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CSIR-JRF/NET (June. 2014)

MATHEMATICAL SCIENCE

Subject Code – 4 **Booklet Code – B**

2014 (I)

TEST BOOKLET

(22 June. 2014)

Time Allowed: Three Hours **Maximum Marks: 200**

INSTRUCTIONS

1. **You have opted for English as medium of Question Paper.** This Test Booklet contains one hundred and twenty (20 Part 'A' + 40 Part 'B' + 60 Part 'C') Multiple Choice Questions (MCQs). You are required to answer a maximum of 15, 25 and 20 questions from part 'A', 'B' and 'C' respectively. If more than required number of questions are answered, only first 15, 25 and 20 questions in Part 'A', 'B' and 'C' respectively, will be taken up for evaluation.
2. Answer sheet has been provided separately. Before you start filling up your particulars, please ensure that the booklet contains requisite number of pages and that these are not torn or mutilated. If it is so, you may request the invigilator to change the booklet. Likewise, check the answer sheet also. Sheets for rough work have been appended to the test booklet.
3. Write your Roll No., Name, Your address and Serial Number and this Test Booklet on the Answer sheet in the space provided on the side 1 of Answer sheet. Also put your signatures in the space identified.

Used theorem :-

Let $a, b \in \mathbb{Q}$. Such $\sqrt{a} \notin \mathbb{Q}$ and $\sqrt{b} \notin \mathbb{Q}$ prove that If $\sqrt{a} \in \mathbb{Q}(\sqrt{b})$, then $a = bc^2$ for some $c \in \mathbb{Q}$.

Now $\sqrt{2} \in \mathbb{Q}(\sqrt{-1}) = \mathbb{Q}(i)$

$$2 = \frac{(\sqrt{2})^2}{(\sqrt{-1})^2} \times (\sqrt{-1})^2$$

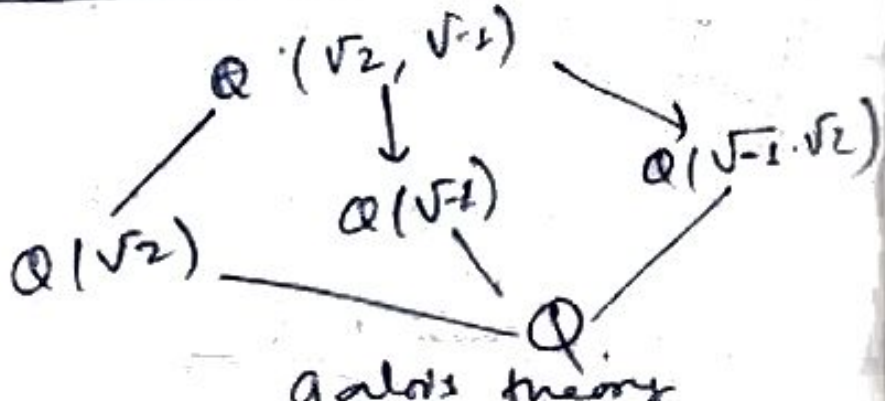
$$2 = \left(\frac{\sqrt{2}}{i}\right)^2 \cdot i^2 = b \cdot c^2$$

But $(\sqrt{2}/i) \notin \mathbb{Q}$

Hence it contradicts.

$\mathbb{Q}(\sqrt{2}) \neq \mathbb{Q}(i)$

Option (c) and (d) are true by Galois Extension theory



Thank U ~ Sonia gupta

Q88

Note: $F_n = 2^{2^n} + 1$ and $G_m = 2^{2^n} - 1$.

check at $n = 2, m = 1$

check at $n = 1, m = 2$

option (a) is true. $5 \mid 15$ $m > n$.

(b) is false ($\gcd(5, 15) \neq 1$)

(c) c is true.

take $n = 2, m = 1$.

$F_n = 2^{2^n} + 1 = 17$

$F_m = 2^{2^m} + 1 = 5$

$\gcd(17, 5) = 1$

(d) it is false.

Q89 → part A question

Q90 (a) $\dim(\mathbb{Q}(\sqrt{2})) = 2 = \dim(\mathbb{Q}(i))$

$\dim(\mathbb{Q}(\sqrt{2}) = \dim \mathbb{Q}(\sqrt{-1})$

$2 = 2$

option (a) is correct.

(b) option is false

For option (d) is true

If G is a finite subgroups.

for every $R \in G$, $|R| = 1$.

since If $|R| \neq 1, 1 \ldots R^n \ldots n \in N$

is infinite.

Let N be the order of G, $R^N = 1$.

This implies that $R^{Nic} = 1$

where $c = P\left(\frac{2\pi}{N}\right)$

we conclude that this a subgroup generated by $e^{\frac{2\pi i}{N}}$, and G is cyclics

Since G is a finite subgroup cyclics.

so every finite subgroup of G is cyclics.

Ans - (1) and (4)

For option (a) Pf. R is a subgroup of G_1 containing an element a primitive 2^n-th roots of unity, then every 2^{nm} roots of unity is in R

So Pf R contains a primitive 2^n-th roots of unity for large $N > n$, then $G_1 = R$

Otherwise take n to be maximal and R is simply the multiplicative group of 2^n th root of unity which is finite

Also option (a) is Correct

Q86.

Option (a) If $f(a) \neq 0$ obviously no — where vanishing.

True.

Option (b) True by Weierstrass factorization theorem

Option (c) Identity theorem

If f and g are analytic such that

$fg = 0$ on a domain.

then $f \equiv 0$ or $g \equiv 0$ on some domain.

We can claim that it is integral domain.

This show that H must has an element of order 8 and it is cyclic i.e Z_8 49 - cont

Direct formula

$$\left(\frac{F}{\{0\}}, \cdot\right) \cong \left(GF(p^n)\right) = Z_{p^n-1}.$$

$$= Z_{3^2-1}.$$

$$= Z_{9-1} \cong Z_8$$

for option (4) from option (1), (2) and (3) it is correct.

or take $A = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ on Z_3

$F_9 = \{0, I, A, A^2, A^3, A^4, A^5, A^6, A^7\}$

9 elements

$$A^2 = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 0+1 & 1+4 \\ 0+2 & 1+4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 5 \\ 2 & 5 \end{pmatrix}$$

$o(A^2) = o(A) = 2$

so, $H = \frac{Z}{3Z} \times \frac{Z}{3Z} = Z_3 \times Z_3$

$H \cong Z_8$

Q85

option (a) is true

Solution:

F is field and FF is the splitting field of $x^{3^2} - x$ on Z_3.

$$(x^3)^2 - x$$
$$= (x^3 - x)(x^3 + x)$$
$$\downarrow \qquad \downarrow$$
$$Z_3 \times Z_3 \approx G$$

$\text{char}(G) = 3$ (prime)

option (b) is false $\frac{Z}{3Z} \cong Z_3$

$\text{char}(G) = 9$

Characteristic of field must be either 0 or prime number.

so it is not a field. → Contradicts

option (c) $H = \left\{ \frac{F}{\text{Zero}}, \cdot \right) = (F - \text{Zero}, \cdot)$ (cyclic)

~~H has~~ You can take 1, 2, 4 or 8 as an element.

In H any element has order 1, 2, 4 or 8.

Now let us assume that there is no elements of order 8, then all element of H, satisfy

$x^4 = 1, \quad 1^1 = 1$

$2^4 = 16 = 2 \times 8$

$4^4 = 2^8 (2^2 \times 2)^4$

$= 2^3 \times 2^5 = 8 \times 2^5$

Above polynomial has 8 roots.

So it contradicts that there is no elements of order 8, as

It must have an elements of order 8.

Now check

$f(x) = x^4 + 3x^3 - 9x^2 + 7x + 27$

$f(1) \neq 0$, $f(-1) \neq 0$

$f(3) \neq 0$, $f(-3) \neq 0$

$f(\pm 9) \neq 0$, $f(\pm 27) \neq 0$

so $f(x)$ is irreducible over $\mathbb{Q}$.

Option (4) See C.S.I.R December 2012 (same pattern) Q (88)

$f(x)$ is reducible over Z_3

so we cannot claim that $f(x)$ is irreducible over $\mathbb{Z}$ as $Z_3 \subseteq \mathbb{Z}$

~~also option (4) is not true.~~

~~answer is (a), (b)~~ (a), (b)

Here Root of $\mathbb{Z}$ are $\pm 1, \pm 3, \pm 9, \pm 27$

all are not zero when $f(x)|_{x = \pm 1, \pm 3, \pm 9, \pm 27}$

so, $f(x)$ is irreducible,

answer is option (a), (b) and (d).

Q84 $f(x) = x^4 + 3x^3 - 9x^2 + 7x + 27$ option (1) is correct

Sol: $Z_2 = \{0, 1\}$

$f(0) \neq 0$, $f(1) \neq 0$

so, $f_2(x)$ is irreducible over Z_2

option (2) is not correct.

option (3) $Z_3 = \{0, 1, 2\}$.

$f(2) = 2^4 + 3 \cdot 2^3 - 9 \cdot 2^2 + 7 \cdot 2 + 27$

$= 16 + 3 \cdot 8 - 36 + 14 + 27$

$= 81 - 36$

$= 45$

$f(2) = 0$

$f(2) = 45 \pmod 3$

$f(2) = 0$

$$\begin{array}{r} 15 \\ 3 \overline{)\,45} \\ \underline{3} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

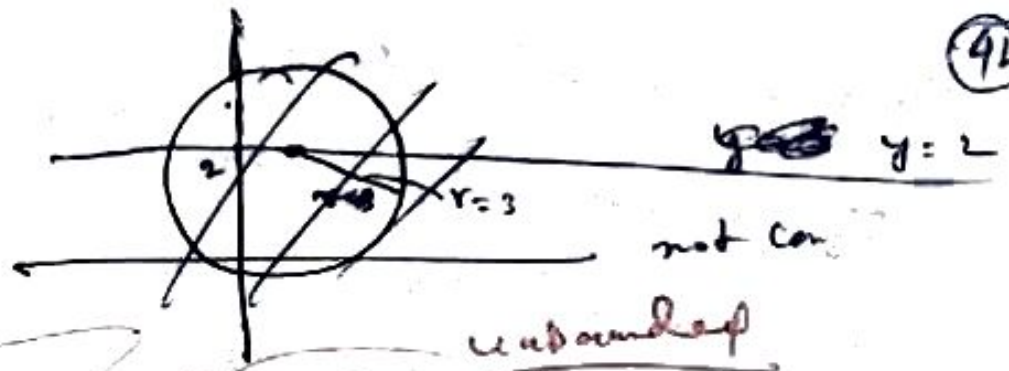
so Z_3 is reducible.

option (4) $f(x) = x^4 + 3x^3 - 9x^2 + 7x + 27$ over $Z[x]$.

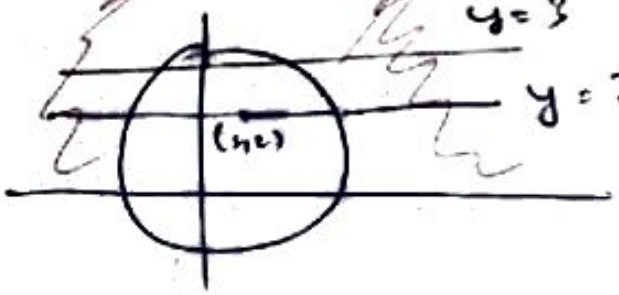
Here Eisenstein is fail.

Q83.

Option (i)



unbounded



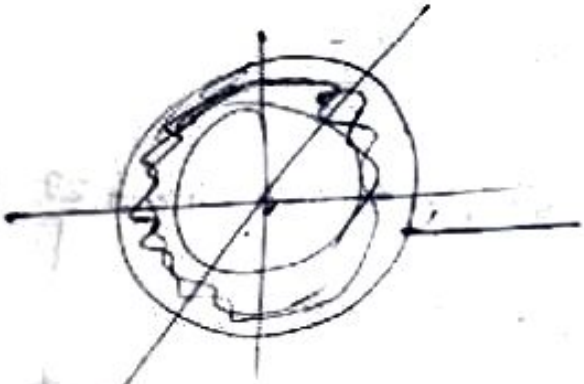
(b) $(0,0)$ is limit point.

so it is closed.

given set is bounded, so it is compact.

(c) is not corrects (hypersolic)

(d)



compacts

Q8)

Soln: $r(f) = f_k$ and ~~$r(g) = f$~~.

$r(g) = g_k$.

By Identity theorem :- $f_k = g_k$ as f and g are entire and have a limit point everywhere.

Now let us consider open ball B_n at centre 0 and radius n, natural number.

then $\mathbb{C} = \cup_{n \in N} B_N$ i.e $K \subseteq \cup_{n \in N} B_N$.

Now if K is countable, then it will contradicts because B_n contains uncountably many complex zero of f, so K must be uncountable.

let us take B_k for some natural number k, Contains uncountably many complex zero of f. since zeros of f has Bounded uncountable subset in B_k. By Bolzano-Weierstrass theorem it has limit point, Now by identity theorem

$f = 0$, i.e if $f_k = g_k \Rightarrow r(f) = r(g)$

By open-mapping theorem option (a) (b) (d) is not done.

Q81 Tricky question (Take $f(z)$ as zero function) (39)

Solution $f_n(z) = \sum_{n=0}^{\infty} c_n (z-a)^n$

a) $f^n(z) = 0$ for infinitely many $n \neq 0$

Take zero function $f(z) = 0$

(b) $f^{2n}(0) = 0$ for ever $n \geq 0$,

$f^0(0) = 0$ but $f^1(0) \neq 0$.

$f^2(0) = 0$ but $f^3(0) \neq 0$

$f^4(0) = 0$ but $f^5(0) \neq 0$

$\vdots$

$f^{2n}(0) = 0$ but $f^{2(n+1)}(0) \neq 0$

so, this option is false.

(c) Same pattern option (2) — false.

(d) Same as option (1) (Correct)

Answer is (1), (4)

Q80

Sol: Sample (By Schwarz lemma)

Take $f(z) = \alpha z$ where $\alpha < 1$.

(See same pattern question C.S.IR June 2018 Q.77.)

Option (1) $f'(z) = \alpha$.

$f'(0) = \frac{1}{2} < 1$ true.

Option (2) $f\left(\frac{1}{3}\right) = \alpha \cdot \frac{1}{3} < 1$. true.

Option (3) $f\left(\frac{1}{3}\right) = \alpha \cdot \frac{1}{3} < 1$. true.

Option (4) $f'(0) = \alpha$

$f'(0) = \sec\pi/6$ ~~sec~~ $(\pi/6)$

$= \frac{1}{\cos\pi/6} = \frac{2}{\sqrt{3}}$

(See C.S.IR June 2018 question also Q.82.)

$= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{2\sqrt{3}}{3} > 1$.

$f'(0) > 1$ It Contradicts that Statement $f'(z) = \alpha < 1$

So answer of option (a), (b) and (c) are correct.

Q79

Soln. for $z \in \mathbb{C}$, $f(z) = \dfrac{e^z}{e^z - 1}$.

option (a) f is entire

at $z = 0$

$f(z) = \dfrac{e^0}{e^0 - 1} = \dfrac{1}{0}$



so, $f(z)$ is not entire.

but $f(z)$ is entire on $\mathbb{C} - \{0\}$

option (b) $f(z) = \dfrac{e^z}{e^z - 1}$

for poles $f(z) \Rightarrow e^z - 1 = 0$

$e^z = 1.$

$e^z = e^{i2n\pi}$, $\forall\ n \in \mathbb{Z}$ (Integer)

$(e^{i2n\pi} = \cos(2n\pi) + i\sin 2n\pi$

$e^{i2n\pi} = 1.)$

$z = i2n\pi$ $\forall\ n \in \mathbb{Z}$

→ infinite.

As poles are living in the y axis.

as $f(z)$ have only one type of singularity and that is called pole.

Here simple meaning order is 1.

$(z-0)^1 = i2n\pi$

Here z has order 1.

option b, c, d are correct.

Q77.

Used simple logic here

(a) Ps $p(x) = a_n x^2 + b_n x + c_n$.

option (1) Ps true. as $\lim_{n \to \infty} p_n(x_0) = A_0$

$\lim_{n \to \infty} p_n(\lambda_0) = A_0$ (Exist)

⋮

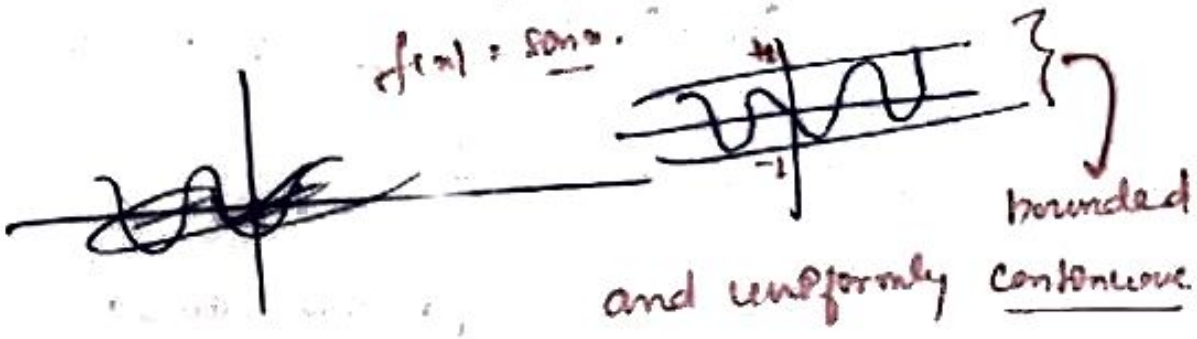
then $\lim_{n \to \infty} p_n\left(\frac{\lambda_0 + \lambda_1 + \lambda_2}{3}\right)$ Ps also exist.

answer option (a) and (b)

Q78.

~~(c)~~ option (a) and (b)

$f(x) = \sin x$.



bounded

and uniformly continuous

Q76.

Sol:- $f(x,y) = (x + 2y + y^2 + |xy|,\ 2x + y + x^2 + |xy|)$

~~Df(x,y) = Jf~~

Here, $f_1(x,y) = x + 2y + y^2 + |xy|$

$f_2(x,y) = 2x + y + x^2 + |xy|$

$$Df(x,y) = \begin{bmatrix} \frac{df_1}{dx} & \frac{df_2}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{bmatrix}$$

Now If $x+y=-1$, then
It doesn't vanish.
i/e option (1) is Correct.

~~D of f~~) $D(f(x,y)) \neq 0$ (Invertible)
one-one & onto.
Infinitely differentiable on A

option (b) and (e) are correct

option (c)

$f(A) = R^2$

Here $f(x,y) = \left(\frac{x}{1+x+y}, \frac{y}{1+x+y}\right)$

$= \left(\frac{x}{1-1}, \frac{y}{1-1}\right)$

$= (\infty, \infty)$ Can doesn't exist.

so It is not Correct.

But $f(A) = R^2 - \{(x+y)=1\}$

~~Is open~~ Then is true.

~~XX option~~ ~~option~~ option ② is true.

option ① is not Correct.

for option ③ and ④

see part B, Question no: 33

as $\{(m,n)\} \in \mathbb{Z}$ is countable.

$R^2 - S$ is uncountable.

Q75.

Sol:

$$f_1(x,y) = \left(\frac{x}{1+x+y}\right), \quad \frac{y}{1+x+y}$$

$$f_2(x,y) = \frac{y}{1+x+y}.$$

$$D(f(x,y)) = \begin{pmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{pmatrix}$$

$$D(f(x,y)) = \begin{pmatrix} \frac{1+y}{(1+x+y)^2} & \frac{-x}{(1+x+y)^2} \\ \frac{-y}{(1+x+y)^2} & \frac{1+x}{(1+x+y)^2} \end{pmatrix}$$

Q73 (Logical question)

Soln: $b_n \uparrow b$ mean b_n decreasing to b as n goes to ∞

$a_n \uparrow a$ ⟶ monotone increasing sequence.

⇒ (a) and (c)

Q74

Soln: $$S = \left\{ \left(m + \frac{1}{2^{|p|}}, n + \frac{1}{2^{|q|}} \right) : m,n \right.$$
$$\left. m, n, p, q \in \mathbb{Z} \right\}$$

Soln ⇒ $\lim_{k \to \infty} S = \lim_{k \to \infty} \left\{ \left(m + \frac{1}{2^k}, n + \frac{1}{2^k} \right) \right\}$

where $k \in \mathbb{Z}$, $m, n \in \mathbb{Z}$

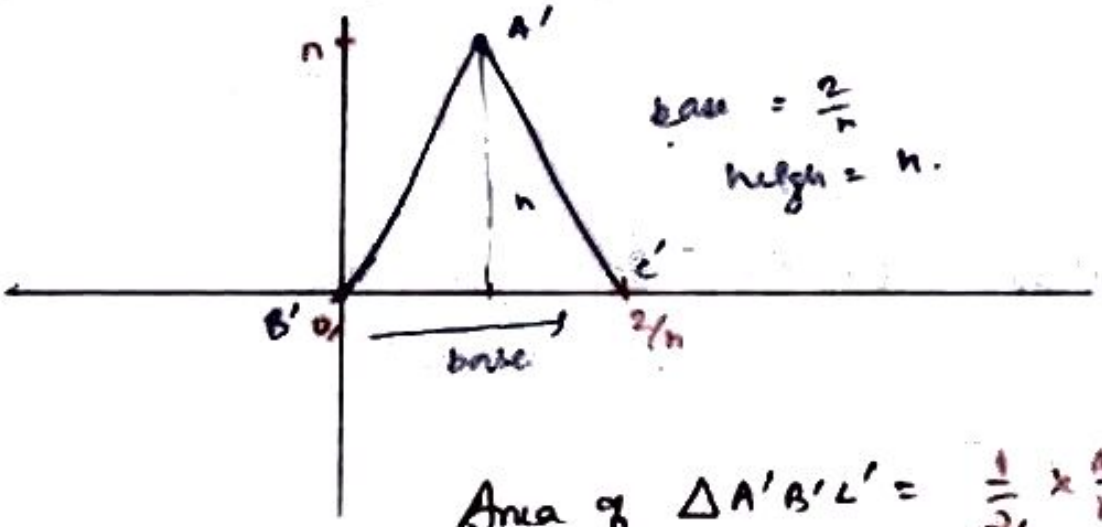
$$S = \{(m, n)\}.$$

$$S = \left\{ m + \frac{1}{\infty}, n + \frac{1}{\infty} \right\}$$

$$S = (m, n)$$

$m, n \in \mathbb{Z}$, take $m = 0$, $n = 0$

$S = (0, 0)$ is not a isolated point, so it can not be discrete.



(30)

Area of $\Delta A'B'C' = \frac{1}{2} \times \frac{2}{n} \times n$

area of $\Delta A'B'C' = 1$

Here $\int_0^\infty f(x)\,dx = 1$ for all n.

So $\lim_{n\to\infty} \int_0^\infty f_n(x)\,dx \neq \int_0^\infty f(x)\,dx$

as $\lim_{n\to\infty} \int_{-\infty}^{\infty} f_n(x)\,dx \neq \int_{-\infty}^{\infty} f(x)\,dx$.

No option (1) is not correct.

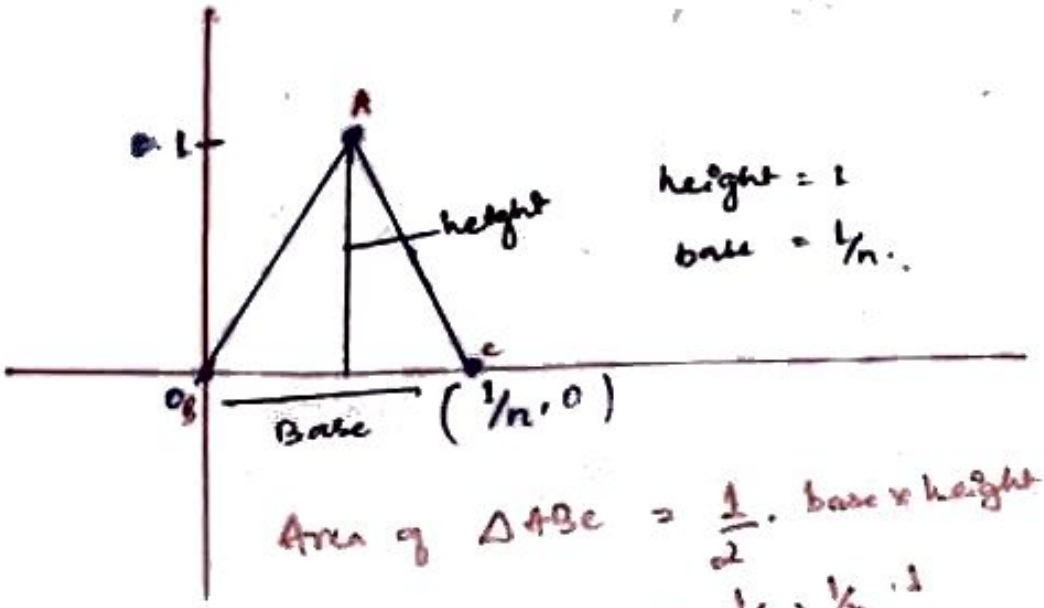
For option (2) and (4) looking similar.

See here :-

Q72

Soln: Take $[0,\infty) \subseteq R$

Here you see R as $[0,\infty)$



height = 1

base = $1/n$.

Area of $\Delta ABC = \frac{1}{2} \cdot \text{base} \times \text{height}$

$= 1/2 \cdot 1/n \cdot 1$

$= \frac{1}{2n}$.

$$\lim_{n\to\infty} \int_0^{\infty} f_n(x)\,dx = \lim_{n\to\infty} \int_0^{\infty} \frac{1}{2n}$$

$$\int_0^{\infty} f_n(x)\,dx = \frac{1}{2n} \quad \text{and} \quad \lim_{n\to\infty} \int_0^{\infty} f_n(x)\,dx$$

$$\lim_{n\to\infty} \frac{1}{2n} \to 0 \text{ as } n \to \infty$$

$$\lim_{n\to\infty} \int_0^{\infty} f_n(x)\,dx = \lim_{n\to\infty} \frac{1}{2n} = \frac{1}{\infty} = 0$$

Now again check at the point (0,0)

$(\frac{1}{n}, n)$ and $(\frac{2}{n}, 0)$

Q7)

Soln $p(x) = a_0 + a_1 x + \cdots + a_n x^n$.

$T(p(x)) = a_0 - a_1 + a_2 x^2 - \cdots + (-1)^n a_n x^n$.

If $p(x) = 1$, $T(1) = 1$.

$p(x) = x$, $T(x) = -x$

$p(x) = x^2$, $T(x^2) = x^2$

$\vdots$

$p(x) = x^n$, $T(x^n) = (-1)^n x^n$.

$$T = \begin{array}{c} \\ 1 \\ x \\ x^2 \\ \vdots \\ x^n \end{array} \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & -1 & 0 & \cdots & 0 \\ & & & & \\ 0 & \cdots & \cdots & \cdots & (-1)^n \end{bmatrix}_{(n+1)\times(n+1)}$$

$\dim P(x) = n+1$.

$\det A = \pm 1$

$$\det A = \begin{cases} -1 & \text{for } n = \text{odd} \\ +1 & n = \text{even}. \end{cases}$$

T is invertible as it is one-one and onto.

option (D) True.

By theorem closed subsets of compacts space is closed.

Q70

⇒ (a) open + open ⟶ is always open.

Infinite union of open set is open.

(b) option (B) is not true.

Take ~~$X = \{-n+1 : n \in \mathbb{N}\}$~~

$X = \{-n + \frac{1}{n} ; n \in \mathbb{N}\}$ and

$Y = \mathbb{N}$

Both x and y are closed but

$X + Y$ has a subset of P

where $P = \{\frac{1}{n} : n \in \mathbb{N}\}$. which has

a limit point at zero but $0 \notin X + Y$.

as $X + Y$ doesnot contain 0. so it

is is not closed.

as $X + Y \supseteq \{-y + \frac{1}{n} + y\}$

$\supseteq \{\frac{1}{n}\}$

$\supseteq 0$ as $n \to \infty$

$P \subseteq X + Y$.

as $n \subseteq \mathbb{N}$

Option (c) True by take X compact and Y compact.

$f(x, y) = x + y$

as continuous image of compact is

Compact ∎

069

Used logic here

$X = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 5\}$. — (1)

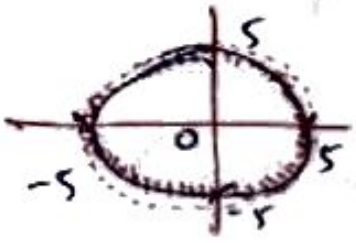
$K = \{(x, y) \in \mathbb{R}^2 : 1 \le x^2 + y^2 \le 2 \rightarrow (2)$ — Compact

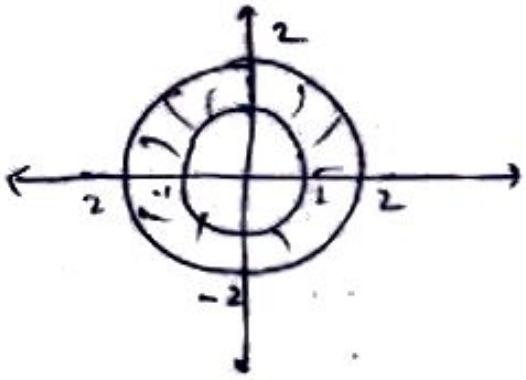
$3 \le x^2 + y^2 \le 4\}$. — (3) — Compact

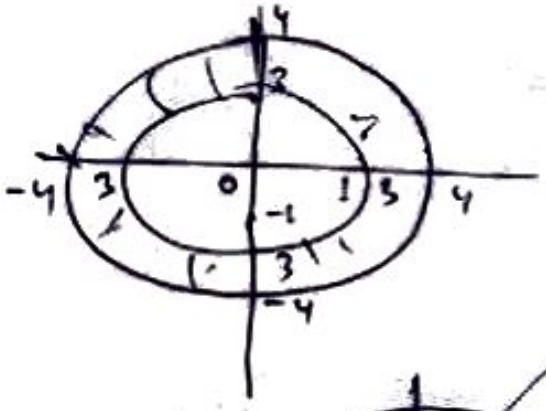
(a) $X \setminus K$ has three connected components.

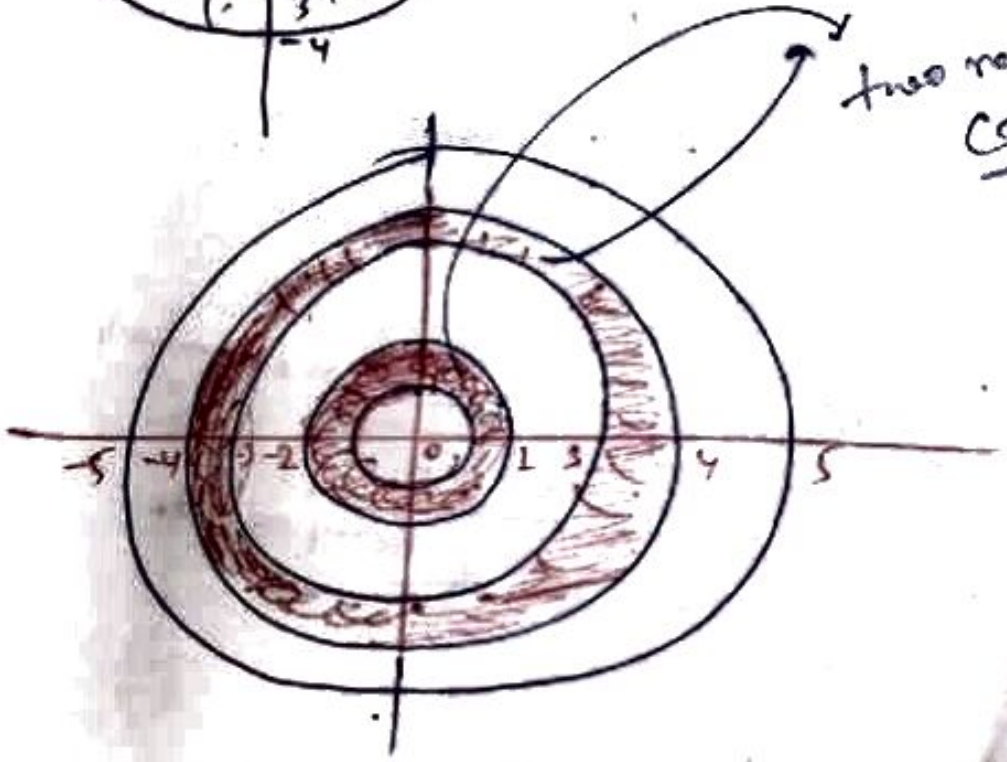
(c) $X \setminus K$ has two relative compacts.

or









Q68.

Soln:

$$f(z) = z^2$$

$$f(z) = (x+iy)^2 \qquad z = x+iy$$

$$f(z) = (x^2 - y^2 + i2xy)$$

$$f(z) = (x^2 - y^2, 2xy) = \phi^{-1} f \phi$$

$$f_1(x,y) = x^2 - y^2, \quad f_2(x,y) = 2xy$$

$$\frac{Df}{d(x,y)} = DF(x,y) = \begin{pmatrix} \frac{df_1}{dx} & \frac{\partial f}{dy} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{dy} \end{pmatrix}$$

$$Df = \begin{bmatrix} 2x & -2y \\ 2y & 2x \end{bmatrix}$$

$$Df = 2\begin{bmatrix} x & -y \\ y & x \end{bmatrix} = T(x,y).$$ represent derivative of f at (x,y).

Option (a) is Correct.

Option (b) is not Correct.

Option (c) $T(z) = 2z = 2\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$

as $f(z) = z^2$, $f'(z) = 2z$

$f'(z) = T(z) = 2z$

Option (d) is clearly false.

There uncountable no. of open/closed ball in (X,T) space.
But (X,T) is not closed as u can say that it is countable.

Inside countable. ball has countable small ball. i.e
Inside (X,T) it must have countable.
ball. but it's contradicts that it has uncountable ball $(0,1)$, $(1,0,1)$, $(1,0)$

so, (X,T) is not compact.

Q9

option → (a) correct
(b) – correct
(c) – not
(d) – correct.

(x_n) ~ 0 → subsequence
~ 1 → subsequence.

subsequence are converges either 0 or 1 but not sequence, sequence is not converge as it is oscillates.

Q66 Don't do this type of question in Exam, and Never, Never. & forever it will kill ur your times, (used Jacobian Matrix)

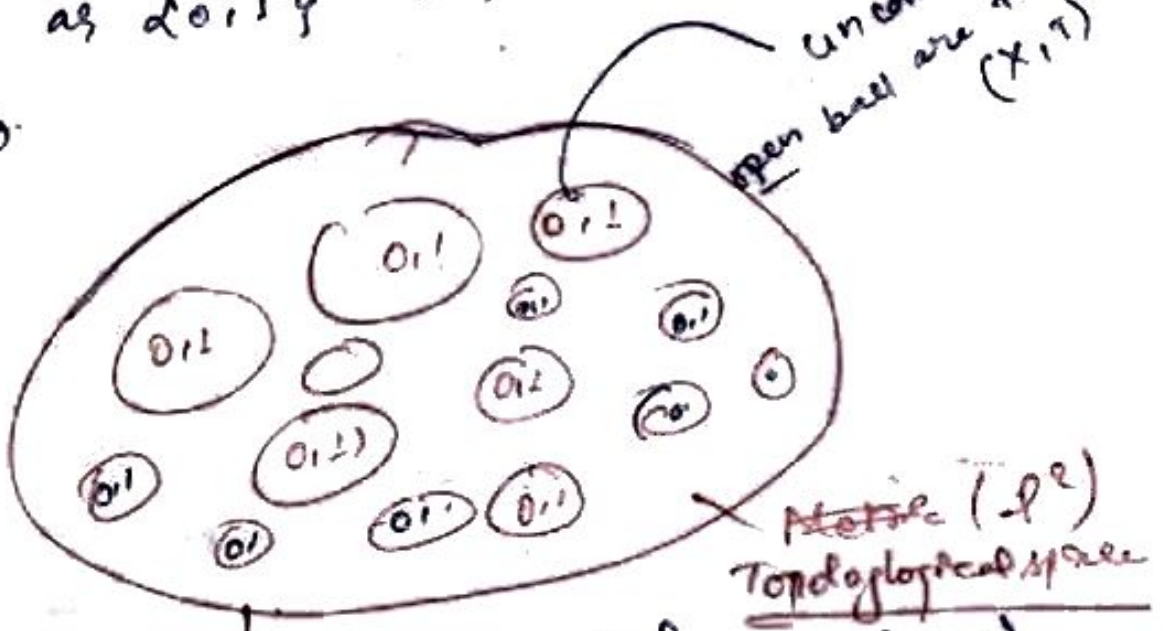
Q67

this U Know that 0,1, 0,1,.....

uncountable.

as $\{0,1\}^{\mathbb{N}}$ → uncountable.

Now



closed & bounded.

But not compact.

Compact has two meaning.

closed/bounded

every ∞ ball has finite subcover.

Q65

option (a) True → clearly visible.

(b) no (False)

$AX = 0$ (homogeneous).

$n \longrightarrow$ Rank.

Rank(m,n) = min Rank(m,n)

Rank A $= m$ as $(n > m)$

Total no. of unknown (variables) $= n$

Aug $[A:0]$ = Aug A

Rank Aug$[A:0]$ = Rank(A) — for solution exist it must satisfy.

$m = m$

So it must have a solution either unique or infinite.

option (c) → same — true. (Read 2 option)

option (iv) In homogeneous case, $AX = 0$

Total no. of solution = $n - m$
(linearly Indepen)

n → no. of variables.

m → Rank

as it denotes the dimension space of given $AX = 0$ i.e $n - m$.

$$A = \begin{pmatrix} x_1 & 0 & \cdots & 0 & \cdots & 0 \\ x_1 & x_2 & & & & \\ \vdots & \vdots & & & & \\ x_1 & x_2 & & x_{n-1} & & \\ x_1 & x_2 & & x_{n-1} & x_n & \end{pmatrix}_{n \times n}$$

determinant of $A = x_1 (x_2) x_3 \cdots x_n$.

$$|A| = \prod_{i=1}^{n} x_i$$ — product symbol.

Now we know if determinant $\neq 0$, then we can say it linearly independent.

→ See option (1) — is correct.

option (2) is correct.

option (3) → span mean generator that is not "spam" — body cell.

→ simple meaning :- if Span $(x_1, x_2 \cdots x_m)$ is equal to V, then we say that $(x_1, x_2 \cdots x_m)$ spans V.

$$\text{Span}(x_1 x_2 \cdots x_m) = \{c_1 x_1 + c_2 x_2 + c_3 x_3 + \cdots + c_m x_m\} \text{ where } c_1, c_2 x_3 \cdots c_n \in F.$$

Here C is linearly independent if

$$\dim(\text{Span}(x_1 x_2 \cdots x_m)) = \dim C$$

option (c) and (d) are true.

Q63

Sol:- Read Norm (see in wikipedia)

Option (1) is true.

$\|P\| = 0$

i.e $P(1) = P(2) = \cdots = P(n+1) = 0$

$\|\alpha P\| = |\alpha| \cdot \|P\|$ it satisfy the property of norms

Option (2) is true. → See - sup norm. (definition)

Option (3) is true → See - (Integral norms (definition)

Option (4) $\|P\| = \sup_{t \in [0,1]} |P'(t)|$, $P \in V$.

$P(t) = 1$.

$P'(t) = 0$

But $\|1\| = 0$ (is not possible)

or we can said that

Let $P(x) = c$ constant. Degree of $P(x) = 0$

and $P(x) \neq 0$ but $\|P\| = \sup_{x \in [0,1]} |P'(x)| = 0$

It violates the rules the "Norm" (breaks)

so option (4) is not true.

Q63

Sol:- Read Norm (see in wikipedia)

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and $P(x) \neq 0$ but $\|P\| = \sup_{x \in [0,1]} |P'(x)| = 0$

It violates the rules the "Norm" (breaks)

so option (4) is not true.

Therefore $c_1 - c_2 - c_3 = 0$

Hence we can conclude that
u, v and w are linearly independent.

or

~~$4c_1 + 0$~~

$4c_1 + 0c_2 + c_3 = 0$

$0c_1 + 4c_2 - c_3 = 0$

$c_1 - c_2 + 4c_3 = 0$

↓ transform into Matrix.

$$\begin{vmatrix} 4 & 0 & 1 \\ 0 & 4 & -1 \\ 1 & -1 & 4 \end{vmatrix} = 4\begin{vmatrix} 4 & -1 \\ -1 & 4 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 4 & -1 \end{vmatrix}$$

$= 4 \cdot 17 - 4 \neq 0$

i/e P.t is linearly independents.

~~Q3.~~

option ⓒ $\langle w, 4v - w \rangle = \langle w, 4v \rangle - \langle w, w \rangle$ (16)

$= 4 \times 1 - 4 = 0$

are orthogonal.

true

option ⓓ . for linearly independent-

we $c_1 = c_2 = c_3$ and now make a linear combination. we ~~ask~~

$c_1 u + c_2 v + c_3 w = 0$, $c_1, c_2, c_3 \in \mathbb{F}$

~~from option ⓒ, we have~~ and same time ~~applied the~~ inner product ~~to~~

~~Here $v = 4u$~~

Now applied the inner product.

$\langle c_1 u, c_1 u + c_2 v + c_3 w \rangle = \langle c_1 u, 0 \rangle$

$\Rightarrow \langle c_1 u, c_1 u \rangle + \langle c_1 u, c_2 v \rangle + \langle c_1 u, c_3 w \rangle = 0$

$\Rightarrow c_1^2 \langle u, u \rangle + c_1 c_2 \langle u, v \rangle + c_1 c_3 \langle u, w \rangle = 0$

$\Rightarrow c_1^2 \|u\|^2 + c_1 c_2 \cdot 0 + c_1 c_3 \cdot 1 = 0$

$\Rightarrow c_1^2 \cdot 4 + 0 + c_1 c_3 = 0$

$\Rightarrow c_1 c_3 + 4 c_1^2 = 0$

$\Rightarrow c_1 (4 c_1 + c_3) = 0 \cdot 0$

Here $c_1 = 0$, as $4 c_1 + c_3 = 0$

$\Rightarrow c_1 = c_3 = 0$

Q62

Soln: $\|u\| = 2, \quad \|v\| = \|w\| = 2$

$\|u\|^2 = \langle u, u \rangle \Rightarrow \|u\| = \sqrt{\langle u, u \rangle}$

Option (a) $\|w+v-u\|^2 = \langle w+v-u, w+v-u \rangle$

$= \langle w, (w+v-u) \rangle + \langle v, (w+v-u) \rangle - \langle u, (w+v-u) \rangle$

$= \langle w, w \rangle + \langle w, v \rangle - \langle w, u \rangle + \langle v, w \rangle + \langle v, v \rangle - \langle v, u \rangle - \langle u, w \rangle - \langle u, v \rangle + \langle u, u \rangle$

Now apply the values given in question.

$= 4 + (-1) - 1 + (-1) + 4 - 0 - 1 - 0 + 4$

$= 8 = \sqrt{2}$

$\|w+v-u\|^2 = 8$

$\|w+v-u\| = 2\sqrt{2}.$

Option (b) $\left\{\frac{1}{2}u, \frac{1}{2}v\right\}$ as here $\left\|\frac{1}{2}u\right\| = \frac{1}{2}\|u\| = 1$

$= \frac{1}{2}\|v\| = 1$

$\frac{1}{2}\langle u, v \rangle \quad \left\langle \frac{1}{2}u, \frac{1}{2}v \right\rangle =$

$= \frac{1}{2}\cdot\frac{1}{2}\langle u, v \rangle = 0$

that mean $\frac{1}{2}u, \frac{1}{2}v$ are orthogonal to each other. that mean forme orthonormal basis.

$\Rightarrow$ (orthogonal $\longrightarrow$ orthonormal.)

option (d)

$$|A - \lambda I|$$

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\lambda & 1 & 0 & 0 \\ 0 & -\lambda & 0 & 0 \\ 0 & 0 & -\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{pmatrix}$$

$$= -\lambda \left(-\lambda \left(-\lambda (1-\lambda) \right) \right.$$

$$= -\lambda^3 (1-\lambda) = \lambda^3 (\lambda - 1)$$

$$= \lambda^4 - \lambda^3 \quad \text{— (1)}$$

put $x = \lambda$ in (1), we get

$$x^4 - x^3$$

so it is true

answer (a), (c) and (d)

part C

Q61

Ans: Take $A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

→ rank(A) = 2

→ A is do not diagonalizable

option (A) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

option (A) is correct.

option (B) $A^2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \neq A \neq 0$

$A^2 \neq A$ simple.

option (B) is false.

option (C) is true. $A \neq 0 \Rightarrow Av \neq 0$

$A^2 \neq 0 \Rightarrow A^2 v = 0$

so it true. take $v = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$.

Note : ~~for~~ ~~is~~ $v = (0, 1, 0, 0)$ or

$$K = \frac{n}{2}\phi(n)$$

It is given that $K = 240n$.

$$240n = \frac{n}{2}\phi(n)$$

$$\Rightarrow \phi(n) = 480$$

Q39.

Sol: total number of non-isomorphic group of order $= 122 = 2 \times 61$.

$$Z_{122} = Z_2 \times Z_{61} \text{ (abelian)}$$

$$D_{2n} = D_{61} \text{ (non-abelian)}$$

So there are two non-isomorphic group $Z_2 \times Z_{61} \cong Z_{122}$

and D_{61}

$$\Rightarrow \frac{Z}{p^2qz} \approx Z_{p^2q} \quad \left(\frac{Z}{nz} = Z_n\right)$$

$Z_{p^2} \times Z_q$. if $\gcd(p^2, q) = 1$.

But here there two distinct prime p and (p) and (q).

Q38.

Sol^n ⇒ Very important ;

number of summands ~~(Arithmetic mean)~~ $\phi(n)$.

For proof

$$\text{let } K = \sum_{1 \le a \le n (a,n)} a \quad \{(a,n) = (n-a, n) \text{ as } \gcd(a,n) = 1\}$$

$$\text{then } K = \sum_{1 \le a \le n (a,n)} (n-a)$$

$$2K = \phi(n)(a + n - a)$$

$$2K = n\,\phi(n)$$

$$\text{Hence } K = \frac{n}{2}\phi(n) \Rightarrow k = \frac{n}{2}\phi(n)$$

Not ~~arithmetic mean~~ may be

$$\frac{K}{\phi(n)} = \frac{\frac{n}{2}\phi(n)}{\phi(n)}$$

$\Rightarrow \lim_{n\to\infty} \left|\frac{p(n+1)}{p(n)}\right| = 1.$

$\Rightarrow$ take $p(n) = n^K$ where K is degree.

$$\left|\frac{n^{K+1}}{n^K}\right| = \left|\frac{1}{n}\right|^{1/n} = 1.$$

$\lim_{n\to\infty} n^{1/n} = 1.$

Therefore radius of given power series $= 1$.

Q36

Sol.

$F_{3^{100}}$ = Galois field (3^{100})

$|\text{Galois field}(3^{100})| = 100$

$\Rightarrow$ ~~Ref~~ $\tau(100)$ = no. of divisors).

$= \tau(2^2 \times 5^2)$

$= (2+1)(2+1)$

$= 3 \times 3 = 9$

Q37. Use simple logic here

p^2q has two distinct element

$(p.p) \longrightarrow$ one q

distinct terms

$1+1 = 2$

$$\boxed{\frac{\mathbb{Z}}{p^2q\mathbb{Z}} = \frac{1}{p^2q}}.$$

Q34

Note: given that f has a zero of order k at $z=a$.

i/e $f(z) = (z-a)^k$

and g has a pole of order m at $z=0$

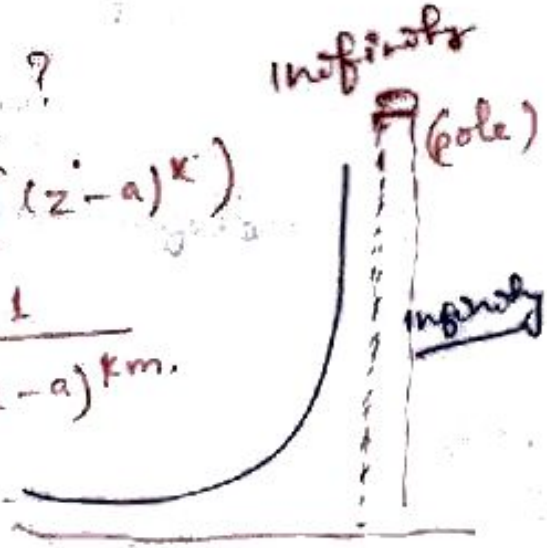
i/e $g(z) = \frac{1}{z^m}$

Now $g\,f(z) = ?$

$g(f(z)) = g\left((z-a)^k\right)$

$= \frac{1}{(z-a)^{km}}$

at $z=a$ it tend $= \frac{1}{0} = \infty$



Inspired from Prof. Rajendra Dubey (Dips academy)

$\therefore$ $g(f(z))$ has a pole of order km at $z=a$.

Q35 take ~~$p(x) = x^n$~~ used above this test also.

$p(x) = (x^n)^{1/n}$.

~~$p(n) = x^n$~~

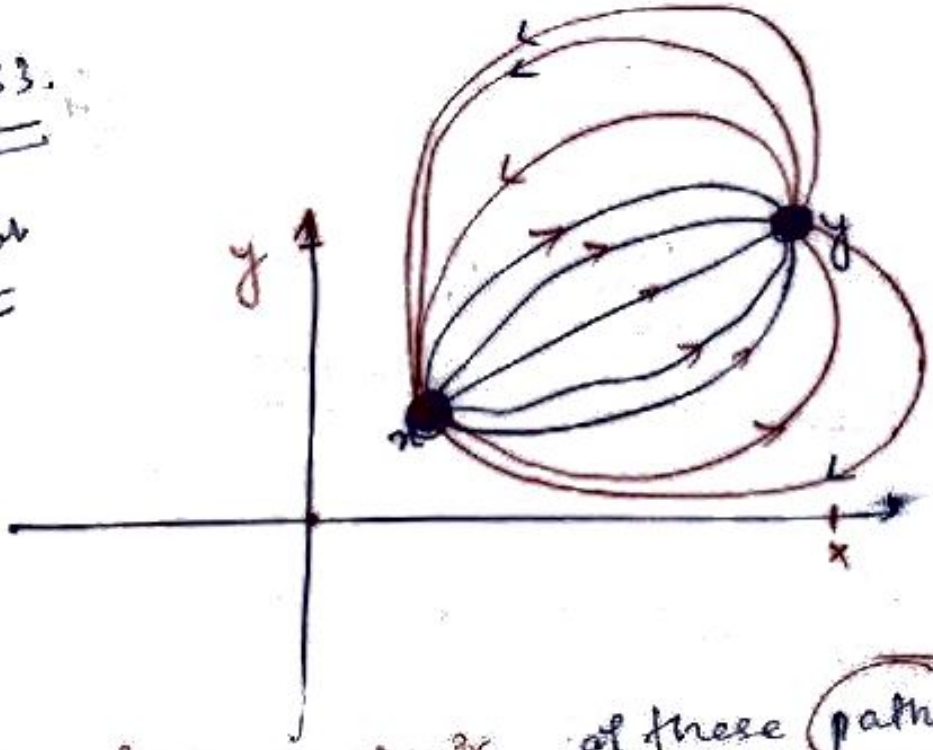
$$R = \frac{a_n}{a_{n+1}} = \left| \frac{p(n) \cdot z^n}{p(n+1) \cdot z^{n+1}} \right|$$

$$= \left| \frac{p(n)}{p(n+1)} \right| \cdot \frac{1}{|z|}$$

Q33.

(8)

Sol



$x, y \in \mathbb{R}^2 - A$

$x \neq y$

10 Since no pair of these paths share any common points other than x and y.

Here Uncountable many paths (lines) are there → Important thing is that they are not intersecting each other. That mean they are disjoints

Here A is a countable and any Countable (many path) can intersection with A i,e non-zero intersection.

8 answer is option (D)

Q34

Sol

Q82

Sol.

$$\lim_{n\to\infty} \frac{p(n)}{2^n} \longrightarrow \frac{\infty}{\infty} \quad \text{if } n \to \infty$$

So we must use L - hospital rule

$$\therefore \lim_{n\to\infty} \frac{p(n)}{2^n}$$

Take $p(n) = n^5$ as degree 5.

$$\lim_{n\to\infty} \frac{n^5}{2^n}$$

$$\frac{5n^4}{n\cdot 2^{n-1}} = \frac{5n^4}{n\cdot 2^{n-1}}, \quad (\text{differentiate 1 time})$$

$$= \frac{4\cdot 5n^3}{n(n-1)\cdot 2^{n-2}} \quad (\text{" differentiate})$$

$$= \frac{4\cdot 5\cdot 3n^2}{n(n-1)\cdot 2^{n-3}} \quad (\text{" " }).$$

$$= \frac{4\cdot 5\cdot 3\cdot 2n}{n(n-1)\cdot 2^{n-2}}$$

$$= \frac{4\cdot 5\cdot 3\cdot 2\cdot 1}{(n-2)n(n-1)\,2^{n-2}} \longrightarrow (\text{Constant})$$

$$= \lim_{n\to\infty} \frac{\text{Constant}}{(n-2)(n-1)n\cdot 2^{n-2}} = \frac{1}{\infty} = 0$$

0 is the correct answer.

Q29.

Option $f(x) = x^2$ is U.C on $[a,b]$ → closed & bounded.

not on $\mathbb{R}$

answer is option (a)

Q30

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 2 & 5 & 4 \end{pmatrix}$$

$$\sigma = (1\,3\,2)(4\,5)$$

$$(3)\quad (2)$$

order of σ = l.c.m (3, 2)

= 6

$\sigma^6 = I$ i.e $A^6 = I$

$A^5 = A^{-1}$

$\Rightarrow \boxed{A^{-5} = A}$ Ans.

Q31

Sol: $\int_{-\infty}^{\infty} \frac{1}{|x|^{pa}} dx = \infty$ is Correct option

$= \int_{-\infty}^{0} \frac{1}{|x|^{pa}} dx + \int_{0}^{\infty} \frac{1}{|x|^{pa}} dx$

0 or ∞ two choices

$= \int_{-\infty}^{0} \frac{1}{(-x^p)^{\alpha}} dx + \int_{0}^{\infty} \frac{1}{(x^p)^{\alpha}} dx$ (II) → 0 or ∞ two choices

But at same time both can't be zero.

$0 + \infty = \infty + 0 = \infty$

1 choice, 2nd choice.

$\int_0^{\infty} \frac{1}{x^p}$ if $p > 1$ = converges

If you don't understand this mail at bairagiraviteja123@gmail.com.

Take $(x_n) \longrightarrow x$. makes a new sequence ⑤

$y_{2n} = x_n$, $y_{2n+1} = x$

so N and u will get $f(y_n) = f(x_0), f(x), f(x_1), f(x)$

as u can write like this also.

~~$d(f(y_{2n}), f(y_{2n+1}))$~~ $d(f(y_{2n}), f(y_{2n+1})) > \varepsilon$

It must be less than ε

so its contradicts the defination of Cauchy

Note:- If u don't understand that concepts mail at (baliraopaculego123@gmail.com)

For option ③ It just opposite ①

If true as ① Read option 1.

For option ④ $\{x_n\}$ is always convergent in X

↓

This statement is true if both X and Y are complete Metric space

but Here it is not given that it is a complete metric space:

so it is false.

Take $x_n = \begin{cases} y_n, & n \text{ even} \\ x, & n \text{ odd} \end{cases}$.

(Counter-Example)

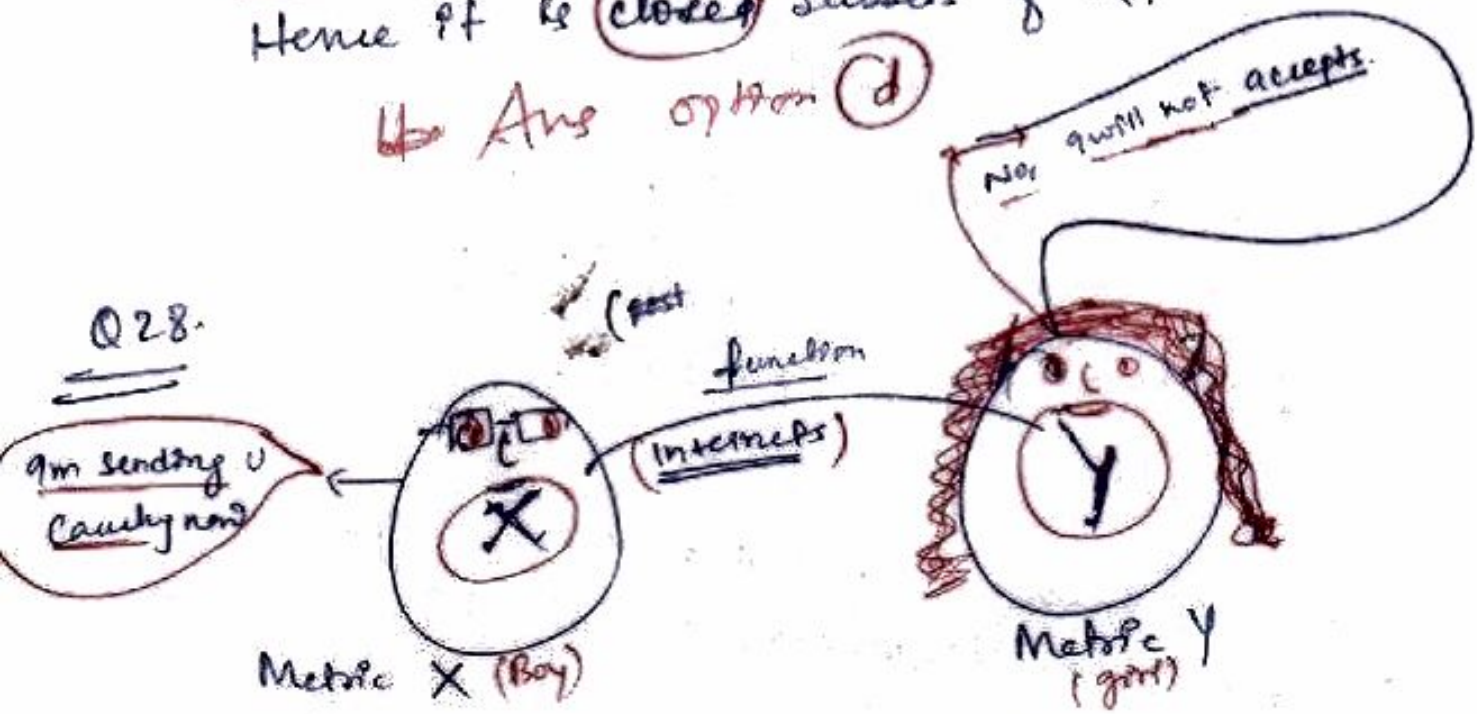
$$f(k) = \lim_{n\to\infty} f(k_n) = \lim_{n\to\infty} 0 .$$ (4)

so $k \in Z(f) = \{x : f(x) = 0\}$.

so $Z(f)$ Therefore $Z(f)$ contain all of its limits points and.

Hence it is closed subsets of $\mathbb{R}$.

Ans option (d)



option (a) Take $f(x) = x^2$ on $\mathbb{R}$

$f(x_n)$ is not uniformly continuous on $\mathbb{R}$

that mean you can say that

$f(x_n)$ is not Cauchy in Y

Note For Cauchy $\rightarrow$ exist in closed and bound interval.

option (b) If $f(x_n)$ is Cauchy, then it is not 100% guarantee. that $f(x_n)$ is convergent in Y

as there are many subsequences are there.

Q25 Use simple logics

(25) Answer (d) Check at 3×3 matrix.

$$J = A \text{ of } A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

~~det A = (1 (1 1)~~

$$J = A - I = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$|J - I| = 0 - 1 \times (-1) + 1 \times 1.$$

$$|J - I| = 2.$$

For 3×3 matrix, $\det A = \det J = 2$

for 10×10 matrix, $\det J = 100$

Q26 → simple take $A = \begin{pmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{pmatrix}_{5\times5}$

$$A^5 = I_{5\times5}$$

$$\text{trace}(I)_{5\times5} = 5$$

Q27 $Z(f) = \{x \in R : f(x) = 0\}$

Let K be an limit point of $Z(f)$.

so there is a sequence $\{k_n\}$ such that $k_n \in \{x : f(x) = 0\}$ for all n and $\lim_{n\to\infty} k_n = k$.

If it is given that function is continuous, then we have.

or

Take $f(x) = 1+\sqrt{3}$ (as Irrational no. is uncountable)

(2)

X = The set of polynomial function on R

Take any function.

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \in \mathbb{Q}^c \end{cases}$$

Y = Same as X (use same logic)

Z = { f(x) : f(x) is any function it may be Constant or non-constant) }

So all W, X, Y and Z has same cardinality.

Q24 $A^6 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}_{2\times 2}$.

use simple logics

option (2) is more clear as

Now $\cos 2\pi = 1$

$\cos n\frac{\pi}{3} = \cos 2\pi$

$\Rightarrow n\frac{\pi}{3} = 2\pi$

$\Rightarrow n = 6$

answer is $n\frac{\pi}{3} = I$

i/e $\cos \pi/3$ Ans

Solution of C.S.I.R - JUNE 2014

Part B

Q21

Sol^n: Using simple trick.

Rank(AB) = 2. ⟶ option (c)

Q22

Sol^n: $\lim_{n\to\infty} \frac{1}{\sqrt{n}} \left(\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \cdots + \frac{1}{\sqrt{2n-1}+\sqrt{2n+1}} \right)$

$\Rightarrow \lim_{n\to\infty} \frac{1}{\sqrt{n}} \left[\frac{(\sqrt{1}-\sqrt{3})}{(\sqrt{1}+\sqrt{3})(\sqrt{1}-\sqrt{3})} + \cdots \frac{\sqrt{3}-\sqrt{5}}{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})} + \cdots \right.$

$\left. + \frac{(\sqrt{2n-1}) - \sqrt{2n+1}}{(\sqrt{2n-1}+\sqrt{2n+1})(\sqrt{2n-1}-\sqrt{2n})} \right)$

Remaining to be derived ??

$= \lim_{n\to\infty} \frac{-1}{2} \times \frac{1}{\sqrt{n}} \times \cdot \sqrt{2n+1}$

$= \lim_{n\to\infty} \frac{-1}{2} \times \frac{1}{\sqrt{n}} \times (-\sqrt{2n+1} \cdot$

$= \lim_{n\to\infty} \sqrt{2+\frac{1}{n}} \times \frac{1}{2}$

$= \frac{1}{\sqrt{2}}$

Q23.

Sol^n ① $W = \{0,1\}^N \longrightarrow$ (uncountable)

$f : R \longrightarrow \{0,1\}^N$

↓ uncountable ⟶ uncountable

same cardinality.