1}=2\sqrt{K_n I_{DQ}}=2\sqrt{(0.5)(0.1)}=0.4472$ mA/V

$$r_{o2}=\frac{1}{(0.02)(0.1)}=500\ \text{k}\Omega$$

Then $(0.4472)(V_1+V_2)+\frac{V_o}{500}=V_s\left[2(0.4472)+\frac{1}{2000}+\frac{1}{500}\right]=V_s(0.8969)$

We have (1) $V_s=(0.4986)(V_1+V_2)+V_o(0.0022299)$

Also (2) $\frac{V_o-V_s}{r_{o2}}+\frac{V_o}{r_{o4}}+g_{m1}V_{gs2}-g_{m1}V_{gs1}=0$

$$V_o\left(\frac{1}{r_{o2}}+\frac{1}{r_{o4}}\right)-\frac{V_s}{r_{o2}}+g_{m1}(V_2-V_s-(V_1-V_s))=0$$

$$V_o\left(\frac{1}{r_{o2}}+\frac{1}{r_{o4}}\right)-\frac{V_s}{r_{o2}}+g_{m1}(V_2-V_1)=0$$

We find $r_{o4}=\frac{1}{(0.03)(0.1)}=333.3\ \text{k}\Omega$

Then $V_o\left(\frac{1}{500}+\frac{1}{333.3}\right)-\frac{1}{500}[(0.4986)(V_1+V_2)+V_o(0.0022299)]+(0.4472)(V_2-V_1)=0$

$$V_o[(0.005)-(0.00000446)]-(0.0009972)(V_1+V_2)+(0.4472)(V_2-V_1)=0$$

(a) Let $V_1=V_d$ and $V_2=0$

$$V_o[0.00499554]-V_d(0.0009972)-V_d(0.4472)=0$$

$$A_d=\frac{V_o}{V_d}=89.72$$

(b) Let $V_1=0$ and $V_2=-V_d$

$$V_o[0.00499554]+V_d(0.0009972)-V_d(0.4472)=0$$

$$A_d=\frac{V_o}{V_d}=89.32$$

(c) Let $V_1=\frac{V_d}{2}$ and $V_2=-\frac{V_d}{2}$

$$V_o[0.00499554]=V_d(0.4472)$$

$$A=\frac{V_o}{V_d}=89.52$$

11.64

a. From symmetry.

$$V_{GS3} = V_{GS4} = V_{DS3} = V_{DS4} = \sqrt{\frac{0.1}{0.1}} + 1$$

Or $\underline{V_{DS3} = V_{DS4} = 2 \text{ V}}$

$$V_{SG1} = V_{SG2} = \sqrt{\frac{0.1}{0.1}} + 1 = 2 \text{ V}$$

$$V_{SD1} = V_{SD2} = V_{SG1} - (V_{DS3} - 10)$$
$$= 2 - (2 - 10)$$

Or $\underline{V_{SD1} = V_{SD2} = 10 \text{ V}}$

b.

$$r_{0n} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.1)} \Rightarrow 1 \text{ M}\Omega$$

$$r_{0p} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.015)(0.1)} \Rightarrow 0.667 \text{ M}\Omega$$

$$g_m = 2K_p (V_{SG} + V_{TP})$$
$$= 2(0.1)(2 - 1) = 0.2 \; mA/V$$

$$A_d = g_m (r_{on} \| r_{op}) = (0.2)(1000 \| 667) \Rightarrow \underline{A_d = 80}$$

(c)

$$I_{D2} = I_{D1} = \frac{I_Q}{2} = 0.1 \; mA$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.01)(0.1)} = 1000 \; k\Omega$$

$$r_{o2} = \frac{1}{\lambda_P I_{D2}} = \frac{1}{(0.015)(0.1)} = 667 \; k\Omega$$

$$R_o = r_{o2} \| r_{o4} = 667 \| 1000 = 400 \; k\Omega$$

11.65

(a) $A_d = g_m (r_{o2} \| r_{o4})$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_n I_{DQ}} = 2\sqrt{\left(\frac{0.1}{2}\right)(8)(0.06)} = 0.3098 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.025)(0.06)} = 666.7 \text{ k}\Omega$$

$$r_{o4} = \frac{1}{(0.04)(0.06)} = 416.7 \text{ k}\Omega$$

$$A_d = (0.3098)(666.7 \| 416.7) = 79.44$$

(b) $i_{D3} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_p (\upsilon_{SG3} + V_{TP})^2$

$$0.06 = \left(\frac{0.04}{2}\right)(10)(\upsilon_{SG3} - 0.3)^2 \Rightarrow \upsilon_{SG3} = 0.8477 \text{ V}$$

$$\upsilon_O = 1.8 - 0.8477 = 0.9523$$

$$\upsilon_{CM}(\max) = \upsilon_O - \upsilon_{DS}(sat) + \upsilon_{GS}$$

$$0.06 = \left(\frac{0.1}{2}\right)(8)(\upsilon_{GS} - 0.3)^2 \Rightarrow \upsilon_{GS} = 0.6873 \text{ V}, \quad \upsilon_{DS}(sat) = 0.3873 \text{ V}$$

$$\upsilon_{CM}(\max) = 0.9523 - 0.3873 + 0.6873 = 1.25\ \text{V}$$

11.66

$$R_{04} = r_{04}\left[1 + g_{m4}\left(R \| r_{\pi 4}\right)\right]$$

$$r_{04} = \frac{80}{0.1} = 800\ \text{K}$$

$$g_{m4} = \frac{0.1}{0.026} = 3.846$$

$$r_{\pi 4} = \frac{(100)(0.026)}{0.1}$$

$$= 26\ \text{K}$$

$$R \| r_{\pi 4} = 1 \| 26 = 0.963\ \text{K}$$

Assume $\beta = 100$

$$r_{\pi 3} = \frac{(100)(0.026)}{0.1} = 26\ \text{k}\Omega$$

$$g_{m3} = \frac{0.1}{0.026} = 3.846\ \text{mA/V}$$

$$R_{04} = 800\left[1 + (3.846)(0.963)\right] \Rightarrow 3.763\ \text{M}\Omega$$

$$\Rightarrow R_0 = 3.763\,\text{M}\Omega$$

Then

$$A_v = -g_m\left(r_{02} \| R_0\right)$$

$$r_{02} = \frac{120}{0.1} = 1200\ \text{k}\Omega$$

$$g_m = \frac{0.1}{0.026} = 3.846\ \text{mA/V}$$

$$A_v = -(3.846)\left[1200 \| 3763\right] \Rightarrow \underline{A_v = -3499}$$

b.

For

$$R = 0, \quad r_{04} = \frac{80}{0.1} = 800\ \text{k}\Omega$$

$$A_v = -g_m\left(r_{02} \| r_{04}\right)$$

$$= -(3.846)\left[1200 \| 800\right] \Rightarrow \underline{A_v = -1846}$$

(c) For part (a), $R_o = (3.763 \| 1.2) = 0.910\ M\Omega$

For part (b), $R_o = (1.2 \| 0.8) = 0.48\ M\Omega$

11.67

$$I_{B5} = \frac{I_{E5}}{1+\beta} = \frac{I_{B3}+I_{B4}}{1+\beta} = \frac{I_{C3}+I_{C4}}{\beta(1+\beta)}$$

Now $I_{C3} + I_{C4} \approx I_Q$

So $I_{B5} \approx \dfrac{I_Q}{\beta(1+\beta)}$

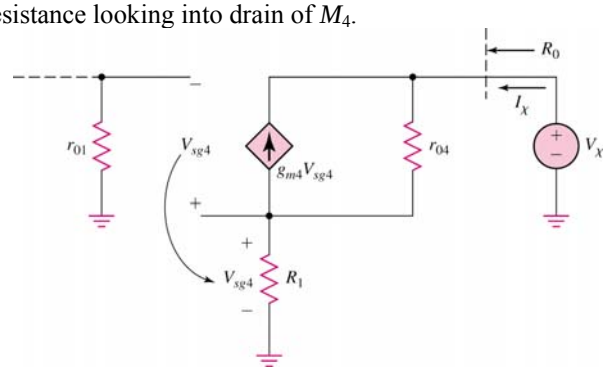
$$I_{B6} = \frac{I_{E6}}{1+\beta} = \frac{I_{Q1}}{\beta(1+\beta)}$$

For balance, we want $I_{B6} = I_{B5}$

So that $\underline{I_{Q1} = I_Q}$

11.68

Resistance looking into drain of M_4.



$$V_{sg4} \cong I_X R_1$$

$$I_X + g_{m4} V_{sg4} = \frac{V_X - V_{sg4}}{r_{o4}}$$

$$I_X \left[1 + g_{m4} R_1 + \frac{R_1}{r_{o4}} \right] = \frac{V_X}{r_{o4}}$$

Or $R_o = r_{o4}\left[1 + g_{m4} R_1 + \dfrac{R_1}{r_{o4}} \right]$

a.

$$A_d = g_{m2}\left(r_{o2}\|R_o\right)$$

$$g_{m2} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.080)(0.1)}$$
$$= 0.179\ mA/V$$

$$r_{o2} = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.015)(0.1)} = 667\ k\Omega$$

$$g_{m4} = 2\sqrt{K_P I_{DQ}} = 2\sqrt{(0.080)(0.1)}$$
$$= 0.179\ mA/V$$

$$r_{o4} = \frac{1}{\lambda_p I_{DQ}} = \frac{1}{(0.02)(0.1)} = 500\ k\Omega$$

$$R_0 = 500\left[1+(0.179)(1)+\frac{1}{500}\right] = 590.5\ \text{k}\Omega$$

$$A_d = (0.179)\left[667\|590.5\right] \Rightarrow \underline{A_d = 56.06}$$

b.

When $R_1 = 0, \quad R_0 = r_{04} = 500\ \text{k}\Omega$

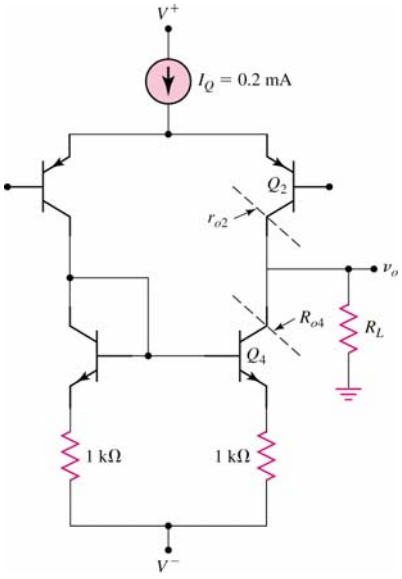
$$A_d = (0.179)\left[667\|500\right] \Rightarrow \underline{A_d = 51.15}$$

(c) For part (a), $R_o = r_{o2}\|R_o = 667\|590.5 \Rightarrow \underline{R_o = 313\ k\Omega}$

For part (b), $R_o = r_{o2}\|r_{o4} = 667\|500 \Rightarrow \underline{R_o = 286\ k\Omega}$

11.69

Let $\beta = 100, \quad V_A = 100\ V$



$$r_{o2} = \frac{V_A}{I_{CQ}} = \frac{100}{0.1} = 1000\ k\Omega$$

$R_{o4} = r_{o4}\left[1 + g_m R_E'\right]$ where $R_E' = r_\pi \| R_E$

Now

$$r_\pi = \frac{(100)(0.026)}{0.1} = 26\ k\Omega$$

$$g_m = \frac{0.1}{0.026} = 3.846\ mA/V$$

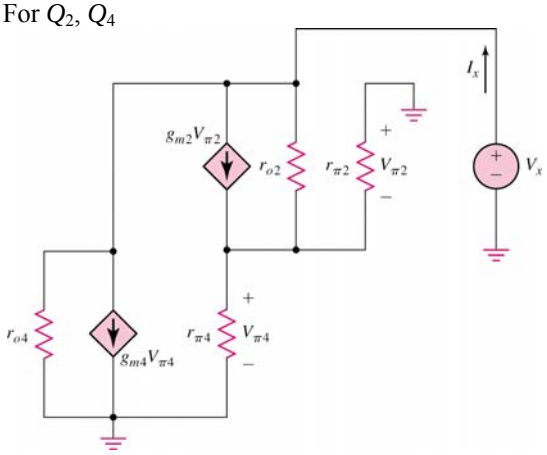
$R_E' = 26 \| 1 = 0.963\ k\Omega$

Then $R_{o4} = 1000\left[1 + (3.846)(0.963)\right] = 4704\ k\Omega$

$A_d = g_m\left(r_{o2} \| R_{o4}\right) = 3.846\left(1000 \| 4704\right) \Rightarrow \underline{A_d = 3172}$

11.70

(a) For Q_2, Q_4



(1) $$I_x = \frac{V_x - V_{\pi 4}}{r_{o2}} + g_{m2}V_{\pi 2} + g_{m4}V_{\pi 4} + \frac{V_x}{r_{o4}}$$

(2) $$g_{m2}V_{\pi 2} + \frac{V_x - V_{\pi 4}}{r_{o2}} = \frac{V_{\pi 4}}{r_{\pi 4} \| r_{\pi 2}}$$

(3) $$V_{\pi 4} = -V_{\pi 2}$$

From (2) $$\frac{V_x}{r_{o2}} = V_{\pi 4}\left[\frac{1}{r_{\pi 4} \| r_{\pi 2}} + \frac{1}{r_{o2}} + g_{m2}\right]$$

Now

$$I_{C4}=\left(\frac{\beta}{1+\beta}\right)\left(\frac{I_Q}{2}\right)=\left(\frac{120}{121}\right)(0.5)=0.496\ mA$$

$$I_{C2}=\left(\frac{I_Q}{2}\right)\left(\frac{1}{1+\beta}\right)\left(\frac{\beta}{1+\beta}\right)=(0.5)\left(\frac{120}{(121)^2}\right)\Rightarrow I_{C2}=0.0041\ mA$$

So

$$r_{\pi 2}=\frac{(120)(0.026)}{0.0041}=761\ k\Omega$$

$$g_{m2}=\frac{0.0041}{0.026}=0.158\ mA/V$$

$$r_{o2}=\frac{100}{0.0041}\Rightarrow 24.4\ M\Omega$$

$$r_{\pi 4}=\frac{(120)(0.026)}{0.496}=6.29\ k\Omega$$

$$g_{m4}=\frac{0.496}{0.026}=19.08\ mA/V$$

$$r_{o4}=\frac{100}{0.496}=202\ k\Omega$$

Now

$$\frac{V_x}{r_{o2}}=V_{\pi 4}\left[\frac{1}{6.29\|761}+\frac{1}{24400}+0.158\right]\Rightarrow \text{which yields } V_{\pi 4}=\frac{V_x}{(0.318)r_{o2}}$$

From (1),

$$I_x=\frac{V_x}{r_{o2}}+\frac{V_x}{r_{o4}}+V_{\pi 4}\left(g_{m4}-g_{m2}-\frac{1}{r_{o2}}\right)$$

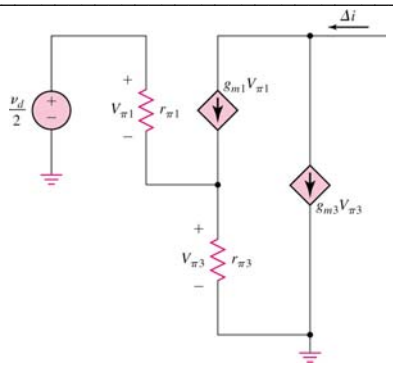
$$\frac{I_x}{V_x}=\left[\frac{1}{24400}+\frac{1}{202}+\frac{\left(19.08-0.158-\frac{1}{24400}\right)}{(0.318)(24400)}\right]\text{ which yields } R_{o2}=\frac{V_x}{I_x}=135\ k\Omega$$

Now $r_{o6}=\dfrac{80}{0.5}=160\ k\Omega$

Then $R_o=R_{o2}\|r_{o6}=135\|160\Rightarrow \underline{R_o=73.2\ k\Omega}$

(b)

$$A_d=g_m^c R_o \text{ where } g_m^c=\frac{\Delta i}{v_d/2}$$



$\Delta i = g_{m1}V_{\pi 1} + g_{m3}V_{\pi 3}$ and $V_{\pi 1} + V_{\pi 3} = \dfrac{v_d}{2}$

Also $\left(\dfrac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1}\right) r_{\pi 3} = V_{\pi 3}$

So $V_{\pi 1}\left(\dfrac{1+\beta}{r_{\pi 1}}\right) r_{\pi 3} = V_{\pi 3}$

Or $V_{\pi 1}\left(\dfrac{121}{761}\right)(6.29) = V_{\pi 3} \cong V_{\pi 1}$

Then $2V_{\pi 1} = \dfrac{v_d}{2} \Rightarrow V_{\pi 1} = \dfrac{v_d}{4}$

So $\Delta i = (g_{m1} + g_{m3})V_{\pi 1} = (0.158 + 19.08)\left(\dfrac{v_d}{4}\right) = 9.62\left(\dfrac{v_d}{2}\right)$

So $g_m^c = \dfrac{\Delta i}{v_d/2} = 9.62 \Rightarrow A_d = (9.62)(73.2) \Rightarrow \underline{A_d = 704}$

Now $R_{id} = 2R_i$ where $R_i = r_{\pi 1} + (1+\beta) r_{\pi 3}$

$R_i = 761 + (121)(6.29) = 1522\ k\Omega$

Then $\underline{R_{id} = 3.044\ M\Omega}$

11.71 - Design Problem

11.72

Input: $-8 \le V_d \le 8$ mV

Output: $-0.8 \le V_o \le 0.8$ V

$$A_d = \frac{V_o}{V_d} = \frac{0.8}{0.008} = 100$$

$$A_d = g_m\left(r_{o2} \| r_{o4}\right)$$

Let $I_Q = 0.5$ mA, $I_{DQ} = 0.25$ mA

$$r_{o2} = \frac{1}{(0.025)(0.25)} = 160\ \text{k}\Omega$$

$$r_{o4} = \frac{1}{(0.04)(0.25)} = 100\ \text{k}\Omega$$

$$100 = g_m(160 \| 100) \Rightarrow g_m = 1.625\ \text{mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_n I_{DQ}}$$

$$1.625 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_n (0.25)} \Rightarrow \left(\frac{W}{L}\right)_n = 52.8$$

11.73

For current source, $I_{REF} = I_Q$

$$P = (2I_Q)(V^+ - V^-)$$

$$0.8 = (2I_Q)(6) \Rightarrow I_Q \cong 66\,\mu\text{A}$$

$$A_d = g_m(r_{o2} \| r_{o4})$$

$$r_{o2} = \frac{1}{(0.02)\left(\frac{0.066}{2}\right)} = 1515\ \text{k}\Omega$$

$$r_{o4} = \frac{1}{(0.03)\left(\frac{0.066}{2}\right)} = 1010\ \text{k}\Omega$$

$$240 = g_m(1515 \| 1010) \Rightarrow g_m = 0.396\ \text{mA/V}$$

$$g_m = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_n I_{DQ}}$$

$$0.396 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_n (0.033)} \Rightarrow \left(\frac{W}{L}\right)_n = 23.76$$

11.74

$$A_d = g_m(r_{o2} \| R_o)$$

$$\approx g_m r_{o2}$$

$$r_{o2} = \frac{1}{\lambda_n I_D}$$

$$= \frac{1}{(0.015)(0.1)} = 666.7\ \text{K}$$

$$A_d = 400 = g_m(666.7)$$

$$g_m = 0.60\ \text{mA/V}$$

$$= 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right) I_D}$$

$$0.60 = 2\sqrt{\left(\frac{0.08}{2}\right)\left(\frac{W}{L}\right)(0.1)}$$

$$0.090 = 0.004\left(\frac{W}{L}\right)$$

$$\underline{\left(\frac{W}{L}\right)_1 = \left(\frac{W}{L}\right)_2 = 22.5}$$

11.75

$A_d = g_m\left(R_{o4}\|R_{o6}\right)$

where

$R_{o4} = r_{o4} + r_{o2}\left[1 + g_{m4}r_{o4}\right]$

$R_{o6} = r_{o6} + r_{o8}\left[1 + g_{m6}r_{o6}\right]$

We have

$$r_{o2} = r_{o4} = \frac{1}{(0.015)(0.040)} = 1667\ k\Omega$$

$$r_{o6} = r_{o8} = \frac{1}{(0.02)(0.040)} = 1250\ k\Omega$$

$$g_{m4} = 2\sqrt{\left(\frac{0.060}{2}\right)(15)(0.040)} = 0.268\ mA/V$$

$$g_{m6} = 2\sqrt{\left(\frac{0.025}{2}\right)(10)(0.040)} = 0.141\ mA/V$$

Then

$R_{o4} = 1667 + 1667\left[1 + (0.268)(1667)\right] \Rightarrow 748\ M\Omega$

$R_{o6} = 1250 + 1250\left[1 + (0.141)(1250)\right] \Rightarrow 222.8\ M\Omega$

(a)

$R_o = R_{o4}\|R_{o6} = 748\|222.8 \Rightarrow \quad \underline{R_o = 172\ M\Omega}$

(b)

$A_d = g_{m4}\left(R_{o4}\|R_{o6}\right) = (0.268)(172000) \Rightarrow \underline{A_d = 46096}$

11.76

(a) $A_d = g_m\left(r_{o2}\|r_{o4}\right)$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.06)} = 0.2191\ \text{mA/V}$$

$$r_{o2} = r_{o4} = \frac{1}{(0.025)(0.06)} = 666.7\ \text{k}\Omega$$

$$A_d = (0.2191)(666.7\|666.7) = 73.0$$

(b) $R_o = r_{o2}\|r_{o4} = 333.3\ \text{k}\Omega$

(c) $i_{D3} = K_p\left(\upsilon_{SG3} + V_{TP}\right)^2$

$0.06 = 0.2(\upsilon_{SG3} - 0.3)^2 \Rightarrow \upsilon_{SG3} = 0.8477\ \text{V}$

$\upsilon_O = V^+ - \upsilon_{SG3} = 2.8 - 0.8477 = 1.9523\ \text{V}$

$i_{D1} = K_n\left(\upsilon_{GS1} - V_{TN}\right)^2$

$0.06 = 0.2(\upsilon_{GS1} - 0.3)^3 \Rightarrow \upsilon_{GS1} = 0.8477\ \text{V}, \ \Rightarrow \upsilon_{DS1}(sat) = 0.5477\ \text{V}$

$\upsilon_{CM}(\max) = \upsilon_O - \upsilon_{DS1}(sat) + \upsilon_{GS1} = 1.9523 - 0.5477 + 0.8477 = 2.25\ \text{V}$

11.77

(a) $i_{D3} = K_p\left(\upsilon_{SG3} + V_{TP}\right)^2$

$0.25 = 0.25\left(\upsilon_{SG3} - 0.4\right)^2 \Rightarrow \upsilon_{SG3} = 1.4 \text{ V}$

Then $\upsilon_{GS1} = 1.4 \text{ V}, \quad \upsilon_{DS1}(sat) = 1.0 \text{ V}$

$\upsilon_{CM} = \upsilon_O - \upsilon_{DS1}(sat) + \upsilon_{GS1}$

$3 = \left(V^+ - 1.4\right) - 1.0 + 1.4 \Rightarrow V^+ = 4 \text{ V} = -V^-$

(b) $A_d = g_m\left(r_{o2} \| r_{o4}\right)$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.25)(0.25)} = 0.50 \text{ mA/V}$$

$$r_{o2} = r_{o4} = \frac{1}{(0.02)(0.25)} = 200 \text{ k}\Omega$$

$A_d = (0.50)(200 \| 200) = 50$

11.78

(a) For $v_{cm} = +2\text{V} \Rightarrow \underline{V^+ = 2.7 \text{ V}}$

If I_Q is a 2-transistor current source,

$V^- = v_{cm} - 0.7 - 0.7$

$V^- = -3.4 \text{ V} \Rightarrow \underline{V^+ = -V^- = 3.4 \text{ V}}$

(b)

$A_d = g_m\left(r_{o2} \| r_{o4}\right) \quad r_{o2} = \dfrac{100}{0.1} = 1000 \text{ K}$

$$r_{o4} = \frac{60}{0.1} = 600 \text{ K}$$

$$g_m = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$A_d = (3.846)(1000 \| 600) \Rightarrow \underline{A_d = 1442}$

11.79

(a) $V^+ = -V^- = 3.4 \text{ V}$

(b)

$$r_{o2} = \frac{75}{0.06} = 1250 \text{ K}$$

$$r_{o4} = \frac{40}{0.06} = 666.7 \text{ K}$$

$$g_m = \frac{0.06}{0.026} = 2.308 \text{ mA/V}$$

$A_d = (2.308)(1250 \| 666.7)$

$\underline{A_d = 1004}$

11.80

(a) $I_{E2} = 0.25$ mA, $I_{B2} = 0.001656$ mA, $I_{C2} = 0.2483$ mA

$$I_{D1} = 0.25 + 0.001656 = 0.25166 \text{ mA}$$

$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.25166)} = 0.4487 \text{ mA/V}$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{0.2483}{0.026} = 9.55 \text{ mA/V}$$

$$r_{\pi_2} = \frac{(150)(0.026)}{0.2483} = 15.71 \text{ k}\Omega$$

$$g_m^C = \frac{(0.4487)[1 + (9.55)(15.71)]}{1 + (0.4487)(15.71)} = 8.42 \text{ mA/V}$$

(b) $I_{E2} = 0.45$ mA, $I_{B2} = 0.002980$ mA, $I_{C2} = 0.4470$ mA

$$I_{D1} = 0.05 + 0.00298 = 0.05298 \text{ mA}$$

$$g_{m1} = 2\sqrt{(0.2)(0.05298)} = 0.2059 \text{ mA/V}$$

$$g_{m2} = \frac{0.4470}{0.026} = 17.19 \text{ mA/V}$$

$$r_{\pi_2} = 8.725 \text{ k}\Omega$$

$$g_m^C = \frac{(0.2059)[1 + (17.19)(8.725)]}{1 + (0.2059)(8.725)} = 11.12 \text{ mA/V}$$

11.81

$$r_0(M_2) = \frac{1}{\lambda_n I_{DQ}} = \frac{1}{(0.01)(0.2)} = 500 \text{ k}\Omega$$

$$r_0(Q_2) = \frac{V_A}{I_{CQ}} = \frac{80}{0.2} = 400 \text{ k}\Omega$$

$$g_m(M_2) = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.2)(0.2)}$$

$$= 0.4 \text{ mA/V}$$

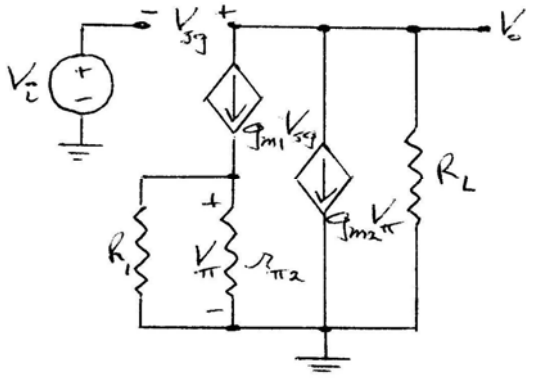
$$A_d = g_m(M_2)\left[r_0(M_2) \| r_0(Q_2)\right]$$

$$= 0.4[500 \| 400] \Rightarrow \underline{A_d = 88.9}$$

If the I_Q current source is ideal, $\underline{A_{cm} = 0}$ and $\underline{CMRR_{dB} = \infty}$

11.82

(a)



(b) $I_{R1} = \dfrac{0.7}{8} = 0.0875\,\text{mA}$

$I_Q = 0.5 = I_{R1} + I_{E2} = 0.0875 + I_{E2} \Rightarrow I_{E2} = 0.4125\,\text{mA}$

$I_{B2} = 0.002279\,\text{mA}, \quad I_{C2} = 0.41022\,\text{mA}$

$I_{D1} = I_{R1} + I_{B2} = 0.0875 + 0.002279 = 0.08978\,\text{mA}$

$g_{m1} = 2\sqrt{(0.7)(0.08978)} = 0.5014\,\text{mA/V}$

$g_{m2} = \dfrac{0.41022}{0.026} = 15.78\,\text{mA/V}$

$r_{\pi_2} = 11.41\,\text{k}\Omega$

(c) $V_o = -\left(g_{m1}V_{sg} + g_{m2}V_\pi\right)R_L$

$V_\pi = \left(g_{m1}V_{sg}\right)\left(R_1 \| r_{\pi 2}\right)$

$V_o = -\left[g_{m1}V_{sg} + g_{m2}g_{m1}V_{sg}\left(R_1 \| r_{\pi 2}\right)\right]R_L$

$-V_o = V_{sg}\left[g_{m1} + g_{m2}g_{m1}\left(R_1 \| r_{\pi 2}\right)\right]R_L$

Now $V_{sg} = V_o - V_i$

So $V_o\left[1 + g_{m1}\left(1 + g_{m2}\left(R_1 \| r_{\pi 2}\right)\right)R_L\right] = V_i\, g_{m1}\left[1 + g_{m2}\left(R_1 \| r_{\pi 2}\right)\right]R_L$

$$A_\upsilon = \frac{V_o}{V_i} = \frac{g_{m1}\left[1 + g_{m2}\left(R_1 \| r_{\pi 2}\right)\right]R_L}{1 + g_{m1}\left[1 + g_{m2}\left(R_1 \| r_{\pi 2}\right)\right]R_L}$$

$$A_\upsilon = \frac{(0.5014)\left[1 + (15.78)(8 \| 11.41)\right]R_L}{1 + (0.5014)\left[1 + (15.78)(8 \| 11.41)\right]R_L} = \frac{37.71R_L}{1 + 37.71R_L}$$

(i) For $R_L = 10\,\text{k}\Omega$,

$$A_\upsilon = \frac{(37.71)(10)}{1 + (37.71)(10)} = 0.99736$$

(ii) For $R_L = 100\,\text{k}\Omega$,

$$A_\upsilon = \frac{(37.71)(100)}{1 + (37.71)(100)} = 0.99973$$

11.83

(a) $I_{R1} = \dfrac{0.7}{8} = 0.0875\,\text{mA}$

$I_Q = 0.8 = I_{R1} + I_{E2} = 0.0875 + I_{E2} \Rightarrow I_{E2} = 0.7125\,\text{mA}$

$I_{B2} = 0.003936\,\text{mA}, \quad I_{C2} = 0.7086\,\text{mA}$

$I_{D1} = I_{R1} + I_{B2} = 0.0875 + 0.003936 = 0.091436\,\text{mA}$

$g_{m1} = 2\sqrt{(0.7)(0.091436)} = 0.506\,\text{mA/V}$

$g_{m2} = \dfrac{0.7086}{0.026} = 27.25\,\text{mA/V}$

$r_{\pi 2} = 6.605\,\text{k}\Omega$

(b) $I_x = g_{m2}V_\pi + g_{m1}V_{sg}$

$V_\pi = (g_{m1}V_{sg})(R_1 \| r_{\pi 2})$

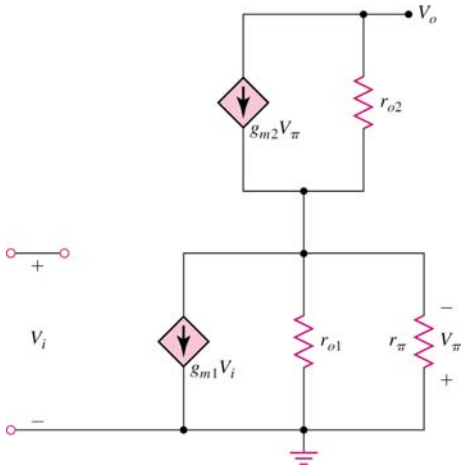
Now $V_{sg} = V_x$

$$\frac{I_x}{V_x} = \frac{1}{R_o} = g_{m1} + g_{m2}g_{m1}(R_1 \| r_{\pi 2}) = 0.506 + (0.506)(27.25)(8 \| 6.605) = 50.39\,\text{mA/V}$$

Or $R_o = 19.8\,\Omega$

11.84

(a)



(1) $$g_{m2}V_\pi + \frac{V_o - (-V_\pi)}{r_{o2}} = 0$$

(2) $$g_{m2}V_\pi + \frac{V_o - (-V_\pi)}{r_{o2}} = g_{m1}V_i + \frac{-V_\pi}{r_{o1}} + \frac{-V_\pi}{r_\pi} \quad \text{or} \quad 0 = g_{m1}V_i - V_\pi\left(\frac{1}{r_{o1}} + \frac{1}{r_\pi}\right)$$

$$V_\pi = \frac{g_{m1}V_i}{\left(\dfrac{1}{r_{o1}} + \dfrac{1}{r_\pi}\right)}$$

Then

From (1)

$$\left(g_{m2}+\frac{1}{r_{o2}}\right)V_\pi+\frac{V_o}{r_{o2}}=0$$

$$V_o=-r_{o2}\left(g_{m2}+\frac{1}{r_{o2}}\right)V_\pi=-r_{o2}g_{m1}V_i\frac{\left(g_{m2}+\frac{1}{r_{o2}}\right)}{\left(\frac{1}{r_{o1}}+\frac{1}{r_\pi}\right)}$$

$$A_v=\frac{V_o}{V_i}=\frac{-g_{m1}r_{o2}\left(g_{m2}+\frac{1}{r_{o2}}\right)}{\left(\frac{1}{r_{o1}}+\frac{1}{r_\pi}\right)}$$

Now

$$g_{m1}=2\sqrt{K_nI_Q}=2\sqrt{(0.25)(0.025)}=0.158\ mA/V$$

$$g_{m2}=\frac{I_Q}{V_T}=\frac{0.025}{0.026}=0.9615\ mA/V$$

$$r_{o1}=\frac{1}{\lambda I_Q}=\frac{1}{(0.02)(0.025)}=2000\ k\Omega$$

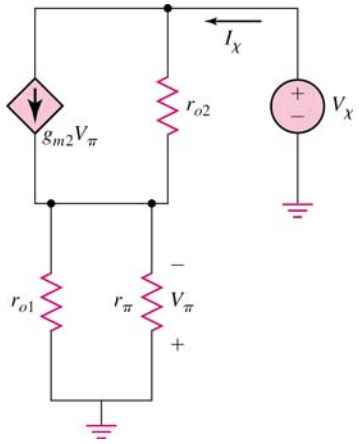
$$r_{o2}=\frac{V_A}{I_Q}=\frac{50}{0.025}=2000\ k\Omega$$

$$r_\pi=\frac{\beta V_T}{I_Q}=\frac{(100)(0.026)}{0.025}=104\ k\Omega$$

Then

$$A_v=\frac{-(0.158)(2000)\left(0.9615+\frac{1}{2000}\right)}{\left(\frac{1}{2000}+\frac{1}{104}\right)}\Rightarrow \underline{A_v=-30039}$$

To find R_o; set $V_i=0\Rightarrow g_{m1}V_i=0$



$$I_x = g_{m2}V_\pi + \frac{V_x - (-V_\pi)}{r_{o2}}$$

$$V_\pi = -I_x\left(r_{o1} \| r_\pi\right)$$

Then

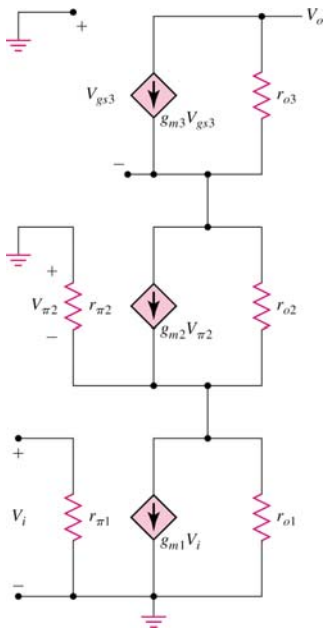
$$I_x = \left(g_{m2} + \frac{1}{r_{o2}}\right)(-I_x)\left(r_{o1} \| r_\pi\right) + \frac{V_x}{r_{o2}}$$

Combining terms,

$$R_o = \frac{V_x}{I_x} = r_{o2}\left[1 + \left(r_{o1} \| r_\pi\right)\left(g_{m2} + \frac{1}{r_{o2}}\right)\right]$$

$$= 2000\left[1 + (2000 \| 104)\left(0.9615 + \frac{1}{2000}\right)\right] \Rightarrow \underline{R_o = 192.2\ M\Omega}$$

(b)



(1) $$g_{m3}V_{gs3} + \frac{V_o - (-V_{gs3})}{r_{o3}} = 0$$

(2) $$g_{m3}V_{gs3} + \frac{V_o - (-V_{gs3})}{r_{o3}} = g_{m2}V_{\pi 2} + \frac{-V_{gs3} - (-V_{\pi 2})}{r_{o2}} \text{ or } 0 = V_{\pi 2}\left(g_{m2} + \frac{1}{r_{o2}}\right) - \frac{V_{gs3}}{r_{o2}}$$

(3) $$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} + \frac{-V_{gs3} - (-V_{\pi 2})}{r_{o2}} = g_{m1}V_i + \frac{(-V_{\pi 2})}{r_{o1}}$$

$$V_{\pi 2} = \frac{V_{gs3}}{r_{o2}\left(g_{m2} + \frac{1}{r_{o2}}\right)}$$

From (2),

Then

(3) $$V_{\pi 2}\left(\frac{1}{r_{\pi 2}} + g_{m2} + \frac{1}{r_{o2}} + \frac{1}{r_{o1}}\right) = g_{m1}V_i + \frac{V_{gs3}}{r_{o2}}$$

or

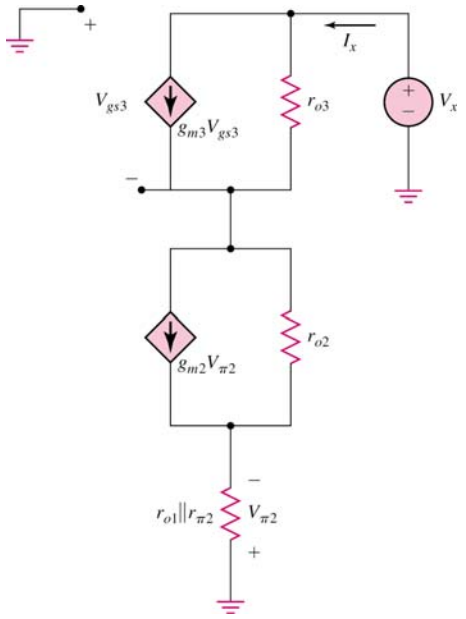
$$\frac{V_{gs3}}{r_{o2}\left(g_{m2}+\frac{1}{r_{o2}}\right)}\left[\frac{1}{r_{\pi 2}}+g_{m2}+\frac{1}{r_{o2}}+\frac{1}{r_{o1}}\right]=g_{m1}V_i+\frac{V_{gs3}}{r_{o2}}$$

$$\frac{V_{gs3}}{2000\left(0.9615+\frac{1}{2000}\right)}\left[\frac{1}{104}+0.9615+\frac{1}{2000}+\frac{1}{2000}\right]=0.9615V_i+\frac{V_{gs3}}{2000}$$

Then $V_{gs3}=1.83\times10^5V_i$

From (1), $\left(g_{m3}+\frac{1}{r_{o3}}\right)V_{gs3}=\frac{-V_o}{r_{o3}}$ or $V_o=-2000\left(0.158+\frac{1}{2000}\right)\left(1.83\times10^5\right)V_i$

$$A_v=\frac{V_o}{V_i}=-5.80\times10^7$$



To find R_o

(1) $$I_x=g_{m3}V_{gs3}+\frac{V_x-\left(-V_{gs3}\right)}{r_{o3}}$$

(2) $$g_{m3}V_{gs3}+\frac{V_x-\left(-V_{gs3}\right)}{r_{o3}}=g_{m2}V_{\pi 2}+\frac{-V_{gs3}-\left(-V_{\pi 2}\right)}{r_{o2}}$$

(3) $$V_{\pi 2}=-I_x\left(r_{o1}\|r_{\pi 2}\right)$$

From (1) $I_x=V_{gs3}\left(g_{m3}+\frac{1}{r_{o3}}\right)+\frac{V_x}{r_{o3}}$

$$I_x=V_{gs3}\left(0.158+\frac{1}{2000}\right)+\frac{V_x}{2000}$$

So $V_{gs3}=\frac{I_x-\frac{V_x}{2000}}{0.1585}$

From (2),

$$V_{gs3}\left[g_{m3}+\frac{1}{r_{o3}}+\frac{1}{r_{o2}}\right]+\frac{V_x}{r_{o3}}=V_{\pi 2}\left(g_{m2}+\frac{1}{r_{o2}}\right)$$

$$V_{gs3}\left[0.158+\frac{1}{2000}+\frac{1}{2000}\right]+\frac{V_x}{2000}=V_{\pi 2}\left(0.9615+\frac{1}{2000}\right)$$

Then $$\left[\frac{I_x-V_x/2000}{0.1585}\right](0.159)+\frac{V_x}{2000}=-I_x\left(2000\|104\right)(0.962)$$

We find $$R_o=\frac{V_x}{I_x}=6.09\times 10^{10}\ \Omega$$

11.85

Assume emitter of Q_1 is capacitively coupled to signal ground.

$$I_{CQ}=0.2\left(\frac{80}{81}\right)=0.1975\ mA$$

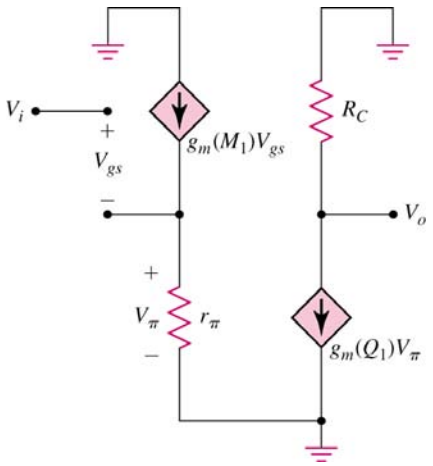
$$I_{DQ}=\frac{0.2}{81}=0.00247\ mA$$

$$r_\pi=\frac{(80)(0.026)}{0.1975}=10.5\ k\Omega$$

$$g_m(Q_1)=\frac{0.1975}{0.026}=7.60\ mA/V$$

$$g_m(M_1)=2\sqrt{K_nI_D}=2\sqrt{(0.2)(0.00247)}$$

$$g_m(M_1)=0.0445\ mA/V$$



$V_i = V_{gs} + V_\pi$ and $V_\pi = g_m(M_1)V_{gs}r_\pi$ or $V_{gs} = \dfrac{V_\pi}{g_m(M_1)r_\pi}$

Then $V_i = V_\pi\left(1+\dfrac{1}{g_m(M_1)r_\pi}\right)$ or $V_\pi = \dfrac{V_i}{\left(1+\dfrac{1}{g_m(M_1)r_\pi}\right)}$

$V_o = -g_m(Q_1)V_\pi R_C \Rightarrow A_v = \dfrac{V_o}{V_i} = \dfrac{-g_m(Q_1)R_C}{\left(1+\dfrac{1}{g_m(M_1)r_\pi}\right)}$

Then $A_v = \dfrac{-(7.60)(20)}{\left(1+\dfrac{1}{(0.0445)(10.5)}\right)} \Rightarrow \underline{A_v = -48.4}$

11.86

$$r_{\pi 11} = \frac{(120)(0.026)}{0.2} = 15.6\ \text{k}\Omega$$

$$R'_E = R_3 \| r_{\pi 11} = 0.2 \| 15.6 = 0.1975\ \text{k}\Omega$$

$$g_{m11} = \frac{0.2}{0.026} = 7.692\ \text{mA/V}$$

$$r_{o11} = \frac{V_{A11}}{I_{C11}} = \frac{120}{0.2} = 600\ \text{k}\Omega$$

$$R_{C11} = r_{o11}(1 + g_{m11}R'_E) = 600[1 + (7.692)(0.1975)] = 1512\ \text{k}\Omega$$

$$R_{C7} = r_{o7} = \frac{V_{A7}}{I_{C7}} = \frac{60}{0.2} = 300\ \text{k}\Omega$$

$$Z = R_{C7} \| R_{C11} = 300 \| 1512 = 250\ \text{k}\Omega$$

$$r_{\pi 8} = \frac{(120)(0.026)}{1} = 3.12\ \text{k}\Omega$$

$$I_{C9} = \frac{1}{120}\left(\frac{120}{121}\right) = 0.008264\ \text{mA}$$

$$r_{\pi 9} = \frac{(120)(0.026)}{0.008264} = 377.5\ \text{k}\Omega$$

Now $R_o = R_4 \left\| \dfrac{r_{\pi 8} + \left(\dfrac{r_{\pi 9} + Z}{121}\right)}{121} = 5 \right\| \dfrac{3.12 + \left(\dfrac{377.5 + 250}{121}\right)}{121} = 5 \| 0.06864$

Or $R_o = 67.7\ \Omega$

11.87

$$R_i = r_{\pi 1} + (1+\beta) r_{\pi 2}$$

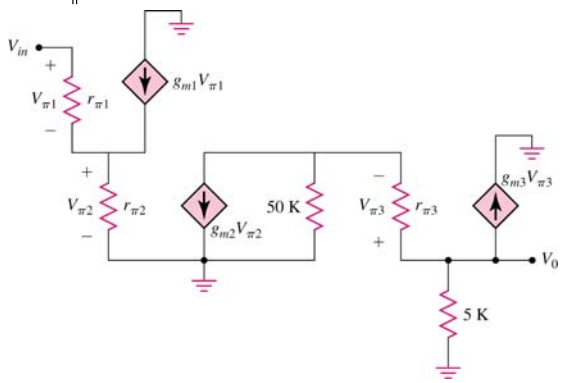
$$r_{\pi 2} = \frac{(100)(0.026)}{0.5} = 5.2 \text{ k}\Omega$$

$$r_{\pi 1} = \frac{(100)(0.026)}{(0.5/100)} = \frac{(100)^2 (0.026)}{0.5} = 520 \text{ k}\Omega$$

$$R_i = 520 + (101)(5.2) \Rightarrow \underline{R_i \cong 1.05 \text{ M}\Omega}$$

$$R_0 = 5 \left\| \frac{r_{\pi 3} + 50}{101}, \quad r_{\pi 3} = \frac{(100)(0.026)}{1} = 2.6 \text{ k}\Omega \right.$$

$$R_0 = 5 \left\| \frac{2.6 + 50}{101} = 5 \right\| 0.521 \Rightarrow \underline{R_0 = 0.472 \text{ k}\Omega}$$



$$V_0 = -\left(\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3} V_{\pi 3} \right)(5)$$

$$V_0 = -V_{\pi 3} \left(\frac{1+\beta}{r_{\pi 3}} \right)(5) \qquad (1)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} = g_{m2} V_{\pi 2} + \frac{(V_0 - V_{\pi 3})}{50}$$

$$g_{m2} V_{\pi 2} = V_{\pi 3} \left(\frac{1}{r_{\pi 3}} + \frac{1}{50} \right) - \frac{V_0}{50} \qquad (2)$$

$$V_{\pi 2} = \left(\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1} V_{\pi 1} \right) r_{\pi 2}$$

$$= V_{\pi 1} \left(\frac{1+\beta}{r_{\pi 1}} \right) r_{\pi 2} \qquad (3)$$

and

$$V_{in} = V_{\pi 1} + V_{\pi 2} \qquad (4)$$

$$g_{m2} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

Then

$$V_0 = -V_{\pi 3} \left(\frac{101}{2.6} \right)(5) \Rightarrow V_{\pi 3} = -V_0 (0.005149) \qquad (1)$$

And

$$19.23V_{\pi 2} = -V_0\left(0.005149\right)\left(\frac{1}{2.6}+\frac{1}{50}\right)-\frac{V_0}{50}$$

$$= -V_0\left(0.02208\right)$$

$$\text{Or } V_{\pi 2} = -V_0\left(0.001148\right) \qquad (2)$$

And

$$V_{\pi 1} = V_{in} - V_{\pi 2} = V_{in} + V_0\left(0.001148\right) \qquad (4)$$

So

$$-V_0\left(0.001148\right) = \left[V_{in} + V_0\left(0.001148\right)\right]\left(\frac{101}{520}\right)(5.2) \qquad (3)$$

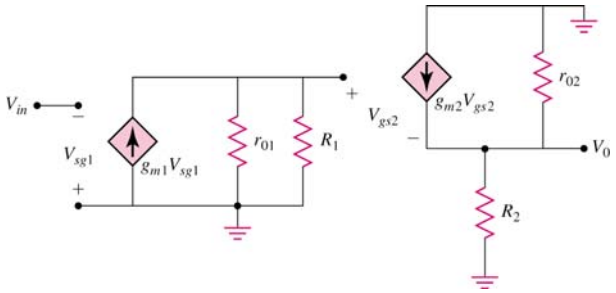
$$-V_0\left(0.001148\right) - V_0\left(0.001159\right) = V_{in}\left(1.01\right) \Rightarrow \underline{A_v = \frac{V_0}{V_{in}} = -438}$$

11.88

$$I_2 = \frac{5}{5} = 1 \text{ mA}$$

$$V_{GS2} = \sqrt{\frac{1}{0.5}} + 0.8 = 2.21 \text{ V}$$

$$I_1 = \frac{2.21-(-5)}{35} = 0.206 \text{ mA}$$



$V_0 = \left(g_{m2}V_{gs2}\right)\left(R_2 \| r_{02}\right)$

$V_{gs2} = \left(g_{m1}V_{sg1}\right)\left(r_{01} \| R_1\right) - V_0$ and $V_{sg1} = -V_{in}$

So $V_{gs2} = -\left(g_{m1}V_{in}\right)\left(r_{01} \| R_1\right) - V_0$

Then

$V_0 = g_{m2}\left(R_2 \| r_{02}\right)\left[-\left(g_{m1}V_{in}\right)\left(r_{01} \| R_1\right) - V_0\right]$

$$A_v = \frac{V_0}{V_{in}} = \frac{-g_{m2}\left(R_2 \| r_{02}\right)g_{m1}\left(r_{01} \| R_1\right)}{1 + g_{m2}\left(R_2 \| r_{02}\right)}$$

$g_{m2} = 2\sqrt{K_{n2}I_{D2}} = 2\sqrt{(0.5)(1)} = 1.414\ mA/V$

$g_{m1} = 2\sqrt{K_{p1}I_{D1}} = 2\sqrt{(0.2)(0.206)} = 0.406\ mA/V$

$$r_{01} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.01)(0.206)} = 485\ \text{k}\Omega$$

$$r_{02} = \frac{1}{\lambda_2 I_{D2}} = \frac{1}{(0.01)(1)} = 100\ \text{k}\Omega$$

$R_2 \| r_{02} = 5 \| 100 = 4.76\ \text{k}\Omega$

$R_1 \| r_{01} = 35 \| 485 = 32.6\ \text{k}\Omega$

Then $A_v = \dfrac{-(1.414)(4.76)(0.406)(32.6)}{1 + (1.414)(4.76)}$

So $\Rightarrow \underline{A_v = -11.5}$

Output Resistance—From the results for a source follower in Chapter 4.

$$R_0 = \frac{1}{g_{m2}} \| R_2 \| r_{02} = \frac{1}{1.414} \| 5 \| 100$$

$$= 0.707 \| 4.76$$

So $\underline{R_0 = 0.616\ \text{k}\Omega}$

11.89

(a) $R_2 = \dfrac{5-0}{1} = 5\ \text{k}\Omega$

$I_2 = K_p\left(V_{SG} + V_{TP}\right)^2 \Rightarrow 1 = 1\left(V_{SG} - 0.8\right)^2 \Rightarrow V_{SG} = 1.8\ \text{V}$

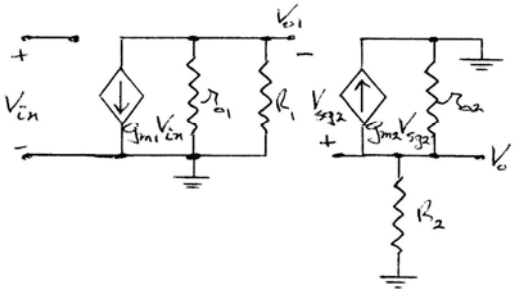
$$R_1 = \frac{5 - (-1.8)}{0.25} = 27.2\ \text{k}\Omega$$

(b) $g_{m1} = 2\sqrt{(0.5)(0.25)} = 0.7071\ \text{mA/V}$

$g_{m2} = 2\sqrt{(1)(1)} = 2\ \text{mA/V}$

$r_{o1} = \dfrac{1}{(0.02)(0.25)} = 200\ \text{k}\Omega$, $r_{o2} = \dfrac{1}{(0.02)(1)} = 50\ \text{k}\Omega$

(c)



$$V_{o1} = -g_{m1}V_{in}\left(r_{o1}\|R_1\right)$$
$$V_{sg2} = V_o - V_{o1}$$

$$\frac{V_o}{R_2} + \frac{V_o}{r_{o2}} + g_{m2}V_{sg2} = 0$$
$$V_o\left(\frac{1}{R_2} + \frac{1}{r_{o2}}\right) + g_{m2}\left[V_o + g_{m1}V_{in}\left(r_{o1}\|R_1\right)\right] = 0$$
$$V_o\left(\frac{1}{R_2} + \frac{1}{r_{o2}} + g_{m2}\right) = -g_{m1}g_{m2}\left(r_{o1}\|R_1\right)V_{in}$$
$$A_\upsilon = \frac{V_o}{V_{in}} = \frac{-g_{m1}g_{m2}\left(r_{o1}\|R_1\right)}{\left(\frac{1}{R_2} + \frac{1}{r_{o2}} + g_{m2}\right)} = \frac{-(0.7071)(2)(200\|27.2)}{\left(\frac{1}{5} + \frac{1}{50} + 2\right)} = -15.25$$

(d) $$I_x = \frac{V_x}{r_{o2}} + \frac{V_x}{R_2} + g_{m2}V_x$$
$$R_o = r_{o2}\|R_2\left\|\frac{1}{g_{m2}}\right. = 50\|5\left\|\frac{1}{2}\right. = 4.545\|0.5$$
$$R_o = 0.450\,\text{k}\Omega$$

11.90

(a) $$R = \frac{5-0}{0.25} = 20\,\text{k}\Omega$$
$$R_{E1} = \frac{-0.7-(-5)}{0.25} = 17.2\,\text{k}\Omega$$
$$R_C = \frac{5-0.7}{0.25} = 17.2\,\text{k}\Omega$$
$$R_{E2} = \frac{0-(-5)}{2} = 2.5\,\text{k}\Omega$$

(b) $$A_{d1} = \frac{\upsilon_{o2}}{\upsilon_d} = \frac{g_{m1}}{2}\left(R\|r_{\pi 3}\right)$$

$$g_{m1} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_{d1} = \frac{(9.615)}{2}(20 \| 12.48) = 36.94$$

$$A_3 = -g_{m3}(R_C \| R_{i4})$$

$$R_{i4} = r_{\pi 4} + (1+\beta)R_{E2}$$

$$g_{m3} = \frac{0.25}{0.026} = 9.615 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(120)(0.026)}{2} = 1.56 \text{ k}\Omega$$

$$R_{i4} = 1.56 + (121)(2.5) = 304 \text{ k}\Omega$$

$$A_3 = -(9.615)(17.2 \| 304) = -156.5$$

$$A_4 = \frac{(1+\beta)R_{E2}}{r_{\pi 4} + (1+\beta)R_{E2}} = \frac{(121)(2.5)}{1.56 + (121)(2.5)} = 0.995$$

Now $A_d = \dfrac{v_o}{v_d} = A_{d1} \cdot A_3 \cdot A_4 = (36.94)(-156.5)(0.995) = -5752$

(c) $$A_{cm1} = \frac{-g_{m1}(R \| r_{\pi 3})}{1 + \dfrac{2(1+\beta)R_o}{r_{\pi 1}}}$$

$$r_{\pi 1} = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(9.615)(20 \| 12.48)}{1 + \dfrac{2(121)(200)}{12.48}} = -0.01905$$

$$A_{cm} = A_{cm1} \cdot A_3 \cdot A_4 = (-0.01905)(-156.5)(0.995) = 2.966$$

$$CMRR_{dB} = 20 \log_{10}\left(\frac{5752}{2.966}\right) = 65.8 \text{ dB}$$

11.91

a.

$$R_{C1} = \frac{10 - v_{01}}{I_{C1}} = \frac{10-2}{0.1} \Rightarrow \underline{R_{C1} = 80 \text{ k}\Omega}$$

$$R_{C2} = \frac{10 - v_{04}}{I_{C4}} = \frac{10-6}{0.2} \Rightarrow \underline{R_{C2} = 20 \text{ k}\Omega}$$

b.

$$A_{d1} = \frac{v_{01} - v_{02}}{v_d} = -g_{m1}\left(R_{C1} \| r_{\pi 3}\right)$$

$$g_{m1} = \frac{0.1}{0.026} = 3.846 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(180)(0.026)}{0.2} = 23.4 \text{ k}\Omega$$

$$A_{d1} = -(3.846)(80 \| 23.4) \Rightarrow \underline{A_{d1} = -69.6}$$

$$A_{d2} = \frac{v_{04}}{v_{01} - v_{02}} = \frac{1}{2} g_{m4} R_{C2}$$

$$g_{m4} = \frac{0.2}{0.026} = 7.692 \text{ mA/V}$$

$$A_{d2} = \frac{1}{2}(7.692)(20) = 76.9$$

$$\text{Then } A_d = (76.9)(-69.6) \Rightarrow \underline{A_d = -5352}$$

11.92

a. Neglect the effect of r_0 in determining the differential-mode gain.

$$A_{d1} = \frac{v_{02}}{v_d} = \frac{1}{2} g_{m2}\left(R_C \| R_{i3}\right) \text{ where } R_{i3} = r_{\pi 3} + (1+\beta) R_E$$

$$A_2 = \frac{-\beta R_{C2}}{r_{\pi 3} + (1+\beta) R_E}$$

$$I_1 = \frac{12 - 0.7 - (-12)}{R_1} = \frac{23.3}{12} = 1.94 \text{ mA} \approx I_{C5}$$

$$g_{m2} = \frac{\frac{1}{2} \cdot (1.94)}{0.026} = 37.3 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(200)(0.026)}{I_{C3}}$$

$$v_{02} = 12 - \frac{1}{2}(1.94)(8) = 4.24 \text{ V}$$

$$I_{C3} = \frac{4.24 - 0.7}{3.3} = 1.07 \text{ mA}$$

$$r_{\pi 3} = \frac{(200)(0.026)}{1.07} = 4.86 \text{ k}\Omega$$

$$R_{i3} = 4.86 + (201)(3.3) = 668 \text{ k}\Omega$$

$$A_{d1} = \frac{1}{2}(37.3)\left[8 \| 668\right] = 147.4$$

Then

$$A_d = A_{d1} \cdot A_2 = (147.4)(-1.197) \Rightarrow \underline{A_d = -176}$$

$$R_0 = r_{05} = \frac{V_A}{I_{C5}} = \frac{80}{1.94} = 41.2 \text{ k}\Omega$$

$$A_{cm1} = \frac{-g_{m2}\left(R_C \| R_{i3}\right)}{1 + \dfrac{2(1+\beta)R_0}{r_{\pi 2}}}$$

$$r_{\pi 2} = \frac{(200)(0.026)}{\frac{1}{2}\cdot(1.94)} = 5.36 \text{ k}\Omega$$

$$A_{cm1} = \frac{-(37.3)(8 \| 668)}{1 + \dfrac{2(201)(41.2)}{5.36}} = -0.09539$$

$$A_2 = -1.197$$

$$A_{cm} = (-0.09539)(-1.197) \Rightarrow \underline{A_{cm} = 0.114}$$

b.

$$v_d = v_1 - v_2 = 2.015\sin\omega t - 1.985\sin\omega t$$

$$v_d = 0.03\sin\omega t \,(\text{V})$$

$$v_{cm} = \frac{v_1 + v_2}{2} = 2.0\sin\omega t$$

$$v_{03} = A_d v_d + A_{cm} v_{cm}$$

$$= (-176)(0.03) + (0.114)(2)$$

Or $\underline{v_{03} = -5.052\sin\omega t}$

Ideal, $A_{cm} = 0$

So

$$v_{03} = A_d v_d = (-176)(0.03)$$

$$\underline{v_{03} = -5.28\sin\omega t}$$

c.

$$R_{id} = 2r_{\pi 2} = 2(5.36) \Rightarrow \underline{R_{id} = 10.72 \text{ k}\Omega}$$

$$2R_{icm} \cong 2(1+\beta)R_0 \left\| (1+\beta)r_0 \right.$$

$$r_0 = \frac{V_A}{I_{C2}} = \frac{80}{\frac{1}{2}\cdot(1.94)} = 82.5 \text{ k}\Omega$$

$$2R_{icm} = \left[2(201)(41.2)\right] \left\| \left[(201)(82.5)\right] \right.$$

$$= 16.6 \text{ M}\Omega \| 16.6 \text{ M}\Omega$$

So $\Rightarrow \underline{R_{icm} = 4.15 \text{ M}\Omega}$

11.93

a.

$$I_1 = \frac{24 - V_{GS4}}{R_1} = k_n \left(V_{GS4} - V_{Th}\right)^2$$

$$24 - V_{GS4} = (55)(0.2)\left(V_{GS4} - 2\right)^2$$

$$24 - V_{GS4} = 11\left(V_{GS4}^2 - 4V_{GS4} + 4\right)$$

$$11V_{GS4}^2 - 43V_{GS4} + 20 = 0$$

$$V_{GS4} = \frac{43 \pm \sqrt{(43)^2 - 4(11)(20)}}{2(11)} = 3.37 \text{ V}$$

$$I_1 = \frac{24 - 3.37}{55} = 0.375 \text{ mA} = I_Q$$

$$v_{02} = 12 - \left(\frac{0.375}{2}\right)(40) = 4.5 \text{ V}$$

$$\frac{v_{02} - V_{GS3}}{R_S} = I_{D3} = k_n \left(V_{GS3} - V_{Th}\right)^2$$

$$4.5 - V_{GS3} = (0.2)(6)\left(V_{GS3}^2 - 4V_{GS3} + 4\right)$$

$$1.2V_{GS3}^2 - 3.8V_{GS3} + 0.3 = 0$$

$$V_{GS3} = \frac{3.8 \pm \sqrt{(3.8)^2 - 4(1.2)(0.3)}}{2(1.2)} = 3.09 \text{ V}$$

$$I_{D3} = \frac{4.5 - 3.09}{6} = 0.235 \text{ mA}$$

$$g_{m2} = 2\sqrt{K_n I_{D2}} = 2\sqrt{(0.2)\left(\frac{0.375}{2}\right)}$$
$$= 0.387 \text{ mA/V}$$
$$A_{d1} = \frac{1}{2} g_{m2} R_D = \frac{1}{2}(0.387)(40) \Rightarrow A_{d1} = 7.74$$
$$A_2 = \frac{-g_{m3} R_{D2}}{1 + g_{m3} R_5}$$
$$g_{m3} = 2\sqrt{K_n I_{D3}} = 2\sqrt{(0.2)(0.235)}$$
$$= 0.434 \text{ mA/V}$$
$$A_2 = \frac{-(0.434)(4)}{1 + (0.434)(6)} = -0.482$$
$$\text{So } A_d = A_{d1} \cdot A_2 = (7.74)(-0.482) \Rightarrow \underline{A_d = -3.73}$$
$$R_0 = r_{05} = \frac{1}{\lambda I_Q} = \frac{1}{(0.02)(0.375)} = 133 \text{ k}\Omega$$
$$A_{cm1} = \frac{-g_{m2} R_D}{1 + 2 g_{m2} R_0} = \frac{-(0.387)(40)}{1 + 2(0.387)(133)}$$
$$= -0.149$$
$$A_{cm} = (-0.149)(-0.482) \Rightarrow \underline{A_{cm} = 0.0718}$$

b.

$$v_d = v_1 - v_2 = 0.3 \sin \omega t$$
$$v_{cm} = \frac{v_1 + v_2}{2} = 2 \sin \omega t$$
$$v_{03} = A_d v_d + A_{cm} v_{cm}$$
$$= (-3.73)(0.3) + (0.0718)(2) \Rightarrow \underline{v_{03} = -0.975 \sin \omega t \,(\text{V})}$$

Ideal, $A_{cm} = 0$

$$v_{03} = A_d v_d = (-3.73)(0.3)$$

Or

$$\Rightarrow \underline{v_{03} = -1.12 \sin \omega t \,(\text{V})}$$

11.94

(a) $$A_d = \frac{\beta R_C}{r_\pi + R_B}$$

Assuming $I_{CQ} \cong I_{EQ}$, $\quad r_\pi = \dfrac{(150)(0.026)}{0.4} = 9.75 \text{ k}\Omega$

$$A_d = \frac{(150)(10)}{9.75 + 0.5} = 146$$

(b) $C_M \cong C_\mu\left(1+g_m R_C\right)$

$$g_m = \frac{0.4}{0.026} = 15.38 \text{ mA/V}$$

$$C_M = (0.2)[1+(15.38)(10)] = 30.96 \text{ pF}$$

(c) $$f_H = \frac{1}{2\pi\left(r_\pi \| R_B\right)\left(C_\pi + C_M\right)} = \frac{1}{2\pi(9.75\|0.5)\times 10^3 (1.2+30.96)\times 10^{-12}}$$

$$f_H = 10.4 \text{ MHz}$$

11.95

(a) $$f_Z = \frac{1}{2\pi R_o C_o} = \frac{1}{2\pi\left(10\times 10^6\right)\left(0.4\times 10^{-12}\right)} \Rightarrow f_Z = 39.8 \text{ kHz}$$

(b) $$r_\pi = \frac{(150)(0.026)}{0.4} = 9.75 \text{ k}\Omega$$

$$R_{eq} = \frac{R_o\left(1+\dfrac{R_B}{r_\pi}\right)}{1+\dfrac{R_B}{r_\pi}+\dfrac{2(1+\beta)R_o}{r_\pi}} = \frac{(10)\left(1+\dfrac{0.5}{9.75}\right)}{1+\dfrac{0.5}{9.75}+\dfrac{2(151)(10{,}000)}{9.75}}$$

Or $R_{eq} = 33.94\,\Omega$

$$f_P = \frac{1}{2\pi R_{eq} C_o} = \frac{1}{2\pi(33.94)\left(0.4\times 10^{-12}\right)}$$

$$f_P = 11.7 \text{ GHz}$$

11.96

a. From Equation (7.73), $$f_T = \frac{g_m}{2\pi\left(C_\pi + C_\mu\right)}$$

$$g_m = \frac{1}{0.026} = 38.46 \text{ mA/V}$$

Then $800\times10^6 = \dfrac{38.46\times10^{-3}}{2\pi\left(C_\pi + C_\mu\right)}$

Or $C_\pi + C_\mu = 7.65\times10^{-12}$ F = 7.65 pF

And $C_\pi = 6.65$ pF

$$C_M = C_\mu\left(1+g_m R_C\right) = 1\left[1+(38.46)(10)\right] = 386 \text{ pF}$$

$$f_H = \frac{1}{2\pi\left[r_\pi \| R_B\right](C_\pi + C_M)}$$

$$r_\pi = \frac{(120)(0.026)}{1} = 3.12 \text{ k}\Omega$$

$$f_H = \frac{1}{2\pi\left[3.12\|1\right]\times10^3\times(6.65+386)\times10^{-12}}$$

Or $\underline{f_H = 535 \text{ kHz}}$

b. From Equation (11.140),
$$f_Z = \frac{1}{2\pi R_0 C_0} = \frac{1}{2\pi\left(10\times10^6\right)\left(10^{-12}\right)}$$

Or $\underline{f_Z = 15.9 \text{ kHz}}$

11.97

(a) $$A_\upsilon = \frac{\frac{1}{2}\beta R_C}{r_\pi + (1+\beta)R_E}$$

$$r_\pi = \frac{(120)(0.026)}{0.25} = 12.48 \text{ k}\Omega$$

$$A_\upsilon = \frac{\frac{1}{2}(120)(8)}{12.48 + (121)(0.1)} = 19.5$$

(b) $$A_\upsilon = \frac{\frac{1}{2}(120)(8)}{12.48 + (121)(0.25)} = 11.2$$

Chapter 12

12.1

(a) $A_f = \dfrac{A}{1+A\beta} \Rightarrow 100 = \dfrac{5\times10^4}{1+\left(5\times10^4\right)\beta} \Rightarrow \beta = 9.98\times10^{-3}$

(b) $80 = \dfrac{A}{1+A(0.012)} \Rightarrow A = 2000$

12.2

(a) $A_f = \dfrac{A}{1+A\beta} \Rightarrow -80 = \dfrac{-10^5}{1+\left(-10^5\right)\beta} \Rightarrow \beta = -0.01249$

(b) $A_f = \dfrac{-5\times10^4}{1+\left(-5\times10^4\right)(-0.015)} = -66.58$

12.3

(a) $A_f = \dfrac{A}{1+A\beta} \quad \beta = 0.15$

$T = A\beta$

(i) $T = \infty$

(ii) $A = 80\,\text{dB} \Rightarrow A = 10^4 \Rightarrow T = 1.5\times10^3$

(iii) $T = 15$

(i) $A_f = \dfrac{1}{\beta} = 6.667$

(ii) $A_f = 6.662$

(iii) $A_f = 6.25$

(b)

(i) $T = \infty$

(ii) $T = 2.5\times10^3$

(iii) $T = 25$

(i) $A_f = \dfrac{1}{\beta} = 4.00$

(ii) $A_f = 3.9984$

(iii) $A_f = 3.846$

12.4

(a)

$A_f = \dfrac{A}{1+A\beta} \cong \dfrac{1}{\beta} = 125$

$\beta = 0.0080$

(b)

$$A_f = (125)(0.9975) = 124.6875$$

$$124.6875 = \frac{A}{1+(0.008)A}$$

$$124.6875[1+(0.008)A] = \text{A}$$

$$124.6875 = A[1-0.9975]$$

$$A = 49{,}875$$

12.5

(a) $$A_f = \frac{A}{1+A\beta} \Rightarrow -80 = \frac{-2\times10^4}{1+\left(-2\times10^4\right)\beta} \Rightarrow \beta = -0.01245$$

(b) $$\frac{dA_f}{A_f} = \frac{1}{(1+A\beta)}\cdot\frac{dA}{A}$$

$$0.01 = \frac{1}{\left[1+\left(-2\times10^4\right)(-0.01245)\right]}\cdot\frac{dA}{A} \Rightarrow \frac{dA}{A} = 2.5\%$$

12.6

$$\frac{dA_f}{A_f} = \frac{1}{1+\beta A}\cdot\frac{dA}{A}$$

$$0.001 = \frac{1}{1+\beta A}\cdot(0.10) \Rightarrow 1+\beta A = 100$$

$$\beta = \frac{100-1}{5\times10^4} \Rightarrow \beta = 1.98\times10^{-3}$$

Now $$A_f = \frac{A}{1+\beta A} = \frac{5\times10^4}{1+\left(1.98\times10^{-3}\right)\left(5\times10^4\right)} \Rightarrow A_f = 500$$

12.7

(a) $$A_{vf} = \left(\frac{A_1}{1+A_1\beta_1}\right)\left(\frac{A_2}{1+A_2\beta_1}\right)$$

$$50 = \left(\frac{200}{1+200\beta_1}\right)\left(\frac{10}{1+10\beta_1}\right)$$

$$(1+200\beta_1)(1+10\beta_1) = \frac{(200)(10)}{50} = 40$$

Then $$2000\beta_1^2 + 210\beta_1 - 39 = 0 \Rightarrow \beta_1 = 0.096685$$

$$A_{vf} = \frac{A_1A_2}{1+A_1A_2\beta_2}$$

$$50 = \frac{2000}{1+2000\beta_2} \Rightarrow \beta_2 = 0.0195$$

(b) From circuit (a):

$$A_{vf} = \left(\frac{200}{1+(200)(0.096685)}\right)\left(\frac{9}{1+(9)(0.096685)}\right) = 47.33 \Rightarrow (-5.43\%)$$

For circuit (b):

$$A_{vf} = \frac{1800}{1+(1800)(0.0195)} = 49.86 \Rightarrow (-0.28\%)$$

12.8

$$V_O = (-10)(-15)(-20)V_\varepsilon = -3000V_\varepsilon$$

(a) $V_\varepsilon = \beta V_O + V_S$

So $V_O = -3000(\beta V_O + V_S)$

We find

$$A_{vf} = \frac{V_O}{V_S} = \frac{-3000}{1+3000\beta}$$

$$A_{vf} = -120 = \frac{-3000}{1+3000\beta} \Rightarrow \underline{\beta = 0.008}$$

For

(b) Now $V_O = (-9)(-13.5)(-18)V_\varepsilon = -2187V_\varepsilon$

Then

$$A_{vf} = \frac{-2187}{1+2187\beta} = \frac{-2187}{1+2187(0.008)} = -118.24$$

$$\%\text{ change} = \frac{120-118.24}{120}\times 100 \Rightarrow \underline{1.47\%\text{ change}}$$

12.9

(a) $A_v f_H = A_{vf} f_C$

$$(5\times 10^4)(10) = (25) f_C \Rightarrow f_C = 20\ \text{kHz}$$

(b) $$A_{vf} = \frac{A_v f_H}{f_C} = \frac{(10^5)(8)}{20\times 10^3} = 40$$

12.10

(a) $$f_C = \frac{(5\times 10^4)(10)}{50} \Rightarrow f_C = 10\,\text{kHz}$$

(b) $$f_C = \frac{(10^4)(10)}{50} \Rightarrow f_C = 2\,\text{kHz}$$

12.11

(a) (i) $$A_v = \frac{A_{vf} f_C}{f_H} = \frac{(75)(35\times 10^3)}{5} = 5.25\times 10^5$$

(ii) $$75 = \frac{5.25\times 10^5}{1+(5.25\times 10^5)\beta} \Rightarrow \beta = 0.01333$$

(b) $A = (5.25\times10^5)(0.9) = 4.725\times10^5$

$$A_{vf} = \frac{4.725\times10^5}{1+(4.725\times10^5)(0.01333)} = 74.99$$

$$f_C = \frac{A_v f_H}{A_{vf}} = \frac{(4.725\times10^5)(5)}{74.99} \Rightarrow f_C = 31.5 \text{ kHz}$$

12.12

$$A_f = \frac{A_0}{1+A_0\beta}$$

Low freq.

$$100 = \frac{5000}{1+(5000)\beta} \Rightarrow \beta = 0.0098$$

Freq. response

$$A_f = \frac{A}{1+A\beta} = \frac{\dfrac{5000}{\left(1+j\dfrac{f}{f_1}\right)\left(1+j\dfrac{f}{f_2}\right)}}{1+\dfrac{(5000)(0.0098)}{\left(1+j\dfrac{f}{f_1}\right)\left(1+j\dfrac{f}{f_2}\right)}}$$

$$= \frac{5000}{\left(1+j\dfrac{f}{f_1}\right)\left(1+j\dfrac{f}{f_2}\right)+49}$$

$$= \frac{5000}{1+j\dfrac{f}{f_1}+j\dfrac{f}{f_2}+\left(\dfrac{jf}{f_1}\right)\left(\dfrac{jf}{f_2}\right)+49}$$

$$= \frac{5000}{50+j\dfrac{f}{f_1}+j\dfrac{f}{f_2}+\left(\dfrac{jf}{f_1}\right)\left(\dfrac{jf}{f_2}\right)}$$

Also

$$A_f = \frac{A_{f0}}{\left(1+j\dfrac{f}{f_A}\right)\left(1+j\dfrac{f}{f_B}\right)} = \frac{100}{1+j\dfrac{f}{f_A}+j\dfrac{f}{f_B}+\left(j\dfrac{f}{f_A}\right)\left(j\dfrac{f}{f_B}\right)}$$

So

$$\frac{100}{1+j\dfrac{f}{f_A}+j\dfrac{f}{f_B}+\left(j\dfrac{f}{f_A}\right)\left(j\dfrac{f}{f_B}\right)} = \frac{100}{1+j\dfrac{f}{50f_1}+j\dfrac{f}{50f_2}+\dfrac{1}{50}\left(\dfrac{jf}{f_1}\right)\left(\dfrac{jf}{f_2}\right)}$$

Then

$$\frac{1}{f_A}+\frac{1}{f_B} = \frac{1}{50f_1}+\frac{1}{50f_2}$$

and

$$\frac{1}{f_A f_B} = \frac{1}{50 f_1 f_2}$$

$f_1 = 10$ and $f_2 = 2000$

$$\frac{1}{f_A}+\frac{1}{f_B} = \frac{1}{50(10)}+\frac{1}{50(2000)} = 0.002+0.000010 = 0.002010$$

and

$$\frac{1}{f_A f_B} = \frac{1}{(50)(10)(2000)} \Rightarrow \frac{1}{f_A} = \frac{f_B}{10^6}$$

$$\frac{f_B}{10^6}+\frac{1}{f_B}=0.002010$$

Then

$$10^{-6}f_B^2+1=2.01+10^{-3}f_B$$

$$10^{-6}f_B^2-2.01\times10^{-3}f_B+1=0$$

$$f_B=\frac{2.01\times10^{-3}\pm\sqrt{4.0401\times10^{-6}-4(10^{-6})(1)}}{2(10^{-6})}$$

$$f_B=\frac{2.01\times10^{-3}\pm2.0025\times10^{-4}}{2(10^{-6})}$$

+ sign $\quad f_B=1.105\times10^3$ Hz

+ sign $\quad f_A=9.05\times10^2$ Hz

12.13

(a) Fig. P12.7(a)

$$A_f=\left[\frac{\left(\frac{200}{1+j\frac{f}{f_1}}\right)}{1+\left(\frac{200}{1+j\frac{f}{f_1}}\right)(0.1126)}\right]\left[\frac{10}{1+(10)(0.1126)}\right]$$

$$=\left[\frac{200}{\left(1+j\frac{f}{f_1}\right)+22.52}\right](4.704)$$

$$=\frac{940.73}{23.52+j\frac{f}{f_1}}=\frac{940.73}{23.52}\cdot\frac{1}{1+j\frac{f}{(23.52)f_1}}$$

$$=\frac{40}{1+\frac{jf}{(23.52)f_1}}\qquad f_{-3dB}=(23.52)(100)\Rightarrow\underline{2.352\text{ kHz}}$$

Fig P12.7(b)

$$A_f=\frac{\frac{(200)(10)}{1+j\frac{f}{f_1}}}{1+\frac{(0.0245)(200)(10)}{1+j\frac{f}{f_1}}}=\frac{2000}{1+j\frac{f}{f_1}+49}$$

$$=\frac{2000}{50}\cdot\frac{1}{1+j\frac{f}{(50)f_1}}\qquad f_{-3dB}=(50)(100)\Rightarrow5\text{ KHz}$$

(b) Overall feedback $\Rightarrow$ wider bandwidth.

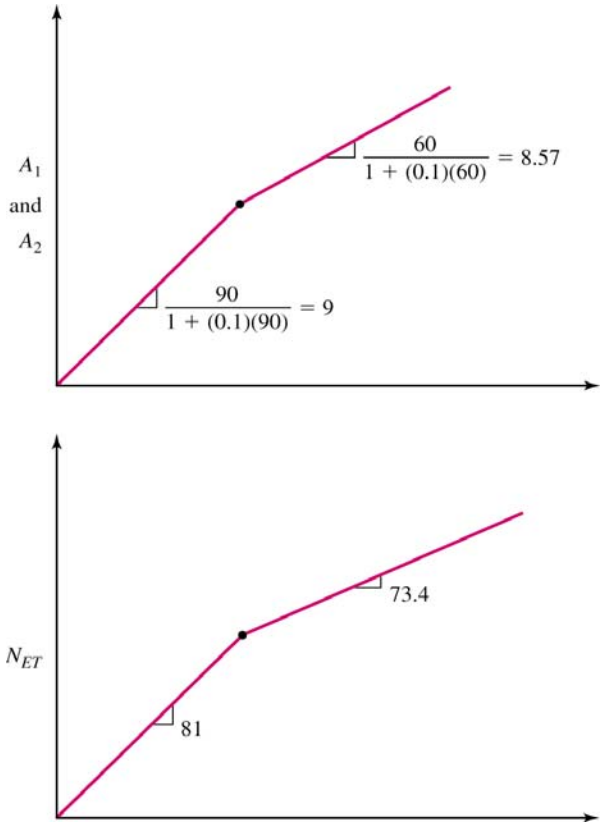
12.14

$$v_0=A_1A_2v_i+A_1v_n$$

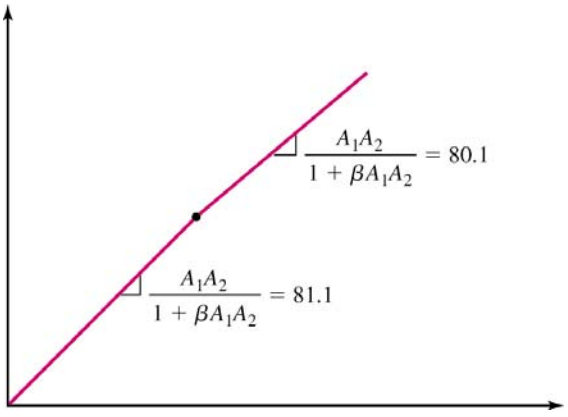
$$v_0=(100)v_i+(1)v_n=(100)(10)+(1)(1)\Rightarrow\underline{\frac{S_0}{N_0}=\frac{1000}{1}=1000}$$

12.15

(a)



(b)



Circuit (b) – less distortion

12.16

$$A_{vf} = \frac{A_v}{1+\beta_v A_v} = \frac{5\times10^3}{1+(0.0080)(5\times10^3)} \Rightarrow A_{vf} = 121.95$$

$$R_{if} = R_i(1+\beta_v A_v) = (10)\left[1+(0.0080)(5\times10^3)\right] \Rightarrow R_{if} = 410\ \text{k}\Omega$$

$$R_{of} = \frac{R_o}{1+\beta_v A_v} = \frac{1\times10^3}{1+(0.0080)(5\times10^3)} \Rightarrow R_{of} = 24.4\,\Omega$$

12.17

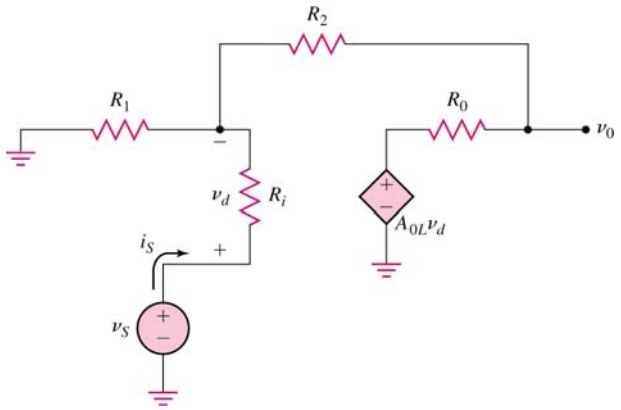
$$V_{fb} = \beta_v V_o = (0.0096)(2.5) \Rightarrow V_{fb} = 24\,\text{mV}$$

$$V_\in = V_i - V_{fb} = 25 - 24 = 1\,\text{mV}$$

$$V_o = A_v V_\in \Rightarrow A_v = \frac{V_o}{V_\in} = \frac{2.5}{0.001} = 2.5\times10^3\ \text{V/V}$$

$$A_{vf} = \frac{V_o}{V_i} = \frac{2.5}{0.025} = 100\ \text{V/V}$$

12.18



$$A_{vf} \approx \left(1+\frac{R_2}{R_1}\right) = 20 \Rightarrow \frac{R_2}{R_1} = 19$$

$$v_d = i_S R_i$$

$$i_s = \frac{v_S - v_d}{R_1} + \frac{(v_s - v_d) - v_0}{R_2} \qquad (1)$$

$$\frac{v_0 - A_{0L}v_d}{R_0} + \frac{v_0 - (v_s - v_d)}{R_2} = 0 \qquad (2)$$

$$v_0\left(\frac{1}{R_0}+\frac{1}{R_2}\right) = \frac{A_{0L}v_d}{R_0} + \frac{(v_S - v_d)}{R_2}$$

$$v_0 = \frac{\dfrac{A_{0L}v_d}{R_0} + \dfrac{(v_S - v_d)}{R_2}}{\left(\dfrac{1}{R_0}+\dfrac{1}{R_2}\right)}$$

From (1):

$$i_S = \frac{v_S - v_d}{R_1} + \frac{v_S - v_d}{R_2} - \frac{\frac{1}{R_2}\cdot\left[\frac{A_{0L}v_d}{R_0} + \frac{(v_S - v_d)}{R_2}\right]}{\left(\frac{1}{R_0} + \frac{1}{R_2}\right)}$$

$$i_S = v_S\left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{\frac{1}{R_2}}{1 + \frac{R_2}{R_0}}\right) - v_d\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{\frac{A_{0L}}{R_0} - \frac{1}{R_2}}{1 + \frac{R_2}{R_0}}\right)$$

$$v_d = i_S R_i$$

$$i_S\left\{1 + \frac{R_i\left[\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(1 + \frac{R_2}{R_0}\right) + \frac{A_{0L}}{R_0} - \frac{1}{R_2}\right]}{1 + \frac{R_2}{R_0}}\right\} = v_S\left[\frac{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(1 + \frac{R_2}{R_0}\right) - \frac{1}{R_2}}{1 + \frac{R_2}{R_0}}\right]$$

$$i_S\left\{1 + \frac{R_2}{R_0} + R_i\left[\frac{1}{R_1} + \frac{R_2}{R_1}\cdot\frac{1}{R_0} + \frac{1}{R_0} + \frac{A_{0L}}{R_0}\right]\right\} = v_S\left[\frac{1}{R_1} + \frac{R_2}{R_1}\cdot\frac{1}{R_0} + \frac{1}{R_0}\right]$$

$$i_S\left\{R_0 + R_2 + R_i\left[\frac{R_0}{R_1} + \left(1 + \frac{R_2}{R_1}\right) + A_{0L}\right]\right\} = v_S\left[\frac{R_0}{R_1} + \left(1 + \frac{R_2}{R_1}\right)\right] \quad (1)$$

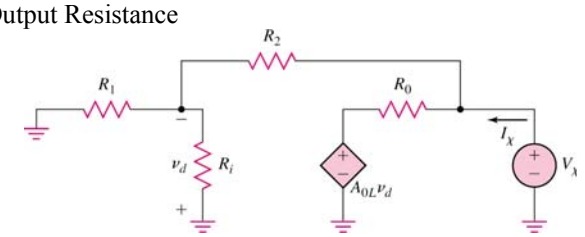
Let $R_2 = 190\ \text{k}\Omega,\ \ R_1 = 10\ \text{k}\Omega$

$$i_S\left\{0.1 + 190 + 100\cdot\left[\frac{0.1}{10} + 20 + 10^5\right]\right\} = v_S\left[\frac{0.1}{10} + 20\right]$$

$$i_S(1.000219\times10^7) = v_S(20.01)$$

$$R_{if} = \frac{v_S}{i_S} \cong 5\times10^5\ \text{k}\Omega \Rightarrow \underline{R_{if} \cong 500\ \text{M}\Omega}$$

Output Resistance



$$I_X = \frac{V_X - A_{0L}v_d}{R_0} + \frac{V_X}{R_2 + R_1 \| R_i}$$

$$v_d = \frac{-R_1 \| R_i}{R_1 \| R_i + R_2}\cdot V_X$$

$$\frac{I_X}{V_X} = \frac{1}{R_{0f}} = \frac{1}{R_0} + \frac{A_{0L}\cdot R_1 \| R_i}{R_0(R_1 \| R_i + R_2)} + \frac{1}{R_2 + R_1 \| R_i}$$

$$R_1 \| R_i = 10 \| 100 = 9.09$$

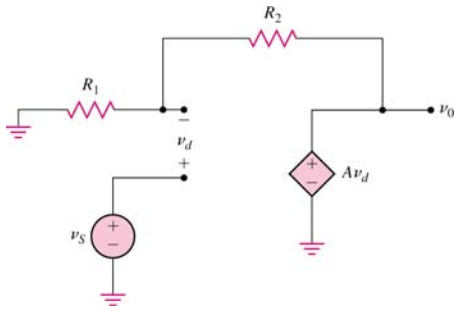
$$\frac{1}{R_{0f}} = \frac{1}{0.1} + \frac{10^5}{0.1}\cdot\left(\frac{9.09}{9.09 + 190}\right) + \frac{1}{190 + 9.09}$$

$$= 10 + 4.566\times10^4 + 0.00502$$

$$R_{0f} = 2.19\times10^{-5}\ \text{k}\Omega \Rightarrow \underline{R_{0f} = 0.0219\ \Omega}$$

12.19

a.



$$\frac{v_S - v_d}{R_1} = \frac{v_0 - (v_S - v_d)}{R_2} \quad \text{and} \quad v_d = \frac{v_0}{A}$$

$$\frac{v_S}{R_1} + \frac{v_S}{R_2} = \frac{v_0}{R_2} + v_d\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$= \frac{v_0}{R_2} + \frac{v_0}{A}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$v_S\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v_0}{R_2}\left[1 + \frac{1}{A}\left(1 + \frac{R_2}{R_1}\right)\right]$$

$$\frac{v_0}{v_S} = \frac{\left(1 + \frac{R_2}{R_1}\right)}{1 + \frac{1}{A}\left(1 + \frac{R_2}{R_1}\right)}$$

which can be written as

$$A_{vf} = \frac{v_0}{v_S} = \frac{A}{1 + \left[A \Big/ \left(1 + \frac{R_2}{R_1}\right)\right]}$$

b. $$\beta = \frac{1}{1 + \frac{R_2}{R_1}}$$

c. $$20 = \frac{10^5}{1 + (10^5)\beta}$$

So $$\beta = \frac{\frac{10^5}{20} - 1}{10^5} \Rightarrow \underline{\beta = 0.04999}$$

Then $$\frac{R_2}{R_1} = \frac{1}{\beta} - 1 = \frac{1}{0.04999} - 1 \Rightarrow \underline{\frac{R_2}{R_1} = 19.004}$$

d. $A \to 9 \times 10^4$

$$A_f = \frac{9 \times 10^4}{1 + (9 \times 10^4)(0.04999)} = 19.99956$$

$$\frac{\Delta A_f}{A_f} = \frac{-4.444 \times 10^{-4}}{20} = -2.222 \times 10^{-3}\% \Rightarrow \underline{\frac{\Delta A_f}{A_f} = -0.005\%}$$

12.20

$$I_{\in} = I_i - I_{fb} = 20 - 19 = 1\,\mu\text{ A}$$

$$I_{fb} = \beta_i I_o \Rightarrow I_o = \frac{19}{0.0095} \Rightarrow I_o = 2\text{ mA}$$

$$A_i = \frac{I_o}{I_{\in}} = \frac{2\times 10^{-3}}{1\times 10^{-6}} = 2\times 10^3\text{ A/A}$$

$$A_{if} = \frac{I_o}{I_i} = \frac{2}{0.020} = 100\text{ A/A}$$

$$R_{if} = \frac{R_i}{1+\beta_i A_i} = \frac{500}{1+(0.0095)(2\times 10^3)} = 25\,\Omega$$

$$R_{of} = (1+\beta_i A_i)R_o = [1+(0.0095)(2\times 10^3)](20\times 10^3) \Rightarrow R_{of} = 400\text{ k}\,\Omega$$

12.21

$$I_{fb} = I_i - I_{\in} = 25 - 0.8 = 24.2\,\mu\text{ A}$$

$$I_o = A_{if} I_i = (125)(25) \Rightarrow I_o = 3.125\text{ mA}$$

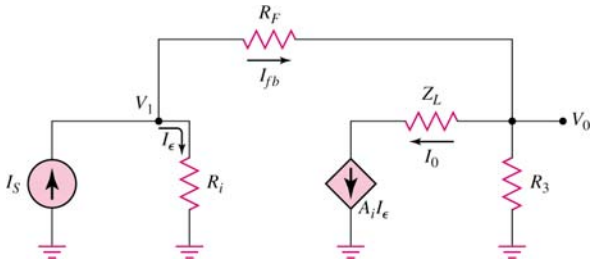
$$\beta_i = \frac{I_{fb}}{I_o} = \frac{24.2}{3125} = 0.007744\text{ A/A}$$

$$A_{if} = \frac{A_i}{1+\beta_i A_i}$$

$$125 = \frac{A_i}{1+(0.007744)A_i} \Rightarrow A_i = 3906\text{ A/A}$$

12.22

a.



Assume that V_1 is at virtual ground.

$$V_0 = -I_{fb}R_F$$

Now

$$I_{fb} = I_0 + \frac{V_0}{R_3} = I_0 - \frac{I_{fb}R_F}{R_3}$$

$$I_{fb} = I_S - I_\varepsilon$$

and

$$I_0 = A_i I_\varepsilon = \frac{I_0}{A_i}$$

so

$$I_{fb} = I_S - \frac{I_0}{A_i}$$

From above

$$I_{fb}\left(1+\frac{R_F}{R_3}\right) = I_0$$

$$\left(I_S - \frac{I_0}{A_i}\right)\left(1+\frac{R_F}{R_3}\right) = I_0$$

$$I_S\left(1+\frac{R_F}{R_3}\right) = I_0\left[1+\frac{1}{A_i}\left(1+\frac{R_F}{R_3}\right)\right]$$

or

$$A_{if} = \frac{I_0}{I_S} = \frac{\left(1+\frac{R_F}{R_3}\right)}{\left[1+\frac{1}{A_i}\left(1+\frac{R_F}{R_3}\right)\right]}$$

$$= \frac{A_i}{1+\frac{A_i}{\left(1+\frac{R_F}{R_3}\right)}} = A_{if}$$

b.
$$\beta_i = \frac{1}{\left(1+\frac{R_F}{R_3}\right)}$$

c.
$$25 = \frac{10^5}{1+(10^5)\beta_i}$$

so
$$\beta_i = \frac{\frac{10^5}{25}-1}{10^5} \Rightarrow \underline{\beta_i = 0.03999}$$

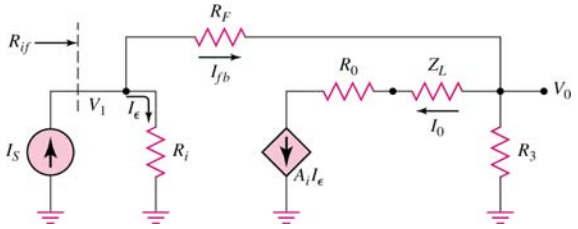
so
$$\frac{R_F}{R_3} = \frac{1}{\beta_i} - 1 = \frac{1}{0.03999} - 1 \Rightarrow \underline{\frac{R_F}{R_3} = 24.0}$$

d.
$$A_i = 10^5 - (0.15)(10^5) = 8.5\times10^4$$

so
$$A_{if} = \frac{8.5\times10^4}{1+(8.5\times10^4)(0.03999)} = 24.9989$$

so
$$\frac{\Delta A_{if}}{A_{if}} = -\frac{1.10\times10^{-3}}{25} = -4.41\times10^{-5} \Rightarrow \underline{-4.41\times10^{-3}\%}$$

12.23



$$I_S = I_\varepsilon + I_{fb}, V_1 = I_\varepsilon R_i$$

$$I_{fb} = I_0 + \frac{V_0}{R_3} \text{ and } V_0 = V_1 - I_{fb}R_F$$

$$I_0 = A_i I_\varepsilon \Rightarrow I_\varepsilon = \frac{I_0}{A_i}$$

Now

$$I_{fb} = A_i I_\varepsilon + \frac{1}{R_3}(V_1 - I_{fb}R_F)$$

$$I_{fb}\left[1 + \frac{R_F}{R_3}\right] = A_i I_\varepsilon + \frac{V_1}{R_3}$$

$$I_{fb} = I_S - I_e$$

$$(I_S - I_e)\left[1 + \frac{R_F}{R_3}\right] = A_i I_e + \frac{V_1}{R_3}$$

$$I_S\left[1 + \frac{R_F}{R_3}\right] = I_e\left[\left(1 + \frac{R_F}{R_3}\right) + A_i\right] + \frac{V_1}{R_3}$$

$$I_\varepsilon = \frac{V_1}{R_i}$$

$$I_S\left[1 + \frac{R_F}{R_3}\right] = V_1\left\{\frac{1}{R_i}\cdot\left[\left(1 + \frac{R_F}{R_3}\right) + A_i\right] + \frac{1}{R_3}\right\}$$

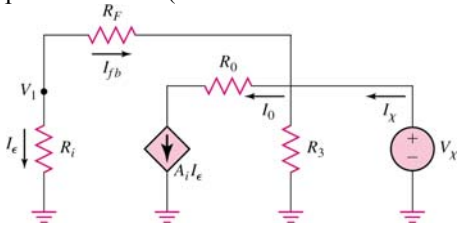
$$R_{if} = \frac{V_1}{I_S} = \frac{\left(1 + \frac{R_F}{R_3}\right)}{\left\{\frac{1}{R_i}\cdot\left[\left(1 + \frac{R_F}{R_3}\right) + A_i\right] + \frac{1}{R_3}\right\}}$$

The $1/R_3$ term in the denominator will be negligible. Then

$$R_{if} = \frac{25}{\left\{\frac{1}{2}\left[(25) + 10^5\right]\right\}}$$

$$R_{if} \cong 5\times 10^{-4} \text{ k}\Omega \Rightarrow \underline{R_{if} = 0.5\ \Omega}$$

Output Resistance (Let $Z_L = 0$)



$$I_X = \frac{V_X}{R_3} + A_i I_\varepsilon + \frac{V_X}{R_F + R_i}$$

$$I_\varepsilon = \frac{V_X}{R_F + R_i}$$

so

$$\frac{I_X}{V_X} = \frac{1}{R_{0f}} = \frac{1}{R_3} + \frac{A_i + 1}{R_F + R_i}, \frac{R_F}{R_3} = 24$$

Let $R_F = 240\ \text{k}\Omega,\ \ R_3 = 10\ k\Omega$

$$\frac{1}{R_{of}} = \frac{1}{10} + \frac{10^5+1}{240+2}$$

$$R_{0f} \approx \frac{R_F + R_i}{A_i + 1} = \frac{240+2}{10^5+1} \Rightarrow R_{0f} \approx 2.42\times10^{-3}\ \text{k}\Omega \text{ or } \underline{R_{0f} \approx 2.42\ \Omega}$$

so

12.24

$$V_\in = V_i - V_{fb} = 0.2 - 0.195 \Rightarrow V_\in = 5\ \mu\text{V}$$

$$\beta_z = \frac{V_{fb}}{I_o} = \frac{0.195\times10^{-3}}{5\times10^{-3}} = 0.039\ \text{V/A}$$

$$A_g = \frac{I_o}{V_\in} = \frac{5\times10^{-3}}{5\times10^{-6}} = 1000\ \text{A/V}$$

$$A_{gf} = \frac{A_g}{1 + A_g\beta_z} = \frac{1000}{1+(1000)(0.039)} = 25\ \text{A/V}$$

$$R_{if} = R_i\left(1+\beta_z A_g\right) = \left(20\times10^3\right)\left[1+(0.039)(1000)\right] \Rightarrow R_{if} = 800\ \text{k}\Omega$$

$$R_{of} = R_o\left(1+\beta_z A_g\right) = \left(10\times10^3\right)\left[1+(0.039)(1000)\right] \Rightarrow R_{of} = 400\ \text{k}\Omega$$

12.25

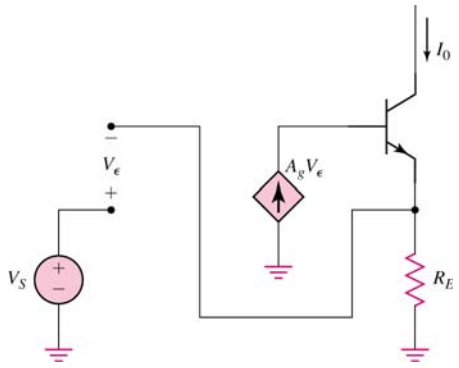
$$A_{gf} = \frac{A_g}{1+\beta_z A_g} = \frac{2000}{1+(2000)(0.0245)} = 40\ \text{A/V}$$

$$I_o = A_{gf} V_i = (40)\left(150\times10^{-6}\right) \Rightarrow I_o = 6\ \text{mA}$$

$$V_{fb} = \beta_z I_o = (0.0245)\left(6\times10^{-3}\right) \Rightarrow V_{fb} = 147\ \mu\text{V}$$

$$V_\in = V_i - V_{fb} = 150 - 147 \Rightarrow V_\in = 3\ \mu\text{V}$$

12.26



$$I_E = \frac{(1+h_{FE})}{h_{FE}} \cdot I_0 = \frac{V_S - V_\varepsilon}{R_E}$$

Also $I_0 = h_{FE}(A_g V_\varepsilon)$ so $V_\varepsilon = \dfrac{I_0}{h_{FE} A_g}$

Then

$$\frac{1+h_{FE}}{h_{FE}}\cdot I_0 = \frac{V_S}{R_E} - \frac{I_0}{h_{FE}A_g R_E}$$

$$\left[\frac{1+h_{FE}}{h_{FE}} + \frac{1}{h_{FE}A_g R_E}\right] I_0 = \frac{V_S}{R_E}$$

$$\left[\frac{A_g(1+h_{FE})R_E + 1}{h_{FE}A_g R_E}\right] I_0 = \frac{V_S}{R_E}$$

$$\frac{I_0}{V_S} = \frac{1}{R_E}\cdot\left[\frac{h_{FE}A_g R_E}{1+A_g(1+h_{FE})R_E}\right] \Rightarrow \underline{\frac{I_0}{V_S} \approx \frac{h_{FE}A_g}{1+(h_{FE}A_g)R_E}}$$

b. $\beta_z = R_E$

c. $$10 = \frac{5\times10^5}{1+(5\times10^5)\beta_z}$$

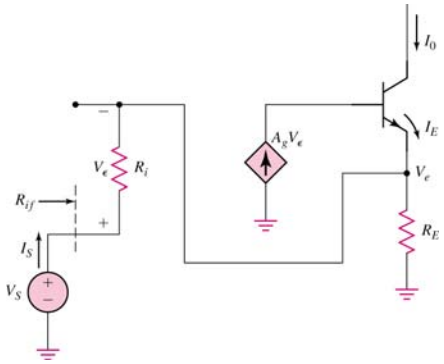
$$\beta_z = \frac{\frac{5\times10^5}{10} - 1}{5\times10^5} \Rightarrow \underline{\beta_z = R_E = 0.099998\ \text{k}\Omega}$$

d. If $A_g \to 5.5\times10^5$ then

$$A_{gf} = \frac{5.5\times10^5}{1+(5.5\times10^5)(0.099998)} = 10.0000182$$

$$\frac{\Delta A_{gf}}{A_{gf}} = \frac{1.82\times10^{-5}}{10} \Rightarrow \underline{1.82\times10^{-4}\%}$$

12.27



$$I_E = (1+h_{FE})A_g V_\varepsilon, \quad I_E = \frac{V_\varepsilon}{R_E} - I_S \quad \text{and} \quad V_\varepsilon = I_S R_i, \quad V_\varepsilon = V_S - V_\varepsilon = V_S - I_S R_i$$

$$(1+h_{FE})A_g I_S R_i = \frac{1}{R_E}\cdot(V_S - I_S R_i) - I_S$$

Now

$$\left[(1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1\right] I_S = \frac{V_S}{R_E}$$

$$R_{if} = \frac{V_S}{I_S} = R_E\left[(1+h_{FE})A_g R_i + \frac{R_i}{R_E} + 1\right]$$

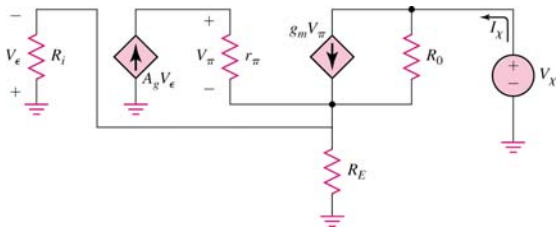
We have

$$(1+h_{FE})A_g \approx h_{FE}A_g = 5\times10^5\ \text{mS}$$

$$R_E \approx 0.1\ \text{k}\Omega$$

so $$R_{if} = (0.1)\left[(5\times10^5)(20) + \frac{20}{0.1} + 1\right]$$

or $\underline{R_{if} = 10^6 \text{ k}\Omega}$



$$\frac{V_\pi}{r_\pi} = A_g V_\varepsilon$$

$$I_X = g_m V_\pi + \frac{V_X - (-V_\varepsilon)}{R_0} \qquad (1)$$

$$V_\varepsilon = -(I_X + A_g V_\varepsilon)(R_E \parallel R_i) \qquad (2)$$

or $V_\varepsilon\left[1 + A_g(R_\varepsilon \parallel R_i)\right] = -I_X(R_E \parallel R_i)$

Now:

$$I_X = g_m A_g r_\pi V_\varepsilon + \frac{V_X}{R_0} + \frac{V_\varepsilon}{R_0} \qquad (1)$$

$$I_X = \left(g_m A_g r_\pi + \frac{1}{R_0}\right)\left[\frac{-I_X(R_E \parallel R_i)}{1 + A_g(R_E \parallel R_i)}\right] + \frac{V_X}{R_0}$$

$$R_{0f} = \frac{V_X}{I_X}$$

$$= R_0\left\{1 + \left(g_m A_g r_\pi + \frac{1}{R_0}\right)\left[\frac{(R_E \parallel R_i)}{1 + A_g(R_E \parallel R_i)}\right]\right\}$$

$g_m r_\pi A_g = h_{FE} A_g = 5\times 10^5$ mS

Let $h_{FE} = 100$ so $A_g = 5\times 10^3$ mS

$R_E \parallel R_i = 0.1 \parallel 20 \approx 0.1\,\text{k}\Omega$

Then

$$R_{0f} = 50\left\{1 + \left(5\times 10^5 + \frac{1}{50}\right)\left[\frac{0.1}{1 + (5\times 10^3)(0.1)}\right]\right\}$$

or $\underline{R_{0f} = 5.04 \text{ M}\Omega}$

12.28

$$A_{zf} = \frac{A_z}{1 + \beta_g A_z}$$

$$0.20\times 10^6 = \frac{A_z}{1 + \left(4.25\times 10^{-6}\right)A_z} \Rightarrow A_z = 1.333 \text{ V/}\mu\text{A}$$

$$R_{if} = \frac{R_i}{1 + \beta_g A_z} = \frac{500}{1 + \left(4.25\times 10^{-6}\right)\left(1.333\times 10^6\right)} \Rightarrow R_{if} = 75\,\Omega$$

$$R_{of} = \frac{R_o}{1 + \beta_g A_z} = 75\,\Omega$$

12.29

$$I_\varepsilon = I_i - I_{fb} = 40 - 38 = 2\ \mu\text{A}$$

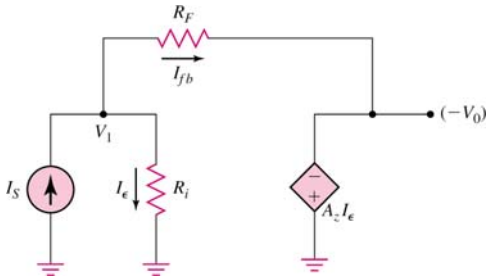
$$A_z = \frac{V_o}{I_\varepsilon} = \frac{8}{2} = 4\frac{\text{V}}{\mu\text{A}}$$

$$\beta_g = \frac{I_{fb}}{V_o} = \frac{38}{8} = 4.75\ \frac{\mu\text{A}}{\text{V}}$$

$$A_{zf} = \frac{V_o}{I_i} = \frac{8}{40} = 0.2\frac{\text{V}}{\mu\text{A}}$$

12.30

a.



Assuming V_1 is at virtual ground

$$(-V_0) = -I_{fb}R_F \quad \text{and} \quad (-V_0) = -A_z I_\varepsilon \Rightarrow I_\varepsilon = \frac{V_0}{A_z}$$

$$I_{fb} = I_S - I_\varepsilon$$

$$V_0 = (I_S - I_\varepsilon)R_F = I_S R_F - \left(\frac{V_0}{A_z}\right)R_F$$

So

$$V_0\left[1 + \frac{R_F}{A_z}\right] = I_S R_F$$

$$A_{zf} = \frac{V_0}{I_S} = \frac{R_F}{\left[1 + \frac{R_F}{A_z}\right]} = \frac{A_z R_F}{A_z + R_F}$$

so

$$A_{zf} = \frac{A_z}{1 + A_z\left(\frac{1}{R_F}\right)} = \frac{A_z}{1 + A_z\beta_g}$$

or

$$\beta_g = \frac{1}{R_F}$$

b.

$$5\times10^4 = \frac{5\times10^6}{1 + (5\times10^6)\beta_g}$$

c.

$$\beta_g = \frac{\frac{5\times10^6}{5\times10^4} - 1}{5\times10^6} \Rightarrow \underline{\beta_g = 1.98\times10^{-5}}$$

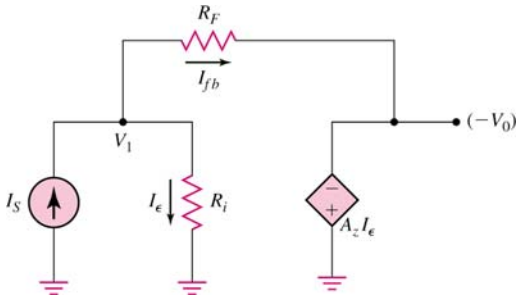
$$R_F = \frac{1}{\beta_g} \Rightarrow \underline{R_F = 50.5\ \text{k}\Omega}$$

d. $A_z = (0.9)(5\times10^6) = 4.5\times10^6$

$$A_{zf} = \frac{4.5\times10^6}{1+(4.5\times10^6)(1.98\times10^{-5})} = 4.994\times10^4$$

$$\frac{\Delta A_{zf}}{A_{zf}} = -\frac{55.4939}{5\times10^4} = -1.11\times10^{-3} \Rightarrow \underline{-0.111\%}$$

12.31



$V_1 = I_\varepsilon R_i, -V_0 = -A_z I_\varepsilon \Rightarrow V_0 = A_x I_\varepsilon$

$I_{fb} = I_S - I_\varepsilon$ and $-V_0 = V_1 - I_{fb}R_F$

$-A_z I_\varepsilon = V_1 - (I_S - I_\varepsilon)R_F$

$$-A_z\left(\frac{V_1}{R_i}\right) = V_1 - I_S R_F + \left(\frac{V_1}{R_i}\right)R_F$$

$$I_S R_F = V_1\left[1+\frac{A_z}{R_i}+\frac{R_F}{R_i}\right]$$

$$R_{if} = \frac{V_1}{I_S} = \frac{R_F}{\left[1+\frac{A_z}{R_i}+\frac{R_F}{R_i}\right]}$$

We have

$$R_{if} = \frac{50.5\times10^3}{\left[1+\frac{5\times10^6}{10\times10^3}+\frac{50.5\times10^3}{10\times10^3}\right]}$$

$$= \frac{50.5\times10^3}{[1+500+5.05]} \Rightarrow \underline{R_{if} = 99.79\ \Omega}$$

12.32

(a) Low input $R \Rightarrow$ Shunt input
Low output $R \Rightarrow$ Shunt output
Or a Shunt-Shunt circuit

(b) High input $R \Rightarrow$ Series input
High output $R \Rightarrow$ Series output
Or a series-Series circuit

(c) Shunt-Series circuit

(d) Series-Shunt circuit

12.33

(a) $R_i(\max) = R_i(1+T) = 10(1+10^4) \Rightarrow \underline{R_i(\max) \cong 10^5\ k\Omega}$

$$R_i(\min) = \frac{R_i}{1+T} = \frac{10}{1+10^4} \cong 10^{-3}\ k\Omega$$

Or $\underline{R_i(\min) = 1\Omega}$

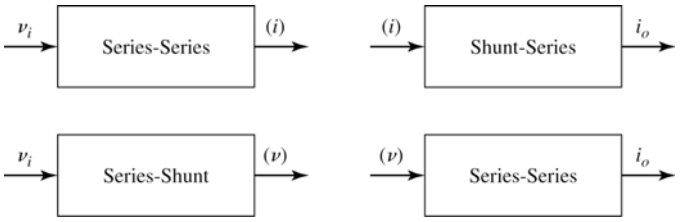
(b) $R_o(\max) = R_o(1+T) = 1(1+10^4) \Rightarrow \underline{R_o(\max) \cong 10^4\, k\Omega}$

$$R_o(\min) = \frac{R_o}{1+T} = \frac{1}{1+10^4} \cong 10^{-4}\, k\Omega$$

Or $\underline{R_o(\min) = 0.1\Omega}$

12.34

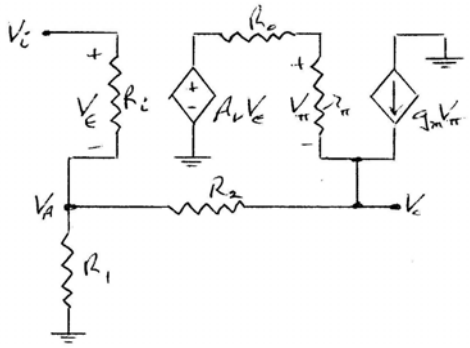
Overall Transconductance Amplifier, $A_g = \dfrac{i_o}{v_i}$ Series output = current signal and Shunt input = current signal. Also, Shunt output = voltage signal and Series input = voltage signal. Two possible solutions are shown.



12.35

$$I_{CQ} = 0.2\ \text{mA},\quad r_\pi = \frac{(140)(0.026)}{0.2} = 18.2\ \text{k}\Omega\ ,\quad g_m = \frac{0.2}{0.026} = 7.692\ \text{mA/V}$$

(a)



(1) $$\frac{V_\pi}{r_\pi} + g_m V_\pi = \frac{V_o - V_A}{R_2} = V_\pi\left(\frac{1+h_{FE}}{r_\pi}\right)$$

(2) $$\frac{V_\pi}{r_\pi} = \frac{A_v V_\epsilon - V_o}{R_o + r_\pi}$$

(3) $$\frac{V_A - V_o}{R_2} + \frac{V_A}{R_1} + \frac{V_A - V_i}{R_i} = 0$$

(4) $$V_\epsilon = V_i - V_A$$

From (1), $\dfrac{V_o - V_A}{R_2} = \left(\dfrac{1+h_{FE}}{r_\pi}\right)\left[r_\pi\left(\dfrac{A_\upsilon V_\epsilon - V_o}{R_o + r_\pi}\right)\right] = (1+h_{FE})\left(\dfrac{A_\upsilon V_\epsilon - V_o}{R_o + r_\pi}\right)$

Using (4), $\dfrac{V_o - V_i + V_\epsilon}{R_2} = (1+h_{FE})\left(\dfrac{A_\upsilon V_\epsilon - V_o}{R_o + r_\pi}\right)$ (Eq. 5)

From (3), $V_A\left(\dfrac{1}{R_2}+\dfrac{1}{R_1}+\dfrac{1}{R_i}\right) = \dfrac{V_o}{R_2}+\dfrac{V_i}{R_i}$

$$(V_i - V_\epsilon)\left(\frac{1}{R_2}+\frac{1}{R_1}+\frac{1}{R_i}\right) = \frac{V_o}{R_2}+\frac{V_i}{R_i}$$

$$(V_i - V_\epsilon)\left(\frac{1}{10}+\frac{1}{1}+\frac{1}{30}\right) = \frac{V_o}{10}+\frac{V_i}{30}$$

$$(V_i - V_\epsilon)(1.1333) = V_o(0.10) + V_i(0.0333)$$

We find, $V_\epsilon = V_i(0.9706) - V_o(0.08824)$

From Eq. (5) above,

$$\frac{V_o - V_i}{R_2} + \frac{(1+h_{FE})V_o}{R_o + r_\pi} = \frac{(1+h_{FE})}{R_o + r_\pi}\cdot A_\upsilon V_\epsilon - \frac{V_\epsilon}{R_2}$$

$$\frac{V_o - V_i}{10} + \frac{141}{0.5+18.2}\cdot V_o = \frac{141}{0.5+18.2}(10^5)V_\epsilon - \frac{V_\epsilon}{10}$$

$$7.640V_o - V_i(0.1) = 7.540\times 10^5 V_\epsilon = 7.540\times 10^5\left[V_i(0.9706) - V_o(0.08824)\right]$$

$$6.653705\times 10^4 V_o = V_i(7.318235\times 10^5)$$

Then $\dfrac{V_o}{V_i} = 11.0$

(b) From (2), $V_\pi = r_\pi\left(\dfrac{A_\upsilon V_\epsilon - V_o}{R_o + r_\pi}\right)$

From (1), $V_o = V_A + R_2 V_\pi\left(\dfrac{1+h_{FE}}{r_\pi}\right)$

$$V_o = (V_i - V_\epsilon) + R_2(1+h_{FE})\left(\frac{A_\upsilon V_\epsilon - V_o}{R_o + r_\pi}\right)$$

$$V_o\left[1+\frac{R_2(1+h_{FE})}{R_o + r_\pi}\right] = V_i + V_\epsilon\left[\frac{R_2(1+h_{FE})A_\upsilon}{R_o + r_\pi} - 1\right] \quad \text{(Eq. 6)}$$

From (3),

$$V_A\left(\frac{1}{R_2}+\frac{1}{R_1}+\frac{1}{R_i}\right) = \frac{V_o}{R_2}+\frac{V_i}{R_i}$$

$$(V_i - V_\epsilon)\left(\frac{1}{R_2}+\frac{1}{R_1}+\frac{1}{R_i}\right) = \frac{V_o}{R_2}+\frac{V_i}{R_i}$$

Then, using Eq. (6),

$$V_i\left(\frac{1}{R_2}+\frac{1}{R_1}\right) - V_\epsilon\left(\frac{1}{R_2}+\frac{1}{R_1}+\frac{1}{R_i}\right) = \frac{1}{R_2}\left\{\frac{V_i + V_\epsilon\left[\dfrac{R_2(1+h_{FE})A_\upsilon}{R_o + r_\pi} - 1\right]}{1+\dfrac{R_2(1+h_{FE})}{R_o + r_\pi}}\right\}$$

$$V_i(1.10) - V_\epsilon(1.1333) = \frac{1}{10}\left\{\frac{V_i + V_\epsilon(7.540106\times10^6)}{75.401}\right\}$$

$$V_i(1.098674) = V_\epsilon(1.1333 + 1.0\times10^4)$$

So $\dfrac{V_i}{V_\epsilon} = 9.103\times10^3$

Then $R_{if} = R_i\left(\dfrac{V_i}{V_\epsilon}\right) = (30\times10^3)(9.103\times10^3) \Rightarrow R_{if} = 273\ \text{M}\Omega$

(c) $I_x + g_m V_\pi + \dfrac{V_\pi}{r_\pi} = \dfrac{V_x - V_A}{R_2}$

From (2), $V_\pi = r_\pi\left(\dfrac{A_\upsilon V_\epsilon - V_x}{R_o + r_\pi}\right)$

Then, $I_x + \dfrac{(1+h_{FE})}{R_o + r_\pi}(A_\upsilon V_\epsilon - V_x) = \dfrac{V_x - V_A}{R_2}$

Now, $V_A = -V_\epsilon$ and $V_\epsilon = -V_x(0.08824)$

So, $I_x + \dfrac{141}{0.5+18.2}\left[10^5(-0.08824V_x) - V_x\right] = \dfrac{V_x}{10} - \dfrac{V_x(0.08824)}{10}$

$$I_x = V_x(0.091176 + 6.654\times10^4)$$

$$R_{of} = \frac{V_x}{I_x} = 15\ \mu\ \Omega$$

12.36

a. Neglecting base currents

$I_{C2} = 0.5$ mA, $V_{C2} = 12 - (0.5)(22.6) = 0.7$ V

$I_{C1} = 0.5$ mA $\Rightarrow \underline{v_0 = 0}$

Then $I_{C3} = 2$ mA

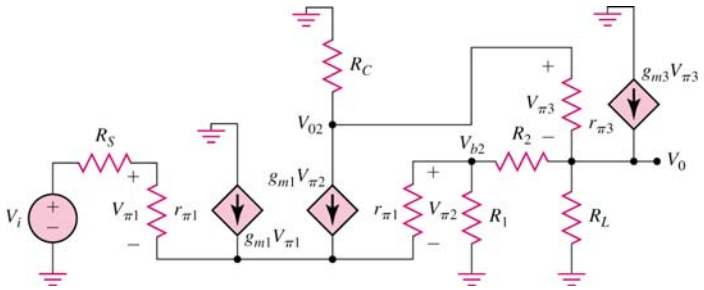
b.

$$r_{\pi1} = r_{\pi2} = \frac{h_{FE}\cdot V_T}{I_{C1}} = \frac{(100)(0.026)}{0.5} = 5.2\ \text{k}\Omega$$

$$g_{m1} = g_{m2} = \frac{0.5}{0.026} = 19.23\ \text{mA/V}$$

$$r_{\pi3} = \frac{(100)(0.026)}{2} = 1.3\ \text{k}\Omega$$

$$g_{m3} = \frac{2}{0.026} = 76.92\ \text{mA/V}$$



$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} + g_{m1}V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0$$

$$(V_{\pi 1} + V_{\pi 2})\left(\frac{1}{r_{\pi 1}} + g_{m1}\right) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2} \qquad (1)$$

$$V_i = \frac{V_{\pi 1}}{r_{\pi 1}}(R_S + r_{\pi 1}) - V_{\pi 2} + V_{b2}$$

$$V_i = V_{\pi 1}\left(1 + \frac{R_S}{r_{\pi 1}}\right) - V_{\pi 2} + V_{b2}$$

or

But $V_{\pi 2} = -V_{\pi 1}$

so

$$V_i = V_{\pi 1}\left(2 + \frac{R_S}{r_{\pi 1}}\right) + V_{b2} \qquad (2)$$

$$\frac{V_{02}}{R_C} + g_{m1}V_{\pi 2} + \frac{V_{02} - V_0}{r_{\pi 3}} = 0 \qquad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_0}{R_L} + \frac{V_0 - V_{b2}}{R_2}$$

$$V_{\pi 3} = V_{02} - V_0$$

so

$$(V_{02} - V_0)\left(\frac{1 + h_{FE}}{r_{\pi 3}}\right) = V_0\left(\frac{1}{R_L} + \frac{1}{R_2}\right) - \frac{V_{b2}}{R_2} \qquad (4)$$

$$\frac{V_{b2} - V_0}{R_2} + \frac{V_{b2}}{R_1} + \frac{V_{\pi 2}}{r_{\pi 1}} = 0 \qquad (5)$$

Substitute numbers into (2), (3), (4) and (5):

$$V_i = -V_{\pi 2}\left(2 + \frac{1}{5.2}\right) + V_{b2}$$

$$V_i = -V_{\pi 2}(2.192) + V_{b2} \qquad (2)$$

$$V_{02}\left(\frac{1}{22.6} + \frac{1}{1.3}\right) + (19.23)V_{\pi 2} - V_0\left(\frac{1}{1.3}\right) = 0$$

$$V_{02}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_0 = 0 \qquad (3)$$

$$V_{02}\left(\frac{101}{1.3}\right) = V_0\left(\frac{101}{1.3} + \frac{1}{4} + \frac{1}{50}\right) - V_{b2}\left(\frac{1}{50}\right)$$

$$V_{02}(77.69) = V_0(77.96) - V_{b2}(0.02) \qquad (4)$$

$$V_{b2}\left(\frac{1}{50} + \frac{1}{10}\right) - V_0\left(\frac{1}{50}\right) + V_{\pi 2}\left(\frac{1}{5.2}\right) = 0$$

$$V_{b2}(0.120) - V_0(0.020) + V_{\pi 2}(0.1923) = 0 \qquad (5)$$

From (2): $V_{b2} = V_i + V_{\pi 2}(2.192).$ Substitute in (4) and (5) to obtain:

$$V_{02}(77.69) = V_0(77.96) - [V_i + V_{\pi 2}(2.192)](0.02) \qquad (4')$$

$$[V_i + V_{\pi 2}(2.192)](0.120) - V_0(0.020) + V_{\pi 2}(0.1923) = 0 \qquad (5')$$

So we now have the following three equations:

$$V_{02}(0.8135) + (19.23)V_{\pi 2} - (0.7692)V_0 = 0 \qquad (3)$$

$$V_{02}(77.69) = V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384) \qquad (4')$$

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0 \qquad (5')$$

From (3): $V_{02} = V_0(0.9455) - V_{\pi 2}(23.64).$ Substitute for V_{02} in (4′) to obtain:

$$(77.69)[V_o(0.9455) - V_{\pi 2}(23.64)] = V_0(77.96) - V_i(0.02) - V_{\pi 2}(0.04384)$$

or

$$0 = V_0(4.504) - V_i(0.02) + V_{\pi 2}(1836.5)$$

Next, solve $(5')$ for $V_{\pi 2}$:

$$(0.120)V_i + V_{\pi 2}(0.4553) - V_0(0.020) = 0$$

$$V_{\pi 2} = V_0(0.04393) - V_i(0.2636)$$

Finally,

$$0 = V_0(4.504) - V_i(0.02) + (1836.5)[V_0(0.04393) - V_i(0.2636)]$$

$$0 = V_0(85.18) - V_i(484.12)$$

So

$$A_{vf} = \frac{V_0}{V_i} = \frac{484.12}{85.18} \Rightarrow \underline{A_{vf} = 5.68}$$

12.37

a. $R_{TH} = R_1 \,||\, R_2 = 400 \,||\, 75 = 63.2 \text{ k}\Omega$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{75}{75 + 400}\right)(10) = 1.579 \text{ V}$$

$$I_{BQ1} = \frac{1.579 - 0.7}{63.2 + (121)(0.5)} = 0.007106 \text{ mA}$$

$$I_{CQ1} = 0.853 \text{ mA}$$

$$V_{C1} = 10 - (0.853)(8.8) = 2.49 \text{ V}$$

$$I_{C2} \approx \frac{2.49 - 0.7}{3.6} = 0.497 \text{ mA}$$

$$V_{C2} = 10 - (0.497)(13) = 3.54 \text{ V}$$

$$I_{C3} \approx \frac{3.54 - 0.7}{1.4} = 2.03 \text{ mA}$$

Then

$$r_{\pi 1} = \frac{(120)(0.026)}{0.853} = 3.66 \text{ k}\Omega$$

$$g_{m1} = \frac{0.853}{0.026} = 32.81 \text{ mA/V}$$

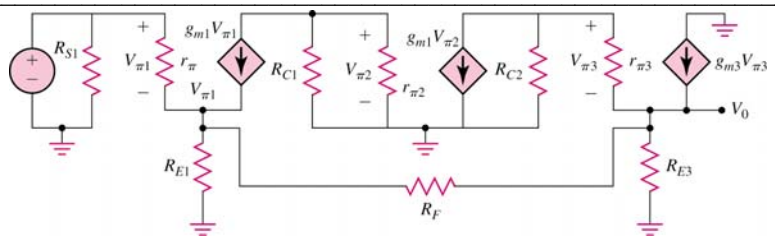
$$r_{\pi 2} = \frac{(120)(0.026)}{0.497} = 6.28 \text{ k}\Omega$$

$$g_{m2} = \frac{0.497}{0.026} = 19.12 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(120)(0.026)}{2.03} = 1.54 \text{ k}\Omega$$

$$g_{m3} = \frac{2.03}{0.026} = 78.08 \text{ mA/V}$$

b.



$$V_i = V_{\pi 1} + V_{\varepsilon 1} \Rightarrow V_{\varepsilon 1} = V_i - V_{\pi 1} \quad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_{\varepsilon 1}}{R_{E1}} + \frac{V_{\varepsilon 1} - V_0}{R_F} \quad (2)$$

$$V_{\pi 2} = -(g_{m1}V_{\pi 1})(R_{C1} \,||\, r_{\pi 2}) \quad (3)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3} + V_0}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \quad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{\varepsilon 1}}{R_F} \quad (5)$$

Substitute numbers in (2), (3), (4) and (5):

$$V_{\pi 1}\left(\frac{1}{3.66} + 32.81\right) = (V_i - V_{\pi 1})\left(\frac{1}{0.5} + \frac{1}{10}\right) - \frac{V_0}{10}$$

or $V_{\pi 1}(35.18) = V_i(2.10) - V_0(0.10)$ (2)

$$V_{\pi 2} = -(32.81)V_{\pi 1}(88 \,||\, 6.28)$$

or $V_{\pi 2} = -V_{\pi 1}(120.2)$ (3)

$$(19.12)V_{\pi 2} + \frac{V_{\pi 3}}{13} + \frac{V_0}{13} + \frac{V_{\pi 3}}{1.54} = 0$$

or

$$V_{\pi 2}(19.12) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0 \quad (4)$$

$$V_{\pi 3}\left(\frac{1}{1.54} + 78.08\right) = V_0\left(\frac{1}{1.4} + \frac{1}{10}\right) - \frac{V_i - V_{\pi 1}}{10}$$

or

$$V_{\pi 3}(78.73) = V_0(0.8143) - V_i(0.10) + V_{\pi 1}(0.10) \quad (5)$$

Now substituting $V_{\pi 2} = -V_{\pi 1}(120.2)$ in (4):

$$(19.12)[-V_{\pi 1}(120.2)] + V_{\pi 3}(0.7263) + V_0(0.07692) = 0$$

or

$$-V_{\pi 1}(2298.2) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0$$

Then

$$V_{\pi 3} = V_{\pi 1}(3164.3) - V_0(0.1059)$$

Substituting $V_{\pi 3} = V_{\pi 1}(3164.3) - V_0(0.1059)$ in (5):

$$(78.73)[V_{\pi 1}(3164.3) - V_0(0.1059)] = V_0(0.8143) - V_i(0.10) + V_{\pi 1}(0.10)$$

or $V_{\pi 1}(2.49 \times 10^5) - V_0(9.152) = -V_i(0.10)$

Then

$$V_{\pi 1} = V_0(3.674 \times 10^{-5}) - V_i(4.014 \times 10^{-7})$$

Now substituting $V_{\pi 1} = V_0(3.674 \times 10^{-5})$

$-V_i(4.014 \times 10^{-7})$ in (2):

$$(35.18)[V_0(3.674) \times 10^{-5}) - V_i(4.014 \times 10^{-7})]$$

$$= V_i(2.10) - V_0(0.10)$$

or $V_0(0.1013) = V_i(2.10)$

So $\dfrac{V_0}{V_i} = 20.7$

c. $R_{if} = \dfrac{V_i}{I_i}$ and $I_i = I_{RB1} + I_{b1}$

$$I_{RB1} = \frac{V_i}{R_{B1}}$$

$$I_{b1} = \frac{V_{\pi 1}}{r_{\pi 1}}$$

Now

$$V_{\pi 1} = (20.7V_i)(3.674\times 10^{-5}) - V_i(4.014\times 10^{-7})$$

$$V_{\pi 1} = V_i(7.60\times 10^{-4})$$

Then

$$R_{if} = \frac{V_i}{\dfrac{V_i}{63.2} + \dfrac{V_i(7.60\times 10^{-4})}{3.66}}$$

$$= \frac{1}{0.01582 + 2.077\times 10^{-4}}$$

or $\underline{R_{if} = 62.4\ \text{k}\Omega}$

d. To determine R_{0f}:

Equation (1) is modified to $V_{\pi 1} + V_{e1} = 0 \quad (V_i = 0)$ Equation (5) is modified to:

$$V_{\pi 3}(78.73) + I_X = V_0(0.8143) + V_{\pi 1}(0.10) \quad (5)$$

Now

$$V_{\pi 1}(35.18) = -V_0(0.10) \quad (2)$$

$$V_{\pi 2} = -V_{\pi 1}(120.2) \quad (3)$$

$$V_{\pi 2}(19.12) + V_{\pi 3}(0.7263) + V_0(0.07692) = 0 \quad (4)$$

Now

$$V_{\pi 1} = -V_0(0.002843)$$

so

$$V_{\pi 2} = -(-V_0)(0.002843)(120.2)$$

$$V_{\pi 2} = V_0(0.3417)$$

Then

$$V_0(0.3417)(19.12) + V_{\pi 3} + (0.7263) + V_0(0.07692) = 0$$

or $V_{\pi 3} = -V_0(9.101) \quad (4)$

So then

$$-V_0(9.101)(78.73) + I_X = V_0(0.8143) + (0.10)(-V_0)(0.002843)$$

or

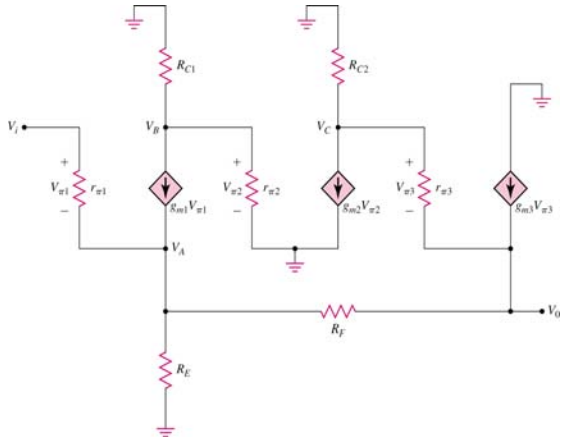
$$I_X = V_0(717.3) \quad (5)$$

or

$$R_{0f} = \frac{V_0}{I_X} = 0.00139\ \text{k}\Omega \Rightarrow \underline{R_{0f} = 1.39\ \Omega}$$

12.38

(a)



(1) $$\frac{V_i - V_A}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_A}{R_E} + \frac{V_A - V_O}{R_F}$$

(2) $$\frac{V_B}{R_{C1}} + g_{m1}V_{\pi 1} + \frac{V_B}{r_{\pi 2}} = 0$$

(3) $$\frac{V_C}{R_{C2}} + g_{m2}V_{\pi 2} + \frac{V_C - V_o}{r_{\pi 3}} = 0$$

(4) $$g_{m3}V_{\pi 3} + \frac{V_C - V_O}{r_{\pi 3}} = \frac{V_O - V_A}{R_F}$$

$$V_{\pi 1} = V_i - V_A \quad V_{\pi 2} = V_B \quad V_{\pi 3} = V_C - V_O$$

(1) $$(V_i - V_A)\left(\frac{1}{r_{\pi 1}} + g_{m1}\right) = \frac{V_A}{R_E} + \frac{V_A - V_O}{R_F}$$

(2) $$V_B\left(\frac{1}{R_{C1}} + \frac{1}{r_{\pi 2}}\right) + g_{m1}(V_i - V_A) = 0$$

(3) $$V_C\left(\frac{1}{R_{C2}} + \frac{1}{r_{\pi 3}}\right) + g_{m2}V_B - \frac{V_O}{r_{\pi 3}} = 0$$

(4) $$(V_C - V_O)\left(g_{m3} + \frac{1}{r_{\pi 3}}\right) = \frac{V_O - V_A}{R_F}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{14.3} = 0.182 \text{ K} \quad g_{m1} = \frac{14.3}{0.026} = 550 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{4.62} = 0.563 \text{ K} \quad g_{m2} = \frac{4.62}{0.026} = 178 \text{ mA/V}$$

$$r_{\pi 3} = \frac{(100)(0.026)}{4.47} = 0.582 \text{ K} \quad g_{m3} = \frac{4.47}{0.026} = 172 \text{ mA/V}$$

(1) $$(V_i - V_A)\left(\frac{1}{0.182} + 550\right) = \frac{V_A}{0.05} + \frac{V_A - V_O}{1.2}$$

(2) $$V_B\left(\frac{1}{0.3} + \frac{1}{0.563}\right) + (550)(V_i - V_A) = 0$$

(3) $$V_C\left(\frac{1}{0.65}+\frac{1}{0.582}\right)+178\ V_B-\frac{V_O}{0.582}=0$$

(4) $$(V_C-V_O)\left(172+\frac{1}{0.582}\right)=\frac{V_O-V_A}{1.2}$$

(1) $$(V_i-V_A)(555.5)=V_A(20)+(V_A-V_O)(0.8333)$$

(2) $$V_B(5.109)+550(V_i-V_A)=0$$

(3) $$V_C(3.257)+178V_B-V_O(1.718)=0$$

(4) $$(V_C-V_O)(173.7)=(V_O-V_A)(0.8333)$$

(1) $$V_i(555.5)+V_O(0.8333)=V_A(576.3)$$

(2) $$V_B(5.109)+550V_i-V_A(550)=0$$

(3) $$V_C(3.257)+178V_B-V_O(1.718)=0$$

(4) $$V_C(173.7)+V_A(0.8333)=V_O(174.5)$$

From (2) $$V_B=V_A(107.7)-V_i(107.7)$$

From (4) $$V_C=V_O(1.0046)-V_A(0.004797)$$

Substitute into (3)

$$(3.257)\left[V_O(1.0046)-V_A(0.004797)\right]$$
$$+(178)[V_A(107.7)-V_i(107.7)]-V_o(1.718)=0$$
$$V_O(3.272)-V_A(0.01562)+V_A(19170.6)-V_i(19170.6)-V_O(1.718)=0$$
$$V_A(19170.6)=V_i(19170.6)-V_O(1.554)$$
$$V_A=V_i(1.00)-V_O(0.00008106)$$

Substitute into (1)

$$V_i(555.5)+V_o(0.8333)=(576.3)\left[V_i(1.00)-V_o(0.00008106)\right]$$
$$=V_i(576.3)-V_o(0.0467)$$
$$V_o(0.880)=V_i(20.8)$$
$$\frac{V_o}{V_i}=A_{vf}=23.6$$

Ideal

$$A_{vf}=\frac{R_F+R_E}{R_E}=\frac{1.2+0.05}{0.05}=25.0$$

(b) $$R_{if}=\frac{V_i}{I_i}\quad\text{and}\quad I_i=\frac{V_{\pi1}}{r_{\pi1}}=\frac{V_i-V_A}{r_{\pi1}}$$

We have

$$V_A=V_i(1.00)-V_o(0.00008106)$$
$$=V_i(1.00)-(23.6)V_i(0.00008106)$$
$$V_A=V_i(0.99809)$$

Then $$I_i=\frac{V_i(1-0.99809)}{0.182}=V_i(0.01051)$$

$$R_{if}=\frac{V_i}{V_i(0.01051)}\Rightarrow \underline{R_{if}=95.1\ \text{K}}$$

To find R_{of}, set $V_i=0$

$$I_X + g_{m3}V_{\pi 3} + \frac{V_{\pi 3}}{r_{\pi 3}} = \frac{V_x - V_A}{R_F}$$

$$V_{\pi 3} = V_C - V_X$$

$$I_X + (V_C - V_X)(g_{m3} + \frac{1}{r_{\pi 3}}) = \frac{V_X - V_A}{R_F}$$

For $V_i = 0,$ we have

$$V_C = V_X(1.0046) - V_A(0.004797)$$

$$V_A(576.3) = V_X(0.8333)$$

$$V_A = V_X(0.001446)$$

$$V_C = V_X(1.0046) - V_X(0.001446)(0.004797)$$

$$V_C = V_X(1.0046)$$

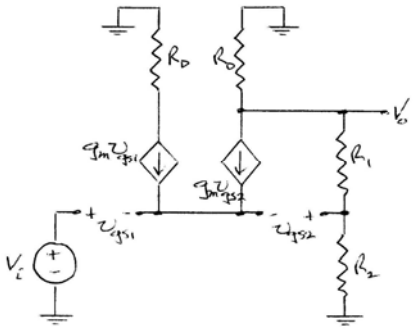
$$I_X + V_X(1.0046 - 1.0)\left(172 + \frac{1}{0.582}\right) = \frac{V_X(1 - 0.004797)}{1.2}$$

$$I_X + V_X(0.7991) = V_X(0.8293)$$

$$I_X = V_X(0.03024)$$

$$R_{of} = \frac{V_X}{I_X} = 33.1 \text{ K}$$

12.39



$$g_m(\upsilon_{gs1} + \upsilon_{gs2}) = 0 \Rightarrow \upsilon_{gs2} = -\upsilon_{gs1}$$

$$V_i = \upsilon_{gs1} - \upsilon_{gs2} + \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_o$$

$$V_i = -2\upsilon_{gs2} + \frac{1}{2}V_o$$

Also $$\frac{V_o}{R_1 + R_2} + g_m\upsilon_{gs2} + \frac{V_o}{R_D} = 0$$

$$\upsilon_{gs2} = -\frac{V_o}{g_m}\left(\frac{1}{R_D} + \frac{1}{R_1 + R_2}\right)$$

Then $$V_i = \frac{2V_o}{g_m}\left(\frac{1}{R_D} + \frac{1}{R_1 + R_2}\right) + \frac{1}{2}V_o$$

We have $R_1 + R_2 >> R_D$

$$V_i \cong V_o\left(\frac{2}{g_m R_D}+\frac{1}{2}\right)$$

$$A_{vf}=\frac{V_o}{V_i}=\frac{1}{\left(\frac{2}{g_m R_D}+\frac{1}{2}\right)}=\frac{g_m R_D}{\left(2+\frac{1}{2}g_m R_D\right)}$$

Now $g_m=2\sqrt{K_n I_{DQ}}=2\sqrt{(0.5)(0.5)}=1.0$ mA/V

$$A_{vf}=\frac{(1)(7)}{\left[2+\frac{1}{2}(1)(7)\right]}=1.273$$

12.40

(a) Neglect base currents

$$\frac{V_{GG}-V_{GS}}{R_L}=I_{D1}+I_{C2}$$

$$I_{D1}=\frac{5-V_D}{R_{D1}}=\frac{5-V_D}{5}$$

$$I_{C2}=\frac{5-(V_D+0.7)}{R_{E2}}=\frac{4.3-V_D}{1.6}$$

$$V_D=5-5I_{D1} \Rightarrow I_{C2}=\frac{4.3-(5-5I_{D1})}{1.6}=-0.4375+3.125I_{D1}$$

Then $\frac{2.5-V_{GS}}{1.2}=I_{D1}+(-0.4375+3.125I_{D1})$

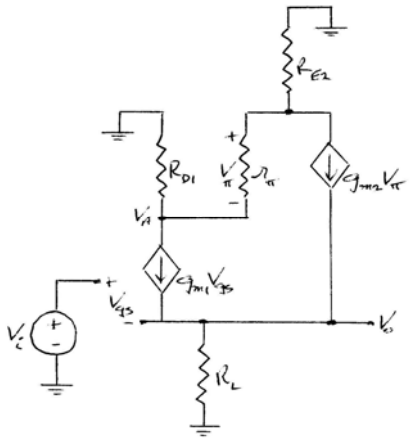
$$3.025=V_{GS}+4.95I_{D1}=V_{GS}+4.95(K_n)(V_{GS}-0.5)^2$$

We find $7.425V_{GS}^2-6.425V_{GS}-1.16875=0 \Rightarrow V_{GS}=1.0197$ V

Then $I_{D1}=(1.5)(1.0197-0.5)^2=0.405$ mA

$$I_{C2}=-0.4375+3.125(0.405)=0.828 \text{ mA}$$

(b)



$$g_{m1}=2\sqrt{K_n I_{D1}}=2\sqrt{(1.5)(0.405)}=1.559 \text{ mA/V}$$

$$g_{m2} = \frac{I_{C2}}{V_T} = \frac{0.828}{0.026} = 31.85 \text{ mA/V}$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{C2}} = \frac{(120)(0.026)}{0.828} = 3.768 \text{ k}\Omega$$

(1) $V_o = (g_{m1}V_{gs} + g_{m2}V_\pi)R_L$

(2) $V_i = V_{gs} + V_o \Rightarrow V_{gs} = V_i - V_o$

(3) $\dfrac{V_A}{R_{D1}} + g_{m1}V_{gs} = \dfrac{V_\pi}{r_\pi}$

(4) $\dfrac{V_A + V_\pi}{R_{E2}} + \dfrac{V_\pi}{r_\pi} + g_{m2}V_\pi = 0$

So $\dfrac{-V_A}{1.6} = V_\pi\left(\dfrac{1}{1.6} + \dfrac{1}{3.768} + 31.85\right) = V_\pi(32.74)$

$$V_A = -52.385V_\pi$$

(3) $\dfrac{-52.385V_\pi}{5} + 1.559V_{gs} = \dfrac{V_\pi}{3.768}$

or $1.559V_{gs} = V_\pi(0.2654 + 10.477)$

so $V_\pi = 0.1451V_{gs} = 0.1451(V_i - V_o)$

(1) $V_o = [1.559(V_i - V_o) + 31.85(0.1451)(V_i - V_o)](1.2)$

Then $A_\upsilon = \dfrac{V_o}{V_i} = 0.8812$

(c)Set $V_i = 0$

$$I_x + g_{m2}V_\pi + g_{m1}V_{gs} = \frac{V_x}{R_L}$$

$$V_{gs} = -V_x$$

$$V_\pi = 0.1451V_{gs} = -0.1451V_x$$

$$I_x = V_x\left[\frac{1}{1.2} + (31.85)(0.1451) + (1.559)\right] = V_x(7.014)$$

$$R_o = \frac{V_x}{I_x} = 143\,\Omega$$

12.41

(a) (i) $A_{\upsilon f} = \dfrac{g_m R_S}{1 + g_m R_S}$

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(1.5)(1.2)} = 2.683 \text{ mA/V}$$

$$A_{\upsilon f} = \frac{(2.683)(1.5)}{1 + (2.683)(1.5)} = 0.801$$

(ii) $R_{of} = \dfrac{1}{g_m}\Big\|R_S = \dfrac{1}{2.683}\|1.5 = 0.3727\|1.5$

$$R_{of} = 299\,\Omega$$

(b) (i) $g_m = 2\sqrt{(2.25)(1.2)} = 3.286$ mA/V

$$A_{vf} = \frac{(3.286)(1.5)}{1+(3.286)(1.5)} = 0.8313 \text{ mA/V}$$

$$\frac{\Delta A_{vf}}{A_{vf}} \times 100\% = +3.78\%$$

(ii) $R_{of} = \dfrac{1}{3.286} \| 1.5 = 0.3043 \| 1.5$

$$R_{of} = 253\,\Omega$$

$$\frac{\Delta R_{of}}{R_{of}} \times 100\% = -15.4\%$$

12.42

dc analysis:

$R_{TH1} = 150 \,||\, 47 = 35.8$ kΩ,

$$V_{TH1} = \left(\frac{47}{47+150}\right)(25) = 5.96 \text{ V}$$

$R_{TH2} = 33 \,||\, 47 = 19.4$ kΩ,

$$V_{TH2} = \left(\frac{33}{33+47}\right)(25) = 10.3 \text{ V}$$

$$I_{B1} = \frac{5.96 - 0.7}{35.8 + (51)(4.8)} = 0.0187 \text{ mA}$$

$$I_{C1} = (50)(0.0187) = 0.935 \text{ mA}$$

$$I_{B2} = \frac{10.3 - 0.7}{19.4 + (51)(4.7)} = 0.03705 \text{ mA}$$

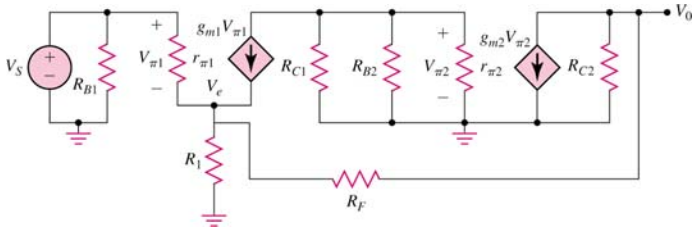
$$I_{C2} = (50)(0.03705) = 1.85 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.935} = 1.39 \text{ kΩ;}$$

$$r_{\pi 2} = \frac{(50)(0.026)}{1.85} = 0.703 \text{ kΩ}$$

$$g_{m1} = \frac{0.935}{0.026} = 35.96 \text{ mA/V}$$

$$g_{m2} = \frac{1.85}{0.026} = 71.15 \text{ mA/V}$$



$$V_S = V_{\pi 1} + V_e \qquad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_e}{R_1} + \frac{V_e - V_0}{R_F} \qquad (2)$$

$$g_{m1}V_{\pi 1}+\frac{V_{\pi 2}}{R_{C1}}+\frac{V_{\pi 2}}{R_{B2}}+\frac{V_{\pi 2}}{r_{\pi 2}}=0 \quad (3)$$

$$g_{m2}V_{\pi 2}+\frac{V_0}{R_{C2}}+\frac{V_0-V_e}{R_F}=0 \quad (4)$$

Substitute numerical values in (2), (3) and (4):

$$V_e=V_S-V_{\pi 1} \quad (1)$$

$$\frac{V_{\pi 1}}{1.39}+(35.96)V_{\pi 1}=(V_S-V_{\pi 1})\left(\frac{1}{0.1}+\frac{1}{4.7}\right)-V_0\left(\frac{1}{4.7}\right)$$

or

$$V_{\pi 1}(46.89)=V_S(10.213)-V_0(0.2128) \quad (2)$$

$$(35.96)V_{\pi 1}+V_{\pi 2}\left(\frac{1}{10}+\frac{1}{19.4}+\frac{1}{0.703}\right)=0$$

or

$$(35.96)V_{\pi 1}+V_{\pi 2}(1.574)=0 \quad (3)$$

$$(71.15)V_{\pi 2}+V_0\left(\frac{1}{4.7}+\frac{1}{4.7}\right)-(V_S-V_{\pi 1})\left(\frac{1}{4.7}\right)=0$$

or

$$(71.15)V_{\pi 2}+V_0(0.4255)-V_S(0.2128)+V_{\pi 1}(0.2128)=0 \quad (4)$$

From (3): $V_{\pi 2}=-V_{\pi 1}(22.85)$

Then substitute in (4):

$$-(71.15)V_{\pi 1}(22.85)+V_0(0.4255)-V_S(0.2128)+V_{\pi 1}(0.2128)=0$$

or

$$-V_{\pi 1}(1625.6)+V_0(0.4255)-V_S(0.2128)=0$$

From (2): $V_{\pi 1}=V_S(0.2178)-V_0(0.004538)$

Then

$$-(1625.6)[V_S(0.2178)-V_0(0.004538)]+V_0(0.4255)-V_S(0.2128)=0$$

or $-V_S(354.3)+V_0(7.802)=0$

Finally

$$\Rightarrow \frac{V_0}{V_S}=45.4$$

12.43

For example, use a 2-stage amplifier. Each stage is shown in Fig. 12.29.

12.44

$$I_{DQ1}=\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GS}-V_{TN})^2$$

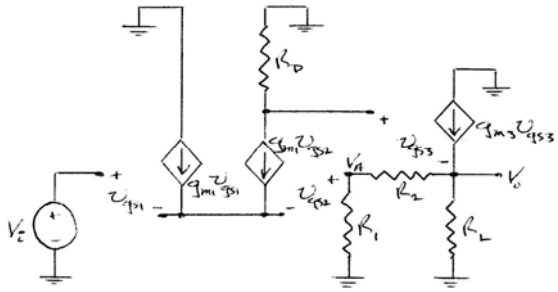
Let all $\left(\frac{W}{L}\right)=20$

$$0.5=\left(\frac{0.1}{2}\right)(20)(V_{GS}-1.5)^2 \Rightarrow V_{GS1,2}=2.207\ \text{V}$$

$$I_{DQ3}=2=\left(\frac{0.1}{2}\right)(20)(V_{GS3}-1.5)^2 \Rightarrow V_{GS3}=2.914\ \text{V}$$

Want $V_{G3}=2.914\ \text{V}$, then

$$R_D = \frac{12 - 2.914}{0.5} = 18.2\,\text{k}\Omega$$



$$g_{m1}\upsilon_{gs1} + g_{m1}\upsilon_{gs2} = 0 \Rightarrow \upsilon_{gs2} = -\upsilon_{gs1}$$

$$V_i = \upsilon_{gs1} - \upsilon_{gs2} + V_A = -2\upsilon_{gs2} + V_A$$

$$V_A = \left(\frac{R_1}{R_1 + R_2}\right)\cdot V_o$$

$$\frac{V_o}{R_L} + \frac{V_o}{R_1 + R_2} = g_{m3}\upsilon_{gs3}$$

$$\upsilon_{gs3} = -\left(g_{m1}\upsilon_{gs2}R_D\right) - V_o$$

$$V_o\left(\frac{1}{R_L} + \frac{1}{R_1 + R_2}\right) = g_{m3}\left(-g_{m1}\upsilon_{gs2}R_D - V_o\right)$$

$$\upsilon_{gs2} = \frac{1}{2}\left(V_A - V_i\right) = \frac{1}{2}\left(\frac{R_1}{R_1 + R_2}\right)\cdot V_o - \frac{1}{2}\cdot V_i$$

$$V_o\left(\frac{1}{R_L} + \frac{1}{R_1 + R_2}\right) = -g_{m1}g_{m3}R_D\left[\frac{1}{2}\left(\frac{R_1}{R_1 + R_2}\right)\cdot V_o - \frac{1}{2}\cdot V_i\right] - g_{m3}V_o$$

$$V_o\left[\frac{1}{R_L} + \frac{1}{R_1 + R_2} + \frac{1}{2}g_{m1}g_{m3}R_D\left(\frac{R_1}{R_1 + R_2}\right) + g_{m3}\right] = \frac{1}{2}g_{m1}g_{m3}R_DV_i$$

$$A_{vf} = \frac{V_o}{V_i} = \frac{\frac{1}{2}g_{m1}g_{m3}R_D}{\left[\frac{1}{R_L} + \frac{1}{R_1 + R_2} + \frac{1}{2}g_{m1}g_{m3}R_D\left(\frac{R_1}{R_1 + R_2}\right) + g_{m3}\right]}$$

$$g_{m1} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.5)} = 1.414\ \text{mA/V}$$

$$g_{m3} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(2)} = 2.828\,\text{mA/V}$$

Then $$8 = \frac{\frac{1}{2}(1.414)(2.828)(18.2)}{\left[\frac{1}{10} + \frac{1}{R_1 + R_2} + \frac{1}{2}(1.414)(2.828)(18.2)\left(\frac{15}{R_1 + R_2}\right) + 2.828\right]}$$

$$8 = \frac{36.389}{\left[2.928 + \dfrac{546.8}{R_1 + R_2}\right]}$$

$$\Rightarrow R_1 + R_2 = 337\,\text{k}\Omega \quad \Rightarrow R_2 = 322\,\text{k}\Omega$$

12.45

$$\frac{I_o}{I_s} = \frac{5}{0.06} = 83.33 = 1 + \frac{R_1}{R_2} \Rightarrow \frac{R_1}{R_2} = 82.33$$

For example, let $R_2 = 3\,\text{k}\Omega$, $R_2 = 247\,\text{k}\Omega$

12.46

(a) (1) $V_A = (I_{D1} + I_{D2})R_{D2}$

(2) $I_{D1}R_{D1} = V_{SG2}$

(3) $\dfrac{(V_G - V_{GS1}) - V_A}{R_F} = I_{D1}$

Now $V_G - V_{GS1} - (I_{D1} + I_{D2})R_{D2} = I_{D1}R_F$

And $V_{GS2} = \sqrt{\dfrac{I_{D2}}{K_p}} - V_{TP} = I_{D1}R_{D1}$

$\sqrt{I_{D2}}(0.3162) + 1 = I_{D1}(0.525)$

$\sqrt{I_{D2}} = I_{D1}(1.660) - 3.162$

$I_{D2} = [I_{D1}(1.660) - 3.162]^2$

Then $V_G - V_{GS1} - [I_{D1}(1.660) - 3.162]^2 R_{D2} = I_{D1}(R_{D2} + R_F)$

$I_{D1} = K_n(V_{GS1} - V_{TN})^2$

$7.6 - V_{GS1} - [I_{D1}(1.660) - 3.162]^2(0.25) = I_{D1}(0.75)$

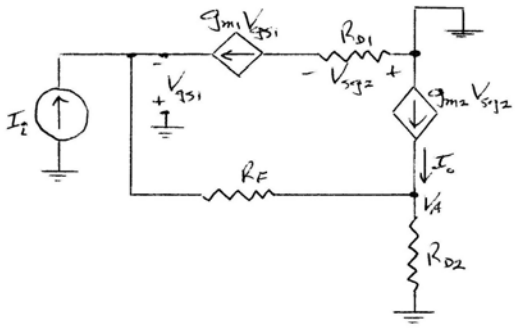
$I_{D1} = 10(V_{GS1} - 1)^2$

By trial and error, $I_{D1} \cong 3.98\,\text{mA}$

Then $V_{SG2} = I_{D1}R_{D1} = (3.98)(0.525) = 2.0895\,\text{V}$

$I_{D1} = (10)(2.0895 - 1)^2 = 11.87\,\text{mA}$

(b)



$$I_o = g_{m2}V_{sg2}$$

$$V_{sg2} = g_{m1}V_{gs1}R_{D1}$$

$$I_o = \frac{V_A}{R_{D2}} + \frac{V_A - \left(-V_{gs1}\right)}{R_F}$$

$$I_i + g_{m1}V_{gs1} + \frac{V_A + V_{gs1}}{R_F} = 0$$

or $$V_A = -R_F\left[I_i + g_{m1}V_{gs1} + \frac{V_{gs1}}{R_F}\right]$$

$$I_o = V_A\left(\frac{1}{R_{D2}} + \frac{1}{R_F}\right) + \frac{V_{gs1}}{R_F}$$

$$I_o = -R_F\left(I_i + g_{m1}V_{gs1} + \frac{V_{gs1}}{R_F}\right)\left(\frac{1}{R_{D2}} + \frac{1}{R_F}\right) + \frac{V_{gs1}}{R_F}$$

Now $$V_{gs1} = \frac{V_{sg2}}{g_{m1}R_{D1}} = \frac{I_o}{g_{m2}} \cdot \frac{1}{g_{m1}R_{D1}}$$

$$I_o = -R_F I_i\left(\frac{1}{R_{D2}} + \frac{1}{R_F}\right) - R_F V_{gs1}\left[\left(g_{m1} + \frac{1}{R_F}\right)\left(\frac{1}{R_{D2}} + \frac{1}{R_F}\right) - \frac{1}{R_F^2}\right]$$

Then $$I_o\left[1 + \frac{R_F}{g_{m1}g_{m2}R_{D1}}\left(\frac{g_{m1}}{R_{D2}} + \frac{g_{m1}}{R_F} + \frac{1}{R_F R_{D2}}\right)\right] = -I_i\left(1 + \frac{R_F}{R_{D2}}\right)$$

So $$A_i = \frac{I_o}{I_i} = \frac{-g_{m2}R_{D1}}{1 + \dfrac{1}{g_{m1}\left(R_F + R_{D2}\right)} + \dfrac{g_{m2}R_{D1}R_{D2}}{R_F + R_{D2}}}$$

(c) $$g_{m1} = 2\sqrt{K_n I_{D1}} = 2\sqrt{(10)(3.98)} = 12.62 \text{ mA/V}$$

$$g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{(10)(11.87)} = 21.79 \text{ mA/V}$$

$$A_i = \frac{-(21.79)(0.525)}{1 + \dfrac{1}{(12.62)(0.75)} + \dfrac{(21.79)(0.525)(0.25)}{0.75}}$$

$$A_i = -2.33$$

12.47

(a) $$I_Q = I_{D1} + I_{D2}$$

$$V_{SG2} = I_{D1}R_D$$

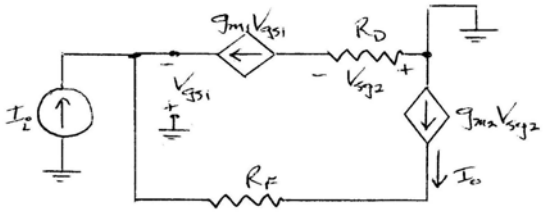
$$I_{D2} = K_p\left(V_{SG2} + V_{TP}\right)^2 = K_p\left(I_{D1}R_D + V_{TP}\right)^2$$

$$I_Q = I_{D1} + K_p\left[I_{D1}^2 R_D^2 - 2I_{D1}R_D + 1\right]$$

We find $$2.756I_{D1}^2 - 9.5I_{D1} - 6 = 0$$

$$\Rightarrow I_{D1} = 3.99 \text{ mA} \text{ and } I_{D2} = 12.01 \text{ mA}$$

(b)



$$I_i + g_{m1}V_{gs1} + I_o = 0$$

$$V_{sg2} = g_{m1}V_{gs1}R_D \Rightarrow V_{gs1} = \frac{V_{sg2}}{g_{m1}R_D}$$

$$I_o = g_{m2}V_{sg2}$$

Then $V_{gs1} = \dfrac{I_o}{g_{m1}g_{m2}R_D}$

$$I_i + g_{m1}\left(\frac{I_o}{g_{m1}g_{m2}R_D}\right) + I_o = 0$$

So that $A_i = \dfrac{I_o}{I_i} = \dfrac{-1}{\left(1 + \dfrac{1}{g_{m2}R_D}\right)} = \dfrac{-g_{m2}R_D}{1 + g_{m2}R_D}$

We find $g_{m2} = 2\sqrt{K_p I_{D2}} = 2\sqrt{(10)(12.01)} = 21.92$ mA/V

$$A_i = \frac{-(21.92)(0.525)}{1 + (21.92)(0.525)} = -0.920$$

12.48

(a) Neglect base currents

$$I_Q = I_{C1} + I_{C2}$$

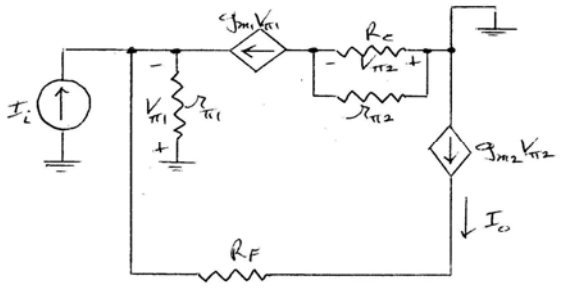
$$V_{EB2} = I_{C1}R_C$$

$$I_Q = I_{C1} + I_{S2}\exp\left(\frac{I_{C1}R_C}{V_T}\right)$$

$$16\times10^{-3} = I_{C1} + (10^{-15})\exp\left[\frac{I_{C1}(200)}{0.026}\right]$$

By trial and error, $I_{C1} = 3.92$ mA and $I_{C2} = 12.08$ mA

(b)



$$I_o = g_{m2}V_{\pi2}$$

$$V_{\pi2} = g_{m1}V_{\pi1}\left(r_{\pi2}\|R_C\right)$$

Then $$I_o = g_{m1}g_{m2}V_{\pi1}\left(r_{\pi2}\|R_C\right)$$

$$I_i + I_o + g_{m1}V_{\pi1} + \frac{V_{\pi1}}{r_{\pi1}} = 0$$

$$I_i + I_o + V_{\pi1}\left(\frac{1+\beta_1}{r_{\pi1}}\right) = 0$$

Now $$V_{\pi1} = \frac{I_o}{g_{m1}g_{m2}\left(r_{\pi2}\|R_C\right)}$$

Then $$I_i + I_o\left[1 + \frac{1}{g_{m1}g_{m2}\left(r_{\pi2}\|R_C\right)}\left(\frac{1+\beta_1}{r_{\pi1}}\right)\right] = 0$$

So $$A_i = \frac{I_o}{I_i} = \frac{-\left(\frac{\beta_1}{1+\beta_1}\right)(g_{m2})(r_{\pi2}\|R_C)}{1+\left(\frac{\beta_1}{1+\beta_1}\right)(g_{m2})(r_{\pi2}\|R_C)}$$

(c) $$g_{m2} = \frac{I_{C2}}{V_T} = \frac{12.08}{0.026} = 464.6\ \text{mA/V}$$

$$r_{\pi2} = \frac{\beta_2 V_T}{I_{C2}} = \frac{(180)(0.026)}{12.08} = 0.3874\ \text{k}\Omega$$

$$r_{\pi2}\|R_C = 387.4\|200 = 131.9\,\Omega$$

$$A_i = \frac{-\left(\frac{180}{181}\right)(464.6)(0.1319)}{1+\left(\frac{180}{181}\right)(464.6)(0.1319)} = -0.984$$

12.49

$$I_{C1} = \left(\frac{h_{FE}}{1+h_{FE}}\right) I_{E1} = \left(\frac{100}{101}\right)(0.2) = 0.198\ mA$$

$$V_{C1} = 10 - (0.198)(40) = 2.08\ V$$

$$I_{E2} = \frac{2.08 - 0.7}{1} = 1.38\ mA$$

$$I_{C2} = \left(\frac{100}{101}\right)(1.38) = 1.37\ mA$$

(a)

For Q_1:

$$r_{\pi 1} = \frac{(100)(0.026)}{0.198} = 13.1\ k\Omega$$

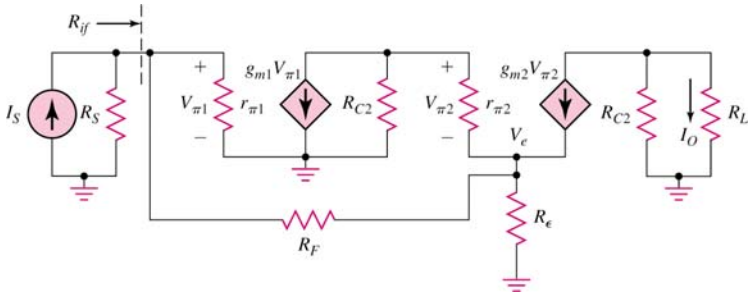
$$g_{m1} = \frac{0.198}{0.026} = 7.62\ mA/V$$

For Q_2:

$$r_{\pi 2} = \frac{(100)(0.026)}{1.37} = 1.90\ k\Omega$$

$$g_{m2} = \frac{1.37}{0.026} = 52.7\ mA/V$$

(b)



$$I_S = \frac{V_{\pi 1}}{R_S} + \frac{V_{\pi 1}}{r_{\pi 1}} + \frac{V_{\pi 1} - V_e}{R_F} \qquad (1)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2} + V_e}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \qquad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_e}{R_E} + \frac{V_e - V_{\pi 1}}{R_F} \qquad (3)$$

Substitute numerical values in (1), (2), and (3):

$$I_S = V_{\pi 1}\left(\frac{1}{10} + \frac{1}{13.1} + \frac{1}{10}\right) - V_e\left(\frac{1}{10}\right)$$

$$I_S = V_{\pi 1}(0.2763) - V_e(0.10) \qquad (1)$$

$$(7.62)V_{\pi 1} + V_{\pi 2}\left(\frac{1}{40} + \frac{1}{1.90}\right) + V_e\left(\frac{1}{40}\right) = 0$$

$$(7.62)V_{\pi 1} + V_{\pi 2}(0.5513) + V_e(0.025) = 0 \qquad (2)$$

$$V_{\pi 2}\left(\frac{1}{1.90} + 52.7\right) = V_e\left(\frac{1}{1} + \frac{1}{10}\right) - V_{\pi 1}\left(\frac{1}{10}\right)$$

$$V_{\pi 2}(53.23) = V_e(1.10) - V_{\pi 1}(0.10) \qquad (3)$$

From (3),

$$V_e = V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)$$

Substituting into (1),

$$I_S = V_{\pi 1}(0.2763) - (0.10)\left[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)\right]$$

or

$$I_S = V_{\pi 1}(0.2672) - V_{\pi 2}(4.839) \quad (1')$$

and substituting into (2),

$$(7.62)V_{\pi 1} + V_{\pi 2}(0.5513) + (0.025)\left[V_{\pi 2}(48.39) + V_{\pi 1}(0.0909)\right] = 0$$

or

$$(7.622)V_{\pi 1} + V_{\pi 2}(1.761) = 0 \Rightarrow V_{\pi 1} = -V_{\pi 2}(0.2310) \quad (2')$$

Then substituting (2′) into (1′), we obtain

$$I_S = (0.2672)(-V_{\pi 2})(0.2310) - V_{\pi 2}(4.839)$$

or

$$I_S = -V_{\pi 2}(4.901)$$

Now

$$I_O = -g_{m2}V_{\pi 2}\left(\frac{R_{C2}}{R_{C2} + R_L}\right)$$

$$= -(52.7)\left(\frac{2}{2 + 0.5}\right)V_{\pi 2} = -(42.16)V_{\pi 2}$$

Then

$$I_O = -(42.16)\left(\frac{-I_S}{4.901}\right)$$

or

$$\underline{A_{if} = \frac{I_O}{I_s} = 8.60}$$

(c) $R_i = \dfrac{V_{\pi 1}}{I_S}$ and $R_i = R_S \,||\, R_{if}$

We had

$$V_{\pi 1} = -V_{\pi 2}(0.2310) \text{ and } I_S = -V_{\pi 2}(4.901)$$

so

$$I_S = -\left(\frac{-V_{\pi 1}}{0.2310}\right)(4.901) = V_{\pi 1}(21.22)$$

Then

$$R_i = \frac{V_{\pi 1}}{I_S} = \frac{1}{21.22} = 0.04713$$

Finally

$$0.04713 = \frac{10R_{if}}{10 + R_{if}} \Rightarrow \underline{R_{if} = 47.4\ \Omega}$$

12.50

(a)

(1) $$I_i = \frac{V_{\pi 1}}{R_S \left\| R_{B1} \right\| r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F}$$

(2) $$g_{m1}V_{\pi 1} + \frac{V_{C1}}{R_{C1} \left\| R_{B2} \right.} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0$$

(3) $$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F}$$

$$I_o = -(g_{m2}V_{\pi 2})\left(\frac{R_{C2}}{R_{C2}+R_L}\right)$$

(4)

Now

$$I_i = \frac{V_{\pi 1}}{R_S \| R_{B1} \| r_{\pi 1} \| R_F} - \frac{V_{e2}}{R_F}$$

(1)′

$$V_{e2} = \left(\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F}\right)V_{\pi 1} - I_i R_F$$

So

Now, from (2)

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2}+V_{e2}}{R_{c1} \| R_{B2}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0$$

$$\left(g_{m1} + \frac{1}{r_{\pi 2}}\right)V_{\pi 1} + \frac{V_{\pi 2}}{R_{C1} \| R_{B2} \| r_{\pi 2}} + \frac{V_{e2}}{R_{C1} \| R_{B2}} = 0$$

(2)′

Also

$$\left(g_{m2} + \frac{1}{r_{\pi 2}}\right)V_{\pi 2} + \frac{V_{\pi 1}}{R_F} = V_{e2}\left(\frac{1}{R_{E2}} + \frac{1}{R_F}\right)$$

(3)′

And

$$V_{\pi 2} = -\left(\frac{I_O}{g_{m2}}\right)\left(\frac{R_{C2}+R_L}{R_{C2}}\right)$$

(4)′

Substitute (1)′ into (2)′ and (3)′

$$\left(g_{m1} + \frac{1}{r_{\pi 2}}\right)V_{\pi 1} + \frac{V_{\pi 2}}{R_{C1} \| R_{B2} \| r_{\pi 2}} + \frac{1}{R_{C1} \| R_{B2}}\left[\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F} - V_{\pi 1} - I_i R_F\right] = 0$$

(2)″

$$\left(g_{m2} + \frac{1}{r_{\pi 2}}\right)V_{\pi 2} + \frac{V_{\pi 1}}{R_F} = \left(\frac{1}{R_{E2}} + \frac{1}{R_F}\right)\left[\frac{R_F}{R_S \| R_{B1} \| r_{\pi 1} \| R_F} - V_{\pi 1} - I_i R_F\right]$$

(3)″

Solve for $V_{\pi 1}$ from (2)″ and substitute into (3)″. Also use Equation (4)′.

(b) $R_{B1} = R_1 \| R_2 = 20 \| 80 = 16\,\text{K}$

$$V_{TH1} = \left(\frac{20}{100}\right)(10) = 2\,\text{V}$$

$$I_{BQ1} = \frac{2-0.7}{16+(101)(1)} = 0.0111\ \text{mA}$$

$$I_{CQ1} = 1.11\ \text{mA}$$

$$R_{TH2} = 15 \| 85 = 12.75\,\text{K}$$

$$V_{TH2} = \left(\frac{15}{100}\right)(10) = 1.5\,\text{V}$$

$$I_{BQ2} = \frac{1.5-0.7}{12.75+(101)(0.5)} = 0.01265\ \text{mA}$$

$$I_{CQ2} = 1.265\ \text{mA}$$

$$g_{m1} = \frac{1.11}{0.026} = 42.69\ \text{mA/V}$$

$$g_{m2} = \frac{1.265}{0.026} = 48.65\ \text{mA/V}$$

$$r_{\pi 1} = \frac{(100)(0.026)}{1.11} = 2.34\,\text{K}$$

$$r_{\pi 2} = \frac{(100)(0.026)}{1.265} = 2.06\,\text{K}$$

Now $R_{C1} \| R_{B2} = 2 \| 12.75 = 1.729\,\text{K}$

$$R_S \| R_{B1} \| r_{\pi 1} \| R_F \cong R_{B1} \| r_{\pi 1} \| R_F = 16 \| 2.34 \| 10 = 1.695\,\text{K}$$

$$R_{C1} \| R_{B2} \| r_{\pi 2} = 1.729 \| 2.06 = 0.940\,\text{K}$$

Now

$$\left(42.69 + \frac{1}{2.06}\right)V_{\pi 1} + \frac{V_{\pi 2}}{0.940} + \frac{1}{1.729}\left[\frac{10}{1.695}\cdot V_{\pi 1} - I_i(10)\right]$$

(2)″ $46.587V_{\pi 1} + 1.064V_{\pi 2} - 5.784I_i = 0$

$$\left(\frac{101}{2.06}\right)V_{\pi 2} + \frac{V_{\pi 1}}{10} = \left(\frac{1}{0.5} + \frac{1}{10}\right)\left[\frac{10}{1.695}\cdot V_{\pi 1} - I_i(10)\right]$$

(3)″ $49.03V_{\pi 2} = 12.29V_{\pi 1} - 21I_i$

From (2)″ $V_{\pi 1} = (0.1242)I_i - (0.02284)V_{\pi 2}$

Then

(3)″ $49.03V_{\pi 2} = 12.29\left[(0.1242)I_i - (0.02284)V_{\pi 2}\right] - 21I_i$

$$49.31V_{\pi 2} = -19.47I_i$$

From (4)′ $$V_{\pi 2} = -\left(\frac{I_o}{48.65}\right)\left(\frac{4+4}{4}\right) = -(0.0411)I_o$$

Then

$$(49.31)\left[-(0.0411)I_o\right] = -19.47I_i$$

$$\frac{I_o}{I_i} = A_i = 9.61$$

12.51

$$R_{TH} = 13.5 \,||\, 38.3 = 9.98\ \text{k}\Omega$$

$$V_{TH} = \left(\frac{13.5}{13.5 + 38.3}\right)(10) = 2.606\ \text{V}$$

$$I_{C1} = \frac{(120)(2.606 - 0.7)}{9.98 + (121)(1)} = 1.75\ \text{mA}$$

a.

$$V_{C1} = 10 - (1.75)(3) = 4.75\ \text{V}$$

$$I_{C2} \approx \frac{4.75 - 0.7}{8.1} = 0.50\ \text{mA}$$

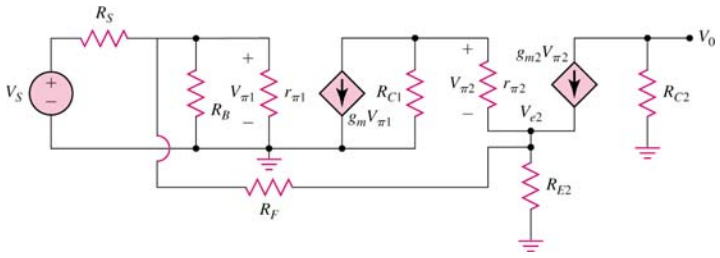
$$r_{\pi 1} = \frac{(120)(0.026)}{1.75} = 1.78\ \text{k}\Omega$$

$$g_{m1} = \frac{1.75}{0.026} = 67.31\ \text{mA/V}$$

$$r_{\pi 2} = \frac{(120)(0.026)}{0.50} = 6.24\ \text{k}\Omega$$

$$g_{m2} = \frac{0.50}{0.026} = 19.23\ \text{mA/V}$$

b.



$$\frac{V_S - V_{\pi 1}}{R_S} = \frac{V_{\pi 1}}{R_B \,||\, r_{\pi 1}} + \frac{V_{\pi 1} - V_{e2}}{R_F} \quad (1)$$

$$g_{m1} V_{\pi 1} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} + \frac{V_{\pi 2}}{r_{\pi 2}} = 0 \quad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2} V_{\pi 2} = \frac{V_{e2}}{R_{E2}} + \frac{V_{e2} - V_{\pi 1}}{R_F} \quad (3)$$

and

$$V_0 = -(g_{m2} V_{\pi 2}) R_{C2} \quad (4)$$

Substitute numerical values in (1), (2), and (3)

$$\frac{V_S}{0.6} = V_{\pi 1}\left[\frac{1}{0.6} + \frac{V_{\pi 1}}{9.98 \,||\, 1.78} + \frac{1}{1.2}\right] - \frac{V_{e2}}{1.2}$$

$$V_S(1.67) = V_{\pi 1}(4.011) - V_{e2}(0.8333) \quad (1)$$

$$(67.31) V_{\pi 1} + V_{\pi 2}\left(\frac{1}{3} + \frac{1}{6.24}\right) + \frac{V_{e2}}{3} = 0$$

or

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + V_{e2}(0.3333) = 0 \quad (2)$$

$$V_{\pi 1}\left(\frac{1}{6.24} + 19.23\right) = \frac{V_{e2}}{8.1} + \frac{V_{e2}}{1.2} - \frac{V_{\pi 2}}{1.2}$$

or

$$V_{\pi 2}(19.39) = V_{e2}(0.9568) - V_{\pi 1}(0.8333) \quad (3)$$

From (1)

$$V_{e2} = V_{\pi 1}(4.813) - V_S(2.00)$$

Then

$$V_{\pi 1}(67.31) + V_{\pi 2}(0.4936) + (0.3333)[V_{\pi 1}(4.813) - V_S(2.00)] = 0$$

or

$$V_{\pi 1}(68.91) + V_{\pi 2}(0.4936) - V_S(0.6666) = 0 \quad (2')$$

and

$$V_{\pi 2}(19.39) = (0.9568)\left[V_{\pi 1}(4.813) - V_S(2.00)\right] - V_{\pi 1}(0.8333)$$

or

$$V_{\pi 2}(19.39) = V_{\pi 1}(3.772) - V_S(1.914) \quad (3')$$

We find

$$V_{\pi 1} = V_S(0.009673) - V_{\pi 2}(0.007163)$$

Then

$$V_{\pi 2}(19.39) = (3.772)[V_S(0.009673) - V_{\pi 2}(0.007163)] - V_S(1.914)$$

$$V_{\pi 2}(19.42) = V_S(-1.878) \text{ or } V_{\pi 2} = -V_S(0.09670)$$

so that

$$V_0 = -(19.23)(4)(-V_S)(0.09670)$$

Then

$$\frac{V_0}{V_S} = 7.44$$

12.52

Using the circuit from Problem 12.51, we have $R_{if} = \dfrac{V_{\pi 1}}{I_S}$.

Where $I_S = \dfrac{V_S - V_{\pi 1}}{R_S}$

From Problem 12.51

$$V_{\pi 1} = V_S(0.009673) - V_{\pi 2}(0.007163)$$
$$= V_S(0.009673) - (0.007163)(-V_S)(0.09670)$$
$$= V_S(0.01037)$$

So

$$R_{if} = \frac{V_S(0.01037)\cdot(0.6)}{V_S - V_S(0.01037)} = 0.00629 \text{ k}\Omega$$

or

$$R_{if} = 6.29\ \Omega$$

12.53

$$R_{TH} = 1.4 \,||\, 17.9 = 1.298 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{1.4}{1.4+17.9}\right)(10) = 0.7254 \text{ V}$$

$$I_{B1} = \frac{0.7254 - 0.7}{1.298} = 0.0196 \text{ mA}$$

$$I_{C1} = (50)(0.0196) = 0.98 \text{ mA}$$

Neglecting dc base currents,

$$V_{B2} = 10 - (0.98)(7) = 3.14 \text{ V}$$

$$I_{E2} = \frac{3.14 - 0.7}{0.25 + 0.5} = 3.25 \text{ mA}$$

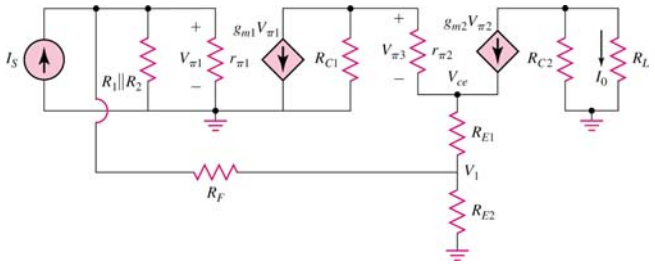
$$I_{C2} = \left(\frac{50}{51}\right)(3.25) = 3.19 \text{ mA}$$

$$r_{\pi 1} = \frac{(50)(0.026)}{0.98} = 1.33 \text{ k}\Omega$$

$$g_{m1} = \frac{0.98}{0.026} = 37.7 \text{ mA/V}$$

$$r_{\pi 2} = \frac{(50)(0.026)}{3.19} = 0.408 \text{ k}\Omega$$

$$g_{m2} = \frac{3.19}{0.026} = 123 \text{ mA/V}$$



$$I_S = \frac{V_{\pi 1}}{R_1 \| R_2 \| r_{\pi 1}} + \frac{V_{\pi 1} - V_1}{R_F} \qquad (1)$$

$$g_{m1}V_{\pi 1} + \frac{V_{\pi 2}}{r_{\pi 2}} + \frac{V_{\pi 2} + V_{e2}}{R_{C1}} = 0 \qquad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}} + g_{m2}V_{\pi 2} = \frac{V_{e2} - V_1}{R_{E1}} \qquad (3)$$

$$\frac{V_{e2} - V_{\pi 1}}{R_{E1}} = \frac{V_1}{R_{E2}} + \frac{V_1 - V_{\pi 1}}{R_F} \qquad (4)$$

Enter numerical values in (1), (2), (3) and (4):

$$I_S = \frac{V_{\pi 1}}{17.9 \| 1.4 \| 1.33} + \frac{V_{\pi 1} - V_1}{5}$$

or

$$I_S = V_{\pi 1}(1.722) - V_1(0.20) \qquad (1)$$

$$(37.7)V_{\pi 1} + \frac{V_{\pi 2}}{0.408} + \frac{V_{\pi 2} + V_{e2}}{7} = 0$$

or

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + V_{e2}(0.1429) = 0 \qquad (2)$$

$$\frac{V_{\pi 2}}{0.408} + (123)V_{\pi 2} = \frac{V_{e2} - V_1}{0.25}$$

or

$$V_{\pi 2}(125.5) = V_{e2}(4) - V_1(4) \qquad (3)$$

$$\frac{V_{e2} - V_1}{0.25} = \frac{V_1}{0.50} + \frac{V_1 - V_{\pi 1}}{5}$$

or

$$V_{e2}(4) = V_1(6.20) - V_{\pi 1}(0.20) \qquad (4)$$

From (4):

$$V_{e2} = V_1(1.55) - V_{\pi 1}(0.05)$$

Then substituting in (3):

$$V_{\pi 2}(125.5) = (4)[V_1(1.55) - V_{\pi 1}(0.05)] - V_1(4)$$

or

$$V_{\pi 2}(125.5) = V_1(2.20) - V_{\pi 1}(0.20) \qquad (3')$$

and substituting in (2):

$$V_{\pi 1}(37.7) + V_{\pi 2}(2.594) + (0.1429)\left[V_1(1.55) - V_{\pi 1}(0.05)\right] = 0$$

or

$$V_{\pi 1}(37.69) + V_{\pi 2}(2.594) + V_1(0.2215) = 0$$

Now

$$V_1 = -V_{\pi 1}(170.16) - V_{\pi 2}(11.71)$$

Then substituting in (1):

$$I_S = V_{\pi 1}(1.722) - (0.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)]$$

or

$$I_S = V_{\pi 1}(35.75) + V_{\pi 2}(2.342)$$

and substituting in (3′):

$$V_{\pi 2}(125.5) = (2.20)[-V_{\pi 1}(170.16) - V_{\pi 2}(11.71)] - V_{\pi 1}(0.20)$$

$$V_{\pi 2}(151.3) = -V_{\pi 1}(374.55)$$

or $V_{\pi 1} = -V_{\pi 2}(0.4040)$ so that

Then

$$I_S = (35.75)[-V_{\pi 2}(0.4040)] + V_{\pi 2}(2.342)$$

$$I_S = -V_{\pi 2}(12.10)$$

$$I_0 = -(g_{m2}V_{\pi 2})\left(\frac{R_{C2}}{R_{C2} + R_L}\right)$$

$$= -(123)\left(\frac{2.2}{2.2+2}\right)V_{\pi 2} = -(64.43)V_{\pi 2}$$

or $V_{\pi 2} = -(0.01552)I_0$

Then

$$\frac{I_0}{I_S} = \frac{1}{(0.01552)(12.10)} \Rightarrow \underline{\frac{I_0}{I_S} = 5.33}$$

12.54

For example, use the circuit shown in Figure P12.49

12.55

(a) $\dfrac{V_i - V_L}{R_1} = \dfrac{V_L - V_O}{R_F}$

$$\frac{V_O}{R_F} = V_L\left(\frac{1}{R_1} + \frac{1}{R_F}\right) - \frac{V_i}{R_1}$$

So (1) $V_O = V_L\left(1 + \dfrac{R_F}{R_1}\right) - \left(\dfrac{R_F}{R_1}\right)V_i$

$$\frac{V_O - V_L}{R_3} = \frac{V_L}{R_L} + \frac{V_L}{R_2}$$

So (2) $V_O = V_L\left(1 + \dfrac{R_3}{R_L} + \dfrac{R_3}{R_2}\right)$

Then, from (1) = (2)

$$V_L\left(1 + \frac{R_F}{R_1}\right) - \left(\frac{R_F}{R_1}\right)V_i = V_L\left(1 + \frac{R_3}{R_L} + \frac{R_3}{R_2}\right)$$

$$V_L\left(1 + \frac{R_F}{R_1} - 1 - \frac{R_3}{R_L} - \frac{R_3}{R_2}\right) = \left(\frac{R_F}{R_1}\right)V_i$$

Now $V_L = I_O R_L$

$$I_O R_L\left(\frac{R_F}{R_1} - \frac{R_3}{R_L} - \frac{R_3}{R_2}\right) = \left(\frac{R_F}{R_1}\right)V_i$$

$$A_{gf} = \frac{I_O}{V_i} = \frac{\dfrac{R_F}{R_1}}{\left(\dfrac{R_L R_F}{R_1} - R_3 - \dfrac{R_L R_3}{R_2}\right)}$$

(b) Set $\dfrac{R_F}{R_1} = \dfrac{R_3}{R_2}$

Then $A_{gf} = \dfrac{R_F / R_1}{-R_3} = \dfrac{-R_F}{R_1 R_3}$

Or $A_{gf} = \dfrac{I_O}{V_i} = \dfrac{-1}{R_2}$

(c) For $\dfrac{I_O}{V_i} = -0.5 \text{ mA/V} = \dfrac{-1}{R_2} \Rightarrow R_2 = 2 \text{ k}\Omega$

For $\dfrac{R_F}{R_1} = \dfrac{R_3}{R_2}$, set $R_3 = 2 \text{ k}\Omega$ and $R_1 = R_F = 10 \text{ k}\Omega$

12.56

(a) $V_{D1} = 3 - (1)(1.6) = 1.40 \text{ V}, \quad V_{SG3} = (1)(1.6) = 1.6 \text{ V}$

$I_{DQ3} = K_p (V_{SG3} + V_{TP})^2 = (10)(1.6 - 0.5)^2 = 12.1 \text{ mA}$

$V_G = (12.1)(0.248) - 3 \cong 0$

(b) $g_{m1} V_{gs1} + g_{m1} V_{gs2} = 0 \Rightarrow V_{gs2} = -V_{gs1}$

$V_{sg3} = g_{m1} V_{gs1} R_D$

$I_o = g_{m3} V_{sg3} = g_{m1} g_{m3} R_D V_{gs1}$

$V_i = V_{gs1} - V_{gs2} + I_o R_L = 2V_{gs1} + I_o R_L$

$V_{gs1} = \dfrac{V_i - I_o R_L}{2}$

$I_o = g_{m1} g_{m3} R_D \left(\dfrac{V_i - I_o R_L}{2}\right)$

$I_o \left(1 + \dfrac{g_{m1} g_{m3} R_D R_L}{2}\right) = \dfrac{g_{m1} g_{m3} R_D}{2} \cdot V_i$

$A_{gf} = \dfrac{I_o}{V_i} = \dfrac{g_{m1} g_{m3} R_D}{2 + g_{m1} g_{m3} R_D R_L}$

(c) $g_{m1} = 2\sqrt{K_n I_{DQ1}} = 2\sqrt{(2)(1)} = 2.828 \text{ mA/V}$

$g_{m3} = 2\sqrt{K_p I_{DQ3}} = 2\sqrt{(10)(12.1)} = 22 \text{ mA/V}$

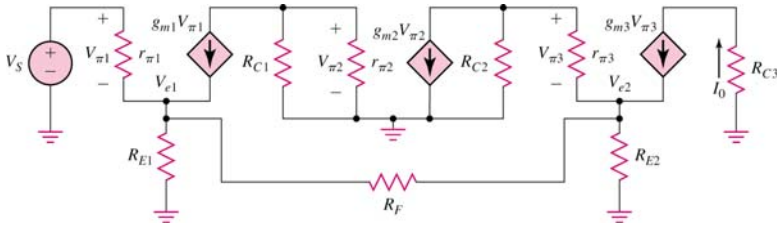
$A_{gf} = \dfrac{(2.828)(22)(1.6)}{2 + (2.828)(22)(1.6)(0.248)} = 3.73 \text{ mA/V}$

12.57

$r_{\pi 1} = 6.24\ \text{k}\Omega,\ r_{\pi 2} = 3.12\ \text{k}\Omega,\ r_{\pi 3} = 1.56\ \text{k}\Omega$

$g_{m1} = 19.23\,\text{mA/V},\ g_{m2} = 38.46\ \text{mA/V},$

$g_{m3} = 76.92\,\text{mA/V}$



$$V_S = V_{\pi 1} + V_{e1} \qquad (1)$$

$$\frac{V_{\pi 1}}{r_{\pi 1}} + g_{m1}V_{\pi 1} = \frac{V_{e1}}{R_{E1}} + \frac{V_{e1} - V_{e3}}{R_F} \qquad (2)$$

$$V_{\pi 2} = -g_{m1}V_{\pi 1}(R_{C1} \,||\, r_{\pi 2}) \qquad (3)$$

$$g_{m2}V_{\pi 2} + \frac{V_{\pi 3} + V_{e3}}{R_{C2}} + \frac{V_{\pi 3}}{r_{\pi 3}} = 0 \qquad (4)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}} + g_{m3}V_{\pi 3} = \frac{V_{e3}}{R_{E2}} + \frac{V_{e3} - V_{e1}}{R_F} \qquad (5)$$

Enter numerical values in (2)-(5):

$$\frac{V_{\pi 1}}{6.24} + (19.23)V_{\pi 1} = V_{e1}\left(\frac{1}{0.1} + \frac{1}{0.8}\right) - V_{e3}\left(\frac{1}{0.8}\right)$$

or

$$V_{\pi 1}(19.39) = V_{e1}(11.25) - V_{e3}(1.25) \qquad (2)$$

$$V_{\pi 2} = -(19.23)V_{\pi 1}(5 \,||\, 3.12) = -(36.94)V_{\pi 1} \qquad (3)$$

$$(38.46)V_{\pi 2} + V_{\pi 3}\left(\frac{1}{2} + \frac{1}{1.56}\right) + V_{e3}\left(\frac{1}{2}\right) = 0$$

or

$$V_{\pi 2}(38.46) + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \qquad (4)$$

$$V_{\pi 3}\left(\frac{1}{1.56} + 76.92\right) = V_{e3}\left(\frac{1}{0.1} + \frac{1}{0.8}\right) - V_{e3}\left(\frac{1}{0.8}\right)$$

or

$$V_{\pi 3}(77.56) = V_{e3}(11.25) - V_{e1}(1.25) \qquad (5)$$

From (1) $V_{\pi 1} = V_S - V_{e1}$

Then

$$(V_S - V_{e1})(19.39) = V_{e1}(11.25) - V_{e3}(1.25)$$

or

$$V_S(19.39) = V_{e1}(30.64) - V_{e3}(1.25) \qquad (2')$$

$$V_{\pi 2} = -V_S(36.94) + V_{e1}(36.94) \qquad (3')$$

$$(38.46)[-V_S(36.94) + V_{e1}(36.94)] + V_{\pi 3}(1.141) + V_{e3}(0.5) = 0 \qquad (4')$$

From (5): $V_{e3} = V_{\pi 3}(6.894) + V_{e1}(0.1111)$

Then

$$V_S(19.39) = V_{e1}(30.64) - (1.25)[V_{\pi 3}(6.894) + V_{e1}(0.1111)]$$

or

$$V_S(19.39) = V_{e1}(30.50) - V_{\pi 3}(8.6175) \qquad (2'')$$

---

and

$$-V_S(1420.7)+V_{e1}(1420.7)+V_{\pi 3}(1.141)+(0.5)[V_{\pi 3}(6.894)+V_{e1}(0.1111)]=0$$

or

$$-V_S(1420.7)+V_{e1}(1420.76)+V_{\pi 3}(4.588)=0 \qquad (4'')$$

From (2″):

$$V_{e1}=V_S(0.6357)+V_{\pi 3}(0.2825)$$

Then substituting in (4″):

$$-V_S(1420.7)+(1420.76)[V_S(0.6357)+V_{\pi 3}(0.2825)]+V_{\pi 3}(4.588)=0$$

$$-V_S(517.5)+V_{\pi 3}(405.95)=0$$

Now

$$I_0=g_{m3}V_{\pi 3}=76.92V_{\pi 3} \quad \text{or} \quad V_{\pi 3}=I_0(0.0130)$$

Then $-V_S(517.5)+I_0(0.0130)(405.95)=0$

or

$$\frac{I_0}{V_S}=98.06 \text{ mA/V}$$

12.58

Computer Analysis

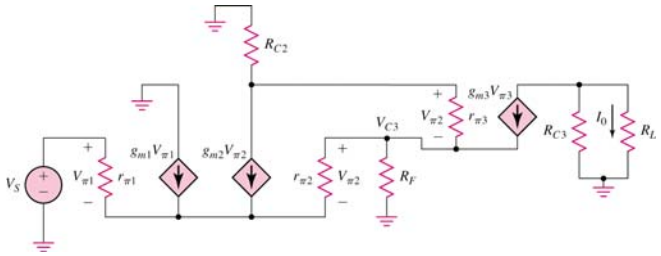
12.59

$$r_{\pi 1}=r_{\pi 2}=\frac{(100)(0.026)}{0.5}=5.2 \text{ k}\Omega$$

$$g_{m1}=g_{m2}=\frac{0.5}{0.026}=19.23 \text{ mA/V}$$

$$r_{\pi 3}=\frac{(100)(0.026)}{2}=1.3 \text{ k}\Omega$$

$$g_{m3}=\frac{2}{0.026}=76.92 \text{ mA/V}$$



$$\frac{V_{\pi 1}}{r_{\pi 1}}+g_{m1}V_{\pi 1}+g_{m2}V_{\pi 2}+\frac{V_{\pi 2}}{r_{\pi 2}}=0 \qquad (1)$$

Since $r_{\pi 1}=r_{\pi 2}$ and $g_{m1}=g_{m2}$, then $V_{\pi 1}=-V_{\pi 2}$

$$V_S=V_{\pi 1}-V_{\pi 2}+V_{e3}=-2V_{\pi 2}+V_{e3} \qquad (2)$$

$$g_{m2}V_{\pi 2}+\frac{V_{\pi 3}}{r_{\pi 3}}+\frac{V_{\pi 3}+V_{e3}}{R_{C2}}=0 \qquad (3)$$

$$\frac{V_{\pi 3}}{r_{\pi 3}}+g_{m3}V_{\pi 3}=\frac{V_{e3}}{R_F}+\frac{V_{\pi 2}}{r_{\pi 2}} \qquad (4)$$

$$I_0 = -\left(\frac{R_{C3}}{R_{C3}+R_L}\right)(g_{m3}V_{\pi 3}) \qquad (5)$$

From (2): $V_{e3} = V_S + 2V_{\pi 2}$

$$V_{\pi 2} + \frac{V_{\pi 3}}{1.3} + \frac{V_{\pi 3}}{18.6} + \frac{1}{18.6}(V_S + 2V_{\pi 2}) = 0 \qquad (19.23)$$

or

$$(19.23)V_{\pi 2} + (0.8230)V_{\pi 3} + (0.05376)V_S = 0 \qquad (3')$$

$$V_{\pi 3}\left(\frac{1}{1.3} + 76.92\right) = \left(\frac{1}{10}\right)(V_S + 2V_{\pi 2}) + \frac{V_{\pi 2}}{5.2}$$

or

$$(77.69)V_{\pi 3} = (0.3923)V_{\pi 2} + (0.1)V_S \qquad (4')$$

$$I_0 = -\left(\frac{2}{2+1}\right)(76.92)V_{\pi 3} = -(51.28)V_{\pi 3} \qquad (5')$$

From (3′):

$$V_{\pi 2} = -(0.04255)V_{\pi 3} - (0.002780)V_S$$

Then

$$(77.69)V_{\pi 3} = (0.3923)[-(0.04255)V_{\pi 3} - (0.002780)V_S] + (0.1)V_S$$

$$(77.71)V_{\pi 3} = (0.0989)V_S$$

or

$$V_{\pi 3} = (0.001273)V_S$$

so that

$$I_0 = -(51.28)(0.001273)V_S$$

or

$$\frac{I_0}{V_S} = -(0.0653)\ \text{mA/V}$$

12.60

(a) $A_{gf} = \dfrac{I_o}{V_i} = \dfrac{1}{R_E} = 3\ \text{mA/V} \Rightarrow R_E = 333.3\,\Omega$

(b) $A_{gf} = \dfrac{I_o}{V_i} = \dfrac{h_{FE}A_g}{1+(h_{FE})A_g R_E} = \dfrac{(80)(1000)}{1+(80)(1000)(0.33333)} = 2.9998875\ \text{mA/V}$

$I_o = A_{gf}(5) = 14.9994377$

$\text{Error} = \dfrac{14.9994377 - 15.0}{15.0} \times 100\% = -0.00375\%$

12.61

(a) $5 = (1+h_{FE})I_{BQ}R_E + V_{EB}(on) + I_{BQ}R_B$

$I_{BQ} = \dfrac{5-0.7}{100+(81)(0.5)} = 0.030605\ \text{mA}$

$I_{CQ} = 2.448\ \text{mA},\ \text{and}\ \ I_{EQ} = 2.479\ \text{mA}$

$V_{ECQ} = 5 - (2.479)(0.5) - (2.448)(1) = 1.31\,\text{V}$

(b) $\dfrac{V_o}{R_C}+\dfrac{V_o}{r_o}+h_{FE}I_b+\dfrac{V_o-V_\pi}{R_F}=0$

$$V_o\left(\frac{1}{R_C}+\frac{1}{r_o}+\frac{1}{R_F}\right)+h_{FE}I_b-\frac{V_\pi}{R_F}=0$$

$$V_\pi=I_b r_\pi$$

$$I_s=\frac{V_\pi}{R_B}+\frac{V_\pi}{r_\pi}+\frac{V_\pi-V_o}{R_F}$$

$$I_s=I_b\left(1+\frac{r_\pi}{R_B}+\frac{r_\pi}{R_F}\right)-\frac{V_o}{R_F}$$

$$I_b=\frac{I_s+\dfrac{V_o}{R_F}}{\left(1+\dfrac{r_\pi}{R_B}+\dfrac{r_\pi}{R_F}\right)}$$

$$V_o\left(\frac{1}{R_C}+\frac{1}{r_o}+\frac{1}{R_F}\right)+\left(h_{FE}-\frac{r_\pi}{R_F}\right)\left[\frac{I_s+\dfrac{V_o}{R_F}}{1+\dfrac{r_\pi}{R_B}+\dfrac{r_\pi}{R_F}}\right]=0$$

$$V_o\left(\frac{1}{R_C}+\frac{1}{r_o}+\frac{1}{R_F}\right)\left(1+\frac{r_\pi}{R_B}+\frac{r_\pi}{R_F}\right)+\left(h_{FE}-\frac{r_\pi}{R_F}\right)\cdot I_s+\left(h_{FE}-\frac{r_\pi}{R_F}\right)\cdot\frac{V_o}{R_F}=0$$

Now $r_o=\dfrac{V_A}{I_{CQ}}=\dfrac{100}{2.448}=40.85\,\text{k}\Omega$

$$r_\pi=\frac{h_{FE}V_T}{I_{CQ}}=\frac{(80)(0.026)}{2.448}=0.8497\,\text{k}\Omega$$

Then $V_o\left(\dfrac{1}{1}+\dfrac{1}{40.85}+\dfrac{1}{10}\right)\left(1+\dfrac{0.8497}{100}+\dfrac{0.8497}{10}\right)+\left(80-\dfrac{0.8497}{10}\right)\cdot I_s+\left(80-\dfrac{0.8497}{10}\right)\cdot\dfrac{V_o}{10}=0$

$$V_o(9.22114)+I_s(79.915)=0$$

$$A_{zf}=\frac{V_o}{I_s}=-8.666\ \text{V/mA}$$

(c) $R_{if}=\dfrac{V_\pi}{I_s}=\dfrac{I_b r_\pi}{I_s}=r_\pi\left(\dfrac{1+\dfrac{V_o}{I_s}\cdot\dfrac{1}{R_F}}{1+\dfrac{r_\pi}{R_B}+\dfrac{r_\pi}{R_F}}\right)=(0.8497)\left(\dfrac{1-\dfrac{8.666}{10}}{1+\dfrac{0.8497}{100}+\dfrac{0.8497}{10}}\right)$

$$R_{if}=0.1037\,\text{k}\Omega$$

(d) $I_x=\dfrac{V_x}{R_C}+\dfrac{V_x}{r_o}+h_{FE}I_b+\dfrac{V_x}{R_F+R_B\|r_\pi}$

$I_b=\dfrac{V_\pi}{r_\pi}$, and $V_\pi=\left(\dfrac{R_B\|r_\pi}{R_B\|r_\pi+R_F}\right)\cdot V_x$

Then $\dfrac{I_x}{V_x}=\dfrac{1}{R_{of}}=\dfrac{1}{R_C}+\dfrac{1}{r_o}+\left(\dfrac{h_{FE}}{r_\pi}\right)\left(\dfrac{R_B\|r_\pi}{R_B\|r_\pi+R_F}\right)+\dfrac{1}{R_B\|r_\pi+R_F}$

$$R_B \| r_\pi = 100 \| 0.8497 = 0.8425 \ \text{k}\Omega$$

$$\frac{1}{R_{of}} = \frac{1}{1} + \frac{1}{40.85} + \left(\frac{80}{0.8497}\right)\left(\frac{0.8425}{0.8425+10}\right) + \frac{1}{0.8425+10}$$

Then $R_{of} = 0.1186 \ \text{k}\Omega$

12.62

(a) (i) $V^+ = I_{DQ}R_D + V_{GS} = K_n R_D (V_{GS} - V_{TN})^2 + V_{GS}$

$$3 = (0.5)(5)(V_{GS}^2 - 0.8V_{GS} + 0.16) + V_{GS}$$

We find $2.5V_{GS}^2 - V_{GS} - 2.6 = 0 \Rightarrow V_{GS} = 1.239$ V

$$I_{DQ} = (0.5)(1.239 - 0.4)^2 = 0.3522 \ \text{mA}$$

(ii) $g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.3522)} = 0.8392$ mA/V

(b) (1) $\dfrac{V_o}{R_D} + g_m V_{gs} + \dfrac{V_o - V_{gs}}{R_F} = 0$

(2) $I_s = \dfrac{V_{gs} - V_o}{R_F} \Rightarrow V_{gs} = I_s R_F + V_o$

Then $V_o\left(\dfrac{1}{R_D} + \dfrac{1}{R_F}\right) + \left(g_m - \dfrac{1}{R_F}\right)(I_s R_F + V_o) = 0$

$$V_o\left(\frac{1}{R_D} + g_m\right) + I_s(g_m R_F - 1) = 0$$

$$A_{zf} = \frac{V_o}{I_s} = \frac{1 - g_m R_F}{\dfrac{1}{R_D} + g_m} = \frac{1 - (0.8392)(25)}{\dfrac{1}{5} + 0.8392}$$

$$A_{zf} = -19.23 \ \text{V/mA}$$

(c) $I_x = \dfrac{V_x}{R_D} + g_m V_{gs}$ and $V_{gs} = V_x$

$$\frac{I_x}{V_x} = \frac{1}{R_{of}} = \frac{1}{R_D} + g_m = \frac{1}{5} + 0.8392$$

$$R_{of} = 0.962 \ \text{k}\Omega$$

12.63

(a) $$A_{zf} = \frac{1 - g_m R_F}{\dfrac{1}{R_D} + g_m} = \frac{\dfrac{1}{g_m} - R_F}{\dfrac{1}{g_m R_D} + 1}$$

As $g_m \to \infty$, $A_{zf} = -R_F$

(b) $-0.95R_F = \dfrac{1-g_m R_F}{\dfrac{1}{R_D}+g_m}$

$$-23.75 = \frac{1-g_m(25)}{0.2+g_m}$$

So that $g_m = 4.6\,\text{mA/V}$

12.64

dc analysis

$$R_{TH} = 24 \,||\, 150 = 20.7\ \text{k}\Omega$$

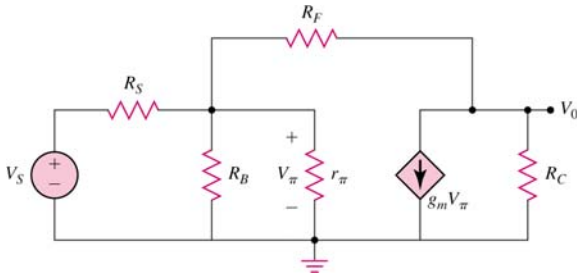
$$V_{TH} = \left(\frac{24}{24+150}\right)(12) = 1.655\ \text{V}$$

$$I_{BQ} = \frac{1.655-0.7}{20.7+(151)(1)} = 0.00556\ \text{mA}$$

so $I_{CQ} = 0.834\ \text{mA}$

$$r_\pi = \frac{(150)(0.026)}{0.834} = 4.68\ \text{k}\Omega$$

$$g_m = \frac{0.834}{0.026} = 32.08\ \text{mA/V}$$



$$\frac{V_S - V_\pi}{R_S} = \frac{V_\pi}{R_B \,||\, r_\pi} + \frac{V_\pi - V_0}{R_F} \qquad (1)$$

$$g_m V_\pi + \frac{V_0}{R_C} + \frac{V_0 - V_\pi}{R_F} = 0 \qquad (2)$$

From (1):

$$\frac{V_S}{5} = V_\pi\left[\frac{1}{5} + \frac{1}{20.7 \,||\, 4.68} + \frac{1}{R_F}\right] - \frac{V_0}{R_F}$$

or

$$V_S(0.20) = V_\pi\left(0.4620 + \frac{1}{R_F}\right) - \frac{V_0}{R_F}$$

From (2):

$$\left(32.08 - \frac{1}{R_F}\right)V_\pi + V_0\left(\frac{1}{6} + \frac{1}{R_F}\right) = 0$$

so

$$V_\pi = \frac{-V_0\left(0.1667 + \dfrac{1}{R_F}\right)}{\left(32.08 - \dfrac{1}{R_F}\right)} \qquad (2)$$

Then

$$V_S(0.20) = \left(0.4620 + \frac{1}{R_F}\right)\left[\frac{-V_0\left(0.1667 + \frac{1}{R_F}\right)}{\left(32.08 - \frac{1}{R_F}\right)}\right] - \frac{V_0}{R_F}$$

Neglect the R_F in the denominator term. Now

$$\frac{V_0}{V_S} = -5 \Rightarrow V_S = -\frac{V_0}{5} = -V_0(0.20)$$

$$-V_0(0.20)(0.20)R_F = (0.4620R_F + 1)\left[\frac{-V_0\left(0.1667R_F + 1\right)}{32.08R_F}\right] - V_0$$

$$-1.283R_F^2 = -(0.4620R_F + 1)(0.1667R_F + 1) - 32.08R_F$$

$$1.206R_F^2 - 32.71R_F - 1 = 0$$

$$R_F = \frac{32.71 \pm \sqrt{(32.71)^2 + 4(1.206)(1)}}{2(1.206)}$$

so that

$$\underline{R_F = 27.2\ \text{k}\Omega}$$

12.65

dc analysis

$$R_{TH} = 4 \,||\, 15 = 3.16\ \text{k}\Omega = R_B$$

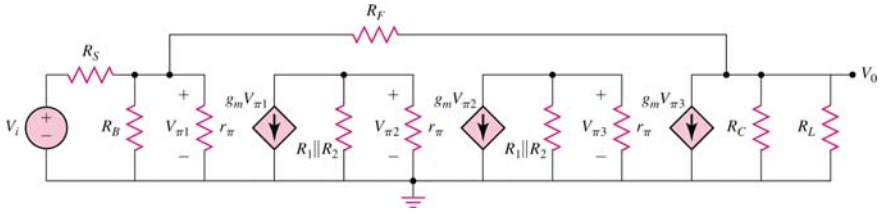
$$V_{TH} = \left(\frac{4}{4+15}\right)12 = 2.526\ \text{V}$$

$$I_{BQ} = \frac{2.526 - 0.7}{3.16 + (181)(4)} = 0.00251$$

$$I_{CQ} = 0.452\ \text{mA}$$

$$r_\pi = \frac{(180)(0.026)}{0.452} = 10.4\ \text{k}\Omega$$

$$g_m = \frac{0.452}{0.026} = 17.4\ \text{mA/V}$$



$$\frac{V_i - V_{\pi 1}}{R_S} = \frac{V_{\pi 1}}{R_B \,||\, r_\pi} + \frac{V_{\pi 1} - V_0}{R_F} \qquad (1)$$

$$g_m V_{\pi 1} + \frac{V_{\pi 2}}{R_C \,||\, R_B \,||\, r_\pi} = 0 \qquad (2)$$

$$g_m V_{\pi 2} + \frac{V_{\pi 3}}{R_C \,||\, R_B \,||\, r_\pi} = 0 \qquad (3)$$

$$g_m V_{\pi 3} + \frac{V_0}{R_C} + \frac{V_0}{R_L} + \frac{V_0 - V_{\pi 1}}{R_F} = 0 \qquad (4)$$

Now

$$R_C \parallel R_B \parallel r_\pi = 8 \parallel 3.16 \parallel 10.4 = 1.86 \text{ k}\Omega$$

$$R_B \parallel r_\pi = 3.16 \parallel 10.4 = 2.42 \text{ k}\Omega$$

Now substituting in (2):

$$(17.4)V_{\pi 1} + \frac{V_{\pi 2}}{1.86} = 0 \quad \text{or} \quad V_{\pi 2} = -(32.36)V_{\pi 1}$$

and substituting in (3):

$$(17.4)V_{\pi 2} + \frac{V_{\pi 3}}{1.86} = 0$$

$$(17.4)[-(32.36)V_{\pi 1}] + \frac{V_{\pi 3}}{1.86} = 0$$

or $V_{\pi 3} = (1047.3)V_{\pi 1}$

Substitute numerical values in (1):

$$\frac{V_i}{10} = V_{\pi 1}\left(\frac{1}{10} + \frac{1}{2.42} + \frac{1}{R_F}\right) - \frac{V_0}{R_F}$$

or

$$V_i(0.10) = V_{\pi 1}\left(0.513 + \frac{1}{R_F}\right) - \frac{V_0}{R_F}$$

Substitute numerical values in (4):

$$(17.4)(1047.3)V_{\pi 1} + V_0\left(\frac{1}{8} + \frac{1}{4} + \frac{1}{R_F}\right) - \frac{V_{\pi 1}}{R_F} = 0$$

$$V_{\pi 1}\left(1.822\times 10^4 - \frac{1}{R_F}\right) + V_0\left(0.375 + \frac{1}{R_F}\right) = 0$$

$$V_{\pi 1} = \frac{-V_0\left(0.375 + \frac{1}{R_F}\right)}{1.822\times 10^4 - \frac{1}{R_F}}$$

so that

$$V_i(0.10) = \left(0.513 + \frac{1}{R_F}\right)\left[\frac{-V_0\left(0.375 + \frac{1}{R_F}\right)}{1.822\times 10^4 - \frac{1}{R_F}}\right] - \frac{V_0}{R_F}$$

We have $\frac{V_0}{V_i} = -80$ or $V_i = -\frac{V_0}{80}$

$$-\frac{(0.10)}{80} = \left(0.513 + \frac{1}{R_F}\right)\left[\frac{-\left(0.375 + \frac{1}{R_F}\right)}{1.822\times 10^4 - \frac{1}{R_F}}\right] - \frac{1}{R_F}$$

Neglect that $1/R_F$ term in the denominator.

$$-(0.00125R_F) = -\frac{(0.513R_F + 1)(0.375R_F + 1)}{1.822\times 10^4 R_F} - 1$$

$$22.775R_F^2 = (0.513R_F + 1)(0.375R_F + 1) + 1.822\times 10^4 R_F$$

We find

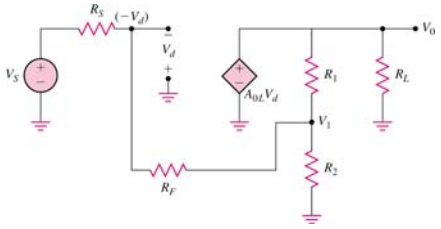
$$22.58R_F^2 - 1.822\times 10^4 R_F - 1 = 0$$

$$R_F = \frac{1.822\times10^4 \pm \sqrt{(1.822\times10^4)^2 + 4(22.58)(1)}}{2(22.58)}$$

or

$$\underline{R_F = 0.807\ \text{M}\Omega}$$

12.66



a.

$$\frac{V_S-(-V_d)}{R_S} = \frac{-V_d - V_1}{R_F}$$

or

$$V_d\left(\frac{1}{R_S}+\frac{1}{R_F}\right)+\frac{V_S}{R_S}+\frac{V_1}{R_F}=0 \qquad (1)$$

$$\frac{V_0-V_1}{R_1} = \frac{V_1}{R_2}+\frac{V_1-(-V_d)}{R_F}$$

or

$$\frac{V_0}{R_1} = V_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_F}\right)+\frac{V_d}{R_F} \qquad (2)$$

and $V_0 = A_{0L}V_d$ or $V_d = \dfrac{V_0}{A_{0L}}$

Substitute numerical values in (1) and (2):

$$\frac{V_0}{10^4}\cdot\left(\frac{1}{5}+\frac{1}{10}\right)+\frac{V_S}{5}+\frac{V_1}{10}=0$$

or

$$V_0(0.3\times10^{-4})+V_S(0.20)+V_1(0.10)=0 \qquad (1)$$

$$\frac{V_0}{50} = V_1\left(\frac{1}{50}+\frac{1}{10}+\frac{1}{10}\right)+\frac{V_0}{10^4}\cdot\left(\frac{1}{10}\right)$$

or

$$V_0(0.02-10^{-5}) = V_1(0.22) \qquad (2)$$

Then $V_1 = V_0\left(\dfrac{0.02-10^{-5}}{0.22}\right)$

and

$$V_0(0.3\times10^{-4})+V_S(0.20)+(0.10)\left[V_0\left(\frac{0.02-10^{-5}}{0.22}\right)\right]=0$$

$$V_0\left[0.3\times10^{-4}-0.4545\times10^{-5}+0.00909\right]+V_S(0.20)=0$$

Then $\dfrac{V_0}{V_S} = \dfrac{-0.20}{9.115\times10^{-3}} \Rightarrow \underline{\dfrac{V_0}{V_S} = -21.94}$

$$R_{if} = \frac{-V_d}{\dfrac{V_S - (-V_d)}{R_S}} = \frac{-V_d \cdot R_S}{V_S + V_d}$$

b.

$$V_d = \frac{V_0}{A_{0L}} = -\frac{21.94 V_S}{10^4}$$

Now

$$R_{if} = \frac{(21.94 \times 10^{-4})(5)}{1 - 21.94 \times 10^{-4}}$$

Then

$$R_{if} = 1.099 \times 10^{-2}\ \text{k}\Omega \Rightarrow \underline{R_{if} = 10.99\ \Omega}$$

or

c. Because of the $A_{0L} V_d$ source,

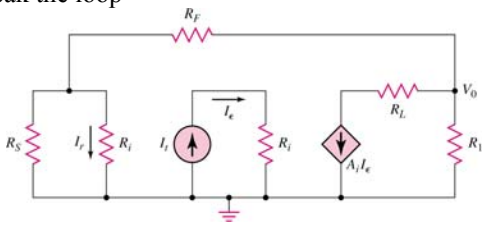
$$\underline{R_{0f} = 0}$$

12.67

For example, use the circuit shown in Figure 12.41

12.68

Break the loop



$$I_t = I_\varepsilon$$

$$A_i I_t + \frac{V_0}{R_1} + \frac{V_0}{R_F + R_S \| R_i} = 0$$

Now

$$I_r = \left(\frac{R_S}{R_S + R_i}\right) \cdot \frac{V_0}{R_F + R_S \| R_i}$$

$$V_0 = I_r \left(\frac{R_S + R_i}{R_S}\right) \cdot (R_F + R_S \| R_i)$$

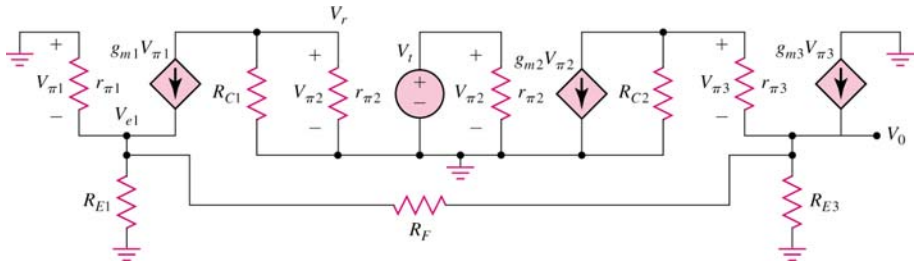
or

Then

$$A_i I_t + \left(\frac{1}{R_1} + \frac{1}{R_F + R_S \| R_i}\right) \times \left[I_r \left(\frac{R_S + R_i}{R_S}\right) (R_F + R_S \| R_i) \right] = 0$$

$$T = -\frac{I_r}{I_t} \Rightarrow T = \underline{\frac{A_i}{\left[\dfrac{1}{R_1} + \dfrac{1}{R_F + R_S \| R_i}\right] \left(\dfrac{R_S + R_i}{R_S}\right) (R_F + R_S \| R_i)}}$$

12.69



$$\frac{V_{\pi1}}{r_{\pi1}} + g_{m1}V_{\pi1} = \frac{V_{\varepsilon1}}{R_{E1}} + \frac{V_{\varepsilon1} - V_0}{R_F} \quad (1)$$

$$g_{m1}V_{\pi1} + \frac{V_r}{R_{C1} \| r_{\pi2}} = 0 \Rightarrow V_r = -(g_{m1}V_{\pi1})(R_{C1} \| r_{\pi2}) \quad (2)$$

$V_{\pi2} = V_t$ so that

$$g_{m2}V_t + \frac{V_{\pi3} + V_0}{R_{C2}} + \frac{V_{\pi3}}{r_{\pi3}} = 0 \quad (3)$$

$$\frac{V_{\pi3}}{r_{\pi3}} + g_{m3}V_{\pi3} = \frac{V_0}{R_{E3}} + \frac{V_0 - V_{\varepsilon1}}{R_F} \quad (4)$$

From (4):

$$V_0\left(\frac{1}{R_{E3}} + \frac{1}{R_F}\right) = V_{\pi3}\left(\frac{1}{r_{\pi3}} + g_{m3}\right) + \frac{V_{\varepsilon1}}{R_F}$$

But $V_{\varepsilon1} = -V_{\pi1}$

$$V_0 = \frac{V_{\pi3}\left(\frac{1}{r_{\pi3}} + g_{m3}\right) - \frac{V_{\pi1}}{R_F}}{\left(\frac{1}{R_{E3}} + \frac{1}{R_F}\right)}$$

so

Then

$$V_{\pi1}\left[\left(\frac{1}{r_{\pi1}} + g_{m1}\right) - \left(\frac{1}{R_{E1}} + \frac{1}{R_F}\right)\right] = \frac{-V_{\pi3}\left(\frac{1}{r_{\pi3}} + g_{m3}\right) + \frac{V_{\pi1}}{R_F}}{R_F \cdot \left(\frac{1}{R_{E3}} + \frac{1}{R_F}\right)} \quad (1')$$

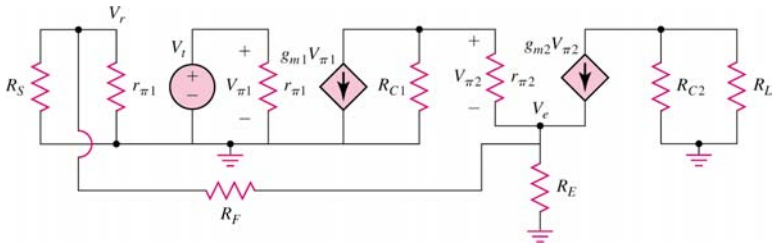
and

$$g_{m2}V_t + V_{\pi3}\left(\frac{1}{R_{C2}} + \frac{1}{r_{\pi3}}\right) + \frac{V_{\pi3}\left(\frac{1}{r_{\pi3}} + g_{m3}\right) - \frac{V_{\pi1}}{R_F}}{R_{C2} \cdot \left(\frac{1}{R_{E3}} + \frac{1}{R_F}\right)} = 0 \quad (3')$$

From (3′), solve for $V_{\pi3}$ and substitute into (1′). Then from (1′), solve for $V_{\pi1}$ and substitute into (2).

Then $T = -\frac{V_r}{V_t}$.

12.70



$$\frac{V_r}{R_S}+\frac{V_r}{r_{\pi 1}}+\frac{V_r-V_e}{R_F}=0 \qquad (1)$$

$$g_{m1}V_t+\frac{V_{\pi 2}+V_e}{R_{C1}}+\frac{V_{\pi 2}}{r_{\pi 2}}=0 \qquad (2)$$

$$\frac{V_{\pi 2}}{r_{\pi 2}}+g_{m2}V_{\pi 2}=\frac{V_e}{R_E}+\frac{V_e-V_r}{R_F} \qquad (3)$$

Assume $r_{\pi 1}=15.8\,\text{k}\Omega$, $g_{m1}=7.62$ mA/V, $r_{\pi 2}=2.28\,\text{k}\Omega$, $g_{m2}=52.7$ mA/V

$$V_r\left(\frac{1}{10}+\frac{1}{15.8}+\frac{1}{10}\right)-\frac{V_e}{10}=0$$

or

$$V_r(0.2633)=V_e(0.10) \qquad (1)$$

$$(7.62)V_t+V_{\pi 2}\left(\frac{1}{40}+\frac{1}{2.28}\right)+\frac{V_e}{40}=0$$

or

$$V_t(7.62)+V_{\pi 2}(0.4636)+V_e(0.025)=0 \qquad (2)$$

$$V_{\pi 2}\left(\frac{1}{2.28}+52.7\right)=V_e\left(\frac{1}{1}+\frac{1}{10}\right)-\frac{V_r}{10}$$

or

$$V_{\pi 2}(53.14)=V_e(1.10)-V_r(0.10)$$

Then

$$V_{\pi 2}=V_e(0.0207)-V_r(0.001882) \qquad (3)$$

Substituting in (2):

$$V_t(7.62)+(0.4636)[V_e(0.0207)-V_r(0.001882)]+V_e(0.025)=0$$

or

$$V_t(7.62)+V_e(0.03460)-V_r(0.0008725)=0$$

From (1) $V_e=V_r(2.633)$

Then

$$V_t(7.62)+V_r(2.633)(0.03460)-V_r(0.0008725)=0$$

$$V_t(7.62)+V_r(0.09023)=0$$

$$\frac{V_r}{V_t}=-84.45$$

or

Now

$$T=-\frac{V_r}{V_t}\Rightarrow \underline{T=84.45}$$

12.71

$$R_{TH} = R_1 \| R_2 = 24 \| 150 = 20.69 \text{ k}\Omega$$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{CC} = \left(\frac{24}{24+150}\right)(12) = 1.655 \text{ V}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE}(on)}{R_{TH} + (1+h_{FE})R_E} = \frac{1.655 - 0.7}{20.69 + (51)(1)} = 0.0133 \text{ mA}$$

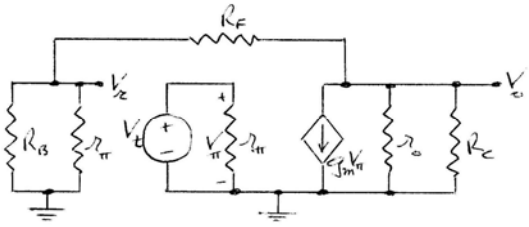
$$I_{CQ} = 0.666 \text{ mA}$$

$$g_m = \frac{0.666}{0.026} = 25.62 \text{ mA/V}$$

$$r_\pi = \frac{(50)(0.026)}{0.666} = 1.951 \text{ k}\Omega$$

$$r_o = \frac{100}{0.666} = 150 \text{ k}\Omega$$

From Problem 12.64, let $R_F = 27.2 \text{ k}\Omega$



We see that $V_\pi = V_t$. Let $R_B = R_1 \| R_2 \| R_S$

(1) $$g_m V_t + \frac{V_o}{r_o \| R_C} + \frac{V_o}{R_F + r_\pi \| R_B} = 0$$

(2) $$V_r = \left(\frac{R_B \| r_\pi}{R_B \| r_\pi + R_F}\right) \cdot V_o$$

Now $R_B = 24 \| 150 \| 5 = 20.69 \| 5 = 4.027 \text{ k}\Omega$; $r_\pi \| R_B = 1.951 \| 4.027 = 1.314 \text{ k}\Omega$

$$r_o \| R_C = 150 \| 6 = 5.769 \text{ k}\Omega$$

(1) $$25.62 V_t + V_o \left[\frac{1}{5.769} + \frac{1}{27.2 + 1.314}\right] = 0 \Rightarrow V_o = -122.93 V_t$$

(2) $$V_r = \left(\frac{1.314}{1.314 + 27.2}\right) \cdot V_o = (0.04608)(-122.93) V_t$$

$$T = -\frac{V_r}{V_t} = 5.66$$

12.72

(a) $$\phi = -\tan^{-1}\left(\frac{f}{10^3}\right) - 2\tan^{-1}\left(\frac{f}{5 \times 10^4}\right)$$

At $f_{180} = 5.1 \times 10^4$ Hz, $\phi \cong -180°$

(b) $$|T| = 0.25 = \frac{\beta(5\times10^4)}{\sqrt{1+\left(\frac{5.1\times10^4}{10^3}\right)^2}\left[1+\left(\frac{5.1\times10^4}{5\times10^4}\right)^2\right]}$$

$$0.25 = \frac{\beta(5\times10^4)}{(51.0098)(2.0404)}$$

$$\beta = 5.2\times10^{-4}$$

12.73

(a) $$\phi = -2\tan^{-1}\left(\frac{f}{5\times10^3}\right) - \tan^{-1}\left(\frac{f}{5\times10^5}\right)$$

At $f_{180} = 7.1\times10^4$ Hz, $\phi \cong -180°$

(b) $$|T| = 1 = \frac{\beta(10^4)}{\left[1+\left(\frac{7.1\times10^4}{5\times10^3}\right)^2\right]\sqrt{1+\left(\frac{7.1\times10^4}{5\times10^5}\right)^2}}$$

$$1 = \frac{\beta(10^4)}{(202.64)(1.01)} \Rightarrow \beta = 0.0205$$

(c) $$A_{vf}(0) = \frac{10^4}{1+(0.0205)(10^4)} = 48.54$$

(d) Smaller

12.74

$$\phi = -\tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{5\times10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right)$$

At $f = 8.1\times10^4$ Hz, $\phi = -180.28°$

Determine $|T(f)|$ at this frequency.

$$|T| = \beta(10^3)\times\frac{1}{\sqrt{1+\left(\frac{8.1\times10^4}{10^4}\right)^2}}\times\frac{1}{\sqrt{1+\left(\frac{8.1\times10^4}{5\times10^4}\right)^2}}\times\frac{1}{\sqrt{1+\left(\frac{8.1\times10^4}{10^5}\right)^2}}$$

$$= \frac{\beta(10^3)}{(8.161)(1.904)(1.287)}$$

a. For $\beta = 0.005$

$|T(f)| = 0.250 < 1 \Rightarrow$ Stable

b. For $\beta = 0.05$

$|T(f)| = 2.50 > 1 \Rightarrow$ Unstable

12.75

(b) Phase margin $= 80° \Rightarrow \phi = -100°$

$$\phi = -100 = -2\tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{5\times10^4}\right)$$

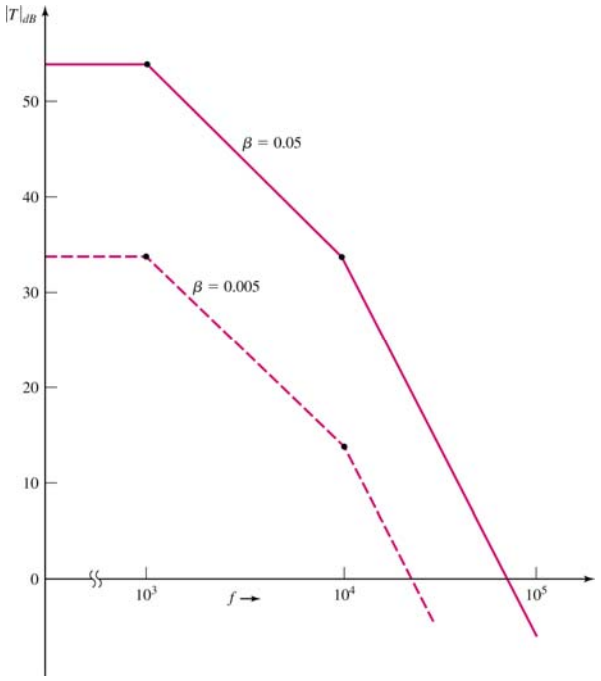
By trial and error, $f = 1.16\times10^3$ Hz

Then

$$|T| = 1 = \frac{\beta(5\times10^3)}{\left(\sqrt{1+\left(\frac{1.16\times10^3}{10^3}\right)^2}\right)^2 \cdot \sqrt{1+\left(\frac{1.16\times10^3}{5\times10^4}\right)^2}}$$

$$= \frac{\beta(5\times10^3)}{(2.35)(1.00)} \Rightarrow \underline{\beta = 4.7\times10^{-4}}$$

12.76



c. For $\beta = 0.005$,

$|T(f)| = 1 (0 \text{ dB})$ at $f \approx 2.10\times10^4$ Hz

Then

$$\phi = -\tan^{-1}\left(\frac{2.10\times10^4}{10^3}\right) - \tan^{-1}\left(\frac{2.10\times10^4}{10^4}\right) - \tan^{-1}\left(\frac{2.10\times10^4}{10^5}\right)$$

$$= -87.27 - 64.54 - 11.86$$

or

$$\phi = -163.7$$

System is stable.

Phase margin $= 16.3°$

For $\beta = 0.05$,

$|T(f)| = 1 (0 \text{ dB})$ at $f \approx 6.44\times10^4$ Hz

Then

$$\phi = -\tan^{-1}\left(\frac{6.44\times10^4}{10^3}\right) - \tan^{-1}\left(\frac{6.44\times10^4}{10^4}\right) - \tan^{-1}\left(\frac{6.44\times10^4}{10^5}\right)$$

$$= -89.11 - 81.17 - 32.78$$

or

$\phi = -203.1° \Rightarrow$ System is unstable.

12.77

(a) $\phi = -\tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{10^7}\right)$

At $f_{180} = 10^6$ Hz, $\phi \cong -180°$

(b) $\phi = -135° = -\tan\left(\frac{f_{135}}{10^3}\right) - \tan^{-1}\left(\frac{f_{135}}{10^5}\right) - \tan^{-1}\left(\frac{f_{135}}{10^7}\right)$

$f_{135} \cong 10^5$ Hz

$$|T(f_{135})| = 1 = \frac{\beta(5\times10^4)}{\sqrt{1+\left(\frac{10^5}{10^3}\right)^2}\sqrt{1+\left(\frac{10^5}{10^5}\right)^2}\sqrt{1+\left(\frac{10^5}{10^7}\right)^2}}$$

$$1 = \frac{\beta(5\times10^4)}{(100)(1.414)(1)} \Rightarrow \beta = 2.83\times10^{-3}$$

(c) $$A_{vf}(0) = \frac{5\times10^4}{1+(2.83\times10^{-3})(5\times10^4)} = 351$$

12.78

(a) $$100 = \frac{10^5}{1+(10^5)\beta} \Rightarrow \beta = 9.99 \pm 10^{-3}$$

$$|T| = \quad 1 = \beta A_v = \frac{(9.99\times10^{-3})(10^5)}{\sqrt{1+\left(\frac{f}{10^3}\right)^2}\sqrt{1+\left(\frac{f}{10^5}\right)^2}}$$

$$1 = \frac{999}{\sqrt{1+\left(\frac{f}{10^3}\right)^2}\sqrt{1+\left(\frac{f}{10^5}\right)^2}}$$

$$f = 3.08\times10^5 \text{ Hz}$$

Phase

$$\phi = -\tan^{-1}\frac{f}{10^3} - \tan^{-1}\frac{f}{10^5}$$

$$= -\tan^{-1}\frac{3.08\times10^5}{10^3} - \tan^{-1}\frac{3.08\times10^5}{10^5}$$

$$= -89.81 - 72.01$$

$$\phi = -161.8$$

Stable

(b) Phase Margin $= 180 - 161.8 = 18.2°$

12.79

(a) $\phi = -\tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{10^6}\right)$

At $f_{180} = 3.33\times10^5$ Hz, $\phi \cong -180°$

(b) (i) $$|T(f_{180})| = \frac{(0.019)(10^3)}{\sqrt{1+(33.3)^2}\sqrt{1+(3.33)^2}\sqrt{1+(0.333)^2}}$$

$$|T(f_{180})| = \frac{(0.019)(10^3)}{(33.315)(3.477)(1.054)} = 0.156$$

(ii) $$|T| = 1 = \frac{(0.019)(10^3)}{\sqrt{1+\left(\frac{f}{10^4}\right)^2}\sqrt{1+\left(\frac{f}{10^5}\right)^2}\sqrt{1+\left(\frac{f}{10^6}\right)^2}} \Rightarrow f = 1.2\times10^5 \text{ Hz}$$

$$\phi = -\tan^{-1}\left(\frac{1.2\times10^5}{10^4}\right) - \tan^{-1}\left(\frac{1.2\times10^5}{10^5}\right) - \tan^{-1}\left(\frac{1.2\times10^5}{10^6}\right)$$

$$= -85.236° - 50.194° - 6.843°$$

$$\phi = -142.3°$$

(c) $$A_f(0) = \frac{10^3}{1+(0.019)(10^3)} = 50$$

12.80

$$\phi = -180 = -\tan^{-1}\frac{f_{180}}{5\times10^3} - 2\tan^{-1}\frac{f_{180}}{10^5}$$

(a) $f_{180} = 1.05\times10^5$ Hz

$$|T| = \frac{(0.0045)(2\times10^3)}{\sqrt{1+\left(\frac{1.05\times10^5}{5\times10^3}\right)^2}\left[1+\left(\frac{1.05\times10^5}{10^5}\right)^2\right]}$$

$$= \frac{9}{(21.02)(2.1025)}$$

(b) $|T| = f_{180}^0 = 0.204$

System is stable

$$|T| = 1 = \frac{9}{\sqrt{1+\left(\frac{f}{5\times10^3}\right)^2}\left[1+\left(\frac{f}{10^5}\right)^2\right]}$$

$$f = 3.88\times10^4 \text{ Hz}$$

$$\phi = -\tan^{-1}\frac{3.88\times10^4}{5\times10^3} - 2\tan^{-1}\frac{3.88\times10^4}{10^5}$$

$$= -82.66 - 42.41$$

$$\phi = -125.1°$$

$$|T| = \frac{(0.15)(2\times10^3)}{\sqrt{1+\left(\frac{1.05\times10^5}{5\times10^3}\right)^2}\left[1+\left(\frac{1.05\times10^5}{10^5}\right)^2\right]}$$

$$= \frac{300}{(21.02)(2.1025)}$$

(c) $|T| = 6.79$

System is unstable

$$|T| = \quad 1 = \frac{300}{\sqrt{1+\left(\frac{f}{5\times10^3}\right)^2}\left[1+\left(\frac{f}{10^5}\right)^2\right]}$$

$$f = 2.33\times10^5 \text{ Hz}$$

$$\phi = -\tan^{-1}\frac{2.33\times10^5}{5\times10^3} - 2\tan^{-1}\frac{2.33\times10^5}{10^5}$$

$$= -88.77 - 133.54$$

$$\phi = -222.3°$$

12.81

Phase Margin $= 45° \Rightarrow \phi = -135°$

$$\phi = -135°$$

$$= -\tan^{-1}\left(\frac{f}{10^3}\right) - \tan^{-1}\left(\frac{f}{10^4}\right) - \tan^{-1}\left(\frac{f}{10^5}\right) - \tan^{-1}\left(\frac{f}{10^6}\right)$$

At $f = 10^4$ Hz, $\phi = -135.6°$

$$|T| = 1$$

$$= \beta(10^3)\times\frac{1}{\sqrt{1+\left(\frac{10^4}{10^3}\right)^2}}\times\frac{1}{\sqrt{1+\left(\frac{10^4}{10^4}\right)^2}}\times$$

$$\times\frac{1}{\sqrt{1+\left(\frac{10^4}{10^5}\right)^2}}\times\frac{1}{\sqrt{1+\left(\frac{10^4}{10^6}\right)^2}}$$

$$1 = \frac{\beta(10^3)}{(10.05)(1.414)(1.005)(1.00)}$$

or

$$\underline{\beta = 0.01428}$$

12.82

$$\phi = -120° = -\tan^{-1}\left(\frac{f_{120}}{f_{PD}}\right) - \tan^{-1}\left(\frac{f_{120}}{4\times10^5}\right) - \tan^{-1}\left(\frac{f_{120}}{4\times10^6}\right) - \tan^{-1}\left(\frac{f_{120}}{4\times10^7}\right)$$

$$f_{120} \cong 2.31\times10^5 \text{ Hz}$$

$$|T(f_{120})| = 1 = \frac{4000}{\sqrt{1+\left(\frac{2.31\times10^5}{f_{PD}}\right)^2}\sqrt{1+\left(\frac{2.31\times10^5}{4\times10^5}\right)^2}\sqrt{1+\left(\frac{2.31\times10^5}{4\times10^6}\right)^2}\sqrt{1+\left(\frac{2.31\times10^5}{4\times10^7}\right)^2}}$$

$$1 = \frac{4000}{\sqrt{1+\left(\frac{2.31\times10^5}{f_{PD}}\right)^2}(1.1548)(1.00167)(1.0)}$$

$$\frac{2.31\times10^5}{f_{PD}} = \frac{4000}{(1.1548)(1.00167)(1.09)}$$

$$f_{PD} = 66.8\,\text{Hz}$$

12.83

(a) $f_{180} \Rightarrow -180° = -2\tan^{-1}\left(\frac{f_{180}}{10^4}\right) - \tan^{-1}\left(\frac{f_{180}}{10^6}\right)$

$$f_{180} = 1.42\times10^5 \text{ Hz}$$

$$|T(f_{180})| = \frac{10^3}{\left[1+(14.2)^2\right]\sqrt{1+(0.142)^2}} = \frac{10^3}{(202.64)(1.01)} = 4.89$$

$$|T(f_{180})| > 1 \Rightarrow \text{Unstable}$$

(b) $\phi = -135° = -\tan^{-1}\left(\frac{f_{135}}{f_{PD}}\right) - 2\tan^{-1}\left(\frac{f_{135}}{10^4}\right) - \tan^{-1}\left(\frac{f_{135}}{10^6}\right)$

$$f_{135} \cong 0.414\times10^4 \text{ Hz}$$

$$|T(f_{135})| = 1 = \frac{10^3}{\sqrt{1+\left(\frac{0.414\times10^4}{f_{PD}}\right)^2}\left[1+\left(\frac{0.414\times10^4}{10^4}\right)^2\right]\sqrt{1+\left(\frac{0.414\times10^4}{10^6}\right)^2}}$$

$$\frac{0.414\times10^4}{f_{PD}} = \frac{10^3}{(1.171)(1)}$$

$$f_{PD} = 4.85\,\text{Hz}$$

12.84

(a) $\phi = -180 = -\tan^{-1}\left(\frac{f_{180}}{10^4}\right) - \tan^{-1}\left(\frac{f_{180}}{5\times10^4}\right) - \tan^{-1}\left(\frac{f_{180}}{10^5}\right)$

$$f_{180} \cong 8.06\times10^4 \text{ Hz}$$

$$|T| = \frac{500}{\sqrt{1+\left(\frac{8.06\times10^4}{10^4}\right)^2}\sqrt{1+\left(\frac{8.06\times10^4}{5\times10^4}\right)^2}\sqrt{1+\left(\frac{8.06\times10^4}{10^5}\right)^2}}$$

$$= \frac{500}{(8.122)(1.897)(1.284)}$$

(b) $|T| = 25.3$

(c)
$$T = \frac{500}{\left(1+j\frac{f}{f_{PD}}\right)\left(1+j\frac{f}{10^4}\right)\left(1+j\frac{f}{5\times10^4}\right)\left(1+j\frac{f}{10^5}\right)}$$

Phase Margin $= 60° \Rightarrow \phi = -120°$

$$-120 = -\tan^{-1}\frac{f}{f_{PD}} - \tan^{-1}\frac{f}{10^4} - \tan^{-1}\frac{f}{5\times10^4} - \tan^{-1}\frac{f}{10^5}$$

Assume $\tan^{-1}\frac{f}{f_{PD}} \cong 90°$

Then $f \cong 4.2\times10^3$ Hz

$$|T| = 1 = \frac{500}{\sqrt{1+\left(\frac{4.2\times10^3}{f_{PD}}\right)^2}\sqrt{1+\left(\frac{4.2\times10^3}{10^4}\right)^2}\sqrt{1+\left(\frac{4.2\times10^3}{5\times10^4}\right)^2}\sqrt{1+\left(\frac{4.2\times10^3}{10^5}\right)^2}}$$

$$1 = \frac{500}{\sqrt{1+\left(\frac{4.2\times10^3}{f_{PD}}\right)^2}(1.085)(1.004)(1.0)}$$

$$\frac{4.2\times10^3}{f_{PD}} \cong \frac{500}{(1.0846)(1.0035)(1.0)}$$

$$f_{PD} = 9.14 \text{ Hz}$$

12.85

(a) $$50 = \frac{10^4}{1+(10^4)\beta} \Rightarrow \beta = 0.0199$$

$$T = \frac{(0.0199)(10^4)}{\left(1+j\frac{f}{f_{PD}}\right)\left(1+j\frac{f}{10^5}\right)}$$

Phase margin $= 45° \Rightarrow \phi = -135°$

$$-135 = -\tan^{-1}\frac{f}{f_{PD}} - \tan^{-1}\frac{f}{10^5}$$

$$f = 10^5 \text{ Hz}$$

$$|T| = 1 = \frac{(0.0199)(10^4)}{\sqrt{1+\left(\frac{10^5}{f_{PD}}\right)^2}\sqrt{1+\left(\frac{10^5}{10^5}\right)^2}}$$

$$\frac{10^5}{f_{PD}} = \frac{(0.0199)(10^4)}{1.414}$$

$$f_{PD} = 711 \text{ Hz}$$

(b) $$20 = \frac{10^4}{1+(10^4)\beta} \Rightarrow \beta = 0.0499$$

$$T = \frac{(0.0499)(10^4)}{\left(1+j\frac{f}{711}\right)\left(1+j\frac{f}{10^5}\right)}$$

$$|T| = 1 = \frac{(0.0499)(10^4)}{\sqrt{1+\left(\frac{f}{711}\right)^2}\sqrt{1+\left(\frac{f}{10^5}\right)^2}}$$

$$f = 1.76\times 10^5 \text{ Hz}$$

$$\phi = -\tan^{-1}\left(\frac{1.76\times 10^5}{711}\right) - \tan^{-1}\left(\frac{1.76\times 10^5}{10^5}\right)$$

$$= -89.77 - 60.40$$

$$\phi = -150.2$$

Phase $\text{Margin} = 180 - 150.2 = 29.8°$

12.86

(a) $$A_O = 100 \text{ dB} \Rightarrow A_O = 10^5$$

$$20 = \frac{10^5}{1+(10^5)\beta} \Rightarrow \beta = 0.04999$$

$$T = \frac{(0.04999)(10^5)}{\left(1+j\frac{f}{f_{PD}}\right)\left(1+j\frac{f}{10^6}\right)\left(1+j\frac{f}{10^7}\right)}$$

Phase $\text{Margin} = 45° \Rightarrow \phi = -135°$

$$-135 = -\tan^{-1}\frac{f}{f_{PD}} - \tan^{-1}\frac{f}{10^6} - \tan^{-1}\frac{f}{10^7}$$

$$f \approx 10^6 \text{ Hz}$$

$$|T| = 1 = \frac{(0.04999)(10^5)}{\sqrt{1+\left(\frac{10^6}{f_{PD}}\right)^2}\sqrt{1+\left(\frac{10^6}{10^6}\right)^2}\sqrt{1+\left(\frac{10^6}{10^7}\right)^2}}$$

$$1 = \frac{(0.04999)(10^5)}{\sqrt{1+\left(\frac{10^6}{f_{PD}}\right)^2}(1.414)(1.005)}$$

$$\frac{10^6}{f_{PD}} = \frac{(0.04999)(10^5)}{(1.414)(1.005)}$$

$$f_{PD} = 2.84 \text{ Hz}$$

(b) $$5 = \frac{10^5}{1+(10^5)\beta} \Rightarrow \beta = 0.19999$$

$$|T| = 1 = \frac{(0.19999)(10^5)}{\sqrt{1+\left(\frac{f}{284}\right)^2}\sqrt{1+\left(\frac{f}{10^6}\right)^2}\sqrt{1+\left(\frac{f}{10^7}\right)^2}}$$

$$f = 2.25\times10^6 \text{ Hz}$$

$$\phi = -\tan^{-1}\left(\frac{2.25\times10^6}{284}\right) - \tan^{-1}\left(\frac{2.25\times10^6}{10^6}\right) - \tan^{-1}\left(\frac{2.25\times10^6}{10^7}\right)$$

$$= -89.99 - 66.04 - 12.68$$

$$\phi = -168.7$$

Phase Margin $= 180 - 168.7 = 11.3°$

12.87

a. $$T(0) = 100 \text{ dB} \Rightarrow T(0) = 10^5$$

$$T(f) = \frac{10^5}{\left(1+j\frac{f}{10}\right)\left(1+j\frac{f}{5\times10^6}\right)\left(1+j\frac{f}{10\times10^6}\right)}$$

$$|T| = 1 =$$

$$= 10^5 \times \frac{1}{\sqrt{1+\left(\frac{f}{10}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{5\times10^6}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{10\times10^6}\right)^2}}$$

By trial and error

$$\underline{f = 0.976 \text{ MHz}}$$

$$\phi = -\tan^{-1}\left(\frac{0.976\times10^6}{10}\right) - \tan^{-1}\left(\frac{0.976}{5}\right) - \tan^{-1}\left(\frac{0.976}{10}\right)$$

$$= -90° - 11.05° - 5.574° = -106.6°$$

Phase Margin $= 180° - 106.6° = \underline{73.4°}$

b. $$f'_{P1} \propto \frac{1}{C_F} \text{ so } \frac{10}{f'_{P1}} = \frac{75}{20}$$

or

$$\underline{f'_{P1} = 2.67 \text{ Hz}}$$

Now

$$|T| = 1 =$$

$$= 10^5 \times \frac{1}{\sqrt{1+\left(\frac{f}{2.67}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{5\times10^6}\right)^2}} \times \frac{1}{\sqrt{1+\left(\frac{f}{10\times10^6}\right)^2}}$$

By trial and error

$$f \approx 2.66\times10^5 \text{ Hz}$$

then

$$\phi = -\tan^{-1}\left(\frac{2.66\times10^5}{2.67}\right) - \tan^{-1}\left(\frac{0.266}{5}\right) - \tan^{-1}\left(\frac{0.266}{10}\right)$$

$$= -90° - 3.045° - 1.524° = -94.57°$$

Phase Margin $= 180° - 94.57° = \underline{85.4°}$

12.88

(a) $$f_P = \frac{1}{2\pi\left(R_{o1}\|R_{i2}\right)C_i} = \frac{1}{2\pi\left(2\|0.75\right)\times10^6\times\left(1.2\times10^{-12}\right)}$$

$$f_P = 243\,\text{kHz}$$

(b) $$f_{PD} = 6 = \frac{1}{2\pi\left(R_{o1}\|R_{i2}\right)\left(C_i + C_M\right)} = \frac{1}{2\pi\left(2\|0.75\right)\times10^6\times\left(C_i + C_M\right)}$$

$$C_i + C_M = 4.863\times10^{-8} = 1.2\times10^{-12} + C_M$$

$$C_M = 0.0486\,\mu\,\text{F}$$

(c) $$C_M = C_F\left(1+|A|\right)$$

$$0.0486\times10^{-6} = C_F\left(1001\right)$$

$$C_F = 48.6\,\text{pF}$$

12.89

$$\phi = -120° = -\tan^{-1}\left(\frac{f_{120}}{f_{PD}}\right) - \tan^{-1}\left(\frac{f_{120}}{4\times10^6}\right) - \tan^{-1}\left(\frac{f_{120}}{4\times10^7}\right)$$

$$f_{120} \cong 2.05\times10^6\ \text{Hz}$$

$$|T(f_{120})| = 1 = \frac{4000}{\sqrt{1+\left(\frac{2.05\times10^6}{f_{PD}}\right)^2}\sqrt{1+\left(\frac{2.05\times10^6}{4\times10^6}\right)^2}\sqrt{1+\left(\frac{2.05\times10^6}{4\times10^7}\right)^2}}$$

$$\frac{2.05\times10^6}{f_{PD}} = \frac{4000}{(1.1237)(1.0013)}$$

$$f_{PD} = 577\,\text{Hz}$$

12.90

(a) $$40 = \frac{5\times10^5}{1+\beta\left(5\times10^5\right)} \Rightarrow \beta = 0.024998$$

(b) $$\phi = -120° = -\tan^{-1}\left(\frac{f_{120}}{f_{PD}}\right) - \tan^{-1}\left(\frac{f_{120}}{5\times10^5}\right) - \tan^{-1}\left(\frac{f_{120}}{10^7}\right)$$

$$f_{120} \cong 2.71\times10^5 \text{ Hz}$$

$$|T(f_{120})| = 1 = \frac{(0.024998)(5\times10^5)}{\sqrt{1+\left(\frac{2.71\times10^5}{f_{PD}}\right)^2}\sqrt{1+\left(\frac{2.71\times10^5}{5\times10^5}\right)^2}\sqrt{1+\left(\frac{2.71\times10^5}{10^7}\right)^2}}$$

$$\frac{2.71\times10^5}{f_{PD}} = \frac{(0.024998)(5\times10^5)}{(1.1374)(1)}$$

$$f_{PD} = 24.66 \text{ Hz}$$

Chapter 13

13.1

(a) $R_{D2} = \dfrac{0-(-3)}{0.2} = 15\ \text{k}\Omega$

$$I_{D3} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2$$

$$0.2 = \left(\frac{0.04}{2}\right)(40)(V_{SG3} - 0.4)^2 \Rightarrow V_{SG3} = 0.9\ \text{V}$$

$$R_{D1} = \frac{0.9}{0.1} = 9\ \text{k}\Omega$$

(b) (i) $A_d = \dfrac{\upsilon_{o1}}{\upsilon_d} = \dfrac{g_{m1}}{2} \cdot R_{D1}$

$$g_{m1} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.1)} = 0.6325\ \text{mA/V}$$

$$A_d = \left(\frac{0.6325}{2}\right)(9) = 2.846$$

(ii) $A_2 = \dfrac{\upsilon_o}{\upsilon_{o1}} = -g_{m3}R_{D2}$

$$g_{m3} = 2\sqrt{\left(\frac{0.04}{2}\right)(40)(0.2)} = 0.8\ \text{mA/V}$$

$$A_2 = -(0.8)(15) = -12$$

(c) $A = A_d \cdot A_2 = (2.846)(-12) = -34.15$

13.2

(a) $R_{C2} = \dfrac{3-0}{0.4} = 7.5\ \text{k}\Omega$

$$V_{B3} = 0.7 + (0.4)(0.5) - 3 = -2.1\ \text{V}$$

$$R_{C1} = \frac{-2.1-(-3)}{0.25} = 3.6\ \text{k}\Omega$$

(b) (i) $A_d = \dfrac{\upsilon_{o1}}{\upsilon_d} = \dfrac{g_{m1}}{2}\left(R_{C1} \| R_{i3}\right)$

$$g_{m1} = \frac{0.25}{0.026} = 9.615\ \text{mA/V},\ r_{\pi 3} = \frac{(180)(0.026)}{0.4} = 11.7\ \text{k}\Omega$$

$$R_{i3} = r_{\pi 3} + (1+\beta_n)R_E = 11.7 + (181)(0.5) = 102.2\ \text{k}\Omega$$

$$A_d = \left(\frac{9.615}{2}\right)(3.6 \| 102.2) = 16.72$$

(ii) $A_2 = \dfrac{\upsilon_o}{\upsilon_{o1}} = \dfrac{-\beta_n R_{C2}}{r_{\pi 3} + (1+\beta_n)R_E} = \dfrac{-(180)(7.5)}{11.7 + (181)(0.5)} = -13.21$

(c) $A = A_d \cdot A_2 = (16.72)(-13.21) = -220.9$

13.3 Computer Analysis

13.4 Computer Analysis

13.5

(a) $A_d = g_{m1}\left(r_{o2} \| r_{o4} \| R_{i6}\right)$

$$g_{m1} = \frac{I_{C1}}{V_T} = \frac{20}{0.026} \Rightarrow 0.769\ mA/V$$

$$r_{o2} = \frac{V_{A2}}{I_{C2}} = \frac{80}{20} = 4\ M\Omega$$

$$r_{o4} = \frac{V_{A4}}{I_{C2}} = \frac{80}{20} = 4\ M\Omega$$

$$R_{i6} = r_{\pi 6} + (1+\beta_n)\left[R_1 \| r_{\pi 7}\right]$$

$$r_{\pi 7} = \frac{(120)(0.026)}{0.2} = 15.6\ k\Omega$$

$$I_{C6} \cong \frac{V_{BE}(on)}{R_1} = \frac{0.6}{20} = 0.030\ mA$$

$$r_{\pi 6} = \frac{(120)(0.026)}{0.030} = 104\ k\Omega$$

Then

$$R_{i6} = 104 + (121)\left[20 \| 15.6\right] \Rightarrow 1.16\ M\Omega$$

Then

$$A_d = 769\left(4 \| 4 \| 1.16\right) \Rightarrow \underline{A_d = 565}$$

Now

$$V_o = -I_{c7} r_{o7} = -(\beta_n I_{b7}) r_{o7} = -\beta_n r_{o7}\left(\frac{R_1}{R_1 + r_{\pi 7}}\right) I_{c6}$$

$$= -\beta_n (1+\beta_n) r_{o7}\left(\frac{R_1}{R_1 + r_{\pi 7}}\right) I_{b6} \text{ and } I_{b6} = \frac{V_{o1}}{R_{i6}}$$

Then

$$A_{v2} = \frac{V_o}{V_{o1}} = \frac{-\beta_n (1+\beta_n) r_{o7}}{R_{i6}}\left(\frac{R_1}{R_1 + r_{\pi 7}}\right)$$

$$r_{o7} = \frac{V_A}{I_{C7}} = \frac{80}{0.2} = 400\ k\Omega$$

So

$$A_{v2} = \frac{-(120)(121)(400)}{1160}\left(\frac{20}{20+15.6}\right) \Rightarrow \underline{A_{v2} = -2813}$$

Overall gain $= A_d \cdot A_{v2} = (565)(-2813) \Rightarrow \underline{A = -1.59 \times 10^6}$

(b) $R_{id} = 2r_{\pi 1}$ and $r_{\pi 1} = \dfrac{(80)(0.026)}{0.020} = 104\ k\Omega$

$\underline{R_{id} = 208\ k\Omega}$

(c) $f_{PD} = \dfrac{1}{2\pi R_{eq} C_M}$ and $C_M = (10)(1+2813) = 28{,}140\ pF$

$R_{eq} = r_{o2} \| r_{o4} \| R_{i6} = 4 \| 4 \| 1.16 = 0.734\ M\Omega$

$f_{PD} = \dfrac{1}{2\pi(0.734\times10^{6})(28{,}140\times10^{-12})} = 7.71\ Hz$

Gain-Bandwidth Product = $(7.71)(1.59\times10^{6}) \Rightarrow \underline{12.3\ MHz}$

13.6

a. Q_3 acts as the protection device.

b. Same as part (a).

13.7

If we assume $V_{BE}(\text{on}) = 0.7\ \text{V}$, then $V_{\text{in}} = 0.7 + 0.7 + 50 + 5$

So breakdown voltage $\approx \underline{56.4\ \text{V}}$.

13.8

(a) $V_{EB12} = V_{BE11} = (0.026)\ln\left(\dfrac{0.5\times10^{-3}}{5\times10^{-16}}\right) = 0.7184\ \text{V}$

$R_5 = \dfrac{15 - 0.7184 - 0.7184 - (-15)}{0.5} = 57.1\ \text{k}\Omega$

$R_4 = \dfrac{(0.026)}{(0.03)}\ln\left(\dfrac{0.5}{0.03}\right) = 2.438\ \text{k}\Omega$

$V_{BE10} = V_{BE11} - I_{C10}R_4 = 0.7184 - (0.03)(2.438) = 0.6453\ \text{V}$

(b) $I_{REF} = \dfrac{15 - 0.6 - 0.6 - (-15)}{57.1} = 0.5044\ \text{mA}$

$I_{C10}(2.438) = (0.026)\ln\left(\dfrac{0.5044}{I_{C10}}\right) \Rightarrow I_{C10} \cong 30.1\,\mu\ \text{A}$

(c) I_{REF}: $\left(\dfrac{0.5044 - 0.5}{0.5}\right)\times100\% = 0.88\%$

I_{C10}: $\left(\dfrac{30.1 - 30}{30}\right)\times100\% = 0.33\%$

13.9

(a) $R_5 = \dfrac{5 - 0.7184 - 0.7184 - (-5)}{0.5} = 17.13\ \text{k}\Omega$

$R_4 = \dfrac{(0.026)}{(0.03)}\ln\left(\dfrac{0.5}{0.03}\right) = 2.438\ \text{k}\Omega$

$V_{EB12} = V_{BE11} = 0.7184\ \text{V}$

$V_{BE10} = 0.6453\ \text{V}$

(b) $I_{REF} = \dfrac{5-0.6-0.6-(-5)}{17.13} = 0.5137 \text{ mA}$

$$I_{C10}(2.438) = (0.026)\ln\left(\frac{0.5137}{I_{C10}}\right) \Rightarrow I_{C10} \cong 30.22\ \mu\text{A}$$

(c) I_{REF}: $\left(\dfrac{0.5137-0.5}{0.5}\right)\times 100\% = 2.74\%$

I_{C10}: $\left(\dfrac{30.22-30}{30}\right)\times 100\% = 0.733\%$

13.10

(a) $I_{REF} = \dfrac{3-V_{EB2}-V_{BE1}-(-3)}{80}$

$$V_{EB2} = V_{BE1} = (0.026)\ln\left(\frac{I_{REF}}{3\times 10^{-15}}\right)$$

By trial and error, $I_{REF} \cong 59.6\ \mu\text{A}$

$$I_3(3.5) = (0.026)\ln\left(\frac{0.0596}{I_3}\right) \Rightarrow I_3 \cong 11.9\ \mu\text{A}$$

$$I_4 = \frac{I_{S4}}{I_{S2}}\cdot I_{REF} = \frac{(3\times 10^{-15})}{(5\times 10^{-15})}\cdot(59.6) = 35.76\ \mu\text{A}$$

$$I_5 = \frac{I_{S5}}{I_{S2}}\cdot I_{REF} = \frac{(1\times 10^{-15})}{(5\times 10^{-15})}\cdot(59.6) = 11.92\ \mu\text{A}$$

(b) $I_{REF} = 59.6\ \mu\text{A}$, $I_3 = 11.9\ \mu\text{A}$

$$I_4 = \frac{I_{S4}}{I_{S2}}\cdot I_{REF} = \frac{(8\times 10^{-15})}{(5\times 10^{15})}\cdot(59.6) = 95.36\ \mu\text{A}$$

$$I_5 = \frac{I_{S5}}{I_{S2}}\cdot I_{REF} = \frac{(2\times 10^{-15})}{(5\times 10^{-15})}\cdot(59.6) = 23.84\ \mu\text{A}$$

13.11

$$I_{REF} = \frac{5-0.6-0.6-(-5)}{40} \Rightarrow \underline{I_{REF} = 0.22 \text{ mA}}$$

$$I_{C10}R_4 = V_T \ln\left(\frac{I_{REF}}{I_{C10}}\right)$$

$$I_{C10}(5) = (0.026)\ln\left(\frac{0.22}{I_{C10}}\right)$$

By trial and error;

$$\underline{I_{C10} \cong 14.2\ \mu\text{A}}$$

$$I_{C6} \cong \frac{I_{C10}}{2} \Rightarrow \underline{I_{C6} = 7.10\mu\text{A}}$$

$$I_{C17} = 0.75I_{REF} \Rightarrow \underline{I_{C17} = 0.165 \text{ mA}}$$

$$I_{C13A} = 0.25I_{REF} \Rightarrow \underline{I_{C13A} = 0.055 \text{ mA}}$$

13.12

$$I_{C8} = I_{C9}$$

$$2I = I_{C8} + \frac{2I_{C9}}{\beta_p} = I_{C9}\left(1 + \frac{2}{\beta_p}\right)$$

$$I_{E1} = I_{E2} = I\left(\frac{1+\beta_n}{\beta_n}\right)$$

$$I_{B3} = I_{B4} = I\left(\frac{1+\beta_n}{\beta_n}\right)\left(\frac{1}{1+\beta_p}\right)$$

$$I_{C10} = I_{C9} + 2I\left(\frac{1+\beta_n}{\beta_n}\right)\left(\frac{1}{1+\beta_p}\right)$$

$$I_{C10} = I_{C9} + I_{C9}\left(1 + \frac{2}{\beta_p}\right)\left(\frac{1+\beta_n}{\beta_n}\right)\left(\frac{1}{1+\beta_p}\right)$$

$$50 = I_{C9}\left[1 + \left(1 + \frac{2}{40}\right)\left(\frac{91}{90}\right)\left(\frac{1}{41}\right)\right] = I_{C9}(1.0259) \Rightarrow I_{C9} = 48.738\,\mu\text{ A}$$

$$I_{C2} = I = \frac{I_{C9}}{2}\left(1 + \frac{2}{\beta_p}\right) = \frac{48.738}{2}\left(1 + \frac{2}{40}\right) = 25.587\,\mu\text{ A}$$

$$I_{C4} = I_{C2}\left(\frac{\beta_p}{1+\beta_p}\right)\left(\frac{1+\beta_n}{\beta_n}\right) = (25.587)\left(\frac{40}{41}\right)\left(\frac{91}{90}\right) = 25.240\,\mu\text{ A}$$

$$I_{B9} = \frac{I_{C9}}{\beta_p} = \frac{48.738}{40} = 1.218\,\mu\text{ A}$$

$$I_{B4} = \frac{I_{C4}}{\beta_p} = \frac{25.240}{40} = 0.631\,\mu\text{ A}$$

13.13

$$V_{B5} - V^- = V_{BE}(\text{on}) + I_{C5}(1)$$
$$= 0.6 + (0.0095)(1) = 0.6095$$

$$I_{C7} = \frac{0.6095}{50} \Rightarrow I_{C7} = 12.2\ \mu\text{A}$$

$$I_{C8} = I_{C9} = 19\ \mu\text{A}$$

$$I_{REF} = 0.72\text{ mA}$$

$$I_{E13} = I_{REF} = 0.72\text{ mA}$$

$$I_{C14} = 138\ \mu\text{A}$$

$$\text{Power} = \left(V^+ - V^-\right)\left[I_{C7} + I_{C8} + I_{C9} + I_{REF} + I_{E13} + I_{C14}\right]$$

$$= 30[0.0122 + 0.019 + 0.019 + 0.72 + 0.72 + 0.138]$$

$$\Rightarrow \underline{\text{Power} = 48.8\text{ mW}}$$

$$\text{Current supplied by } V^+ \text{and } V^- = I_{C7} + I_{C8} + I_{C9} + I_{REF} + I_{E13} + I_{C14}$$
$$= \underline{1.63\text{ mA}}$$

13.14

(a) $v_{cm}(\min) = -15 + 0.6 + 0.6 + 0.6 + 0.6 = -12.6\ V$

$v_{cm}(\max) = +15 - .6 = 14.4\ V$

So $-12.6 \le v_{cm} \le 14.4\ V$

(b) $v_{cm}(\min) = -5 + 4(0.6) = -2.6\ V$

$v_{cm}(\max) = 5 - 0.6 = 4.4\ V$

So $-2.6 \le v_{cm} \le 4.4\ V$

13.15

$$I_{C13A} = (0.25)I_{REF} = (0.25)(0.5) = 0.125\,\text{mA}$$

$$I_{R10} \cong \frac{0.6}{50} = 0.012\ \text{mA}$$

$$I_{C19} \cong I_{C13A} - I_{R10} = 0.125 - 0.012 = 0.113\,\text{mA}$$

$$I_{B19} = \frac{I_{C19}}{\beta_n} = \frac{0.113}{200} \Rightarrow I_{B19} = 0.565\,\mu\text{A}$$

$$I_{C18} = I_{R10} + I_{B19} = 12 + 0.565 = 12.565\,\mu\text{A}$$

$$V_{BE18} = (0.026)\ln\left(\frac{12.565\times10^{-6}}{10^{-14}}\right) = 0.54474\ \text{V}$$

$$V_{BE19} = (0.026)\ln\left(\frac{0.113\times10^{-3}}{10^{-14}}\right) = 0.60185\ \text{V}$$

$$\frac{V_{BB}}{2} = \frac{V_{BE18} + V_{Be19}}{2} = 0.57329\ \text{V}$$

$$I_{C14} = \left(3\times10^{-14}\right)\exp\left(\frac{0.57329}{0.026}\right) \Rightarrow I_{C14} = 113\,\mu\text{A}$$

13.16

(a) $$V_{BB} = 2V_T\ \ln\left(\frac{I_{Bias}}{I_{SD}}\right) = 2(0.026)\ln\left(\frac{80\times10^{-6}}{5\times10^{-15}}\right) = 1.22178\ \text{V}$$

$$I_{CN} = I_{CP} = I_{SQ}\ \exp\left[\frac{1.22178/2}{0.026}\right] \Rightarrow I_{CN} = I_{CP} = 128\,\mu\text{A}$$

(b) For $\upsilon_I = 3\ \text{V},\ \upsilon_O \cong 3\ \text{V},\ i_L \cong \frac{3}{10} = 0.3\,\text{mA}$

First approximation:

$$I_{BN} \cong \frac{0.3}{120} \Rightarrow I_{BN} = 2.5\,\mu\text{A}$$

$$I_D = 80 - 2.5 = 77.5\,\mu\text{A}$$

$$V_{BB} = 2(0.026)\ln\left(\frac{77.5\times10^{-6}}{5\times10^{-15}}\right) = 1.22013\ \text{V}$$

$$V_{BEN} = (0.026)\ln\left(\frac{0.3\times10^{-3}}{8\times10^{-15}}\right) = 0.63304\ \text{V}$$

$$V_{EBP} = V_{BB} - V_{BEN} = 0.58709\ \text{V}$$

$$I_{CP} = \left(8\times10^{-15}\right)\exp\left(\frac{0.58709}{0.026}\right) \Rightarrow I_{CP} = 51.24\ \mu\text{ A}$$

Second approximation:

$$I_{CN} = \left(\frac{120}{121}\right)(0.3 + 0.05124) = 0.34834\text{ mA},\quad I_{BN} = 2.903\,\mu\text{ A}$$

$$I_D = 80 - 2.903 = 77.1\,\mu\text{A}$$

$$V_{BB} = 2(0.026)\ln\left(\frac{77.1\times10^{-6}}{5\times10^{-15}}\right) = 1.219864\text{ V}$$

$$V_{BEN} = (0.026)\ln\left(\frac{0.34834\times10^{-3}}{8\times10^{-15}}\right) = 0.636922\text{ V}$$

$$V_{EBP} = 1.219864 - 0.636922 = 0.582942\text{ V}$$

$$I_{CP} = \left(8\times10^{-15}\right)\exp\left(\frac{0.582942}{0.026}\right) \Rightarrow I_{CP} = 43.7\,\mu\text{ A}$$

13.17

$$R_1 + R_2 = \frac{V_{BB}}{(0.1)I_{Bias}} = \frac{1.160}{(0.1)(0.12)} = 96.67\text{ k}\Omega$$

$$V_{BE} = V_T\ln\left(\frac{I_{CQ}}{I_S}\right) = (0.026)\ln\left(\frac{(0.9)(120\times10^{-6})}{5\times10^{-15}}\right) = 0.6187\text{ V}$$

$$V_{BE} = \left(\frac{R_2}{R_1+R_2}\right)(V_{BB}) \Rightarrow 0.6187 = \left(\frac{R_2}{96.67}\right)(1.16)$$

So $R_2 = 51.56\,\text{k}\Omega$, $R_1 = 45.11\,\text{k}\Omega$

13.18

(a) $A_d = -g_m\left(r_{o4}\|r_{o6}\|R_{i2}\right)$

From example 13.4

$$g_m = \frac{9.5}{0.026} = 365\ \mu A/V,\ r_{o4} = 5.26\ M\Omega$$

Now

$$r_{o6} = r_{o4} = 5.26\ M\Omega$$

Assuming $R_8 = 0$, we find

$$R_{i2} = r_{\pi16} + (1+\beta_n)R_E'$$
$$= 329 + (201)\left(50\|9.63\right) \Rightarrow 1.95\ M\Omega$$

Then

$$A_d = -(365)\left(5.26\|5.26\|1.95\right) \Rightarrow \underline{A_d = -409}$$

(b) From Equation (13.20),

$$A_{v2} = \frac{-\beta_n(1+\beta_n)R_9\left(R_{act2}\|R_{i3}\|R_{017}\right)}{R_{i2}\left\{R_9 + \left[r_{\pi17} + (1+\beta_n)R_g\right]\right\}}$$

For $R_g = 0,\ R_{i2} = 1.95\ M\Omega$

Using the results of Example 13.5

$$A_{v2} = \frac{-200(201)(50)\left(92.6 \| 4050 \| 92.6\right)}{(1950)\{50 + 9.63\}} \Rightarrow \underline{A_{v2} = -792}$$

13.19

Let $I_{C10} = 40\ \mu\text{A},$ then $I_{C1} = I_{C2} = 20\ \mu\text{A}.$ Use the procedure in Example 13.4:

$$R_{i2} = 4.07\ \text{M}\Omega$$

$$r_{\pi 6} = \frac{(200)(0.026)}{0.020} = 260\ \text{k}\Omega$$

$$g_{m6} = \frac{0.020}{0.026} = 0.769\ \text{mA/V}$$

$$r_{06} = \frac{50}{0.02} \Rightarrow 2.5\ \text{M}\Omega$$

Then

$$R_{\text{act1}} = 2.5[1 + (0.769)(1 \| 260)] = 4.42\ \text{M}\Omega$$

$$r_{06} = \frac{50}{0.02} \Rightarrow 2.5\ \text{M}\Omega$$

Then

$$A_d = -\left(\frac{I_{CQ}}{V_T}\right)(r_{04} \| R_{\text{act1}} \| R_{i2})$$

$$= -\left(\frac{20}{0.026}\right)(2.5 \| 4.42 \| 4.07)$$

So

$$\underline{A_d = -882}$$

13.20

From Problem 13.11

$$I_1 = I_2 = 7.10\ \mu\text{A},\ I_{C17} = 0.165\ \text{mA},\ I_{C13A} = 0.055\ \text{mA}$$

$$I_{C16} \approx I_{B17} + \frac{I_{E17}R_8 + V_{BE17}}{R_9} = \frac{0.165}{200} + \frac{(0.165)(0.1) + 0.6}{50}$$

$$= 0.000825 + 0.01233$$

$$I_{C16} = 0.0132\ \text{mA}$$

$$r_{\pi 17} = \frac{(200)(0.026)}{0.165} = 31.5\ \text{K}$$

$$R_E^1 = R_9 \| [r_{\pi 17} + (1 + \beta)R_8] = 50 \| [31.5 + (201)(0.1)]$$

$$= 50 \| 51.6 = 25.4\ \text{K}$$

$$r_{\pi 16} = \frac{(200)(0.026)}{0.0132} = 394\ \text{K}$$

Then

$$R_{i2} = r_{\pi 16} + (1 + \beta)R_E^1 = 394 + (201)(25.4) \Rightarrow 5.50\ \text{M}\Omega$$

Now

$$r_{\pi 6} = \frac{(200)(0.026)}{0.0071} = 732 \text{ K}$$

$$g_{m6} = \frac{0.0071}{0.026} = 0.273 \text{ mA/V}$$

$$r_{o6} = \frac{50}{0.0071} \Rightarrow 7.04 \text{ M}\Omega$$

$$R_{\text{act1}} = r_{o6}[1 + g_{m6}(R \| r_{\pi 6})]$$
$$= 7.04[1 + (0.273)(1 \| 732)] = 8.96 \text{ M}\Omega$$

$$r_{o4} = \frac{50}{0.0071} \Rightarrow 7.04 \text{ M}\Omega$$

Then

$$A_d = -g_{m1}(r_{o4} \| R_{\text{act1}} \| R_{i2})$$
$$= -\left(\frac{7.10}{0.026}\right)(7.04 \| 8.96 \| 5.5)$$

$$\underline{A_d = -627}$$

$$R_{\text{act2}} = \frac{50}{0.165} \Rightarrow 303 \text{ K} \quad R_{o17} = \frac{50}{0.165} = 303 \text{ K}$$

Now

From Eq. (13.20), assuming $R_{i3} \to \infty$

$$A_{v2} \cong -\frac{\beta(1+\beta)R_9(R_{\text{act2}} \| R_{017})}{R_{i2}\{R_9 + [r_{\pi 17} + (1+\beta)R_8]\}}$$
$$= \frac{-(200)(201)(50)(303 \| 303)}{(5500)[50 + 31.5 + (201)(0.1)]} = \frac{-3.045 \times 10^8}{5.588 \times 10^5}$$

$$A_{v2} = -545$$

Overall gain $A_v = (-627)(-545) = 341,715$

13.21

Using results from Problem 13.20

$$R_{i2} = 5.50 \text{ M}\Omega, \; R_{\text{act1}}\left(\frac{100}{0.0071}\right)[1 + (0.273)(1 \| 732)] \Rightarrow 17.93 \text{ M}\Omega$$

$$r_{o4} = \frac{100}{0.0071} \Rightarrow 14.08 \text{ M}\Omega$$

$$A_d = -\left(\frac{7.10}{0.026}\right)(14.08 \| 17.93 \| 5.50)$$

$$A_d = -885$$

Now

$$R_{\text{act2}} = \frac{100}{0.165} = 606 \text{ K} \quad R_{o17} = \frac{100}{0.165} = 606 \text{ K}$$

$$A_{v2} = \frac{-(200)(201)(50)(606 \| 606)}{(5500)[50 + 31.5 + (201)(0.1)]} = \frac{-6.09 \times 10^8}{5.588 \times 10^5}$$

$$A_{v2} = -1090$$

Overall gain

$$A_v = (-885)(-1090) = 964,650$$

13.22

(a) $I_{C18} + I_{C19} = I_{Bias}$

$(0.25)I_{C19} + I_{C19} = 0.18\text{ mA}, \Rightarrow I_{C19} = 0.144\text{ mA}$

$I_{C18} = 0.036\text{ mA}$

$$V_{BE19} = (0.026)\ln\left(\frac{0.144\times10^{-3}}{10^{-14}}\right) = 0.60815\text{ V}$$

$$R_{10} = \frac{0.60815}{0.036} = 16.9\text{ k}\Omega$$

(b) $V_{BE19} = 0.60815\text{ V}$

$$V_{BE18} = (0.026)\ln\left(\frac{0.036\times10^{-3}}{10^{-14}}\right) = 0.5721\text{ V}$$

13.23

(b) $I_{C19} = 0.168\text{ mA}, \quad g_{m19} = \dfrac{0.168}{0.026} = 6.462\text{ mA/V},$

$$r_{\pi19} = \frac{(200)(0.026)}{0.168} = 30.95\text{ k}\Omega, \quad r_{o19} = \frac{50}{0.168} = 298\text{ k}\Omega$$

$I_{C18} = 12.84\,\mu\text{A}, \quad g_{m18} = \dfrac{0.01284}{0.026} = 0.4938\text{ mA/V}$

$$r_{\pi18} = \frac{(200)(0.026)}{0.01284} = 405\text{ k}\Omega, \quad r_{o18} = \frac{50}{0.01284} = 3894\text{ k}\Omega$$

$V_x = V_{\pi18} + V_{\pi19}$

(1) $$I_x = \frac{V_{\pi18}}{r_{\pi18}} + g_{m18}V_{\pi18} + \frac{V_{\pi18}}{r_{o18}} + g_{m19}V_{\pi19} + \frac{V_x}{r_{o19}}$$

(2) $$\frac{V_{\pi18}}{r_{\pi18}} + g_{m18}V_{\pi18} + \frac{V_{\pi18}}{r_{o18}} = \frac{V_{\pi19}}{R_{10}} + \frac{V_{\pi19}}{r_{\pi19}}$$

Then $$V_{\pi18}\left(\frac{1}{405} + 0.4938 + \frac{1}{3894}\right) = V_{\pi19}\left(\frac{1}{50} + \frac{1}{30.95}\right)$$

$$V_{\pi18}(0.49653) = V_{\pi19}(0.05231) \Rightarrow V_{\pi18} = V_{\pi19}(0.10535)$$

Then (1) $$I_x = V_{\pi18}(0.49653) + (6.462)V_{\pi19} + \frac{V_x}{298}$$

$$I_x = V_{\pi19}(0.10535)(0.49653) + (6.462)V_{\pi19} + (0.003356)V_x$$

$$I_x = V_{\pi19}(6.5143) + V_x(0.003356)$$

Now $V_x = V_{\pi18} + V_{\pi19} = V_{\pi19}(0.10535) + V_{\pi19} = (1.10535)V_{\pi19}$

Or $V_{\pi19} = (0.90469)V_x$

Then $I_x = (0.90469)V_x(6.5143) + V_x(0.003356)$

So $$\frac{I_x}{V_x} = \frac{1}{R_{eq}} = 5.8968 \Rightarrow R_{eq} = 170\,\Omega$$

13.24

Now

$$R_{e14} = \frac{r_{\pi 14} + R_{01}}{1+\beta_P} \quad \text{and} \quad R_0 = R_6 + R_{e14}$$

Assume series resistance of Q_{18} and Q_{19} is small. Then

$$R_{01} = r_{013A} \| R_{e22}$$

$$R_{e22} = \frac{r_{\pi 22} + R_{017} \| r_{013B}}{1+\beta_P}$$

where

and $R_{017} = r_{017}[1 + g_{m17}(R_8 \| r_{\pi 17})]$

Using results from Example 13.6,

$r_{\pi 17} = 9.63\ \text{k}\Omega \qquad r_{\pi 22} = 7.22\ \text{k}\Omega$

$g_{m17} = 20.8\ \text{mA/V} \quad r_{017} = 92.6\ \text{k}\Omega$

Then

$$R_{017} = 92.6[1 + (20.8)(0.1 \| 9.63)] = 283\ \text{k}\Omega$$

$$r_{013B} = \frac{50}{0.54} = 92.6\ \text{k}\Omega$$

Then

$$R_{e22} = \frac{7.22 + 283 \| 92.6}{51} = 1.51\ \text{k}\Omega$$

$$R_{01} = r_{013A} \| R_{e22} = 278 \| 1.51 = 1.50\ \text{k}\Omega$$

$$r_{\pi 14} = \frac{(50)(0.026)}{2} = 0.65\ \text{k}\Omega$$

Then

$$R_{e14} = \frac{0.65 + 1.50}{51} = 0.0422\ \text{k}\Omega$$

or

$$R_{e14} = 42.2\ \Omega$$

Then

$$R_0 = 42.2 + 27 \Rightarrow \underline{R_0 = 69.2\ \Omega}$$

13.25

$$R_{id} = 2\left[r_{\pi 1} + (1+\beta_n)\left(\frac{r_{\pi 3}}{1+\beta_P}\right)\right]$$

$\beta_n = 200, \quad \beta_P = 10$

(a)

$I_{C1} = 9.5\ \mu A$

$$r_{\pi 1} = \frac{(200)(0.026)}{0.0095} = 547\ \text{K}$$

$$r_{\pi 3} = \frac{(10)(0.026)}{0.0095} = 27.4\ \text{K}$$

Then

$$R_{id} = 2\left[547 + \frac{(201)(27.4)}{11}\right]$$

$\underline{R_{id} \Rightarrow 2.095\ \text{M}\Omega}$

(b)

$$I_{C1} = 7.10\ \mu\text{A}$$

$$r_{\pi 1} = \frac{(200)(0.026)}{0.0071} = 732\ \text{K}$$

$$r_{\pi 3} = \frac{(10)(0.026)}{0.0071} = 36.6\ \text{K}$$

$$R_{id} = 2\left[732 + \frac{(201)(36.6)}{11}\right]$$

$$\underline{R_{id} \Rightarrow 2.80\ \text{M}\Omega}$$

13.26

We can write

$$A(f) = \frac{A_0}{\left(1 + j\frac{f}{f_{PD}}\right)\left(1 + j\frac{f}{f_1}\right)}$$

$$= \frac{181,260}{\left(1 + j\frac{f}{10.7}\right)\left(1 + j\frac{f}{f_1}\right)}$$

Phase:

$$\phi = -\tan^{-1}\left(\frac{f}{10.7}\right) - \tan^{-1}\left(\frac{f}{f_1}\right)$$

For a Phase margin $= 70°,\ \ \phi = -110°$

So

$$-110° = -\tan^{-1}\left(\frac{f}{10.7}\right) - \tan^{-1}\left(\frac{f}{f_1}\right)$$

Assuming $f >> 10.7\ \text{Hz},$ we have

$$\tan^{-1}\left(\frac{f}{f_1}\right) = 20° \Rightarrow \frac{f}{f_1} = 0.364$$

At this frequency, $|A(f)| = 1,$ so

$$1 = \frac{181,260}{\sqrt{1 + \left(\frac{f}{10.7}\right)^2} \cdot \sqrt{1 + (0.364)^2}}$$

$$= \frac{170,327}{\sqrt{1 + \left(\frac{f}{10.7}\right)^2}}$$

$$\frac{f}{10.7} = 170,327 \Rightarrow f = 1.82\ \text{MHz}$$

or

Then, second pole at

$$f_1 = \frac{f}{0.364} \Rightarrow \underline{f_1 = 5\ \text{MHz}}$$

13.27

(a) $10\times10^6 = f_{PD}\left(10^6\right) \Rightarrow f_{PD} = 10\,\text{Hz}$

(b) $f_{PD} = \dfrac{1}{2\pi R_{eq}C_M} \Rightarrow C_M = \dfrac{1}{2\pi\left(1.2\times10^6\right)(10)} = 1.326\times10^{-8}\,\text{F}$

$$C_M = C_F\left(1+|A|\right) \Rightarrow C_F = \frac{1.326\times10^{-8}}{1001} \Rightarrow C_F = 13.25\,\text{pF}$$

13.28

$$\phi = -110 = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - 2\tan^{-1}\left(\frac{f}{f_{2,3}}\right) = -90 - 2\tan^{-1}\left(\frac{f}{f_{2,3}}\right)$$

So $\left(\dfrac{f}{f_{2,3}}\right) = 0.1763$

$$|A| = 1 = \frac{200{,}000}{\sqrt{1+\left(\dfrac{f}{f_{PD}}\right)^2\left[1+\left(\dfrac{f}{f_{2,3}}\right)^2\right]}} \cong \frac{200{,}000}{\left(\dfrac{f}{10}\right)\left[1+(0.1763)^2\right]} \Rightarrow f = 1.9397\,\text{MHz}$$

Then $f_{2,3} = \dfrac{1.9397\times10^6}{0.1763} \Rightarrow f_{2,3} = 11.0\,\text{MHz}$

13.29

(a) $I_{D3} = \left(\dfrac{k_p'}{2}\right)\left(\dfrac{W}{L}\right)_3\left(V_{SG3}+V_{TP}\right)^2$

$$150 = \left(\frac{40}{2}\right)(50)\left(V_{SG3}-0.4\right)^2 \Rightarrow V_{SG3} = 0.7873\,\text{V}$$

$$R_{D1} = \frac{0.7873}{0.1} = 7.87\,\text{k}\Omega$$

$$I_{D4} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_4\left(V_{GS4}-V_{TN}\right)^2$$

$$200 = \left(\frac{100}{2}\right)(40)\left(V_{GS4}-0.4\right)^2 \Rightarrow V_{GS4} = 0.7162\,\text{V}$$

$$R_{D2} = \frac{0.7162-(-3)}{0.15} = 24.8\,\text{k}\Omega$$

$$R_S = \frac{0-(-3)}{0.15} = 15\,\text{k}\Omega$$

(b) (i) $A_{d1} = \left(\dfrac{g_{m1}}{2}\right)\cdot R_{D1}$, $g_{m1} = 2\sqrt{\left(\dfrac{0.1}{2}\right)(20)(0.1)} = 0.6325\,\text{mA/V}$

$$A_{d1} = \left(\frac{0.6325}{2}\right)(7.87) = 2.49$$

(ii) $A_2 = -g_{m3}R_{D2}$, $g_{m3} = 2\sqrt{\left(\dfrac{0.04}{2}\right)(50)(0.15)} = 0.7746\,\text{mA/V}$

$$A_2 = -(0.7746)(24.8) = -19.21$$

(iii) $A_3 = \dfrac{g_{m4}R_S}{1+g_{m4}R_S}$, $\quad g_{m4} = 2\sqrt{\left(\dfrac{0.1}{2}\right)(40)(0.2)} = 1.265$ mA/V

$$A_3 = \frac{(1.265)(15)}{1+(1.265)(15)} = 0.950$$

(c) $A = A_{d1}\cdot A_2 \cdot A_3 = (2.49)(-19.21)(0.95) = -45.4$

13.30

(a) $I_{D3} = 100 = \left(\dfrac{100}{2}\right)(25)(V_{GS3} - 0.4)^2 \Rightarrow V_{GS3} = 0.6828$ V

$$R_{D1} = \frac{0.6828}{0.05} = 13.66\ \text{k}\Omega$$

$$I_{D4} = 200 = \left(\frac{40}{2}\right)(100)(V_{SG4} - 0.4)^2 \Rightarrow V_{SG4} = 0.7162\ \text{V}$$

$$R_{D3} = \frac{3-(-0.7162)}{0.1} = 37.16\ \text{k}\Omega$$

$$R_S = \frac{3-0}{0.2} = 15\ \text{k}\Omega$$

(b) (i) $A_d = \left(\dfrac{g_{m1}}{2}\right)\cdot R_{D1}$, $\quad g_{m1} = 2\sqrt{\left(\dfrac{0.04}{2}\right)(80)(0.05)} = 0.5657$ mA/V

$$A_d = \left(\frac{0.5657}{2}\right)(13.66) = 3.864$$

(ii) $A_2 = -g_{m3}R_{D3}$, $\quad g_{m3} = 2\sqrt{\left(\dfrac{0.1}{2}\right)(25)(0.1)} = 0.7071$ mA/V

$$A_2 = -(0.7071)(37.16) = -26.28$$

(iii) $A_3 = \dfrac{g_{m4}R_S}{1+g_{m4}R_S}$, $\quad g_{m4} = 2\sqrt{\left(\dfrac{0.04}{2}\right)(100)(0.2)} = 1.265$ mA/V

$$A_3 = \frac{(1.265)(15)}{1+(1.265)(15)} = 0.950$$

(c) $A = A_d \cdot A_2 \cdot A_3 = (3.864)(-26.28)(0.95) = -96.5$

13.31

a. Original g_{m1} and g_{m2}

$$K_{p1} = K_{p2} = \left(\frac{W}{L}\right)\left(\frac{\mu_P C_{ox}}{2}\right) = (12.5)(10)$$

$$= 125\ \mu A/V^2$$

So

$$g_{m1} = g_{m2} = 2\sqrt{K_{p1}\left(\frac{I_Q}{2}\right)} = 2\sqrt{(0.125)(10)}$$

$$= 0.09975\ \text{mA/V}$$

If $\left(\dfrac{W}{L}\right)$ is increased to 50, then

$$K_{p1} = K_{p2} = (50)(10) = 500\ \mu A/V^2$$

So

$$g_{m1} = g_{m2} = 2\sqrt{(0.5)(0.0199)} = 0.1995 \text{ mA/V}$$

b. Gain of first stage

$$A_d = g_{m1}(r_{02} \| r_{04}) = (0.1995)(5025 \| 5025)$$

or

$$\underline{A_d = 501}$$

Voltage gain of second stage remains the same, or

$$A_{v2} = 251$$

Then $A_v = A_d \cdot A_{v2} = (501)(251)$

or

$$\underline{A_d = 125,751}$$

13.32

$$\upsilon_{CM}(\max) = V^+ - \upsilon_{SD6}(\min) - \upsilon_{SG1}$$

$$\upsilon_{SD6}(\min) = \upsilon_{SD}(sat) + 0.2 = 0.9022 - 0.5 + 0.2 = 0.6022 \text{ V}$$

$$i_{D1} = K_{p1}(\upsilon_{SG1} + V_{TP})^2$$

$$20.2 = 250(\upsilon_{SG1} - 0.5)^2 \Rightarrow \upsilon_{SG1} = 0.7843 \text{ V}$$

$$\upsilon_{CM}(\max) = 5 - 0.6022 - 0.7843 = 3.61 \text{ V}$$

$$\upsilon_{CM}(\min) = V^- + \upsilon_{GS3} + \upsilon_{SD1}(\min) - \upsilon_{SG1}$$

$$i_{D3} = K_n(\upsilon_{GS3} - V_{TN})^2, \quad K_n = \left(\frac{0.1}{2}\right)(6.25) = 0.3125 \text{ mA/V}^2$$

$$20.2 = 312.5(\upsilon_{GS3} - 0.5)^2 \Rightarrow \upsilon_{GS3} = 0.7542 \text{ V}$$

$$\upsilon_{SD1}(\min) = 0.7843 - 0.5 + 0.2 = 0.4843 \text{ V}$$

$$\upsilon_{CM}(\min) = -5 + 0.7542 + 0.4843 - 0.7843 = -4.55 \text{ V}$$

So $-4.55 \le \upsilon_{CM} \le 3.61$ V

13.33

(a) $$K_{p5} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_5 = \left(\frac{0.04}{2}\right)(50) = 1.0 \text{ mA/V}^2$$

$$K_{p5}(V_{SG5} + V_{TP})^2 = \frac{V^+ - V_{SG5} - V^-}{R_{set}}$$

$$(1)(50)(V_{SG5}^2 - 1.4V_{SG5} + 0.49) = 10 - V_{SG5}$$

$$50V_{SG5}^2 - 69V_{SG5} + 14.5 = 0 \Rightarrow V_{SG5} = 1.121 \text{ V}$$

$$I_{set} = I_Q = \frac{10 - 1.121}{50} = 0.1776 \text{ mA} = I_{D7}$$

(b) $$A_d = g_{m1}(r_{o2} \| r_{o4}), \quad g_{m1} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)\left(\frac{0.1776}{2}\right)} = 0.5960 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.04)\left(\frac{0.1776}{2}\right)} = 281.5 \text{ k}\Omega, \quad r_{o4} = \frac{1}{(0.02)\left(\frac{0.1776}{2}\right)} = 563.1 \text{ k}\Omega$$

$$A_d = (0.5960)(281.5 \| 563.1) = 111.9$$

$$A_2 = -g_{m7}(r_{o7} \| r_{o8}), \quad g_{m7} = 2\sqrt{\left(\frac{0.1}{2}\right)(30)(0.1776)} = 1.032 \text{ mA/V}$$

$$r_{o7} = \frac{1}{(0.02)(0.1776)} = 281.5 \text{ k}\Omega, \quad r_{o8} = \frac{1}{(0.04)(0.1776)} = 140.8 \text{ k}\Omega$$

$$A_2 = -(1.032)(281.5 \| 140.8) = -96.86$$

$$A = A_d \cdot A_2 = (111.9)(-96.86) = -10{,}839$$

13.34

$$A_d = g_{m1}(r_{o2} \| r_{o4}), \quad g_{m1} = 2\sqrt{\left(\frac{0.04}{2}\right)(50)(0.1)} = 0.6325 \text{ mA/V}$$

$$r_{o2} = \frac{1}{(0.025)(0.1)} = 400 \text{ k}\Omega, \quad r_{o4} = \frac{1}{(0.015)(0.1)} = 666.7 \text{ k}\Omega$$

$$A_d = (0.6325)(400 \| 666.7) = 158.1$$

$$A_2 = -g_{m7}(r_{o7} \| r_{o8}), \quad g_{m7} = 2\sqrt{\left(\frac{0.1}{2}\right)(30)(0.2)} = 1.095 \text{ mA/V}$$

$$r_{o7} = \frac{1}{(0.015)(0.2)} = 333.3 \text{ k}\Omega, \quad r_{o8} = \frac{1}{(0.025)(0.2)} = 200 \text{ k}\Omega$$

$$A_2 = -(1.095)(333.3 \| 200) = -136.9$$

$$A = A_d \cdot A_2 = (158.1)(-136.9) = -21{,}644$$

13.35

$$f_{PD} = \frac{1}{2\pi R_{eq} C_i}$$

where $R_{eq} = r_{04} \| r_{02}$ and $C_i = C_1(1 + |A_{v2}|)$

We can find that

$A_{v2} = 251$ and $r_{04} = r_{02} = 5.025 \text{ M}\Omega$

Now

$$R_{eq} = 5.025 \| 5.025 = 2.51 \text{ M}\Omega$$

and

$$C_i = 12(1 + 251) = 3024 \text{ pF}$$

So

$$f_{PD} = \frac{1}{2\pi(2.51 \times 10^6)(3024 \times 10^{-12})}$$

or

$$\underline{f_{PD} = 21.0 \text{ Hz}}$$

13.36

From Problem 13.33,

$$|A_2| = 96.86, \quad r_{o2} = 281.5\,\text{k}\Omega, \quad r_{o4} = 563.1\,\text{k}\Omega$$

$$f_{PD} = \frac{1}{2\pi(r_{o2} \| r_{o4})C_M}$$

Then $$C_M = \frac{1}{2\pi(8)(281.5 \| 563.1)\times 10^3} = 1.06\times 10^{-7}\,\text{F}$$

$$C_M = C_1(1 + |A_2|) \Rightarrow 1.06\times 10^{-7} = C_1(97.86)$$

Or $$C_1 = 1.08\times 10^{-9}\ \text{F}$$

13.37

$$R_0 = r_{07} \| r_{08}$$

We can find that

$$r_{07} = r_{08} = 2.52\ \text{M}\Omega$$

Then

$$R_0 = 2.52 \| 2.52$$

or

$$\underline{R_0 = 1.26\ \text{M}\Omega}$$

13.38

(a) $$V_{GS2} = \frac{6}{3} = 2\ \text{V}$$

$$I_D = 0.5 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(2 - 0.4)^2 \Rightarrow \left(\frac{W}{L}\right)_{2-5} = 3.91$$

(b) $$A_\upsilon = \frac{g_{m1}(r_{o1} \| r_{o2})}{1 + g_{m1}(r_{o1} \| r_{o2})}, \quad r_{o1} = r_{o2} = \frac{1}{(0.025)(0.5)} = 80\,\text{k}\Omega, \quad r_{o1} \| r_{o2} = 40\,\text{k}\Omega$$

$$0.98 = \frac{g_{m1}(40)}{1 + g_{m1}(40)} \Rightarrow g_{m1} = 1.225\ \text{mA/V}$$

$$g_{m1} = 1.225 = 2\sqrt{\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_1 (0.5)} \Rightarrow \left(\frac{W}{L}\right)_1 = 15$$

(c) $$R_o = \frac{1}{g_{m1}} \| r_{o1} \| r_{o2} = \frac{1}{1.225} \| 40 = 0.8163 \| 40$$

$$R_o = 800\,\Omega$$

13.39

$$I_{Q2} = \left(\frac{80}{2}\right)(20)\left[1.1737 - 0.7\right]^2$$

(a) $$I_{Q2} = 180\ \mu A$$

$$I_{D6}=\left(\frac{80}{2}\right)(25)\left(V_{GS6}-0.7\right)^2=25\Rightarrow V_{GS6}=0.8581\text{ V}$$

$$I_{D7}=\left(\frac{40}{2}\right)(50)\left(V_{SG7}-0.7\right)^2=25\Rightarrow V_{SG7}=0.8581\text{ V}$$

(b)

Set

$$V_{SG8P}=V_{GS8N}=0.8581\text{ V}$$

$$180=\left(\frac{40}{2}\right)\left(\frac{W}{L}\right)_{8P}(0.8581-0.7)^2\Rightarrow\left(\frac{W}{L}\right)_{8P}=360$$

$$180=\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_{8N}(0.8581-0.7)^2\Rightarrow\left(\frac{W}{L}\right)_{8N}=180$$

13.40

$$I_{REF}=I_{Q1}=0.150=\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_{11}\left(V_{GS11}-0.5\right)^2=\left(\frac{0.1}{2}\right)(20)\left(V_{GS11}-0.5\right)^2\Rightarrow V_{GS11}=0.8873\text{ V}$$

$$V_{GS,REF}=5-0.8873=4.1127\text{ V}$$

For three NMOS transistors in series:

$$V_{GS}=\frac{4.1127}{3}=1.3709\text{ V}$$

$$I_{REF}=0.15=\left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(1.3709-0.5)^2\Rightarrow\left(\frac{W}{L}\right)=3.96$$

13.41

(a)

$$I_{Q2}=250\mu A=\left(\frac{80}{2}\right)(5)\left(V_{GS8}-0.7\right)^2$$

$$\Rightarrow V_{GS8}=1.818\text{ V}$$

$$\Rightarrow V_{GS6}=V_{SG7}=\frac{1.818}{2}=0.909\text{ V}$$

$$I_{D6}=I_{D7}=\left(\frac{80}{2}\right)(25)(0.909-0.7)^2=\underline{43.7\ \mu A}$$

(b)

$$g_{m1}=2\sqrt{\left(\frac{80}{2}\right)(15)\left(\frac{250}{2}\right)}\Rightarrow 0.5477\text{ mA/V}$$

$$r_{o2}=\frac{1}{(0.01)(0.125)}=800\,\text{K}$$

$$r_{04}=\frac{1}{(0.015)(0.125)}=533.3\,\text{K}$$

$$A_{d1}=g_{m1}\left(r_{o2}\|r_{o4}\right)=(0.5477)(800\|533.3)$$

$$A_{d1}=175$$

Second stage:

$$A_2 = -g_{m5}(r_{o5} \| r_{o9})$$

$$g_{m5} = 2\sqrt{\left(\frac{40}{2}\right)(80)(250)} \Rightarrow 1.265 \text{ mA/V}$$

$$r_{05} = \frac{1}{(0.015)(0.25)} = 266.7 \text{ K}$$

$$r_{09} = \frac{1}{(0.01)(0.25)} = 400 \text{ K}$$

$$A_2 = -(1.265)(266.7 \| 400)$$

$$A_2 = -202$$

Assume the gain of the output stage ≈ 1, then

$$A_v = A_{d1} \cdot A_2 = (175)(-202)$$

$$A_v = -35,350$$

13.42

(a) $A_d = g_{m1}(R_{o6} \| R_{o8})$

$$g_{m1} = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224 \ \mu A/V$$

$$g_{m1} = g_{m8}$$

$$g_{m6} = 2\sqrt{(0.5)(0.025)} \Rightarrow 224 \ \mu A/V$$

$$r_{o1} = r_{o6} = r_{o8} = r_{o10} = \frac{1}{\lambda I_{DQ}} = \frac{1}{(0.015)(25)} = 2.67 \ M\Omega$$

$$r_{o4} = \frac{1}{\lambda I_{D4}} = \frac{1}{(0.015)(50)} \Rightarrow 1.33 M\Omega$$

Now

$$R_{o8} = g_{m8}(r_{o8} r_{o10}) = (224)(2.67)(2.67) = 1597 \ M\Omega$$

$$R_{o6} = g_{m6}(r_{o6})(r_{o4} \| r_{o1}) = (224)(2.67)(2.67 \| 1.33) \Rightarrow R_{o6} = 531 \ M\Omega$$

Then

$$A_d = (224)(531 \| 1597) \Rightarrow \underline{A_d = 89,264}$$

(b) $R_o = R_{o6} \| R_{o8} = 531 \| 1597 \Rightarrow \underline{R_o = 398 \ M\Omega}$

(c) $$f_{PD} = \frac{1}{2\pi R_o C_L} = \frac{1}{2\pi\left(398\times10^6\right)\left(5\times10^{-12}\right)} \Rightarrow \underline{f_{PD} = 80 \ Hz}$$

$$GBW = (89,264)(80) \Rightarrow \underline{GBW = 7.14 \ MHz}$$

13.43

(a)

$$r_{o1} = r_{o8} = r_{o10} = \frac{1}{\lambda_p I_D} = \frac{1}{(0.02)(25)} = 2\ M\Omega$$

$$r_{o6} = \frac{1}{\lambda_n I_D} = \frac{1}{(0.015)(25)} = 2.67\ M\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(50)} = 1.33\ M\Omega$$

$$g_{m1} = 2\sqrt{\left(\frac{35}{2}\right)\left(\frac{W}{L}\right)_1 (25)} = 41.8\sqrt{\left(\frac{W}{L}\right)_1} = g_{m8}$$

$$g_{m6} = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_6 (25)} = 63.2\sqrt{\left(\frac{W}{L}\right)_6}$$

$$R_o = R_{o6} \| R_{o8} = [g_{m6}(r_{o6})(r_{o4} \| r_{o1})] \| [g_{m8}(r_{o8} r_{o10})]$$

Define $X_1 = \sqrt{\left(\frac{W}{L}\right)_1}$ and $X_6 = \sqrt{\left(\frac{W}{L}\right)_6}$

Then

$$R_o = \left[63.2X_6(2.67)(1.33 \| 2)\right] \| \left[41.8X_1(2)(2)\right]$$

$$= 134.8X_6 \| 167.2X_1 = \frac{22{,}539X_1X_6}{134.8X_6 + 167.2X_1}$$

$$A_d = g_{m1}R_o = (41.8X_1)\left(\frac{22{,}539X_1X_6}{134.8X_6 + 167.2X_1}\right)$$

$$= 10{,}000$$

Now $X_6 = \sqrt{\left(\frac{W}{L}\right)_6} = \sqrt{\frac{1}{2.2}\left(\frac{W}{L}\right)_1} = 0.674X_1$

We then find

$$\underline{X_1^2 = \left(\frac{W}{L}\right)_1 = 4.06 = \left(\frac{W}{L}\right)_p}$$

and

$$\underline{\left(\frac{W}{L}\right)_n = 1.85}$$

13.44

Let $V^+ = 5V,\ V^- = -5V$

$$P = I_T(10) = 3 \Rightarrow I_T = 0.3\ \text{mA} \Rightarrow I_{REF} = 0.1\ mA = 100\ \mu\text{A}$$

$$r_{o1} = r_{o8} = r_{o10} = \frac{1}{(0.02)(50)} = 1\ \text{M}\Omega$$

$$r_{o6} = \frac{1}{(0.015)(50)} = 1.33\ \text{M}\Omega$$

$$r_{o4} = \frac{1}{(0.015)(100)} = 0.667\ M\Omega$$

$$g_{m1} = 2\sqrt{\left(\frac{35}{2}\right)\left(\frac{W}{L}\right)_1 (50)} = 59.2X_1 = g_{m8}$$

where $X_1 = \sqrt{\left(\frac{W}{L}\right)_1}$

Assume all width-to-length ratios are the same.

$$g_{m6} = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)(50)} = 89.4X_1$$

Now

$$R_o = R_{o6} \| R_{o8} = \left[g_{m6}\left(r_{o6}\right)\left(r_{o4} \| r_{o1}\right)\right] \| \left[g_{m8}\left(r_{o8} r_{o10}\right)\right]$$

$$= \left[89.4X_1 (1.33)\left(0.667 \| 1\right)\right] \| \left[59.2X_1 (1)(1)\right]$$

$$= \left[47.6X_1\right] \| \left[59.2X_1\right] = \frac{(47.6X_1)(59.2X_1)}{47.6X_1 + 59.2X_1}$$

So $R_o = 26.4X_1$

Now

$$A_d = g_{m1} R_o = \left(59.2X_1\right)\left(26.4X_1\right) = 25,000$$

So that $X_1^2 = \dfrac{W}{L} = 16$ for all transistors

13.45

(a) $A_d = Bg_{m1}\left(r_{o6} \| r_{o8}\right)$

$$g_{m1} = 2\sqrt{\left(\frac{0.1}{2}\right)(20)(0.06)} = 0.4899 \text{ mA/V}$$

$$r_{o6} = \frac{1}{(0.04)(0.06)(3)} = 138.9 \text{ k}\Omega$$

$$r_{o8} = \frac{1}{(0.02)(0.06)(3)} = 277.8 \text{ k}\Omega$$

$$A_d = (3)(0.4899)(138.9 \| 277.8) = 136.1$$

(b) $R_o = r_{o6} \| r_{o8} = 138.9 \| 277.8 = 92.6 \text{ k}\Omega$

(c) $f_{PD} = \dfrac{1}{2\pi R_o C} = \dfrac{1}{2\pi\left(92.6\times10^3\right)\left(5\times10^{-12}\right)} \Rightarrow f_{PD} = 343.7 \text{ kHz}$

$$GBW = (136.1)\left(343.7\times10^3\right) \Rightarrow GBW = 46.8 \text{ MHz}$$

13.46

$$r_{o6} = \frac{1}{(0.02)(2.5)(40)} = 0.5\ M\Omega$$

$$r_{o8} = \frac{1}{(0.015)(2.5)(40)} = 0.667\ M\Omega$$

$$A_d = Bg_{m1}\left(r_{o6} \| r_{o8}\right)$$

$$400 = (2.5)g_{m1}\left(0.5 \| 0.667\right) \Rightarrow g_{m1} = 560\ \mu A/V$$

$$g_{m1} = 560 = 2\sqrt{\left(\frac{80}{2}\right)\left(\frac{W}{L}\right)(40)} \Rightarrow \underline{\left(\frac{W}{L}\right) = 49}$$

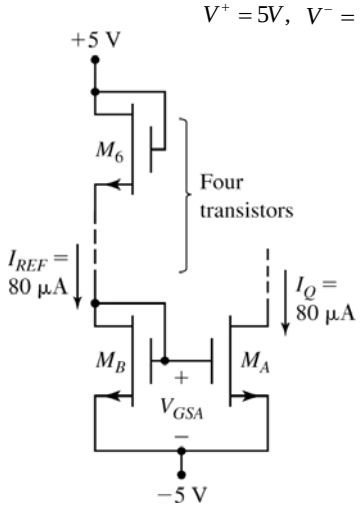
(a)

Assume all (W/L) ratios are the same except for

M_5 and M_6. $\underline{\left(\frac{W}{L}\right)_5 = \left(\frac{W}{L}\right)_6 = 122.5}$

(b) Assume the bias voltages are

$$V^+ = 5V, \quad V^- = -5V.$$



Assume $\left(\frac{W}{L}\right)_A = \left(\frac{W}{L}\right)_B = 49$

$$I_Q = \left(\frac{80}{2}\right)(49)(V_{GSA} - 0.5)^2 = 80 \Rightarrow V_{GSA} = 0.702\ V$$

Then

$$I_{REF} = 80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C (V_{GSC} - 0.5)^2$$

For four transistors

$$V_{GSC} = \frac{10 - 0.702}{4} = 2.325\ V$$

$$80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C (2.325 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_C = 0.60}$$

(c) $$f_{3-dB} = \frac{1}{2\pi R_o C} \quad R_o = 0.5 \| 0.667 = 0.286\ M\Omega$$

$$f_{3-dB} = \frac{1}{2\pi(286\times10^3)(3\times10^{-12})} = 185\ kHz$$

$$GBW = (400)(185\times10^3) \Rightarrow 74\ MHz$$

13.47

(a) From previous results, we can write

$$R_{o10} = g_{m10}(r_{o10} r_{o6})$$

$$R_{o12} = g_{m12}(r_{o12} r_{o8})$$

$$A_d = B g_{m1}\left(R_{o10} \| R_{o12}\right)$$

Now

$$r_{o10} = r_{o6} = \frac{1}{\lambda_P B\left(I_Q/2\right)} = \frac{1}{(0.02)(2.5)(40)} = 0.5\ M\Omega$$

$$r_{o12} = r_{o8} = \frac{1}{\lambda_n B\left(I_Q/2\right)} = \frac{1}{(0.015)(2.5)(40)} = 0.667\ M\Omega$$

Assume all transistors have the same width-to-length ratios except for M_5 and M_6.

Let $\left(\frac{W}{L}\right) = X^2$

Then

$$g_{m10} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_{10}\left(I_{DQ10}\right)} = 2\sqrt{\left(\frac{35}{2}\right)X^2(2.5)(40)}$$

$$= 83.67X$$

$$g_{m12} = 2\sqrt{\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_{12}\left(I_{DQ12}\right)} = 2\sqrt{\left(\frac{80}{2}\right)X^2(2.5)(40)}$$

$$= 126.5\,X$$

$$g_{m1} = 2\sqrt{\left(\frac{80}{2}\right)X^2(40)} = 80\,X$$

Then

$$R_{o10} = (83.67\,X)(0.5)(0.5) = 20.9X\ M\Omega$$

$$R_{o12} = (126.5\,X)(0.667)(0.667) = 56.3X\ M\Omega$$

We want

$$20,000 = (2.5)(80X)[20.9X \,\|\, 56.3X]$$

$$= 200X\left[\frac{(20.9X)(56.3X)}{20.9X + 56.3X}\right] = 3048X^2$$

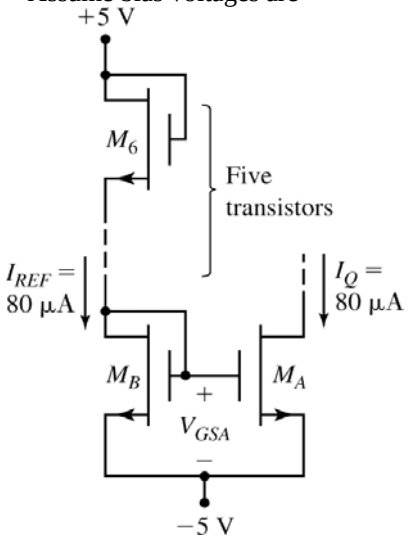
Then

$$X^2 = 6.56 = \left(\frac{W}{L}\right)$$

Then

$$\left(\frac{W}{L}\right)_6 = \left(\frac{W}{L}\right)_5 = (2.5)(6.56) = 16.4$$

(b) Assume bias voltages are $V^+ = 5V,\ V^- = -5V$



Assume $\left(\frac{W}{L}\right)_A = \left(\frac{W}{L}\right)_B = 6.56$

$$I_Q = 80 = \left(\frac{80}{2}\right)(6.56)(V_{GSA} - 0.5)^2 \Rightarrow V_{GSA} = 1.052\ V$$

Need 5 transistors in series

$$V_{GSC} = \frac{10 - 1.052}{5} = 1.79\ V$$

Then

$$I_{REF} = 80 = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_C (1.79 - 0.5)^2 \Rightarrow \underline{\left(\frac{W}{L}\right)_C = 1.20}$$

(c) $f_{3-dB} = \dfrac{1}{2\pi R_o C}$ where $R_o = R_{o10} \| R_{o12}$

Now

$$R_{o10} = 20.9\sqrt{6.56} = 53.5\ M\Omega$$

$$R_{o12} = 56.3\sqrt{6.56} = 144\ M\Omega$$

Then

$$R_o = 53.5 \| 144 = 39\ M\Omega$$

$$f_{3-dB} = \frac{1}{2\pi(39\times10^6)(3\times10^{-12})} = 1.36\ kHz$$

$$GBW = (20,000)(1.36x10^3) \Rightarrow GBW = 27.2\ MHz$$

13.48

(a) $\Delta V_{O1} = 0.7 + (0.3)(0.4) = 0.82\ \text{V}$

$$R_D = \frac{0.82}{0.15} = 5.47\ \text{k}\Omega$$

$$R_C = \frac{3 - 0}{0.3} = 10\ \text{k}\Omega$$

(b) $A_d = \dfrac{g_{m1}}{2}\left(R_D \| R_{iC}\right)$, $g_{m1} = 2\sqrt{\left(\dfrac{0.04}{2}\right)(50)(0.15)} = 0.7746\ \text{mA/V}$

$$R_{iC} = r_\pi + (1+\beta)R_E,\quad r_\pi = \frac{(120)(0.026)}{0.3} = 10.4\ \text{k}\Omega$$

$$R_{iC} = 10.4 + (121)(0.4) = 58.8\ \text{k}\Omega$$

$$A_d = \left(\frac{0.7746}{2}\right)(5.47 \| 58.8) = 1.938$$

(c) $A_2 = \dfrac{-\beta R_C}{r_\pi + (1+\beta)R_E} = \dfrac{-(120)(10)}{10.4 + (121)(0.4)} = -20.41$

(d) $A_v = A_d \cdot A_2 = (1.938)(-20.41) = -39.6$

13.49

(a) $R_S = \dfrac{0 - (-3)}{0.3} = 10\ \text{k}\Omega$

$$I_{D1} = K_n\left(V_{GS1} - V_{TN}\right)^2$$

$$0.3 = 3(V_{GS1} - 0.4)^2 \Rightarrow V_{GS1} = 0.7162\ \text{V}$$

$$R_{C2} = \frac{0.7162 - (-3)}{0.3} = 12.39\ \text{k}\Omega$$

$$\Delta V_{O1} = (0.3)(0.5) + 0.7 = 0.85\ \text{V}$$

$$R_{C1} = \frac{0.85}{0.25} = 3.4\ \text{k}\Omega$$

(b) $$g_{m1} = \frac{0.25}{0.026} = 9.615\ \text{mA/V}, \quad r_{\pi 1} = \frac{(120)(0.026)}{0.25} = 12.48\ \text{k}\Omega$$

$$g_{m3} = \frac{0.3}{0.026} = 11.54\ \text{mA/V}, \quad r_{\pi 3} = \frac{(120)(0.026)}{0.3} = 10.4\ \text{k}\Omega$$

$$A_d = \frac{g_{m1}}{2}\left(R_{C1} \| R_{i3}\right)$$

$$R_{i3} = r_{\pi 3} + (1+\beta)R_E = 10.4 + (121)(0.5) = 70.9\ \text{k}\Omega$$

$$A_d = \frac{9.615}{2}\left(3.4 \| 70.9\right) = 15.60$$

(c) $$A_2 = \frac{-\beta R_{C2}}{r_{\pi 3} + (1+\beta)R_E} = \frac{-(120)(12.39)}{10.4 + (121)(0.5)} = -20.97$$

(d) $$A_3 = \frac{g_{mD}R_S}{1 + g_{mD}R_S}, \quad g_{mD} = 2\sqrt{(3)(0.3)} = 1.897\ \text{mA/V}$$

$$A_3 = \frac{(1.897)(10)}{1 + (1.897)(10)} = 0.950$$

(e) $$A_v = A_d \cdot A_2 \cdot A_3 = (15.6)(-20.97)(0.95) = -310.8$$

13.50

(a) For PMOS:

$$g_{m1} = g_{m2} = 2\sqrt{\left(\frac{0.04}{2}\right)(40)(0.125)} = 0.6325\ \text{mA/V}$$

$$r_{oP} = \frac{1}{\lambda I_{D1}} = \frac{1}{(0.035)(0.125)} = 228.6\ \text{k}\Omega$$

For BJT:

$$r_{o2} = \frac{V_A}{I_C} = \frac{150}{0.125} = 1200\ \text{k}\Omega$$

(b) $$A_d = g_{m2}\left(r_{oP} \| r_{o2}\right) = (0.6325)(228.6 \| 1200) = 121.5$$

13.51

(a) For NMOS:

$$g_{mN} = 2\sqrt{\left(\frac{0.1}{2}\right)(40)(0.125)} = 1.0\ \text{mA/V}$$

$$r_{oN} = \frac{1}{(0.02)(0.125)} = 400\ \text{k}\Omega$$

For BJT:

$$r_{o2} = \frac{100}{0.125} = 800\ \text{k}\Omega$$

(b) $$A_d = g_{mN}\left(r_{oN} \| r_{o2}\right) = (1.0)(400 \| 800) = 266.7$$

13.52

$I_{REF} = 200\ \mu A \qquad K_n = K_p = 0.5\ mA/V^2$

$\lambda_n = \lambda_p = 0.015\ V^{-1}$

(a) $A_d = g_{m1}\left(R_{o6} \| R_{o8}\right)$

where

$R_{o8} = g_{m8}(r_{o8} r_{o10})$

$R_{o6} = g_{m6}(r_{o6})\left(r_{o4} \| r_{o1}\right)$

Now

$g_{m8} = 2\sqrt{K_P I_{D8}} = 2\sqrt{(0.5)(0.1)} = 0.447\ mA/V$

$r_{o8} = \dfrac{1}{\lambda_P I_{D8}} = \dfrac{1}{(0.015)(0.1)} = 667\ k\Omega$

$r_{o10} = \dfrac{1}{\lambda_P I_{D8}} = 667\ k\Omega$

$g_{m6} = \dfrac{I_{C6}}{V_T} = \dfrac{0.1}{0.026} = 3.846\ mA/V$

$r_{o6} = \dfrac{V_A}{I_{C6}} = \dfrac{80}{0.1} = 800\ k\Omega$

$r_{o4} = \dfrac{1}{\lambda_n I_{D4}} = \dfrac{1}{(0.015)(0.2)} = 333\ k\Omega$

$r_{o1} = \dfrac{1}{\lambda_p I_{D1}} = \dfrac{1}{(0.015)(0.1)} = 667\ k\Omega$

$g_{m1} = 2\sqrt{K_P I_{D1}} = 2\sqrt{(0.5)(0.1)} = 0.447\ mA/V$

So

$R_{o8} = (0.447)(667)(667) \Rightarrow 198.9\ M\Omega$

$R_{o6} = (3.846)(800)(333 \| 667) \Rightarrow 683.4\ M\Omega$

Then

$A_d = 447(198.9 \| 683.4) \Rightarrow \underline{A_d = 68{,}865}$

13.53

Assume biased at $V^+ = 10V,\ V^- = -10V.$

$P = 3I_{REF}(20) = 10 \Rightarrow I_{REF} = 167\ \mu A$

$A_d = g_{m1}\left(R_{o6} \| R_{o8}\right) = 25{,}000$

$k_n' = 80\mu A/V^2, k_p' = 35\ \mu A/V^2$

$\lambda_n = 0.015V^{-1}, \lambda_p = 0.02\ V^{-1}$

Assume $\left(\dfrac{W}{L}\right)_p = 2.2\left(\dfrac{W}{L}\right)_n$

$R_{o8} = g_{m8}\left(r_{o8} r_{o10}\right)$

$R_{o6} = g_{m6}(r_{o6})(r_{o4} \| r_{o1})$

$r_{o8} = \dfrac{1}{\lambda_P I_{D8}} = \dfrac{1}{(0.02)(83.3)} = 0.60\ M\Omega$

$r_{o10} = \dfrac{1}{\lambda_P I_{D8}} = 0.60\ M\Omega$

$g_{m8} = 2\sqrt{\left(\dfrac{k_p'}{2}\right)\left(\dfrac{W}{L}\right)_8 I_{D8}} = 2\sqrt{\left(\dfrac{35}{2}\right)(2.2)X^2(83.3)}$

$= 113.3X$

where $X^2 = \left(\frac{W}{L}\right)_n$

$$r_{o6} = \frac{V_A}{I_{C6}} = \frac{80}{83.3} = 0.960\ M\Omega$$

$$r_{o4} = \frac{1}{\lambda_n I_{D4}} = \frac{1}{(0.015)(167)} = 0.40\ M\Omega$$

$$r_{o1} = \frac{1}{\lambda_p I_{D1}} = \frac{1}{(0.02)(83.3)} = 0.60\ M\Omega$$

$$g_{m6} = \frac{I_{C6}}{V_T} = \frac{83.3}{0.026} = 3204\ \mu A/V$$

$$g_{m1} = 2\sqrt{\left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_1 I_{D1}} = 2\sqrt{\left(\frac{35}{2}\right)(2.2)X^2(83.3)}$$

$$= 113.3X$$

Now

$$R_{o6} = (3204)(0.960)\left[0.40 \| 0.60\right] = 738\ M\Omega$$

$$R_{o8} = (113.3X)(0.60)(0.60) = 40.8X\ M\Omega$$

Then

$$A_d = 25{,}000 = (113.3X)\left[738 \| 40.8X\right]$$

$$= (113.3X)\left[\frac{30{,}110X}{738 + 40.8X}\right]$$

which yields $X = 2.48$

or

$$\underline{X^2 = 6.16 = \left(\frac{W}{L}\right)_n}$$

and

$$\underline{\left(\frac{W}{L}\right)_P + (2.2)(6.16) = 12.3}$$

13.54

For v_{cm} (max), assume $V_{CB}(Q_5) = 0.$ Then

$$V_S = 15 - 0.6 - 0.6 = 13.8\ \text{V}$$

$$I_{D9} = I_{D10} = \frac{0.236}{2} = 0.118\ \text{mA}$$

Using parameters given in Example 13.12

$$V_{SG} = \sqrt{\frac{I_{D9}}{K_P}} - V_{TP} = \sqrt{\frac{0.118}{0.20}} + 1.4 = 2.17\ V$$

Then

$$v_{cm}(\text{max}) = 13.8 - 2.17 \Rightarrow v_{cm}(\text{max}) = 11.6\ \text{V}$$

For

$v_{cm}(\text{min})$, assume

$$V_{SD}(M_9) = V_{SD}(sat) = V_{SG} + V_{TP} = 2.17 - 1.4 = 0.77\ \text{V}$$

Now

$$V_{D10} = I_{D10}(0.5) + 0.6 + I_{D10}(0.5) - 15$$

$$= 0.118 + 0.6 - 15 \Rightarrow V_{D10} = -14.28\ \text{V}$$

Then

$$v_{cm}(\min) = -14.28 + V_{SD}(\text{sat}) - V_{SG}$$
$$= -14.28 + 0.77 - 2.17 = -15.68 \text{ V}$$

Then, common-mode voltage range

$$\underline{-15.68 \le v_{cm} \le 11.6}$$

Or, assuming the input is limited to ± 15 V, then

$$\underline{-15 \le v_{cm} \le 11.6 \text{ V}}$$

13.55

$$I_1 = I_2$$

$$K_p\left(V_{SG} + V_{TP}\right)^2 = \frac{V_{SG} - V_{BE7}}{R_1}$$

$$I_2 = 0.15 = \frac{V_{SG} - 0.6}{8} \Rightarrow V_{SG} = 1.8 \text{ V}$$

$$I_1 = 0.15 = K_p(1.8 - 1)^2 \Rightarrow K_p = 0.234 \text{ mA/V}^2$$

13.56

(a) $K_p\left(V_{SG} + V_{TP}\right)^2 = \dfrac{V_{SG} - V_{BE7}}{R_1}$

$$0.15(V_{SG} - 1.2)^2 = \frac{V_{SG} - 0.6}{8}$$

We find $1.2V_{SG}^2 - 3.88V_{SG} + 2.328 = 0$, $\Rightarrow V_{SG} = 2.437$ V

$$I_1 = I_2 = \frac{2.437 - 0.6}{8} = 0.2296 \text{ mA}$$

(b) $V_{C7} = V^+ - 0.6 - 0.6$

$$V_{C6} = V^- + 2.437$$

Set $V_{C6} = V_{C7}$

Then $V_S - 1.2 = -V_S + 2.437 \Rightarrow V_S = 1.82$ V

13.57

$$I_{C5} = I_{C4} = 300 \ \mu\text{A}$$

We have

$$R_{i2} = r_{\pi 13} = \frac{\beta_n V_T}{I_{C13}} = \frac{(200)(0.026)}{0.3} = 17.3 \text{ k}\Omega$$

$$A_d = \sqrt{2K_n I_{Q5}} \cdot (R_{i2}) = \sqrt{2(0.6)(0.3)} \cdot (17.3)$$

or

$$\underline{A_d = 10.38}$$

Now

$$g_{m13} = \frac{I_{C13}}{V_T} = \frac{0.3}{0.026} = 11.5 \text{ mA/V}$$

$$r_{013} = \frac{V_A}{I_{C13}} = \frac{50}{0.3} = 167 \text{ k}\Omega$$

Then

$$|A_{v2}| = g_{m13} \cdot r_{013} = (11.5)(167)$$

or

$$\underline{|A_{v2}| = 1917}$$

Overall gain:

$$\underline{|A_v| = (10.38)(1917) = 19,895}$$

13.58

Assuming the resistances looking into Q_4 and into the output stage are very large, we have

$$|A_{v2}| = \frac{\beta R_{013}}{r_{\pi 13} + (1+\beta)R_{E13}}$$

where $R_{013} = r_{013}\left[1 + g_{m13}\left(R_{E13} \| r_{\pi 13}\right)\right]$

$$I_{C13} = 300\ \mu A,\ r_{013} = \frac{50}{0.3} = 167 \text{ k}\Omega$$

$$g_{m13} = \frac{0.3}{0.026} = 11.5 \text{ mA/V}$$

$$r_{\pi 13} = \frac{(200)(0.026)}{0.3} = 17.3 \text{ k}\Omega$$

So

$$R_{013} = (167)\left[1 + (11.5)\left(1 \| 17.3\right)\right] \Rightarrow 1.98 \text{ M}\Omega$$

Then

$$|A_{v2}| = \frac{(200)(1980)}{17.3 + (201)(1)} = 1814$$

Now

$$C_i = C_1(1 + |A_{v2}|) = 12\left[1 + 1814\right]$$

$$\Rightarrow C_i = 21,780 \text{ pF}$$

$$f_{PD} = \frac{1}{2\pi R_{eq} C_i}$$

$$R_{eq} = R_{i2} \| r_{012} \| r_{010}$$

Neglecting R_3,

$$r_{010} = \frac{1}{\lambda I_{D10}} = \frac{1}{(0.02)(0.15)} = 333 \text{ k}\Omega$$

Neglecting R_5,

$$r_{012} = \frac{50}{0.15} = 333 \text{ k}\Omega$$

$$R_{i2} = r_{\pi 13} + (1+\beta)R_{E13} = 17.3 + (201)(1)$$

$$= 218 \text{ k}\Omega$$

Then

$$f_{PD} = \frac{1}{2\pi\left[218 \| 333 \| 333\right] \times 10^3 \times (21,780) \times 10^{-12}}$$

or

$$\underline{f_{PD} = 77.4 \text{ Hz}}$$

Unity-Gain Bandwidth

Gain of first stage:

$$A_d = \sqrt{2K_n I_{Qs}} \cdot (R_{12} \| r_{o12} \| r_{o10})$$
$$= \sqrt{2(0.6)(0.3)} \cdot (218 \| 333 \| 333)$$
$$= (0.6)(218 \| 333 \| 333)$$

or $A_d = 56.6$

Overall gain:

$$A_v = (56.6)(1814) = 102,672$$

Then unity-gain bandwidth $= (77.4)(102,672)$

$$\Rightarrow \underline{7.95 \text{ MHz}}$$

13.59

Since $V_{GS} = 0$ in J_6, $I_{REF} = I_{DSS}$

$$\Rightarrow \underline{I_{DSS} = 0.8\ mA}$$

13.60

a. $$R_{i2} = r_{\pi 5} + (1+\beta)\left[r_{\pi 6} + (1+\beta)R_E\right]$$

$$r_{\pi 6} = \frac{(100)(0.026)}{0.2} = 13 \text{ k}\Omega$$

$$I_{C5} \cong \frac{I_{C6}}{\beta} = \frac{200\ \mu\text{A}}{100} = 2\ \mu\text{A}$$

So

$$r_{\pi 5} = \frac{(100)(0.026)}{0.002} = 1300 \text{ k}\Omega$$

Then

$$R_{i2} = 1300 + (101)\left[13 + (101)(0.3)\right]$$

or

$$\underline{R_{i2} = 5.67 \text{ M}\Omega}$$

b. $$A_v = g_{m2}\left(r_{02} \| r_{04} \| R_{i2}\right)$$

$$g_{m2} = \frac{2}{V_P} \cdot \sqrt{I_D \cdot I_{DSS}} = \frac{2}{3} \cdot \sqrt{(0.1)(0.2)}$$
$$= 0.0943 \text{ mA/V}$$

$$r_{02} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.1)} = 500 \text{ k}\Omega$$

$$r_{04} = \frac{V_A}{I_{C4}} = \frac{5.0}{0.1} = 500 \text{ k}\Omega$$

Then

$$A_v = (0.0943)[500 \,||\, 500 \,||\, 5670]$$

or

$\underline{A_v = 22.6}$

13.61

a. Need $V_{SD}(Q_E) \geq V_{SD}(\text{sat}) = V_P$ For minimum bias ± 3 V

Set $\underline{V_P = 3 \text{ V}}$ and $\underline{V_{ZK} = 3 \text{ V}}$

$$I_{REF2} = \frac{V_{ZK} - V_{D1}}{R_3}$$

so that $R_3 = \dfrac{3 - 0.6}{0.1} \Rightarrow \underline{R_3 = 24 \text{ k}\Omega}$

Set bias in $Q_E = I_{REF2} + I_{Z2} = 0.1 + 0.1 = 0.2$ mA

Therefore,

$$\underline{I_{DSS} = 0.2 \text{ mA}}$$

b. Neglecting base currents

$$I_{01} = I_{REF1} = 0.5 \text{ mA} = \frac{12 - 0.6}{R_4}$$

so that

$$\underline{R_4 = 22.8 \text{ k}\Omega}$$

13.62

a. We have

$$g_{m2} = \frac{2}{|V_P|} \cdot \sqrt{I_D \cdot I_{DSS}} = \frac{2}{4} \cdot \sqrt{(0.5)(1)}$$

$$= 0.354 \text{ mA/V}$$

$$r_{02} = \frac{1}{\lambda I_D} = \frac{1}{(0.02)(0.5)} = 100 \text{ k}\Omega$$

$$r_{04} = \frac{V_A}{I_D} = \frac{100}{0.5} = 200 \text{ k}\Omega$$

$$g_{m4} = \frac{0.5}{0.026} = 19.23 \text{ mA/V}$$

$$r_{\pi 4} = \frac{(200)(0.026)}{0.5} = 10.4 \text{ k}\Omega$$

So

$$R_{04} = r_{04}\left[1 + g_{m4}\left(r_{\pi 4} \| R_2\right)\right]$$

$$= 200\left[1 + (19.23)(10.4 \| 0.5)\right]$$

$$= 2035 \text{ k}\Omega$$

$$|A_d| = g_{m2}\left(r_{02} \| R_{04} \| R_L\right)$$

$R_L \to \infty$

For $|A_d| = 0.354(100 \,||\, 2035) = 33.7$

With these parameter values, gain can never reach 500.

b. Similarly for this part, gain can never reach 700.

Chapter 14

14.1

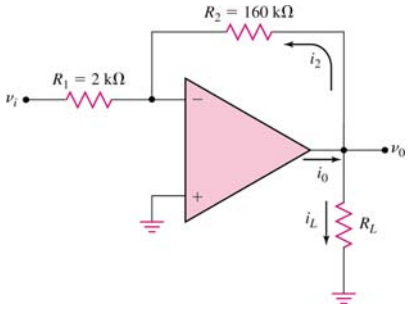
$$A_d = \frac{v_o}{v_i} = -80$$

$$v_o(\text{max}) = 4.5 \Rightarrow v_i(\text{max}) = 56.25 \text{ mV}$$

$$v_i(\text{max})_{rms} = \frac{56.25}{\sqrt{2}} = \underline{\underline{39.77 \text{ mV}}}$$

So

14.2



(a)

$$i_2 = \frac{4.5}{160} = 0.028125 \text{ mA}$$

$$i_L = \frac{4.5}{1} = 4.5 \text{ mA}$$

Output Circuit $= 4.528$ mA

$$v_i = -\frac{v_o}{A} = \frac{-4.5}{80} \Rightarrow v_i = -0.05625 \text{ V}$$

(b)

$$i_o \approx 15 \text{ mA} = \frac{v_o}{R_L} = \frac{4.5}{R_L}$$

$$\Rightarrow \underline{R_L(\text{min}) = 300\,\Omega}$$

14.3

(1) $v_o = 2$ V

(2) $v_2 = 12.5$ mV

(3) $A_{OL} = 2 \times 10^4$

(4) $v_1 = 8\ \mu$V

(5) $A_{OL} = 1000$

14.4

(a) $A_{CL}(\infty) = -\dfrac{R_2}{R_1} = -\dfrac{120}{5.6} = -21.42857$

$$A_{CL} = \frac{-21.42857}{1+\dfrac{22.42857}{10^5}} = -21.42376$$

$$\frac{-21.42376-(-21.42857)}{-21.42857}\times 100\% = -0.0224\%$$

(b) $A_{CL}(\infty) = -\dfrac{R_2}{R_1} = -\dfrac{120}{8.2} = -14.634146$

$$A_{CL} = \frac{-14.634146}{1+\dfrac{15.634146}{10^5}} = -14.63186$$

$$\frac{-14.63186-(-14.634146)}{-14.634146}\times 100\% = -0.0156\%$$

14.5

(a) (i) $$A_{CL} = \frac{1+\dfrac{47}{6.8}}{1+\dfrac{\left(1+\dfrac{47}{6.8}\right)}{2\times 10^4}} = \frac{7.91176}{1+\dfrac{7.91176}{2\times 10^4}} = 7.90863$$

(ii) $$\frac{7.90863-7.91176}{7.91176}\times 100\% = -0.03956\%$$

(b) (i) $$A_{CL} = \frac{7.91176}{1+\dfrac{7.91176}{10^3}} = 7.84966$$

(ii) $$\frac{7.84966-7.91176}{7.91176}\times 100\% = -0.785\%$$

14.6

(a) $$-15.0 = \frac{-R_2/R_1}{1+\dfrac{\left(1+R_2/R_1\right)}{2\times 10^3}} = \frac{-R_2/R_1}{1.0005+5\times 10^{-4}\left(R_2/R_1\right)} \Rightarrow \frac{R_2}{R_1} = 15.12091$$

(b) $$A_{CL} = \frac{-15.12091}{1+\dfrac{16.12091}{5\times 10^4}} = -15.1160$$

14.7

$$(1-0.0001)(90) = \frac{90}{1+\dfrac{90}{A_{OL}}} \Rightarrow A_{OL} = 8.9991\times 10^5$$

14.8

$$A_{CL} = (1-0.0002)(1) = \frac{1}{1+\dfrac{1}{A_{OL}}} \Rightarrow A_{OL} = 4999$$

14.9

(a) $$|A| = +\frac{R_2}{R_1} = \frac{210(1\pm 0.001)}{21(1\pm 0.001)}$$

$$|A|_{\max} = \frac{210.21}{20.979} = 10.02$$

$$|A|_{\min} = \frac{209.79}{21.021} = 9.98$$

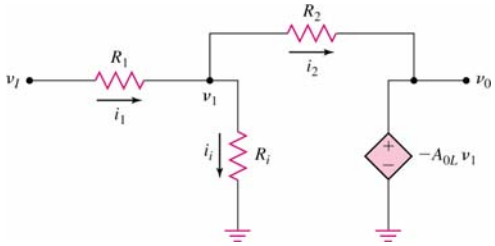
So $9.98 \le |A| \le 10.02$

(b) $$|A|_{\max} = \frac{10.02}{1+\dfrac{11.02}{10^4}} = 10.009$$

$$|A|_{\min} = \frac{9.98}{1+\dfrac{10.98}{10^4}} = 9.969$$

So $9.969 \le |A| \le 10.009$

14.10



$$\frac{v_I - v_1}{R_1} = \frac{v_1 - v_0}{R_2} + \frac{v_1}{R_i} \text{ and } v_0 = -A_{0L}v_1$$

so that $$v_1 = -\frac{v_0}{A_{0L}}$$

$$\frac{v_I}{R_1} + \frac{v_0}{R_2} = v_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_i}\right)$$

So

$$\frac{v_I}{R_1} = -v_0\left[\frac{1}{R_2}+\frac{1}{A_{0L}}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_i}\right)\right]$$

Then

$$\frac{v_0}{v_I} = \frac{-(1/R_1)}{\left[\dfrac{1}{R_2}+\dfrac{1}{A_{0L}}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_i}\right)\right]} = A_{CL}$$

From Equation (14.20) for $R_L = \infty$ and $R_0 = 0$

$$\frac{1}{R_{if}} = \frac{1}{R_i} + \frac{1}{R_2} \cdot \frac{(1 + A_{0L})}{1}$$

a. For $R_i = 1\ \text{k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[\frac{1}{100} + \frac{1}{10^3}\left(\frac{1}{20} + \frac{1}{100} + \frac{1}{1}\right)\right]}$$

$$= \frac{-0.05}{[0.01 + 1.06 \times 10^{-3}]}$$

or

$$\Rightarrow \underline{A_{CL} = -4.52}$$

$$\frac{1}{R_{if}} = \frac{1}{1} + \frac{1 + 10^3}{100} \Rightarrow \underline{R_{if} = 90.8\ \Omega}$$

b. For $R_i = 10\ \text{k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[\frac{1}{100} + \frac{1}{10^3}\left(\frac{1}{20} + \frac{1}{100} + \frac{1}{10}\right)\right]}$$

$$= \frac{-0.05}{[0.01 + 1.6 \times 10^{-4}]}$$

or

$$\Rightarrow \underline{A_{CL} = -4.92}$$

$$\frac{1}{R_{if}} = \frac{1}{10} + \frac{1 + 10^3}{100} \Rightarrow \underline{R_{if} = 98.9\ \Omega}$$

c. For $R_i = 100\ \text{k}\Omega$

$$A_{CL} = \frac{-(1/20)}{\left[\frac{1}{100} + \frac{1}{10^3}\left(\frac{1}{20} + \frac{1}{100} + \frac{1}{100}\right)\right]}$$

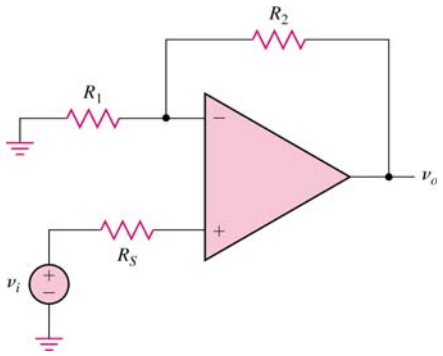
$$= \frac{-0.05}{[0.01 + 7 \times 10^{-5}]}$$

or

$$\Rightarrow \underline{A_{CL} = -4.965}$$

$$\frac{1}{R_{if}} = \frac{1}{100} + \frac{1 + 10^3}{100} \Rightarrow \underline{R_{if} = 99.8\ \Omega}$$

14.11



$$A_{CL} = \frac{v_o}{v_i} = \frac{\left(1+\frac{R_2}{R_1}\right)}{\left[1+\frac{1}{A_{OL}}\left(1+\frac{R_2}{R_1}\right)\right]}$$

For the ideal:

$$\left(1+\frac{R_2}{R_1}\right) = \frac{0.10}{0.002} = 50$$

$$v_o(actual) = (0.10)(1-0.001) = 0.0999$$

So

$$\frac{0.0999}{0.002} = \frac{50}{1+\frac{1}{A_{OL}}(50)} = 49.95$$

which yields

$$\underline{A_{OL} = 1000}$$

14.12

From Equation (14.18)

$$A_{vf1} = \frac{v_{o1}}{v_1} = \frac{-\left(\frac{A_{OL}}{R_o}-\frac{1}{R_2}\right)}{\left(\frac{1}{R_L}+\frac{1}{R_o}+\frac{1}{R_2}\right)}$$

Or

$$v_{o1} = \frac{-\left(\frac{5\times10^3}{1}-\frac{1}{100}\right)}{\left(\frac{1}{10}+\frac{1}{1}+\frac{1}{100}\right)}\cdot v_1 = \frac{-(4.99999\times10^3)}{1.11}\cdot v_1$$

$$v_{o1} = -4.504495\times10^3 \cdot v_1$$

Now

$$\frac{i_1}{v_1} = \frac{v_i - v_1}{R_1 v_1} \equiv K$$

Then

$$v_i - v_1 = KR_1 v_1$$

which yields

$$v_1 = \frac{v_i}{KR_1 + 1}$$

Now, from Equation (14.20)

$$K = \frac{1}{10}+\frac{1}{100}\left[\frac{1+5\times10^3+\frac{1}{10}}{1+\frac{1}{10}+\frac{1}{100}}\right]$$

$$= (0.1)+(0.01)\left[\frac{5.0011\times10^3}{1.11}\right] = 45.15495$$

Then

$$v_1 = \frac{v_i}{(45.15495)(10)+1} = \frac{v_i}{452.5495}$$

We find

$$v_{o1} = -4.504495\times10^{3}\left[\frac{v_i}{452.5495}\right]$$

Or

$$\underline{A_{vf1} = \frac{v_{o1}}{v_i} = -9.9536}$$

For the second stage, $R_L = \infty$

$$v_{o2} = \frac{-\left(\frac{5\times10^3}{1} - \frac{1}{100}\right)}{\left(\frac{1}{1}+\frac{1}{100}\right)}\cdot v_1' = -4.950485\times10^{3}\cdot v_1'$$

$$K \equiv \frac{1}{10}+\frac{1}{100}\left[\frac{1+5\times10^3}{1+\frac{1}{100}}\right] = 49.61485$$

$$v_1' = \frac{v_{o1}}{KR_1+1} = \frac{v_{o1}}{(49.61485)(10)+1} = \frac{v_{o1}}{497.1485}$$

Then

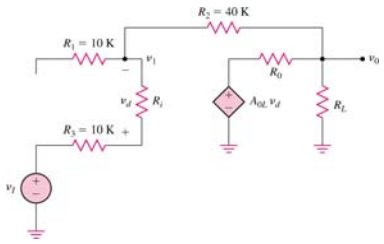
$$\frac{v_{o2}}{v_{o1}} = \frac{-4.950485\times10^{3}}{497.1485} = -9.95776$$

So

$$A_{vf} = \frac{v_{o2}}{v_i} = (-9.9536)(-9.95776) \Rightarrow \underline{A_{vf} = 99.12}$$

14.13

a.



$$\frac{v_1 - v_I}{R_3 + R_i} + \frac{v_1}{R_1} + \frac{v_1 - v_0}{R_2} = 0$$

$$v_1\left[\frac{1}{R_3+R_i}+\frac{1}{R_1}+\frac{1}{R_2}\right] = \frac{v_0}{R_2}+\frac{v_I}{R_3+R_i} \qquad (1)$$

$$\frac{v_0}{R_L} + \frac{v_0 - A_{0L}v_d}{R_0} + \frac{v_0 - v_1}{R_2} = 0$$

$$v_0\left[\frac{1}{R_L}+\frac{1}{R_0}+\frac{1}{R_2}\right] = \frac{v_1}{R_2}+\frac{A_{0L}v_d}{R_0} \qquad (2)$$

or

$$v_d = \left(\frac{v_I - v_1}{R_3 + R_i}\right)\cdot R_i \qquad (3)$$

So substituting numbers:

$$v_1\left[\frac{1}{10+20}+\frac{1}{10}+\frac{1}{40}\right]=\frac{v_0}{40}+\frac{v_I}{10+20} \quad (1)$$

or

$$v_1[0.15833]=v_0[0.025]+v_I[0.03333]$$

$$v_0\left[\frac{1}{1}+\frac{1}{0.5}+\frac{1}{40}\right]=\frac{v_1}{40}+\frac{(10^4)v_d}{0.5} \quad (2)$$

or

$$v_0\left[3.025\right]=v_1\left[0.025\right]+\left(2\times10^4\right)v_d$$

$$v_d=\left(\frac{v_I-v_1}{10+20}\right)\cdot 20=0.6667\left(v_I-v_1\right) \quad (3)$$

So

$$v_0\left[3.025\right]=v_1\left[0.025\right]+\left(2\times10^4\right)\left(0.6667\right)\left(v_I-v_1\right) \quad (2)$$

or

$$v_0\left[3.025\right]=1.333\times10^4 v_I-1.333\times10^4 v_1$$

From (1):

$$v_1=v_0\left(0.1579\right)+v_I\left(0.2105\right)$$

Then

$$v_0\left[3.025\right]=1.333\times10^4 v_I-1.333\times10^4\left[v_0\left(0.1579\right)+v_I\left(0.2105\right)\right]$$

$$v_0\left[2.1078\times10^3\right]=v_I\left[1.0524\times10^4\right]$$

or

$$\underline{A_{CL}=\frac{v_0}{v_I}=4.993}$$

To find R_{if}: Use Equation (14.27)

$$i_I\left(1+\frac{0.5}{1}+\frac{0.5}{40}\right)$$

$$=v_1\left\{\left(\frac{1}{10}+\frac{1}{40}\right)\left(1+\frac{0.5}{1}+\frac{0.5}{40}\right)-\frac{0.5}{(40)^2}\right\}-\frac{\left(10^3\right)v_d}{40}$$

$$i_I(1.5125)=v_1\{(0.125)(1.5125)-0.0003125\}-25v_d$$

or

$$i_I(1.5125)=v_I\{0.18875\}-25v_d$$

Now

$$v_d=i_I R_i=i_I(20) \text{ and } v_1=v_I-i_I(20)$$

So

$$i_I(1.5125)=[v_I-i_I(20)]\cdot[0.18875]-25i_I(20)$$

$$i_I[505.3]=v_I(0.18875)$$

or

$$\frac{v_I}{i_I}=2677\ \text{k}\Omega$$

Now $R_{if}=10+2677\Rightarrow \underline{R_{if}=2.687\ \text{M}\Omega}$

To determine R_{0f}: Using Equation (14.36)

$$\frac{1}{R'_{0f}}=\frac{1}{R_0}\cdot\left[\frac{A_{0L}}{1+\frac{R_2}{R_1\|R_i}}\right]=\frac{1}{0.5}\cdot\left[\frac{10^3}{1+\frac{40}{10\|20}}\right]$$

or $R'_{0f} = 3.5\ \Omega$

Then $R_{0f} = 1\ \text{k}\Omega \| 3.5\ \Omega$

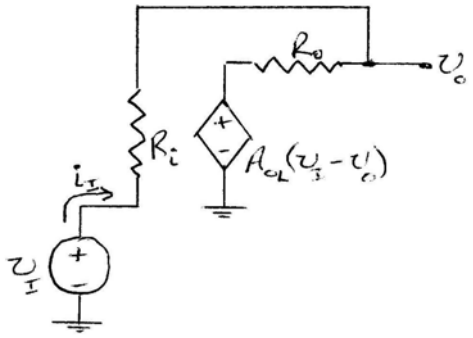
$\Rightarrow \underline{R_{0f} = 3.49\ \Omega}$

b. Using Equation (14.16)

$$\frac{dA_{CL}}{A_{CL}} = (-10)\left(\frac{5}{10^3}\right) \Rightarrow \underline{\frac{dA_{CL}}{A_{CL}} = -(0.05)\%}$$

14.14

(a)



(b) (i) $\dfrac{\upsilon_I - \upsilon_O}{R_i} = \dfrac{\upsilon_O - A_{OL}(\upsilon_I - \upsilon_O)}{R_o}$

$$\frac{\upsilon_I}{R_i} + \frac{A_{OL}\upsilon_I}{R_o} = \upsilon_O\left(\frac{1}{R_i} + \frac{1}{R_o} + \frac{A_{OL}}{R_o}\right)$$

$$\upsilon_I\left(\frac{1}{10} + \frac{5\times10^3}{1}\right) = \upsilon_O\left(\frac{1}{10} + \frac{1}{1} + \frac{5\times10^3}{1}\right)$$

$$\upsilon_I\left(5.0001\times10^3\right) = \upsilon_O\left(5.0011\times10^3\right)$$

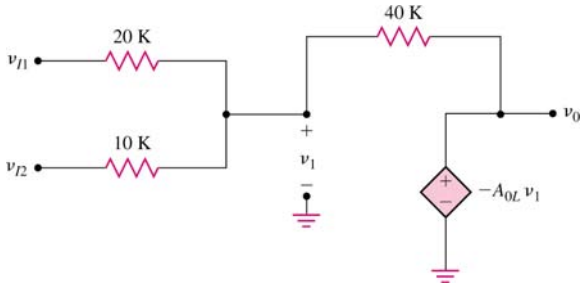
$$\frac{\upsilon_O}{\upsilon_I} = 0.9998$$

(ii) $I_x = \dfrac{V_x - A_{OL}(-V_x)}{R_o} + \dfrac{V_x}{R_i}$

$$\frac{I_x}{V_x} = \frac{1}{R_{of}} = \frac{1 + A_{OL}}{R_o} + \frac{1}{R_i} = \frac{1 + 5\times10^3}{1} + \frac{1}{10}$$

$$R_{of} \cong 0.2\,\Omega$$

14.15



$$\frac{v_{I1}-v_1}{20}+\frac{v_{I2}-v_1}{10}=\frac{v_1-v_0}{40}$$

$$\frac{v_{I1}}{20}+\frac{v_{I2}}{10}+\frac{v_0}{40}=v_1\left[\frac{1}{20}+\frac{1}{10}+\frac{1}{40}\right]$$

and $v_0=-A_{0L}v_1$ so that $v_1=-\dfrac{v_0}{A_{0L}}$

Then

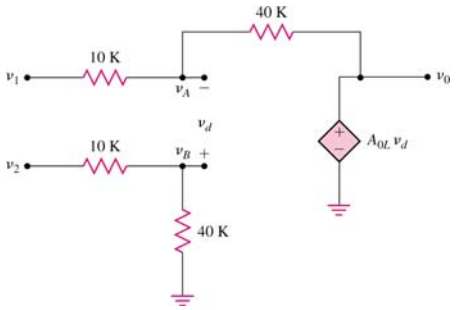
$$v_{I1}(0.05)+v_{I2}(0.10)=-v_0\left\{\frac{1}{40}+\frac{1}{2\times10^3}\cdot\left(\frac{7}{40}\right)\right\}$$

$$=-v_0[2.50875\times10^{-2}]$$

$$\Rightarrow \underline{v_0=-1.993v_{I1}-3.986v_{I2}}$$

$$\frac{\Delta v_0}{v_0}=\frac{2-1.993}{2}\Rightarrow \underline{\frac{\Delta v_0}{v_0}=0.35\%}$$

14.16



$$v_B=\left(\frac{40}{40+10}\right)v_2=\left(\frac{4}{5}\right)v_2=0.8v_2 \qquad (1)$$

$$\frac{v_1-v_A}{10}=\frac{v_A-v_0}{40}$$

$$\frac{v_1}{10}+\frac{v_0}{40}=v_A\left(\frac{1}{10}+\frac{1}{40}\right)$$

$$v_1(0.1)+v_0(0.025)=v_A(0.125) \qquad (2)$$

$$v_0=A_{0L}v_d=A_{0L}(v_B-v_A) \qquad (3)$$

or

$$v_0 = A_{0L}[0.8v_2 - v_A]$$

$$\frac{v_0}{A_{0L}} - 0.8v_2 = -v_A$$

$$\Rightarrow \underline{v_A = 0.8v_2 - \frac{v_0}{A_{0L}}}$$

Then

$$v_1(0.1) + v_0(0.025) = (0.125)\left[0.8v_2 - \frac{v_0}{A_{0L}}\right]$$

$$v_1(0.1) - v_2(0.1) = -v_0\left[0.025 + \frac{0.125}{10^3}\right]$$

$$= -v_0[2.5125 \times 10^{-2}]$$

$$\Rightarrow \underline{A_d = \frac{v_0}{v_2 - v_1} = 3.9801}$$

$$\Rightarrow \frac{\Delta A_d}{A_d} = \frac{0.0199}{4} \Rightarrow \underline{0.4975\%}$$

14.17

a. Considering the second op-amp and Equation (14.20), we have

$$\frac{1}{R_{if2}} = \frac{1}{10} + \frac{1}{0.1} \cdot \left[\frac{1+100}{1+\frac{1}{0.1}}\right] = 0.10 + \frac{101}{(0.1)(11)}$$

So $R_{if2} = 0.0109\ \text{k}\Omega$

The effective load on the first op-amp is then

$$R_{L1} = 0.1 + R_{if2} = 0.1109\ \text{k}\Omega$$

Again using Equation (14.20), we have

$$\frac{1}{R_{if}} = \frac{1}{10} + \frac{1}{1} \cdot \frac{1+100+\frac{1}{0.1109}}{1+\frac{1}{0.1109}+\frac{1}{1}} = 0.10 + \frac{110.017}{11.017}$$

so that

$$\underline{R_{if} = 99.1\ \Omega}$$

b. To determine R_{0f}:

For the first op-amp, we can write, using Equation (14.36)

$$\frac{1}{R_{0f1}} = \frac{1}{R_0} \cdot \left[\frac{A_{0L}}{1+\frac{R_2}{R_1 \,||\, R_i}}\right] = \frac{1}{1} \cdot \left[\frac{100}{1+\frac{40}{1||10}}\right]$$

which yields $R_{0f1} = 0.021\ \text{k}\Omega$

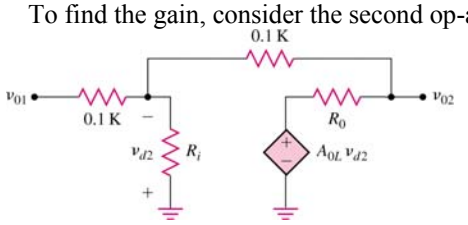
For the second op-amp, then

$$\frac{1}{R_{0f}} = \frac{1}{R_0}\cdot\left[\frac{A_{0L}}{1+\dfrac{R_2}{(R_1+R_{0f1})\,\|\,R_i}}\right]$$

$$= \frac{1}{1}\cdot\left[\frac{100}{1+\dfrac{0.10}{(0.121)\,\|\,10}}\right]$$

or $R_{0f} = 18.4\ \Omega$

c. To find the gain, consider the second op-amp.



$$\frac{v_{01}-(-v_{d2})}{0.1}+\frac{v_{d2}}{R_i}=\frac{-v_{d2}-v_{02}}{0.1} \qquad (1)$$

$$\frac{v_{01}}{0.1}+v_{d2}\left(\frac{1}{0.1}+\frac{1}{10}+\frac{1}{0.1}\right)=-\frac{v_{02}}{0.1}$$

or

$$v_{01}(10)+v_{d2}(20.1)=-v_{02}(10)$$

$$\frac{v_{02}-A_{0L}v_{d2}}{R_0}+\frac{v_{02}-(-v_{d2})}{0.1}=0 \qquad (2)$$

$$\frac{v_{02}}{1}-v_{d2}\left(\frac{100}{1}-\frac{1}{0.1}\right)+\frac{v_{02}}{0.1}=0$$

$$v_{02}(11)-v_{d2}(90)=0$$

or

$$v_{d2}=v_{02}(0.1222)$$

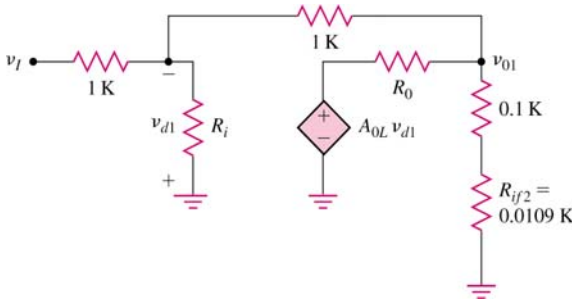
Then Equation (1) becomes

$$v_{01}(10)+v_{02}(0.1222)(20.1)=-v_{02}(10)$$

or

$$v_{01}=-v_{02}(1.246)$$

Now consider the first op-amp.



$$\frac{v_I-(-v_{d1})}{1}+\frac{v_{d1}}{R_i}=\frac{-v_{d1}-v_{01}}{1}$$

$$v_I(1)+v_{d1}\left(\frac{1}{1}+\frac{1}{10}+\frac{1}{1}\right)=-v_{01}(1) \qquad (1)$$

or

$$v_I(1)+v_{d1}(2.1)=-v_{01}(1)$$

$$\frac{v_{01}}{0.1109}+\frac{v_{01}-A_{0L}v_{d1}}{R_0}+\frac{v_{01}-(-v_{d1})}{1}=0$$

$$v_{01}\left(\frac{1}{0.1109}+\frac{1}{1}+\frac{1}{1}\right)-v_{d1}\left(\frac{100}{1}-\frac{1}{1}\right)=0 \qquad (2)$$

$$v_{01}(11.017)-v_{d1}(99)=0$$

or

$$v_{d1}=v_{01}(0.1113)$$

Then Equation (1) becomes

$$v_I(1)+v_{01}(0.1113)(2.1)=-v_{01}$$

or $v_I=-v_{01}(1.234)$

We had $v_{01}=-v_{02}(1.246)$

So $v_I=v_{02}(1.246)(1.234)$

or $\dfrac{v_{02}}{v_I}=0.650$

d. Ideal $\dfrac{v_{02}}{v_I}=1$

So ratio of actual to ideal $=0.650$.

14.18

(a) For the op-amp. $A_{0L}\cdot f_{3dB}=10^6$

$$f_{3dB}=\frac{10^6}{2\times10^4}=50\text{ Hz}$$

For the closed-loop amplifier.

$$f_{3dB}=\frac{10^6}{25}=40\text{ kHz}$$

(b) Open-loop amplifier.

$$A=\frac{2\times10^4}{1+j\dfrac{f}{f_{3dB}}}\Rightarrow|A|=\frac{2\times10^4}{\sqrt{1+\left(\dfrac{f}{f_{3dB}}\right)^2}}$$

$$f=0.25\,f_{3dB}\Rightarrow|A|=\frac{2\times10^4}{\sqrt{1+(0.25)^2}}=1.94\times10^4$$

$$f=5\,f_{3-dB}\Rightarrow|A|=\frac{2\times10^4}{\sqrt{1+(5)^2}}=3.92\times10^3$$

Closed-loop amplifier

$$f=0.25\,f_{3dB}\Rightarrow|A|=\frac{25}{\sqrt{1+(0.25)^2}}=24.25$$

$$f=5\,f_{3-dB}\Rightarrow|A|=\frac{25}{\sqrt{1+(5)^2}}=4.90$$

14.19

$A_o = 100 \text{ dB}, \Rightarrow A_o = 10^5$

$A = 38 \text{ dB}, \quad A = 79.43$

Then $79.43 = \dfrac{10^5}{\sqrt{1+\left(\dfrac{10^4}{f_{PD}}\right)^2}}$

$\dfrac{10^4}{f_{PD}} \cong \dfrac{10^5}{79.43} \Rightarrow f_{PD} = 7.94 \text{ Hz}$

$GBW = (10^5)(7.94) = 7.94\times10^5 \text{ Hz}$

14.20

(a) $A_{CLO} = \left(1+\dfrac{R_2}{R_1}\right) = \left(1+\dfrac{150}{15}\right) = 11$

$f_T = 1.2\times10^6 = (11) f_{3-dB} \Rightarrow f_{3-dB} = 109 \text{ kHz}$

(b) $A_{CLO} = \left[1+\dfrac{(150)(1\pm0.05)}{(15)(1\pm0.05)}\right]$

$A_{CLO}(\max) = 1+\dfrac{157.5}{14.25} = 12.05$

$A_{CLO}(\min) = 1+\dfrac{142.5}{15.75} = 10.05$

Then $10.05 \le A_{CLO} \le 12.05$

$f_T = 1.2\times10^6 = (12.05) f_{3-dB} \Rightarrow f_{3-dB} = 99.6 \text{ kHz}$

$f_T = 1.2\times10^6 = (10.05) f_{3-dB} \Rightarrow f_{3-dB} = 119.4 \text{ kHz}$

Then $99.6 \le f_{3-dB} \le 119.4 \text{ kHz}$

14.21

The open loop gain can be written as

$$A_{0L}(f) = \frac{A_0}{\left(1+j\cdot\dfrac{f}{f_{PD}}\right)\left(1+j\cdot\dfrac{f}{5\times10^6}\right)}$$

where $A_0 = 2\times10^5$.

The closed-loop response is

$$A_{CL} = \frac{A_{0L}}{1+\beta A_{0L}}$$

At low frequency,

$$100 = \frac{2\times10^5}{1+\beta(2\times10^5)}$$

So that $\beta = 9.995\times10^{-3}$.

Assuming the second pole is the same for both the open-loop and closed-loop, then

$$\phi = -\tan^{-1}\left(\frac{f}{f_{PD}}\right) - \tan^{-1}\left(\frac{f}{5\times10^6}\right)$$

For a phase margin of $80°$, $\phi = -100°$.

So

$$-100 = -90 - \tan^{-1}\left(\frac{f}{5\times10^6}\right)$$

or

$$f = 8.816\times10^5 \text{ Hz}$$

Then

$$|A_{0L}| = 1$$

$$= \frac{2\times10^5}{\sqrt{1+\left(\frac{8.816\times10^5}{f_{PD}}\right)^2}\sqrt{1+\left(\frac{8.816\times10^5}{5\times10^6}\right)^2}}$$

or

$$\frac{8.816\times10^5}{f_{PD}} \cong 1.9696\times10^5$$

or

$$\underline{f_{PD} = 4.48 \text{ Hz}}$$

14.22

(a) 1st stage

(10) $f_{3-dB} = 1\ MHz \Rightarrow f_{3-dB} = 100\ kHz$

2nd stage

(50) $f_{3-dB} = 1\ MHz \Rightarrow f_{3-dB} = 20\ kHz$

Bandwidth of overall system $\cong 20\ kHz$

(b) If each stage has the same gain, so

$K^2 = 500 \Rightarrow K = 22.36$

Then bandwidth of each stage

(22.36) $f_{3-dB} = 1\ MHz \Rightarrow f_{3-dB} = 44.7\ kHz$

14.23

(a) $$A_{CLO} = \frac{-R_2/R_1}{1+\frac{\left(1+R_2/R_1\right)}{A_O}} = \frac{-10.0}{1+\frac{11}{5\times10^4}} = -9.9978$$

$$f_T = 1.5\times10^6 = (9.9978) f_{3-dB} \Rightarrow f_{3-dB} = 150.033 \text{ kHz}$$

(b) $$A_{CLO} = (-9.9978)^3 = -999.34$$

At f_{3-dB}; $\Rightarrow |A_{CL}| = \frac{999.34}{\sqrt{2}} = 706.64$

Then $706.64 = \dfrac{999.34}{\left[\sqrt{1+\left(\dfrac{f_{3-dB}}{150.033\times10^3}\right)^2}\right]^3}$

$$\left[1+\left(\frac{f_{3-dB}}{150.033\times10^3}\right)^2\right]^3 = \left(\frac{999.34}{706.64}\right)^2 \Rightarrow f_{3-dB} = 76.49\text{ kHz}$$

14.24

$(5\times10^4)\,f_{PD} = 10^6 \Rightarrow \underline{f_{PD} = 20\ Hz}$

$(25)\,f_{3-dB}10^6 \Rightarrow \underline{f_{3-dB} = 40\ kHz}$

$$A_v = \frac{A_{vo}}{1+j\dfrac{f}{f_{3-dB}}} \Rightarrow |A_v| = \frac{25}{\sqrt{1+\left(\dfrac{f}{40\times10^3}\right)^2}}$$

At $f = 0.5\,f_{3-dB} = 20\ kHz$

$$|A_v| = \frac{25}{\sqrt{1+(0.5)^2}} = 22.36$$

At $f = 2\,f_{3-dB} = 80\ kHz$

$$|A_v| = \frac{25}{\sqrt{1+(2)^2}} = 11.18$$

14.25

$(20\times10^3)\cdot|A_{vf}|_{MAX} = 10^6 \Rightarrow \underline{|A_{vf}|_{MAX} = 50}$

14.26

(a) $f_{\max} = \dfrac{SR}{2\pi V_{PO}} = \dfrac{5\times10^6}{2\pi(5)} \Rightarrow f_{\max} = 159\text{ kHz}$

(b) $f_{\max} = \dfrac{5\times10^6}{2\pi(1.5)} \Rightarrow f_{\max} = 530.5\text{ kHz}$

(c) $f_{\max} = \dfrac{5\times10^6}{2\pi(0.4)} \Rightarrow f_{\max} = 1.99\text{ MHz}$

14.27

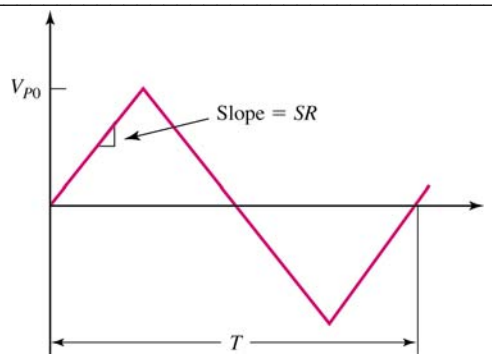
a. Using Equation (14.55),

$$V_{P0} = \frac{8\times10^6}{2\pi(250\times10^3)}$$

or

$\underline{V_{P0} = 5.09\text{ V}}$

b.



Period $T = \dfrac{1}{f} = \dfrac{1}{250\times10^3} = 4\times10^{-6}$ s

One-fourth period $= 1\ \mu$s

$$\text{Slope} = \frac{V_{P0}}{1\mu s} = SR = 8\ \text{V}/\mu\text{s}$$

$$\Rightarrow \underline{V_{P0} = 8\ \text{V}}$$

14.28

$$f_{\max} = \frac{SR}{2\pi V_{PO}}$$

$$SR = 2\pi(10)\left(12\times10^3\right) = 7.54\times10^5\ \text{V/s}$$

Or $SR = 0.754\ \text{V}/\mu\text{s}$

14.29

(a) $f_{\max} = 20\times10^3 = \dfrac{0.63\times10^6}{2\pi V_{PO}} \Rightarrow V_{PO} = 5.0\ \text{V}$

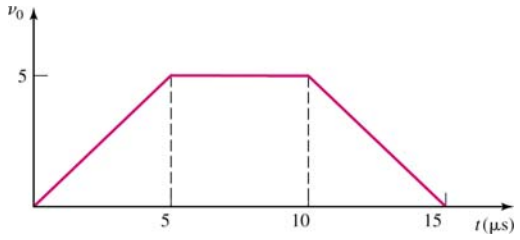
(b) $V_{PO} = \dfrac{3\times10^6}{2\pi\left(20\times10^3\right)} = 23.87\ \text{V}$

14.30

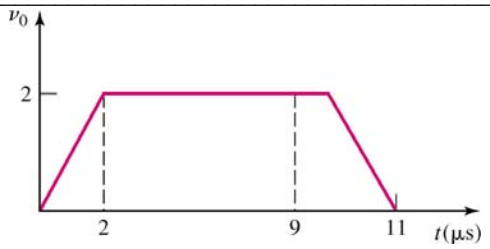
For input (a), maximum output is 5 V.

$SR = 1\ \text{V}/\mu\text{s}$

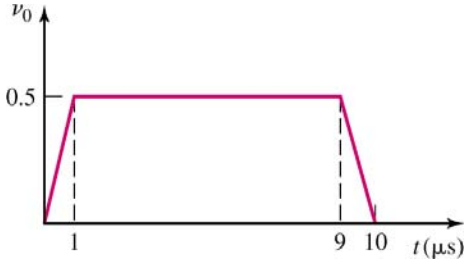
so



For input (b), maximum output is 2 V.

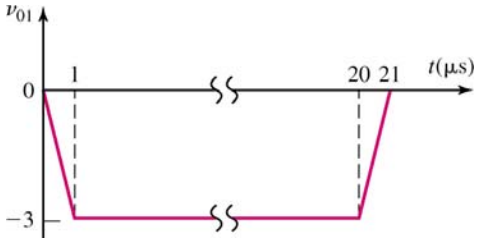


For input (c), maximum output is 0.5 V so the output is

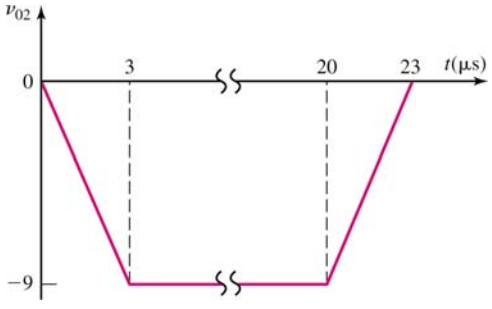


14.31

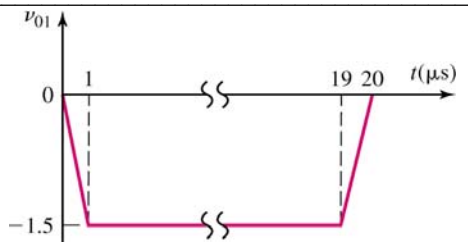
For input (a), $\max|v_{01}| = 3$ V.



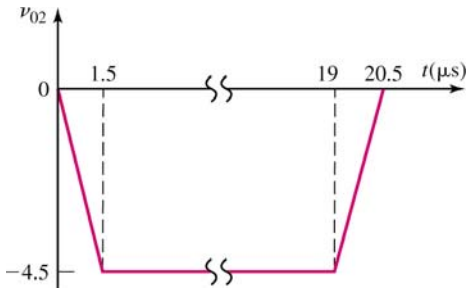
Then $|v_{02}|_{max} = 3(3) = 9$ V



For input (b), $\max|v_{01}| = 1.5$ V.



Then $|v_{02}|_{max} = 3(1.5) = 4.5\,\text{V}$



14.32

$$I_1 = I_{S1}\exp\left(\frac{V_{BE1}}{V_T}\right),\quad I_2 = I_{S2}\exp\left(\frac{V_{BE2}}{V_T}\right)$$

Want $I_1 = I_2$, so

$$\frac{I_1}{I_2} = 1 = \frac{5\times10^{-14}(1+x)\exp\left(\frac{V_{BE1}}{V_T}\right)}{5\times10^{-14}(1-x)\exp\left(\frac{V_{BE2}}{V_T}\right)}$$

$$= \frac{(1+x)}{(1-x)}\exp\left(\frac{V_{BE1}-V_{BE2}}{V_T}\right)$$

Or

$$\frac{1+x}{1-x} = \exp\left(\frac{V_{BE2}-V_{BE1}}{V_T}\right) = \exp\left(\frac{V_{OS}}{V_T}\right)$$

$$= \exp\left(\frac{0.0025}{0.026}\right) = 1.10$$

Now

$$1+x = (1-x)(1.10) \Rightarrow$$

$$x = 0.0476 \Rightarrow 4.76\%$$

14.33

(a) Balanced circuit, $I_{S4} = 5\times10^{-15}\,\text{A}$

(b) From Eq. (14.62), $\upsilon_{CE1} = 5\,\text{V}$, $\upsilon_{CE2} = 5.6 - 1.2 = 4.4\,\text{V}$

$$\frac{1+\frac{5}{120}}{1+\frac{0.6}{80}}=\frac{I_{S3}}{I_{S4}}\cdot\frac{\left(1+\frac{4.4}{120}\right)}{\left(1+\frac{1.2}{80}\right)}$$

$$\frac{1.041667}{1.0075}=\frac{I_{S3}}{I_{S4}}\cdot\frac{(1.036667)}{(1.015)}$$

$$\frac{I_{S3}}{I_{S4}}=1.0123\Rightarrow I_{S4}=4.939\times10^{-15}\ \text{A}$$

(c) $\upsilon_{CE1}=5\,\text{V},\ \upsilon_{CE2}=5.6-2.5=3.1\,\text{V}$

$$\frac{1+\frac{5}{120}}{1+\frac{0.6}{80}}=\frac{I_{S3}}{I_{S4}}\cdot\frac{\left(1+\frac{3.1}{120}\right)}{\left(1+\frac{2.5}{80}\right)}$$

$$\frac{1.041667}{1.0075}=\frac{I_{S3}}{I_{S4}}\cdot\frac{(1.025833)}{(1.03125)}$$

$$\frac{I_{S3}}{I_{S4}}=1.03937\Rightarrow I_{S4}=4.811\times10^{-15}\ \text{A}$$

14.34

$$K_n=150\,\mu\,\text{A/V}^2$$

$$\Delta K_n=150(1+x)-150(1-x)=300x\,\mu\,\text{A/V}^2$$

$$|V_{OS}|=\frac{1}{2}\sqrt{\frac{I_Q}{2K_n}}\left(\frac{\Delta K_n}{K_n}\right)$$

$$15\times10^{-3}=\frac{1}{2}\sqrt{\frac{200}{2(150)}}\left(\frac{300x}{150}\right)=0.8165x\Rightarrow x=0.01837$$

14.35

(a) $\upsilon_O=-30(10\pm2)\times10^{-3}=(-300\pm60)\times10^{-3}\ \text{V}$

So $-0.360\le\upsilon_O\le-0.240\ \text{V}$

(b) $\upsilon_O=-30(100\pm2)\times10^{-3}=-3\pm0.06\ \text{V}$

So $-3.06\le\upsilon_O\le-2.94\ \text{V}$

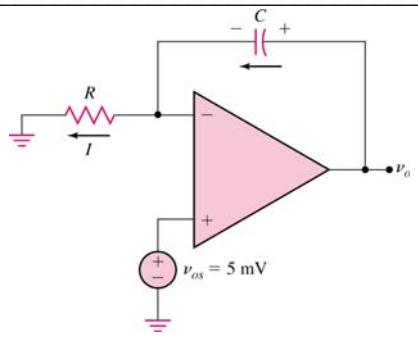
14.36

$$\upsilon_O=-30(25\sin\omega t\pm2)\,\text{mV}$$

$$\upsilon_O=-0.75\sin\omega t\pm0.06\ \text{V}$$

So $(-0.75\sin\omega t-0.06)\le\upsilon_O\le(-0.75\sin\omega t+0.06)\ \text{V}$

14.37



$$I = \frac{0.5\times10^{-3}}{10^4} = 5\times10^{-8}\ A$$

Also

$$I = C\frac{dV_o}{dt} \Rightarrow V_o = \frac{1}{C}\int_0^i Idt = \frac{I}{C}\cdot t$$

Then

$$5 = \frac{5\times10^{-8}}{10\times10^{-6}}t \Rightarrow \underline{t = 10^3\ s}$$

14.38

(a) $\upsilon_{O1} = \left(1+\frac{100}{10}\right)(\pm 3)\ \text{mV},\ -33 \le \upsilon_{O1} \le 33\ \text{mV}$

$$\upsilon_{O2} = -\left(\frac{50}{10}\right)(\pm 33 \pm 3)\ \text{mV},\ -180 \le \upsilon_{O2} \le 180\ \text{mV}$$

(b) $\upsilon_{O1} = (11)(10 \pm 3)\ \text{mV},\ \Rightarrow 77 \le \upsilon_{O1} \le 143\ \text{mV}$

$\upsilon_{O2} = -5(143+3) = -730\ \text{mV}$

$\upsilon_{O2} = -5(77-3) = -370\ \text{mV}$

So $-0.73 \le \upsilon_{O2} \le -0.37\ \text{V}$

(c) $\upsilon_{O1} = (11)(100 \pm 3)\ \text{mV}$

$1.067 \le \upsilon_{O1} \le 1.133\ \text{V}$

$\upsilon_{O2} = -5(1.133+0.003) = -5.68\ \text{V}$

$\upsilon_{O2} = -5(1.067-0.003) = -5.32\ \text{V}$

So $-5.68 \le \upsilon_{O2} \le -5.32\ \text{V}$

14.39

v_0 due to v_I

$$v_0 = (0.5)\left(1+\frac{1}{1.1}\right) = 0.9545\ \text{V}$$

Wiper arm at $V^+ = 10\ \text{V}$, (using superposition)

$$v_1 = \left(\frac{R_1 \,||\, R_5}{R_1 \,||\, R_5 + R_4}\right)(10) = \left(\frac{0.0909}{0.0909 + 10}\right)(10)$$
$$= 0.090$$

$$v_{01} = -\left(\frac{1}{1}\right)(0.090) = -0.090$$

Then

Wiper arm in center, $v_1 = 0$ and $v_{02} = 0$

Wiper arm at V^- $= -10$ V, $v_1 = -0.090$

So

$$v_{03} = 0.090$$

Finally, total output v_0: (from superposition)

Wiper arm at V^+,

$$\underline{v_0 = 0.8645 \text{ V}}$$

Wiper arm in center,

$$\underline{v_0 = 0.9545 \text{ V}}$$

Wiper arm at V^-,

$$\underline{v_0 = 1.0445 \text{ V}}$$

14.40

a. $R_1' = R_2' = 0.5 \,||\, 25 = 0.490$ kΩ

or

$$\underline{R_1' = R_2' = 490\ \Omega}$$

b. From Equation (14.75),

$$(0.026)\ln\left(\frac{125\times10^{-6}}{2\times10^{-14}}\right) + (0.125)R_1'$$
$$= (0.026)\ln\left(\frac{125\times10^{-6}}{2.2\times10^{-14}}\right) + (0.125)R_2'$$

$$0.586452 + (0.125)R_1' = 0.583974 + (0.125)R_2'$$
$$0.002478 = (0.125)(R_2' - R_1')$$

So $R_2' - R_1' = 0.0198$ kΩ $\Rightarrow 19.8\ \Omega$

Then

$$\frac{R_2(1-x)R_x}{R_2 + (1-x)R_x} - \frac{R_1 \times R_x}{R_1 + xR_x} = 0.0198$$
$$\frac{(0.5)(1-x)(50)}{(0.5) + (1-x)(50)} - \frac{(0.5)(50)x}{(0.5) + x(50)} = 0.0198$$
$$\frac{25(1-x)}{50.5 - 50x} - \frac{25x}{0.5 + 50x} = 0.0198$$
$$\frac{(0.5+50x)(25-25x) - (25x)(50.5-50x)}{(50.5-50x)(0.5+50x)} = 0.0198$$

$$25\left\{0.5-0.5x+50x-50x^2-50.5x+50x^2\right\}=0.0198\left\{25.25+2525x-25x-2500x^2\right\}$$

$$25\left\{0.5-x\right\}=0.0198\left\{25.25+2500x-2500x^2\right\}$$

$$0.5-x=0.019998+1.98x-1.98x^2$$

$$1.98x^2-2.98x+0.48=0$$

$$x=\frac{2.98\pm\sqrt{(2.98)^2-4(1.98)(0.48)}}{2(1.98)}$$

So

$$\underline{x=0.183}$$

and

$$\underline{1-x=0.817}$$

14.41

$$R_1'=R_1\,||\,15=0.5\,||\,15=0.4839\ \text{k}\Omega$$

$$R_2'=R_2\,||\,35=0.5\,||\,35=0.4930\ \text{k}\Omega$$

From Equation (14.75),

$$(0.026)\ln\left(\frac{i_{C1}}{I_{S3}}\right)+i_{C1}R_1'=(0.026)\ln\left(\frac{i_{C2}}{I_{S4}}\right)+i_{C2}R_2'$$

$$(0.026)\ln\left(\frac{i_{C1}}{i_{C2}}\right)=i_{C2}R_2'-i_{C1}R_1'$$

$$(0.026)\ln\left(\frac{i_{C1}}{i_{C2}}\right)=i_{C2}R_2'\left[1-\frac{i_{C1}}{i_{C2}}\cdot\frac{R_1'}{R_2'}\right]$$

$$(0.026)\ln\left(\frac{i_{C1}}{i_{C2}}\right)=i_{C2}(0.4930)\left[1-(0.9815)\left(\frac{i_{C1}}{i_{C2}}\right)\right]$$

By trial and error:

$$\underline{i_{C1}=252\ \mu\text{A}}\ \text{and}\ \underline{i_{C2}=248\ \mu\text{A}}$$

or

$$\underline{\frac{i_{C1}}{i_{C2}}=1.0155}$$

14.42

(a) $\upsilon_O\big|_{(1\,\mu A)}=\left(10^{-6}\right)\left(200\times10^3\right)=0.2\ \text{V}$

Insert resistor R_3

$$\upsilon_O\big|_{(2\,\mu A)}=-0.2=-\left(2\times10^{-6}\right)R_3\left(1+\frac{200}{20}\right)\Rightarrow R_3=9.09\ \text{k}\,\Omega$$

(b) $\upsilon_O\big|_{(0.8\,\mu A)}=\left(0.8\times10^{-6}\right)\left(200\times10^3\right)=0.16\ \text{V}$

$$\upsilon_O\big|_{(0.5\,\mu A)}=-0.16=-\left(0.5\times10^{-6}\right)R_3\left(1+\frac{200}{20}\right)\Rightarrow R_3=29.09\ \text{k}\,\Omega$$

14.43

(a) $v_O = -I_B R_2 = -\left(2\times10^{-6}\right)\left(150\times10^{3}\right) = -0.3\text{ V}$

(b) $v_O = -\dfrac{150}{15}(0.02) - 0.3 = -0.5\text{ V}$

(c) $v_O = -\dfrac{150}{15}(-0.02) - 0.3 = -0.1\text{ V}$

(d) $v_O = -\dfrac{150}{15}(0.1) - 0.3 = -1.3\text{ V}$

14.44

(a) $v_O = \left(0.6\times10^{-6}\right)\left(250\times10^{3}\right) = 0.15\text{ V}$

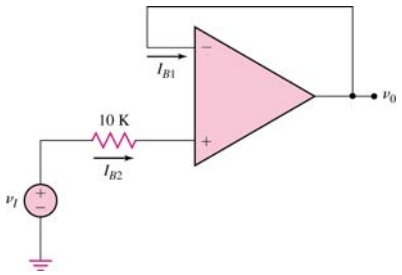
(b) $v_O = (41)(0.008) + 0.15 = 0.478\text{ V}$

(c) $v_O = (41)(-0.0035) + 0.15 = 0.0065\text{ V}$

(d) $v_O = (41)(0.005\sin\omega t) + 0.15 = 0.205\sin\omega t + 0.15\ \text{(V)}$

14.45

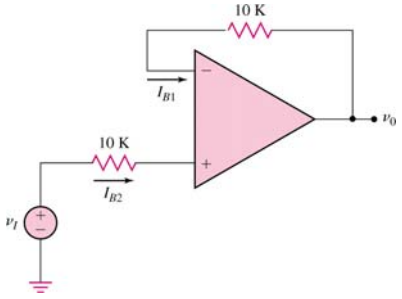
a.



For $I_{B2} = 1\ \mu\text{A},$ then $v_0 = -\left(10^{-6}\right)\left(10^{4}\right)$

or $\underline{v_0 = -0.010\text{ V}}$

b. If a $10\ \text{k}\Omega$ resistor is included in the feedback loop



Now $v_0 = -I_{B2}(10) + I_{B1}(10) = 0$

Circuit is compensated if $I_{B1} = I_{B2}.$

14.46

From Equation (14.83), we have

$$v_0 = R_2 I_{0S}$$

where $R_2 = 40\ \text{k}\Omega$ and $I_{0S} = 3\ \mu\text{A}$.

Then

$$v_0 = \left(40\times10^3\right)\left(3\times10^{-6}\right)$$

or

$$\underline{v_0 = 0.12\ \text{V}}$$

14.47

a. Assume all bias currents are in the same direction and into each op-amp.

$$v_{01} = I_{B1}\left(100\ \text{k}\Omega\right) = \left(10^{-6}\right)\left(10^5\right) \Rightarrow \underline{v_{01} = 0.1\ \text{V}}$$

Then

$$\begin{aligned} v_{02} &= v_{01}\left(-5\right) + I_{B1}\left(50\ \text{k}\Omega\right) \\ &= \left(0.1\right)\left(-5\right) + \left(10^{-6}\right)\left(5\times10^4\right) \\ &= -0.5 + 0.05 \end{aligned}$$

or

$$\underline{v_{02} = -0.45\ \text{V}}$$

b. Connect $R_3 = 10 \,||\, 100 = 9.09\ \text{k}\Omega$ resistor to noninverting terminal of first op-amp, and $R_3 = 10 \,||\, 50 = 8.33\ \text{k}\Omega$ resistor to noninverting terminal of second op-amp.

14.48

a. For a constant current through a capacitor.

$$v_0 = \frac{1}{C}\int_0^t I\ dt$$

or

$$v_0 = \frac{0.1\times10^{-6}}{10^{-6}}\cdot t \Rightarrow \underline{v_0 = (0.1)t}$$

b. At $t = 10\ \text{s}$, $v_0 = 1\ \text{V}$

c. Then

$$v_0 = \frac{100\times10^{-12}}{10^{-6}}\cdot t \Rightarrow v_0 = (10^{-4})t$$

At $t = 10\ \text{s}$, $\underline{v_0 = 1\ \text{mV}}$

14.49

(a) $\upsilon_{O1} = \left(3\times10^{-6}\right)\left(50\times10^3\right) = 0.15\ \text{V}$

$\upsilon_{O2} = 0.15\ \text{V}$

$$\upsilon_{O3} = -\frac{20}{20}(0.15) + \left(3\times10^{-6}\right)\left(20\times10^3\right) = -0.09\ \text{V}$$

(b) $R_A = 10 \,\|\, 50 = 8.33\ \text{k}\Omega$

$R_B = 20 \,\|\, 20 = 10\ \text{k}\Omega$

(c) $\upsilon_{O1} = \pm\left(50\times10^3\right)\left(0.3\times10^{-6}\right) = \pm0.015\ \text{V}$

$\upsilon_{O2} = \pm0.015\ \text{V}$

$\upsilon_{O3} = \pm\left(20\times10^3\right)\left(0.3\times10^{-6}\right) \pm 0.015 = \pm0.021\ \text{V}$

14.50

a. Using Equation (14.79),
Circuit (a),

$$v_0 = \left(0.8\times10^{-6}\right)\left(50\times10^{3}\right) - \left(0.8\times10^{-6}\right)\left(25\times10^{3}\right)\left(1+\frac{50}{50}\right)$$

or

$$\underline{v_0 = 0}$$

Circuit (b),

$$v_0 = \left(0.8\times10^{-6}\right)\left(50\times10^{3}\right) - \left(0.8\times10^{-6}\right)\left(10^{3}\right)\left(1+\frac{50}{50}\right)$$

$$= 4\times10^{-2} - 1.6$$

or

$$\underline{v_0 = -1.56\text{ V}}$$

b. Assume $I_{B1} = 0.7\ \mu\text{A}$ and $I_{B2} = 0.9\ \mu\text{A}$, then using Equation (14.79):
Circuit (a),

$$v_0 = \left(0.7\times10^{-6}\right)\left(50\times10^{3}\right) - \left(0.9\times10^{-6}\right)\left(25\times10^{3}\right)\left(1+\frac{50}{50}\right)$$

$$= 0.035 - 0.045$$

or $\underline{v_0 = -0.010\text{ V}}$

Circuit (b),

$$v_0 = \left(0.7\times10^{-6}\right)\left(50\times10^{3}\right) - \left(0.9\times10^{-6}\right)\left(10^{6}\right)\left(1+\frac{50}{50}\right)$$

$$= 0.035 - 1.8$$

or $\underline{v_0 = -1.765\text{ V}}$

14.51

(a) For V_{OS}: $\upsilon_O = \left(1+\frac{100}{10}\right)(\pm 3) = \pm 33\text{ mV}$

For I_B: $\upsilon_O(\max) = \left(0.43\times10^{-6}\right)\left(100\times10^{3}\right) = 0.043\text{ V}$

$\upsilon_O(\max) = \left(0.37\times10^{-6}\right)\left(100\times10^{3}\right) = 0.037\text{ V}$

So $4 \le \upsilon_O \le 76\text{ mV}$

(b) For V_{OS}: $\upsilon_O = \pm 33\text{ mV}$

For I_{OS}: $\upsilon_O = \pm\left(0.06\times10^{-6}\right)\left(100\times10^{3}\right) = \pm 0.006\text{ V}$

So $-39 \le \upsilon_O \le 39\text{ mV}$

(c) $\upsilon_O = \left(1+\frac{100}{10}\right)(0.2) \pm 0.039$

So $2.161 \le \upsilon_O \le 2.239\text{ V}$

14.52

a. $\left(\frac{R_i}{R_i + R_2}\right)(15) = 0.010 \text{ V}$

$$\frac{15}{15 + R_2} = 0.0006667$$

$$15(1 - 0.0006667) = 0.0006667\ R_2$$

Then

$$\underline{R_2 = 22.48 \text{ M}\Omega}$$

b. $R_1 = R_i \,||\, R_F = 15 \,||\, 10 \Rightarrow \underline{R_1 = 6 \text{ k}\Omega}$

14.53

a. Assume the offset voltage polarities are such as to produce the worst case values, but the bias currents are in the same direction.

Use superposition:

Offset voltages

$$|v_{01}| = \left(1 + \frac{100}{10}\right)(10) = \underline{110 \text{ mV} = |v_{01}|}$$

$$|v_{02}| = (5)(110) + \left(1 + \frac{50}{10}\right)(10)$$

$$\Rightarrow \underline{|v_{02}| = 610 \text{ mV}}$$

Bias Currents:

$$v_{01} = I_B(100 \text{ k}\Omega) = (2 \times 10^{-6})(100 \times 10^3) = 0.2 \text{ V}$$

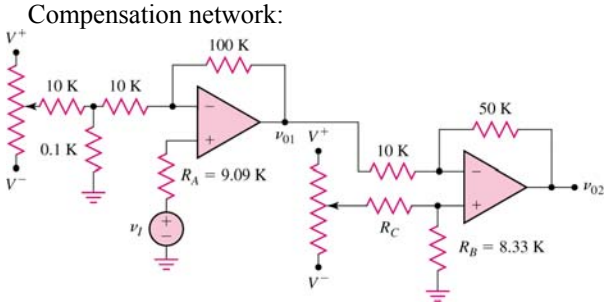
Then

$$v_{02} = (-5)(0.2) + (2 \times 10^{-6})(50 \times 10^3) = -0.9 \text{ V}$$

Worst case: v_{01} is positive and v_{02} is negative, then

$\underline{v_{01} = 0.31 \text{ V}}$ and $\underline{v_{02} = -1.51 \text{ V}}$

b. Compensation network:



If we want

$$\left(\frac{R_B}{R_B + R_C}\right)V^+ = 20 \text{ mV and } V^+ = 10 \text{ V}$$

$$\left(\frac{8.33}{8.33 + R_C}\right)(10) = 0.020$$

or $\underline{R_C \cong 4.15 \text{ M}\Omega}$

14.54

(a) Offset voltage:

$$\upsilon_{O1} = \left(1+\frac{50}{10}\right)(\pm 2) = \pm 12\,\text{mV}$$

$$\upsilon_{O2} = \pm 12 \pm 2 = \pm 14\ \text{mV}$$

$$\upsilon_{O3} = \left(-\frac{20}{20}\right)(\pm 12) + (2)(\pm 2) = \pm 16\ \text{mV}$$

Bias current:

$$\upsilon_{O1} = \left(0.21\times 10^{-6}\right)\left(50\times 10^{3}\right) = 0.0105\ \text{V}$$

or $\upsilon_{O1} = \left(0.19\times 10^{-6}\right)\left(50\times 10^{3}\right) = 0.0095\ \text{V}$

$$\upsilon_{O2} = \upsilon_{O1}$$

$$\upsilon_{O3} = (-1)(\upsilon_{O1}) + \left(0.21\times 10^{-6}\right)\left(20\times 10^{3}\right) = -\upsilon_{O1} + 0.0042$$

or $\upsilon_{O3} = -\upsilon_{O1} + \left(0.19\times 10^{-6}\right)\left(20\times 10^{3}\right) = -\upsilon_{O1} + 0.0038$

By superposition

$$-2.5 \le \upsilon_{O1} \le 22.5\,\text{mV}$$

$$-4.5 \le \upsilon_{O2} \le 24.5\,\text{mV}$$

$$-22.3 \le \upsilon_{O3} \le 10.7\,\text{mV}$$

(b) Bias currents:

$$\upsilon_{O1} = \pm I_{OS}\left(50\times 10^{3}\right) = \pm\left(0.02\times 10^{-6}\right)\left(50\times 10^{3}\right) \Rightarrow \upsilon_{O1} = \pm 1\,\text{mV}$$

$$\upsilon_{O3} = \pm I_{OS}\left(20\times 10^{3}\right) = \pm\left(0.02\times 10^{-6}\right)\left(20\times 10^{3}\right) \Rightarrow \upsilon_{O3} = \pm 0.4\,\text{mV}$$

By superposition: $\upsilon_{O3} = \pm\upsilon_{O1} \pm 2(2) \pm 0.4$

$$-13 \le \upsilon_{O1} \le 13\,\text{mV}$$

$$-15 \le \upsilon_{O2} \le 15\,\text{mV}$$

$$-17.4 \le \upsilon_{O3} \le 17.4\,\text{mV}$$

14.55

For circuit (a), effect of bias current:

$$v_0 = (50\times 10^{3})(100\times 10^{-9}) \Rightarrow 5\ \text{mV}$$

Effect of offset voltage

$$v_0 = (2)\left(1+\frac{50}{50}\right) = 4\ \text{mV}$$

So net output voltage is $\underline{v_0 = 9\ \text{mV}}$

For circuit (b), effect of bias current:

Let $I_{B2} = 550\ \text{nA},\ I_{B1} = 450\ \text{nA},$ then from Equation (14.79),

$$v_0 = (450\times 10^{-9})(50\times 10^{3}) - (550\times 10^{-9})(10^{6})\left(1+\frac{50}{50}\right)$$

$$= 2.25\times 10^{-2} - 1.1$$

or

$$v_0 = -1.0775\ \text{V}$$

If the offset voltage is negative, then

$$v_0 = (-2)(2) = -4\ \text{mV}$$

So the net output voltage is

$$\underline{v_0 = -1.0815\ \text{V}}$$

14.56

a. At $T = 25°C,\ V_{0S} = 2\ \text{mV}$ so the output voltage for each circuit is

$$\underline{v_0 = 4\ \text{mV}}$$

b. For $T = 50°C,$ the offset voltage for is

$$V_{0S} = 2\ \text{mV} + (0.0067)(25) = 2.1675\ \text{mV}$$

so the output voltage for each circuit is

$$\underline{v_0 = 4.335\ \text{mV}}$$

14.57

a. At $T = 25°C,\quad V_{0S} = 1\ \text{mV},$ then

$$v_{01} = (1)\left(1 + \frac{50}{10}\right) \Rightarrow \underline{v_{01} = 6\ \text{mV}}$$

and

$$v_{02} = v_{01}\left(1 + \frac{60}{20}\right) + (1)\left(1 + \frac{60}{20}\right)$$

$$= 6(4) + (1)(4) \Rightarrow \underline{v_{02} = 28\ \text{mV}}$$

b. At $T = 50°C,\ V_{0S} = 1 + (0.0033)(25) = 1.0825\ \text{mV},$ then

$$v_{01} = (1.0825)(6) \Rightarrow \underline{v_{01} = 6.495\ \text{mV}}$$

and

$$v_{02} = (6.495)(4) + (1.0825)(4)$$

or

$$\underline{v_{02} = 30.31\ \text{mV}}$$

14.58

$$25°C;\ I_B = 500\ \text{nA}, I_{0S} = 200\ \text{nA}$$

$$50°C,\ I_B = 500\ \text{nA} + (8\ \text{nA}/°C)(25°C) = 700\ \text{nA}$$

$$I_{0S} = 200\ \text{nA} + (2\ \text{nA}/°C)(25°C) = 250\ \text{nA}$$

a. Circuit (a): For $I_B,$ bias current cancellation, $v_0 = 0$

Circuit (b): For $I_B,$ Equation (14.79),

$$v_0 = (500\times10^{-9})(50\times10^3) - (500\times10^{-9})(10^6)\left(1 + \frac{50}{50}\right)$$

$$= 0.025 - 1.00 \Rightarrow \underline{v_0 = -0.975\ \text{V}}$$

b. Due to offset bias currents.

Circuit (a):

$$v_0 = (200\times10^{-9})(50\times10^3) \Rightarrow \underline{v_0 = 0.010\ \text{V}}$$

Circuit (b):

Let $I_{B2} = 600\ \text{nA}$

$$I_{B1} = 400\ \text{nA}$$

Then

$$v_0 = (400\times10^{-9})(50\times10^3) - (600\times10^{-9})(10^6)\left(1 + \frac{50}{50}\right)$$

$$= 0.020 - 1.20 \Rightarrow \underline{v_0 = -1.18\ \text{V}}$$

c. Circuit (a): Due to $I_B,\ \underline{v = 0}$

Circuit (b): Due to $I_B,$

$$v_0 = (700\times10^{-9})(50\times10^3) - (700\times10^{-9})(10^6)\left(1+\frac{50}{50}\right)$$
$$= 0.035 - 1.40 \Rightarrow \underline{v_0 = -1.365 \text{ V}}$$

Circuit (a): Due to I_{0S},

$$v_0 = (250\times10^{-9})(50\times10^3) \Rightarrow \underline{v_0 = 0.0125 \text{ V}}$$

Circuit (b): Due to I_{0S},

Let $I_{B2} = 825$ nA

$I_{B1} = 575$ nA

Then

$$v_0 = (575\times10^{-9})(50\times10^3) - (825\times10^{-9})(10^6)\left(1+\frac{50}{50}\right)$$
$$= 0.02875 - 1.65 \Rightarrow \underline{v_0 = -1.62 \text{ V}}$$

14.59

$$25°\text{C}; I_B = 2\ \mu\text{A}, \quad I_{0S} = 0.2\ \mu\text{A}$$
$$50°\text{C}, I_B = 2\ \mu\text{A} + (0.020\ \mu\text{A}/°\text{C})(25°\text{C}) = 2.5\ \mu\text{A}$$
$$I_{0S} = 0.2\ \mu\text{A} + (0.005\ \mu\text{A}/°\text{C})(25°\text{C}) = 0.325\ \mu\text{A}$$

a. Due to I_B: (Assume bias currents into op-amp).

$$v_{01} = I_B(50\ \text{k}\Omega) = (2\times10^{-6})(50\times10^3)$$
$$\Rightarrow \underline{v_{01} = 0.10 \text{ V}}$$
$$v_{02} = v_{01}\left(1+\frac{60}{20}\right) + I_B(60\ \text{k}\Omega) - I_B(50\ \text{k}\Omega)\left(1+\frac{60}{20}\right)$$
$$= (0.1)(4) + (2\times10^{-6})(60\times10^3) - (2\times10^{-6})(60\times10^3)4$$

or $\underline{v_{02} = 0.12 \text{ V}}$

b. Due to I_{0S}:

1st op-amp. Let $I_{B1} = 2.1\ \mu$A

2nd op-amp. Let $I_{B1} = 2.1\ \mu$A

$I_{B2} = 1.9\ \mu$A

$$v_{01} = I_{B1}(50\ \text{k}\Omega) = (2.1\times10^{-6})(50\times10^3)$$
$$\Rightarrow \underline{v_{01} = 0.105 \text{ V}}$$
$$v_{02} = v_{01}\left(1+\frac{60}{20}\right) + I_{B1}(60\ \text{k}\Omega) - I_{B2}(50\ \text{k}\Omega)\left(1+\frac{60}{20}\right)$$
$$= (0.105)(4) + (2.1\times10^{-6})(60\times10^3) - (1.9\times10^{-6})(50\times10^3)(4)$$

or

$\underline{v_{02} = 0.166 \text{ V}}$

c. Due to I_B:

$$v_{01} = (2.5\times10^{-6})(50\times10^3) \Rightarrow \underline{v_{01} = 0.125 \text{ V}}$$
$$v_{01} = v_{02}\left(1+\frac{60}{20}\right) + I_B(60\ \text{k}\Omega) - I_B(50\ \text{k}\Omega)\left(1+\frac{60}{20}\right)$$
$$= (0.125)(4) + (2.5\times10^{-6})(60\times10^3) - (2.5\times10^6)(50\times10^3(4)$$

or $\underline{v_{02} = 0.15 \text{ V}}$

Due to I_{0S}:

Let $I_{B1} = 2.625\ \mu$A

$I_{B2} = 2.3375\ \mu$A

$$v_{01} = I_{B1}(50\ \text{k}\Omega) = (2.6625\times10^{-6})(50\times10^{3})$$

$$\Rightarrow \underline{v_{01} = 1.133\ \text{V}}$$

$$v_{02} = v_{01}\left(1+\frac{60}{20}\right) + I_{B1}(60\ \text{k}\Omega) - I_{B2}(50\ \text{k}\Omega)\left(1+\frac{60}{20}\right)$$

$$= (0.133)(4) + (2.6625\times10^{-6})(60\times10^{3}) - (2.3375\times10^{-6})(50\times10^{3})(4)$$

or

$$\underline{v_{02} = 0.224\ \text{V}}$$

14.60

(a) $A_d = \dfrac{50}{10} = 5.0$

For common-mode, $\upsilon_{I1} = \upsilon_{I2}$

From Chapter 9,

$$A_{cm} = \frac{\left(1+\dfrac{R_2}{R_1}\right)}{\left(1+\dfrac{R_3}{R_4}\right)} - \frac{R_2}{R_1}$$

If $R_2 = 50(1.015) = 50.75$, $R_1 = 10(1-0.015) = 9.85$

$R_3 = 10(1-0.015) = 9.85$, $R_4 = 50(1.015) = 50.75$

Then $$A_{cm} = \frac{1+\dfrac{50.75}{9.85}}{1+\dfrac{9.85}{50.75}} - \frac{50.75}{9.85} = \frac{6.15228}{1.19409} - 5.15228 = 5.046\times10^{-6}$$

If $R_3 = 10(1.015) = 10.15$, $R_4 = 50(1-0.015) = 49.25$

Then $$A_{cm} = \frac{1+\dfrac{50.75}{9.85}}{1+\dfrac{10.15}{49.25}} - \frac{50.75}{9.85} = \frac{6.15228}{1.20609} - 5.15228 = -0.051268$$

If $R_2 = 49.25$, $R_1 = 10.15$

Then $$A_{cm} = \frac{1+\dfrac{49.25}{10.15}}{1+\dfrac{9.85}{50.75}} - \frac{49.25}{10.15} = \frac{5.85222}{1.19409} - 4.85222 = +0.04877$$

Now $$CMRR_{dB}(\min) = 20\log_{10}\left(\frac{5}{0.051268}\right) = 39.8\ \text{dB}$$

(b) $R_2 = 50(1.03) = 51.5$, $R_1 = 10(0.97) = 9.70$

$R_4 = 50(0.97) = 48.5$, $R_3 = 10(1.03) = 10.3$

$$A_{cm} = \frac{1+\dfrac{51.5}{9.70}}{1+\dfrac{10.3}{48.5}} - \frac{51.5}{9.70} = \frac{6.30928}{1.21237} - 5.30928 = -0.10519$$

$$CMRR_{dB} = 20\log_{10}\left(\frac{5}{0.10519}\right) = 33.5 \text{ dB}$$

14.61

(a) $CMRR_{dB} = 50 \text{ dB} \Rightarrow CMRR = 316.2$

$$\left|A_{cm}\right| = \frac{5}{316.2} = 0.01581$$

From Problem 14.60,

$$A_{cm} = -0.01581 = \frac{1+\dfrac{50(1+x)}{10(1-x)}}{1+\dfrac{10(1+x)}{50(1-x)}} - \frac{50(1+x)}{10(1-x)}$$

x is small, so that $\dfrac{1}{1-x} \cong 1+x$

Then $-0.01581 = \dfrac{1+5(1+x)(1+x)}{1+0.2(1+x)(1+x)} - 5(1+x)(1+x)$

Neglect x^2,

$$-0.01581 \cong \frac{1+5(1+2x)}{1+0.2(1+2x)} - 5(1+2x)$$

$$(-0.01581)[1+0.2(1+2x)] = 1+5(1+2x) - 5(1+2x)[1+0.2(1+2x)]$$

We find $(1+2x)^2 - 0.003162(1+2x) - 1.01581 = 0 \Rightarrow (1+2x) = 1.009456$

Then $x = 0.004728 \Rightarrow x = 0.4728\%$

(b) $CMRR_{dB} = 75 \text{ dB}, \Rightarrow CMRR = 5623.4$

$$A_{cm} = \frac{5}{5623.4} = 0.00088914$$

Then $(-0.00088914)[1+0.2(1+2x)] = 1-(1+2x)^2$

$$(1+2x)^2 - 0.000177828(1+2x) - 1.00088914 = 0 \Rightarrow (1+2x) = 1.0005334 \Rightarrow x = 0.0267\%$$

Chapter 15

15.1

(a) Noninverting amplifier

$$8 = 1 + \frac{R_2}{R_1} \Rightarrow \frac{R_2}{R_1} = 7 \Rightarrow R_2 = 210\,\text{k}\Omega,\ R_1 = 30\,\text{k}\Omega$$

At noninverting terminal

$$RC = \frac{1}{2\pi f} = \frac{1}{2\pi\left(30\times10^3\right)} = 5.305\times10^{-6}$$

Let input $C = 0.001\,\mu\text{F}$, then $R = 5.305\,\text{k}\Omega$

(b) Set $R_1 = 15\,\text{k}\Omega$, $R_2 = 300\,\text{k}\Omega$ for inverting amplifier

$$R_1C = \frac{1}{2\pi f} = \frac{1}{2\pi\left(20\times10^3\right)} = 7.958\times10^{-6}$$

Put C in series with R_1

$$C = \frac{7.958\times10^{-6}}{15\times10^3} \Rightarrow C = 530.5\,\text{pF}$$

15.2

(a) $$|T| = \frac{1}{\sqrt{1+(1.5)^4}} = 0.4061 \Rightarrow -7.83\,\text{dB}$$

(b) $$|T| = \frac{1}{\sqrt{1+(1.5)^6}} = 0.2841 \Rightarrow -10.93\,\text{dB}$$

(c) $$|T| = \frac{1}{\sqrt{1+(1.5)^8}} = 0.1938 \Rightarrow -14.25\,\text{dB}$$

(d) $$|T| = \frac{1}{\sqrt{1+(1.5)^{10}}} = 0.1306 \Rightarrow -17.68\,\text{dB}$$

15.3

$$|T| = -6\,\text{dB},\ \Rightarrow |T| = 0.50$$

$$0.50 = \frac{1}{\sqrt{1+\left(\frac{1}{0.9}\right)^{2N}}} = \frac{1}{\sqrt{1+(1.111)^{2N}}}$$

or $$(1.111)^{2N} = \left(\frac{1}{0.5}\right)^2 - 1 = 3$$

For N=6, $(1.111)^{(2)(6)} = 3.54$, $\Rightarrow$ 6-pole filter

---

15.4

(a) From Figure 15.8(a)

$$RC = \frac{1}{2\pi f_{3-dB}} = \frac{1}{2\pi\left(25\times10^3\right)} = 6.366\times10^{-6}$$

Let $C = 0.001\,\mu$F, then $R = 6.366\,\text{k}\Omega$

And $R_3 = (0.707)R = 4.50\,\text{k}\Omega$

$R_4 = (1.414)R = 9.0\,\text{k}\Omega$

(b) (i) $|T| = \dfrac{1}{\sqrt{1+\left(\dfrac{25}{22}\right)^4}} = 0.6123 \Rightarrow -4.26\,\text{dB}$

(ii) $|T| = \dfrac{1}{\sqrt{1+\left(\dfrac{25}{25}\right)^4}} = 0.707 \Rightarrow -3\,\text{dB}$

(iii) $|T| = \dfrac{1}{\sqrt{1+\left(\dfrac{25}{28}\right)^4}} = 0.7819 \Rightarrow -2.14\,\text{dB}$

15.5

(a) $RC = \dfrac{1}{2\pi f_{3-dB}} = \dfrac{1}{2\pi(20\times10^3)} = 7.958\times10^{-6}$

Let $R = 20\,\text{k}\Omega$, then $C = 397.9\,\text{pF}$

And $C_1 = (3.546)C = 1411\,\text{pF}$

$C_2 = (1.392)C = 553.9\,\text{pF}$

$C_3 = (0.2024)C = 80.5\,\text{pF}$

(b) (i) $|T| = \dfrac{1}{\sqrt{1+\left(\dfrac{10}{20}\right)^6}} = 0.9923 \Rightarrow -0.0673\,\text{dB}$

(ii) $|T| = \dfrac{1}{\sqrt{1+\left(\dfrac{15}{20}\right)^6}} = 0.9214 \Rightarrow -0.711\,\text{dB}$

(iii) $|T| = \dfrac{1}{\sqrt{1+\left(\dfrac{20}{20}\right)^6}} = 0.7071 \Rightarrow -3.0\,\text{dB}$

(iv) $|T| = \dfrac{1}{\sqrt{1+\left(\dfrac{25}{20}\right)^6}} = 0.4557 \Rightarrow -6.83\,\text{dB}$

(v) $|T| = \dfrac{1}{\sqrt{1+\left(\dfrac{30}{20}\right)^6}} = 0.2841 \Rightarrow -10.9\,\text{dB}$

15.6

From Equation (15.7).

$$T(s)=\frac{Y_1Y_2}{Y_1Y_2+Y_4(Y_1+Y_2+Y_3)}$$

For a high-pass filter, let $Y_1=Y_2=sC$,

$Y_3=\dfrac{1}{R_3}$, and $Y_4=\dfrac{1}{R_4}$

Then

$$T(s)=\frac{s^2C^2}{s^2C^2+\dfrac{1}{R_4}\left(sC+sC+\dfrac{1}{R_3}\right)}$$

$$=\frac{1}{1+\dfrac{1}{sR_4C}\left(2+\dfrac{1}{sR_3C}\right)}$$

Define $\tau_3=R_3C$ and $\tau_4=R_4C$

$$T(s)=\frac{1}{1+\dfrac{1}{s\tau_4}\left(2+\dfrac{1}{s\tau_3}\right)}$$

Set $s=j\omega$

$$T(j\omega)=\frac{1}{1+\dfrac{1}{j\omega\tau_4}\left(2+\dfrac{1}{j\omega\tau_3}\right)}$$

$$=\frac{1}{1-\dfrac{j}{\omega\tau_4}\left(2-\dfrac{j}{\omega\tau_3}\right)}$$

$$=\frac{1}{\left(1-\dfrac{1}{\omega^2\tau_3\tau_4}\right)-\dfrac{2j}{\omega\tau_4}}$$

$$|T(j\omega)|=\left\{\left(1-\frac{1}{\omega^2\tau_3\tau_4}\right)^2+\frac{4}{\omega^2\tau_4^2}\right\}^{-1/2}$$

For a maximally flat filter, we want

$$\left.\frac{d|T|}{d\omega}\right|_{\omega\to\infty}=0$$

Taking the derivative, we find

$$\frac{d|T(j\omega)|}{d\omega}=-\frac{1}{2}\left\{\left(1-\frac{1}{\omega^2\tau_3\tau_4}\right)^2+\frac{4}{\omega^2\tau_4^2}\right\}^{-3/2}\times\left[2\left(1-\frac{1}{\omega^2\tau_3\tau_4}\right)\left(\frac{2}{\omega^3\tau_3\tau_4}\right)+\frac{4(-2)}{\omega^3\tau_4^2}\right]$$

or

$$\left.\frac{d|T(j\omega)|}{d\omega}\right|_{\omega\to\infty} = 0$$

$$= \left[\left(\frac{4}{\omega^3\tau_3\tau_4}\right)\left(1-\frac{1}{\omega^2\tau_3\tau_4}\right)-\frac{8}{\omega^3\tau_4^2}\right]$$

$$= \frac{4}{\omega^3}\left[\frac{1}{\tau_3\tau_4}\left(1-\frac{1}{\omega^2\tau_3\tau_4}\right)-\frac{2}{\tau_4^2}\right]$$

Then

$$\left[\frac{1}{\tau_3\tau_4}\left(1-\frac{1}{\omega^2\tau_3\tau_4}\right)-\frac{2}{\tau_4^2}\right]_{\omega\to\infty} = 0$$

So that $\frac{1}{\tau_3} = \frac{2}{\tau_4} \Rightarrow 2\tau_3 = \tau_4$

Then the transfer function can be written as:

$$|T(j\omega)| = \left\{\left[1-\frac{1}{\omega^2\left(2\tau_3^2\right)}\right]^2 + \frac{4}{\omega^2\left(4\tau_3^2\right)}\right\}^{-1/2}$$

$$= \left\{1-\frac{1}{\omega^2\tau_3^2}+\frac{1}{4\left(\omega^2\tau_3^2\right)^2}+\frac{1}{\omega^2\tau_3^2}\right\}^{-1/2}$$

$$= \left\{1+\frac{1}{4\left(\omega^2\tau_3^2\right)^2}\right\}^{-1/2}$$

$3-dB$ frequency

$$2\omega^2\tau_3^2 = 1 \text{ or } \omega = \frac{1}{\sqrt{2}(\tau_3)} = \frac{1}{\sqrt{2}(R_3C)}$$

Define

$$\omega = \frac{1}{RC}$$

So that

$$\underline{R_3 = \frac{R}{\sqrt{2}}}$$

We had $2\tau_3 = \tau_4$ or $2(R_3C) = R_4C \Rightarrow R_4 = 2R_3$

So that $\underline{R_4 = \sqrt{2}\cdot R}$

15.7

$$-14\,\text{dB} \Rightarrow |T| = 0.1995$$

$$0.1995 = \frac{1}{\sqrt{1+(1.2)^{2N}}} \Rightarrow (1.2)^{2N} = \left(\frac{1}{0.1995}\right)^2 - 1 = 24.1$$

N=9, 9th order filter

15.8

$-12\text{ dB} \Rightarrow |T| = 0.2512$

$$0.2512 = \frac{1}{\sqrt{1+\left(\frac{4}{3}\right)^{2N}}} \Rightarrow (1.333)^{2N} = \left(\frac{1}{0.2512}\right)^2 - 1 = 14.85$$

N=5, 5th order filter

15.9

$$|T| = \frac{1}{\sqrt{1+\left(\frac{f}{f_{3dB}}\right)^{2N}}}$$

At $f = 12$ kHz, $|T| = 0.9$

$$0.9 = \frac{1}{\sqrt{1+\left(\frac{f}{f_{3dB}}\right)^{2N}}} = \frac{1}{\sqrt{1+\left(\frac{12}{f_{3dB}}\right)^{2N}}}$$

$$\left(\frac{12}{f_{3dB}}\right)^{2N} = \frac{1}{(0.9)^2} - 1 = 0.2346$$

Also

$$0.01 = \frac{1}{\sqrt{1+\left(\frac{14}{f_{3dB}}\right)^{2N}}}$$

$$\left(\frac{14}{f_{3dB}}\right)^{2N} = \frac{1}{(0.01)^2} - 1 = 9999$$

$$\frac{\left(\frac{14}{f_{3dB}}\right)^{2N}}{\left(\frac{12}{f_{3dB}}\right)^{2N}} = \left(\frac{14}{12}\right)^{2N} = \frac{9999}{0.2346} = 4.262\times 10^4$$

$$(1.16667)^{2N} = 4.262\times 10^4$$

$\underline{N = 35}$

Then

$$0.9 = \frac{1}{\sqrt{1+\left(\frac{12}{f_{3dB}}\right)^{2N}}}$$

$$\left(\frac{12}{f_{3dB}}\right)^{2N} = 0.2346$$

$$\left(\frac{12}{f_{3dB}}\right) = (0.2346)^{\left(\frac{1}{2N}\right)} = (0.2346)^{0.014286}$$

$$= 0.9795$$

So

$$\underline{f_{3dB} = 12.25 \text{ kHz}}$$

15.10

(a) $$|T| = \frac{1}{\sqrt{1+\left(\frac{1}{0.8}\right)^{2N}}} = \frac{1}{\sqrt{1+(1.25)^{2N}}}$$

For N=3, $$|T| = \frac{1}{\sqrt{1+(1.25)^{6}}} = 0.4557 \Rightarrow -6.83 \text{ dB}$$

(b) For N=5, $$|T| = \frac{1}{\sqrt{1+(1.25)^{10}}} = 0.3114 \Rightarrow -10.1 \text{ dB}$$

(c) For N=7, $$|T| = \frac{1}{\sqrt{1+(1.25)^{14}}} = 0.2053 \Rightarrow -13.8 \text{ dB}$$

15.11

(a) $$|T| = \frac{1}{\sqrt{1+(1.4)^{2N}}}$$

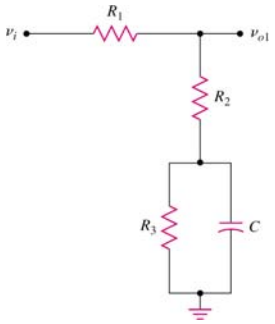
For N=3, $$|T| = \frac{1}{\sqrt{1+(1.4)^{6}}} = 0.3424 \Rightarrow -9.31 \text{ dB}$$

(b) For N=5, $$|T| = \frac{1}{\sqrt{1+(1.4)^{10}}} = 0.1828 \Rightarrow -14.8 \text{ dB}$$

(c) For N=7, $$|T| = \frac{1}{\sqrt{1+(1.4)^{14}}} = 0.0944 \Rightarrow -20.5 \text{ dB}$$

15.12

Consider

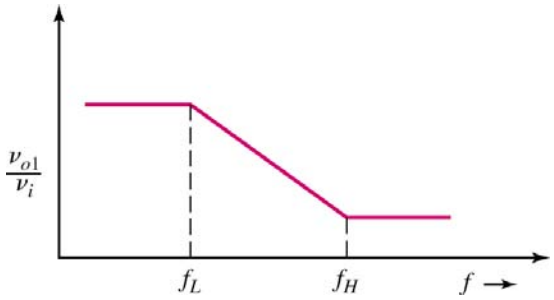


For low-frequency: $$\frac{v_o}{v_i} = \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

For high-frequency: $$\frac{v_o}{v_i} = \frac{R_2}{R_1 + R_2}$$

So we need

$$\frac{R_2+R_3}{R_1+R_2+R_3}=25\left(\frac{R_2}{R_1+R_2}\right)$$

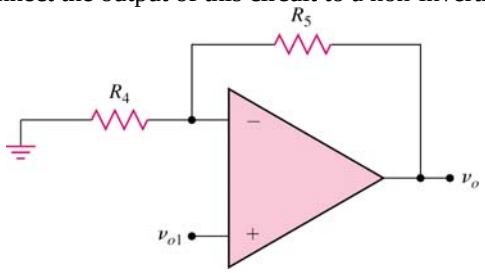


Let $R_1+R_2=50\ k\Omega$ and $\underline{R_2=1.5\ k\Omega \Rightarrow R_1=48.5\ k\Omega}$

Then

$$\frac{1.5+R_3}{50+R_3}=25\left(\frac{1.5}{50}\right)\Rightarrow \underline{R_3=144\,k\Omega}$$

Connect the output of this circuit to a non-inverting op-amp circuit.



At low-frequency:

$$v_{o1}=\frac{R_2+R_3}{R_1+R_2+R_3}\cdot v_i=\frac{1.5+144}{48.5+1.5+144}\cdot v_i=0.75v_i$$

Need to have $v_o=25.$

$$v_o=25=\left(1+\frac{R_5}{R_4}\right)\cdot v_{o1}=\left(1+\frac{R_5}{R_4}\right)(0.75)v_i\Rightarrow \frac{R_5}{R_4}=32.3$$

To check at high-frequency.

$$v_{o1}=\frac{R_2}{R_1+R_2}v_i=\frac{1.5}{1.5+48.5}v_i=0.03v_i$$

$$v_o=(1+32.3)v_{o1}=(33.3)(0.03)v_i=(1.0)v_i$$

which meets the design specification

Consider the frequency response.

$$\frac{v_{o1}}{v_i}=\frac{R_2+R_3\left\|\frac{1}{sC}\right.}{R_1+R_2+R_3\left\|\frac{1}{sC}\right.}$$

Now

$$R_3\left\|\frac{1}{sC}\right.=\frac{R_3}{1+sR_3C}$$

Then, we find

$$\frac{v_{o1}}{v_i} = \frac{R_3 + R_2(1+sR_3C)}{R_3 + (R_1+R_2)(1+sR_3C)}$$

which can be rearranged as

$$\frac{v_{o1}}{v_i} = \frac{(R_2+R_3)\left(1+s(R_2 \,\|\, R_3)C\right)}{(R_1+R_2+R_3)\left(1+s\left(R_3 \,\|\, (R_1+R_2)\right)C\right)}$$

So

$$f_L \cong \frac{1}{2\pi(R_2 \,\|\, R_3)C} = \frac{1}{2\pi(1.5 \,\|\, 144)\times 10^3 C} = \frac{1}{(9.33\times 10^3)C}$$

$$f_H \cong \frac{1}{2\pi\left(R_3 \,\|\, (R_1+R_2)\right)C} = \frac{1}{2\pi(144 \,\|\, 50)\times 10^3 C}$$

$$= \frac{1}{(2.33\times 10^5)C}$$

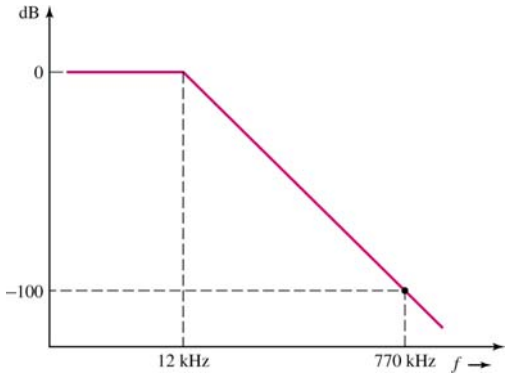
Set

$$25\ kHz = \frac{f_L + f_H}{2} = \frac{1}{2}\left[\frac{1}{(9.33\times 10^3)C} + \frac{1}{(2.33\times 10^5)C}\right]$$

Which yields

$$\underline{C = 2.23\ nF}$$

15.13



$$|A_v| = \frac{1}{\sqrt{1+\left(\frac{f}{f_{3-dB}}\right)^{2N}}} - 100\ dB \Rightarrow 10^{-5}$$

So

$$10^{-5} = \frac{1}{\sqrt{1+\left(\frac{770}{12}\right)^{2N}}}$$

or

$$1+(64.2)^{2N} = \left(\frac{1}{10^{-5}}\right)^2 = 10^{10}$$

or

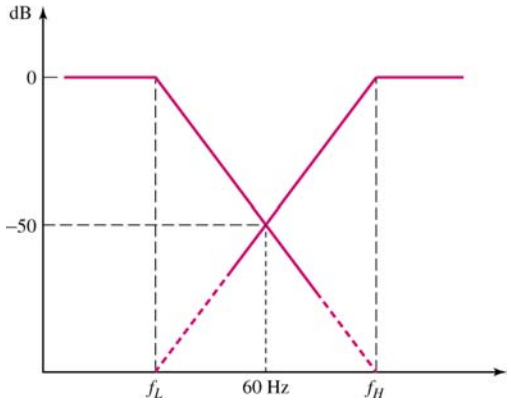
$$(64.2)^{2N} \cong 10^{10}$$

Now

N	*Left Side*
1	4.112×10^{3}
2	1.7×10^{7}
3	7×10^{10}

So, we need a 3rd order filter.

15.14



Low-pass: $-50\ dB \Rightarrow 3.16\times10^{-3}$
Then

$$3.16\times10^{-3} = \frac{1}{\sqrt{1+\left(\frac{f}{f_L}\right)^4}} = \frac{1}{\sqrt{1+\left(\frac{60}{f_L}\right)^4}}$$

We find $f_L = 3.37\ Hz$
High Pass:

$$3.16\times10^{-3} = \frac{1}{\sqrt{1+\left(\frac{f_H}{f}\right)^4}} = \frac{1}{\sqrt{1+\left(\frac{f_H}{60}\right)^4}}$$

We find $f_H = 1067\ Hz$

Bandwidth: $BW = f_H - f_L = 1067 - 3.37 \Rightarrow$

$\underline{BW \cong 1064\ Hz}$

15.15

a.

$$\frac{v_I}{R_4} = -\frac{v_{02}}{R_3} - \frac{v_0}{R_1 \left\| \left(\frac{1}{sC}\right)\right.} \qquad (1)$$

$$\frac{v_0}{R_2} = -\frac{v_{01}}{\left(\frac{1}{sC}\right)} \qquad (2)$$

$$\frac{v_{01}}{R_5} = -\frac{v_{02}}{R_5} \Rightarrow v_{01} = -v_{02} \qquad (3)$$

Then

$$\frac{v_0}{R_2} = +\frac{v_{02}}{\left(\frac{1}{sC}\right)} \quad \text{or} \quad v_{02} = v_0\left(\frac{1}{sR_2C}\right) \qquad (2)$$

And

$$\frac{v_I}{R_4} = -\frac{v_0}{R_3}\cdot\left(\frac{1}{sR_2C}\right) - \frac{v_0}{R_1 \left\| \left(\frac{1}{sC}\right)\right.}$$

$$= -v_0\left[\frac{1}{R_3(sR_2C)} + \frac{1}{\frac{R_1\cdot(1/sC)}{R_1+(1/sC)}}\right] \qquad (1)$$

$$= -v_0\left[\frac{1}{R_3(sR_2C)} + \frac{1+sR_1C}{R_1}\right]$$

$$= -v_0\left[\frac{R_1+(1+sR_1C)(sR_2R_3C)}{(sC)R_1R_2R_3}\right]$$

Then

$$\frac{v_0}{v_I} = -\frac{1}{R_4}\left[\frac{(sC)(R_1R_2R_3)}{R_1+sR_2R_3C+s^2R_1R_2R_3C^2}\right]$$

or

$$A_v(s) = \frac{v_0}{v_I} = \frac{-\frac{1}{R_4}}{\frac{1}{R_1}+sC+\frac{1}{sCR_2R_3}}$$

b.

$$A_v(j\omega) = \frac{-\frac{1}{R_4}}{\frac{1}{R_1}+j\omega C+\frac{1}{j\omega CR_2R_3}}$$

or

$$A_v(j\omega) = \frac{-\frac{1}{R_4}}{\frac{1}{R_1} + j\left[\omega C - \frac{1}{\omega C R_2 R_3}\right]}$$

$$= -\frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + j\left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right]\right\}}$$

$$|A_v(j\omega)| = \frac{R_1}{R_4} \cdot \frac{1}{\left\{1 + \left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right]^2\right\}^{-1/2}}$$

$|A_v|_{max}$ when $\left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right] = 0$

Then

$$|A_v|_{max} = \frac{R_1}{R_4} = \frac{85}{3} \Rightarrow \underline{|A_v|_{max} = 28.3}$$

Now

$$\omega R_1 C\left[1 - \frac{1}{\omega^2 C^2 R_2 R_3}\right] = 0 \quad \text{or} \quad \omega = \frac{1}{C\sqrt{R_2 R_3}}$$

Then

$$f = \frac{1}{2\pi C\sqrt{R_2 R_3}} = \frac{1}{2\pi(0.1\times10^{-6})\sqrt{(300)^2}}$$

So

$$\underline{f = 5.305 \text{ kHz}}$$

To find the two $3-dB$ frequencies,

$$\left[\omega R_1 C - \frac{R_1}{\omega C R_2 R_3}\right] = \pm 1$$

$$\omega^2 R_1 R_2 R_3 C^2 - R_1 = \pm\omega R_2 R_3 C$$

$$\omega^2 (85\times10^3)(300)^2 (0.1\times10^{-6})^2 - 85\times10^3 = \pm\omega(300)^2(0.1\times10^{-6})$$

$$\omega^2 (7.65\times10^{-5}) - 85\times10^3 = \pm\omega(9\times10^{-3})$$

$$\omega^2 (7.65\times10^{-5}) \pm \omega(9\times10^{-3}) - 85\times10^3 = 0$$

$$\omega = \frac{\pm(9\times10^{-3})}{2(7.65\times10^{-5})} \pm \frac{\sqrt{(9\times10^{-3})^2 + 4(7.65\times10^{-5})(85\times10^{-3})}}{2(7.65\times10^{-5})}$$

We find $f = 5.315$ kHz and $f = 5.296$ kHz

15.16

a.

$$\frac{v_I - v_A}{R} = \frac{v_A}{\left(\frac{1}{sC}\right)} \qquad (1)$$

$$\frac{v_I - v_B}{R} = \frac{v_B - v_0}{R} \qquad (2)$$

and $v_A = v_B$

So

$$\frac{v_I}{R} = v_A\left(\frac{1}{R} + sC\right) = v_A\left(\frac{1+sRC}{R}\right) \qquad (1)$$

or

$$v_A = \frac{v_I}{1+sRC}$$

Then

$$v_I + v_0 = 2v_B = 2v_A = \frac{2v_I}{1+sRC} \qquad (2)$$

$$v_0 = v_I\left[\frac{2}{1+sRC} - 1\right] = v_I\left[\frac{1-sRC}{1+sRC}\right]$$

Now

$$\frac{v_0}{v_I} = A(j\omega) = \frac{1-j\omega RC}{1+j\omega RC}$$

$$|A| = \frac{\sqrt{1+\omega^2R^2C^2}}{\sqrt{1+\omega^2R^2C^2}} \Rightarrow |A| = 1$$

Phase:

$$\phi = -2\ \tan^{-1}(\omega RC)$$

b. $RC = (10^4)(15.9\times10^{-9}) = 1.59\times10^{-4}$

f	ϕ
0	0
10^2	−11.4
5×10^3	−53.1
$1/2\pi RC = 10^3$ Hz	−90°
5×10^3	−157
10^4	−169

15.17

a.

$$\frac{V_i}{R_1}+\frac{V_i-V_0}{R_2 \,||\, (1/sC)}=0$$

$$\frac{V_i}{R_1}+\frac{V_i-V_0}{\left[\dfrac{R_2}{1+sR_2C}\right]}=0$$

$$\frac{R_2}{R_1}\cdot\frac{1}{1+sR_2C}(V_i)+V_i=V_0$$

$$\frac{V_0}{V_i}=\frac{R_2+R_1(1+sR_2C)}{R_1(1+sR_2C)}=\frac{(R_2+R_1)\left[1+s(R_1 \,||\, R_2)C\right]}{R_1(1+sR_2C)}$$

$$\Rightarrow \underline{\frac{V_0}{V_i}=\left(1+\frac{R_2}{R_1}\right)\left[\frac{1+s(R_1 \,||\, R_2)C}{(1+sR_2C)}\right]}$$

$$\Rightarrow \underline{f_{3dB_1}=\frac{1}{2\pi R_2C}}$$

$$\Rightarrow \underline{f_{3dB_2}=\frac{1}{2\pi(R_1 \,||\, R_2)C}}$$

b.

$$\frac{V_i}{R_1 \,||\, (1/sC)}+\frac{V_i-V_0}{R_2}=0$$

$$\frac{V_i}{\left(\dfrac{R_1}{1+sR_1C}\right)}+\frac{V_i}{R_2}=\frac{V_0}{R_2}$$

$$V_i\left[\frac{R_2}{R_1}\cdot(1+sR_1C)+1\right]=V_0$$

$$\frac{V_i}{R_1}\cdot[R_2+R_1+sR_1R_2C]=V_0$$

$$\frac{V_0}{V_i}=\frac{R_2+R_1}{R_1}\cdot\left[1+s(R_1 \,||\, R_2)C\right] \Rightarrow \underline{\frac{V_0}{V_i}=\left(1+\frac{R_2}{R_1}\right)\left[1+s(R_1 \,||\, R_2)C\right]} \Rightarrow \underline{f_{3dB}=\frac{1}{2\pi(R_1 \,||\, R_2)C}}$$

15.18

a.

$$\frac{V_i}{R_1+(1/sC_1)}=\frac{-V_0}{R_2 \,||\, (1/sC_2)}$$

$$V_i\left(\frac{sC_1}{1+sR_1C_1}\right)=-V_0\left(\frac{1+sR_2C_2}{sC_2}\right)$$

$$\frac{V_0}{V_i}=\frac{-sR_2C_1}{(1+sR_1C_1)(1+sR_2C_2)}=\frac{-sR_2C_1}{1+sR_1C_1+sR_2C_2+s^2R_1R_2C_1C_2}$$

$$\frac{V_0}{V_i}=-\frac{R_2}{R_1}\times\left[\frac{sC_1}{\dfrac{1}{R_1}+sC_1\left(1+\dfrac{R_2}{R_1}\cdot\dfrac{C_2}{C_1}\right)+s^2R_2C_1C_2}\right]$$

or

$$T(s)=\frac{V_0}{V_i}=-\frac{R_2}{R_1}\cdot\left[\frac{1}{\frac{1}{sR_1C_1}+\left(1+\frac{R_2}{R_1}\cdot\frac{C_2}{C_1}\right)+sR_2C_2}\right]$$

b.

$$|T(j\omega)|=-\frac{R_2}{R_1}\times\frac{1}{\left\{\left(1+\frac{R_2}{R_1}\cdot\frac{C_2}{C_1}\right)^2+\left(\omega R_2C_2-\frac{1}{\omega R_1C_1}\right)^2\right\}^{1/2}}$$

when $\left(\omega R_2C_2-\frac{1}{\omega R_1C_1}\right)=0,$ we want

$$|T(j\omega)|=50=\frac{R_2}{R_1}\cdot\frac{1}{\left(1+\frac{R_2}{R_1}\cdot\frac{C_2}{C_1}\right)}$$

At the $3-dB$ frequencies, we want

$$\left(\omega R_2C_2-\frac{1}{\omega R_1C_1}\right)=\pm\left(1+\frac{R_2}{R_1}\cdot\frac{C_2}{C_1}\right)$$

For $f=5$ kHz, use $+$ sign and for $f=200$ Hz, use $-$ sign.

$$\omega_1=2\pi(200)=1257$$

$$\omega_2=2\pi(5\times10^3)=3.142\times10^4$$

Define $\tau_2=R_2C_2$ and $\tau_1=R_1C_1$

Then

$$50=\frac{R_2}{R_1}\cdot\frac{1}{1+\frac{\tau_2}{\tau_1}} \qquad (1)$$

$$\left(\omega_2\tau_2-\frac{1}{\omega_2\tau_1}\right)=+\left(1+\frac{\tau_2}{\tau_1}\right) \qquad (2)$$

$$\left(\omega_1\tau_2-\frac{1}{\omega_1\tau_1}\right)=-\left(1+\frac{\tau_2}{\tau_1}\right) \qquad (3)$$

From (2)

$$\frac{\omega_2^2\tau_1\tau_2-1}{\omega_2\tau_1}=\frac{\tau_1+\tau_2}{\tau_1}$$

or

$$\omega_2\tau_1\tau_2-\frac{1}{\omega_2}=\tau_1+\tau_2$$

$$\tau_1\left(\omega_2\tau_2-1\right)=\frac{1}{\omega_2}+\tau_2$$

So

$$\tau_1=\frac{\frac{1}{\omega_2}+\tau_2}{\omega_2\tau_2-1}$$

Substituting into (3), we find

$$\omega_1\tau_2 - \frac{1}{\omega_1\cdot\dfrac{\left(\dfrac{1}{\omega_2}+\tau_2\right)}{(\omega_2\tau_2-1)}} = -\left[1+\frac{\tau_2(\omega_2\tau_2-1)}{\dfrac{1}{\omega_2}+\tau_2}\right]$$

$$\omega_1\tau_2\left[\frac{1}{\omega_2}+\tau_2\right]-\frac{1}{\omega_1}(\omega_2\tau_2-1) = -\left[\left(\frac{1}{\omega_2}+\tau_2\right)+\tau_2(\omega_2\tau_2-1)\right]$$

$$\frac{\omega_1}{\omega_2}\cdot\tau_2+\omega_1\tau_2^2-\frac{\omega_2}{\omega_1}\cdot\tau_2+\frac{1}{\omega_1} = -\frac{1}{\omega_2}-\tau_2-\omega_2\tau_2^2+\tau_2$$

$$(\omega_1+\omega_2)\tau_2^2+\left(\frac{\omega_1}{\omega_2}-\frac{\omega_2}{\omega_1}\right)\tau_2+\left(\frac{1}{\omega_1}+\frac{1}{\omega_2}\right)=0$$

$$\left(3.2677\times10^4\right)\tau_2^2-24.96\tau_2+8.273\times10^{-4}=0$$

$$\tau_2=\frac{24.96\pm\sqrt{(24.96)^2-4\left(3.2677\times10^4\right)\left(8.273\times10^{-4}\right)}}{2\left(3.2677\times10^4\right)}$$

Since ω_2 is large, τ_2 should be small so use minus sign:

$$\tau_2 = 3.47\times10^{-5}\ \text{s}$$

Then

$$\tau_1=\frac{3.18\times10^{-5}+3.47\times10^{-5}}{9.09\times10^{-2}} \Rightarrow \tau_1 = 7.32\times10^{-4}\ \text{s}$$

Now

$$50=\frac{R_2}{R_1}\cdot\frac{1}{1+\dfrac{3.47\times10^{-5}}{7.32\times10^{-4}}}$$

Then

$$\frac{R_2}{R_1}=52.37 \quad \text{or} \quad \underline{R_2 = 524\ \text{k}\Omega}$$

Also

$\tau_1 = R_1C_1$ so that $C_1 = 0.0732\,\mu\text{F}$

$\tau_2 = R_2C_2$ so that $C_2 = 66.3\,\text{pF}$

15.19

Two noninverting amplifiers,

$$\left(1+\frac{R_2}{R_1}\right)^2 = 30\ \text{dB} \Rightarrow = 31.62$$

which gives $\dfrac{R_2}{R_1}=4.62$, then $R_2 = 250\,\text{k}\Omega$, $R_1 = 54.1\,\text{k}\Omega$

For high-pass filter:

$$RC=\frac{1}{2\pi\left(20\times10^3\right)}=7.958\times10^{-6}$$

Set $R = 250\,\text{k}\Omega$, then $C = 31.8\,\text{pF}$

For low-pass filter:

$$RC = \frac{1}{2\pi(150)} = 1.061\times10^{-3}$$

Set $R = 250\ \text{k}\Omega$, then $C = 0.00424\ \ \mu\text{F}$

15.20

(a) $R_{eq} = \dfrac{1}{(50\times10^{3})(0.5\times10^{-12})} \Rightarrow R_{eq} = 40\ \text{M}\Omega$

(b) $R_{eq} = \dfrac{1}{(50\times10^{3})(2\times10^{-12})} \Rightarrow R_{eq} = 10\ \text{M}\Omega$

(c) $R_{eq} = \dfrac{1}{(50\times10^{3})(10\times10^{-12})} \Rightarrow R_{eq} = 2\ \text{M}\Omega$

15.21

a. From Equation (15.28),

$$Q = \frac{V_1 - V_2}{R_{eq}}\cdot T_C$$

and $f_C = 100\ \text{kHz}$ so that $T_C = \dfrac{1}{100\times10^{3}} \Rightarrow 10\ \mu\text{s}$

Now

$$R_{eq} = \frac{1}{f_C C} = \frac{1}{(100\times10^{3})(10\times10^{-12})} \Rightarrow 1\ \text{M}\Omega$$

So

$$Q = \frac{(2-1)(10\times10^{-6})}{10^{6}} = 10\times10^{-12}\ \text{C}$$

or

$$\underline{Q = 10\ \text{pC}}$$

b. $I_{eq} = \dfrac{Q}{T_C} = \dfrac{10\times10^{-12}}{10\times10^{-6}}$ or $\underline{I_{eq} = 1\ \mu\text{A}}$

c. $Q = CV$ so find the time that V_0 reaches 99% of its full value.

$$V_o = V_1\left(1 - e^{-t/\tau}\right) \text{ where } \tau = RC$$

Then $0.99 = 1 - e^{-t/\tau}$ or $e^{-t/\tau} = 0.01$

or $t = \tau\ln(100)$

$$\tau = RC = (10^{3})(10\times10^{-12}) = 10^{-8}\ \text{s}$$

Then

$$\underline{t = 4.61\times10^{-8}\ \text{s}}$$

15.22

$$\text{gain} = -10 \Rightarrow \frac{C_1}{C_2} = 10$$

Low frequency

$$f_{3dB} = 10\times10^{3}\ \text{Hz} = \frac{f_C C_2}{2\pi C_F}$$

Set

$f_C = 10 f_{3dB} = 100$ kHz

Then

$$\frac{C_2}{C_F} = \frac{2\pi(10\times10^3)}{100\times10^3} = 0.628$$

The largest capacitor is C_1, so let

$$\underline{C_1 = 30 \text{ pF}}$$

Then

$$\underline{C_2 = 3 \text{ pF}}$$

and

$$\underline{C_F = 4.78 \text{ pF}}$$

15.23

a. Time constant $= \tau = R_{eq}C_F$ where

$$R_{eq} = \frac{1}{f_C C_1} = \frac{1}{(100\times10^3)(5\times10^{-12})} = 2\times10^6 \ \Omega$$

Then

$$\tau = \left(2\times10^6\right)\left(30\times10^{-12}\right)$$

or

$$\tau = 60 \ \mu\text{s}$$

b. $$v_o = -\frac{1}{\tau}\int v_I \, dt$$

or

$$\Delta v_o = \frac{(1)(T_C)}{\tau} \text{ and } T_C = \frac{1}{f_C}$$

So

$$\Delta v_0 = \frac{1}{(60\times10^{-6})(100\times10^3)}$$

or

$$\underline{\Delta v_0 = 0.167 \text{ V}}$$

c. Now $\Delta v_0 = 13 = N(0.167)$

or

$$\underline{N = 78} \text{ clock pulses}$$

15.24

(a) $$f_o = \frac{1}{2\pi\sqrt{3}RC} = \frac{1}{2\pi\sqrt{3}\left(20\times10^3\right)\left(0.001\times10^{-6}\right)}$$

$f_o = 4.59$ kHz, $R_2 = 8R = 160 \text{ k}\Omega$

(b) $$C = \frac{1}{2\pi\sqrt{3}R f_o} = \frac{1}{2\pi\sqrt{3}\left(20\times10^3\right)\left(25\times10^3\right)}$$

$C = 184$ pF, $R_2 = 160 \text{ k}\Omega$

15.25

a.

$$v_1 = \frac{R}{R + (1/sC_V)} \cdot v_0 = \left(\frac{sRC_V}{1 + sRC_V}\right) \cdot v_0$$

$$v_2 = \frac{R}{R + \dfrac{1}{sC}} \cdot v_1 = \left(\frac{sRC}{1 + sRC}\right) \cdot v_1$$

$$v_3 = \frac{R}{R + \dfrac{1}{sC}} \cdot v_2 = \left(\frac{sRC}{1 + sRC}\right) \cdot v_2$$

$$v_0 = -\frac{R_2}{R} \cdot v_3$$

Then

$$v_0 = -\frac{R_2}{R}\left(\frac{sRC}{1 + sRC}\right)^2 \left(\frac{sRC_V}{1 + sRC_V}\right) v_0$$

Set $s = j\omega$

$$1 = -\frac{R_2}{R}\left(\frac{-\omega^2 R^2 C^2}{1 + 2j\omega RC - \omega^2 R^2 C^2}\right)\left(\frac{j\omega RC_V}{1 + j\omega RC_V}\right)$$

The real part of the denominator must be zero.

$$1 - \omega^2 R^2 C^2 - 2\omega^2 R^2 C C_V = 0$$

so

$$\underline{\omega_0 = \frac{1}{R\sqrt{C(C + 2C_V)}}}$$

b.

$$f_{0,\max} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11} + 2[10^{-11}])}}$$

$$f_{0,\max} = 919 \text{ kHz}$$

$$f_{0,\min} = \frac{1}{2\pi(10^4)\sqrt{(10^{-11})(10^{-11} + 2[50 \times 10^{-12}])}}$$

$$f_{0,\min} = 480 \text{ kHz}$$

15.26

(a) $$f_o = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}\left(12 \times 10^3\right)\left(150 \times 10^{-12}\right)}$$

$$f_o = 36.1 \text{ kHz}, \quad R_2 = 29R = 348 \text{ k}\Omega$$

(b) $$R = \frac{1}{2\pi\sqrt{6}C f_o} = \frac{1}{2\pi\sqrt{6}\left(0.001 \times 10^{-6}\right)\left(22 \times 10^3\right)}$$

$$R = 2.95 \text{ k}\Omega, \quad R_2 = 29R = 85.6 \text{ k}\Omega$$

15.27

$$\frac{v_0 - v_1}{\frac{1}{sC}} = \frac{v_1}{R} + \frac{v_1 - v_2}{\frac{1}{sC}} \qquad (1)$$

$$\text{or } (v_0 - v_1)sC = \frac{v_1}{R} + (v_1 - v_2)sC$$

$$\frac{v_1 - v_2}{\frac{1}{sC}} = \frac{v_2}{R} + \frac{v_2}{\frac{1}{sC} + R} \qquad (2)$$

$$\text{or } (v_1 - v_2)sC = \frac{v_2}{R} + \frac{v_2(sC)}{1 + sRC}$$

$$\frac{v_2}{\frac{1}{sC} + R} = -\frac{v_0}{R_2} \qquad (3)$$

$$\text{or } \frac{v_2 sC}{1 + sRC} = -\frac{v_0}{R_2}$$

so

$$v_2 = \frac{-v_0}{sR_2C}(1 + sRC)$$

From (2)

$$v_1(sC) = v_2\left[sC + \frac{1}{R} + \frac{sC}{1 + sRC}\right]$$

or

$$v_1 = -\frac{v_0(1 + sRC)}{sR_2C} \cdot \left[1 + \frac{1}{sRC} + \frac{1}{1 + sRC}\right]$$

From (1)

$$v_0(sC) = v_1\left[sC + \frac{1}{R} + sC\right] - v_2(sC)$$

Then

$$v_0 = \left[2 + \frac{1}{sRC}\right]\left[\frac{-v_0(1 + sRC)}{sR_2C}\right] \times \left[\frac{1 + sRC}{sRC} + \frac{1}{1 + sRC}\right] + \frac{v_0}{sR_2C} \cdot (1 + sRC)$$

$$-1 = \left[\frac{1 + 2sRC}{sRC}\right]\left[\frac{1 + sRC}{sR_2C}\right]\left[\frac{(1 + sRC)^2 + sRC}{(sRC)(1 + sRC)}\right] - \frac{1 + sRC}{sR_2C}$$

$$-1 = \frac{(1 + 2sRC)(1 + 2sRC + s^2R^2C^2 + sRC)}{(sRC)^2(sR_2C)} - \frac{(1 + sRC)(sRC)^2}{(sRC)^2(sR_2C)}$$

Set $s = j\omega$, then $-1 = \dfrac{(1 + 2j\omega RC)(1 + 3j\omega RC + \omega^2R^2C^2)}{(-\omega^2R^2C^2)(j\omega R_2C)} - \dfrac{(1 + j\omega RC)(-\omega^2R^2C^2)}{(-\omega^2R^2C^2)(j\omega R_2C)}$

The real part of the numerator must be zero.

$$1 - \omega^2R^2C^2 - 6\omega^2R^2C^2 + \omega^2R^2C^2 = 0$$

$$6\omega^2R^2C^2 = 1$$

so that

$$\omega_0 = \frac{1}{\sqrt{6}RC}$$

Condition for oscillation:

$$-1=\frac{2j\omega RC+3j\omega RC-2j\omega^3R^3C^3+j\omega^3R^3C^3}{(-\omega^3R^2C^2)(j\omega R_2C)}$$

$$1=\frac{5-\omega^2R^2C^2}{(\omega RC)(\omega R_2C)}$$

But

$$\omega=\omega_0=\frac{1}{\sqrt{6}RC}$$

Then

$$1=\frac{5-\frac{1}{6}}{\frac{(RC)(R_2C)}{6R^2C^2}}=\frac{\left(5-\frac{1}{6}\right)(6R^2C^2)}{RR_2C^2}$$

$$1=\frac{\left(\frac{29}{6}\right)(6R)}{R_2}\text{ or }\underline{\frac{R_2}{R}=29}$$

15.28

Let $R_{F1}=R_{F2}=R_{F3}\equiv R_F$

$$\upsilon_{o1}=\left(1+\frac{R_F}{R}\right)\left(\frac{1}{1+sRC}\right)\cdot\upsilon_o$$

$$\upsilon_{o2}=\left(1+\frac{R_F}{R}\right)\left(\frac{1}{1+sRC}\right)\cdot\upsilon_{o1}$$

$$\frac{\upsilon_{o3}-\upsilon_{o2}}{R}+\frac{\upsilon_{o3}}{1/sC}+\frac{\upsilon_{o3}}{R}=0$$

$$\upsilon_{o3}\left(\frac{2}{R}+sC\right)=\frac{\upsilon_{o2}}{R}$$

$$\upsilon_{o3}=\left(\frac{1}{2+sRC}\right)\cdot\upsilon_{o2}$$

$$\upsilon_o=-\frac{R_F}{R}\cdot\upsilon_{o3}$$

$$\upsilon_o=-\frac{R_F}{R}\left(\frac{1}{2+sRC}\right)\left(1+\frac{R_F}{R}\right)\left(\frac{1}{1+sRC}\right)\left(1+\frac{R_F}{R}\right)\left(\frac{1}{1+sRC}\right)\cdot\upsilon_o$$

$$1=-\frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2\left(\frac{1}{2+sRC}\right)\left(\frac{1}{1+sRC}\right)\left(\frac{1}{1+sRC}\right)$$

Let $s=j\omega$

$$1=-\frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2\left(\frac{1}{2+j\omega RC}\right)\left(\frac{1}{1+j\omega RC}\right)\left(\frac{1}{1+j\omega RC}\right)$$

$$=-\frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2\left(\frac{1}{2+j\omega RC}\right)\left(\frac{1}{1+2j\omega RC-\omega^2R^2C^2}\right)$$

$$=-\frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2\left(\frac{1}{2+4j\omega RC-2\omega^2R^2C^2+j\omega RC-2\omega^2R^2C^2-j\omega^3R^3C^3}\right)$$

$$1 = -\frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2\left[\frac{1}{2-4\omega^2R^2C^2+5j\omega RC-j\omega^3R^3C^3}\right]$$

Imaginary Term must be zero

$$5j\omega_0 RC - j\omega_0^3R^3C^3 = 0$$

$$5 - j\omega_0^2R^2C^2 = 0$$

$$\omega_0 = \frac{\sqrt{5}}{RC}$$

Then

$$1 = -\frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2\left[\frac{1}{2-\dfrac{4R^2C^2-5}{R^2C^2}}\right]$$

$$1 = -\frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2\left[\frac{1}{2-20}\right] = \frac{1}{18}\cdot\frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2$$

$$18 = \frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2 \Rightarrow \underline{\underline{\frac{R_F}{R} = 2}}$$

15.29

(a) 1st stage:

$$\upsilon_{O1} = \frac{\dfrac{1}{j\omega C}}{\dfrac{1}{j\omega C}+R_V}\cdot\left(1+\frac{R_F}{R}\right)\cdot\upsilon_O = \left(\frac{1}{1+j\omega R_V C}\right)\left(1+\frac{R_F}{R}\right)\cdot\upsilon_O$$

2nd stage:

$$\upsilon_{O2} = \left(\frac{1}{1+j\omega RC}\right)\left(1+\frac{R_F}{R}\right)\cdot\upsilon_{O1}$$

At node of C_3:

$$\frac{\upsilon_A-\upsilon_{O2}}{R}+\upsilon_A(j\omega C)+\frac{\upsilon_A}{R} = 0 \Rightarrow \upsilon_A = \frac{\upsilon_{O2}}{2+j\omega RC}$$

3rd stage:

$$\upsilon_O = -\frac{R_F}{R}\cdot\upsilon_A = -\frac{R_F}{R}\cdot\frac{\upsilon_{O2}}{(2+j\omega RC)}$$

Now

$$\upsilon_O = -\frac{R_F}{R}\cdot\frac{1}{(2+j\omega RC)}\left(\frac{1}{1+j\omega RC}\right)\left(1+\frac{R_F}{R}\right)\left(\frac{1}{1+j\omega R_V C}\right)\left(1+\frac{R_F}{R}\right)\cdot\upsilon_O$$

$$1 = -\frac{R_F}{R}\left(1+\frac{R_F}{R}\right)^2\cdot\frac{1}{(2+j\omega RC)}\cdot\frac{1}{(1+j\omega RC)}\cdot\frac{1}{(1+j\omega R_V C)}$$

From the denominator, we have:

$$(2+j\omega RC)(1+j\omega RC)(1+j\omega R_V C) = (2+3j\omega RC-\omega^2R^2C^2)(1+j\omega R_V C)$$

$$[(2-\omega^2R^2C^2)+3j\omega RC](1+j\omega R_V C)$$

$$= (2-\omega^2R^2C^2)+3j\omega RC + j\omega R_V C(2-\omega^2R^2C^2)-3\omega^2RR_VC^2$$

For oscillation, imaginary part must be zero.

$$3\omega_o RC + \omega_o R_V C\left(2 - \omega_o^2 R^2 C^2\right) = 0$$

$$3RC + 2R_V C - \omega_o^2 R_V R^2 C^3 = 0$$

So that $\omega_o = \dfrac{1}{RC}\sqrt{\dfrac{3R + 2R_V}{R_V}}$

(b) $1 = -\dfrac{R_F}{R}\left(1 + \dfrac{R_F}{R}\right)^2 \cdot \dfrac{1}{\left(2 - \omega_o^2 R^2 C^2 - 3\omega_o^2 R R_V C^2\right)}$

Consider the term:

$$2 - \omega_o^2\left(R^2C^2 + 3RR_V C^2\right) = 2 - \frac{1}{R^2C^2}\left(\frac{3R + 2R_V}{R_V}\right)\left(R^2C^2 + 3RR_V C^2\right)$$

$$= 2 - \left(\frac{3R + 2R_V}{R_V}\right)\left(1 + \frac{3R_V}{R}\right) = \frac{2RR_V - \left(3R + 2R_V\right)\left(R + 3R_V\right)}{RR_V}$$

$$= \frac{2RR_V - \left(3R^2 + 9RR_V + 2RR_V + 6R_V^2\right)}{RR_V} = -\frac{\left(3R^2 + 9RR_V + 6R_V^2\right)}{RR_V}$$

Then

$$1 = \frac{R_F}{R}\left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{RR_V}{\left(3R^2 + 9RR_V + 6R_V^2\right)}$$

For $R = R_V$

$$1 = \frac{R_F}{R}\left(1 + \frac{R_F}{R}\right)^2 \cdot \frac{R^2}{18R^2}$$

Or $18 = \dfrac{R_F}{R}\left(1 + \dfrac{R_F}{R}\right)^2 \Rightarrow \dfrac{R_F}{R} = 2$

(c) For $R_V = 15\,\text{k}\Omega$,

$$f_o = \frac{1}{2\pi\left(25\times10^3\right)\left(0.001\times10^{-6}\right)}\sqrt{\frac{3(25) + 2(15)}{15}} \Rightarrow f_o = 16.84\,\text{kHz}$$

For $R_V = 30\,\text{k}\Omega$,

$$f_o = \frac{1}{2\pi\left(25\times10^3\right)\left(0.001\times10^{-6}\right)}\sqrt{\frac{3(25) + 2(30)}{30}} \Rightarrow f_o = 13.5\,\text{kHz}$$

So $13.5 \le f_o \le 16.84\,\text{kHz}$

15.30

(a) $\dfrac{\upsilon_{O1} - \upsilon_O}{R} + \upsilon_{O1}\left(j\omega C\right) + \dfrac{\upsilon_{O1} - \upsilon_{O2}}{R} = 0$

$$\upsilon_{O1}\left(\frac{2}{R} + j\omega C\right) = \frac{\upsilon_O}{R} + \frac{\upsilon_{O2}}{R}$$

Then (1) $\upsilon_{O1}\left(2 + j\omega RC\right) = \upsilon_O + \upsilon_{O2}$

$$\frac{\upsilon_{O2} - \upsilon_{O1}}{R} + \upsilon_{O2}\left(j\omega C\right) + \frac{\upsilon_{O2} - \upsilon_{O3}}{R} = 0$$

So (2) $\upsilon_{O2}\left(2 + j\omega RC\right) = \upsilon_{O1} + \upsilon_{O3}$

$$\frac{\upsilon_{O3} - \upsilon_{O2}}{R} + \upsilon_{O3}\left(j\omega C\right) + \frac{\upsilon_{O3}}{R} = 0$$

And (3) $\upsilon_{O3}(2+j\omega RC)=\upsilon_{O2}$

Also (4) $\upsilon_O=-\dfrac{R_F}{R}\cdot\upsilon_{O3}$

From (1) $\upsilon_{O1}(2+j\omega RC)=\upsilon_O+\upsilon_{O3}(2+j\omega RC)\Rightarrow\upsilon_{O1}=\dfrac{\upsilon_O}{(2+j\omega RC)}+\upsilon_{O3}$

From (2) $\upsilon_{O3}(2+j\omega RC)^2=\upsilon_{O1}+\upsilon_{O3}\Rightarrow\upsilon_{O3}(2+j\omega RC)^2=2\upsilon_{O3}+\dfrac{\upsilon_O}{(2+j\omega RC)}$

$$\upsilon_{O3}(4+4j\omega RC-\omega^2R^2C^2-2)=\frac{\upsilon_O}{(2+j\omega RC)}$$

$$\upsilon_{O3}=\frac{\upsilon_O}{(2+j\omega RC)(2-\omega^2R^2C^2+4j\omega RC)}$$

Using (4) $$\upsilon_O=-\frac{R_F}{R}\cdot\frac{\upsilon_O}{(2+j\omega RC)(2-\omega^2R^2C^2+4j\omega RC)}$$

$$1=-\frac{R_F}{R}\cdot\frac{1}{\left[(2)(2-\omega^2R^2C^2)+8j\omega RC+j\omega RC(2-\omega^2R^2C^2)-4\omega^2R^2C^2\right]}$$

For oscillation, denominator must be real, so

$$8\omega_oRC+\omega_oRC(2-\omega_o^2R^2C^2)=0\Rightarrow\omega_o=\frac{\sqrt{10}}{RC}$$

(b) $$1=-\frac{R_F}{R}\cdot\frac{1}{\left[(2)(2-\omega_o^2R^2C^2)-4\omega_o^2R^2C^2\right]}$$

$$1=-\frac{R_F}{R}\cdot\frac{1}{\left[4-2\omega_o^2R^2C^2-4\omega_o^2R^2C^2\right]}=-\frac{R_F}{R}\cdot\frac{1}{\left[4-6\left(\dfrac{10}{R^2C^2}\right)R^2C^2\right]}=-\frac{R_F}{R}\cdot\frac{1}{(-56)}$$

Or $\dfrac{R_F}{R}=56$

(c) $$C=\frac{\sqrt{10}}{2\pi f_oR}=\frac{\sqrt{10}}{2\pi(22\times10^3)(20\times10^3)}\Rightarrow C=0.00114\,\mu\text{F}$$

$R_F=(56)R\Rightarrow R_F=1.12\ \text{M}\Omega$

15.31

(a) $$\frac{\upsilon_{O1}-\upsilon_O}{R_V}+\upsilon_{O1}(j\omega C)+\frac{\upsilon_{O1}-\upsilon_{O2}}{R}=0$$

$$\upsilon_{O1}\left(\frac{1}{R_V}+\frac{1}{R}+j\omega C\right)=\frac{\upsilon_O}{R_V}+\frac{\upsilon_{O2}}{R}$$

$$\upsilon_{O1}\left(\frac{1}{R\|R_V}+j\omega C\right)=\frac{\upsilon_O}{R_V}+\frac{\upsilon_{O2}}{R}$$

Then (1) $\upsilon_{O1}(1+j\omega(R\|R_V)C)=(R\|R_V)\left(\dfrac{\upsilon_O}{R_V}+\dfrac{\upsilon_{O2}}{R}\right)$

From Problem 15.30,

(2) $\upsilon_{O2}(2+j\omega RC)=\upsilon_{O1}+\upsilon_{O3}$

(3) $\upsilon_{O3}(2+j\omega RC)=\upsilon_{O2}$

(4) $\upsilon_O = -\dfrac{R_F}{R}\cdot\upsilon_{O3}$

From (2), $\upsilon_{O3}(2+j\omega RC)^2 = \upsilon_{O1}+\upsilon_{O3} \Rightarrow \upsilon_{O1} = \upsilon_{O3}\left[(2+j\omega RC)^2-1\right]$

Then (1) $\upsilon_{O3}\left[(2+j\omega RC)^2-1\right]\left(1+j\omega\left(R\|R_V\right)C\right) = \left(R\|R_V\right)\left[\dfrac{\upsilon_O}{R_V}+\dfrac{\upsilon_{O3}}{R}(2+j\omega RC)\right]$

$$\upsilon_{O3}\left\{\left[(2+j\omega RC)^2-1\right]\left(1+j\omega\left(R\|R_V\right)C\right)-\frac{R\|R_V}{R}(2+j\omega RC)\right\} = \left(R\|R_V\right)\cdot\frac{\upsilon_O}{R_V}$$

Then, $$\upsilon_O = -\frac{R_F}{R}\cdot\frac{\left(R\|R_V\right)}{R_V}\cdot\upsilon_O\cdot\frac{1}{\left\{\left[(2+j\omega RC)^2-1\right]\left(1+j\omega\left(R\|R_V\right)C\right)-\dfrac{R\|R_V}{R}(2+j\omega RC)\right\}}$$

Consider the denominator:

$$\left[4+4j\omega RC-\omega^2R^2C^2-1\right]\left(1+j\omega\left(R\|R_V\right)C\right)-\frac{R\|R_V}{R}(2+j\omega RC)$$

$$\left(3-\omega^2R^2C^2+4j\omega RC\right)\left(1+j\omega\left(R\|R_V\right)C\right)-\frac{R\|R_V}{R}(2+j\omega RC)$$

For oscillation, the denominator must be real, so

$$4j\omega_oRC+j\omega_o\left(R\|R_V\right)C\left[3-\omega_o^2R^2C^2\right]-\frac{R\|R_V}{R}\cdot j\omega_oRC = 0$$

$$4R+\left(R\|R_V\right)\left[3-\omega_o^2R^2C^2\right]-R\|R_V = 0$$

$$4R+2\left(R\|R_V\right) = \omega_o^2\left(R\|R_V\right)R^2C^2$$

So that $\omega_o = \dfrac{1}{RC}\sqrt{4\left(\dfrac{R}{R\|R_V}\right)+2}$

(b) For $R_V = 15\,\text{k}\Omega$,

$$f_o = \frac{1}{2\pi\left(25\times10^3\right)\left(0.001\times10^{-6}\right)}\sqrt{\frac{4(25)}{25\|15}+2} \Rightarrow f_o = 22.66\,\text{kHz}$$

For $R_V = 30\,\text{k}\Omega$,

$$f_o = \frac{1}{2\pi\left(25\times10^3\right)\left(0.001\times10^{-6}\right)}\sqrt{\frac{4(25)}{25\|30}+2} \Rightarrow f_o = 19.45\,\text{kHz}$$

So $19.45 \le f_o \le 22.66\,\text{kHz}$

15.32

a. We can write

$$v_A = \left(\frac{R_1}{R_1+R_2}\right)v_0 \quad \text{and} \quad v_B = \left(\frac{Z_p}{Z_p+Z_s}\right)v_0$$

where $$Z_p = R_B\left\|\frac{1}{sC_B}\right. = \frac{R_B}{1+sR_BC_B}$$

and $$Z_s = R_A+\frac{1}{sC_A} = \frac{1+sR_AC_A}{sC_A}$$

Setting $v_A = v_B$, we have

$$\frac{R_1}{R_1+R_2}=\frac{\dfrac{R_B}{1+sR_BC_B}}{\dfrac{R_B}{1+sR_BC_B}+\dfrac{1+sR_AC_A}{sC_A}}$$

$$\frac{R_1}{R_1+R_2}=\frac{sR_BC_A}{sR_BC_A+(1+sR_AC_A)(1+sR_BC_B)} \qquad (1)$$

To find the frequency of oscillation, set $s=j\omega$ and set the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is

$$j\omega R_BC_A+(1+j\omega R_AC_A)(1+j\omega R_BC_B)$$

or

$$j\omega R_BC_A+1+j\omega R_AC_A+j\omega R_BC_B-\omega^2R_AR_BC_AC_B \qquad (2)$$

Then from (2), we must have

$$1-\omega_0^2R_AR_BC_AC_B=0$$

or

$$\underline{f_0=\frac{1}{2\pi\sqrt{R_AR_BC_AC_B}}}$$

b. To find the condition for sustained oscillation, combine Equations (1) and (2). Then

$$\frac{R_1}{R_1+R_2}=\frac{j\omega R_BC_A}{j\omega R_BC_A+j\omega R_AC_A+j\omega R_BC_B)}$$

or

$$1+\frac{R_2}{R_1}=1+\frac{R_A}{R_B}+\frac{C_B}{C_A}$$

Then

$$\underline{\frac{R_2}{R_1}=\frac{R_A}{R_B}+\frac{C_B}{C_A}}$$

15.33

a. We can write

$$v_A=\left(\frac{R_1}{R_1+R_2}\right)v_0$$

and

$$v_B=\left(\frac{R\,||\,sL}{R\,||\,sL+R+sL}\right)v_0$$

Setting $v_A=v_B$, we have

$$\frac{R_1}{R_1+R_2}=\left[\frac{\dfrac{sRL}{R+sL}}{\dfrac{sRL}{R+sL}+R+sL}\right]\cdot v_0$$

$$\frac{R_1}{R_1+R_2}=\frac{sRL}{sRL+(R+sL)^2} \qquad (1)$$

To find the frequency of oscillation, set $s=j\omega$ and se the real part of the denominator on the right side of Equation (1) equal to zero.

The denominator term is:

$$j\omega RL + (R + j\omega L)^2$$

or

$$j\omega RL + R^2 + 2j\omega RL - \omega^2 L^2 \qquad (2)$$

Then

$$R^2 - \omega_0^2 L^2 = 0$$

or

$$\underline{f_0 = \frac{1}{2\pi}\cdot\sqrt{\frac{R}{L}}}$$

b. To find the condition for sustained oscillations, combine Equations (1) and (2).

$$\frac{R_1}{R_1 + R_2} = \frac{j\omega RL}{j\omega RL + 2j\omega RL} = \frac{1}{3}$$

Then

$$1 + \frac{R_2}{R_1} = 3$$

so that

$$\underline{\frac{R_2}{R_1} = 2}$$

15.34

$$f_o = \frac{1}{2\pi RC}$$

$$RC = \frac{1}{2\pi f_o} = \frac{1}{2\pi\left(35\times10^3\right)} = 4.547\times10^{-6}$$

Let $C = 0.001\,\mu\text{F}$, then $R = 4.55\,\text{k}\Omega$

Set $\dfrac{R_2}{R_1} = 2$

15.35

$$g_m = 2\sqrt{K_n I_{DQ}} = 2\sqrt{(0.7)(0.8)} = 1.497\,\text{mA/V}$$

$$\frac{C_2}{C_1} = g_m R = (1.497)(2) = 2.993$$

$$C_2 \cong 3(0.02) = 0.06\,\mu\text{F}$$

$$2\pi f_o = \frac{1}{\sqrt{L\left(\dfrac{C_1 C_2}{C_1 + C_2}\right)}}$$

Then $L\left[\dfrac{(0.02)(0.06)\times10^{-6}}{0.02 + 0.06}\right] = \left(\dfrac{1}{2\pi\left(350\times10^3\right)}\right)^2 \Rightarrow L = 13.8\,\mu\text{H}$

15.36

$$V_\pi = -V_0$$

$$\frac{V_0}{\left(\frac{1}{sC_2}\right)} + \frac{V_0}{R_L} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} = g_m V_\pi = -g_m V_0$$

$$V_0\left[sC_2 + sC_1 + \frac{1}{R_L} + g_m\right] = V_1(sC_1) \qquad (1)$$

$$\frac{V_1}{sL} + \frac{V_0 - V_1}{\left(\frac{1}{sC_1}\right)} + g_m V_\pi = 0 \qquad (2)$$

$$V_1\left(\frac{1}{sL} + sC_1\right) = V_0(sC_1 + g_m)$$

$$V_1 = \frac{V_0(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1\right)}$$

Then

$$V_0\left[s(C_1 + C_2) + \frac{1}{R_L} + g_m\right] = \frac{V_0(sC_1)(sC_1 + g_m)}{\left(\frac{1}{sL} + sC_1\right)}$$

$$\left[s(C_1 + C_2) + \frac{1}{R_L} + g_m\right]\left(\frac{1}{sL} + sC_1\right) = sC_1(sC_1 + g_m)$$

$$\frac{C_1 + C_2}{L} + s^2C_1(C_1 + C_2) + \frac{1}{sR_LL} + \frac{sC_1}{R_L} + sg_mC_1 + \frac{g_m}{sL} = s^2C_1^2 + sg_mC_1$$

$$\frac{C_1 + C_2}{L} + s^2C_1C_2 + \frac{1}{sR_LL} + \frac{sC_1}{R_L} + \frac{g_m}{sL} = 0$$

Set $s = j\omega$

$$\frac{C_1 + C_2}{L} - \omega^2C_1C_2 + \frac{1}{j\omega R_LL} + \frac{j\omega C_1}{R_L} + \frac{g_m}{j\omega L} = 0$$

Then

$$\omega^2 = \frac{C_1 + C_2}{C_1C_2L} \Rightarrow \omega_0 = \sqrt{\frac{C_1 + C_2}{C_1C_2L}}$$

and

$$\frac{g_m}{\omega L} + \frac{1}{\omega R_LL} = \frac{\omega C_1}{R_L}$$

Then

$$\frac{g_m}{L} + \frac{1}{R_LL} = \frac{(C_1 + C_2)C_1}{C_1C_2LR_L}$$

$$g_m + \frac{1}{R_L} = \frac{C_1 + C_2}{C_2R_L}$$

$$g_mR_L + 1 = \frac{C_1}{C_2} + 1 \text{ or } \underline{\frac{C_1}{C_2} = g_mR_L}$$

15.37

a.

$$\frac{V_0}{sL_1}+\frac{V_0}{R}+g_mV_\pi+\frac{V_0}{\frac{1}{sC}+sL_2}=0 \qquad (1)$$

$$V_\pi=\left(\frac{sL_2}{\frac{1}{sC}+sL_2}\right)V_0 \qquad (2)$$

Then

$$V_0\left\{\frac{1}{sL_1}+\frac{1}{R}+\frac{sC}{1+s^2L_2C}+\frac{g_m(s^2L_2C)}{1+s^2L_2C}\right\}=0$$

$$\left\{\frac{R(1+s^2L_2C)+(sL_1)(1+s^2L_2C)}{(sRL_1)(1+s^2L_2C)}+\frac{s^2RL_1C+g_m(sRL_1)(s^2L_2C)}{(sRL_1)(1+s^2L_2C)}\right\}=0$$

Set $s=j\omega$. Both real and imaginary parts of the numerator must be zero.

$$R(1-\omega^2L_2C)+j\omega L_1(1-\omega^2L_2C)-\omega^2RL_1C+(j\omega g_mRL_1)(-\omega^2L_2C)=0$$

Real part:

$$R(1-\omega^2L_2C)-\omega^2RL_1C=0$$

$$R=\omega^2RC(L_1+L_2)$$

or

$$\underline{\omega_0=\frac{1}{\sqrt{C(L_1+L_2)}}}$$

b. Imaginary part:

$$j\omega L_1\left(1-\omega^2L_2C\right)-j\omega g_mRL_1\left(\omega^2L_2C\right)=0$$

$$L_1=\omega^2L_1L_2C+g_mRL_1\left(\omega^2L_2C\right)$$

Now $\omega^2=\dfrac{1}{(L_1+L_2)}$

$$1=\frac{1}{C(L_1+L_2)}[L_2C+g_mRL_2C]$$

$$1=\frac{L_2}{L_1+L_2}(1+g_mR)\Rightarrow\frac{L_1}{L_2}=(1+g_mR)$$

or

$$\underline{\frac{L_1}{L_2}=g_mR}$$

15.38

(a) (1) $g_mV_\pi+\dfrac{V_C}{R}+\dfrac{V_C}{j\omega L_1}+\left(V_C-V_\pi\right)\left(j\omega C\right)=0$

(2) $V_\pi=\left(V_C-V_\pi\right)\left(j\omega C\right)\left(j\omega L_2\right)$

$$V_\pi\left(1-\omega^2CL_2\right)=-V_C\left(\omega^2CL_2\right)$$

$$V_\pi = \frac{V_C\left(\omega^2 L_2 C\right)}{\omega^2 L_2 C - 1}$$

Then (1) $V_\pi\left(g_m - j\omega C\right) + V_C\left(\frac{1}{R} + \frac{1}{j\omega L_1} + j\omega C\right) = 0$

$$V_C \cdot \frac{\left(\omega^2 L_2 C\right)\left(g_m - j\omega C\right)}{\omega^2 L_2 C - 1} + V_C \cdot \left(\frac{1}{R} - \frac{j}{\omega L_1} + j\omega C\right) = 0$$

$$\frac{g_m \omega^2 L_2 C}{\omega^2 L_2 C - 1} + \frac{1}{R} + j\left[\omega C - \frac{1}{\omega L_1} - \frac{\omega C\left(\omega^2 L_2 C\right)}{\omega^2 L_2 C - 1}\right] = 0$$

Set the imaginary part equal to zero.

$$\frac{\omega^2 L_1 C - 1}{\omega L_1} = \frac{\omega C\left(\omega^2 L_2 C\right)}{\omega^2 L_2 C - 1}$$

$$\left(\omega^2 L_1 C - 1\right)\left(\omega^2 L_2 C - 1\right) = \left(\omega^2 L_1 C\right)\left(\omega^2 L_2 C\right)$$

$$\omega^4 L_1 L_2 C^2 - \omega^2 L_2 C - \omega^2 L_1 C + 1 = \omega^4 L_1 L_2 C^2$$

$$\omega_o^2\left(L_1 + L_2\right)C = 1$$

So $\omega_o = \frac{1}{\sqrt{\left(L_1 + L_2\right)C}}$

(b) Set the real part equal to zero,

$$\frac{g_m \omega_o^2 L_2 C}{\omega_o^2 L_2 C - 1} + \frac{1}{R} = 0$$

$$g_m R \omega_o^2 L_2 C = 1 - \omega_o^2 L_2 C$$

$$\omega_o^2 L_2 C\left(g_m R + 1\right) = 1$$

Then $\frac{L_2 C\left(g_m R + 1\right)}{\left(L_1 + L_2\right)C} = 1 \Rightarrow g_m R + 1 = \frac{\left(L_1 + L_2\right)C}{L_2 C}$

We find $g_m R = \frac{L_1}{L_2}$

(c) $C = \frac{1}{\left(2\pi f_o\right)^2\left(L_1 + L_2\right)} = \frac{1}{\left[2\pi\left(750\times10^3\right)\right]^2\left(100\times10^{-6}\right)} \Rightarrow C = 450\ \text{pF}$

$$g_m R = \frac{L_1}{L_2} = 1 \Rightarrow R = \frac{1}{g_m} = \frac{1}{30} \Rightarrow R = 33.3\,\Omega$$

15.39

$$\frac{v_0 - v_1}{\left(\frac{1}{sC}\right)} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \qquad (1)$$

and

$$\frac{v_B}{\left(\frac{1}{sC}\right)} + \frac{v_B - v_1}{R} = 0 \qquad (2)$$

or

$$v_B\left(sC + \frac{1}{R}\right) = \frac{v_1}{R} \Rightarrow v_1 = v_B(1 + sRC)$$

From (1)

$$v_0(sC) = v_1\left(sC + \frac{2}{R}\right) - \frac{v_B}{R}$$

or

$$v_0(sRC) = v_B(1+sRC)(2+sRC) - v_B = v_B\left[(1+sRC)(2+sRC) - 1\right]$$

Now

$$T(s) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{sRC}{(1+sRC)(2+sRC)-1}\right] = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{sRC}{2 + 3sRC + s^2R^2C^2 - 1}\right]$$

or

$$\underline{T(s) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{sRC}{s^2R^2C^2 + 3sRC + 1}\right]}$$

$$T(j\omega) = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{j\omega RC}{1 - \omega^2R^2C^2 + 3j\omega RC}\right]$$

Frequency of oscillation:

$$\underline{f_0 = \frac{1}{2\pi RC}}$$

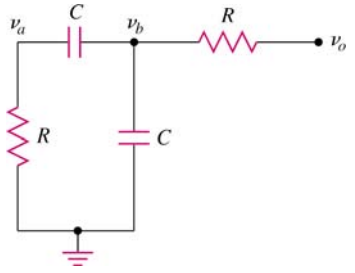
Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1}\right)\left[\frac{j\omega RC}{3j\omega RC}\right]$$

or

$$\underline{\frac{R_2}{R_1} = 2}$$

15.40



$$\frac{v_b - v_o}{R} + \frac{v_b}{\frac{1}{sC}} + \frac{v_b - v_a}{\frac{1}{sC}} = 0$$

(1) $$\frac{v_b - v_o}{R} + 2v_b \cdot sC - v_a \cdot sC = 0$$

$$\frac{V_a - V_b}{\frac{1}{sC}} + \frac{V_a}{R} = 0$$

(2) $$V_a\left(sC + \frac{1}{R}\right) = v_b \cdot sC \Rightarrow v_b = v_a\left(\frac{1+sRC}{sRC}\right)$$

From (1)

$$v_b\left(\frac{1}{R} + 2sC\right) = \frac{v_o}{R} + v_a \cdot sC$$

Substitute (2) into (1)

$$v_a\left(\frac{1+sRC}{sRC}\right)\left(\frac{1+2sRC}{R}\right) = \frac{v_o}{R} + v_a \cdot sC$$

$$v_a\left[\frac{(1+sRC)(1+2sRC)}{(sRC)\cdot R} - sC\right] = \frac{v_o}{R}$$

$$v_a\left[\frac{(1+sRC)(1+2sRC)}{sRC} - sRC\right] = v_o$$

$$\frac{v_o}{v_a} = \frac{(1+sRC)(1+2sRC) - s^2R^2C^2}{sRC}$$

$$\frac{v_a}{v_o} = \frac{sRC}{1+3sRC+2(sRC)^2 - s^2R^2C^2}$$

$$\frac{v_a}{v_o} = \frac{sRC}{1+3sRC+(sRC)^2}$$

$$T(s) = \left(1+\frac{R_2}{R_1}\right)\cdot\frac{sRC}{1+3sRC+(sRC)^2}$$

$$T(j\omega) = \left(1+\frac{R_2}{R_1}\right)\left[\frac{j\omega RC}{1-\omega^2R^2C^2+3j\omega RC}\right]$$

So $1-\omega_0^2R^2C^2 = 0$

So $f_O = \frac{1}{2\pi RC}$

Also $1 = \left(1+\frac{R_2}{R_1}\right)\left(\frac{1}{3}\right)$

So $\frac{R_2}{R_1} = 2$

15.41

$$\frac{v_0 - v_1}{sL} = \frac{v_1}{R} + \frac{v_1 - v_B}{R} \qquad (1)$$

$$v_B = \left(\frac{sL}{R+sL}\right) v_1 \qquad (2)$$

or

$$v_1 = \left(\frac{R+sL}{sL}\right) v_B$$

Then

$$\frac{v_0}{sL} = v_1\left(\frac{1}{sL} + \frac{2}{R}\right) - \frac{v_B}{R}$$

or

$$\frac{v_0}{sL} = \left(\frac{R+sL}{sL}\right)\left(\frac{1}{sL} + \frac{2}{R}\right) v_B - \frac{v_B}{R} \qquad (1)$$

$$= v_B\left\{\left(\frac{R+sL}{sL}\right)\left(\frac{R+2sL}{sRL}\right) - \frac{1}{R}\right\}$$

Then

$$v_B = \frac{v_0}{sL} \cdot \frac{1}{\left\{\dfrac{(R+sL)(R+2sL) - (sL)^2}{(sL)(sRL)} \cdot\right\}}$$

Now

$$T(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{sRL}{R^2 + 3sRL + 2s^2L^2 - s^2L^2}\right)$$

or

$$\underline{T(s) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{sRL}{s^2L^2 + 3sRL + R^2}\right)}$$

And

$$T(j\omega) = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{j\omega RL}{R^2 - \omega^2L^2 + 3j\omega RL}\right)$$

Frequency of oscillation: $\underline{f_0 = \dfrac{R}{2\pi L}}$

Condition for oscillation:

$$1 = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{1}{3}\right)$$

or

$$\underline{\frac{R_2}{R_1} = 2}$$

15.42

$$\upsilon_I = -\frac{R_2}{R_1}\cdot V_{REF} = -\frac{(R_{2f} + R_{2v})}{R_1}\cdot V_{REF}$$

Set $R_{2f} = 20\,\text{k}\Omega$

For $\upsilon_I = -2$ V, set $R_{2v} = 0$

$$-2 = -\frac{20}{R_1}(5) \Rightarrow R_1 = 50\,\text{k}\Omega$$

Then

$$-4 = -\frac{(20 + R_{2v})}{50}(5) \Rightarrow R_{2v} = 20\,\text{k}\Omega$$

15.43

For $\upsilon_O = 10$ V, $\upsilon_I = -5$ V

$$i = 0.2\,\text{mA} = \frac{10 - (-5)}{R_1 + R_2} \Rightarrow R_1 + R_2 = 75\,\text{k}\Omega$$

$$(V_{TH} - V_{TL}) = \frac{R_1}{R_2}(V_H - V_L)$$

$$0.4 = \frac{R_1}{R_2}[10 - (-10)] \Rightarrow R_2 = 50R_1$$

$$R_1 + R_2 = R_1 + 50R_1 = 75 \Rightarrow R_1 = 1.47\,\text{k}\Omega, \quad R_2 = 73.53\,\text{k}\Omega$$

15.44

(a) $$V_{TH} - V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)(V_H - V_L)$$

$$0.2 = \left(\frac{R_1}{R_1 + R_2}\right)[9 - (-9)] = 18\left(\frac{R_1}{R_1 + R_2}\right)$$

Set $R_1 = 2\,\text{k}\Omega$

Then $$2 + R_2 = \frac{(18)(2)}{0.2} \Rightarrow R_2 = 178\,\text{k}\Omega$$

(b) $$i = \frac{9}{R_1 + R_2} = \frac{9}{2 + 178} \Rightarrow i = 50\,\mu\text{A}$$

15.45

(a) $$V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)\cdot V_H = \left(\frac{2}{2 + 48}\right)(10) = 0.4\text{ V}$$

$$V_{TL} = \left(\frac{R_1}{R_1 + R_2}\right)\cdot V_L = \left(\frac{2}{2 + 48}\right)(-10) = -0.4\text{ V}$$

(b) For $33.33 \le t \le 41.67$ ms, $\sin[2\pi(60)t] \Rightarrow$ positive half cycle

At $\upsilon_I = 0$, $\upsilon_O = +10$ V

At $\upsilon_I = +0.4 = 10\sin[2\pi(60)t_1] \Rightarrow t_1 = +0.106 + 33.333 = 33.439$ ms

So, for $33.333 \le t \le 33.439$ ms, $\upsilon_O = +10$ V

$33.439 \le t \le 41.77$ ms, $\upsilon_O = -10$ V

$41.77 \le t \le 50$ ms, $\upsilon_O = +10$ V

15.46

a. Upper crossover voltage when $v_0 = +V_P$,

Now

$$v_B = \left(\frac{R_1}{R_1 + R_2}\right)(+V_P)$$

and

$$v_A = \left(\frac{R_A}{R_A + R_B}\right)V_{REF} + \left(\frac{R_B}{R_A + R_B}\right)V_{TH}$$

$v_A = v_B$ so that

$$\left(\frac{R_1}{R_1 + R_2}\right)V_P = \left(\frac{R_A}{R_A + R_B}\right)V_{REF} + \left(\frac{R_B}{R_A + R_B}\right)V_{TH}$$

or

$$V_{TH} = \left(\frac{R_A + R_B}{R_1 + R_2}\right)\left(\frac{R_1}{R_B}\right)V_P - \left(\frac{R_A}{R_B}\right)V_{REF}$$

Lower crossover voltage when $v_0 = -V_P$

So

$$V_{TL} = -\left(\frac{R_A + R_B}{R_1 + R_2}\right)\left(\frac{R_1}{R_B}\right)V_P - \left(\frac{R_A}{R_B}\right)V_{REF}$$

b.

$$V_{TH} = \left(\frac{10 + 20}{5 + 20}\right)\left(\frac{5}{20}\right)(10) - \left(\frac{10}{20}\right)(2)$$

or

$$\underline{V_{TH} = 2 \text{ V}}$$

and

$$V_{TL} = -\left(\frac{10 + 20}{5 + 20}\right)\left(\frac{5}{20}\right)(10) - 1 \Rightarrow \underline{V_{TL} = -4 \text{ V}}$$

15.47

a.

$$\frac{v_B}{R_1} = \frac{V_{REF} - v_B}{R_3} + \frac{v_0 - v_B}{R_2}$$

$$v_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V_{REF}}{R_3} + \frac{v_0}{R_2}$$

$V_{TH} = v_B$ when $v_0 = +V_P$ and $V_{TL} = v_B$ when $v_0 = -V_P$

So

$$V_{TH} = \frac{\frac{V_{REF}}{R_3} + \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

and

$$V_{TL} = \frac{\frac{V_{REF}}{R_3} - \frac{V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

b.

$$V_S = \frac{V_{REF}}{R_3\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$-5 = \frac{-10}{10\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{10}\right)}$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{5} - \frac{1}{10} = 0.10$$

$$\Delta V_T = V_{TH} - V_{TL} = \frac{\frac{2V_P}{R_2}}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)}$$

$$0.2 = \frac{2(12)}{R_2(0.10 + 0.10)}$$

So $\underline{R_2 = 600\ \text{k}\Omega}$

Then

$$\frac{1}{R_1} + \frac{1}{R_2} = 0.10$$

$$\frac{1}{R_1} + \frac{1}{600} = 0.10 \Rightarrow \underline{R_1 = 10.17\ \text{k}\Omega}$$

c.

$$V_{TH} = -5 + 0.1 = -4.9$$

$$V_{TL} = -5 - 0.1 = -5.1$$

15.48

a. If the saturated output voltage is $|V_P| < 6.2$ V, then the circuit behaves as a comparator where $|v_0| < 6.2$ V.

If the saturated output voltage is $|V_P| > 6.2$ V, the output will flip to either $+V_P$ or $-V_P$ and the input has no control.

b. Same as part (a) except the curve at $v_I \approx 0$ will have a finite slope.

c. Circuit works as a comparator as long as $v_{01} < 8.7$ V and $v_{02} > -3.7$ V. Otherwise the input has no control.

15.49

(a) When $\upsilon_O = 0$, $\upsilon_+ = V_S = \left(\frac{R_2}{R_1 + R_2}\right) \cdot V_{REF}$

When $\upsilon_O = V_H$, $\upsilon_+ = V_{TH}$

$$\frac{V_{REF} - V_{TH}}{R_1} = \frac{V_{TH} - V_H}{R_2}$$

$$\frac{V_{REF}}{R_1} + \frac{V_H}{R_2} = V_{TH}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = V_{TH}\left(\frac{R_1 + R_2}{R_1 R_2}\right)$$

$$V_{TH} = V_{REF}\left(\frac{R_2}{R_1 + R_2}\right) + V_H\left(\frac{R_1}{R_1 + R_2}\right) = V_S + V_H\left(\frac{R_1}{R_1 + R_2}\right)$$

$$V_{TL} = V_S + V_L\left(\frac{R_1}{R_1 + R_2}\right)$$

(b) $V_S = -1.75\ \text{V}$, $R_1 = 4\ \text{k}\Omega$

$$V_{TH} = -1.5 = -1.75 + (12)\left(\frac{4}{4 + R_2}\right) \Rightarrow R_2 = 188\ \text{k}\Omega$$

$$-1.75 = \left(\frac{188}{188 + 4}\right) \cdot V_{REF} \Rightarrow V_{REF} = -1.787\ \text{V}$$

(c) (i) For $\upsilon_O = 12\ \text{V}$, $|i| = \frac{12 - (-1.787)}{R_1 + R_2} = \frac{13.787}{4 + 188} \Rightarrow |i| = 71.8\ \mu\text{A}$

(ii) For $\upsilon_O = -12\ \text{V}$, $|i| = \frac{12 - 1.787}{192} \Rightarrow |i| = 53.2\ \mu\text{A}$

15.50

a. Switching point when $v_0 = 0$.

Now

$$v_+ = V_{REF} = \left(\frac{R_2}{R_1 + R_2}\right) v_I \quad \text{where } v_I = V_S.$$

Then

$$V_S = \left(\frac{R_1 + R_2}{R_2}\right) V_{REF} = \left(1 + \frac{R_1}{R_2}\right) V_{REF}$$

Now upper crossover voltage for v_1 occurs when $v_0 = V_L$ and $v_+ = V_{REF}$. Then

$$\frac{V_{TH} - V_{REF}}{R_1} = \frac{V_{REF} - V_L}{R_2}$$

$$\text{or } V_{TH} = -\frac{R_1}{R_2} \cdot V_L + V_{REF}\left(1 + \frac{R_1}{R_2}\right)$$

$$\text{or } V_{TH} = V_S - \frac{R_1}{R_2} \cdot V_L$$

Lower crossover voltage for v_I occurs when $v_0 = V_H$ and $v_I = V_{REF}$. Then

$$\frac{V_H - V_{REF}}{R_2} = \frac{V_{REF} - V_{TL}}{R_1}$$

$$\text{or } V_{TL} = -\frac{R_1}{R_2}\cdot V_H + V_{REF}\left(1+\frac{R_1}{R_2}\right)$$

$$\text{or } V_{TL} = V_S - \frac{R_1}{R_2}\cdot V_H$$

b. For $V_{TH} = -1$ and $V_{TL} = -2,\ V_S = -1.5$ V. Then $V_{TL} = V_S - \frac{R_1}{R_2}\cdot V_H \Rightarrow -2 = -1.5 - \frac{R_1}{20}(12)$

so that $\underline{R_1 = 0.833\ \text{k}\Omega}$

Now

$$V_S = \left(1+\frac{R_1}{R_2}\right)V_{REF}$$

$$-1.5 = \left(1+\frac{0.833}{20}\right)V_{REF}$$

which gives

$$\underline{V_{REF} = -1.44\ \text{V}}$$

15.51

(a) $V_{TH} = \left(\frac{R_1}{R_1+R_3}\right)\cdot V_\gamma = \left(\frac{25}{25+75}\right)(0.7) = 0.175\ \text{V}$

$$V_{TL} = \left(\frac{R_1}{R_1+R_3}\right)(-V_\gamma) = -0.175\ \text{V}$$

(c) (i) $\upsilon_I = 2\ \text{V},\ \upsilon_O = -0.7\ \text{V}$

$$I_{D1} = 0$$

$$I_{R2} = -\left(\frac{-0.7-(-10)}{20}\right) = -0.465\ \text{mA}$$

$$I_{R3} = +\left(\frac{-0.175-(-0.7)}{75}\right) \Rightarrow I_{R3} = 7\ \mu\text{A}$$

$$I_{R3} + I_{R2} + I_{D2} = 0 \Rightarrow I_{D2} = -I_{R3} - I_{R2} = -0.007 - (-0.465) = 0.458\ \text{mA}$$

(ii) $\upsilon_I = -2\ \text{V},\ \upsilon_O = +0.7\ \text{V}$

$$I_{D2} = 0$$

$$I_{R2} = \left(\frac{10-0.7}{20}\right) = 0.465\ \text{mA}$$

$$I_{R3} = \left(\frac{0.175-(0.7)}{75}\right) \Rightarrow I_{R3} = -7\ \mu\text{A}$$

$$I_{D1} = I_{R2} + I_{R3} = 0.465 - 0.007 = 0.458\ \text{mA}$$

15.52

(a) $V_\gamma + V_Z = 0.7 + 5.6 = 6.3\,\text{V}$

$$\frac{V_{TH}}{R_1} = \frac{0-(-6.3)}{R_2} \Rightarrow V_{TH} = \frac{R_1}{R_2}(6.3)$$

$$V_{TL} = -\frac{R_1}{R_2}(6.3)$$

$$V_{TH} - V_{TL} = 0.6 = \frac{R_1}{R_2}[6.3-(-6.3)] = (12.6)\frac{R_1}{R_2}$$

Then $\dfrac{R_2}{R_1} = 21$, Set $R_1 = 4\,\text{k}\Omega$, then $R_2 = 84\,\text{k}\Omega$

(b) Maximum current in R_2,

$$i_{R2} = \frac{6.3}{R_2} = \frac{6.3}{84} = 0.075\,\text{mA}$$

$$i_R = \frac{10-6.3}{R} = 0.8 + 0.075 = 0.875\,\text{mA}$$

$$R = \frac{10-6.3}{0.875} = 4.23\,\text{k}\Omega$$

15.53

a.

$$v_0 = V_{REF} + 2V_\gamma$$

$$5 = V_{REF} + 2(0.7)$$

or

$$\underline{V_{REF} = 3.6\ \text{V}}$$

b.

$$V_{TH} = \left(\frac{R_1}{R_1+R_2}\right)(V_{REF} + 2V_\gamma)$$

$$0.5 = \left(\frac{R_1}{R_1+R_2}\right)(5)$$

$$\text{or } 1 + \frac{R_2}{R_1} = 10 \Rightarrow \frac{R_2}{R_1} = 9$$

For example, let $R_2 = 90\,\text{k}\Omega$ and $R_1 = 10\,\text{k}\Omega$

c. For $v_I = 10\ \text{V}$, and v_0 is in its low state. D_1 is on and D_2 is off.

$$\frac{v_1 - (v_1 + 0.7)}{100} + \frac{V_{REF} - v_1}{1} = \frac{v_1 - v_0}{1}$$

For $v_1 = -0.7$, then

$$\frac{10-0}{100} + \frac{3.6-(-0.7)}{1} = \frac{-0.7 - v_0}{1}$$

or

$$\underline{v_0 = -5.1\ \text{V}}$$

15.54

For $v_0 = \text{High} = (V_{REF} + 2V_\gamma)$. Then switching point is when.

$$v_I = v_B = \left(\frac{R_1}{R_1 + R_2}\right) v_0$$

$$\text{or } V_{TH} = \left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_\gamma)$$

Lower switching point is when

$$v_1 = v_B = \left(\frac{R_1}{R_1 + R_2}\right) v_0 \quad \text{and} \quad v_0 = -(V_{REF} + 2V_\gamma)$$

so

$$V_{TL} = -\left(\frac{R_1}{R_1 + R_2}\right)(V_{REF} + 2V_\gamma)$$

15.55

By symmetry, inverting terminal switches about zero.

Now, for v_0 low, upper diode is on.

$$V_{REF} - v_1 = v_1 - v_0$$

$$v_0 = 2v_1 - V_{REF} \quad \text{where} \quad v_1 = -V_\gamma$$

so

$$\underline{v_0 = -(V_{REF} + 2V_\gamma)}$$

Similarly, in the high state

$$\underline{v_0 = (V_{REF} + 2V_\gamma)}$$

Switching occurs when non-inverting terminal is zero.

So for v_0 low.

$$\frac{V_{TH} - 0}{R_1} = \frac{0 - \left[-\left(V_{REF} + 2V_\gamma\right)\right]}{R_2}$$

$$\text{or } V_{TH} = \frac{R_1}{R_2} \cdot \left(V_{REF} + 2V_\gamma\right)$$

By symmetry

$$V_{TL} = -\frac{R_1}{R_2} \cdot (V_{REF} + 2v_\gamma)$$

15.56

(a) $\upsilon_+ = \left(\frac{R_1}{R_1 + R_2}\right) \cdot \upsilon_O = \left(\frac{10}{10 + 20}\right)(5) = 1.667 \text{ V}$

$$\upsilon_X = 5 + (-1.667 - 5)\exp\left(-\frac{t}{\tau_X}\right) = 5 - 6.667\exp\left(-\frac{t}{\tau_X}\right), \text{ for } 0 < t < t_1$$

$$\upsilon_X = -5 + (1.667 - (-5))\exp\left(-\left(\frac{t - t_1}{\tau_X}\right)\right) = -5 + 6.667\exp\left(-\frac{(t - t_1)}{\tau_X}\right), \text{ for } t_1 < t < T$$

(b) $1.667 = 5 - 6.667\exp\left(-\dfrac{t_1}{\tau_X}\right)$

$$6.667\exp\left(-\frac{t_1}{\tau_X}\right) = 3.333$$

$$\exp\left(+\frac{t_1}{\tau_X}\right) = 2 \Rightarrow t_1 = \tau_X \ln(2)$$

By symmetry, $T - t_1 = \tau_X \ln(2)$

Then $T = 2\tau_X \ln(2)$

$$f = \frac{1}{T} = \frac{1}{2\tau_X \ln(2)}, \quad \tau_X = R_X C_X = \left(40\times10^3\right)\left(0.02\times10^{-6}\right) = 8\times10^{-4}\text{ s}$$

$$f = \frac{1}{2\left(8\times10^{-4}\right)\ln(2)} = 902\text{ Hz}$$

Duty cycle = 50%

15.57

(a) $\upsilon_+(+) = \left(\dfrac{R_1}{R_1+R_2}\right)(5) = \left(\dfrac{10}{10+20}\right)(5) = 1.667\text{ V}$

$$\upsilon_+(-) = \left(\frac{R_1}{R_1+R_2}\right)(-10) = \left(\frac{10}{10+20}\right)(-10) = -3.333\text{ V}$$

$$\upsilon_X = 5 + (-3.333-5)\exp\left(-\frac{t}{\tau_X}\right) = 5 - 8.333\exp\left(-\frac{t}{\tau_X}\right), \quad \text{for } 0 < t < t_1$$

$$\upsilon_X = -10 + (1.667-(-10))\exp\left(-\frac{(t-t_1)}{\tau_X}\right) = -10 + 11.667\exp\left(-\frac{(t-t_1)}{\tau_X}\right), \text{ for } t_1 < t < T$$

(b) $1.667 = 5 - 8.333\exp\left(-\dfrac{t_1}{\tau_X}\right)$

$$8.333\exp\left(\frac{-t_1}{\tau_X}\right) = 3.333 \Rightarrow \exp\left(+\frac{t_1}{\tau_X}\right) = 2.5$$

$$t_1 = \tau_X \ln(2.5)$$

Also $-3.333 = -10 + 11.667\exp\left(-\dfrac{(T-t_1)}{\tau_X}\right)$

$$11.667\exp\left(-\frac{(T-t_1)}{\tau_X}\right) = 6.667 \Rightarrow \exp\left(+\frac{(T-t_1)}{\tau_X}\right) = 1.75$$

$$T - t_1 = \tau_X \ln(1.75)$$

Now $T = \tau_X\left[\ln(2.5) + \ln(1.75)\right] = \tau_X \ln\left[(2.5)(1.75)\right]$

$$f = \frac{1}{T}, \quad \tau_X = R_X C_X = 8\times10^{-4}\text{ s}$$

$$f = \frac{1}{\left(8\times10^{-4}\right)\ln\left[(2.5)(1.75)\right]} = 847\text{ Hz}$$

15.58

$$f = \frac{1}{2.2 R_X C_X}$$

$$R_X C_X = \frac{1}{2.2 f} = \frac{1}{(2.2)(12\times10^3)}$$

$$R_X C_X = 3{,}788\times10^{-5}$$

$$R_X = 56 \text{ K}$$

For example, Let $C_X = 680$ pF

Within $\frac{1}{2}$ of 1% of design specification.

15.59

$$t_1 = 1.1 R_X C_X = (1.1)(10^4)(0.1\times10^{-6}) \Rightarrow t_1 = 1.1 \text{ ms}$$

$$0 < t < t_1, \quad \upsilon_Y = 10\left(1 - e^{-t/\tau_Y}\right)$$

$$\tau_Y = R_Y C_Y = (2\times10^3)(0.02\times10^{-6})$$

$$= 4\times10^{-5} \text{ s}$$

Now $\frac{t_1}{\tau_Y} = 2.75$

$\Rightarrow C_Y$ completely charges during each cycle.

15.60

a. Switching voltage

$$v_X = \left(\frac{R_1 + R_3}{R_1 + R_3 + R_2}\right)\cdot V_P = \left(\frac{10+10}{10+10+10}\right)(\pm10)$$

So $v_X = \pm6.667$ V

Using Equation (15.83(a))

$$v_X = V_P + \left(-\frac{2}{3}V_P - V_P\right)e^{-t_1/\tau_X} = \frac{2}{3}V_P$$

Then $1 - \frac{5}{3}\cdot e^{-t_1/\tau_X} = \frac{2}{3}$

$$\frac{1}{3} = \frac{5}{3}\cdot e^{-t_1/\tau_X} \text{ or } t_1 = \tau_X \ln(5)$$

$$t_1 = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2(500)} \Rightarrow t_1 = 0.001 \text{ s}$$

$$10^{-3} = \tau_X \ln(5) \Rightarrow \tau_X = 6.21\times10^{-4} = R_X(0.01\times10^{-6})$$

So $R_X = 62.1 \text{ k}\Omega$

b. Switching voltage

$$v_X = \left(\frac{R_1}{R_1 + R_3 + R_2}\right)(\pm V_P)$$

$$= \left(\frac{10}{10+10+10}\right)(\pm V_P) = \frac{1}{3}\cdot(\pm V_P)$$

Using Equation (15.83(a))

$$v_X = V_P + \left(-\frac{1}{3}V_P - V_P\right)e^{-t_1/\tau_X} = \frac{1}{3}V_P$$

Then $1 - \frac{4}{3}e^{-t_1/\tau_X} = \frac{1}{3}$

$$\frac{2}{3} = \frac{4}{3}\cdot e^{-t_1/\tau_X}$$

$$t_1 = \tau_X \ln(2) = (6.21\times10^{-4})\ln(2) = 4.30\times10^{-4}\ \text{s}$$

$$T = 2t_1 = 8.6\times10^{-4}\ \text{s}$$

$$f = \frac{1}{T} \Rightarrow f = 1.16\ \text{kHz}$$

15.61

(a) $\beta = \dfrac{R_1}{R_1 + R_2} = \dfrac{20}{20+12} = 0.625$

$$T = 250\times10^{-6} = \tau_X \ln\left[\frac{1+\frac{0.7}{10}}{1-0.625}\right] = \tau_X(1.0485)$$

So $\tau_X = 2.384\times10^{-4} = R_X C_X$

Set $C_X = 0.01\,\mu\text{F}$, then $R_X = 23.84\,\text{k}\Omega$

(b) For $t < 0$, $\upsilon_X = 0.7$ V

So υ_Y must be $< \upsilon_X \Rightarrow \upsilon_I < 0$

(c) $\upsilon_X = V_\gamma = 10 + [-\beta(10) - 10]\exp\left(-\dfrac{(T'-T)}{\tau_X}\right)$

$$0.7 = 10 - 16.25\exp\left(-\frac{(T'-T)}{\tau_X}\right) \Rightarrow 16.26\exp\left(-\frac{(T'-T)}{\tau_X}\right) = 9.3$$

$$\exp\left(+\frac{(T'-T)}{\tau_X}\right) = 1.747$$

$$T'-T = \tau_X \ln(1.747) = 2.384\times10^{-4}\ln(1.747)$$

$$T'-T = 133\,\mu\text{s}$$

15.62

(a) $T = \tau_X \ln\left[\dfrac{1+\frac{V_\gamma}{V_P}}{1-\beta}\right]$, $V_\gamma = 0.7$ V, $V_P = 5$ V, $\beta = 0.5$

$$\tau_X = R_X C_X = (20\times10^3)(1.2\times10^{-6}) = 2.4\times10^{-2}\ \text{s}$$

$$T = (2.4\times10^{-2})\ln\left[\frac{1+\frac{0.7}{5}}{1-0.5}\right] \Rightarrow T = 19.78\ \text{ms}$$

(b) $T' - T \cong 0.4\tau_X = 9.6\,\text{ms}$

15.63

a. From Equation (15.95)

$T = 1.1\,RC$

For $T = 60\text{ s} = 1.1\,\text{RC}$

then $RC = 54.55\text{ s}$

For example, let

$\underline{C = 50\ \mu\text{F}}$ and $\underline{R = 1.09\ \text{M}\Omega}$

b. Recovery time: capacitor is discharged by current through the discharge transistor.

If $V^+ = 5\text{ V},$ then $I_B \cong \dfrac{5 - 0.7}{100} = 0.043\text{ mA}$

If $\beta = 100,\ \ I_C = 4.3\text{ mA}$

$$V_C = \frac{1}{C}\int I_C dt = \frac{Ic}{C}\cdot t$$

Capacitor has charged to $\dfrac{2}{3}\cdot V^+ = 3.33\text{ V}$

So that $t = \dfrac{V_C \cdot C}{I_C \cdot} = \dfrac{(3.33)(50\times10^{-6})}{4.3\times10^{-3}}$

So recovery time $\underline{t \approx 38.7\text{ ms}}$

15.64

$T = 1.1\,RC$

$5\times10^{-6} = 1.1\,RC$

so $RC = 4.545\times10^{-6}\text{ s}$

For example, let

$\underline{C = 100\text{ pF}}$ and $\underline{R = 45.5\text{ k}\Omega}$

From Problem (15.63), recovery time

$$t \cong \frac{V_C \cdot C}{I_C} = \frac{(3.33)(100\times10^{-12})}{4.3\times10^{-3}}$$

or

$\underline{t = 77.4\text{ ns}}$

15.65

$$\text{Duty cycle } = 60\% = \frac{R_A + R_B}{R_A + 2R_B}\times100\%$$

$$0.60 = \frac{25 + R_B}{25 + 2R_B} \Rightarrow R_B = 50\,\text{k}\,\Omega$$

$$f = 80\times10^3 = \frac{1}{(0.693)(25 + 2(50))\times10^3 C} \Rightarrow C = 144.3\text{ pF}$$

15.66

$$f = \frac{1}{(0.693)(R_A + 2R_B)C}$$

$R_A = R_1 = 10\ \text{k}\Omega$, $R_B = R_2 + xR_3$

So $10\ \text{k}\Omega \le R_B \le 110\ \text{k}\Omega$

$$f_{\min} = \frac{1}{(0.693)(10 + 2(110)) \times 10^3 \times (0.01 \times 10^{-6})} = 627\ \text{kHz}$$

$$f_{\max} = \frac{1}{(0.693)(10 + 2(10)) \times 10^3 \times (0.01 \times 10^{-6})} = 4.81\ \text{kHz}$$

So $627\ \text{Hz} \le f \le 4.81\ \text{kHz}$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

Now

$$\frac{10 + 10}{10 + 2(10)} \times 100\% = \underline{66.7\%}$$

and

$$\frac{10 + 110}{10 + 2(110)} \times 100\% = \underline{52.2\%}$$

So $52.2 \le \text{Duty cycle} \le 66.7\%$

15.67

$1\ \text{k}\Omega \le R_A \le 51\ \text{k}\Omega$

$1\ \text{k}\Omega \le R_B \le 51\ \text{k}\Omega$

$$f_{\min} = \frac{1}{(0.693)(1 + 2(51)) \times 10^3 \times (0.01 \times 10^{-6})} = 1.40\ \text{Hz}$$

$$f_{\max} = \frac{1}{(0.693)(51 + 2(1)) \times 10^3 \times (0.01 \times 10^{-6})} = 2.72\ \text{kHz}$$

or $1.40\ \text{kHz} \le f \le 2.72\ \text{kHz}$

$$\text{Duty cycle} = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

$$\frac{1 + 51}{1 + 2(51)} \times 100\% = 50.5\%$$

or

$$\frac{51 + 1}{51 + 2(1)} \times 100\% = \underline{98.1\%}$$

or

$50.5\% \le \text{Duty cycle} \le 98.1\%$

15.68

a.

$$I_{E3} = I_{E4} = \frac{V^+ - 3V_{EB}}{R_{1A} + R_{1B}}$$

Assume $V_{EB} = 0.7$

$$I_{E3} = I_{E4} = \frac{22 - 3(0.7)}{25 + 25} = 0.398 \text{ mA}$$

Now

$$I_{C3} = I_{C4} = I_{C5} = I_{C6} = \left(\frac{20}{21}\right)(0.398)$$

$$\underline{I_{C3} = I_{C4} = I_{C5} = I_{C6} = 0.379 \text{ mA}}$$

$$I_{C1} = I_{C2} = \frac{0.398}{21}\left(\frac{20}{21}\right) \Rightarrow \underline{I_{C1} = I_{C2} = 0.018 \text{ mA}}$$

b. $\underline{I_D = 0.398 \text{mA},}$ current in D_1 and D_2

$$V_{BB} = 2V_D = 2V_T \ln\left(\frac{I_D}{I_S}\right) = 2(0.026) \ln\left(\frac{0.398 \times 10^{-3}}{10^{-13}}\right)$$

or

$$V_{BB} = 1.149 \text{ V} = V_{BE7} + V_{EB8}$$

Now

$$I_{C7} \approx I_{C4} + I_{C9} + I_{E8}$$

$$I_{C4} = 0.379 \text{ mA}$$

$$I_{B9} = I_{C8} = \left(\frac{20}{21}\right) I_{E8}$$

So

$$I_{E8} = 1.05 I_{B9} = 1.05\left(\frac{I_{C9}}{100}\right)$$

$$I_{C7} = I_{C4} + \left(\frac{100}{1.05}\right) I_{E8} + I_{E8} = I_{C4} + (96.24)\left(\frac{21}{20}\right) I_{C8}$$

So $I_{C7} = 0.379 \text{ mA} + 101 I_{C8}$

and

$$V_{BE7} = V_T \ln\left(\frac{I_{C7}}{I_S}\right); V_{EB8} = V_T \ln\left(\frac{I_{C8}}{I_S}\right)$$

Then

$$1.149 = 0.026\left[\ln\left(\frac{I_{C7}}{I_S}\right) + \ln\left(\frac{I_{C8}}{I_S}\right)\right]$$

$$44.19 = \ln\left[\frac{I_{C8}(0.379\times10^{-3}) + 101 I_{C8}}{(10^{-13})^2}\right]$$

$$(10^{-13})^2 \exp(44.19) = 101 I_{C8}^2 + 3.79\times10^{-4} I_{C8}$$

$$1.554\times10^{-7} = 101 I_{C8}^2 + 3.79\times10^{-4} I_{C8}$$

$$I_{C8} = \frac{-3.79\times10^{-4}}{2(101)} \pm \frac{\sqrt{(3.79\times10^{-4})^2 + 4(101)(1.554\times10^{-7})}}{2(101)}$$

$$\underline{I_{C8} = 37.4\ \mu\text{A}}$$

$$I_{C7} = 0.379 + 101(0.0374) \Rightarrow \underline{I_{C7} = 4.16\ \text{mA}}$$

$$I_{C9} = 4.16 - 0.379 - 0.0374\left(\frac{21}{20}\right)$$

$$\underline{I_{C9} = 3.74\ \text{mA}}$$

c. $P = (0.398 + 0.398 + 4.16)(22) \Rightarrow \underline{P = 109\ \text{mW}}$

15.69

a. From Figure 15.44, 3.7 W to the load

b. $\underline{V^+ \approx 19\ \text{V}}$

c. $\overline{P} = \frac{1}{2}\frac{V_P^2}{R_L}$

or

$$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(10)(3.7)} \Rightarrow \underline{V_P = 8.6\ \text{V}}$$

15.70

(a) $\left(1 + \frac{R_2}{R_1}\right) = 12 \Rightarrow \frac{R_2}{R_1} = 11$, and $\frac{R_4}{R_3} = 12$

(b) $\overline{P}_L = \frac{V_L^2}{2R_L} \Rightarrow V_L = \sqrt{2R_L\overline{P}_L} = \sqrt{2(12)(15)} = 18.97\ \text{V}$

$$\left|v_{O1}\right|_{\max} = \left|v_{O2}\right|_{\max} = 9.49\ \text{V}$$

$$I_L = \frac{V_L}{R_L} = \frac{18.97}{12} = 1.58\ \text{A}$$

(c) (i) $\overline{P}_L = \left(\frac{I_L}{\sqrt{2}}\right)\left(\frac{V_L}{\sqrt{2}}\right) = \left(\frac{0.8}{\sqrt{2}}\right)\left(\frac{24}{\sqrt{2}}\right) = 9.6\ \text{W}$

(ii) $R_L = \frac{V_L}{I_L} = \frac{24}{0.8} = 30\ \Omega$

15.71

a. $v_{01} = iR_2 + v_I$ where $i = \dfrac{v_I}{R_1}$

Then

$$v_{01} = v_I\left(1 + \frac{R_2}{R_1}\right)$$

Now

$$v_{02} = -iR_3 = -v_I\left(\frac{R_3}{R_1}\right)$$

So

$$v_L = v_{01} - v_{02} = v_I\left(1 + \frac{R_2}{R_1}\right) - \left[-v_I\left(\frac{R_3}{R_1}\right)\right]$$

$$\underline{A_v = \frac{v_L}{v_I} = 1 + \frac{R_2}{R_1} + \frac{R_3}{R_1}}$$

b. Want $A_v = 10 \Rightarrow \dfrac{R_2}{R_1} + \dfrac{R_3}{R_1} = 9$

Also want $\left(1 + \dfrac{R_2}{R_1}\right) = \dfrac{R_3}{R_1}$

Then $\dfrac{R_2}{R_1} + \left(1 + \dfrac{R_2}{R_1}\right) = 9$ so $\dfrac{R_2}{R_1} = 4$

For $R_1 = 50\ \text{k}\Omega$, $\underline{R_2 = 200\ \text{k}\Omega}$

and

$\dfrac{R_3}{R_1} = 5$ so $\underline{R_3 = 250\ \text{k}\Omega}$

c. $\overline{P} = \dfrac{1}{2}\dfrac{V_P^2}{R_L}$

or

$$V_P = \sqrt{2R_L\overline{P}} = \sqrt{2(20)(10)} = 20\ \text{V}$$

So peak values of output voltages are

$|v_{01}| = |v_{02}| = 10\ \text{V}$

Peak load current $= \dfrac{20}{20} = 1\ \text{A}$

15.72

(a) $\upsilon_{O1} = \left(1 + \dfrac{R_2}{R_1}\right)\cdot\upsilon_I$

$$\upsilon_{O2} = -\frac{R_4}{R_3}\cdot\upsilon_{O1} = -\frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right)\cdot\upsilon_I$$

$$\upsilon_L = \upsilon_{O1} - \upsilon_{O2}$$

$$A_\upsilon = \frac{\upsilon_L}{\upsilon_I} = \left(1 + \frac{R_2}{R_1}\right)\left(1 + \frac{R_4}{R_3}\right)$$

(b) $\left(1+\frac{R_2}{R_1}\right)=12 \Rightarrow \frac{R_2}{R_1}=11$, Set $R_2=120\,\text{k}\Omega$, then $R_1=10.9\,\text{k}\Omega$

$\frac{R_4}{R_3}=1$, Set $R_3=R_4=120\,\text{k}\Omega$

(c) (i) $\overline{P}_L=\frac{V_L^2}{2R_L}=\frac{(16)^2}{2(25)}=5.12\,\text{W}$

(ii) $I_L=\frac{V_L}{R_L}=\frac{16}{25}=0.64\,\text{A}$

15.73

(a) From Problem 15.72

$\upsilon_{O2}=-\frac{R_4}{R_3}\cdot\upsilon_{O1}$, Set $\frac{R_4}{R_3}=1$

$A_\upsilon=\left(1+\frac{R_2}{R_1}\right)\left(1+\frac{R_4}{R_3}\right)$

$25=\left(1+\frac{R_2}{R_1}\right)(2)\Rightarrow\frac{R_2}{R_1}=11.5$

Set $R_2=R_3=R_4=100\,\text{k}\Omega$, then $R_1=8.69\,\text{k}\Omega$

(b) (i) $\overline{P}_L=\left(\frac{V_L}{\sqrt{2}}\right)\left(\frac{I_L}{\sqrt{2}}\right)=\left(\frac{24}{\sqrt{2}}\right)\left(\frac{1.2}{\sqrt{2}}\right)=14.4\,\text{W}$

(ii) $R_L=\frac{V_L}{I_L}=\frac{24}{1.2}=20\,\Omega$

(c) $\overline{P}_L=\frac{V_L^2}{2R_L'}=\frac{(24)^2}{2(40)}=7.2\,\text{W}$

15.74

$$\text{Line regulation}=\frac{\Delta V_0}{\Delta V^+}$$

Now

$\Delta I=\frac{\Delta V^+}{R_1}$ and $\Delta V_Z=r_Z\cdot\Delta I$ and $\Delta V_0=10\Delta V_Z$

So

$\Delta V_0=10\cdot r_Z\cdot\frac{\Delta V^+}{R_1}$

So

$$\text{Line regulation}=\frac{\Delta V_0}{\Delta V^+}=\frac{10(15)}{9300}\Rightarrow 1.61\%$$

15.75

(a) $R_{of} = \dfrac{|\Delta V_O|}{|\Delta I_O|} = \dfrac{8}{2} \Rightarrow R_{of} = 4\ \text{m}\Omega$

(b) $\Delta V_O = R_{of} \cdot \Delta I_O = (10)(1.2) \Rightarrow \Delta V_O = 12\ \text{mV}$

15.76

For $V_0 = 8\ \text{V}$

$$V^+(\min) = V_0 + I_0(\max)R_{11} + V_{BE11} + V_{BE10} + V_{EB5}$$

This assumes $V_{BC5} = 0.$

Then

$$V^+(\min) = 8 + (0.1)(1.9) + 0.6 + 0.6 + 0.6$$

$$\underline{V^+(\min) = 9.99\ \text{V}}$$

15.77

a.

$$I_{C3} = I_{C5} = \frac{V_Z - 3V_{BE}(\text{npn})}{R_1 + R_2 + R_3}$$

$$I_{C3} = I_{C5} = \frac{6.3 - 3(0.6)}{0.576 + 3.4 + 3.9} = 0.571\ \text{mA}$$

$$I_{C8} = \frac{1}{2}\left(\frac{0.6}{2.84}\right) = 0.106\ \text{mA}$$

Neglecting current in Q_9, total collector current and emitter current in Q_5 is

$$0.571 + 0.106 = 0.677$$

Now

$$I_{Z2}R_4 + V_{EB4} = V_{EB5}$$

$$V_{EB4} = V_T\ \ln\left(\frac{I_{Z2}}{I_5}\right)$$

$$V_{EB5} = V_T\ \ln\left(\frac{I_{C5}}{2I_S}\right)$$

$$I_{Z2}R_4 = V_T\ \ln\left(\frac{I_{C5}}{2I_{Z2}}\right)$$

Then

$$R_4 = \frac{0.026}{0.25} \cdot \ln\left(\frac{0.677}{2(0.25)}\right)$$

or

$$\underline{R_4 = 31.5\ \Omega}$$

b. From Example 15.16, $V_{B7} = 3.43\ V$. Then

$$\left(\frac{R_{13}}{R_{12}+R_{13}}\right)V_0 = V_{B8} = V_{B7}$$

or

$$\left(\frac{2.23}{2.23+R_{12}}\right)(12) = 3.43$$

$$3.43(2.23+R_{12}) = (2.23)(12)$$

which yields

$$\underline{R_{12} = 5.57\ \text{k}\Omega}$$

15.78

$$\text{Line regulation} = \frac{\Delta V_0}{\Delta V^+}$$

Now

$$\Delta V_{B7} = \Delta I_{C3}\cdot R_1$$

and

$$\left(\frac{R_{13}}{R_{12}+R_{13}}\right)(\Delta V_0) = \Delta V_{B7} = \Delta I_{C3}R_1$$

and

$$\Delta I_{C3} = \frac{\Delta V_Z}{R_1+R_2+R_3} = \frac{\Delta I_Z\cdot r_Z}{R_1+R_2+R_3}$$

and

$$\Delta I_Z = \frac{\Delta V^+}{r_0} \quad \text{where} \quad r_0 = \frac{V_A}{I_Z}$$

Then

$$(0.4288)(\Delta V_0) = \Delta I_{C3}(3.9) = (3.9)\Delta I_Z\left(\frac{0.015}{7.876}\right)$$

$$r_0 = \frac{50}{0.571} = 87.6\ \text{k}\Omega$$

Then

$$(0.4288)(\Delta V_0) = (0.00743)\left(\frac{\Delta V^+}{87.6}\right)$$

So

$$\underline{\frac{\Delta V_0}{\Delta V^+} = 0.0198\%}$$

15.79

(a) $R_1 + r_z = \dfrac{V^+}{I_Z} = \dfrac{25}{12} = 2.0833\,\text{k}\,\Omega$

$$R_1 = 2.0833 - 0.012 = 2.0713\,\text{k}\,\Omega$$

(b) $V^+ = 5.6 + (0.012)(12) = 5.744\ \text{V}$

For $x = 0$,

$$5.744 = \left(\frac{R_3+R_4}{R_2+R_3+R_4}\right)\cdot V_O \Rightarrow V_O = \left(\frac{1+2+1}{2+1}\right)(5.744) = 7.659\ \text{V}$$

For $x = 1$,

$$5.744 = \left(\frac{R_4}{R_2 + R_3 + R_4}\right)\cdot V_O \Rightarrow V_O = \left(\frac{1+2+1}{1}\right)(5.744) = 22.976 \text{ V}$$

So $7.659 \le V_O \le 22.976$ V

(c) For $x = 1$, $V_1 = \frac{1}{4}V_O$

$$\upsilon_d = V_{REF} - V_1 = V_{REF} - \frac{1}{4}V_O$$

$$V_O = A_{OL}\upsilon_d - V_{BE} = A_{OL}\upsilon_d - V_T \ln\left(\frac{I_O}{I_S}\right)$$

$$V_O = A_{OL}\left(V_{REF} - \frac{1}{4}V_O\right) - V_T \ln\left(\frac{I_O}{I_S}\right)$$

$$V_O\left(1 + \frac{1}{4}A_{OL}\right) = A_{OL}V_{REF} - V_T \ln\left(\frac{I_O}{I_S}\right)$$

$$V_O = \frac{A_{OL}V_{REF} - V_T \ln\left(\frac{I_O}{I_S}\right)}{1 + \frac{1}{4}A_{OL}}$$

$$\text{Load regulation} = \frac{V_O(NL) - V_O(FL)}{V_O(NL)}$$

$$= \frac{\dfrac{A_{OL}V_{REF} - V_T \ln[I_O(NL)/I_S]}{1 + (1/4)A_{OL}} - \left[\dfrac{A_{OL}V_{REF} - V_T \ln[I_O(FL)/I_S]}{1 + (1/4)A_{OL}}\right]}{\dfrac{A_{OL}V_{REF} - V_T \ln[I_O(NL)/I_S]}{1 + (1/4)A_{OL}}}$$

$$= \frac{V_T\left[\ln\left(\frac{I_O(FL)}{I_S}\right) - \ln\left(\frac{I_O(NL)}{I_S}\right)\right]}{A_{OL}V_{REF} - V_T \ln\left(\frac{I_O(NL)}{I_S}\right)} = \frac{V_T \ln\left(\frac{I_O(FL)}{I_O(NL)}\right)}{A_{OL}V_{REF} - V_T \ln\left(\frac{I_O(NL)}{I_S}\right)}$$

Let $I_O(FL) = 5$ A, $I_O(NL) = 1\% = 0.05$ A

Let $V_T \ln\left(\frac{I_O(NL)}{I_S}\right) \cong 0.7$ V

$$\text{Load regulation} = \frac{(0.026)\ln\left(\frac{5}{0.05}\right)}{(5\times 10^3)(5.744) - 0.7} = \frac{0.11973}{2.8719\times 10^4}$$

$$\text{Load regulation} = 4.169\times 10^{-4}\%$$

15.80

(a) $I_E = \dfrac{V_Z}{R_2} = \dfrac{6.8}{5} = 1.36\text{ mA}$

$$I_O = \left(\frac{\beta}{1+\beta}\right)\cdot I_E = \left(\frac{80}{81}\right)(1.36) = 1.343\text{ mA}$$

For $V_{BC} = 0$, $V_O = 20 - V_Z - V_{EB} = 20 - 6.8 - 0.6 = 12.6\text{ V}$

$$R_L(\max) = \frac{V_O}{I_O} = \frac{12.6}{1.343} = 9.38\text{ k}\Omega$$

So $0 \le R_L \le 9.38\text{ k}\Omega$

(b) $V_+ = \left(\dfrac{R_1}{R_1 + r_Z}\right)(V^+ - V_{ZO}) = \left(\dfrac{10^4}{10^4 + 20}\right)(V^+ - 6.8)$

For $V^+ = 20\text{ V}$, $V_+ = \left(\dfrac{10^4}{10^4 + 20}\right)(13.2) = 13.17365\text{ V}$

$$I_O = \left(\frac{80}{81}\right)\left(\frac{20 - 13.17365}{5}\right) = 1.3484\text{ mA}$$

For $V^+ = 16\text{ V}$, $V_+ = \left(\dfrac{10^4}{10^4 + 20}\right)(16 - 6.8) = 9.181637\text{ V}$

$$I_O = \left(\frac{80}{81}\right)\left(\frac{16 - 9.181637}{5}\right) = 1.3468\text{ mA}$$

So $1.3468 \le I_O \le 1.3484\text{ mA}$

Chapter 16

16.1

(a) $\upsilon_O = V_{DD} - \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)R_D\left[2(\upsilon_I - V_{TN})\upsilon_O - \upsilon_O^2\right]$

$0.1 = 3.3 - \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)(40)\left[2(3.3-0.5)(0.1) - (0.1)^2\right]$

$0.1 = 3.3 - \left(\frac{W}{L}\right)(1.1) \Rightarrow \left(\frac{W}{L}\right) = 2.91$

(b) $\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)R_D V_{Ot}^2 + V_{Ot} - V_{DD} = 0$

$\left(\frac{0.1}{2}\right)(2.91)(40)V_{Ot}^2 + V_{Ot} - 3.3 = 0$

$5.82V_{Ot}^2 + V_{Ot} - 3.3 = 0 \Rightarrow V_{Ot} = 0.672 \text{ V}$

$V_{Ot} = V_{It} - V_{TN} \Rightarrow V_{It} = 1.172 \text{ V}$

(c) $i_{D,\max} = \frac{3.3 - 0.1}{40} \Rightarrow i_{D,\max} = 80\,\mu\text{A}$

$P_{D,\max} = (80)(3.3) = 264\,\mu\text{W}$

16.2

(a) (i) $K_n R_D V_{Ot}^2 + V_{Ot} - V_{DD} = 0$

$(0.05)(100)V_{Ot}^2 + V_{Ot} - 3.3 = 0 \Rightarrow V_{Ot} = 0.7185 \text{ V}$

$\Rightarrow V_{It} = 1.219 \text{ V}$

(ii) $\upsilon_O = 3.3 - (0.05)(100)\left[2(3.3-0.5)\upsilon_O - \upsilon_O^2\right]$

We find $5\upsilon_O^2 - 29\upsilon_O + 3.3 = 0 \Rightarrow \upsilon_O = 0.116 \text{ V}$

(b) (i) $(0.05)(30)V_{Ot}^2 + V_{Ot} - 3.3 = 0 \Rightarrow V_{Ot} = 1.187 \text{ V}$

$\Rightarrow V_{It} = 1.687 \text{ V}$

(ii) $\upsilon_O = 3.3 - (0.05)(30)\left[2(3.3-0.5)\upsilon_O - \upsilon_O^2\right]$

Or $1.5\upsilon_O^2 - 9.4\upsilon_O + 3.3 = 0 \Rightarrow \upsilon_O = 0.373 \text{ V}$

(c) (i) $(0.05)(5)V_{Ot}^2 + V_{Ot} - 3.3 = 0 \Rightarrow V_{Ot} = 2.147 \text{ V}$

$\Rightarrow V_{It} = 2.647 \text{ V}$

(ii) $\upsilon_O = 3.3 - (0.05)(5)\left[2(3.3-0.5)\upsilon_O - \upsilon_O^2\right]$

Or $0.25\upsilon_O^2 - 2.4\upsilon_O + 3.3 = 0 \Rightarrow \upsilon_O = 1.663 \text{ V}$

16.3

(a) $P = IV$

$$0.25 = I(3.3) \Rightarrow I = 75.76\ \mu\text{A}$$

$$R = \frac{3.3 - 0.15}{0.07576} = 41.6\ \text{k}\Omega$$

$$I = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left[2(V_{GS} - V_{TN})V_{DS} - V_{DS}^2\right]$$

$$75.76 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)\left[2(3.3 - 0.5)(0.15) - (0.15)^2\right] \Rightarrow \left(\frac{W}{L}\right) = 1.85$$

(b) $$I_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)(V_{GS} - V_{TN})^2 = \frac{V_{DD} - V_{DS}(sat)}{R}$$

$$\left(\frac{0.1}{2}\right)(1.85)V_{DS}^2(sat) = \frac{3.3 - V_{DS}(sat)}{41.6}$$

Or $3.848V_{DS}^2(sat) + V_{DS}(sat) - 3.3 = 0 \Rightarrow V_{DS}(sat) = 0.805$ V

$$V_{DS}(sat) = V_{GS} - V_{TN}$$

$$0.805 = V_{GS} - 0.5 \Rightarrow V_{GS} = 1.305\ \text{V}$$

Then $0.5 \le V_{GS} \le 1.305$ V

16.4

(a) From Equation (16.21)

$$\frac{K_D}{K_L} = \frac{(W/L)_D}{(W/L)_L} = \frac{(V_{DD} - \upsilon_O - V_{TNL})^2}{2(\upsilon_I - V_{TND})\upsilon_O - \upsilon_O^2}$$

$$= \frac{(1.8 - 0.08 - 0.4)^2}{2(1.4 - 0.4)(0.08) - (0.08)^2} = \frac{1.7424}{0.1536}$$

$$\frac{(W/L)_D}{(W/L)_L} = 11.34$$

$$P_{D,\max} = i_{D,\max} \cdot V_{DD}$$

$$0.3 = i_{D,\max}(1.8) \Rightarrow i_{D,\max} = 0.1667\ \text{mA}$$

$$i_{D,\max} = 0.1667 = \left(\frac{0.1}{2}\right)\left(\frac{W}{L}\right)_D\left[2(1.4 - 0.4)(0.08) - (0.08)^2\right]$$

Which yields $\left(\frac{W}{L}\right)_D = 21.7$ and $\left(\frac{W}{L}\right)_L = 1.91$

(b) $$V_{It} = \frac{1.8 - 0.4 + (0.4)\left(1 + \sqrt{11.34}\right)}{1 + \sqrt{11.34}} = 0.7206\ \text{V}$$

$0.4 \le V_I \le 0.7206$ V

16.5

(a) $$\frac{K_D}{K_L}\left[2(3 - 0.5)(0.25) - (0.25)^2\right] = \left[3 - 0.25 - 0.5\right]^2$$

$$\frac{K_D}{K_L}(1.1875) = (5.0625) \Rightarrow \frac{K_D}{K_L} = 4.26$$

(b) $\dfrac{K_D}{K_L}\left[2(2.5-0.5)(0.25)-(0.25)^2\right]=\left[3-0.25-0.5\right]^2$

$\dfrac{K_D}{K_L}(0.9375)=(5.0625)\Rightarrow \dfrac{K_D}{K_L}=5.4$

(c) $i_D=K_L\left(V_{GSL}-V_{TN}\right)^2=\left(\dfrac{0.1}{2}\right)(1)(3-0.25-0.5)^2=0.253\text{ mA}$

$P=i_D\cdot V_{DD}=(0.253)(3)=0.759\text{ mW}$

16.6

(a) $\dfrac{K_D}{K_L}=\dfrac{\left(V_{DD}-\upsilon_O-V_{TNL}\right)^2}{2\left(\upsilon_I-V_{TND}\right)\upsilon_O-\upsilon_O^2}$

$\upsilon_I=\text{Logic 1}=V_{DD}-V_{TNL}=3-0.5=2.5\text{ V}$

$\dfrac{K_D}{K_L}=\dfrac{(W/L)_D}{(W/L)_L}=\dfrac{(3-0.1-0.5)^2}{2(2.5-0.5)(0.1)-(0.1)^2}=\dfrac{5.76}{0.39}$

$\dfrac{(W/L)_D}{(W/L)_L}=14.77$

$P_{D,\max}=i_{D,\max}\cdot V_{DD}$

$0.4=i_{D,\max}(3)\Rightarrow i_{D,\max}=0.1333\text{ mA}$

$i_{D,\max}=0.1333=\left(\dfrac{0.1}{2}\right)\left(\dfrac{W}{L}\right)_D\left[2(2.5-0.5)(0.1)-(0.1)^2\right]$

Which yields $\left(\dfrac{W}{L}\right)_D=6.84$ and $\left(\dfrac{W}{L}\right)_L=0.463$

(b) $V_{It}=\dfrac{3-0.5+(0.5)\left(1+\sqrt{14.77}\right)}{1+\sqrt{14.77}}=1.016\text{ V}$

$V_{Ot}=1.016-0.5=0.516\text{ V}$

16.7

We have

$\dfrac{K_D}{K_L}\left[2\left(v_I-V_{TND}\right)v_O-v_O^2\right]=\left(V_{DD}-v_O-V_{TNL}\right)^2$

$\dfrac{(W/L)_D}{(W/L)_L}\left[2\left(V_{DD}-V_{TN}-V_{TN}\right)\left(0.08V_{DD}\right)-\left(0.08V_{DD}\right)^2\right]=\left(V_{DD}-0.08V_{DD}-V_{TN}\right)^2$

$\dfrac{(W/L)_D}{(W/L)_L}\left[2\left(V_{DD}-2(0.2)V_{DD}\right)\left(0.08V_{DD}\right)-0.0064V_{DD}^2\right]=\left[(0.92-0.2)V_{DD}\right]^2=0.5184V_{DD}^2$

$\dfrac{(W/L)_D}{(W/L)_L}[0.096]=0.5184\Rightarrow \underline{\dfrac{(W/L)_D}{(W/L)_L}=5.4}$

16.8

$V_{OH} = V_B - V_{TN} = \text{Logic 1}$

So

(a) $V_B = 4\,V \Rightarrow V_{OH} = 3\,V$

(b) $V_B = 5\,V \Rightarrow V_{OH} = 4\,V$

(c) $V_B = 6\,V \Rightarrow V_{OH} = 5\,V$

(d) $V_B = 7\,V \Rightarrow V_{OH} = 5\,V, \text{since } V_{DS} = 0$

For $v_I = V_{OH}$

$$K_D\left[2(v_I - V_T)v_O - v_O^2\right] = K_L\left[V_B - v_O - V_T\right]^2$$

Then

(a) $(1)\left[2(3-1)V_{OL} - V_{OL}^2\right] = (0.4)\left[4 - V_{OL} - 1\right]^2 \Rightarrow \underline{V_{OL} = 0.657\,V}$

(b) $(1)\left[2(4-1)V_{OL} - V_{OL}^2\right] = (0.4)\left[5 - V_{OL} - 1\right]^2 \Rightarrow \underline{V_{OL} = 0.791\,V}$

(c) $(1)\left[2(5-1)V_{OL} - V_{OL}^2\right] = (0.4)\left[6 - V_{OL} - 1\right]^2 \Rightarrow \underline{V_{OL} = 0.935\,V}$

(d) Load in non-sat region

$$i_{DD} = i_{OL}$$

$$(1)\ \left[2(5-1)V_{OL} - V_{OL}^2\right] = (0.4)\left[2(7 - V_{OL} - 1)(5 - V_{OL}) - (5 - V_{OL})^2\right]$$

$$8V_{OL} - V_{OL}^2 = (0.4)\left[2(6 - V_{OL})(5 - V_{OL}) - \left(25 - 10V_{OL} + V_{OL}^2\right)\right]$$

$$= (0.4)\left[2\left(30 - 11V_{OL} + V_{OL}^2\right) - 25 + 10V_{OL} - V_{OL}^2\right]$$

$$= (0.4)\left[60 - 22V_{OL} + 2V_{OL}^2 - 25 + 10V_{OL} - V_{OL}^2\right]$$

$$8V_{OL} - V_{OL}^2 = 14 - 4.8V_{OL} + 0.4V_{OL}^2$$

$$1.4V_{OL}^2 - 12.8V_{OL} + 14 = 0$$

$$V_{OL} = \frac{12.8 \pm \sqrt{163.84 - 4(1.4)(14)}}{2(1.4)}$$

$$V_{OL} = 1.27V$$

For load

$$V_{DS}(\text{sat}) = 7 - 1.27 - 1 = 4.73V$$

$$V_{DS} = 5 - 1.27 = 3.73 \text{ non-sat}$$

16.9

(a) $$\sqrt{\frac{K_D}{K_L}}\left(V_{It} - V_{TND}\right) = -V_{TNL}$$

$$\sqrt{\frac{500}{100}}\left(V_{It} - 0.5\right) = -\left(-0.8\right) \Rightarrow V_{It} = 0.8578 \text{ V}$$

For Driver: $V_{Ot} = V_{It} - V_{TND} = 0.8578 - 0.5 = 0.3578$ V

For Load: $V_{Ot} = V_{DD} + V_{TNL} = 3.3 + (-0.8) = 2.5$ V

(b) $$\frac{K_D}{K_L}\left[2(\upsilon_I - V_{TND})\upsilon_O - \upsilon_O^2\right] = \left(-V_{TNL}\right)^2$$

$$\left(\frac{500}{100}\right)\left[2(3.3 - 0.5)\upsilon_O - \upsilon_O^2\right] = \left[-(-0.8)\right]^2$$

We find $5\upsilon_O^2 - 28\upsilon_O + 0.64 = 0 \Rightarrow \upsilon_O = 0.0230$ V

(c) $i_{D,\max} = K_L\left(-V_{TNL}\right)^2 = 100\left[-\left(-0.8\right)\right]^2 = 64\,\mu\text{A}$

$P_{D,\max} = i_{D,\max}\cdot V_{DD} = (64)(3.3) = 211\,\mu\text{W}$

16.10

$$\left(\frac{500}{50}\right)\left[2(3-0.5)(0.1)-(0.1)^2\right] = \left(-V_{TNL}\right)^2$$

So

$$\left(-V_{TNL}\right)^2 = 4.9 \Rightarrow \underline{V_{TNL} = -2.21\,V}$$

16.11

(a) $P_{D,\max} = i_{D,\max}\cdot V_{DD}$

$$80 = i_{D,\max}(1.8) \Rightarrow i_{D,\max} = 44.44\,\mu\text{A}$$

$$i_{D,\max} = 44.44 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_L\left[-\left(-0.6\right)\right]^2 \Rightarrow \left(\frac{W}{L}\right)_L = 2.47$$

$$\frac{K_D}{K_L}\left[2(1.8-0.3)(0.06)-(0.06)^2\right] = \left[-\left(-0.6\right)\right]^2$$

$$\frac{K_D}{K_L}(0.1764) = (0.36) \Rightarrow \frac{K_D}{K_L} = 2.04 = \frac{(W/L)_D}{(W/L)_L}$$

Then $\left(\frac{W}{L}\right)_D = 5.04$

(b) $$\sqrt{\frac{K_D}{K_L}}\left(V_{It} - V_{TND}\right) = -V_{TNL}$$

$$\sqrt{\frac{5.04}{2.47}}\left(V_{It} - 0.3\right) = \left[-\left(-0.6\right)\right] \Rightarrow V_{It} = 0.720\,\text{V}$$

For Driver: $V_{Ot} = 0.720 - 0.3 = 0.420\,\text{V}$

For Load: $V_{Ot} = 1.8 - 0.6 = 1.2\,\text{V}$

(c) $P_{D,\max} = 80\,\mu\text{W}$

$$2\left(\frac{5.04}{2.47}\right)\left[2(1.8-0.3)\upsilon_O - \upsilon_O^2\right] = \left[-\left(-0.6\right)\right]^2$$

We find $4.08\upsilon_O^2 - 12.24\upsilon_O + 0.36 = 0 \Rightarrow \upsilon_O = 0.0297$ V

16.12

a. From Equation (16.27(b)):

$$\left(\frac{W}{L}\right)_D\left[2(2.5-0.5)(0.05)-(0.05)^2\right] = \left(\frac{W}{L}\right)_L\left[-\left(-1\right)\right]^2$$

$$\left(\frac{W}{L}\right)_L = 1$$

Then $\underline{\left(\frac{W}{L}\right)_D = 5.06}$

b. $i_D = \left(\frac{80}{2}\right)(1)\left[-(-1)\right]^2$

or $\underline{i_D = 40\ \mu\text{A}}$

$P = i_D \cdot V_{DD} = (40)(2.5) \Rightarrow \underline{P = 100\ \mu\text{W}}$

16.13

a. i. $v_I = 0.5\text{ V} \Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$

ii. $v_I = 5\text{ V},$ From Equation (16.11),

$v_0 = 5 - (0.1)(20)\left[2(5 - 1.5)v_0 - v_0^2\right]$

$2v_0^2 - 15v_0 + 5 = 0$

$v_0 = \frac{15 \pm \sqrt{(15)^2 - 4(2)(5)}}{2(2)} \Rightarrow \underline{v_0 = 0.35\text{ V}}$

$i_D = \frac{5 - 0.35}{20} = 0.2325\text{ mA}$

$P = i_D \cdot V_{DD} = (0.2325)(5) \Rightarrow \underline{P = 1.16\text{ mW}}$

b. i. $v_I = 0.25\text{ V} \Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$

ii. $v_I = 4.3\text{ V},$ From Equation (16.21),

$100\left[2(4.3 - 0.7)v_0 - v_0^2\right] = 10\left[5 - v_0 - 0.7\right]^2$

$10\left[7.2v_0 - v_0^2\right] = 18.49 - 8.6v_0 + v_0^2$

Then

$11v_0^2 - 80.6v_0 + 18.49 = 0$

$v_0 = \frac{80.6 \pm \sqrt{(80.6)^2 - 4(11)(18.49)}}{2(11)} \Rightarrow \underline{v_0 = 0.237\text{ V}}$

Then

$i_D = 10\left[5 - 0.237 - 0.7\right]^2 = 165\ \mu\text{A}$

$P = i_D \cdot V_{DD} = (165)(5) \Rightarrow \underline{P = 825\ \mu\text{W}}$

c. i. $v_I = 0.03\text{ V} \Rightarrow i_D = 0 \Rightarrow \underline{P = 0}$

$v_I = 5\text{ V}$

$i_D = K_L\left(-V_{TNL}\right)^2 = (10)\left[-(-2)\right]^2 = 40\ \mu A$

ii. $P = i_D \cdot V_{DD} = (40)(5) \Rightarrow \underline{P = 200\ \mu\text{W}}$

16.14

(a) $\upsilon_I = V_{DD} - V_{TNL} = 5 - 0.5 = 4.5\text{ V}$

$K_D\left[2(\upsilon_I - V_{TND})\upsilon_{O1} - \upsilon_{O1}^2\right] = K_L\left[V_{DD} - \upsilon_{O1} - V_{TNL}\right]^2$

$10\left[2(4.5 - 0.5)\upsilon_{O1} - \upsilon_{O1}^2\right] = (1)\left[5 - \upsilon_{O1} - 0.5\right]^2$

We find $11\upsilon_{O1}^2 - 89\upsilon_{O1} + 20.25 = 0 \Rightarrow \upsilon_{O1} = 0.234\text{ V}$

$\upsilon_{O2} = V_{DD} - V_{TNL} = 4.5\text{ V}$

(b) $\upsilon_I = 0.234\text{ V}, \Rightarrow \upsilon_{O1} = 4.5\text{ V}$

From part (a), $\upsilon_{O2} = 0.234\text{ V}$

16.15

(a) $\dfrac{K_D}{K_L}\left[2(\upsilon_I - V_{TND})\upsilon_O - \upsilon_O^2\right] = (-V_{TNL})^2$

$$\left(\frac{4}{1}\right)\left[2(5-0.8)\upsilon_{O1} - \upsilon_{O1}^2\right] = \left[-(-1.2)\right]^2$$

We find $4\upsilon_{O1}^2 - 33.6\upsilon_{O1} + 1.44 = 0 \Rightarrow \upsilon_{O1} = 0.0431$ V

$\Rightarrow \upsilon_{O2} = 5$ V

(b) For $\upsilon_I = 0.0431$ V, $\upsilon_{O1} = 5$ V

From part (a), $\upsilon_{O2} = 0.0431$ V

16.16

(a) $\upsilon_O = V_{DD} - V_{TNLO} = 2.5 - 0.5 = 2.0$ V

(b) $\upsilon_O = V_{DD} - \left[V_{TNLO} + \gamma\left(\sqrt{2\phi_{fp} + \upsilon_O} - \sqrt{2\phi_{fp}}\right)\right]$

$$\upsilon_O = 2.5 - \left[0.5 + 0.25\left(\sqrt{0.7 + \upsilon_O} - \sqrt{0.7}\right)\right]$$

$$\upsilon_O - 2.209 = -0.25\sqrt{0.7 + \upsilon_O}$$

$$\upsilon_O^2 - 4.418\upsilon_O + 4.88 = 0.0625(0.7 + \upsilon_O)$$

$$\upsilon_O^2 - 4.4805\upsilon_O + 4.836 = 0 \Rightarrow \upsilon_O = 1.81 \text{ V}$$

16.17

(a) $\sqrt{\dfrac{K_D}{K_L}}(\upsilon_I - V_{TND}) = (-V_{TNL})$

Or $\upsilon_I = \left[-(V_{TNL})\right]\sqrt{\dfrac{K_L}{K_D}} + V_{TND} = \left[-(-0.6)\right]\sqrt{\dfrac{20}{100}} + 0.4 = 0.6683$ V

(b) $V_{TNL} = V_{TNLO} + \gamma\left(\sqrt{2\phi_{fp} + \upsilon_O} - \sqrt{2\phi_{fp}}\right)$

$$= -0.6 + 0.25\left(\sqrt{0.7 + 1.25} - \sqrt{0.7}\right) = -0.460 \text{ V}$$

$$\upsilon_I = \left[-(-0.460)\right]\sqrt{\frac{20}{100}} + 0.4 = 0.6057 \text{ V}$$

6.18

(a) $\dfrac{K_D}{K_L}\left[2(\upsilon_I - V_{TND})\upsilon_O - \upsilon_O^2\right] = (-V_{TNL})^2$

$$\frac{K_D}{K_L}\left[2(1.8-0.4)(0.1) - (0.1)^2\right] = \left[-(-0.6)\right]^2$$

$$\frac{K_D}{K_L}(0.27) = 0.36 \Rightarrow \frac{K_D}{K_L} = 1.33$$

(b) $2\left(\frac{K_D}{K_L}\right)\left[2(\upsilon_I - V_{TND})\upsilon_O - \upsilon_O^2\right] = \left[-V_{TNL}\right]^2$

$2(1.333)\left[2(1.8-0.4)\upsilon_O - \upsilon_O^2\right] = \left[-(-0.6)\right]^2$

We find $2.667\upsilon_O^2 - 7.467\upsilon_O + 0.36 = 0 \Rightarrow \upsilon_O = 49.1$ mV

(c) $i_{D,\max} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L (-V_{TNL})^2 = \left(\frac{0.1}{2}\right)(1)\left[-(-0.6)\right]^2 \Rightarrow i_{D,\max} = 18\,\mu\text{A}$

$P = i_{D,\max} \cdot V_{DD} = (18)(1.8) = 32.4\,\mu\text{W}$

6.19

(a) One input high,

$\frac{K_D}{K_L}\left[2(3-0.5)(0.1) - (0.1)^2\right] = \left[-(-1)\right]^2 \Rightarrow \frac{K_D}{K_L} = 2.04$

(b) $P = i_D \cdot V_{DD}$

$0.1 = i_D(3) \Rightarrow i_D = 33.33\,\mu\text{A}$

$i_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_L \left[-V_{TNL}\right]^2$

$33.33 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_L (1) \Rightarrow \left(\frac{W}{L}\right)_L = 0.667$

Then $\left(\frac{W}{L}\right)_D = (2.04)(0.667) = 1.36$

(c) $3(2.04)\left[2(3-0.5)\upsilon_O - \upsilon_O^2\right] = \left[-(-1)\right]^2$

$6.12\upsilon_O^2 - 30.6\upsilon_O + 1 = 0 \Rightarrow \upsilon_O = 0.0329$ V

16.20

(a) $\frac{K_D}{K_L}\left[2(2.5-0.4)(0.05) - (0.05)^2\right] = \left[-(-0.6)\right]^2$

$\frac{K_D}{K_L}(0.2075) = 0.36 \Rightarrow \frac{K_D}{K_L} = 1.735$

(b) $P = i_{D,\max} \cdot V_{DD}$

$50 = i_{D,\max}(2.5) \Rightarrow i_{D,\max} = 20\,\mu\text{A}$

$i_{D,\max} = 20 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_L \left[-(-0.6)\right]^2$

We find $\left(\frac{W}{L}\right)_L = 1.11$ and $\left(\frac{W}{L}\right)_D = 1.93$

(c) (i) $2(1.735)\left[2(2.5-0.4)\upsilon_O - \upsilon_O^2\right] = 0.36$

$3.47\upsilon_O^2 - 14.574\upsilon_O + 0.36 = 0 \Rightarrow \upsilon_O = 24.9$ mV

(ii) $3(1.735)\left[2(2.5-0.4)\upsilon_O - \upsilon_O^2\right] = 0.36$

$5.205\upsilon_O^2 - 21.861\upsilon_O + 0.36 = 0 \Rightarrow \upsilon_O = 16.5$ mV

(iii) $4(1.735)\left[2(2.5-0.4)\upsilon_O-\upsilon_O^2\right]=0.36$

$$6.94\upsilon_O^2-29.148\upsilon_O+0.36=0\Rightarrow\upsilon_O=12.4\ \text{mV}$$

16.21

a.

$$P=i_D\cdot V_{DD}$$

$$250=i_D(5)\Rightarrow i_D=50\ \mu\text{A}$$

$$i_D=\left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_{ML1}\left[-V_{TNL1}\right]^2$$

$$50=\left(\frac{60}{2}\right)\left(\frac{W}{L}\right)_{ML1}\left[-(-2)\right]^2$$

So that $\underline{\left(\frac{W}{L}\right)_{ML1}=0.417}$

$$\frac{K_D}{K_L}\left[2(v_I-V_{TND})v_O-v_O^2\right]=\left[-V_{TNL}\right]^2$$

$$\frac{K_D}{K_L}\left[2(5-0.8)(0.15)-(0.15)^2\right]=\left[-(-2)\right]^2$$

or $\underline{\frac{K_D}{K_L}=3.23\Rightarrow\left(\frac{W}{L}\right)_{MD1}=1.35}$

b. For $v_X=v_Y=0\Rightarrow v_{01}=5$ and $v_{03}=4.2$

Then

$$K_{D2}\left[2(v_{O1}-V_{TND})v_{O2}-v_{O2}^2\right]+K_{D3}\left[2(v_{O3}-V_{TND})v_{O2}-v_{O2}^2\right]=K_{L2}\left[-V_{TNL2}\right]^2$$

$$K_{D2}\propto 8, K_{D3}\propto 8, K_{L2}\propto 1$$

$$8\left[2(5-0.8)v_{02}-v_{02}^2\right]+8\left[2(4.2-0.8)v_{02}-v_{02}^2\right]=(1)\left[-(-2)\right]^2$$

$$67.2v_{02}-8v_{02}^2+54.4v_{02}-8v_{02}^2=4$$

Then

$$16v_0^2-121.6v_0+4=0$$

$$v_{02}=\frac{121.6\pm\sqrt{(121.6)^2-4(16)(4)}}{2(16)}$$

So $\underline{v_{02}=0.0330\ \text{V}}$

16.22

(a) $i_{D,\max}=\left(\frac{100}{2}\right)(1)\left[V_{DD}-\upsilon_O-V_{TN}\right]^2=50\left[3.3-\upsilon_O-0.4\right]^2=50\left[2.9-\upsilon_O\right]^2$

The output is small, so neglect υ_O^2.

Then $i_{D,\max}\cong 50\left[8.41-5.8\upsilon_O\right]$ (Eq. 1)

Also,

$$i_{D,\max}=\left(\frac{100}{2}\right)(12)\left[2(\upsilon_{GSX}-V_{TN})\upsilon_{DSX}-\upsilon_{DSX}^2\right]$$

$$=600\left[2(2.9-0.4)\upsilon_{DSX}-\upsilon_{DSX}^2\right]\cong 3000\upsilon_{DSX}$$ (Eq. 2)

And

$$i_{D,\max} = \left(\frac{100}{2}\right)(12)\left[2(\upsilon_{GSY} - V_{TN})\upsilon_{DSY} - \upsilon_{DSY}^2\right]$$

We have $\upsilon_{GSY} = \upsilon_Y - \upsilon_{DSX}$; $\upsilon_{DSY} = \upsilon_O - \upsilon_{DSX}$

$$i_{D,\max} = 600\left[2(2.9 - \upsilon_{DSX} - 0.4)(\upsilon_O - \upsilon_{DSX}) - (\upsilon_O - \upsilon_{DSX})^2\right]$$

$$\cong 600\left[2(2.5 - \upsilon_{DSX})(\upsilon_O - \upsilon_{DSX})\right] \qquad \text{(Eq.3)}$$

Now $3000\upsilon_{DSX} = 50\left[8.41 - 5.8\upsilon_O\right]$

$$\upsilon_{DSX} = 0.016667\left[8.41 - 5.8\upsilon_O\right] \qquad \text{(Eq. 4)}$$

Also $3000\upsilon_{DSX} = 600\left[2(2.5 - \upsilon_{DSX})(\upsilon_O - \upsilon_{DSX})\right] \cong 1200\left[2.5\upsilon_O - 2.5\upsilon_{DSX}\right]$

$$2.5\upsilon_{DSX} = 2.5\upsilon_O - 2.5\upsilon_{DSX}$$

Or $\upsilon_{DSX} \cong 0.5\upsilon_O$

Then from Eq. 4,

$$0.5\upsilon_O = 0.140 - 0.09667\upsilon_O \Rightarrow \upsilon_O \cong 0.2346 \text{ V}$$

(b) $\upsilon_{DSX} \cong 0.5\upsilon_O \cong 0.1173 \text{ V}$

$\upsilon_{GSX} = 2.9 \text{ V}$

$\upsilon_{GSY} \cong 2.9 - 0.1173 \cong 2.783 \text{ V}$

$\upsilon_{DSY} = \upsilon_O - \upsilon_{DSX} \cong 0.1173 \text{ V}$

16.23

a. We can write

$$K_x\left[2(v_X - V_{TNX})v_{DSX} - v_{DSX}^2\right] = K_y\left[2(v_Y - v_{DSX} - V_{TNY})v_{DSY} - v_{DSY}^2\right] = K_L\left[-V_{TNL}\right]^2$$

From the first and third terms, (neglect v_{DSX}^2),

$$4\left[2(5 - 0.8)v_{DSX}\right] = (1)\left[-(-1.5)\right]^2$$

or $\underline{v_{DSX} = 0.067 \text{ V}}$

From the second and third terms, (neglect v_{DSY}^2),

$$4\left[2(5 - 0.067 - 0.8)v_{DSY}\right] = (1)\left[-(-1.5)\right]^2$$

or $\underline{v_{DSY} = 0.068 \text{ V}}$

Now

$$v_{GSX} = 5, v_{GSY} = 5 - 0.067 \Rightarrow v_{GSY} = 4.933 \text{ V}$$

and $v_0 = v_{DSX} + v_{DSY} \Rightarrow \underline{v_0 = 0.135 \text{ V}}$

Since v_0 is close to ground potential, the body-effect has little effect on the results.

16.24

(a) $$\frac{1}{4}\left(\frac{K_D}{K_L}\right)\left[2(\upsilon_I - V_{TND})\upsilon_O - \upsilon_O^2\right] = (-V_{TNL})^2$$

$$\frac{1}{4}\left(\frac{K_D}{K_L}\right)\left[2(3.3 - 0.4)(0.1) - (0.1)^2\right] = \left[-(-0.6)\right]^2$$

$$\left(\frac{K_D}{K_L}\right)(0.1425) = 0.36 \Rightarrow \left(\frac{K_D}{K_L}\right) = 2.53$$

(b) $P = i_{D,\max} \cdot V_{DD}$

$100 = i_{D,\max}(3.3) \Rightarrow i_{D,\max} = 30.3\,\mu\text{A}$

$$i_{D,\max} = 30.3 = \left(\frac{100}{2}\right)\left(\frac{W}{L}\right)_L \left[-(-0.6)\right]^2$$

Which yields $\left(\frac{W}{L}\right)_L = 1.68$ and $\left(\frac{W}{L}\right)_D = 4.26$

16.25

$\overline{Y} = [A \text{ OR } (B \text{ AND } C)] \text{ AND } D$

16.26

Considering a truth table, we find

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

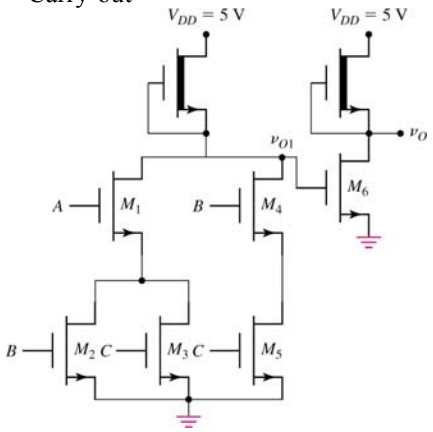
which shows that the circuit performs the exclusive-OR function.

16.27

$$\overline{(A+B)(C+D)}$$

16.28

(a) Carry-out $= A \bullet (B+C) + B \bullet C$



(b) For $v_{O1} = Low = 0.2\,V$

$$\frac{K_D}{K_L}\left[2(5-0.8)(0.2)-(0.2)^2\right] = \left[-(-1.5)\right]^2 \Rightarrow$$

For $\left(\frac{W}{L}\right)_L = 1,$ then $\left(\frac{W}{L}\right)_D = 1.37$

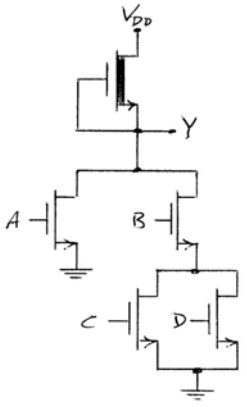
So, for $M_6 : \left(\frac{W}{L}\right)_6 = 1.37$

To achieve the required composite conduction parameter,

For $M_1 - M_5 : \underline{\left(\frac{W}{L}\right)_{1-5} = 2.74}$

16.29

(a)



(b) $$\frac{(W/L)_A}{1}\left[2(2.5-0.4)(0.05)-(0.05)^2\right]=\left[-(-0.6)\right]^2$$

$$\left(\frac{W}{L}\right)_A(0.2075)=0.36 \Rightarrow \left(\frac{W}{L}\right)_A = 1.735$$

$$\Rightarrow \left(\frac{W}{L}\right)_{B,C,D} = 2\left(\frac{W}{L}\right)_A = 3.47$$

16.30 Design Problem

16.31

(a) By symmetry, $V_{It} = \frac{V_{DD}}{2} = \frac{2.5}{2} = 1.25$ V

$$V_{OPt} = 1.25 - (-0.4) = 1.65 \text{ V}$$

$$V_{ONt} = 1.25 - 0.4 = 0.85 \text{ V}$$

(c) For $\upsilon_I = 1.1$ V, NMOS in saturation, PMOS in nonsaturation

$$K_n\left[\upsilon_I - V_{TN}\right]^2 = K_p\left[2(V_{DD} - \upsilon_I + V_{TP})(V_{DD} - \upsilon_O) - (V_{DD} - \upsilon_O)^2\right]$$

$$(1.1-0.4)^2 = 2(2.5-1.1-0.4)(V_{DD} - \upsilon_O) - (V_{DD} - \upsilon_O)^2$$

$$(V_{DD} - \upsilon_O)^2 - 2(V_{DD} - \upsilon_O) + 0.49 = 0 \Rightarrow (V_{DD} - \upsilon_O) = 0.2859 \text{ V}$$

Or $\upsilon_O = 2.5 - 0.2859 = 2.214$ V

For $\upsilon_I = 1.4$ V, NMOS in nonsaturation, PMOS in saturation

$$K_n\left[2(\upsilon_I - V_{TN})\upsilon_O - \upsilon_O^2\right] = K_p(V_{DD} - \upsilon_I + V_{TP})^2$$

$$2(1.4-0.4)\upsilon_O - \upsilon_O^2 = (2.5-1.4-0.4)^2$$

$$\upsilon_O^2 - 2\upsilon_O + 0.49 = 0 \Rightarrow \upsilon_O = 0.286 \text{ V}$$

16.32

(a) $K_n = \left(\frac{100}{2}\right)(2) = 100\,\mu\text{A/V}^2$; $K_p = \left(\frac{40}{2}\right)(5) = 100\,\mu\text{A/V}^2$

(i) By symmetry

$$V_{It} = \frac{V_{DD}}{2} = \frac{3.3}{2} = 1.65\text{ V}$$

$$\Rightarrow V_{OPt} = 1.65 - (-0.4) = 2.05\text{ V}$$

$$\Rightarrow V_{ONt} = 1.65 - 0.4 = 1.25\text{ V}$$

(iii) For $\upsilon_O = 0.25$ V; NMOS in nonsaturation, PMOS in saturation

$$2(\upsilon_I - V_{TN})\upsilon_O - \upsilon_O^2 = (V_{DD} - \upsilon_I + V_{TP})^2$$

$$2(\upsilon_I - 0.4)(0.25) - (0.25)^2 = (3.3 - \upsilon_I - 0.4)^2$$

$$0.5\upsilon_I - 0.2 - 0.0625 = 8.41 - 5.8\upsilon_I + \upsilon_I^2$$

$$\upsilon_I^2 - 6.3\upsilon_I + 8.6725 = 0 \Rightarrow \upsilon_I = 2.03\text{ V}$$

For $\upsilon_O = 3.05$ V; NMOS in saturation, PMOS in nonsaturation

$$(\upsilon_I - V_{TN})^2 = 2(V_{DD} - \upsilon_I + V_{TP})(V_{DD} - \upsilon_O) - (V_{DD} - \upsilon_O)^2$$

$$(\upsilon_I - 0.4)^2 = 2(3.3 - \upsilon_I - 0.4)(3.3 - 3.05) - (3.3 - 3.05)^2$$

$$\upsilon_I^2 - 0.8\upsilon_I + 0.16 = 0.5(2.9 - \upsilon_I) - 0.0625$$

$$\upsilon_I^2 - 0.3\upsilon_I - 1.2275 = 0 \Rightarrow \upsilon_I = 1.27\text{ V}$$

(b) $K_n = \left(\frac{100}{2}\right)(4) = 200\,\mu\text{A/V}^2$; $K_p = \left(\frac{40}{2}\right)(5) = 100\,\mu\text{A/V}^2$

(i) $$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{200}{100}}(0.4)}{1 + \sqrt{\frac{200}{100}}} = 1.436\text{ V}$$

$$\Rightarrow V_{OPt} = 1.436 + 0.4 = 1.836\text{ V}$$

$$\Rightarrow V_{ONt} = 1.436 - 0.4 = 1.036\text{ V}$$

(iii) For $\upsilon_O = 0.25$ V; NMOS in nonsaturation, PMOS in saturation

$$200\left[2(\upsilon_I - 0.4)(0.25) - (0.25)^2\right] = 100(3.3 - \upsilon_I - 0.4)^2$$

$$2(0.5\upsilon_I - 0.2 - 0.0625) = 8.41 - 5.8\upsilon_I + \upsilon_I^2$$

$$\upsilon_I^2 - 6.8\upsilon_I + 8.935 = 0 \Rightarrow \upsilon_I = 1.78\text{ V}$$

For $\upsilon_O = 3.05$ V ; NMOS in saturation, PMOS in nonsaturation

$$200(\upsilon_I - 0.4)^2 = 100\left[2(3.3 - \upsilon_I - 0.4)(3.3 - 3.05) - (3.3 - 3.05)^2\right]$$

$$2(\upsilon_I^2 - 0.8\upsilon_I + 0.16) = 0.5(2.9 - \upsilon_I) - 0.0625$$

$$2\upsilon_I^2 - 1.1\upsilon_I - 1.0675 = 0 \Rightarrow \upsilon_I = 1.06\text{ V}$$

16.33

(a) $K_n = \left(\frac{100}{2}\right)(4) = 200\,\mu\text{A/V}^2$; $K_p = \left(\frac{40}{2}\right)(12) = 240\,\mu\text{A/V}^2$

(i) $$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{200}{240}}(0.4)}{1 + \sqrt{\frac{200}{240}}} = 1.707 \text{ V}$$

$$\Rightarrow V_{OPt} = 1.707 + 0.4 = 2.107 \text{ V}$$

$$\Rightarrow V_{ONt} = 1.707 - 0.4 = 1.307 \text{ V}$$

(ii) For $\upsilon_O = 3.1\,\text{V}$; NMOS in saturation, PMOS in nonsaturation

$$200(\upsilon_I - 0.4)^2 = 240\left[2(3.3 - \upsilon_I - 0.4)(3.3 - 3.1) - (3.3 - 3.1)^2\right]$$

$$\upsilon_I^2 - 0.8\upsilon_I + 0.16 = 1.2\left[0.4(2.9 - \upsilon_I) - 0.04\right]$$

$$\upsilon_I^2 - 0.32\upsilon_I - 1.184 = 0 \Rightarrow \upsilon_I = 1.26 \text{ V}$$

(iii) For $\upsilon_O = 0.2$ V ; NMOS in nonsaturaton, PMOS in saturation

$$200\left[2(\upsilon_I - 0.4)(0.2) - (0.2)^2\right] = 240(3.3 - \upsilon_I - 0.4)^2$$

$$0.4\upsilon_I - 0.2 = 1.2\left(8.41 - 5.8\upsilon_I + \upsilon_I^2\right)$$

$$1.2\upsilon_I^2 - 7.36\upsilon_I + 10.292 = 0 \Rightarrow \upsilon_I = 2.157 \text{ V}$$

(b) $$K_n = \left(\frac{100}{2}\right)(6) = 300\,\mu\text{ A/V}^2 \;;\; K_p = \left(\frac{40}{2}\right)(4) = 80\,\mu\text{ A/V}^2$$

(i) $$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{300}{80}}(0.4)}{1 + \sqrt{\frac{300}{80}}} = 1.25 \text{ V}$$

(ii) For $\upsilon_O = 3.1\,\text{V}$; NMOS in saturation, PMOS in nonsaturation

$$300(\upsilon_I - 0.4)^2 = 80\left[2(3.3 - \upsilon_I - 0.4)(3.3 - 3.1) - (3.3 - 3.1)^2\right]$$

$$3.75\left(\upsilon_I^2 - 0.8\upsilon_I + 0.16\right) = 0.4(2.9 - \upsilon_I) - 0.04$$

$$3.75\upsilon_I^2 - 2.6\upsilon_I - 0.52 = 0 \Rightarrow \upsilon_I = 0.855 \text{ V}$$

(iii) For $\upsilon_O = 0.2\,\text{V}$; NMOS in nonsaturation, PMOS in saturation

$$300\left[2(\upsilon_I - 0.4)(0.2) - (0.2)^2\right] = 80(3.3 - \upsilon_I - 0.4)^2$$

$$3.75(0.4\upsilon_I - 0.2) = 8.41 - 5.8\upsilon_I + \upsilon_I^2$$

$$\upsilon_I^2 - 7.3\upsilon_I + 9.16 = 0 \Rightarrow \upsilon_I = 1.61 \text{ V}$$

16.34

a. For $v_{O1} = 0.6 < V_{TN} \Rightarrow \underline{v_{O2} = 5\,V}$

N_1 in nonsaturation and P_1 in saturation. From Equation (16.43),

$$\left[2(v_I - 0.8)(0.6) - (0.6)^2\right] = \left[5 - v_I - 0.8\right]^2$$

$$1.2v_I - 1.32 = 17.64 - 8.4v_I + v_I^2$$

or

$$v_I^2 - 9.6v_I + 18.96 = 0$$

$$v_I = \frac{9.6 \pm \sqrt{(9.6)^2 - 4(1)(18.96)}}{2}$$

or

$\underline{v_I = 2.78 \text{ V}}$

b. $V_{0Nt} \le v_{02} \le V_{0Pt}$

From symmetry, $V_{It} = 2.5 \text{ V}$

$V_{0Pt} = 2.5 + 0.8 = 3.3 \text{ V}$

and $V_{0Nt} = 2.5 - 0.8 = 1.7 \text{ V}$

So $\underline{1.7 \le v_{02} \le 3.3 \text{ V}}$

16.35

a. $V_{0Nt} \le v_{01} \le V_{0Pt}$

By symmetry, $V_{It} = 2.5 \text{ V}$

$V_{0Pt} = 2.5 + 0.8 = 3.3 \text{ V}$

and $V_{0Nt} = 2.5 - 0.8 = 1.7 \text{ V}$

So $\underline{1.7 \le v_{01} \le 3.3 \text{ V}}$

b. For $v_{O2} = 0.6 < V_{TN} \Rightarrow \underline{v_{O3} = 5 V}$

N_2 in nonsaturation and P_2 in saturation. From Equation (16.43),

$$\left[2(v_{I2} - 0.8)(0.6) - (0.6)^2\right] = \left[5 - v_{I2} - 0.8\right]^2$$

$$1.2v_{I2} - 1.32 = 17.64 - 8.4v_{I2} + v_{I2}^2$$

or

$$v_{I2}^2 - 9.6v_{I2} + 18.96 = 0$$

So $\underline{v_{I2} = v_{01} = 2.78 \text{ V}}$

For $v_{01} = 2.78$, both N_1 and P_1 in saturation. Then

$\underline{v_I = 2.5 \text{ V}}$

16.36

(a) $V_{It} = \dfrac{2.5}{2} = 1.25 \text{ V}$

For $0.4 \le \upsilon_I \le 1.25 \text{ V}$; $i_D = 120(\upsilon_I - 0.4)^2 \ \mu \text{ A}$

$$i_{D,peak} = 120(1.25 - 0.4)^2 = 86.7 \ \mu \text{ A}$$

For $1.25 \le \upsilon_I \le 2.1 \text{V}$; $i_D = 120(2.5 - \upsilon_I - 0.4)^2 \ \mu \text{ A}$

(b) $V_{It} = \left(\dfrac{1.8}{2}\right) = 0.9 \text{ V}$

For $0.4 \le \upsilon_I \le 0.9 \text{ V}$; $i_D = 120(\upsilon_I - 0.4)^2 \ \mu \text{ A}$

$$i_{D,peak} = 120(0.9 - 0.4)^2 = 30 \ \mu \text{ A}$$

For $0.9 \le \upsilon_I \le 1.4 \text{ V}$; $i_D = 120(1.8 - \upsilon_I - 0.4)^2 \ \mu \text{ A}$

16.37

(a) $K_n = \left(\dfrac{80}{2}\right)(2) = 80 \ \mu \text{ A/V}^2$, $K_p = \left(\dfrac{40}{2}\right)(4) = 80 \ \mu \text{ A/V}^2$

$$V_{It} = \frac{1.8}{2} = 0.9 \text{ V}$$

$$i_{D,peak} = 80(0.9 - 0.35)^2 = 24.2\ \mu\text{A}$$

(b) $K_n = \left(\frac{80}{2}\right)(2) = 80\ \mu\text{A/V}^2,\ K_p = \left(\frac{40}{2}\right)(6) = 120\ \mu\text{A/V}^2$

$$V_{It} = \frac{1.8 - 0.35 + \sqrt{\frac{80}{120}}(0.35)}{1 + \sqrt{\frac{80}{120}}} = 0.9556 \text{ V}$$

$$i_{D,peak} = 80(0.9556 - 0.35)^2 = 29.34\ \mu\text{A}$$

(c) $K_n = \left(\frac{80}{2}\right)(4) = 160\ \mu\text{A/V}^2,\ K_p = \left(\frac{40}{2}\right)(4) = 80\ \mu\text{A/V}^2$

$$V_{It} = \frac{1.8 - 0.35 + \sqrt{\frac{160}{80}}(0.35)}{1 + \sqrt{\frac{160}{80}}} = 0.8057 \text{ V}$$

$$i_{D,peak} = 160(0.8057 - 0.35)^2 = 33.23\ \mu\text{A}$$

16.38

(a) $K_n = \left(\frac{100}{2}\right)(3) = 150\ \mu\text{A/V}^2,\ K_p = \left(\frac{40}{2}\right)(7.5) = 150\ \mu\text{A/V}^2$

$$V_{It} = \frac{3.3}{2} = 1.65 \text{ V}$$

$$i_{D,peak} = 150(1.65 - 0.4)^2 = 234\ \mu\text{A}$$

(b) $K_n = \left(\frac{100}{2}\right)(4) = 200\ \mu\text{A/V}^2,\ K_p = \left(\frac{40}{2}\right)(4) = 80\ \mu\text{A/V}^2$

$$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{200}{80}}(0.4)}{1 + \sqrt{\frac{200}{80}}} = 1.369 \text{ V}$$

$$i_{D,peak} = 200(1.369 - 0.4)^2 = 188\ \mu\text{A}$$

(c) $K_n = \left(\frac{100}{2}\right)(3) = 150\ \mu\text{A/V}^2,\ K_p = \left(\frac{40}{2}\right)(12) = 240\ \mu\text{A/V}^2$

$$V_{It} = \frac{3.3 - 0.4 + \sqrt{\frac{150}{240}}(0.4)}{1 + \sqrt{\frac{150}{240}}} = 1.796 \text{ V}$$

$$i_{D,peak} = 150(1.796 - 0.4)^2 = 292\mu\text{A}$$

16.39

(a) $P = fC_L V_{DD}^2 = (10^7)(0.2\times10^{-12})(2.5)^2 \Rightarrow P = 12.5\,\mu\text{ W}$

(b) $P = fC_L V_{DD}^2 = (10^7)(0.2\times10^{-12})(1.8)^2 \Rightarrow P = 6.48\,\mu\text{ W}$

16.40

(a) $P = (4\times10^6)(150\times10^6)(0.12\times10^{-12})(1.8)^2 \Rightarrow P = 233\text{ W}$

(b) $P = 233.28 = (4\times10^6)(300\times10^6)(0.12\times10^{-12})V_{DD}^2 \Rightarrow V_{DD} = 1.27\text{ V}$

16.41

(a) $P = \dfrac{3}{10^7} = \underline{3\times10^{-7}\text{ W}}$

(b) $P = fC_L V_{DD}^2 \Rightarrow C_L = \dfrac{P}{fV_{DD}^2}$

(i) $C_L = \dfrac{3\times10^{-7}}{(5\times10^6)(5)^2} \Rightarrow \underline{C_L = 0.0024\,\text{pF}}$

(ii) $C_L = \dfrac{3\times10^{-7}}{(5\times10^6)(3.3)^2} \Rightarrow \underline{C_L = 0.00551\,\text{pF}}$

(iii) $C_L = \dfrac{3\times10^{-7}}{(5\times10^6)(1.5)^2} \Rightarrow \underline{C_L = 0.0267\,\text{pF}}$

16.42

(a) $P = \dfrac{10}{5\times10^6} = 2\times10^{-6}\text{ W}$

(b) $C_L = \dfrac{P}{fV_{DD}^2}$

(i) $C_L = \dfrac{2\times10^{-6}}{(8\times10^6)(5)^2} \Rightarrow \underline{C_L = 0.01\,\text{pF}}$

(ii) $C_L = \dfrac{2\times10^{-6}}{(8\times10^6)(3.3)^2} \Rightarrow \underline{C_L = 0.023\,\text{pF}}$

(iii) $C_L = \dfrac{2\times10^{-6}}{(8\times10^6)(1.5)^2} \Rightarrow \underline{C_L = 0.111\,\text{pF}}$

16.43

(a) For $\upsilon_I \cong V_{DD}$, NMOS in nonsaturation

$i_D = K_n\left[2(\upsilon_I - V_{TN})\upsilon_{DS} - \upsilon_{DS}^2\right]$ and υ_{DS} very small

So $\dfrac{1}{r_{ds}} = \dfrac{di_D}{d\upsilon_{DS}} \cong K_n\left[2(\upsilon_I - V_{TN})\right]$

Or $r_{ds} \cong \dfrac{1}{\left(\dfrac{k_n'}{2}\right)\left(\dfrac{W}{L}\right)_n [2(V_{DD} - V_{TN})]} = \dfrac{1}{k_n'\left(\dfrac{W}{L}\right)_n (V_{DD} - V_{TN})}$

For $\upsilon_I \cong 0$, PMOS in nonsaturation

$i_D = K_p\left[2(V_{DD} - \upsilon_I + V_{TP})\upsilon_{SD} - \upsilon_{SD}^2\right]$ and υ_{SD} very small

So $\dfrac{1}{r_{sd}} = \dfrac{di_D}{d\upsilon_{SD}} = \left(\dfrac{k_p'}{2}\right)\left(\dfrac{W}{L}\right)_p [2(V_{DD} - \upsilon_I + V_{TP})] \cong k_p'\left(\dfrac{W}{L}\right)_p (V_{DD} + V_{TP})$

Or $r_{sd} = \dfrac{1}{k_p'\left(\dfrac{W}{L}\right)_p (V_{DD} + V_{TP})}$

(b) Let $\left(\dfrac{W}{L}\right)_n = 2$, $\left(\dfrac{W}{L}\right)_p = 4$, and $V_{DD} = 5$ V

For NMOS:

$$r_{ds} = \frac{1}{(0.1)(2)(5 - 0.5)} \Rightarrow r_{ds} = 1.11\,\text{k}\Omega$$

$$i_d = \frac{\upsilon_{ds}}{r_{ds}} = \frac{0.5}{1.11} = 0.45\,\text{mA}$$

For PMOS:

$$r_{sd} = \frac{1}{(0.04)(4)(5 - 0.5)} \Rightarrow r_{sd} = 1.39\,\text{k}\Omega$$

$$i_d = \frac{\upsilon_{sd}}{r_{sd}} = \frac{0.5}{1.39} = 0.36\,\text{mA}$$

16.44

(a) $V_{IL} = V_{TN} + \dfrac{3}{8}(V_{DD} + V_{TP} - V_{TN}) = 0.5 + \dfrac{3}{8}(3.3 - 0.5 - 0.5)$

$$\Rightarrow V_{IL} = 1.3625\,\text{V}$$

$$V_{IH} = V_{TN} + \frac{5}{8}(V_{DD} + V_{TP} - V_{TN}) = 0.5 + \frac{5}{8}(3.3 - 0.5 - 0.5)$$

$$\Rightarrow V_{IH} = 1.9375\,\text{V}$$

(b) $V_{OHU} = \dfrac{1}{2}[2(1.3625) + 3.3 - 0.5 + 0.5] = 3.0125$ V

$$V_{OLU} = \frac{1}{2}[2(1.9375) - 3.3 - 0.5 + 0.5] = 0.2875\,\text{V}$$

Then $NM_L = 1.3625 - 0.2875 = 1.075$ V

$NM_H = 3.0125 - 1.9375 = 1.075$ V

16.45

(a) $\dfrac{K_n}{K_p} = \dfrac{100}{50} = 2$

$$V_{IL} = 0.35 + \frac{(2.5 - 0.35 - 0.35)}{(2-1)}\left[2\sqrt{\frac{2}{2+3}} - 1\right]$$

$$\Rightarrow V_{IL} = 0.8268 \text{ V}$$

$$V_{IH} = 0.35 + \frac{(2.5 - 0.35 - 0.35)}{(2-1)}\left[\frac{2(2)}{\sqrt{3(2)+1}} - 1\right]$$

$$\Rightarrow V_{IH} = 1.2713 \text{ V}$$

(b) $$V_{OHU} = \frac{1}{2}[(1+2)(0.8268) + 2.5 - (2)(0.35) + 0.35]$$

$$\Rightarrow V_{OHU} = 2.3152 \text{ V}$$

$$V_{OLU} = \frac{(1.2713)(1+2) - 2.5 - (2)(0.35) + 0.35}{2(2)}$$

$$\Rightarrow V_{OLU} = 0.2410 \text{ V}$$

Then $NM_L = 0.8268 - 0.2410 = 0.5858$ V

$NM_H = 2.3152 - 1.2713 = 1.0439$ V

16.46

(a) $$K_n = \left(\frac{100}{2}\right)(2) = 100\,\mu \text{ A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(5) = 100\,\mu \text{ A/V}^2$$

$$V_{IL} = 0.4 + \frac{3}{8}(3.3 - 0.4 - 0.4) = 1.3375 \text{ V}$$

$$V_{IH} = 0.4 + \frac{5}{8}(3.3 - 0.4 - 0.4) = 1.9625 \text{ V}$$

$$V_{OHU} = \frac{1}{2}[2(1.3375) + 3.3 - 0.4 + 0.4] = 2.9875 \text{ V}$$

$$V_{OLU} = \frac{1}{2}[2(1.9625) - 3.3 - 0.4 + 0.4] = 0.3125 \text{ V}$$

Then $NM_L = 1.3375 - 0.3125 = 1.025$ V

$NM_H = 2.9875 - 1.9625 = 1.025$ V

(b) $$K_n = \left(\frac{100}{2}\right)(4) = 200\,\mu \text{ A/V}^2, \quad K_p = \left(\frac{40}{2}\right)(12) = 240\,\mu \text{ A/V}^2$$

$$\frac{K_n}{K_p} = \frac{200}{240} = 0.8333$$

$$V_{IL} = 0.4 + \frac{(3.3 - 0.4 - 0.4)}{(0.8333 - 1)}\left[2\sqrt{\frac{0.8333}{3.8333}} - 1\right] = 1.4127 \text{ V}$$

$$V_{IH} = 0.4 + \frac{(3.3 - 0.4 - 0.4)}{(0.8333 - 1)}\left[\frac{2(0.8333)}{\sqrt{3(0.8333)+1}} - 1\right] = 2.0370 \text{ V}$$

Now $$V_{OHU} = \frac{1}{2}[(1 + 0.8333)(1.4127) + 3.3 - (0.8333)(0.4) + 0.4]$$

$$\Rightarrow V_{OHU} = 2.9783 \text{ V}$$

$$V_{OLU} = \frac{2.0370(1+0.8333) - 3.3 - (0.8333)(0.4) + 0.4}{2(0.8333)}$$

$$\Rightarrow V_{OLU} = 0.3007 \text{ V}$$

Then $NM_L = 1.4127 - 0.3007 = 1.1112$ V

$NM_H = 2.9783 - 2.0370 = 0.9413$ V

16.47

a. $v_A = v_B = 5$ V

N_1 and N_2 on, so $v_{DS1} \approx v_{DS2} \approx 0$ V

P_1 and P_2 off

So we have a $P_3 - N_3$ CMOS inverter. By symmetry, $v_C = 2.5$ V (Transition Point).

b. For $v_A = v_B = v_C \equiv v_I$

Want $K_{n,eff} = K_{p,eff}$

$$\frac{k_n'}{2} \cdot \left(\frac{W}{3L}\right)_n = \frac{k_p'}{2} \cdot \left(\frac{3W}{L}\right)_P$$

With $k_n' = 2k_p'$, then

$$\frac{2}{2} \cdot \frac{1}{3} \cdot \left(\frac{W}{L}\right)_n = \frac{1}{2} \cdot 3 \cdot \left(\frac{W}{L}\right)_P$$

Or $\underline{\left(\frac{W}{L}\right)_n = \frac{9}{2} \cdot \left(\frac{W}{L}\right)_P}$

c. We have

$$K_n = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_n = \left(\frac{2k_p'}{2}\right)\left(\frac{9}{2}\right)\left(\frac{W}{L}\right)_p$$

$$K_p = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_p$$

Then from Equation (16.41)

$$V_{It} = \frac{5 + (-0.8) + \sqrt{\frac{K_n}{K_p}} \cdot (0.8)}{1 + \sqrt{\frac{K_n}{K_p}}}$$

Now

$$\frac{K_n}{K_p} = (2)\left(\frac{9}{2}\right) = 9$$

Then

$$V_{It} = \frac{5 + (-0.8) + 3(0.8)}{1+3} \Rightarrow \underline{V_{It} = 1.65\,V}$$

16.48

By definition, NMOS is on if gate voltage is 5 V and is off if gate voltage is 0 V.

State	N_1	N_2	N_3	N_4	N_5	v_0
1	off	on	off	on	on	0

2	off	off	on	on	off	0
3	on	on	off	off	on	5
4	on	on	off	on	on	0

Logic function $(v_X \text{ OR } v_Y) \otimes (v_X \text{ AND } v_Z)$

Exclusive OR of $(v_X \text{ OR } v_Y)$ with $(v_X \text{ AND } v_Z)$

16.49

NMOS in Parallel $\Rightarrow \left(\frac{W}{L}\right)_n = 2$

4-PMOS in series $\Rightarrow \left(\frac{W}{L}\right)_p = 4(4) = 16$

(b) C_L doubles $\Rightarrow$ current must double to maintain switching speed.

$\Rightarrow \left(\frac{W}{L}\right)_n = 4$

$\left(\frac{W}{L}\right)_p = 32$

16.50

4-NMOS in series $\left(\frac{W}{L}\right)_n = 4(2) = 8$

4-PMOS in parallel $\left(\frac{W}{L}\right)_p = 4$

(b) $\left(\frac{W}{L}\right)_n = 16$

$\left(\frac{W}{L}\right)_p = 8$

16.51

(a) NMOS in parallel $\Rightarrow \left(\frac{W}{L}\right)_n = 2$

3-PMOS in series $\Rightarrow \left(\frac{W}{L}\right)_P = 3(4) = 12$

(b) $\left(\frac{W}{L}\right)_n = 4$

$\left(\frac{W}{L}\right)_p = 24$

16.52

(a) 3-NMOS in series $\left(\frac{W}{L}\right)_n = 3(2) = 6$

3-PMOS in parallel $\left(\frac{W}{L}\right)_p = 4$

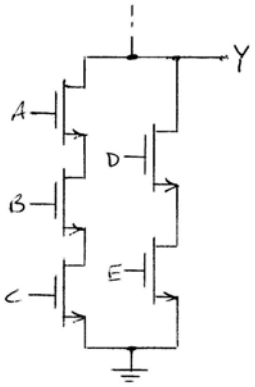
(b) $\left(\frac{W}{L}\right)_n = 12$

$\left(\frac{W}{L}\right)_p = 8$

16.53

(a) $Y = \overline{ABC + DE}$

(b)



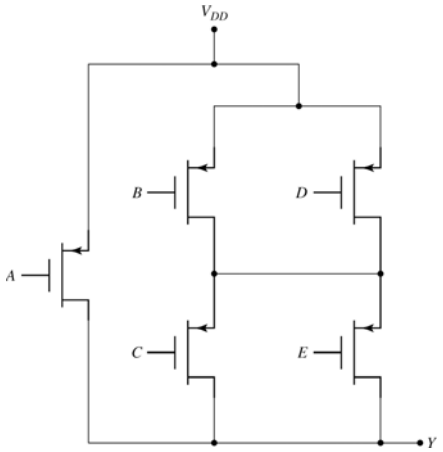
(c) $(W/L)_{An,Bn,Cn} = 6$, $(W/L)_{Dn,En} = 4$

All $(W/L)_p = 8$

16.54

(a) $Y = \overline{A(BD + CE)}$

(b)



(c) NMOS: 3 transistors in series for pull down mode.

For twice the speed: $\left(\frac{W}{L}\right)_n = 2(3)(2) = 12$

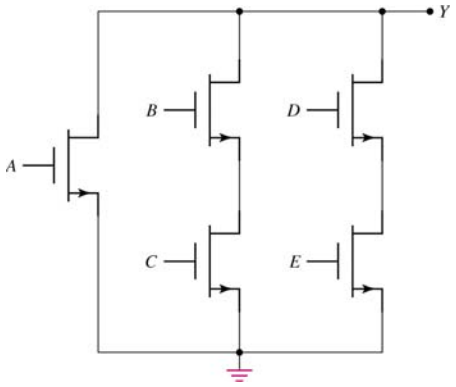
PMOS: $\left(\frac{W}{L}\right)_{P,A} = 2(4) = 8$

$\left(\frac{W}{L}\right)_{P,B,C,D,E} = 2(2)(4) = 16$

16.55

(a) $Y = \overline{A + BC + DE}$

(b)



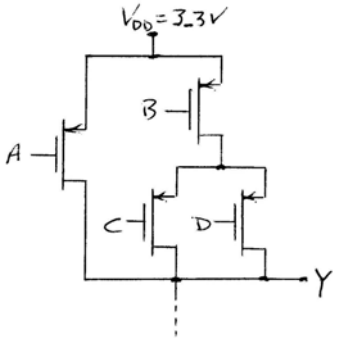
(c) NMOS: $\left(\frac{W}{L}\right)_{n,A} = 2$ $\left(\frac{W}{L}\right)_{n,B,C,D,E} = 4$

PMOS: 3 transistors in series for the pull-up mode

$\left(\frac{W}{L}\right)_p = 3(4) = 12$

16.56

(a) $Y = \overline{A[B + CD]}$



(b)

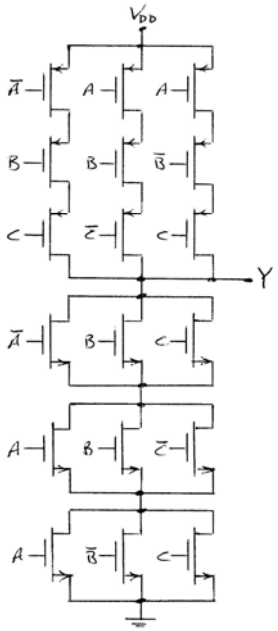
(c) $(W/L)_{An,Bn} = 4$, $(W/L)_{Cn,Dn} = 8$

$(W/L)_{Ap} = 4$, $(W/L)_{Bp,Cp,Dp} = 8$

16.57

(a) For $Y = A\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C}$

We have $\overline{Y} = \overline{A\overline{B}\overline{C} + \overline{A}\overline{B}C + \overline{A}B\overline{C}} = \left(\overline{A} + B + C\right)\left(A + B + \overline{C}\right)\left(A + \overline{B} + C\right)$

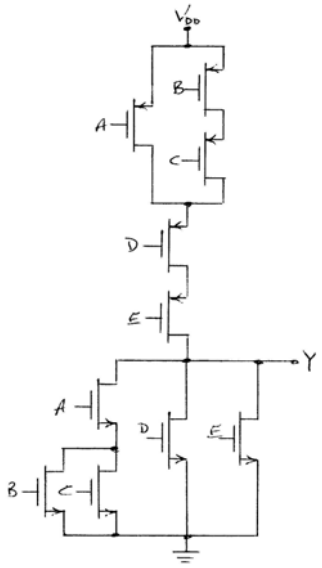


(b) All $(W/L)_n = 3$

All $(W/L)_p = 6$

16.58

(a)



(b) $(W/L)_{An,Bn,Cn} = 2$, $(W/L)_{Dn,En} = 1$

$(W/L)_{Ap,Dp,Ep} = 6$, $(W/L)_{Bp,Cp} = 12$

16.59

(a) $\overline{Y} = C(A+B)$

(b) All $(W/L)_n = 4$

$(W/L)_{Cp} = 4$, $(W/L)_{Ap,Bp} = 8$

16.60

(a) All $(W/L)_n = 2$

All $(W/L)_p = 20$

(b) All $(W/L)_n = 10$

All $(W/L)_p = 4$

16.61

By definition:

NMOS off if gate voltage $= 0$

NMOS on if gate voltage $= 5$ V

PMOS off if gate voltage $= 5$ V

PMOS on if gate voltage $= 0$

State	N_1	P_1	N_A	N_B	N_C	v_{01}	N_2	P_2	v_{02}
1	off	on	off	off	off	5	on	off	0
2	on	off	on	off	off	5	on	off	0
3	off	on	off	off	off	5	on	off	0
4	on	off	off	off	on	5	on	off	0
5	off	on	off	off	off	5	on	off	0
6	on	off	off	on	on	0	off	on	5

Logic function is

$v_{02} = (v_A \text{ OR } v_B) \text{ AND } v_C$

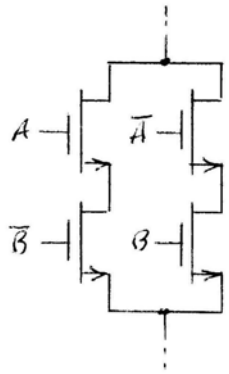
16.62

State	v_{01}	v_{02}	v_{03}
1	5	5	0
2	0	0	5
3	5	5	0
4	5	0	5
5	5	5	0
6	0	5	0

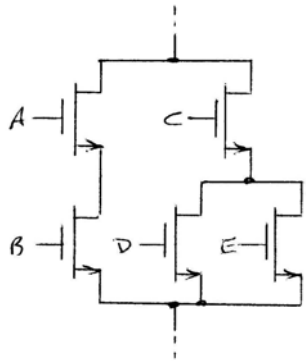
Logic function:

$v_{03} = (v_X \text{ OR } v_Z) \text{ AND } v_Y$

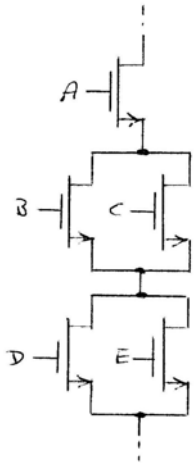
16.63



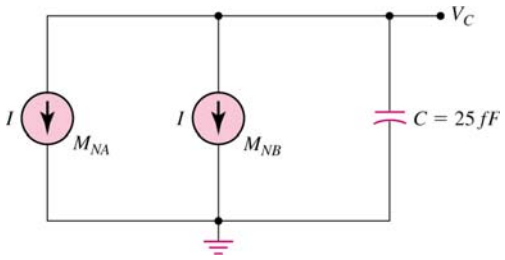
16.64



16.65



16.66



$$2I = -C\frac{dV_C}{dt}$$

So

$$\Delta V_C = -\frac{1}{C}(2I)\cdot t$$

For $\Delta V_C = -0.5\,V$

$$-0.5 = -\frac{2\left(2\,x10^{-12}\right)\cdot t}{25\,x10^{-15}} \Rightarrow \underline{t = 3.125\ ms}$$

16.67

(a) (i) $\upsilon_O = 0$

(ii) $\upsilon_O = \phi - V_{TN} = 3.3 - 0.4 = 2.9\ \text{V}$

(iii) $\upsilon_O = 2.5\ \text{V}$

(b) (i) $\upsilon_O = 0$

(ii) $\upsilon_O = \phi - V_{TN} = 1.8 - 0.4 = 1.4\ \text{V}$

(iii) $\upsilon_O = \phi - V_{TN} = 1.8 - 0.4 = 1.4\ \text{V}$

16.68

(a) (i) $\upsilon_O = 0$

(ii) $\upsilon_O = \phi - V_{TN} = 2.5 - 0.5 = 2\ \text{V}$

(iii) $\upsilon_O = 1.8\ \text{V}$

(b) (i) $\upsilon_O = 0$

(ii) $\upsilon_O = \phi - V_{TN} = 2 - 0.5 = 1.5\ \text{V}$

(iii) $\upsilon_O = \phi - V_{TN} = 2 - 0.5 = 1.5\ \text{V}$

16.69

(a) $\upsilon_{O1} = 2.5 - 0.4 = 2.1\ \text{V}$

$\upsilon_{O2} = 2.5\ \text{V}$

(b) $\upsilon'_{I1} = 2.5 - 0.4 = 2.1\,\text{V}$

$$\frac{(W/L)_1}{(W/L)_2}\left[2(2.1-0.4)(0.1)-(0.1)^2\right]=(2.5-0.1-0.4)^2$$

$$(W/L)_1[0.33]=4 \quad \Rightarrow \left(\frac{W}{L}\right)_1=12.1$$

$\upsilon'_{I3}=2.1\,\text{V}$

$$\frac{(W/L)_3}{(W/L)_4}\left[2(2.1-0.4)(0.1)-(0.1)^2\right]=\left[-(-0.6)\right]^2$$

$$(W/L)_3[0.33]=0.36 \quad \Rightarrow \left(\frac{W}{L}\right)_3=1.09$$

16.70

A	B	Y	
0	0	1	
0	1	0	
1	0	0	
1	1	0.1	$\Rightarrow$ indeterminate

Without the top transistor, the circuit performs the exclusive-NOR function.

16.71

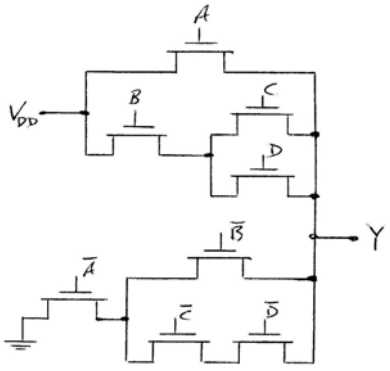
A	$\bar{A}$	B	$\bar{B}$	Y	Z
0	1	0	1	0	1
0	1	1	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0

$Y=A+\bar{A}B=A+B$

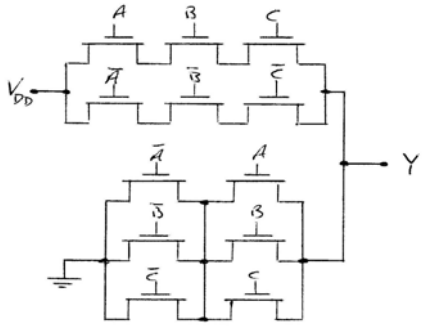
$Z=\bar{Y}$ or $Z=\overline{\bar{A}\bar{B}}$

16.72

(a)



(b)



16.73

(a) (i) $Y = 0$
(ii) $Y = 2.5$ V
(iii) $Y = 0$
(iv) $Y = 2.5$ V

(b) (i) $Y = 0$
(ii) $Y = 0$
(iii) $Y = 2.5$ V
(iv) $Y = 2.5$ V

(c) For $\phi = 1$, $\bar{\phi} = 0$; then $Y = B$

For $\phi = 0$, $\bar{\phi} = 1$; then $Y = A$

A multiplexer

16.74

$$Y = AC + B\bar{C}$$

16.75

(a) (i) $Y = 0$
(ii) $Y = 2.5$ V
(iii) $Y = 2.5$ V
(iv) $Y = 0$

(b) $Y = A\bar{B} + \bar{A}B = A \otimes B$

16.76

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	0

Exclusive-OR function.

16.77

This circuit is referred to as a two-phase ratioed circuit. The same width-to-length ratios between the driver and load transistors must be maintained as discussed previously with the enhancement load inverter.

When ϕ_1 is high, v_{01} becomes the complement of v_I. When ϕ_2 goes high, then v_0 becomes the complement of v_{01} or is the same as v_I. The circuit is a shift register.

16.78

Want $\overline{Q}$ to be the transition point of $M_5 - M_6$.

From Equation (16.26(b)),

$$\sqrt{\frac{K_5}{K_6}}\left(V_{It} - V_{TN5}\right) = -V_{TN6}$$

$$\sqrt{\frac{100}{40}}\left(V_{It} - 0.4\right) = \left[-\left(-0.6\right)\right] \Rightarrow V_{It} = \overline{Q} = 0.7795 \text{ V}$$

This is region where M_1 and M_3 are biased in the saturation region.

$$\sqrt{\frac{K_1}{K_3}}\left(V_{It} - V_{TN1}\right) = -V_{TN3}$$

$$\sqrt{\frac{150}{40}}\left(V_{It} - 0.4\right) = \left[-\left(-0.6\right)\right] \Rightarrow V_{It} = S = 0.7098 \text{ V}$$

This analysis neglected the effect of M_2 starting to conduct.

16.79

$$v_{It} = \frac{3.3 + (-0.4) + 0.5}{1+1} = 1.7 \text{ V}$$

$v_I = 1.5$ V NMOS Sat; PMOS Non Sat

$$\left(v_I - 0.5\right)^2 = \left[2\left(3.3 - v_I - 0.4\right)\left(3.3 - v_{o1}\right) - \left(3.3 - v_{o1}\right)^2\right] \Rightarrow v_{o1} = 2.88 \text{ V}$$

$v_I = 1.6$ V $\quad v_{o1} = 2.693$ V

$v_I = 1.7$ V $\quad v_{o1}$ = variable (switching region)

$v_I = 1.8$ V NMOS Non Sat; PMOS Sat

$$\left(3.3 - V_I - 0.4\right)^2 = \left[2\left(v_I - 0.5\right)v_{o1} - v_{o1}^2\right] \Rightarrow v_{o1} = 0.607 \text{ V}$$

Now

$v_I = 1.5$ V, $v_{o1} = 2.88$ V $\Rightarrow v_o \approx 0$V

$v_I = 1.6$ V, $v_{o1} = 2.693$ V

NMOS Non Sat; PMOS Sat

$$\left(3.3 - v_{o1} - 0.4\right)^2 = \left[2\left(v_{o1} - 0.5\right)v_o - v_o^2\right]$$

$$v_o = 0.00979 \text{ V}$$

$v_I = 1.7$ V, $\quad v_{o1} =$ Switching Mode $\Rightarrow v_0 =$ Switching Mode.

$v_I = 1.8$ V, $\quad v_{o1} = 0.607$ V NMOS Sat; PMOS Non Sat

$$\left(v_{01} - 0.5\right)^2 = \left[2\left(3.3 - v_{01} - 0.4\right)\left(3.3 - v_0\right) - \left(3.3 - v_0\right)^2\right] \Rightarrow v_0 = 3.298 \text{ V}$$

16.80

For $R = \phi = V_{DD}$ and $S = 0 \Rightarrow Q = 0,\ \bar{Q} = 1$

For $S = \phi = V_{DD}$ and $R = 0 \Rightarrow Q = 1,\ \bar{Q} = 1$

The signal ϕ is a clock signal.

For $\phi = 0,$ The output signals will remain in their previous state.

16.81

a. Positive edge triggered flip-flop when CLK $= 1,$ output of first inverter is $\bar{D}$ and then $Q = \bar{\bar{D}} = D$.

b. For example, put a CMOS transmission gate between the output and the gate of M_1 driven by a $\overline{\text{CLK}}$ pulse.

16.82

For $J = 1,\ K = 0,$ and CLK $= 1;$ this makes $Q = 1$ and $\bar{Q} = 0$.

For $J = 0,\ K = 1,$ and CLK $= 1$, and if $Q = 1,$ then the circuit is driven so that $Q = 0$ and $\bar{Q} = 1.$

If initially, $Q = 0,$ then the circuit is driven so that there is no change and $Q = 0$ and $\bar{Q} = 1.$

$J = 1,\ K = 1,$ and CLK $= 1,$ and if $Q = 1,$ then the circuit is driven so that $Q = 0.$

If initially, $Q = 0$, then the circuit is driven so that $Q = 1.$

So if $J = K = 1,$ the output changes state.

16.83

For $J = v_X = 1,\ K = v_Y = 0,$ and CLK $= v_Z = 1,$ then $v_0 = 0.$

For $J = v_X = 0,\ K = v_Y = 1,$ and CLK $= v_Z = 1,$ then $v_0 = 1.$

Now consider $J = K = \text{CLK} = 1.$ With $v_X = v_Z = 1,$ the output is always $v_0 = 0,$ So the output does not change state when $J = K = \text{CLK} = 1.$ This is not actually a $J - K$ flip-flop.

16.84

(a) 256K $\Rightarrow$ 262,144 cells $\Rightarrow$ 512×512

Each decoder $\Rightarrow$ 9 inputs

(b) (i) Row 52, address = 000110011

(ii) Row 129, address = 010000000

(iii) Row 241, address = 011110000

(c) (i) Column 24, address = 000010111

(ii) Column 165, address = 010100100

(iii) Column 203, address = 011001010

16.85

1-Megabit memory $\Rightarrow$

(a) $= 1,048,576 \Rightarrow 1024 \times 1024$

Number of input row and column decodes lines necessary $= 10$

(b) $250\text{K} \times 4$ bits $\Rightarrow 262,144 \times 4$ bits $\Rightarrow 512 \times 512$

For 512 lines $\Rightarrow$ 9 row and column decoder lines necessary.

16.86

32 rows × 16 columns
Each column contains 8 bits

16.87

Assume the address line is initially uncharged, then

$$I = C\frac{dV_C}{dt} \quad \text{or} \quad V_C = \frac{1}{C}\int I dt = \frac{I}{C}\cdot t$$

Then $$t = \frac{V_C \cdot C}{I} = \frac{(2.7)\left(5.8\times10^{-12}\right)}{250\times10^{-6}} \Rightarrow$$

$$\underline{t = 6.26\times10^{-8}\, s \Rightarrow 62.6\, ns}$$

16.88

(a) $$\frac{V_{DD}-Q}{R} = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)_D\left[2\left(V_{GSD}-V_{TN}\right)Q - Q^2\right]$$

$$\frac{2.5-0.02}{1} = \left(\frac{80}{2}\right)\left(\frac{W}{L}\right)_D\left[2(2.5-0.4)(0.02)-(0.02)^2\right]$$

$$0.062 = \left(\frac{W}{L}\right)_D(0.0836) \Rightarrow \left(\frac{W}{L}\right)_D = 0.74$$

(b) 16K $\Rightarrow$ 16,384 cells

$$i_D \cong \frac{V_{DD}}{2} = \frac{1.2}{1} = 1.2\,\mu\text{ A}$$

$$P = (1.2)(1.2)(16{,}384) \Rightarrow P = 23.6\,\text{mW}$$

16.89

$$16\ K \Rightarrow 16{,}384 \text{ cells}$$

$$P_T = 200\ mW \Rightarrow \text{Power per cell} = \frac{200}{16{,}384} \Rightarrow 12.2\ \mu W$$

$$i_D = \frac{P}{V_{DD}} = \frac{12.2}{2.5} = 4.88\ \mu A \cong \frac{V_{DD}}{R} = \frac{2.5}{R} \Rightarrow \underline{R = 0.512\ M\Omega}$$

If we want $v_O = 0.1V$ for a logic 0, then

$$i_D = \left(\frac{k_n'}{2}\right)\left(\frac{W}{L}\right)\left[2\left(V_{DD}-V_{TN}\right)v_O - v_O^2\right]$$

$$4.88 = \left(\frac{35}{2}\right)\left(\frac{W}{L}\right)\left[2(2.5-0.7)(0.1)-(0.1)^2\right]$$

So $$\underline{\left(\frac{W}{L}\right) = 0.797}$$

16.90

$\overline{D} = V_{DD} = 2.5$ V

Assume M_{P3} in saturation; M_{NA}, M_{N1} in nonsaturation

$$I_{DP3} = I_{DNA} = I_{DN1}$$

$$I_{DP3} = \left(\frac{k_p'}{2}\right)\left(\frac{W}{L}\right)_3 (V_{SG3} + V_{TP})^2 = \left(\frac{35}{2}\right)(1)(2.5-0.4)^2 = 77.175\,\mu\text{A}$$

$$I_{DP3} = I_{DN1}$$

$$77.175 = \left(\frac{80}{2}\right)(2)\left[2(2.5-0.4)Q - Q^2\right]$$

We find $Q^2 - 4.2Q + 0.9647 = 0 \Rightarrow Q = 0.244\,\text{V}$

$$I_{DP3} = I_{DNA}$$

$$77.175 = \left(\frac{80}{2}\right)(1)\left[2(2.5-0.244-0.4)(D-0.244) - (D-0.244)^2\right]$$

$$1.9294 = \left[3.712(D-0.244) - (D-0.244)^2\right]$$

$$= 3.712D - 0.9057 - D^2 + 0.488D - 0.05954$$

We find $D^2 - 4.2D + 2.895 = 0 \Rightarrow D = 0.869\,\text{V}$

2nd approximation, M_{P3} in nonsaturation

Assume $V_{DS} = V_{DD} - D \cong 2.5 - 0.869 = 1.631\,\text{V}$

$$I_{DP3} = \left(\frac{35}{2}\right)(1)\left[2(2.5-0.4)(1.631) - (1.631)^2\right] = 73.33\,\mu\text{A}$$

$$I_{DP3} = I_{DN1}$$

$$73.33 = \left(\frac{80}{2}\right)(2)\left[2(2.5-0.4)Q - Q^2\right]$$

$$Q^2 - 4.2Q + 0.9166 = 0 \Rightarrow Q = 0.231\,\text{V}$$

$$I_{DP3} = I_{DNA}$$

$$73.33 = \left(\frac{80}{2}\right)(1)\left[2(2.5-0.231-0.4)(D-0.231) - (D-0.231)^2\right]$$

$$1.833 = \left[3.738(D-0.231) - \left(D^2 - 0.462D + 0.05336\right)\right]$$

We find $D^2 - 4.2D + 2.75 = 0 \Rightarrow D = 0.812\,\text{V}$

16.91

Approximation, M_{N2} cutoff

$I_{DP2} = I_{DB}$, assume $\overline{D} = 0$

Both M_{P2} and M_B in nonsaturation

$$\left(\frac{35}{2}\right)(4)\left[2(2.5-0.4)(2.5-\overline{Q}) - (2.5-\overline{Q})^2\right]$$

$$= \left(\frac{80}{2}\right)(1)\left[2(2.5-0.4)\overline{Q} - \overline{Q}^2\right]$$

$$70\left[4.2(2.5-\overline{Q}) - \left(6.25 - 5\overline{Q} + \overline{Q}^2\right)\right] = 40\left[4.2\overline{Q} - \overline{Q}^2\right]$$

We find $0.75\overline{Q}^2 + 2.8\overline{Q} - 7.4375 = 0 \Rightarrow \overline{Q} = 1.794\ \text{V}$

Approximation, M_{P1} cutoff; assume $D = 2.5\,\text{V}$

$$I_{DNA} = I_{DN1}$$

Both M_A and M_{N1} in nonsaturation

$$\left(\frac{80}{2}\right)(1)\left[2(2.5-Q-0.4)(2.5-Q)-(2.5-Q)^2\right]$$

$$=\left(\frac{80}{2}\right)(2)\left[2(2.5-0.4)Q-Q^2\right]$$

$$2(2.1-Q)(2.5-Q)-(2.5-Q)^2=2\left(4.2Q-Q^2\right)$$

$$2\left(5.25-2.1Q-2.5Q+Q^2\right)-\left(6.25-5Q+Q^2\right)=8.4Q-2Q^2$$

We find $3Q^2-12.6Q+4.25=0 \Rightarrow Q=0.370$ V

16.92

For Logic 1, v_1:

$(5)(0.05)+(4)(1)=(1+0.05)v_1 \Rightarrow \underline{v_1=4.0476\,V}$

v_2:

$(5)(0.025)+(4)(1)=(1+1.025)v_2 \Rightarrow \underline{v_2=4.0244\,V}$

For Logic 0, v_1:

$(0)(0.05)+(4)(1)=(1+0.05)v_1 \Rightarrow \underline{v_1=3.8095\,V}$

v_2:

$(0)(0.025)+(4)(1)=(1+0.025)v_2 \Rightarrow \underline{v_2=3.9024\,V}$

16.93

Design Problem

16.94

Design Problem

16.95

Design Problem

16.96

(a) Quantization error $=\frac{1}{2}LSB \le 1\% \le 0.05$ V

Or $LSB \le 0.10$ V

For a 6-*bit word*, $LSB=\frac{5}{64}=0.078125$ V

(b) $1-LSB=\frac{5}{64}=0.078125$ V

(c) $\frac{3.5424}{5}\times 64=45.34 \Rightarrow n=45$

Digital Output $=101101$

$\frac{45\times 5}{64}=3.515625$

$\Delta=3.5424-3.515625=0.026775<\frac{1}{2}LSB.$

16.97

(a) Quantization error $= \frac{1}{2}\text{LSB} \le 0.5\% \le 0.0165\text{ V}$

$1 - \text{LSB} = 0.033\text{ V}$

For a 7-bit word, $\text{LSB} = \frac{3.3}{128} = 0.0258\text{ V}$

(b) 1-LSB = 0.02578125 V

(c) $\frac{2.5321}{3.3} \times 128 = 98.2 \Rightarrow n = 98$

Digital output = 1100010

Now $\frac{(98)(3.3)}{128} = 2.5265625\text{ V}$

$\Delta = 2.5321 - 2.5265625 = 0.0055375\text{ V} < \frac{1}{2}\text{LSB}$

16.98

(a) $v_O = \left(\frac{0}{2} + \frac{1}{4} + \frac{1}{8} + \frac{0}{16}\right)(5) = 1.875\text{ V}$

(b) $v_O = \left(\frac{1}{2} + \frac{0}{4} + \frac{0}{8} + \frac{1}{16}\right)(5) = 2.8125\text{ V}$

16.99

$$LSB = \left(\frac{1}{16}\right)(5) = 0.3125\text{ V}$$

(a) $\frac{1}{2}LSB = 0.15625\text{ V}$

$$v_o = \left(\frac{10}{20 + \Delta R_1}\right)\quad (5)$$

Now

For $v_o = 2.5 + 0.15625 = 2.65625\text{ V}$

$$20 + \Delta R_1 = \frac{(10)(5)}{2.65625} \Rightarrow \Delta R_1 = -1.176\text{ K}$$

For $v_o = 2.5 - 0.15625 = 2.34375\text{ V}$

$$20 + \Delta R_1 = \frac{(10)(5)}{2.34375\text{ V}}$$

$$\Delta R_1 = +1.333\text{ K}$$

For $|\Delta R_1| = 1.176\text{ K} \Rightarrow \Delta R_1 = 5.88\%$

(b) For $R_4 : v_o = \left(\frac{10}{160 + \Delta R_4}\right)(5)$

$v_o = 0.3125 + 0.15625 = 0.46875$ V

$160 + \Delta R_4 = \dfrac{(10)(5)}{0.46875} \Rightarrow \Delta R_4 = -53.33\,\text{K}$

Or $v_o = 0.3125 - 0.15625 = 0.15625$ V

$160 + \Delta R_4 = \dfrac{(10)(5)}{0.15625} \Rightarrow \Delta R_4 = 160\,\text{K}$

For $|\Delta R_4| = 53.33\,\text{K} \Rightarrow \underline{\Delta R_4 = 33.33\%}$

16.100

$R_5 = 320\ \text{k}\Omega$

$R_6 = 640\ \text{k}\Omega$

$R_7 = 1280\ \text{k}\Omega$

(a) $R_8 = 2560\ \text{k}\Omega$

(b) $v_o = \left(\dfrac{10}{2560}\right)(5) = 0.01953125$ V

16.101

(a)

$I_1 = \dfrac{V_{REF}}{2R} = \dfrac{-5}{10} \Rightarrow I_1 = -0.50$ mA

$I_2 = \dfrac{I_1}{2} = -0.25$ mA

$I_3 = \dfrac{I_2}{2} = -0.125$ mA

$I_4 = \dfrac{I_3}{2} = -0.0625$ mA

$I_5 = \dfrac{I_4}{2} = -0.03125$ mA

$I_6 = \dfrac{I_5}{2} = -0.015625$ mA

$\Delta v_o = |I_6| R_F = (0.015625)(5)$

(b) $\Delta v_o = 0.078125$ V

$v_o = -[I_2 + I_5 + I_6] R_F = [0.25 + 0.03125 + 0.015625](5)$

(c) $\underline{v_o = 1.484375\ \text{V}}$

(d) For 101010; $v_o = (0.50 + 0.125 + 0.03125)(5) = 3.28125$ V

For 010101; $v_o = (0.25 + 0.0625 + 0.015625)(5) = 1.640625$ V

$\underline{\Delta v_o = 1.640625\ \text{V}}$

16.102

$\dfrac{1}{2}\,\text{LSB} = \left(\dfrac{V_{REF}}{8R}\right)\left(\dfrac{R}{2}\right) = \dfrac{3.3}{16} = 0.20625\,\text{V}$

Ideal

υ_A for 101 $\Rightarrow \left(\dfrac{5V_{REF}}{8R}\right)(R) = \dfrac{5}{8}(3.3) = 2.0625\,\text{V}$

Range of $\upsilon_A = 2.0625 \pm \frac{1}{2}\text{LSB}$

$1.85625 \le \upsilon_A \le 2.26875\ \text{V}$

16.103

6-bits $\Rightarrow 2^6 = 64$ resistors

$2^6 - 1 = 63$ comparators

16.104

(a) 10- bit output $\Rightarrow 1024$ clock periods

1 clock period $= \frac{1}{f} = \frac{1}{10^6} = 1\ \mu\text{S}$

Maximum conversion time $= 1024\ \mu\text{S} = \underline{1.024\ \text{mS}}$

(b)

$$\frac{1}{2}LSB = \frac{1}{2}\left(\frac{5}{1024}\right) = 0.002441406\ \text{V}$$

$$v_A' = (128+16+2)\left(\frac{5}{1024}\right) = 0.712890625\ \text{V}$$

So range of $v_A = v_A' \pm \frac{1}{2}LSB$

$0.710449219 \le v_A \le 0.715332031\ \text{V}$

(c) $0100100100 \Rightarrow 256+32+4 = \underline{292\ \text{clock pulses}}$

16.105

(a) $3.125 = \frac{N \times 5}{1024} \Rightarrow N = 640 \Rightarrow 512+128$

Output $= \underline{1010000000}$

(b)

$1.8613 = \frac{N \times 5}{1024} \Rightarrow N = 381.19 \Rightarrow N = 381 \Rightarrow 256+64+32+16+8+4+1$

Output $= \underline{0101111101}$

Chapter 17

17.1

(a) $R_C = \dfrac{0-(-0.2)}{0.1} = 2\,\text{k}\Omega$

(b) (i) $\upsilon_1 = -1\,\text{V}$, Q_1 off, Q_2 on

$\upsilon_{O2} = 0-(0.2)(2) = -0.4\,\text{V}$

$\upsilon_{O1} = 0$

(ii) $\upsilon_1 = -0.4\,\text{V}$, Q_1 on, Q_2 off

$\upsilon_{O1} = 0-(0.2)(2) = -0.4\,\text{V}$

$\upsilon_{O2} = 0$

(c) For (i) and (ii)

$P = (I_Q)(0-V^-) = (0.2)(1.8) = 0.36\,\text{mW}$

17.2

(a) $i_E = \dfrac{-1-0.7-(-2.5)}{R_E} = 0.08\,\text{mA}, \Rightarrow R_E = 10\,\text{k}\Omega$

$R_C = \dfrac{0-(-0.25)}{0.04} = 6.25\,\text{k}\Omega$

(b) (i) $\upsilon_1 = -1.3\,\text{V}$, Q_1 off, Q_2 on

$i_E = \dfrac{-1-0.7-(-2.5)}{10} = 0.08\,\text{mA}$

$\upsilon_{O2} = 0-(0.08)(6.25) = -0.50\,\text{V}$

$\upsilon_{O1} = 0$

(ii) $\upsilon_1 = -0.7\,\text{V}$, Q_1 on, Q_2 off

$i_E = \dfrac{-0.7-0.7-(-2.5)}{10} = 0.11\,\text{mA}$

$\upsilon_{O1} = 0-(0.11)(6.25) = -0.6875\,\text{V}$

$\upsilon_{O2} = 0$

(c) (i) $i_E = 0.08\,\text{mA}$, $P = (0.08)(2.5) = 0.2\,\text{mW}$

(ii) $i_E = 0.11\,\text{mA}$, $P = (0.11)(2.5) = 0.275\,\text{mW}$

17.3

(a) $i_{C2} = I_Q = 0.5 = \dfrac{3-0}{R_{C2}} \Rightarrow \underline{R_{C2} = 6\,\text{K}}$

(b) $i_{C1} = I_Q = 0.5 = \dfrac{3-1}{R_{C1}} \Rightarrow \underline{R_{C1} = 4\,\text{K}}$

$$\frac{i_{C1}}{I_Q}=\frac{I_S\exp\left(\frac{V_{BE1}}{V_T}\right)}{I_S\left[\exp\left(\frac{V_{BE1}}{V_T}\right)+\exp\left(\frac{V_{BE2}}{V_T}\right)\right]}$$

$$=\frac{1}{1+\exp\left(\frac{V_{BE2}-V_{BE1}}{V_T}\right)}$$

(c) $v_I=V_{BE1}-V_{BE2}$

So

$$\frac{i_{C1}}{I_Q}=\frac{1}{1+\exp\left(\frac{-v_I}{V_T}\right)}$$

$$\frac{0.1}{0.5}=\frac{1}{1+\exp\left(\frac{-v_I}{V_T}\right)}=0.2$$

$$\exp\left(\frac{-v_I}{V_T}\right)=\frac{1}{0.2}-1=4$$

$$(-v_I)=(0.026)\ln(4)$$

$$\underline{v_I=-0.0360\text{ V}}$$

17.4

(a) $v_I=0.5$ V, Q_1 on, Q_2 off $=v_{02}=3$ V

$v_{01}=3-(1)(0.5)=2.5$ V

(b) $v_I=-0.5$ V Q_1 off, Q_2 on $\Rightarrow v_{01}=3$ V

$v_{02}=3-(1)(0.5)=2.5$ V

17.5

(a) Q_2 on, $v_E=-1.2-0.7=-1.9V$

$$i_E=i_{C2}=\frac{-1.9-(-5.2)}{2.5}=1.32\ mA$$

$$v_2=-1V=-i_{C2}R_{C2}=-(1.32)(R_{C2})$$

$$\underline{R_{C2}=0.758\ k\Omega}$$

(b) Q_1 on, $v_E=-0.7-0.7=-1.40V$

$$i_E=i_{C1}=\frac{-1.4-(-5.2)}{2.5}=1.52\ mA$$

$$v_1=-1V=-i_{C1}R_{C1}=-(1.52)(R_{C1})$$

$$\underline{R_{C1}=0.658\ k\Omega}$$

(c) For $v_{in}=-0.7V$, Q_1 on, Q_2 off

$$\underline{\Rightarrow v_{O1}=-0.70V}$$

$$v_{O2}=-1-0.7\Rightarrow\underline{v_{O2}=-1.7V}$$

For $v_{in}=-1.7V$, Q_1 off, Q_2 on

$$\Rightarrow v_{O2}=-0.7V$$

$$v_{O1}=-1-0.7\Rightarrow\underline{v_{O1}=-1.7V}$$

(d) (i) For $v_{in} = -0.7V,\ i_E = 1.52\ mA$

$$i_{C4} = \frac{-1.7-(-5.2)}{3} = 1.17\ mA$$

$$i_{C3} = \frac{-0.7-(-5.2)}{3} = 1.5\ mA$$

$$P = (i_E + i_{C4} + i_{C3})(5.2) = (1.52+1.17+1.5)(5.2)$$

or $\underline{P = 21.8\ mW}$

(ii) For $v_{in} = -1.7V, i_E = 1.32\ mA$

$$i_{C4} = \frac{-0.7-(-5.2)}{3} = 1.5\ mA$$

$$i_{C3} = \frac{-1.7-(-5.2)}{3} = 1.17\ mA$$

$$P = (1.32+1.5+1.17)(5.2)$$

or $\underline{P = 20.7\ mW}$

17.6

a. $$I_3 = \frac{3.7-0.7}{0.67+1.33} = 1.5\ \text{mA}$$

$$V_R = I_3 R_4 + V_\gamma = (1.5)(1.33) + 0.7$$

or

$$\underline{V_R = 2.70\ \text{V}}$$

b. logic 1 level $= 3.7 - 0.7 \Rightarrow \underline{3.0\ \text{V}}$

For $v_X = v_Y = \text{logic 1.}$

$$i_E = \frac{3-0.7}{0.8} = 2.875\ \text{mA} = i_{RC1}$$

$$v_{B3} = 3.7 - (2.875)(0.21) = 3.10\ \text{V}$$

$$\Rightarrow \underline{v_{01}(\text{logic 0}) = 2.4\ \text{V}}$$

For $v_X = v_Y = \text{logic 0}, Q_R$ on

$$i_E = \frac{2.7-0.7}{0.8} = 2.5\ \text{mA} = i_{RC2}$$

$$v_{B4} = 3.7 - (2.5)(0.24) = 3.1\ \text{V}$$

$$\Rightarrow \underline{v_{02}(\text{logic 0}) = 2.4\ \text{V}}$$

17.7

(a) $$R_1 = \frac{0.7-0.7-(-2.1)}{0.20} = 10.5\ \text{k}\Omega$$

(b) $$R_5 = R_6 = \frac{0-(-2.1)}{0.12} = 17.5\ \text{k}\Omega$$

(c) $I_Q = I_{REF} = 0.20\ \text{mA}$

$\upsilon_{O1} = -0.7\ \text{V},\ \upsilon_{C2} = 0$

$$R_{C1} = \frac{0.7-0}{0.2} = 3.5\ \text{k}\Omega$$

(d) $I_Q = I_{REF} = 0.20\,\text{mA}$

$\upsilon_{O2} = -0.7\,\text{V},\ \ \upsilon_{CR} = 0$

$$R_{C2} = \frac{0.7 - 0}{0.20} = 3.5\,\text{k}\Omega$$

17.8

$V_R = -0.5\,\text{V}$

$$i_E = \frac{-0.5 - 0.7 - (-3)}{R_E} = 0.4\,\text{mA},\ \ R_E = 4.5\,\text{k}\Omega$$

$$R_5 = \frac{-0.5 - (-3)}{0.4} = 6.25\,\text{k}\Omega$$

$$V_{B2} = -0.5 + 0.7 = 0.2\,\text{V},\ \ R_1 = \frac{0.7 - 0.2}{0.4} = 1.25\,\text{k}\Omega$$

$$R_2 = \frac{0.2 - 0.7 - 0.7 - (-3)}{0.4} = 4.5\,\text{k}\Omega$$

$$R_3 = R_4 = \frac{0 - (-3)}{0.8} = 3.75\,\text{k}\Omega$$

$\upsilon_{OR} = -1\,\text{V},\ \Rightarrow \upsilon_{CR} = -0.3\,\text{V}$

$$R_{C2} = \frac{0.7 - (-0.3)}{0.4} = 2.5\,\text{k}\Omega$$

$\upsilon_I = 0$, $i_E = \dfrac{-0.7 - (-3)}{4.5} = 0.511\,\text{mA}$

$\upsilon_{C1} = -1 + 0.7 = -0.3\,\text{V}$

$$R_{C1} = \frac{0.7 - (-0.3)}{0.511} = 1.957\,\text{k}\Omega$$

17.9

$\upsilon_O = \text{logic 1} = 1.8\,\text{V},\ \text{logic 0} = 1.2\,\text{V}$

For $\upsilon_I = \text{logic 1} = 1.8\,\text{V}$

$$i_E = 0.8 = \frac{1.8 - 0.7}{R_E} \Rightarrow R_E = 1.375\,\text{k}\Omega$$

$$\upsilon_{C1} = 1.2 + 0.7 = 1.9\,\text{V},\ \Rightarrow R_{C1} = \frac{2.5 - 1.9}{0.8} = 0.75\,\text{k}\Omega$$

For $\upsilon_I = \text{logic 0}$, Q_R on; $i_E = \dfrac{1.5 - 0.7}{R_E} = \dfrac{0.8}{1.375} = 0.5818\,\text{mA}$

$\upsilon_{CR} = 1.2 + 0.7 = 1.9\,\text{V}$

$$R_{C2} = \frac{2.5 - 1.9}{0.5818} = 1.031\,\text{k}\Omega$$

$$R_2 = R_3 = \frac{1.8}{0.8} = 2.25\,\text{k}\Omega$$

17.10

Neglecting base currents:

(a) $\underline{I_{E1} = 0, I_{E3} = 0}$

$$I_{E5} = \frac{5 - 0.7}{2.5} \Rightarrow \underline{I_{E5} = 1.72\ mA}$$

$\underline{Y = 0.7\ V}$

$$I_{E1} = \frac{5 - 0.7}{18} \Rightarrow \underline{I_{E1} = 0.239\ mA}$$

$\underline{I_{E3} = 0}$

$$I_{E5} = \frac{5 - 0.7}{2.5} \Rightarrow \underline{I_{E5} = 1.72\ mA}$$

(b) $\underline{Y = 0.7\ V}$

(c) $$I_{E1} = I_{E3} = \frac{5 - 0.7}{18} \Rightarrow \underline{I_{E1} = I_{E3} = 0.239\ mA}$$

$\underline{I_{E5} = 0, Y = 5\ V}$

(d) Same as (c).

17.11

(a) $V_R = -(1)(1) - 0.7 \Rightarrow V_R = -1.7\ V$

(b) Q_R off, then $v_{O1} = Logic\ 1 = -0.7\ V$

Q_R on, then $v_{O1} = -(1)(2) - 0.7 \Rightarrow$

$\underline{v_{O1} = Logic\ 0 = -2.7\ V}$

Q_A / Q_B – off, then $\underline{v_{O2} = \text{Logic } 1 = -0.7\ V}$

Q_A / Q_B – on, then $v_{O2} = -(1)(2) - 0.7 \Rightarrow$

$\underline{v_{O2} = Logic\ 0 = -2.7\ V}$

(c) $A = B = Logic\ 0 = -2.7\ V$, Q_R on,

$V_E = -1.7 - 0.7 \Rightarrow \underline{V_E = -2.4\ V}$

$A = B = Logic\ 1 = -0.7\ V$, Q_A / Q_B on,

$V_E = -0.7 - 0.7 \Rightarrow \underline{V_E = -1.4\ V}$

(d) $A = B = Logic\ 1 = -0.7\ V$, Q_A / Q_B on,

$$i_{C3} = \frac{-2.7 - (-5.2)}{1.5} = 1.67\ mA$$

$$i_{C2} = \frac{-0.7 - (-5.2)}{1.5} = 3\ mA$$

$$P = (1.67 + 1 + 1 + 1 + 3)(5.2) \Rightarrow \underline{P = 39.9\ mW}$$

$A = B = Logic\ 0 = -2.7\ V$

$i_{C3} = 3\ mA,\ i_{C2} = 1.67\ mA$

$\underline{P = 39.9\ mW}$

17.12

a. AND logic function

b. $\underline{\text{logic } 0 = 0 \text{ V}}$

Q_3 on, $i = \dfrac{5-(1.6+0.7)}{1.2} = 2.25 \text{ mA}$

$V_2 = (2.25)(0.8) \Rightarrow \underline{\text{logic } 1 = 1.8 \text{ V}}$

$i_{E1} = \dfrac{5-0.7}{2.6} \Rightarrow \underline{i_{E1} = 1.65 \text{ mA}}$

$i_{E2} = \dfrac{5-(0.7+0.7)}{1.2} \Rightarrow \underline{i_{E2} = 3 \text{ mA}}$

$\underline{i_{C3} = 0}, \quad \underline{i_{C2} = i_{E2} = 3 \text{ mA}}$

c. $\underline{V_2 = 0}$

$i_{E1} = \dfrac{5-(1.8+0.7)}{2.6} \Rightarrow \underline{i_{E1} = 0.962 \text{ mA}}$

$i_{E2} = \dfrac{5-(1.6+0.7)}{1.2} \Rightarrow \underline{i_{E2} = 2.25 \text{ mA}}$

$\underline{i_{C2} = 0}, \quad \underline{i_{C3} = i_{E2} = 2.25 \text{ mA}}$

d. $\underline{V_2 = 1.8 \text{ V}}$

17.13

(a) $\upsilon_R = (\text{logic } 0 + \text{logic } 1)/2 - 0.7 = \dfrac{3.5+3.1}{2} - 0.7 = 2.6 \text{ V}$

(b) For $\upsilon_X = \upsilon_Y = \text{logic } 1 = 3.5 \text{ V}$

$\upsilon_{E1} = 3.5 - 0.7 - 0.7 = 2.1 \text{ V}$

$i_{E1} = \dfrac{2.1-0}{12} = 0.175 \text{ mA}$

$i_{RC1} = \dfrac{1}{3} \cdot i_{E1} = 0.05833 \text{ mA} = \dfrac{0.4}{R_{C1}} \Rightarrow R_{C1} = 6.86 \text{ k}\Omega$

(c) For $\upsilon_X = \upsilon_Y = \text{logic } 0 = 3.1 \text{ V}$

$\upsilon_{E1} = V_R - 0.7 = 2.6 - 0.7 = 1.9 \text{ V}$

$i_{E2} = \dfrac{1.9}{12} = 0.1583 \text{ mA}$

$i_{RC2} = \dfrac{1}{3} \cdot i_{E2} = 0.05277 \text{ mA} = \dfrac{0.4}{R_{C2}} \Rightarrow R_{C2} = 7.58 \text{ k}\Omega$

(d) For $\upsilon_X = \upsilon_Y = \text{logic } 0 = 3.1 \text{ V}$

$i_E = 0.1583 \text{ mA}$

$i_{R1} = \dfrac{3.1-0.7}{8} = 0.3 \text{ mA}$

$P = (0.1583 + 0.3)(3.5) = 1.60 \text{ mW}$

17.14

Assume $V_\gamma = 0.4$ V

(a) Logic 1 = 0.2 V, Logic 0 = –0.2 V

(b) $i_E = \dfrac{0 - 0.7 - (-3.10)}{R_E} = 0.25 \Rightarrow R_E = 9.6\ \text{k}\Omega$

(c) $i_{D1} + i_{R1} = i_E$

$2i_{R1} + i_{R1} = 3i_{R1} = 0.25 \Rightarrow i_{R_1} = 0.08333$ mA

$R_1 = \dfrac{0.4}{0.08333} = 4.8\ \text{k}\Omega$

(d) $i_E = \dfrac{0.2 - 0.7 - (-3.10)}{9.6} = 0.2708$ mA

$i_{R_2} = \dfrac{0.4}{R_2} = \dfrac{0.4}{4.8} = 0.0833$ mA

$i_{D2} = i_E - i_{R2} = 0.2708 - 0.0833 = 0.1875$ mA

(e) $i_E = 0.2708$ mA

$i_{R4} = \dfrac{-0.2 - (-3.10)}{3.3} = 0.8788$ mA

$i_{R3} = \dfrac{0.2 - (-3.10)}{3.3} = 1.0$ mA

$P = (i_E + i_{R3} + i_{R4})[0.9 - (-3.10)] = (0.2708 + 1.0 + 0.8788)(4) = 8.6$ mW

17.15

a.

$i_1 = \dfrac{(-0.9 - 0.7) - (-3)}{1} \Rightarrow \underline{i_1 = 1.4\ \text{mA}}$

$i_3 = \dfrac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow \underline{i_3 = 0.14\ \text{mA}}$

$i_4 = \dfrac{(-0.2 - 0.7) - (-3)}{15} \Rightarrow \underline{i_4 = 0.14\ \text{mA}}$

$i_2 + i_D = i_1 + i_3 = 1.4 + 0.14 = 1.54$ mA

$i_2 = \dfrac{0.4}{0.5} \Rightarrow \underline{i_2 = 0.8\ \text{mA}}$

$\underline{i_D = 0.74\ \text{mA}}$

$\underline{v_0 = -0.4\ \text{V}}$

b. $\underline{i_1 = 1.4\ \text{mA}}$

$i_3 = \dfrac{(0 - 0.7) - (-3)}{15} \Rightarrow \underline{i_3 = 0.153\ \text{mA}}$

$i_4 = i_3 \Rightarrow \underline{i_4 = 0.153\ \text{mA}}$

$i_2 + i_D = i_4 \Rightarrow \underline{i_2 = 0.153\ \text{mA}}$

$\underline{i_D = 0}$

$v_0 = -(0.153)(0.5) \Rightarrow \underline{v_0 = -0.0765\ \text{V}}$

c. $i_1 = \dfrac{(0-0.7-0.7)-(-3)}{1} \Rightarrow \underline{i_1 = 1.6\text{ mA}}$

$i_3 = \dfrac{(-0.2-0.7)-(-3)}{15} \Rightarrow \underline{i_3 = 0.14\text{ mA}}$

$i_4 = i_3 \Rightarrow \underline{i_4 = 0.14\text{ mA}}$

$i_2 + i_D = i_3 \Rightarrow \underline{i_2 = 0.14\text{ mA}}$

$\underline{i_D = 0.0}$

$v_0 = -(0.14)(0.5) \Rightarrow \underline{v_0 = -0.07\text{ V}}$

d. $i_1 = \dfrac{(0-0.7-0.7)-(-3)}{1} \Rightarrow \underline{i_1 = 1.6\text{ mA}}$

$i_3 = \dfrac{(0-0.7)-(-3)}{15} \Rightarrow \underline{i_3 = 0.153\text{ mA}}$

$i_4 = i_3 \Rightarrow \underline{i_4 = 0.153\text{ mA}}$

$i_2 + i_D = i_1 + i_4 = 1.6 + 0.153 = 1.753\text{ mA}$

$i_2 = \dfrac{0.4}{0.5} \Rightarrow \underline{i_2 = 0.8\text{ mA}}$

$\underline{i_D = 0.953\text{ mA}}$

$\underline{v_0 = -0.40\text{ V}}$

17.16

(a) (i) $A = B = C = D = 0\text{ V}$, Q_1, Q_2, Q_3, Q_4 off; Q_5, Q_6 on

$$2.5 = i_{R1}(2) + 0.7 + \frac{1}{2}\left(\frac{i_{R1}}{91}\right)(15)$$

$$1.8 = i_{R1}(2 + 0.0824) \Rightarrow i_{R1} = 0.8644\text{ mA}$$

$$Y = 2.5 - i_{R1}(2) \Rightarrow Y = 0.771\text{ V}$$

(ii) $A = B = 0\text{ V}$, $C = D = 2.5\text{ V}$; Q_1, Q_2, Q_6 off; Q_3, Q_4, Q_5 on

$$2.5 = i_{R1}(2) + 0.7 + \left(\frac{i_{R1}}{91}\right)(15)$$

$$1.8 = i_{R1}(2 + 0.1648) \Rightarrow i_{R1} = 0.8315\text{ mA}$$

$$Y = 2.5 - i_{R1}(2) \Rightarrow Y = 0.837\text{ V}$$

(iii) $A = C = 2.5\text{ V}$, $B = D = 0\text{ V}$; Q_1, Q_3 on, Q_5, Q_6 off

$$i_{R1} = 0 \Rightarrow Y = 2.5\text{ V}$$

(b) Y=(A OR B) AND (C OR D)

(c) (i) $P = i_{R1}(2.5) = (0.8644)(2.5) = 2.16\text{ mW}$

(ii) $i_{R3} = \dfrac{2.5 - 0.7}{15} = 0.12\text{ mA}$

$$P = (i_{R1} + i_{R3})(2.5) = (0.8315 + 0.12)(2.5) = 2.38\text{ mW}$$

(iii) $i_{R2} = i_{R3} = \dfrac{2.5 - 0.7}{15} = 0.12\text{ mA}$

$$P = (i_{R2} + i_{R3})(2.5) = (0.24)(2.5) = 0.60\text{ mW}$$

17.17

a. logic $1 = 0$ V
logic $0 = -0.4$ V

b. $v_{01} = \overline{A \text{ OR } B}$
$v_{02} = \overline{C \text{ OR } D}$
$v_{03} = \overline{v_{01} \text{ OR } v_{02}}$
or
$v_{03} = (A \text{ OR } B) \text{ AND } (C \text{ OR } D)$

17.18

a. For CLOCK = high, I_{DC} flows through the left side of the circuit.. If D is high, I_{DC} flows through the left R resistor pulling $\overline{Q}$ low. If D is low. I_{DC} flows through the right R resistor pulling Q low.
For CLOCK = low, I_{DC} flows through the right side of the circuit maintaining Q and $\overline{Q}$ in their previous state.

b. $P = \left(I_{DC} + 0.5I_{DC} + 0.1I_{DC} + 0.1I_{DC}\right)(3)$
$P = 1.7I_{DC}(3) = (1.7)(50)(3) \Rightarrow \underline{P = 255\ \mu\text{W}}$

17.19

(a) (i) For $\upsilon_I = 0.1$ V
$\upsilon_1 = 0.8$ V
$i_1 = \dfrac{2.5 - 0.8}{12} = 0.1417$ mA
$i_2 = i_3 = 0$, $\upsilon_O = 2.5$ V

(ii) For $\upsilon_I = 2.5$ V
$\upsilon_1 = 0.7 + 0.8 = 1.5$ V
$i_1 = i_2 = \dfrac{2.5 - 1.5}{12} = 0.0833$ mA
$\upsilon_O = 0.1$ V
$i_3 = \dfrac{2.5 - 0.1}{12} = 0.20$ mA

(b) (i) $\upsilon_1 = 0.7 + 0.7 = 1.4$ V
$\upsilon_I = \upsilon_1 - 0.7 = 0.7$ V

(ii) $\upsilon_1 = 0.7 + 0.8 = 1.5$ V
$\upsilon_I = \upsilon_1 - 0.7 = 0.8$ V

17.20

(a)

$v_I = 0 \Rightarrow \underline{V_1 = 0.7 \text{ V}}$

$i_1 = \dfrac{3.3 - 0.7}{6} = 0.433 \text{ mA}$

$i_B = i_C = 0$

$v_o = 3.3 \text{ V}$

(b)

$v_I = 3.3 \text{ V} \quad v_1 = 0.7 + 0.8 = 1.5 \text{ V}$

$i_1 = \dfrac{3.3 - 1.5}{6} = 0.3 \text{ mA}$

$i_R = \dfrac{0.8}{20} = 0.04 \text{ mA}$

$i_B = 0.3 - 0.04 = 0.26 \text{ mA}$

$i_C = \dfrac{3.3 - 0.1}{4} = 0.8 \text{ mA}$

$v_o = 0.1 \text{ V}$

17.21

i. For $v_X = v_Y = 0.1 \text{ V} \Rightarrow \underline{v' = 0.8 \text{ V}}$

$i_1 = \dfrac{5 - 0.8}{8} \Rightarrow \quad \underline{i_1 = 0.525 \text{ mA}}$

$\underline{i_3 = i_4 = 0}$

ii. For $v_X = v_Y = 5 \text{ V}$,

$v' = 0.8 + 0.7 + 0.7 \Rightarrow\Rightarrow \underline{v' = 2.2 \text{ V}}$

$i_1 = \dfrac{5 - 2.2}{8} \Rightarrow \underline{i_1 = 0.35 \text{ mA}}$

$i_4 = i_1 - \dfrac{0.8}{15} \Rightarrow \underline{i_4 = 0.297 \text{ mA}}$

$i_3 = \dfrac{5 - 0.1}{2.4} \Rightarrow \underline{i_3 = 2.04 \text{ mA}}$

17.22

(i) $\upsilon_X = \upsilon_Y = 0.1 \text{ V}$

$\upsilon' = 0.8 \text{ V}$

$i_1 = \dfrac{3.3 - 0.8}{8} = 0.3125 \text{ mA}$

$i_3 = i_4 = 0$

(ii) $\upsilon_X = \upsilon_Y = 3.3 \text{ V}$

$\upsilon' = 0.8 + 0.7 + 0.7 = 2.2 \text{ V}$

$i_1 = \dfrac{3.3 - 2.2}{8} = 0.1375 \text{ mA}$

$i_4 = 0.1375 - \dfrac{0.8}{15} = 0.08417 \text{ mA}$

$i_3 = \dfrac{3.3 - 0.1}{2.4} = 1.333 \text{ mA}$

17.23

a. For $v_X = v_Y = 5$ V, both Q_1 and Q_2 driven into saturation.

$v_1 = 0.8 + 0.7 + 0.8 \Rightarrow \underline{v_1 = 2.3 \text{ V}}$

$i_1 = \dfrac{5 - 2.3}{4} \Rightarrow \underline{i_1 = i_{B1} = 0.675 \text{ mA}}$

$i_2 = \dfrac{5 - (0.8 + 0.7 + 0.1)}{2} \Rightarrow \underline{i_2 = 1.7 \text{ mA}}$

$i_4 = i_{B1} + i_2 \Rightarrow \underline{i_4 = 2.375 \text{ mA}}$

$i_5 = \dfrac{0.8}{10} \Rightarrow \underline{i_5 = 0.08 \text{ mA}}$

$i_{B2} = i_4 - i_5 \Rightarrow \underline{i_{B2} = 2.295 \text{ mA}}$

$i_3 = \dfrac{5 - 0.1}{4} \Rightarrow \underline{i_3 = 1.225 \text{ mA}}$

$\underline{v_0 = 0.1 \text{V}}$

$i'_L = \dfrac{5 - (0.1 + 0.7)}{4} = 1.05 \text{ mA}$

b. $i_C(\max) = \beta i_{B2} = N i'_L + i_3$

$(20)(2.295) = N(1.05) + 1.225$

So

$\underline{N = 42}$

17.24

(a) $\upsilon_1 = 0.8 + 0.7 + 0.8 = 2.3 \text{ V}$

$i_1 = \dfrac{3.3 - 2.3}{4} = 0.25 \text{ mA} = i_{B1}$

$\upsilon_{C1} = 0.8 + 0.7 + 0.1 = 1.6 \text{ V}$

$i_2 = \dfrac{3.3 - 1.6}{2} = 0.85 \text{ mA}$

$i_4 = i_{B1} + i_2 = 0.25 + 0.85 = 1.10 \text{ mA}$

$i_5 = \dfrac{0.8}{10} = 0.08 \text{ mA}$

$i_{B2} = i_4 - i_5 = 1.10 - 0.08 = 1.02 \text{ mA}$

$i_3 = \dfrac{3.3 - 0.1}{4} = 0.80 \text{ mA}$

(b) $i_{Co}(\max) = \beta i_{B2} = i_3 + N i'_L$

$i'_L = \dfrac{3.3 - (0.1 + 0.7)}{4} = 0.625 \text{ mA}$

$(20)(1.02) = 0.8 + N(0.625) \Rightarrow N = 31.36 \Rightarrow N = 31$

17.25

D_X and D_Y off, Q_1 forward active mode

$v_1 = 0.8 + 0.7 + 0.7 = 2.2V$

$5 = i_1 R_1 + i_2 R_2 + v_1$ and $i_1 = (1+\beta) i_2$

So $5 - 2.2 = i_2 \left[(1+\beta) R_1 + R_2\right]$

Assume $\beta = 25$

$$i_2 = \frac{5 - 2.2}{(26)(1.75) + 2} \Rightarrow \underline{i_2 = 0.0589\ mA}$$

$$i_1 = (1+\beta) i_2 = (26)(0.05895) \Rightarrow \underline{i_1 = 1.53\ mA}$$

$$i_3 = \beta i_2 \Rightarrow \underline{i_3 = 1.47\ mA}$$

$$i_{Bo} = i_2 + i_3 - \frac{0.8}{5} = 0.0589 + 1.47 - 0.16 \Rightarrow$$

$\underline{i_{Bo} = 1.37\ mA}$

Q_o in saturation

$$i_{Co} = \frac{5 - 0.1}{6} \Rightarrow \underline{i_{Co} = 0.817\ mA}$$

17.26

(a) $v_I = 0$ V, Q_1 forward active

$$i_B = \frac{5 - 0.7}{6} = 0.717 \text{ mA}$$

$$i_C = (25)(0.71667) = 17.9 \text{ mA}$$

$$i_E = (26)(0.71667) = 18.6 \text{ mA}$$

(b) $V_I = 0.8$ V

$$i_B = \frac{5 - (0.8 + 0.7)}{6} = 0.583 \text{ mA}$$

Because of the relative doping levels of the Emitter and collector, and because of the difference in B-C and B-E areas, we have $-i_C \approx i_B = 0.583$ mA and $i_E = \text{small}$ value.

(c) $v_I = 3.6$ Q_1 inverse active.

$$i_B = \frac{5 - (0.8 + 0.7)}{6} = 0.583 \text{ mA}$$

$$i_E = -\beta_R i_E = -(0.5)(0.583) = -0.292 \text{ mA}$$

$$i_C = -i_B - i_E = -0.583 - 0.292 \Rightarrow i_C = -0.875 \text{ mA}$$

17.27

(a) (i) For $\upsilon_I = 0.1\,\text{V}$, $\upsilon_1 = 0.1 + 0.8 = 0.9\,\text{V}$, and $\upsilon_O = 2.5\,\text{V}$

$$i_1 = \frac{2.5 - 0.9}{12} = 0.1333\,\text{mA}$$

$$i_2 = i_3 = 0$$

(ii) For $\upsilon_I = 2.5\,\text{V}$, $\upsilon_1 = 0.8 + 0.7 = 1.5\,\text{V}$, and $\upsilon_O = 0.1\,\text{V}$

$$i_1 = \frac{2.5 - 1.5}{12} = 0.0833\,\text{mA}$$

$$i_2 = i_1(1 + 0.1) = (0.0833)(1.1) = 0.09167\,\text{mA}$$

$$i_3 = \frac{2.5 - 0.1}{12} = 0.20\,\text{mA}$$

(b) (i) $\upsilon_1 = 0.7 + 0.7 = 1.4\,\text{V}$

$$\upsilon_I = 1.4 - 0.8 = 0.6\,\text{V}$$

(ii) $\upsilon_1 = 0.8 + 0.7 = 1.5\,\text{V}$

$$\upsilon_I = 1.5 - 0.8 = 0.7\,\text{V}$$

17.28

a. i. $v_X = v_Y = 0.1\,\text{V}$, so Q_1 in saturation.

$$i_1 = \frac{5 - (0.1 + 0.8)}{6} \Rightarrow \underline{i_1 = 0.683\,\text{mA}}$$

$$\Rightarrow \underline{i_{B2} = i_2 = i_4 = i_{B3} = i_3 = 0}$$

ii. $v_X = v_Y = 5\,\text{V}$, so Q_1 in inverse active mode.

Assume Q_2 and Q_3 in saturation.

$$i_1 = \frac{5 - (0.8 + 0.8 + 0.7)}{6} \Rightarrow \underline{i_1 = i_{B2} = 0.45\,\text{mA}}$$

$$i_2 = \frac{5 - (0.8 + 0.1)}{2} \Rightarrow \underline{i_2 = 2.05\,\text{mA}}$$

$$i_4 = \frac{0.8}{1.5} \Rightarrow \underline{i_4 = 0.533\,\text{mA}}$$

$$i_{B3} = (i_{B2} + i_2) - i_4 = 0.45 + 2.05 - 0.533$$

or

$$\underline{i_{B3} = 1.97\,\text{mA}}$$

$$i_3 = \frac{5 - 0.1}{2.2} \Rightarrow \underline{i_3 = 2.23\,\text{mA}}$$

b. For Q_3:

$$\frac{i_3}{i_{B3}} = \frac{2.23}{1.97} = 1.13 < \beta$$

For Q_2:

$$\frac{i_2}{i_{B2}} = \frac{2.05}{0.45} = 4.56 < \beta$$

Since $(I_C / I_B) < \beta$, then each transistor is in saturation.

17.29

(a) (i) $\upsilon_X = 0.1\,\text{V},\ \upsilon_Y = 3.3\,\text{V}$

$$\upsilon' = 0.1 + 0.7 = 0.8\,\text{V}$$

$$i_i = \frac{3.3 - 0.8}{16} = 0.156\,\text{mA}$$

$$i_3 = i_4 = 0$$

(ii) $\upsilon_X = \upsilon_Y = 3.3\,\text{V}$

$$\upsilon' = 0.8 + 0.7 + 0.7 = 2.2\ \text{V}$$

$$i_1 = \frac{3.3 - 2.2}{16} = 0.06875\ \text{mA}$$

$$i_4 = 0.06875 - \frac{0.8}{20} = 0.02875\ \text{mA}$$

$$i_3 = \frac{3.3 - 0.1}{6} = 0.5333\,\text{mA}$$

(b) $i_{C1}(\max) = \beta i_4 = i_3 + N i'_L$

$$i'_L = \frac{3.3 - (0.1 + 0.7)}{16} = 0.15625\ \text{mA}$$

$$(50)(0.02875) = 0.5333 + N(0.15625) \Rightarrow N = 5.8 \Rightarrow N = 5$$

(c) $i_{C1}(\max) = \beta i_4 = (50)(0.02875) = 1.44\,\text{mA} < 5\,\text{mA},\ \Rightarrow N = 5$

17.30

a. For $v_X = v_Y = 5\ \text{V}$, Q, in inverse active mode.

$$i_{B1} = \frac{5 - (0.8 + 0.8 + 0.7)}{6} = 0.45\ \text{mA}$$

$$i_{B2} = i_{B1} + 2\beta_R i_{B1} = 0.45(1 + 2[0.1]) = 0.54\ \text{mA}$$

$$i_{C2} = \frac{5 - (0.8 + 0.1)}{2} = 2.05\ \text{mA}$$

$$i_{B3} = (i_{B2} + i_{C2}) - \frac{0.8}{1.5} = 0.54 + 2.05 - 0.533$$

or

$$\underline{i_{B3} = 2.06\ \text{mA}}$$

Now

$$i'_L = \frac{5 - (0.1 + 0.8)}{6} = 0.683\,\text{mA}$$

Then

$$i_{C3}(\max) = \beta_F i_{B3} = N i'_L$$

or $(20)(2.06) = N(0.683)$

$$\Rightarrow \underline{N = 60}$$

b. From above, for v_0 high, $I'_L = (0.1)(0.45) = 0.045$ mA. Now

$$I'_L(\max) = (1 + \beta_F)\left(\frac{5 - 4.9}{R_2}\right) = \frac{(21)(0.1)}{2}$$

$$= 1.05\ \text{mA}$$

So

$$I_L(\max) = N I'_L$$

or $1.05 = N(0.045)$

$$\Rightarrow \underline{N = 23}$$

17.31

(a) (i) $V_{in} = 0.1\text{ V}$

$$i_{RB} = \frac{5-(0.1+0.8)}{4} = 1.025\text{ mA}$$

$$i_{RCP} = i_{Bo} = 0,\quad V_{out} = 5\text{ V}$$

(ii) $V_{in} = 5\text{ V}$

$$i_{RB} = \frac{5-(0.7+0.8+0.7)}{4} = 0.70\text{ mA}$$

$$V_{out} = 0.7 + 0.1 = 0.8\text{ V}$$

$$i_{RCP} = \frac{5-0.8}{1} = 4.2\text{ mA}$$

$$i_{BS} = (1.1)(0.7) = 0.77\text{ mA}$$

$$i_{Co} = \beta i_{Bo},\quad i_{CS} = 4.2 - \beta i_{Bo},\quad i_{ES} = 0.77 + (4.2 - \beta i_{Bo})$$

$$i_{Bo} = i_{ES} - \frac{0.7}{1}$$

$$i_{Bo} = 0.77 + 4.2 - \beta i_{Bo} - 0.7$$

$$(1+\beta) i_{Bo} = 4.27 \Rightarrow i_{Bo} = \frac{4.27}{51} = 0.0837\text{ mA}$$

(b) (i) $V_{in} = 0.1\text{ V},\ V_{out} = \text{High},$

$$i'_L = 5\beta_R i_{RB} = 5(0.1)(0.7) = 0.35\text{ mA}$$

$$V_{out} = 5 - (0.35)(1) = 4.65\text{ V}$$

$$P = (5-0.1)(1.025) + (0.35)(5-4.65) = 5.145\text{ mW}$$

(ii) $i_L = 5(1.025) = 5.125\text{ mA}$

$$P = (0.77 + 4.2)(5) + (5.125)(0.1) = 25.4\text{ mW}$$

17.32

a. $v_X = v_Y = v_Z = 0.1\text{ V}$

$$i_{B1} = \frac{5-(0.1+0.8)}{3.9} \Rightarrow \underline{i_{B1} = 1.05\text{ mA}}$$

Then

$$\underline{i_{C1} = i_{B2} = i_{C2} = i_{B3} = i_{C3} = 0}$$

b. $v_X = v_Y = v_Z = 5\text{ V}$

$$i_{B1} = \frac{5-(0.8+0.8+0.7)}{3.9} \Rightarrow \underline{i_{B1} = 0.692\text{ mA}}$$

Then

$$i_{C1} = i_{B2} = i_{B1}(1+3\beta_R) = (0.692)(1+3[0.5])$$

$$\Rightarrow \underline{i_{C1} = i_{B2} = 1.73\text{ mA}}$$

$$i_{C2} = \frac{5-(0.1+0.8)}{2} \Rightarrow \underline{i_{C2} = 2.05\text{ mA}}$$

$$i_{B3} = i_{B2} + i_{C2} - \frac{0.8}{0.8} = 1.73 + 2.05 - 1.0$$

$$\Rightarrow \underline{i_{B3} = 2.78\text{ mA}}$$

$$i_{R3} = \frac{5-0.1}{2.4} = 2.04\,\text{mA}$$
$$i_L' = \frac{5-(0.1+0.8)}{3.9} = 1.05\ \text{mA}$$
$$i_{C3} = i_{R3} + 5i_L' = 2.04 + (5)(1.05)$$
$$\Rightarrow \underline{i_{C3} = 7.29\ \text{mA}}$$

17.33

(a) (i) $I_L' = 5\,\mu\text{A},\ \ i_{B4} = \dfrac{5}{51} = 0.098\,\mu\text{A}$

$$V_{B5} = 5 - (0.000098)(2) \cong 5\ \text{V}$$
$$\upsilon_O = 3.6\ \text{V}$$

(ii) $I_L' = 5\,\text{mA},\ i_{B4} = \dfrac{5}{51} = 0.098\,\text{mA}$

$$V_{B5} = 5 - (0.098)(2) = 4.804\ \text{V}$$
$$\upsilon_O = 4.804 - 1.4 = 3.404\ \text{V}$$

(iii) Q_4 in saturation

$$I_B = \frac{5-V_B}{2} = \frac{5-(V_E+0.8)}{2}$$
$$I_C = \frac{5-V_C}{0.13} = \frac{5-(V_E+0.1)}{0.13}$$
$$I_B + I_C = I_E = I_L = 25 = \frac{5-(V_E+0.7)}{2} + \frac{5-(V_E+0.1)}{0.13}$$
$$25 = 2.10 - V_E(0.5) + 37.69 - V_E(7.692) \Rightarrow V_E = 1.81\ \text{V}$$
$$\upsilon_O = 1.81 - 0.7 = 1.11\ \text{V}$$

(b) $V_{B4} = 0.7 + 0.8 = 1.5\ \text{V}$

$$I_{B4} = \frac{5-1.5}{2} = 1.75\ \text{mA}$$
$$V_{C4} = 0.7 + 0.1 = 0.8\ \text{V},\ \ I_{C4} = \frac{5-0.8}{0.13} = 32.31\,\text{mA}$$
$$I_L = 1.75 + 32.31 = 34.06\ \text{mA}$$

17.34

a. $v_X = v_Y = v_Z = 2.8$ V, Q_1 biased in the inverse active mode.

$$i_{B1} = \frac{2.8-(0.8+0.8+0.7)}{2} \Rightarrow \underline{i_{B1} = 0.25 \text{ mA}}$$

$$i_{B2} = i_{B1}(1+3\beta_R) = 0.25(1+3[0.3])$$

$$\Rightarrow \underline{i_{B2} = 0.475 \text{ mA}}$$

$$v_{C2} = 0.8 + 0.1 = 0.9 \text{ V}$$

$$i_{B4} = \frac{0.9-(0.7+0.1)}{(1+\beta_F)(0.5)} = \frac{0.1}{(101)(0.5)}$$

$$= 0.00198 \text{ mA (Negligible)}$$

$$i_{R2} = \frac{5-0.9}{0.9} = 4.56 \text{ mA}$$

$$\Rightarrow \underline{i_{C2} = 4.56 \text{ mA}}$$

$$i_{B3} = i_{B2} + i_{C2} - \frac{0.8}{1} = 0.475 + 4.56 - 0.8$$

$$\Rightarrow \underline{i_{B3} = 4.235 \text{ mA}}$$

b. $v_X = v_Y = v_Z = 0.1$ V

$$i_{B1} = \frac{5-(0.1+0.8)}{2} \Rightarrow \underline{i_{B1} = 2.05 \text{ mA}}$$

From part (a),

$$i'_L = \beta_R \cdot i_{B1} = (0.3)(0.25) = 0.075 \text{ mA}$$

Then

$$i_{B4} = \frac{5i'_L}{1+\beta_F} = \frac{5(0.075)}{101} \Rightarrow \underline{i_{B4} = 0.00371 \text{ mA}}$$

17.35

a. $v_X = v_Y = v_Z = 0.1$ V

$$i_{B1} = \frac{2-(0.1+0.8)}{R_{B1}} + i_{B3}$$

where

$$i_{B3} = \frac{(2-0.7)-(0.9)}{R_{B2}} = \frac{0.4}{1}$$

$$\Rightarrow \underline{i_{B3} = 0.4 \text{ mA}}$$

Then

$$i_{B1} = \frac{1.1}{1} + 0.4 \Rightarrow \underline{i_{B1} = 1.5 \text{ mA}}$$

$$\underline{i_{B2} = 0 = i_{C2}}$$

Q_3 in saturation $i_{C3} = 5i'_L$ For v_0 high,

$$v'_{B1} = 0.8 + 0.7 = 1.5 \text{ V} \Rightarrow Q'_3 \text{ off}$$

$$i'_{B1} = \frac{2-1.5}{1} = 0.5 \text{ mA}$$

$$i'_L = \beta_R i'_{B1} = (0.2)(0.5) = 0.1 \text{ mA}$$

Then

$$\underline{i_{C3} = 0.5 \text{ mA}}$$

b. $v_X = v_Y = v_Z = 2\text{ V}$

From part (a),

$\Rightarrow \underline{i_{B1} = 0.5\text{ mA}}$

$\underline{i_{B3} = 0 = i_{C3}}$

$i_{B2} = i_{B1}(1+3\beta_R) = (0.5)(1+3[0.2])$

$\underline{i_{B2} = 0.8\text{ mA}}$

$i_{C2} = 5i'_{L.}$ and from part (a), $i'_L = 1.5\text{ mA}$

So

$\underline{i_{C2} = 7.5\text{ mA}}$

17.36

(a) $I_B + I_D = \dfrac{5.8-0.7}{10} = 0.51\ mA$

$I_C - I_D = \dfrac{5-(0.7-0.3)}{1} = 4.6\ mA$

Now

$I_D = 0.51 - I_B = 0.51 - \dfrac{I_C}{\beta} = 0.51 - \dfrac{I_C}{50}$

Then

$I_C - I_D = I_C - \left(0.51 - \dfrac{I_C}{50}\right) = I_C\left(1+\dfrac{1}{50}\right) - 0.51 = 4.6$

So $\underline{I_C = 5.01\ mA}$

$I_B = \dfrac{I_C}{\beta} = \dfrac{5.01}{50} \Rightarrow \underline{I_B = 0.1002\ mA}$

$I_D = 0.51 - 0.1002 \Rightarrow \underline{I_D = 0.4098\ mA}$

$\underline{V_{CE} = 0.4\ V}$

(b) $\underline{I_D = 0, V_{CE} = V_{CE}(sat) = 0.1\ V}$

$I_B = \dfrac{5.8-0.8}{10} \Rightarrow \underline{I_B = 0.5\ mA}$

$I_C = \dfrac{5-0.1}{1} \Rightarrow \underline{I_C = 4.9\ mA}$

17.37

(a) (i) $\upsilon_I = 0$, $\upsilon_1 = 0.3\text{ V}$

$i_1 = \dfrac{1.5-0.3}{1} = 1.2\text{ mA}$

$i_B = i_C = 0$, $\upsilon_O = 1.5\text{ V}$

(ii) $\upsilon_I = 1.5\text{ V}$, $\upsilon_1 = 0.7 + 0.3 = 1.0\text{ V}$

$i_1 = \dfrac{1.5-1.0}{1} = 0.5\text{ mA}$

$i_B = 0.5 - \dfrac{0.7}{20} = 0.465\text{ mA}$

$$i_C = \frac{1.5 - 0.4}{1.2} = 0.9167 \text{ mA}, \quad \upsilon_O = 0.4 \text{ V}$$

(b) (i) $\upsilon_1 = 0.7 + 0.3 = 1.0 \text{ V}, \quad \upsilon_I = 0.7 \text{ V}$

$$i_B = i_C = 0$$

(ii) $\upsilon_1 = 1.0 \text{ V}, \ \upsilon_I = 0.7 \text{ V}$

$$i_C = \frac{1.5 - 0.4}{1.2} = 0.9167 \text{ mA}$$

$$i_B = \frac{i_C}{\beta} = \frac{0.9167}{25} = 0.03667 \text{ mA}$$

(c) $i_{Co}(\max) = \beta i_B = i_C + N i'_L$

$$i'_L = \frac{1.5 - (0.4 + 0.3)}{1} - \frac{0.4}{20} = 0.78 \text{ mA}$$

$$(25)(0.465) = (0.9167) + N(0.78) \Rightarrow N = 13.7 \Rightarrow N = 13$$

17.38

a. $v_X = v_Y = 0.4 \text{ V}$

$$v_{B1} = 0.4 + 0.7 \Rightarrow \underline{v_{B1} = 1.1 \text{ V}}$$

$$i_{B1} = \frac{5 - 1.1}{2.8} \Rightarrow \underline{i_{B1} = 1.39 \text{ mA}}$$

$$v_{B2} = 0.4 + 0.4 \Rightarrow \underline{v_{B2} = 0.8 \text{ V}}$$

$$i_{B2} = i_{C2} = i_{B0} = i_{C0} = i_{B5} = i_{C5}$$
$$= i_{B3} = i_{C3} = 0 \ (\text{No load})$$

$$5 = i_{B4} R_2 + V_{BE} + (1 + \beta) i_{B4} R_4$$

$$i_{B4} = \frac{5 - 0.7}{0.76 + (31)(3.5)} \Rightarrow \underline{i_{B4} = 0.0394 \text{ mA}}$$

$$i_{C4} = \beta_F i_{B4} \Rightarrow \underline{i_{C4} = 1.18 \text{ mA}}$$

$$v_{B4} = 5 - (0.0394)(0.76) \Rightarrow \underline{v_{B4} = 4.97 \text{ V}}$$

b. $v_X = v_Y = 3.6 \text{ V}$

$$v_{B1} = 0.7 + 0.7 + 0.3 \Rightarrow \quad \underline{v_{B1} = 1.7 \text{ V}}$$
$$\underline{v_{B2} = 1.4 \text{ V}}$$
$$\underline{v_{B0} = 0.7 \text{ V}}$$
$$\underline{v_{C2} = 1.1 \text{ V}}$$

$$i_{B1} = \frac{5 - 1.7}{2.8} \Rightarrow \underline{i_{B1} = 1.1786 \text{ mA}}$$

$$i_{B2} = i_{B1}(1 + 2\beta_R) = 1.18(1 + 2\,[0.1])$$
$$\underline{i_{B2} = 1.41 \text{ mA}}$$

$$i_{B4} = \frac{1.1 - 0.7}{(31)(3.5)} \Rightarrow \underline{i_{B4} = 0.00369 \text{ mA}}$$

$$i_{R2} = \frac{5 - 1.1}{0.76} = 5.13 \text{ mA} \Rightarrow \underline{i_{C2} \approx 5.13 \text{ mA}}$$

$$i_{B0} \approx i_{B2} + i_{C2}$$
$$\underline{i_{B0} = 6.54 \text{ mA}}$$

17.39

(a) For the load, $i_{RB1} = \dfrac{2.5-(0.4+0.3)}{R_{B1}} = \dfrac{1}{2}(0.2) \Rightarrow R_{B1} = 18\ \text{k}\Omega$

For $\upsilon_X = \upsilon_Y = \upsilon_Z = \text{logic 1}$

$$i_{RB1} = \frac{2.5-(0.7+0.8)}{18} = 0.05556\ \text{mA}$$

$$i_{RC1} = \frac{2.5-(0.7+0.1)}{R_{C1}}$$

$$i_{B2} = 0.1 = 0.05556 + \frac{1.7}{R_{C1}} - \frac{0.7}{0.7} \Rightarrow R_{C1} = 1.63\ \text{k}\Omega$$

(b) $\upsilon_X = 0.4\ \text{V},\ \upsilon_{B1} = 0.7\ \text{V},\ \upsilon_{B2} = 0$

$\upsilon_O \cong 2.5 - 0.7 = 1.8\ \text{V}$

All $i_B = 0$, All $i_C = 0$

(c) $\upsilon_{B1} = 1.5\ \text{V},\ \upsilon_{B2} = 0.7\ \text{V}$

$$i_{B1} = \frac{2.5-1.5}{18} = 0.0556\ \text{mA}$$

$$i_{C1} = \frac{2.5-(0.7+0.1)}{1.63} = 1.043\ \text{mA}$$

$$i_{B2} = 0.10\ \text{mA}$$

$$i_{C2} = \frac{4[2.5-(0.4+0.3)]}{18} = 0.40\ \text{mA}$$

$\upsilon_O = 0.4\ \text{V}$

(d) $i_{C2}(\text{max}) = \beta i_{B2} = N i'_L$

$i'_L = 0.1\ \text{mA}$

$(20)(0.1) = N(0.1) \Rightarrow N = 20$

17.40

a. For $v_X = v_Y = 3.6\ \text{V}$

$$v_{B1} = 3(0.7) = 2.1 \Rightarrow i_{B1} = \frac{5-2.1}{10} = 0.29\ \text{mA}$$

$$v_{C1} = 0.7 + 0.7 + 0.4 = 1.8\ \text{V} \Rightarrow i_{C1} = \frac{5-1.8}{10} = 0.32\ \text{mA}$$

$$i_{B2} = i_{B1} + i_{C1} - \frac{1.4}{15} = 0.29 + 0.32 - 0.0933$$

So

$$i_{B2} = 0.517\ \text{mA}$$

$$v_{C2} = 0.7 + 0.4 = 1.1\ \text{V}$$

$$i_{C2} = \frac{5-1.1}{4.1} = 0.951\ \text{mA}$$

$$i_{B5} = i_{B2} + i_{C2} - \frac{0.7}{4} = 0.517 + 0.951 - 0.175$$

or $i_{B5} = 1.293\ \text{mA}$

For $v_0 = 0.4\ \text{V},\ v'_{B1} = 0.4 + 0.7 = 1.1\ \text{V}$

Then

$$i'_{B1} = \frac{1.1 - 0.7}{(31)(15)} = 0.00086 \text{ mA}$$

$$i'_L = \frac{5 - 1.1}{10} - 0.00086 \text{ or } i'_L \approx 0.39 \text{ mA}$$

So $i_{C5}(\max) = \beta i_{B5} = N i'_L$

$$(30)(1.293) = N(0.39) \Rightarrow \underline{N = 99}$$

b. $$P = (0.29 + 0.32 + 0.951)(5) + (99)(0.39)(0.4)$$

$$P = 7.805 + 15.444 \text{ or } \underline{P = 23.2 \text{ mW}}$$

(Assumming 99 load circuits which is unreasonably large.)

17.41

a. Assume no load. For $v_X = \text{logic } 0 = 0.4 \text{ V}$

$$i_{E1} = \frac{5 - (0.4 + 0.7)}{40} = 0.0975 \text{ mA}$$

Essentially all of this current goes to ground from V_{CC}.

$$P = i_{E1} \cdot V_{CC} = (0.0975)(5) \Rightarrow \underline{P = 0.4875 \text{ mW}}$$

b. $$i_{R1} = \frac{5 - (3)(0.7)}{40} = 0.0725 \text{ mA}$$

$$i_{R2} = \frac{5 - (0.7 + 0.7 + 0.4)}{50} = 0.064 \text{ mA}$$

$$i_{R3} = \frac{5 - (0.7 + 0.4)}{15} = 0.26 \text{ mA}$$

$$P = (0.0725 + 0.064 + 0.26)(5)$$

$$\underline{P = 1.98 \text{ mW}}$$

c. For $v_0 = 0, \; v_{C7} = 0.7 + 0.4 = 1.1 \text{ V}$

$$i_{R7} = \frac{5 - 1.1}{0.050} \Rightarrow i_{R7} = \underline{78 \, mA \approx i_{SC}}$$

17.42

(a) $$i_E = \frac{3 - (0.7 + 0.3)}{2} = 1.0 \text{ mA}$$

(b) $$i_E = \frac{2.4 - 0.7}{2} = 0.85 \text{ mA}$$

$$R_C = \frac{3 - 2.4}{0.85} = 0.706 \text{ k}\Omega$$

(c) (i) $P = (1.0)(3) = 3.0 \text{ mW}$

(ii) $P = (0.85)(3) = 2.55 \text{ mW}$

17.43

(a) $v_I = v_O = 2.5\,\text{V}$; A transient situation

$v_{DS}(M_N) = 2.5 - 0.7 = 1.8\,V$

$v_{GS}(M_N) = 2.5 - 0.7 = 1.8\,V \Rightarrow M_N$ in saturation

$v_{SD}(M_P) = 5 - (2.5 + 0.7) = 1.8\,V$

$v_{SG}(M_P)5 - 2.5 = 2.5\,V \Rightarrow M_P$ in saturation

$i_{DN} = K_n(v_{GSN} - V_{TN})^2 = (0.1)(1.8 - 0.8)^2 \Rightarrow \underline{i_{DN} = 0.1\,mA}$

$i_{DP} = K_P(v_{SGP} + V_{TP})^2 = (0.1)(2.5 - 0.8)^2 \Rightarrow \underline{i_{DP} = 0.289\,mA}$

$i_{C1} = \beta i_{DP} = (50)(0.289) \Rightarrow \underline{i_{C1} = 14.45\,mA}$

$i_{C2} = \beta i_{DN} = (50)(0.1) \Rightarrow \underline{i_{C2} = 5\,mA}$

Difference between i_{E1} and $i_{DN} + i_{C2}$ is a load current.

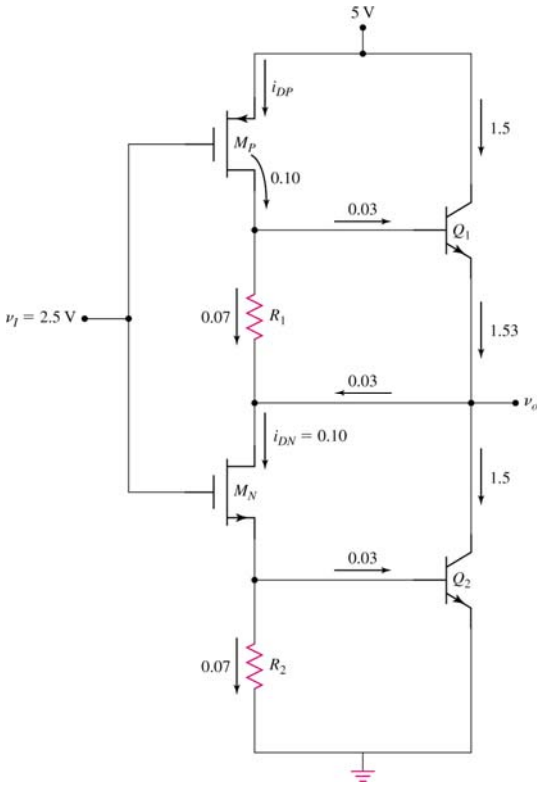
(b) Assume $i_{C1} = 14.45\,mA$ is a constant

$$V_C = \frac{1}{C}\int i_{C1}dt = \frac{i_{C1} \cdot t}{C} \Rightarrow t = \frac{(V_C)(C)}{i_{C1}}$$

$$t = \frac{(5)(15\times10^{-12})}{14.45\times10^{-3}} \Rightarrow \underline{t = 5.19\,ns}$$

(c) $$t = \frac{(5)(15\times10^{-12})}{0.289\times10^{-3}} \Rightarrow \underline{t = 260\,ns}$$

17.44



(a) Assume $R_1 = R_2 = 10\ \text{k}\Omega;\quad \beta = 50$

Then $i_{R1} = i_{R2} = \dfrac{0.7}{10} = 0.07\ \text{mA}$

NMOS in saturation region; $v_{GSN} = 2.5 - 0.7 = 1.8\ \text{V}$

$$i_{DN} = K_n\left(v_{GSN} - V_{TN}\right)^2 = (0.1)(1.8 - 0.8)^2$$

$$i_{DN} = 0.10\ \text{mA}$$

Then $i_{B2} = 0.03 \Rightarrow i_{C2} = (50)(0.03) = 1.5\ \text{mA}$

$$i_{E1} = 1.53\ \text{mA} \Rightarrow i_{B1} = 0.03\ \text{mA} \Rightarrow i_{C1} = 1.5\ \text{mA}$$

So $i_{DP} = 0.10\ \text{mA}$

Now, M_P biased in non-saturation region

$$v_{SGP} = 2.5\ \text{V}$$

$$i_{DP} = 0.10 = 0.10\left[2(2.5 - 0.8)v_{SD} - v_{SD}^2\right]$$

$$0.10\,v_{SD}^2 - 0.34\,v_{SD} + 0.10 = 0$$

$$v_{SD} = \frac{0.34 \pm \sqrt{(0.34)^2 - 4(0.10)(0.10)}}{2(0.10)}$$

Or

$$v_{SD} = 0.325\ \text{V}$$

Then $v_o = 5 - 0.325 - 0.7$

$$v_o = 3.975\ \text{V}$$

(b) $$v = \frac{1}{C}\int i\,dt = \frac{i \cdot t}{C}$$

$$t = \frac{Cv}{i} = \frac{(15\times10^{-12})(5)}{1.53\times10^{-3}}$$

$$t = 49\ \text{ns}$$

(c)

$$t = \frac{Cv}{i} = \frac{(15\times10^{-12})(5)}{0.1\times10^{-3}}$$

$$t = 0.75\ \mu\text{s}$$