Ethiopian University Entrance Examination (EUEE) Mathematics for Natural Science Ginbot 2010/June 2018

BOOKLET CODE:25 **SUBJECT CODE:02**

Number Of Items:65 **Time Allowed:3hours**

DIRECTIONS: for each of the following problems. Choose the best answer from the given alternative and carefully **blacken** the letter of your best choice on the separate answer sheet provided.

1. Which one of the following is true about signum, absolute value and greatest integer functions?

   (A) $sgn\ (x) = \pm\ |x|$, for all x € ℝ.

   (B) $|x|, = xsgn\ (x)$, for all x € ℝ .

   (C) $sgn(x) < \lfloor x \rfloor$, for all $x \leq 0$.

   (D) $sgn(x) \leq \lfloor X \rfloor$, for all $x \geq 0$.

$$|x| = x\ sgn\ for\ all\ real\ number\ x\ sgn\ x\ \leq\ |x|\ for\ all\ x\ \geq 0$$

Ans. B and D

2. What is the equation of the line that passes through (1,1) and is parallel to line $3y - x = 1$?

   (A) $3x - y = 2$

   (B) $x + 3y = 4$

   (C) $3y - x + 2 = 0$

   (D) $x - 3y + 2 = 0$

$39 - x = 1 \Rightarrow 39 = +1$

$9 = \frac{1}{3}x + 1$ *efuation of a line that pass through* (1.1) *and parallel to* $9 = \frac{1}{3}x + 1$

$Is\ \frac{y-1}{x-1} = \frac{1}{3} \Rightarrow 39 - 3 = x1$

$39 - x - 3 + 1 = 0$

$39 - x - 2 = 0$

3. Which one of the following is an equation of the circle whose endpoints of a diameter are (0,-2) and (2, 2)?

   (A) $x^2 + y^2 - 2x - 4 = 0$

   (B) $X^2 + y^2 = 4$

   (C) $(x-1)^2 + y^2 = 4$

   (D) $x^2 + y^2 - 2y - 4 = 0$

The end paints of diameter are (0,-2) and (2-2) by mid point formula center is (1,0) and radius is $\sqrt{5}$
⇨Equation of circle is $x^2 + y^2 - 2x - 4 = 0$
Ans. A

4. What are the greatest lower bound and least upper bound of the sequence $\{(-1)''(1 + \frac{1}{n})\}$, respectively?

   (A) -2and 2

   (B) -2 and 3/2

   (C) – 3/2 and 2

   (D) -2 and – 3/2

$$if\ x = 1\ \left\{(-1)^n(1 + \frac{1}{n})\right\} is -2$$
$$if\ x = 2, \left\{(-1)^n(1 + \frac{1}{n})\right\}\ is\ \frac{3}{32}$$
$$if\ x = 3, \left\{(-1)^n\ (1 + \frac{1}{n})\right\}\ is -\frac{4}{3}$$
$$if\ x = 4 \left\{(-1)^n(1 + \frac{1}{n})\right\} is\ \frac{5}{4}$$

⇨GLb is -2 and L.U.b is $\frac{3}{2}$
Ans. B

5. What is the maximum value of the function f (x)= $x^4 - 2x^2$ on [-2,1]?

   (A) 8

   (B) 12

   (C) 24

   (D) 40

$f(x) = x^4 - 2x^2$

$f'(x) = 4x^3 - 4x$

$f'(x) = 0 \Rightarrow 4x^{3-4x=0}$

$4x(x^{2-}1) = 0$

$4x = 0 \; or \; x = \pm 1$

Critical numbers are -2,0,1,-1

f(-2) = 8 f(1)=-1

f(0)= 0 f(-1)=1

8 is maximum value at (2)

Ans. A

6. If the truth value of $(p \wedge \neg p) \Leftrightarrow [(q \vee \neg q) \Rightarrow r]$ is True, then which one of the following must be True?

   (A) p

   (B) q

   (C) ¬ q

   (D) ¬ r

(Pn ¬ p)⇔$[(qu \neg q) \Rightarrow r]$ *is true*

⇨Pn ¬ p is false since por ¬ p is false

⇨(qu ¬ q) is false since (pn ¬ p) is false and by property of implication

⇨(qu ¬) is ture since of or f is ture

⇨ ¬ r must be true to make the proposition true

Ans. D

7. Let A and B be two events. Suppose that the probability that neither event occur is 3/8. What is the probability that at least one of the events occur?

   (A) 5/8

   (B) ¼

   (C) 1/8

   (D) ¾

P (AUB)' = $\frac{3}{8}$

$P(AUB) = 1 - p(AUB)$

$= 1 - \frac{3}{8}$

$= \frac{5}{8}$

Ans. A

8. What is the value of $\sum_{i=0}^{n} (\frac{1}{n-1} - \frac{1}{n})$?

(A) $\frac{17}{20}$

(B) $\frac{21}{20}$

(C) $\frac{19}{20}$

(D) $\frac{23}{20}$

$$\sum_{n=1}^{20} (\frac{n}{n-1} - \frac{1}{n} = \left\{\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} + \frac{1}{4}\right) + \cdots 19 - \frac{1}{20}\right\}$$

$$= \left\{1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} \cdots \cancel{19} - \frac{1}{20}\right\}$$

$$= \left\{1 - \frac{1}{20}\right\} = \frac{19}{20}$$

Ans. C

9. Which one of the following is a convergent sequence?

(A) $\left\{\frac{(-1)''}{2}\right\}$

(B) $\left\{\frac{1}{n} + \sin(n)\right\}$

(C) $\left\{\frac{1-3^n}{2^n}\right\}$

(D) $\left\{\frac{1-2^n}{3^n}\right\}$

$if\ n = 1, \left\{\frac{1+2^n}{3^n}\right\} is\ 1$

Take $\left\{\frac{1+2^n}{3^n}\right\}\ if\ n = 2, \left\{\frac{1+2^n}{3^n}\right\}\ is\ \frac{5}{9}$

$if\ n = 3, \left\{\frac{1+2^n}{3^n}\right\}\ is\ \frac{9}{27}$

$if\ n = 4, \left\{\frac{1+2^n}{3^n}\right\} is\ \frac{17}{18}$

If is converges to zero the others are diver gent.

Ans. D

10. If a function f is differentiable at a, than what is the value of $\lim_{x\to a} f(x)$

(A) $f'\alpha)$

(B) $f(\alpha)$

(C) $f'(\alpha) - f(\alpha)$

(D) 0

To differentiate the function at a means find f(a). the derivative of f(x) at a is also ex pressed as

f(a)= lim $\frac{f(x)-f(a)}{x-a}$

Ans. A

11. If $f'(x) = e^{x-1}+3x^2 - \frac{1}{x}$ and $f(1)$=5, what is$f(x)$?

(A) $f(x) = e^{x-1}+ x^2 - lnx + 3$

(B) $f(x) = e^{x-1} - x^3 - lnx + 5$

(C) $f(x) = e^{x-1} + 3x^{2-} - \frac{1}{x} + \mathbf{5}$

F(x)= $e^{x-1} + 3x^2 - \frac{1}{x}, f(1) = 5$

$\int f'(x) = f(x) = e^{x-1} + x^3 - 1nx + c$

F(1)=5⇨$e^x + x^3 - 1nx + c = 5$

⇨1+1-0+C=5

⇨C= 3

$\therefore the\ equation\ of\ f(x) is\ e^x + x^3 - 1nx + 3$

Ans. A

12. If A is a square matrix of order 3 and det (A)=5, then $what\ is\ the\ value\ of\ det$ (A.adj(A))?

(A) 3

(B) 5+

(C) 25

(D) 125

A is squat matrix with order 3x3
Det (A and A)= det (A)(A)(det adj A)$^{n-1}$
= det (A)(A.det(adj A)$^{3-1}$
=def(A) (A. det(adj(A)^2
= 5 x 5^2 = 125
Ans. D

13. A salesman sold items x_1, x_2 and x_3, with different rates of commission as shown in the table below.

Month	Sales of units			Total commission (in Birr)
	x_1	x_2	x_3	
February	90	100	20	800
March	130	50	40	900
April	60	50	30	850

What are the rates of commission on items x_1, x_2 and x_3, respectively.

(A) 4,2 and 11

(B)4,11 and 2

(C)2,4 and 11

(D)11,2 and 4

Soln from the a huve table $\int \begin{matrix} 90c + 100x2 + 20x3 = 800 \\ 130x1 + 50x2 + 40x3 = 90 \\ 60x1 + 100x2 + 30x3 = 850 \end{matrix}$

Solve by remodels rule D = $\begin{vmatrix} 9 & 10 & 2 \\ 13 & 5 & 4 \\ 6 & 10 & 3 \end{vmatrix}$ = -175 DC $\begin{vmatrix} 80 & 10 & 2 \\ 90 & 5 & 4 \\ 85 & 10 & 3 \end{vmatrix}$ =-350 Dy $\begin{vmatrix} 9 & 80 & 2 \\ 13 & 90 & 4 \\ 6 & 85 & 3 \end{vmatrix}$ = -700

Dz $\begin{vmatrix} 9 & 10 & 80 \\ 13 & 5 & 90 \\ 6 & 10 & 80 \end{vmatrix}$ = -1925

⇨ x = $\frac{Dx}{D} = \frac{-350}{-175} = 2$

Y= $\frac{Dy}{D} = \frac{-700}{-175}$ =4

Z = $\frac{-1925}{-175} = 11$

The rate of commission x_1, x_2 and x_3 = x,y and 2 = 4,4 and 11respefivly

Ans. C

14. If $= -3 + 4i \ \ and \ w = 1 + 2i$, then what is the value of $\frac{2z}{w} + \bar{w}$?

(A) 2+3i

(B) 3+2i

(C) 3+5i

(D) 3-2i

Z = -3+4i w = 1+2i then

$\frac{2z}{w} + \bar{w} = \frac{2(-3+4i)}{1+2i} + 1 - 2i$

$= \frac{-6+8i}{1+2i} x \frac{1-2i}{1-2i} + 1 - 2i$

$= \frac{-6+12i+8i+16}{1+4} + 1 - 2i$

$= \frac{10+20i}{5} + 1 - 2i$

=2+4i+1-2i

= 3+2i

Ans. B

15. The population of a certain city is increasing at a rate of 3% per year. If the population was 100,000 in 2012 E.C., then what will be the population in 2020 E.C.?

(Given: $(1.03)^9 = 1.30$, $(1.03)^{10} = 1.34$, $(1.3)^9 = 10.60$, $(1.3)^{10} = 13.78$)

(A) 130,000

(B) 134,000

(C) 1,060,000

(D) 1,378,000

Given P =100,000
R = 0.03
T = 10 year
$A = P(1+r)^t$ ⇨ $A = 100{,}000\ (1+0.03)^{10}$
$= 100{,}000\ (1.03)^{10}$
= 134,000

Ans. B

16. What is the slope of the tangent line to the graph of $f(x) = 3e^x + \sin x + 2\ at\ (0,5)$?

(A) 2

(B) 3

(C) 4

(D) 5

F(x) = 3ex+sinx+2 at (0.5)
F'(x) = 3ex+cosx
F(0) = slope of tangent line = 3e0+ cist 0
= 3+1= 4

Ans. C

17. What is the value of $\int_1^2 \frac{x+4}{x(x+2)}$

(A) 2
(B) $\ell n2$
(C) $\ell n3$
(D) $\ell n4 - \ell n3$

$\int \frac{x+4}{x(x+2)}$ x by partial fraction

$$A + B = 1$$
$$2 + B = 1$$
$$B = -1$$

$\frac{x+4}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$

$\frac{x+4}{x(x+2)} = \frac{A(x+2)+Bx}{x(x+2)}$

$\frac{A}{x} + \frac{B}{x+2} = \frac{2}{x} - \frac{1}{x+2}$

X+4 = Ax+2A+bx

X+4 = (A + B) x + 2A

$\int \frac{2}{x} - \frac{1}{x+2}$ abc = $\mu nx - \mu n(x\rho 4)2$

⇨A+B = 1

2A = 4 ⇨A = 2

⇨21n2 -1n4-(0-(n3)

= 1n3

Ans. C

18. What is the area of the region enclosed by the graph of $x^2 = x + 1$ and $y^2 = -x + 1$?

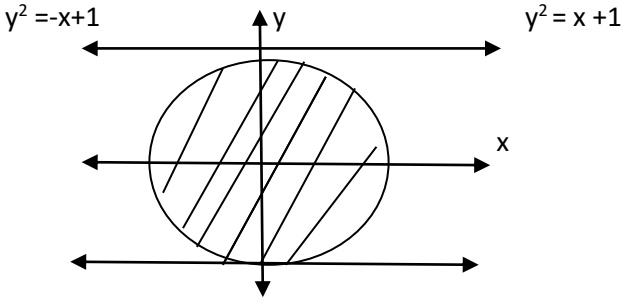
(A) $\frac{8}{3}\ sq.units$

(B) $\frac{4}{3}\ sq.units$

(C) $\frac{3}{8}\ sq.units$

(D) $\frac{3}{4}\ sq.units$

The area of a region enclosed by the groups of 9^2 = x+1 and 9^2 = -x+1



The region is symmetric with respect to 9-axis and we can find the area in terms of 9

A = $2_0 \int -9^2+1-(9^2-1)$ dy

A = $2_0 \int (-9^2+1-(9^2-1)$ dy

A = $2_0 \int i - 29^2+2)$ dy

= 2 $(-\frac{29^3}{3} + 29)\Big|_0^1$

= 2 $(\frac{-2}{3} + 2)$

A = $\frac{8}{3}$ squ. Unit

Ans

19. What is the partial fraction decomposition of $\frac{x^2+x+1}{(x+2)(x^2+1)}$?

(A) $\frac{5}{3(x+2)}+\frac{2x+1}{3(x^2+2)}$

(B) $\frac{3}{5(x+2)}+\frac{2x+1}{5(x^2+1)}$

(C) $\frac{2}{5(x+2)}+\frac{3x+1}{5(x^2+1)}$

(D) $\frac{2}{3(x+2)}+\frac{2x+1}{3(x^2+1)}$

Partial fraction of $\frac{x^2+x+1}{(x+2)(x^2+1)} let\ \frac{A}{x+2}+\frac{Bx+C}{x^2+1}$

$\frac{x^2+x+1}{(x+2)(x^2+1)}+\frac{A(x^2+1)+(Bx+C)(x+C)}{(x+2)(x^2+1)}$

x^2+x+1 = Ax^2+A+Bx^2+2Bx+Cx+2C

x^2+x+1 = (A+B) x^2 + (2B+C) x + A +2C

$A+B=1$ $\quad if x=-2$ $\quad A+B=1$

$2B+C=1$ $\quad 4-2+1=5A$ $\quad \frac{3}{5}+B=1$

$A+2C=2$ $\quad 3=5A$ $\quad B=1-\frac{3}{5}$

B = $\frac{2}{5}$

2B+C =1

$\frac{4}{5}+C=1$

C = $\frac{1}{5}$

$$\frac{A}{x+2}+\frac{Bx+C}{x^2+1}=\frac{3}{5(x+2)}+\frac{\frac{2}{5}x+\frac{1}{5}}{x^2+1}$$

$$=\frac{3}{5(x+2)}+\frac{2x+1}{5(x^2+1)}$$

Ans. B

20. What is the area of the triangle (in sq. units) formed by the lines joining the vertex of the parabola $x^2=36y$ to the end points of the latus rectum?

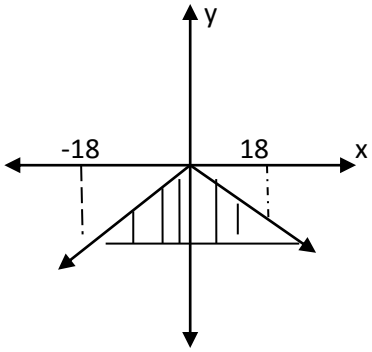
(A) 126

(B) 162

(C) 216

(D) 261

$x^2 = -369$



-4P = -36

P =9

It is isosceles triangle with base =length of latusrctlem of parabola. As shown in the figure length of latusretum (base)

= $|4P| = |-36| = 36$

A = $\frac{1}{2}$ b,h

A = $-x36x9$ ($wher\ h = p$)

A=160 squ unit

Ans. B

21. The time needed to type a sample of 8 business letters in an office is 7, 8, 6, 8, 9, 7, 5, 6 minutes. What are the mean($\bar{x}$) and the standard deviation (s) of the data in minute?

(A) $\bar{x}$ = 7, s =$\sqrt{2}$

(B) $\bar{x}$ = 7, s =$\sqrt{1.5}$

(C) $\bar{x}$ = 8, s =$\sqrt{2}$

(D) $\bar{x}$ = 8, s =$\sqrt{1.5}$

The data is 7, 8, 6, 8, 9, 7, 5, 6,

$\bar{x} = \frac{7+8+6+8+9+7+5+6}{8} = \frac{56}{8} = \underline{7}$

$5^2 = \frac{(7-7)^2+(6-7)^2+(8-7)^2+(8-7)^2+(9-7)^2+(7+7)^2+(5-7)^2+(6-7)^2}{8}$

$= \frac{1+1+1+4+4+1}{8} = \frac{3}{2} = 1.5$

$5 = \sqrt{52} = \sqrt{1.5}$

Ans. B

22.Let A be a 3x3 invertible matrix and be any 3x3 matrix. If $|A| = a$ and, $|B| = b$, then which one of the following is **NOT** true?

(A) $|A^T A| = a^2$

(B) $|A^T A| = a^2$

(C) $|kA| = k^2|A|$, for any k$\in \mathbb{R}$

(D) $|A^{-1} B| = ab$

Two matrices A and B are 3x3 matrices then $|A^T A| = |A^T||A| = a.a = a^2$. $|KA| = K^2 |A| since\ if\ is\ 3x3\ matrix\ but\ |\bar{A}B| \neq ab$

Ans. D

23. Let $z = \left(\frac{1-i}{1+i}\right)^{18}$. Then what is the value of ?

(A) i

(B) -1

(C) –i

(D) 1-i

$z = \left(\frac{1-i}{1+i}\right)^{18}$. $\Rightarrow z = \left(\frac{1-i}{1+i}\right)^{2)9}$.

$\Rightarrow z = \left(\frac{-2i}{2i}\right)^9 = 1^9 = -1$

Ans. B

24. If $f(x) = \frac{|x|}{x}$ and $g(x) = \frac{x+2}{x^3-4x}$, then what is the value of $lim_{x\to -2}\ (f(x) + g(x))$?

(A) $\frac{-9}{8}$

(B) $\frac{-7}{8}$

(C) $\frac{9}{8}$

(D) ∞

F(x) = $\frac{|x|}{x}$ and 9(x) = $\frac{x+2}{x^3-4x}$

⇨ Lim (f(x) + 9 (x) = $\underset{x \to -2}{Lim} f(x) + \underset{x \to 2}{Lim} \frac{x+2}{x^3-4x}$

= $\underset{x \to -2}{Lim} \frac{|x|}{x} + \underset{x \to -2}{Lim} \frac{x+2}{x^3{}_{4x}}$

= - 1 + $\underset{x \to -2}{Lim} \frac{x \neq 2}{x(x \neq 2)(x-2)}$

= 1 + $\underset{x \to -2}{Lim} \frac{1}{x(x-2)}$

= -1 + $\frac{1}{8} = \frac{-7}{8}$

Ans. B

25. What is the value of $\lim_{x \to 0} \frac{1}{x^2} sin^2\left(\frac{x}{2}\right)$?

(A) ¼

(B) ½

(C) 2

(D) 4

(E)

$\underset{x \to 0}{Lim} \frac{1}{x^2} sin^2\left(\frac{x}{2}\right) = \underset{x \to 0}{Lim} \frac{sin^2 \frac{x}{2}}{x^2}$

= $\underset{x \to 0}{Lim} \frac{sin^2 \frac{x}{2}}{x2} \frac{x^{\frac{1}{4}}}{x^{\frac{1}{4}}}$

= $\underset{x \to 0}{\frac{1}{4}} Lim \frac{\sin \frac{(x)^2}{2}}{\frac{(x)^2}{2}} = \frac{1}{4}, 1 = \frac{1}{4}$

Ans. A

26. Which one of the following is NOT true about the function $f(x) = 3x^4 - 4x^3$?,

(A) (0,0) is point of inflection of f.

(B) 0 and 1 are critical number of f

(C) f Is decreasing on $(-\infty, 1)$and increasing on and increasing on $(1, \infty)$

(D) f is concave up ward on and concave down ward on $(\infty, 0)$ and on $(\frac{2}{3}, \infty)$

F(x) = $3x^4$ - $4x^3$

F'(x) = $12x^3$ - $12x^2$ ⇨ f' (x) = 0 ⇨ 12 x (x^2) = 0

F'' (x) = $36x^2 - 24x$ x =0, x = -1, x = 1 are critical numbers

F'' (x) = 0 ⇨ $36x^2 - 24x = 0$

6x (6x-4) =0

X = 0, x = $\frac{2}{3}$ aye in flection points is true the others are not true

Ans. A

27. What is the value of $\int x\sqrt{1-x^2}$ dx?

(A) $\frac{1}{2}\sqrt{1-x^2}$ + C

(B) $\frac{-1}{2}(1-x^2)^{3/2}$ +C

(C) $(1-x^2)^{3/2}$ +C

(D) $-2x\sqrt{1-x^2}$ + C

$\int x\sqrt{1-x^2}$

⇨ $\int x\sqrt{u}.\frac{du}{-2x}$

⇨ $-\frac{1}{2}.\frac{u}{\frac{3}{2}}\frac{3}{2}+c$

⇨ $-\frac{1}{2}\int u\ \frac{1}{2}\ du$ ⇨ $\frac{1}{2}.\frac{2\ u^{\frac{3}{3}}}{3}+c$

$= -\frac{1}{3}(1-x^2)-+\ c$

let $u = 1-x^2$

$du = -2x\ dx$

$dx = \frac{du}{-2x}$

Ans. B

28. What is the value of $\int_1^9 e\frac{\sqrt{x}}{\sqrt{x}}$ dx?

(A) $e^3 - e$

(B) $\frac{e^3}{3} - e$

(C) $2(e^3 - e)$

(D) $e(e^2 - \frac{1}{3})$

$\int_1^9 \frac{e^{\sqrt{x}}}{\sqrt{x}}$ dx = Let u = $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}$ dx

$dx = 2\sqrt{x}\, du$

$\int_1^9 \frac{e^u}{u}\, 2u\, du = \int_1^9 2e^u$ du

$= 2\,(e^u)\Big|_1^9$

$= 2\left(e^{\sqrt{x}}\right)\Big|_1^9$

= 2 ($\underline{\underline{e^3 - e}}$)

Ans. C

29. A running a race- course noted that the sum of the distance from the two flag posts form him is always 10 meters and the distance between the flag posts is 8 meters. What is the equation of the path traced by the man?

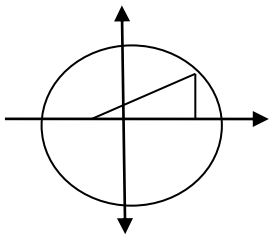
(A) $\frac{x^2}{9} + \frac{y^2}{25} = 1$

(B) $\frac{x^2}{64} + \frac{y^2}{100} = 1$

(C) $\frac{x^2}{100} + \frac{y^2}{64} = 1$

(D) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

the man running in shep of ellipse. the sum distance form the two flags means $2a = 10, a = 5$



the distance $\frac{b}{n}$ *flages means* $\frac{d}{c}\frac{b}{n}$ *foci* $= \begin{matrix} 2c = 8 \\ C = 4 \end{matrix}$

$b^2 = a^2 - c^2$

$b^2 = 25 - 9 \Rightarrow b = 3$

The equation path is $\frac{x^2}{a^2} + \frac{9^2}{b^2} = 2$ ⇨ $\frac{x^2}{25} + \frac{9^2}{9} = 2$

Ans. D

30. Suppose the following are the premises of an argument.

He is healthy and he is not angry.

He is angry or his plan fails.

His plan does not fail if he does not travel abroad.Type equation here.

Given that an the premises are true, which one of the following be conclusion that makes the argument valid?

(A) He travels abroad.

(B) His plan fails and he is angry.

(C) His plan does not fail.

(D) His plan fails and he does not travel abroad.

$Let\ \ p: he\ is\ healthe\ y$
$q: He\ is\ angrg$
$r: His\ plan\ fails$
$S: He\ travel\ abroad$

The premises are P$n\ q, qvr\ \neg s\ \Rightarrow \neg r$ the conclusion that the argument true is he travel abroad (r)

Ans. B

31. The age distribution of students in a certain class is given below:

Age	10-14	15-19	20-24	25-29
No. of students	2	10	6	7

What is the modal value of the distribution?

(A) 17.38

(B) 17.83

(C) 18.37

(D) 18.73

Age	10-14	15-19	20-24	25-29
No stu	2	10	6	7

The medal class is the largest number (class 2)

$$mo = LB + W\left(\frac{d1}{d1 + d2}\right) \qquad where\ d1 = 10 - 2 = 8,\ d2 = 10 - 6 = 4,\ w = 5$$

$$= 14.5 + 5\left(\frac{8}{8 + 4}\right)$$

$$= 14.5 + 3.33$$

$$mo = 17.83$$

Ans. B

32. Let $A = \{1,2,3,4,5,6,7\}, B = \{8,9,10\}$. If one of the numbers is deleted randomly from each of these sets, what is the probability that all the three deleted numbers are even or all are multiples of 3?

(A) $\frac{1}{9}$

(B) $\frac{2}{21}$

(C) $\frac{8}{63}$

(D) $\frac{4}{5}$

Son let P(E_1)- even and p(E_2)- multiple of 3

In Set A	**Set B**	**In set C**
P(E_1) = $\frac{3}{7}$	P(E_1) = $\frac{1}{3}$	P(E_1) = $\frac{2}{3}$
P(E_2) = $\frac{2}{3}$	P(E_2) = $\frac{1}{3}$	P(E_2) = $\frac{1}{3}$
P($E_1 n E_2$) = 0	P($E_1 n E_2$) = 0	P($E_1 n E_2$) = 0

P($E_1 n E_1 n E_1$) = $\frac{3}{7} x \frac{1}{3} x \frac{2}{3} = \frac{2}{2}$

P($E_2 n E_2 n E_2$) = $\frac{2}{3} x \frac{1}{3} x \frac{1}{3} = \frac{2}{63}$

P($E_3 n E_3 n E_3$) = $\frac{1}{7} x 0 x 0 = 0$

P(even or multiple of 3 = P(even) + p (multi 3)

$$\frac{2}{2} + \frac{2}{63} = \frac{8}{63}$$

Ans. C

33. Let A be a 3x3 matrix and $|A| = -2$ .Then what is the value of $|adj(A)|$?

(A) 4

(B) -2

(C) $-1/2$

(D) -8

Let A be 3x3 matrix and /A/ = -2

$|adj\ (A)| = |A|n - 1$

$|adj\ (A)| = (-2) \qquad n = 3$

$= \underline{4}$

Ans. A

34. If the sum of the three consecutive terms of an arithmetic progression $\{A_n\}$, with $A_n > 0$ for n , is 9 and the sum of their squares is 35, then what is the sum S_n of the first n terms?

(A) $n^2 + 1$

(B) $n^2 - 1$

(C) $2n^2 + 1$

(D) n^2

(An) be writhe metric sequence with An > 0 $a_1 + a_2 + a_3 = 9 \Rightarrow 5_3 = \frac{3}{2}(a_1 + a_3) =$

$9 \Rightarrow a_1 + a_3 = 6\ a_1^2 + a_2^2 + a_3^2 = 35 \Rightarrow a_1^2 + a_3^2 = 26$

$\Rightarrow a_1 + a_3 + a_2 = 9 \Rightarrow 6 + a_2 = \Rightarrow a_2 = 3 \Rightarrow (a_1 + a_3)^2 = a_1^2 + 2a1a3 + a_3^2 = 36\ a_1^2 +$

$a_2^3 = 36 - 2a_1a3\ \ 26 + 2a_1a3 = 36 \Rightarrow 2a_1a3 = 10$

$\Rightarrow a_1 + 3 + \frac{5}{a1} = 9 \Rightarrow a + \frac{5}{a1} = 6 \Rightarrow a_1^2 - 6a1 + s = 0\ a_1 = \frac{6 \pm \sqrt{36 - 4x5}}{2} = \frac{6 \pm 4}{2} = 5\ or\ 1\ a_3 =$

$\frac{5}{5} = 1\ or\ a_3 = \frac{5}{1} = 5$

If $a_1 = 1, d = 2\ ther\ an = a_1 + (n-1) if\ a_n = 1 + (n-1)2 = 2n - 1\ \ 5n = \frac{n}{2}(1 + 2n - n) =$ n^2 the sum of the 1st terms is 5n = n^2

Ans. D

35. Let $\{a_n\}$ be a sequence with $a_1 = a,\ a_2 = f(a_1) = f(a),\ = a_3\ f(a_2) = f\big(f(a)\big), \ldots ,$ $a_{n+1=}f(a_n)$, where f is a continuous function ? If $\lim_{n\to\infty} n_a$=5, what is $f(5)$?

(A) 5^n

(B) 5

(C) 5^{n+1}

(D) 1

Soln $\{a_n\}$ be sequence with

$a_1 = a, a_2 = f(a_1) = f(x)$

$a_3 = f(a_2) = f\ (f(a))$

An+1 = f (a_n) where f is continuous function

$\lim_{n \to \infty}$ =5, f (5) = 5

Ans. B

36. If $f(x) = \frac{1}{3}x^3 + cx^2ax + 5$ has a local minimum value at x=1, then which one of the following true about the possible value of $and\ c$?

$(A)\ a = 3, c = -2$

$(B) a = -2c - 1, c < -1$

$(C) a = -2c - 1, c > -1$

$(D) a = -2c - 1, c$ any real number.

F(x) = $\frac{1}{3}x^3 + Cx^2 + ax + 5$

F'(x) = $x^2 + 2cx + a$

F'(1) = 0 ⇨ 1+2c+a=0

A = -2 c-1

If < > -1, f (x) has minimum value at x = 1

Ans. C

37. What is the value of $\int \frac{1}{x}(\ell nx + x^2e^{-x})$dx?

(A)$\frac{1}{2}\ \ell n^2x - (x+1)e^{-x} + C$

(B)$\frac{1}{2}\ \ell n^2x + (2-x)e^{-x} + C$

(C)$\frac{1}{2}\ x^2\ell nx + (2-x)e^{-x} + C$

(D)$\frac{1}{2}\ x^2\ell nx - (x-1)e^{-x} + C$

$\int \frac{1}{x}$(inx + $x^2\ e^{-x}$) dx =$\int(\frac{inx}{x}$ + x $xe^{-x})dx$

= $\int(\frac{inx\ dx}{x} + \int xe^{-x}$ dx by parts

$let\ u = /nx \quad let\ u = x, du = dx$

$= \ du = \frac{1}{x\ dx} \quad dv = \ e^{-x}\ v = \ -e^{-x}$

$dx = x\ du$

= $\int \frac{u}{x}$ x du + uv - $\int vdv$

=$\int udu + x\ (-e^x\ du)$

=$\frac{u^2}{2} + c_1 - xe^{-x} - e^{-x} + C_2$

= $\frac{(/nx)^2}{2} - e^{-x}\ (x+1) + C$

Ans. A

38. Let $f(x)=\frac{3x+1}{x-2}$, then what is the range of $f(x)$?

(A) $\mathbb{R} \setminus \{2\}$

(B) $\mathbb{R}$

(C) $\mathbb{R} \setminus \left\{-\frac{1}{3}\right\}$

(D) $\mathbb{R} \setminus \{3\}$

F(x) = $\frac{3x+1}{x-2}$ Domain = R /{2} *and this graph has horzontal asgmptat at*

$9 = 3$ *and the graph never class this horixantal asymptot*

⇨Range = R /{3}

<u>Ans</u>. D

39. Which one of the following is true about the graph of $(x)\frac{2x^3 2x^2\ 3x}{x^2+x}$?

(A) The graph has y-intercept at (0,3)

(B) The vertical asymptotes of the graph are at x=0 and x=-1 and its oblique asymptote is y=2x.

(C) The graph has at least one x- intercept.

(D) The vertical asymptotes of the graph are at x=0 and x=-1 but it has no horizontal asymptote.

F(x) = $\frac{2x^2+2x^2+3x}{x^2+x} = \frac{x(2x^2+2\,x+3)}{x\,(x+1)}$

$= \frac{2x^2+2\,x+3)}{x\,+1)}$

The graph has vertical asymptote at x = -1 and x= 0 is a hole. This graph as also oblique asymptote we can fid by long division

x^2 x 2x

$2x^3 + 2x^2 + 3x$

$2x^3 + 2x^2 + 3x$

⇨ y = 2x is oblique the others are not correct

<u>Ans</u>. B

40. A private college has 1000 students. 60 % of these students are males. 45% of these students pay their payment by credit card including 175 females. What is the probability that the student is a male or a credit card user?

(A) 0.675

(B) 0.225

(C) 0.775

(D) 0.325

Let m be male students and c be students that uses card

P(m) = $\frac{600}{100}, p(c) = \frac{450}{100}, p(mnc) = \frac{275}{100}\ (450 - 175)$

P(muc) = p(m) + p(c) – p(mnc)

= $\frac{600}{100} + \frac{450}{100} - \frac{275}{100} = 0.775$

Ans. C

41. If $z = \left(1 + \sqrt{3}\, i\right)(1 + i)$, then which one of the following is the polar representation of z ?

(A) $z = 2\sqrt{2}\ \left(\cos(105^0) i \sin(105^0)\right)$

(B) $z = 2\sqrt{2}\ \left(\cos(15^0) + i \sin(15^0)\right)$

(C) $z = 4\left(\cos(105^0) + i \sin(105^0)\right)$

(D) $z = 4\left(\cos(75^0) + i \sin(75^0)\right)$

Z = $(1+\sqrt{3i})\ (1+i)$ ⇨ $z = (1 + i + \sqrt{3i} - \sqrt{3})$

$z = 1 - \sqrt{3} + \left(1 + \sqrt{3}\right)i$

R = $\sqrt{(1 - \sqrt{3})^2 + (1 + \sqrt{3})^2}$

R = $\sqrt{1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3} + 3}$

R = $\sqrt{8} = 2\sqrt{2}$

$\theta = +an^{-1}\ (\frac{1+\sqrt{3}}{1-\sqrt{3}})$

$\theta\ is\ on\ the\ 2nd\ quadrant\ and\ \theta = 150^0$

Z = $2\sqrt{2}$ (cos (150^0) + I sin (150^0) is polar representation

Ans. A

42. What is the sum of the series $\sum_{n=1}^{\infty} 3^n\, 4^{-n}$?

(A) ∞

(B) 3/16

(C) 4

(D) 3

$$if\ n = 1, \quad \sum_{n-1}^{\infty} 3^n.4^{-n}\ is\ \frac{3}{4}$$

$$\sum_{n=1}^{\infty} 3^n.4^{-n}\ i \qquad if\ n = 2, \sum_{n-1}^{\infty} 3^n.4^{-n} = \frac{9}{16}$$

$$if\ n = 3, \quad \sum_{n-1}^{\infty} 3^n.4^{-n} = \frac{27}{64}$$

It is geometric progration with G1 = $\frac{3}{4}$ and r = $\frac{3}{4}$

$$\Rightarrow \sum_{n-1}^{\infty} 3^n.4^{-n} = 5_n = \frac{G1}{1-r} = \frac{\frac{3}{4}}{1-\frac{3}{4}} = 4$$

Ans. C

43. Which one of the following is equal to $\frac{d}{dx}\log_2 \sqrt{6x}$?

(A) $\frac{3x}{2\ell n(2)}$

(B) $\frac{1}{2x\ell n(2)}$

(C) $\frac{3}{2x\ell n(2)}$

(D) $\frac{1}{6x\ell n(2)}$

$$\frac{d}{dx}(log_{2\sqrt{6x}}) = \frac{1}{\sqrt{6x\ell n2}}, (\sqrt{6}) = \frac{1}{\sqrt{6x\ell n2}}, \frac{1}{2\sqrt{6x}}.6 = \frac{6}{2x6x\ell n2} = \frac{1}{2x\ell n2}$$

Ans. B

44. Let $f(x)$ ln($x\sqrt{x}$. Then what is $f'(x)$ equal to ?

(A) $^{2x}/_{3}$

(B) $^{\sqrt{x}}/_{2}$

(C) $^{3}/$

F(x) = ln ($x\sqrt{x}$) ⇨ f'(x) $= \frac{1}{x\sqrt{x}}.(x\sqrt{x^{2'}dx} = \frac{1}{x\sqrt{x}}, \left(x\frac{3}{2}\right)dx = \frac{1}{x\sqrt{x}}\frac{2}{3}x^{\frac{1}{2}}$

45. What is the maximum possible area of a rectangle in square units with diagonal of length 16 units ?

(A) 128

(B) 64

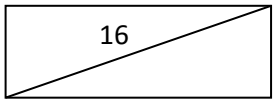
(C) 48

(D) 256

Soln area = x y

$x^2 + Y^2 = (16)^2 = 256$

Y= $\sqrt{256} - x^2$



Area: A (x)= x$\sqrt{\sqrt{256}x^2} = \sqrt{256x^2 - x^4}$

Let 9(x) = $256x^2 - x^4$ *and* $9'(x) = 512\,x - 4x^3$

9'(x) =⇨512x - $4x^3$=0

4x (128-x^2)=0

X=0 or x$\sqrt{128}$ = 8$\sqrt{8^2}$

The maximum area = $(8\sqrt{2})(8\sqrt{2}) = 128$=

<u>Ans</u>. A

46. Let $U = \mathbb{N}$ (the set of natural numbers) be the universe. Which one of the following proposition is true?

(A) $(\exists x)(x+= x)$

(B) $(\forall x)(\exists y)(x - y = x)$

(C) $(\forall x)(\exists y)(y < x)$

(D) $(\forall x)(\exists y)(x \div y = y \div x$

Let u =N The true pro position is

(Ux) ($\exists y$) (x $\div\, y \div x$)

(Ux) ($\exists y$) ($\frac{x}{y} = \frac{y}{x}$)

47. What are the values of a and b so that the function $f(x)\begin{cases} x+1 & x<1 \\ ax+b, & 1\le x<2 \\ 3x, & x\ge 2 \end{cases}$

is continuous every where ?

(A) $a=4, b=2$

(B) $a=4, b=-2$

(C) $a=-4, b=-2$

(D) $a=-4, b=2$

F(x) = $\begin{cases} x+1, x<1 \\ ax+b, 1\le x<2 \\ 3x, x\ge ,2 \end{cases}$

F(x) is continuous at x = 1

⇨ $\lim\limits_{x\to 2}(x+1)=f(1)$ ⇨ $\begin{matrix} Lim\ x+1=ax+b \\ x\to 1\ 2=a+b \end{matrix}$

F(x) is continuous on x = 2

⇨ $\lim\limits_{x\to 2}(a+b)=f(2)$ ⇨ $\begin{matrix} Lim\ ax+b=3x \\ 2a+b=b\ldots.2 \end{matrix}$

Solve 1 and simulation eagle

$\begin{cases} a+b=2 \\ \underline{2a+b=6} \\ -a=-4 \end{cases}$ ⇨ $\begin{matrix} a=4 \\ a+b=2 \\ 4+b=2 \end{matrix}$

B = -2

F(x) is continuous when a = 4 and 6 = -2

Ans. B

48. A cylindrical tank whose inner diameter is 2m contains 4π m^3 oil. If the oil is discharged from the tank at the rate of $\frac{2\pi}{3}$ m^3/ min, then how long (in min) does it take for the tank to be empty?

(A) $\frac{4}{3}$

(B) 4

(C) 6

(D) 12

√ = Abh = πv^2 since it is cylinlerical tank given diameter = 2m and radius = 1m volu
m = 4^3

$\frac{dv}{dt} = \frac{2\pi}{2} m^3/min$

V = πr^2h

$4\pi m^3 = \pi(1)^2_m 2h$

H = 4m = distance where 5 vt

$\frac{dv}{dt} = \sqrt{} = \frac{2}{3}\ m/min$

⇨t = $\frac{5}{v}$

= $\frac{4m}{\frac{2}{3}m/m} = 6\,\text{min} = t$

Ans. C

49. Let $f(x) = x - x^2$ and $g(x) = \frac{1}{x}$. Then what is $\left(f(\frac{1}{x})\right)$ equals to

(A) $\frac{x^2}{x-1}$

(B) $x - x^2$

(C) $\frac{1}{x^2-x}$

(D) $\frac{x-1}{x^2}$

F(x) = x-x^2, $9(x) = \frac{1}{x}$

$= 9(\frac{1}{x} - \frac{1}{x^2})$

9(f($\frac{1}{x}$)) $= 9\frac{x-1}{x^2}$

$= \frac{1}{\frac{x-1}{x^2}} = \frac{x^2}{x-1}$

Ans. D

50. variance of 20 observations is 5. If each observation is multiplied by 2, the what is the variance of the resulting observations?

(A) 5

(B) 10

(C) 40

(D) 20

If x is the variance of x, y, z and t then the variance 2x, 29, 2z and 2z is $2^2\ z$

⇨The new variance = $2^2 x5 = 20$

Ans. D

51. If $f(x) = klnx + e^{sinx}$ and $f''(\pi) = -1$, then what is the value of k?

(A) π

(B) $2\pi^2$

(C) π^2

F(x)= Kinx +e^{sinx}

F'(x) = $\frac{k}{x} + \cos x\ e^{sinx} dx$

F''(x) = $\frac{k}{x^2} - sinx\ e^{sinx} + \cos x e^{sinx} cosx$

= $\frac{k}{x^2} - sinx\ e^{sinx} + cos2_{xe^{sinx}}$

F'' (π) = -1 $\frac{-k}{\pi^2} - 0 + 1 = -1$

$= \frac{-k}{\pi^2} = -2$

$= k = 2\pi^2$

Ans. C

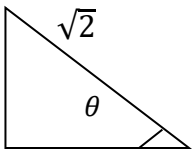
52. If θ is a fourth quadrant angle and $\sec\theta = \sqrt{2}$ then what is $\csc\theta$ equal to ?

(A) $^1/_{\sqrt{2}}$

(B) $^{-1}/_{\sqrt{2}}$

(C) $-\sqrt{2}$

(D) $\sqrt{2}$



$\sec\theta\ \frac{hyp}{ads} = \sqrt{2}$

⇨$\sec\theta = \frac{hyp}{app} = \sqrt{2}\ bat\ it\ is$

Quadrant 4 ($\sec\theta = negative$) $\sec\theta = -\sqrt{2}$

Ans. C

53. Consider the following assertion:

$p + 2''$ is an odd number for any prime p and any $n \in \mathbb{N}$.

Which one of the following is correct about a prove or disprove of the assertion?

(A) There is a counter example that disproves the assertion.

(B) The assertion can be proved by direct method; because p is odd and $2^n = 2(2^{n-1})$ is even imply that $p + 2^n$ is odd since the sun of odd and even is odd.

(C) The assertion can be proved by the indirect method; because if $n \notin \mathbb{N}$,then $2^n \notin \mathbb{N}$ and hence $p + 2^n$ is not odd.

(D) The assertion can be disproved by the method of contradiction.

Solve. p+2^n is an add number

p+ 2^n*isadd numbe for any prime p and only nEN The asser tian can be proved by indirect proo of becouse there is a* counter example that disprove the assertion p= 2 Is only even proem Number p+$\underline{2^n \textit{ is even.}}$

Ans. C

54. What is the standard equation of the line passing through the point (2,3) and parallel to the line given by $\begin{cases} x = 1 + 2\lambda \\ y = -2 - \lambda \end{cases}$, $\lambda \in \mathbb{R}$?

(A) $\frac{x-2}{-1} = \frac{y-3}{2}$

(B) $\frac{x-1}{1} = \frac{y-3}{-2}$

(C) $\frac{x-1}{2} = \frac{y-3}{-2}$

(D) $\frac{x-2}{2} = \frac{y-3}{-1}$

Solve The standard equation

(2, 3) parallel to $\begin{cases} x = 1 + 2\lambda \\ y = -2 - \lambda \end{cases}$

Use water equation of the line x = x0+ λv

$$\lambda = \frac{x-1}{2} = \frac{y+2}{-1} \; and \; parallel \; vator \; v = (1_2 - 1)$$

$$r = ro + \lambda v \Rightarrow r = (x, y) = (3,3) + \lambda \quad (2, -1)$$

$\therefore x = 2 + 2\lambda \quad and \; 9 = 3 - \lambda$

$\lambda = \frac{x-2}{2}$ and $\lambda = \frac{y-3}{-1}$

Ans. D

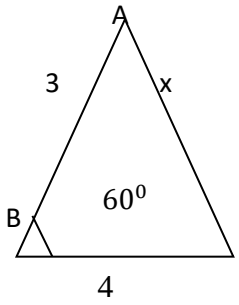
55. If in $\Delta ABC, AB = 3, BC = 4\ and\ m(< B) = 60^0$, then what are the length of AC and the cosine of $<A$, respectively?

(A) $\sqrt{13}$ and $\frac{6}{5\sqrt{13}}$

(B) $\sqrt{13}$ and $\frac{1}{\sqrt{13}}$

(C) $\sqrt{13}$ and $\frac{-1}{\sqrt{13}}$

(D) $\sqrt{13}$ and $\frac{-6}{5\sqrt{13}}$



Using cosine low

$x^2 = 3^2 + 4^2 - 2\ (3x4)cosB$

= 9+16-24 (cos 60^0)

= 25-29 $(\frac{1}{2})$

x^2= 25-12 =13

X = $\sqrt{13}$

And using sin law

$$\frac{4}{\sin A} = \frac{\sqrt{13}}{sin60^0}$$

$$\frac{4}{Sin\ A} = \frac{\sqrt{13}}{\sqrt{\frac{3}{2}}}$$

$\sqrt{13}\ sinA = (\frac{\sqrt{3}}{2})$

$\sin A = \frac{2\sqrt{3}}{\sqrt{13}}$

bat $\sin 2_\theta$ + $\cos 2_\theta$

$\cos 2_\theta$ =- $\sin 2_\theta$

$=1 - (\frac{2\sqrt{3}}{\sqrt{13}})$

$= 1 - \frac{12}{13} = \frac{1}{13}$

Cos = $\frac{1}{\sqrt{13}}$

<u>Ans</u>. B

56. Let $p(n)$ be an open proposition on the set of natural numbers (ℕ). Which one of the following is a correct application of the principle of mathematical induction?

(A) If $P(1)$ is true; for $n = 1$; and if both $P(n)$and $P(n + 1)$are true for certain $n \in \mathbb{N}$, then $P(n)$is true for all $n \in \mathbb{N}$.

(B) If $P(10)$ is true; and if $P(n)is$ true for implies that $P(n + 1)$ is true, then $P(n)$is true for all $n \in \mathbb{N}$.

(C) If $P(10)$ is true; and assuming $P(n) is$ true for any $n > 10$ if it follows that $P(n+1)$is true , then $P(n)$ is true for all $n \geq 10$.

(D) If $P(1) is$ true; and $P(n) \Longrightarrow P(n+1)$ is true for any $n \in \mathbb{N}$, then $P(n)$ is true for all $n \in \mathbb{N}$.

If P(1) is true for n = 1

If P(n) is true for n ∈ ℕ and P (n +1) is true for n+1 ∈ ℕ. $Then\ P(n) is\ true\ for\ n \in \mathbb{N}$

Ans. A

57. If the image of the line 2x-3y=7 under a translation is 2x-3y=0, which one of the following is a translation vector of the translation line?

(A) $u = (-2,1)$

(B) $u = (1,-2)$

(C) $u = (-1,2)$

(D) $u = (2,-1)$

2x-3y=7 $\xrightarrow{v\,(a,b)}$ $2x - 39 = 0$

⇨P(x, y) $\xrightarrow{v\,(a,b)}$ (x', y')

X + a = x' ⇨ x = (x'-y)

Y + b = y' ⇨ y = (y'-b)

$\therefore 2(x-a) - 3\,(y-b) = 7 \rightarrow 2x - 3y = 0$

2x -2a -3y+3b = $\rightarrow 2x - 3y = 0$

2x-3y= 7+2a-3b

⇨$\frac{7+2a-3b=0}{2a-3b=-7}$ ⇨$a = -\frac{7+3b}{2}$

Ans. A

58. Let P= $(1, a, a)$ and Q=$(a-1,1,1)$ be points in space and the distance between P and Q is 3. Then what is the value (S) of a? (M)

(A) $a = -3, a = {}^{1}/_{3}$

(B) $a = 1, a = -9$

(C) $a = -1, a = 9$

(D) $a = -3, a = {}^{-1}/_{3}$

D = $\sqrt{(a-1)-1)^2} + a - 1^2 +)a-1)^2$

d^2 =(a-2)$^2 + a^2 - 2a + 1 + a^2 - 2a + 1$

⇨$a^2 - 4a + 4 + a^2 - 2a + 1 + a^2 - 2a + 1 = 9$

⇨$3a^2 - 8a - 3 = 0$

⇨$3a^2 - 9a + a - 3 = 0$

⇨3a(a-3)+1(a-3) =3

3a+1=0 or a-3=0

A= -$\frac{1}{3}$ $or\ a = 3$

<u>Ans</u>. D

59. What is the image of the circle $x^2 + y^2 - 4x - 6y + 12 = 0$ when it is reflected with respect to the line $y = -x$?

(A) $(x-2)^2 + (y-3)^2 = 1$

(B) $(x+3)^2 + (y+2)^2 = 1$

(C) $(x+2)^2 + (y+3)^2 = 1$

(D) $(x-3)^2 + (y-2)^2 = 1$

Soln. The image of circle $x^2 - 4x + y^2 - 6y + 12 = 0$

$$x^2 - 4x + 4 + y^2 - 6y + a = 4 + 9 + 12$$

The center of circle is (1,3)

(2,3) $\xrightarrow{reqlected\ y=-x}$ (-3,-2)

$\therefore image\ (x+3)^2$ +(y+2)^2 =1

<u>Ans</u>. B

60. If the dot products of a vector $\vec{A}$ with the vectors $i - j + k, 2i + j - 3k\ and\ i + j + k\ are$ 4,0 and 2, respectively, what is $\vec{A}$?

(A) $\vec{A} = (2,1,1)$

(B) $\vec{A} = (-2,1,-1)$

(C) $\vec{A} = (2,-1,1)$

(D) $\vec{A} = (-2,-1,1)$

ASoln. let u = i-j+ki u = 2i+j-3kiw = i+5+k

Let A = xi+yi+2k

A.u = x-y+z=4…..1

A.u = u ⇨2x +y -3z = 0…..2

A.w = 2⇨ x + y + z = 2…….3

From 1 and 2 and 1 and 3 respectively

$\begin{cases} 3x - 2k = 4 \Rightarrow 5x = 10 \\ 2x + 2z = 6 \Rightarrow x = 2 \end{cases}$

Z = 3 –x ⇨z = 3-2 =1

Y = x+z -4

Y = 2+1 -4 = -1

Vector A is (x,y,z) = (2,-1,1)

<u>Ans</u>. C

61. In order to measure the height a of a tower, suppose a surveyor takes two sighting from a transit 1 meter high which are positioned d meters apart on the same ground level as in the figure below.

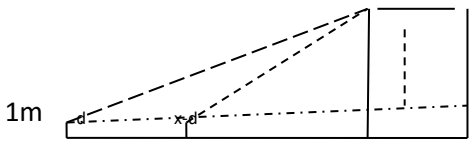
If the first measured angle of elevation is a and the second is β(see, the figure), then what is the height of the tower (in meter) in terms of $a, \beta \; and \; d$?

(A) $\left(\frac{\tan a \; tan \; \beta}{\tan a - tan\beta}\right) d + 1$

(B) $\frac{\tan a \; tan\beta}{(\tan a - \tan \beta) d}$+1

(C) $\left(\frac{\tan a - tan \; \beta}{\tan a \; tan\beta}\right) d + 1$

(D) $\left(\frac{\tan a \;\; tan \; \beta}{\tan a + tan\beta}\right) d + 1$



Let = h1 +1m

Hbe the hight of twer x be d/c b/n the end servey or sighting

Tana =$\frac{h}{x-d}$ ⇨$h1 = (x - d) + an1$ … … 1

tanB= − ⇨$h1 = x \tan B$ … … 2

from 1 and 2, x tanB = xtana –tana

⇨x tan B+ d tana = x tan θ

X(tana-tabB = dtaba ⇨x =$\frac{dtana}{tana - tanB}$

H = hr +1m $\frac{dtaba}{taba - tanB}$.tanB+1

H= ($\frac{tana - tanB}{tana - tanB}$) d+1

<u>Ans</u>. A

62. Let l be the line given by the vector equation $(x, y) = (-2,1) + \lambda(1,1), \lambda \in \mathbb{R}$.which one of the following is the equation of the image of l after being translated by the vector $u = (2, -1)$ followed by a rotation through 45^0 about the origin?

(A) $y = -2\sqrt{2x}$

(B) $y = \sqrt{2}$

(C) $y = \sqrt{2x}$

(D) $x = 0$

Soln. vector equation (x,y) = (-2,1) + λ (1,1), λER

X + 2 = x y-1 ⇨y =x +1

Y = x+3 $\xrightarrow{translated\ by\ (2,-1)}$ y =x take (1.1) be a

Point on y = x, then (1,1) $\xrightarrow{0=45^0}$

X' = x cosθ –y sin θ

X' =1 cosθ 45^0 -1 sinθ

Y' = x sin$\theta + y$ sinθ

Y' = 1sinθ $45^0 + 1\ cos45^2 = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\therefore The\ image\ is\ y = \sqrt{2}$

Ans. B

63. What is the period (P) and the rang of $f(x) = 5\ sin\left(\frac{1}{3}x + 2\right) + 3$?

(A) $P = 6\pi$, range=$[-5,5]$

(B) $P = 6\pi$, range=$[-2,8]$

(C) $P = \frac{2\pi}{3}$, range=$[-5,5]$

(D) $P = \frac{2\pi}{3}$, range=$[-2,8]$

Soln. f(x) = 5 sin x ($\frac{1}{3}x + 2$)+3

Period = $\frac{2\pi}{1/3} = 6\pi$

Range = [-a + c, a+c] = {-5+3, 5+ 3] = (-2,8)

The period and range of f is $6\pi\ and\ [-2\ ,8] respectivly$

Ans. B

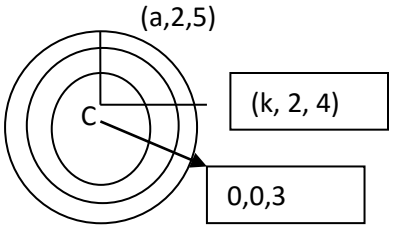
64. Let A$(a, 2,5), for\ a > 0,$ be a point on the sphere $x^2 + y^2 + z^2 + 6z = 0$ and C be the center of the sphere. If $P(k, 2,4)$ is a point in space such that $\overrightarrow{PA}$ is perpendicular to $\overrightarrow{CA}$, what is the cosine of the angle between $\overrightarrow{PA}$ and $\overrightarrow{PC}$?

(A) $\frac{5}{\sqrt{35}}$

(B) $\frac{7}{\sqrt{35}}$

(C) $\frac{5}{\sqrt{70}}$

(D) $\frac{7}{}$



Soln.

A= (a, 2, 5) for a>0 be point on the sphere $x^2 + y^2 + z^2 - 6z0$

⇨$x^2 + y^2 + z^2 - 6z + 9 = 9$ ⇨$x^2 + y^2 + (z-3)^2 = 9\ and\ r = 3$

C be center of the sphere i.e (0,0,3)

P (K, 2,4) is a point the sphere

$\overrightarrow{PA}1\ \overrightarrow{CA}, \overrightarrow{CA} = (1, 2, 3)$

R= $\overrightarrow{\ \ } = \sqrt{a^2 + 2^2 + 2^2} = 3$

$a^2 + 4 + 4 = 9$⇨$a^2 - 1$⇨$a = 1$

$\overline{PA} = (1 - k, 0,1)$

$\overline{PA}, \overline{CA}$ =u ⇨1-k + 0 +2 = 0 ⇨ k =3

Cosθ = $\frac{\overline{PA}, \overline{CP}}{|PA||CP|}$

$\overline{PC}$ (-3, -2, -1) $|\overline{CP}| = \sqrt{3^2 + 2^2 + 1^2} = \sqrt{14}$

$\overline{PA}$ (-2, 0, 1) $|\overline{PA}| = \sqrt{4 + 0 + 1} = \sqrt{5}$

cos$\theta = \frac{\overline{PA}, \overline{CP}}{|PA||CP|} = \frac{6+0+(-1)}{\sqrt{5}\sqrt{14}} = \frac{5}{\sqrt{5}\sqrt{14}} = \frac{5}{\sqrt{70}}$

Ans. C

65. What is the value of $\lim_{n\to\infty} \sum_{i=0}^{n} \frac{2}{n}\left[\left(\frac{2k}{n}\right)^2 + 5\left(\frac{2k}{n}\right)\right]$

(A) 14

(B) 10

(C) 4

(D) 18

Soln. $\lim_{n \to \infty} \sum_{k=1}^{n} \frac{2}{n} [(\frac{2k}{n})^3 + 5 \left(\frac{2k}{n}\right)]$

$$\lim_{n \to \infty} \frac{16}{n^4}\sum_{k=1}^{n} k^3 + \frac{20}{n^2}\sum_{k-1}^{\infty} k$$

$$\lim_{n \to \infty} \frac{16}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) + \lim \frac{20(n(n+1)}{2n^2}$$

$\frac{16}{4} \lim_{n \to \infty} \left(\frac{n^u+2n^2+n^2}{n4}\right) + \frac{20}{2} \lim_{n \to \infty} \frac{n^2+n}{n^2}$ = 4+10 =14

Ans. A