

Jimma University
College of Natural Science
Mathematics Department

Mathematics for Natural Science Mid Examination

Total Mark: 30%

Time Allowed: 2:00 hrs

2015EC



Part I: Multiple Choice Items

(12pts)

Choose the best answer from the given four alternatives

B

1. Given complex number $z = 2 - 3i$, which one of the following is **not true** about z ?

- A) $\bar{z} = 2 + 3i$ B) $|z| = 13$ C) $\text{Im}(z) = -3$ D) $|z| = |\bar{z}|$

C

2. What is the simplified form of $\frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}$?

- A. $-1 + 5i$ B. $-3 + 5i$ C. $3 - 7i$ D. $-3 + 7i$

D

3. Which one of the following is **not true** for a set $A = \{1, \{a\}, \emptyset, \{\emptyset\}\}$?

- A. $\{\emptyset\} \in A$ B. $\{\emptyset\} \subseteq A$ C. $\emptyset \subseteq A$ D. $\{\emptyset, \{\emptyset\}\} \in A$

D

4. Which one of the following is not necessarily **true** for any sets A and B ?

- A. If $A \subseteq A'$, then $A = \emptyset$ C. $A \cap B = A \setminus (A \setminus B)$
 B. If $A \setminus B = B \setminus A$, then $A = B$ D. $B \setminus A = B' \cap A$

A

5. Suppose $2 \in A$ but $-2 \notin A$ and $-2 \in B$ but $2 \notin B$. Which one of the following is necessarily **true**?

- A. $\{-2, 2\} \subseteq A' \cup B'$ B. $A \setminus B = \{2\}$ C. $A \subseteq B$ D. $A \cap B = \{-2, 2\}$

D

6. Which one of the following is **true**?

- A. $(\forall x \in \mathbb{N})(x < x^2)$ B. $(\forall x \in \mathbb{R})(x < x^2)$ C. $(\forall x \in \mathbb{N})\left(\frac{x-1}{x-1} = 1\right)$ D. $(\forall x \in \mathbb{N})(\sqrt{x^2} = x)$

C

7. The greatest lower bound of $\left\{(-1)^n \left(\frac{1}{4} - \frac{2}{n}\right)\right\}$ is:

- A. $-\frac{1}{4}$ B. $\frac{1}{4}$ C. $-\frac{3}{4}$ D. $\frac{7}{4}$

D

8. What are the solution set of $z^2 - (5 + i)z + 18 - i = 0$?

A. $\{2 - 3i, -3 - 4i\}$ B. $\{2 - 3i, -3 + 4i\}$ C. $\{2 - 3i, 3 - 4i\}$ D. $\{2 - 3i, 3 + 4i\}$

Part II: Short Answer Item

(6pts)

Write the most simplified answer on the space provided.

9. If $z = \frac{\sqrt{12}i - 2}{3 - 3i}$, then Principal argument of $z = \frac{11\pi}{12}$

10. If $GCF(x, 600, 900) = 20$ & $LCM(x, 600, 900) = 25200$, then $x = 560$

11. Suppose $A = \{x \mid x \in \mathbb{N}, x^2 - 3x + 2 = 0 \Leftrightarrow x^2 - 2x - 8 > 0\}$.

$$P(A) = \{\emptyset, \{3\}, \{4\}, \{3, 4\}\}$$

Part III: Workout Item

(12pts)

Show all the necessary steps clearly and neatly!

12. Prove that $2(7^n) + 3(5^n) - 5$ is divisible by 24 by applying Mathematical induction.

13. Check whether the following argument form is valid or invalid by using formal proof.

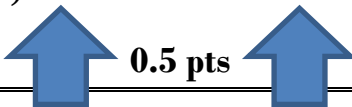
$$(p \Rightarrow q) \wedge (r \Rightarrow s), (q \Rightarrow t) \wedge (s \Rightarrow w), \neg(t \wedge w), p \Rightarrow r \vdash p$$

14. Show that $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \wedge (p \wedge \neg r)$ is contradiction by using logical equivalency.

THE END

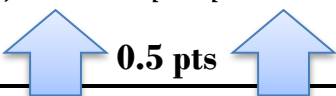
12.**Proof: Method 1****Step 1:** Check for $n = 1$ ($\because$ since there is no restriction on n , start from 1 by default).

$$2(7^1) + 3(5^1) - 5 = 24 \rightarrow \text{which is divisible by 24}$$


0.5 pts
Step 2: Assume the given statement is true for $n = k$

$$2(7^k) + 3(5^k) - 5 \text{ is divisible by 24}$$

$$2(7^k) + 3(5^k) - 5 = 24q, \quad q \in \mathbb{N}$$


0.5 pts
Step 3: Verify the given statement is true for $n = k + 1$

$$\begin{aligned} 2(7^{k+1}) + 3(5^{k+1}) - 5 &= 2(7 \cdot 7^k) + 3(5 \cdot 5^k) - 5 \\ &= 14(7^k) + 15(5^k) - 5 \\ &= 2(7^k) + 12(7^k) + 3(5^k) + 12(5^k) - 5 \\ &= 2(7^k) + 3(5^k) - 5 + 12(7^k) + 12(5^k) \\ &= 24q + 12(7^k + 5^k) \end{aligned} \quad (*)$$


1 point
It is also possible to prove that $7^n + 5^n$ is divisible by 2 using PMI.**Step 1:** Check for $n = 1$

$$7^1 + 5^1 = 12 \rightarrow \text{which is divisible by 2}$$

Step 2: Assume it is true for $n = k$

$7^k + 5^k \rightarrow$ which is divisible by 2

$$7^k + 5^k = 2p$$

Step 3: Verify it is true for $n = k + 1$

$$\begin{aligned} 7^{k+1} + 5^{k+1} &= 7 \cdot 7^k + 5 \cdot 5^k \\ &= (2 + 5)7^k + 5 \cdot 5^k \\ &= 2 \cdot 7^k + 5 \cdot 7^k + 5 \cdot 5^k \\ &= 2 \cdot 7^k + 5(7^k + 5^k) \\ &= 2 \cdot 7^k + 5(2p) \\ &= 2(7^k + 5p) \\ &= 2n, \quad n = 7^k + 5p \end{aligned}$$

Hence, $7^n + 5^n$ is divisible by 2



1point

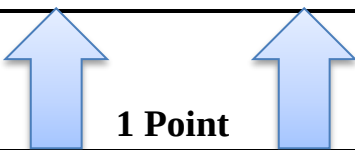
$$7^k + 5^k = 2r \quad (**)$$

Substituting **(**)** into **(*)** gives:

$$\begin{aligned} 2(7^{k+1}) + 3(5^{k+1}) - 5 &= 24q + 12(7^k + 5^k) \\ &= 24q + 12(2r) \\ &= 24q + 24r \\ &= 24(q + r) \end{aligned}$$

$2(7^{k+1}) + 3(5^{k+1}) - 5$ is divisible by 24 (It works for $n = k + 1$)

Hence, $2(7^n) + 3(5^n) - 5$ is divisible by 24 by PMI.

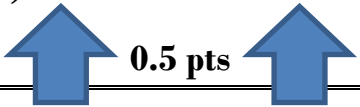


1 Point

12.

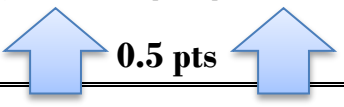
Proof: Method2**Step 1:** Check for $n = 1$ ($\because$ since there is no restriction on n , start from 1 by default).

$$2(7^1) + 3(5^1) - 5 = 24 \rightarrow \text{which is divisible by 24}$$


 0.5 pts
Step 2: Assume the given statement is true for $n = k$

$$2(7^k) + 3(5^k) - 5 \text{ is divisible by 24}$$

$$2(7^k) + 3(5^k) - 5 = 24q, \quad q \in \mathbb{N}$$


 0.5 pts
Step 3: Verify the given statement is true for $n = k + 1$

$$\begin{aligned} 2(7^{k+1}) + 3(5^{k+1}) - 5 &= 2(7 \cdot 7^k) + 3(5 \cdot 5^k) - 5 \\ &= 7(2(7^k)) + 5(3(5^k)) - 5 \end{aligned}$$

$$\text{From step 2, } 2(7^k) = 24q + 5 - 3(5^k)$$

$$\begin{aligned} 2(7^{k+1}) + 3(5^{k+1}) - 5 &= 2(7 \cdot 7^k) + 3(5 \cdot 5^k) - 5 \\ &= 7(24q + 5 - 3(5^k)) + 5(3(5^k)) - 5 \\ &= 24 \times 7m + 35 - 21(5^k) + 15(5^k) - 5 \\ &= 24 \times 7m - 6(5^k) + 30 \end{aligned}$$

$$2(7^{k+1}) + 3(5^{k+1}) - 5 = 24 \times 7m - 6(5^k - 5) \quad (*)$$


 1 point
It is also possible to prove that $5^n - 5$ is divisible by 4 using PMI.**Step1:** Check for $n = 1$: $5^1 - 5 = 0 \rightarrow$ which is divisible by 4

Step 2: Assume it is true for $n = k$

$5^k - 5 \rightarrow$ which is divisible by 4

$$5^k - 5 = 4p$$

Step 3: Verify it is true for $n = k + 1$

$$\begin{aligned} 5^{k+1} - 5 &= 5 \cdot 5^k - 5 \\ &= (4 + 1)5^k - 5 \\ &= 4 \times 5^k + 5^k - 5 \\ &= 4 \times 5^k + 4p \\ &= 4(5^k + p) \\ &= 4r, r = 5^k + p \end{aligned}$$

Hence, $5^n - 5$ is divisible by 4



1 point



$$5^k - 5 = 4r$$

(**)

Substituting (**) into (*) gives:

$$\begin{aligned} 2(7^{k+1}) + 3(5^{k+1}) - 5 &= 24 \times 7m - 6(5^k - 5) \\ &= 24 \times 7m - 6(4r) \\ &= 24 \times 7m - 24r \\ &= 24(7m - r) \end{aligned}$$

$2(7^{k+1}) + 3(5^{k+1}) - 5$ is divisible by 24 (It works for $n = k + 1$)

Hence, $2(7^n) + 3(5^n) - 5$ is divisible by 24 by PMI.



1 point



Remark: Other ways are also possible, Check it if the students use other techniques.

13.

Statement**Reason**

1. $(p \Rightarrow q) \wedge (r \Rightarrow s)$ is True	Premises
2. $(q \Rightarrow t) \wedge (s \Rightarrow w)$ is True	Premises
3. $\neg(t \wedge w)$ is True	Premises
4. $p \Rightarrow r$ is True	Premises
5. $p \Rightarrow q$ is True	From Step 1
6. $r \Rightarrow s$ is True	From Step 1
7. $q \Rightarrow t$ is True	From Step 2
8. $s \Rightarrow w$ is True	From Step 2
9. $\neg t \vee \neg w$ is True	De-Morgan's Law on Step3
10. $t \Rightarrow \neg w$ is True	From Step10
11. $p \Rightarrow t$ is True	Principle of Syllogism on step 5 & 7 respectively
12. $r \Rightarrow w$ is True	Principle of Syllogism on step 6 & 8 respectively
13. $p \Rightarrow \neg w$ is True	Principle of Syllogism on step 11&10 respectively
14. $\neg w \Rightarrow \neg r$ is True	Contrapositive of Step 12
15. $p \Rightarrow \neg r$ is True	Principle of Syllogism on step 13&14 respectively
16. $\neg r \Rightarrow \neg p$ is True	Contrapositive of Step 4
17. $p \Rightarrow \neg p$ is True	Principle of Syllogism on step 15&16 respectively
18. $\neg p \vee \neg p$ is True	From Step 17
19. $\neg p$ is True	From Step 18
20. p is False	From Step 19

Hence, the given argument form is Invalid.

Step1-Step4:	1 point
Step5-Step10:	1 point
Step11-Step16:	1 point
Step17-Step20:	1 point

14

$$\begin{aligned}
 [(p \Rightarrow q) \wedge (q \Rightarrow r)] \wedge (p \wedge \neg r) &\equiv [(\neg p \vee q) \wedge (\neg q \vee r)] \wedge (p \wedge \neg r) \\
 &\equiv (\neg p \vee q) \wedge (\neg q \vee r) \wedge (p \wedge \neg r) \quad \text{(1 Point)} \\
 &\equiv (\neg p \vee q) \wedge (\neg q \vee r) \wedge p \wedge \neg r \\
 &\equiv p \wedge (\neg p \vee q) \wedge \neg r \wedge (\neg q \vee r)
 \end{aligned}$$

$$\begin{aligned}
 &\equiv [p \wedge (\neg p \vee q)] \wedge [\neg r \wedge (\neg q \vee r)] \\
 &\equiv [(p \wedge \neg p) \vee (p \wedge q)] \wedge [(\neg r \wedge \neg q) \vee (\neg r \wedge r)] \quad \text{(1.5Point)} \\
 &\equiv [F \vee (p \wedge q)] \wedge [(\neg r \wedge \neg q) \vee F]
 \end{aligned}$$

$$\begin{aligned}
 &\equiv (p \wedge q) \wedge (\neg r \wedge \neg q) \\
 &\equiv p \wedge q \wedge \neg r \wedge \neg q \\
 &\equiv p \wedge q \wedge \neg q \wedge \neg r \quad \text{(1.5Point)} \\
 &\equiv p \wedge F \wedge \neg r \\
 &\equiv F
 \end{aligned}$$

Therefore, $[(p \Rightarrow q) \wedge (q \Rightarrow r)] \wedge (p \wedge \neg r)$ is Contradiction.

THE END.