

Module 4

POINT SOURCES AND ARRAYS

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4.0 Introduction

As of now the antenna was treated aperture, in this chapter it is formally considered as point source and later the concept extended to the formation of arrays of point sources. The pattern of any antenna can be regarded as produced by an array of point sources.

Here we discuss the array arrays confined to isotropic point sources, which may represent different kind of antennas.

Point Sources:

* Antenna that doesn't have any specified shape is called "point source".

Consider an antenna and observation circle as shown in fig.2.1 where the radiated fields of antenna transverse radially at a sufficient distance id far field whereas near fields have actual variation ignored.

Provided that observation made at the sufficient distance, any antenna regardless of size or complexity can be represented as single point source. Far field is considered because power flow and fields are radiated outwards at this region properly.

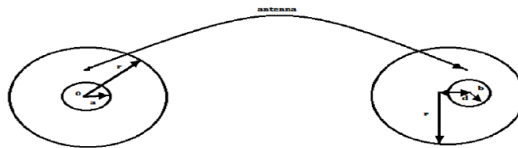


Fig 4.1. Antenna and the observation

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Field measurements can be done either by fixing antenna or fixing observation point but both effects are approximately same.

If in case the center of the antenna is displaced by distance 'd' as shown in fig.1, the distance between two centers are negligible effect on the field pattern at observation circle provided that

$$\mathbf{R} \gg \mathbf{d}, \mathbf{R} \gg \mathbf{b} \text{ and } \mathbf{R} \gg \lambda$$

As we discussed complete description of the far field of a source requires 3 components.

1. $E_\theta(\theta, \phi)$
2. $E_\phi(\theta, \phi)$
3. $\delta(\theta, \phi)$

Power Patterns:

Let transmitting antenna in free space by point source radiation located at origin of the co-ordinates as shown in fig.4.2.

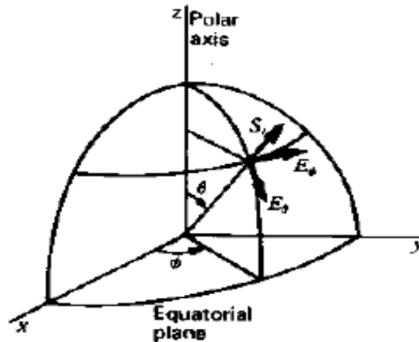


Fig 4.2 Point source at origin.

The radiated energy streams from the source radial lines.

The time rate of energy flow per unit area is "**Poynting vector or power flow density**". The magnitude of Poynting vector is equal to radial component ($|S| = S_r$)

* A source that radiates energy uniformly in all directions is called "**isotropic antenna**".

A graph of S_r at constant radius as a function of angle is Poynting vector, power density, pattern but usually called "**power pattern**".

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$$2. \mathbf{E}_\phi(\theta, \phi)$$

$$3. \mathbf{S}(\theta, \phi)$$

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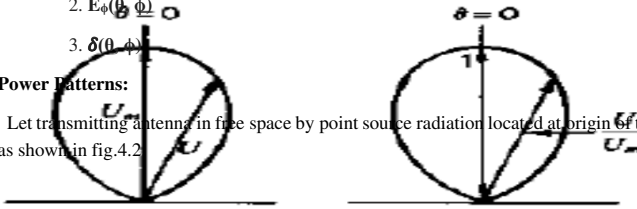


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$$P = S_r \oint ds$$

$$S_r = \frac{P}{4\pi r^2}$$

This equation indicates that the magnitude of Poynting vector varies inversely proportional to the square of the distance from point source radiator.

Radiation Intensity:

It is defined as the power radiated by an antenna per unit solid angle. Denoted by u and unit is W/Sr.

Power Theorem for Radiation Intensity:

The total power radiated by an antenna is given by integral of radiation intensity over solid angle of the sphere.

i.e.,
$$P = \oint u \, d\Omega$$

For an isotropic source radiation intensity remains same at any point or the surface of the sphere.

Let u_0 be the radiation of isotropic source then

$$P = \oint u_0 \, d\Omega$$

$$P = \oint u_0 \sin \theta \, d\theta \, d\phi$$

$$P = u_0 \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} d\phi$$

$$P = 2\pi u_0 (-\cos \theta) \Big|_0^\pi$$

$$P = u_0 (4\pi)$$

$$u_0 = \frac{P}{4\pi}$$

Therefore, the relation between Poynting vector and radiation intensity as follows

W.K.T.
$$S_r = \frac{P}{4\pi r^2}$$

$$S_r r^2 = \frac{P}{4\pi}$$

$$S_r r^2 = u_0$$

$$u_0 = S_r r^2$$

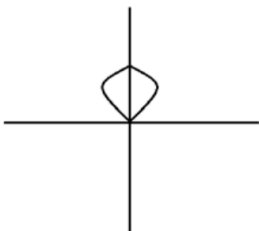
Examples Of Power Patterns:**1. Unidirectional Cosine Pattern**

The radiation intensity of unidirectional cosine pattern is given as

$$u = u_m \cos \theta$$

where u_m is the maximum radiation intensity and u is having value in upper hemisphere.

$$\text{i.e., } u = \begin{cases} u_m \cos \theta & ; 0 < \theta < \frac{\pi}{2} \\ 0 & ; \frac{\pi}{2} < \theta < \pi \\ 0 & ; \text{Elsewhere} \end{cases}$$

**2. Bidirectional Cosine Pattern**

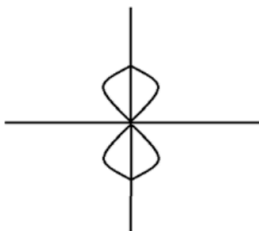
The radiation intensity of unidirectional cosine pattern is given as

$$u = u_m \cos \theta$$

and has the value in both the hemisphere

$$\text{i.e., } u = \begin{cases} u_m \cos \theta & ; 0 < \theta < \pi \\ 0 & ; \text{Elsewhere} \end{cases}$$

It is also known as groundnut pattern because of its appearance as shown.



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The power pattern and the field patterns are inter-related:

$$P(\theta, \phi) = (1/\eta) * |E(\theta, \phi)|^2 = \eta * |H(\theta, \phi)|^2$$

P = power

E = electrical field component vector

H = magnetic field component vector

$\eta = 377$ ohm (free-space impedance)

The power pattern is the measured (calculated) and plotted received power: $|P(\theta, \phi)|$ at a constant (large) distance from the antenna

The amplitude field pattern is the measured (calculated) and plotted electric (magnetic) field intensity, $|E(\theta, \phi)|$ or $|H(\theta, \phi)|$ at a constant (large) distance from the antennas

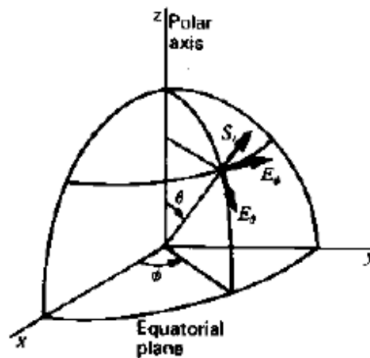


Fig 4.3: Relation of Poynting vector S and 2 electric field components of a far field

4.4 ARRAY OF TWO POINT SOURCES

ARRAY is an assembly of antennas in an electrical and geometrical of such a nature that the radiation from each element add up to give a maximum field intensity in a particular direction & cancels in other directions. An important characteristic of an array is the change of its radiation pattern in response to different excitations of its antenna elements.

Here let us consider the different cases of two isotropic sources placed $\lambda/2$ apart with different scenarios.

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1. Obtain the field pattern for 2 isotropic point sources with equal amplitude and opposite phase. Assume distance between 2 sources is 'd'.

Sol :

This case is identical with the previous but two sources are in opposite phase instead of same phase let the two sources 1 and 2 are located symmetrically with respect to origin of -ve coordinates consider a observation point p at distance 'r', the angle θ is measured clockwise from positive x-axis

if origin is considered as reference, the field from source 1 is related by $(dr/2)\cos\theta$ and field from source 2 is advanced by

$$(dr/2)\cos\theta \quad \text{wr} \quad dr = \beta d = 2\pi/2jE_o * d \dots\dots\dots 1$$

then total electric field in the direction at a large distance r is given

$$E = 2E_o [\exp(j*\Psi/2) - \exp(-j*\Psi/2)]$$

From which

$$E = 2jE_o[(\exp(j*\Psi/2) - \exp(-j*\Psi/2))/2] \dots\dots\dots 2$$

j indicates the phase reversal of one source and it is not in portent

$$E = 2jE_o \sin(\Psi/2)$$

$$E = 2jE_o \sin((dr/2)\cos\theta) \dots\dots\dots 3$$

Normalize eq3 $2jE_o = 1$, for $d = \pi$

$$E = \sin((\pi/2)\cos\theta) \dots\dots\dots 4$$

Since $dr = \beta d = 2\pi/\lambda * (\lambda/2) = \pi$

In above eq

For $\theta = 0$, $E = 1$

$\theta = 30$, $E = 0.977$

$\theta = 60$, $E = 0.70$

$\theta = 90$, $E = 0$

$\theta = 120$, $E = 0.70$

$\theta = 150$, $E = 0.97$

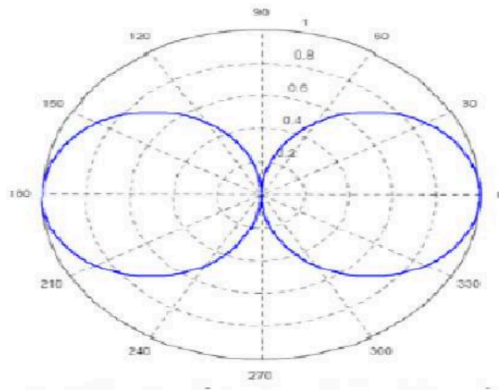
$\theta = 180$, $E = -1$

$\theta = 210$, $E = -0.97$

$\theta = 240$, $E = -0.70$

$\theta = 270$, $E = 0$

$\theta = 300$, $E = 0.70$



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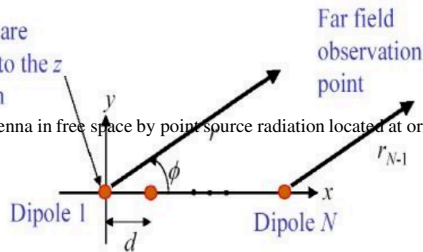


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Antennas and Propagation

4.5 CASE 1: LINEAR BROAD SIDE ARRAY

An array is said to be broadside if the phase angle is such that it makes maximum radiation perpendicular to the line of array i.e. 90° & 270°

For broad side array $\Psi=0$ & $\delta=0$

$$\text{Therefore } \Psi = d r^* \cos \Phi + \delta = \beta d \cos \Phi + 0 = 0 \quad \Phi = \pm 90^\circ$$

$$\text{therefore } \Phi_{\max} = 90^\circ \text{ \& } 270^\circ$$

Broadside array example for $n=4$ and $d=\lambda/2$

By previous results we have $\Phi_{\max} = 90^\circ \text{ \& } 270^\circ$

Direction of pattern maxima:

$$E = (1/n) (\sin(n\Psi/2)) / \sin(\Psi/2)$$

This is maximum when numerator is maximum i.e. $\sin(n\Psi/2)=1$ $n\Psi/2 = \pm(2k+1)\pi/2$
where $k=0,1,2,\dots$

$K=0$ major lobe maxima

$$K=1 \quad n\Psi/2 = \pm 3\pi/2 \quad \Psi = \pm 3\pi/4$$

$$\text{Therefore } d r^* \cos \Phi = 2\pi/\lambda * d * \cos \Phi = \pm 3\pi/4 \quad \cos \Phi = \pm 3/4$$

$$\Phi = (\Phi_{\max})_{\text{minor lobe}} = \cos^{-1}(\pm 3/4) = \pm 41.40^\circ \text{ or } \pm 138.60^\circ$$

At $K=2$, $\phi = \cos^{-1}(\pm 5/4)$ which is not possible

Direction of pattern minima or nulls

It occurs when numerator=0 i.e. $\sin(n\Psi/2)=0$ $n\Psi/2 = \pm k\pi$

where $k=1,2,3,\dots$ now using condition $\delta=0$

$$\Psi = \pm 2k\pi/n = \pm k\pi/2 \quad d r^* \cos \Phi = 2\pi/\lambda * d/2 * \cos \Phi$$

$$\text{Substituting for } d \text{ and rearranging the above term } \pi \cos \Phi = \pm k\pi/2 \quad \cos \Phi = \pm k/2$$

$$\text{Therefore } \Phi_{\min} = \cos^{-1}(\pm k/2)$$

$$\mathbf{k=1, \Phi_{\min} = \cos^{-1}(\pm 1/2) = \pm 60^\circ \text{ or } \pm 120^\circ}$$

$$\mathbf{k=2, \Phi_{\min} = \cos^{-1}(\pm 2/2) = \pm 0^\circ \text{ or } 180^\circ}$$

