

The State of Eritrea
Ministry of Education

Mathematics for Grade 8

Textbook

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Introduction

This Mathematics textbook for Grade 8 is prepared by the ministry of education and is designed to make use of apply the interactive approach of the new curriculum, so as to make you active participants in the learning process.

This new approach helps the students widen their thinking ability, the knowledge of core subject matter of mathematics and solve mathematical problems and relate this with their daily activities. To do these, it starts from solving simple basic problems and proceed to a bit difficult and then go to solve difficult problems and acquire a deep knowledge and skill of what is discussed in the class or out side the class. This is done step by step so that you can understand it easily and apply it in your daily activities.

By the end of grade 8, students can: describe proportional relationship involving numbers, geometry and statistics with the set of real numbers; operate decimals and fractions with the four mathematical operations under the set of real numbers. The can select and use appropriate operations to solve problems and use them to present real life situations; describe and draw angles; measure and calculate surface area and volume of solid figures. They can present data on graphs and calculate the three averages and probability of a given data.

The book consists of 12 chapters. Most of them are continuations of the prior grades. This deal with numbers of different patterns, Algebraic relationships, geometry of lines and angles, geometric figures (plane figure): their perimeters and areas, solids and their volumes and data handling (statistics and probability). Moreover, some new chapters such as congruency and similarity of geometric figures, Business mathematics (profit and loss, discount and comission, simple interest and direct and indirect vatiations) and elementary concept of transformation geometry are introduced.

At the beginning of each chapter there are expected learning outcomes of the chapter. These will identify the knowledge, skills, values and attitudes that you will be able to achieve after learning the chapter.

You will practise different activities in each chapter so as to clearly understand the main concepts and to enhance the application of the contents of each chapter.

Moreover, to know whether you have achieved the intended objectives or not you will be assessed continuously using different methods.

The assessment may be in a form of: classwork, Homework, Assignment, Project work, Quizzes, Tests and Exams. These will help you to evaluate how far you have understood the concept and identify any mis-conception and find remedial measures.

It is wise to recommend parents and interested individuals to play their role in the process so as to meet the target.

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Chapter 1

Real Number System

By the end of the chapter you will be able to:

- identify and describe different types of numbers; such as rational and irrational numbers; and
- describe real number system;

1.1 Rational Numbers

Activity 1.1

- A teacher asked his students to mention some sets of numbers. Can you mention some?

Dawud mentioned the following sets of numbers.

The set of *positive integers*, the set of *negative integers* and zero. Dawit said all of these are called the set of *integers*.

There are also *fractions* which are not integers such as: $\frac{1}{2}$, $\frac{3}{4}$,

$\frac{5}{6}$ etc. The set that contains such numbers is called the set of

rational numbers.



A rational number is any number that can be written in the form of $\frac{a}{b}$ where a and b are integers and $b \neq 0$

- The following are examples of rational numbers.

7 which can be expressed as $\frac{7}{1}$

4 which can be expressed as $\frac{4}{1}$

-5 which can be expressed as $-\frac{5}{1}$

0.25 which can be expressed as $\frac{1}{4}$

3. Write each of the following as ratios of two integers (in the form of $\frac{a}{b}$).

- a) 0 b) -9 c) 0.45 d) 7 e) 3.5 f) -2.3

4. Can you give some more examples of rational numbers?

5. Are all integers rational numbers?

6. Is there an integer which is not a rational number?

Relationship of Decimals and Fractions:

Rational numbers can be expressed in a decimal form by dividing the numerator by the denominator. For example

7. Abdu divides 3 kilogram of sugar to 8 people.

What is the share of each one?

Solution:

The share of each one = $\frac{3}{8}$ kg

We can change $\frac{3}{8}$ of a kilogram into decimal form by dividing 3 by 8.

$$\begin{array}{r} 30 \div 8 \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\frac{3}{8} = 0.375 \text{ Kg}$$

8. A woman had 11 Nakfa and shared among 4 of her sons.

What was the share of each son?

We can also write in a short form as $\frac{2}{3} = \overline{0.6}$ or $\overline{0.6} = 0.6666\dots$

10. Similarly, the fraction $\frac{7}{11}$ can be written as

$$\begin{array}{r}
 70 \div \underline{11} \\
 \underline{66} 0.636363\dots \\
 40 \\
 \underline{33} \\
 70 \\
 \underline{66} \\
 40 \\
 \underline{33} \\
 7
 \end{array}$$

In this case the number $\overline{63}$ repeats again and again and never ends.

We write $0.636363\dots$ as $0.\overline{63}$. The segment over the number means the numbers are repeating.

Activity 1.2

1. Change the following fractions into a decimal form.

a) $\frac{3}{4}$

b) $\frac{2}{5}$

c) $\frac{7}{8}$

d) $\frac{5}{8}$

e) $\frac{9}{4}$

f) $\frac{7}{5}$

g) $3\frac{2}{5}$

h) $5\frac{1}{4}$

2. Change each of the following fractions into a decimal form.

a) $\frac{2}{3}$

b) $\frac{5}{6}$

c) $\frac{1}{3}$

d) $\frac{5}{9}$

e) $\frac{7}{11}$

In what way are the answers in exercises 1 and 2 different ?



The answers in question number 1 terminate while those in question number 2 do not terminate .

Do you agree ?



Decimal numbers such as $\frac{2}{3} = 0.666\dots$, $\frac{7}{11} = 0.636363\dots$ are called repeating decimals or recurring decimals. The decimals such as $\frac{3}{4} = 0.75$ and $\frac{11}{4} = 2.75$ are called terminating decimals.



From the above examples a rational number may be expressed as a *terminating decimal* or as a *repeating decimal*.

Activity 1.3

1. Classify the following numbers as terminating decimals or as repeating decimals

a) $\frac{1}{8}$

b) $\frac{5}{12}$

c) $\frac{7}{20}$

d) $\frac{13}{8}$

e) $\frac{51}{6}$

f) $\frac{4}{9}$

2. a) Is there any short method by which we can determine whether a given fraction is repeating or terminating decimal?
b) Discuss with your partner and tell if there is.
3. Factorize the denominator of each fraction and observe the prime factors of each denominator.
a) What did you observe?
b) Do you recall what prime numbers are?
4. Discuss with your classmates how to find the prime factorization of a composite number.

The prime factors of the denominators of terminating decimals consists of factors 2 or 5 or both 2 and 5 only. Do you agree with the given statement?

5. How do you change repeating decimals into fractions? In grade 6, you learned how to convert numbers from decimals into fractions. You did like this.

a) $0.5 = \frac{5}{10} = \frac{1}{2}$

b) $0.25 = \frac{25}{100} = \frac{1}{4}$

c) $0.321 = \frac{321}{1000}$

d) $3.5 = \frac{35}{10} = \frac{7}{2}$.

6. Is the fraction $\frac{7}{20}$ repeating or terminating decimal?

Solution:

The prime factorization of $20 = 2 \times 2 \times 5$.

Since the prime factorization of 20 consist factors of 2 and 5 only, the fraction is terminating decimal.

7. Is the fraction $\frac{7}{12}$ repeating or terminating decimal?

Solution:

The prime factors of $12 = 2 \times 2 \times 3$.

Since the prime factors of 12 consist factors of 3, the fraction is non terminating (it is repeating decimal).

Can we convert the decimals 0.6..., 0.2323..., 4.3555... by the same method ?



No, we have another method of conversion for recurring decimals. But first we should know how to multiply them.

8. First observe how repeating decimals are multiplied by 10, 100, 1000

$$\text{a) } 0.6 \dots \times 10 = 6.6\dots = 6.\dot{6}$$

$$0.6 \dots \times 100 = 66.6\dots = 66.\dot{6}$$

$$0.6 \dots \times 1000 = 666.6\dots = 666.\dot{6}$$

$$\text{b) } 0.2323\dots \times 10 = 2.323\dots$$

$$0.2323\dots \times 100 = 23.23\dots$$

$$0.2323\dots \times 1000 = 232.323\dots$$

$$\text{c) } 0.345\dots \times 10 = 3.45345\dots$$

$$0.345\dots \times 100 = 34.5345\dots$$

$$0.345\dots \times 1000 = 345.345\dots$$

9. a) Did you understand the above examples?

b) Discuss about each example with your classmates.

Haile changed $0.\dot{6}$ into fractional form using the following

steps: First he let the number equal to $x : x = 0.\dot{6}$;
Then he multiplied both sides of the equality by 10
Finally, he subtracted the first equation from the second equation;

$$x = 0.\dot{6}$$

$$x \times 10 = 0.\dot{6} \times 10$$

$$10x = 6.\dot{6}$$

$$10x - x = 6.\dot{6} - 0.\dot{6}$$

$$\text{Let } x = 0.\overline{6}$$

$$10x = 6.\overline{6}$$

$$10x - x = 6.\overline{6} - 0.\overline{6}$$

$$9x = 6$$

$$x = \frac{6}{9}$$

$$= \frac{2}{3}$$

10. Similarly, he can convert $0.\overline{23}$ to fractional form as:

$$\begin{aligned}\text{Let } x &= 0.\overline{23} \\ 100x &= 23.\overline{23} \\ 100x - x &= 23.\overline{23} - 0.\overline{23} \\ 99x &= 23 \\ x &= \frac{23}{99}\end{aligned}$$

What about if the decimal is mixed that is non repeating and repeating like $1.2333\dots$?



First separate the repeating decimal from the non repeating decimal by multiplying either by 10 or by 100 or by 1000 etc. depending on the number of repeating decimals



For example to change $1.2333\dots$, into fraction we follow the following steps.

Step 1. Take the number $x = 1.2333\dots$

Step 2. Multiply by 10 both sides of the equation in step 1.

$$\begin{aligned}10x &= 1.233\dots \times 10 \\ 10x &= 12.333\dots\end{aligned}$$

Step 3. Multiply again by 10 both sides of the equation in step 2.

$$\begin{aligned}10x \times 10 &= 12.333\dots \times 10 \\ 100x &= 123.3\dots\end{aligned}$$

Step4. Subtract Step 2 from Step 3:

$$100x - 10x = 123.3... - 12.3...$$

$$90x = 111 \quad \text{divide by } 90$$

$$x = \frac{111}{90}$$

$$x = \frac{37}{30}$$

Activity 1. 4

1. Determine whether the following are terminating or non-terminating decimals. Use the above methods.

a) $\frac{7}{8}$ b) $\frac{11}{32}$ c) $\frac{31}{40}$ d) $\frac{5}{24}$ e) $\frac{13}{25}$ f) $\frac{9}{11}$

2. Multiply each of the following by 10, 100 and by 1000.

a) $0.\overline{45}$ b) $2.\overline{45}$ c) $0.\overline{123}$

3. Change the following decimals into fractional forms.

a) 0.1515.... b) 0.235235... c) 3.25666.... d) 4.0909...
e) 0.34555.... f) 7.33333..... g) 5.4323232....

4. Can you change the above decimals into fractions using the given method in the examples?

Discuss with your friends.

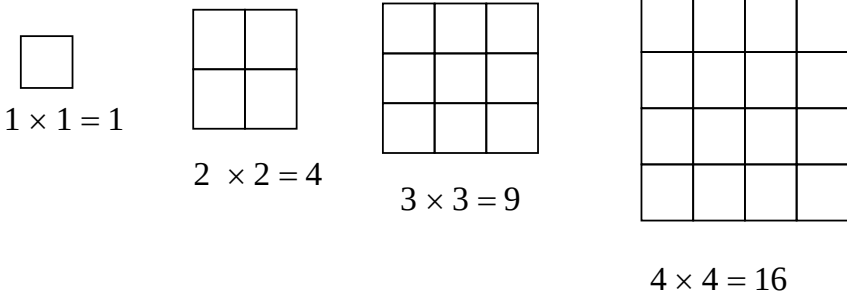
Does it work for all decimals?

1.2 Square and Square Root of Numbers

Activity 1.5

1. To get a better idea about square and square root of numbers, you can explore the connection between the length of a side and the area of a square.

Find the number of square units in the following figures as follows:



2. Find the length of the side of the square, if there are:

- a) 36 square units
- b) 64 square units
- c) 100 square units
- d) 81 square units
- e) 121 square units

Idris calculated the answers for (a) and (b) as follows

$$\sqrt{36} = 6 \text{ since } 6 \times 6 = 36$$

$$\sqrt{64} = 8 \text{ since } 8 \times 8 = 64$$

The symbol $\sqrt{\quad}$ is a sign for square root. 6 is the square root of 36 and 36 is 6 squared.

Numbers such as 9, 16, 25 etc. are called perfect squares, because

$$9 = 3 \times 3$$

$$16 = 4 \times 4$$

25 = 5 x 5, they are the squares of the numbers 3, 4, and 5.

3. Can you give some more examples of numbers which are perfect squares such as:

$$\sqrt{36} = 6 \quad \text{since} \quad 6 \times 6 = 36$$

$$\sqrt{49} = 7 \quad \text{since} \quad 7 \times 7 = 49, \text{ etc}$$

We can find the square root of numbers by prime factorization. Do you know what prime factorization means from grade 6?

For example: prime factorization of

$$\text{a) } 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

$$\text{b) } 225 = 3 \times 3 \times 5 \times 5$$

$$\begin{aligned} \text{a) } \sqrt{144} &= \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3} \\ &= \sqrt{(2 \times 2 \times 3) \times (2 \times 2 \times 3)} \\ &= 2 \times 2 \times 3 \\ &= 12 \end{aligned}$$

$$\begin{aligned} \text{b) } \sqrt{225} &= \sqrt{3 \times 3 \times 5 \times 5} \\ &= \sqrt{(3 \times 5) \times (3 \times 5)} \\ &= 3 \times 5 \\ &= 15 \end{aligned}$$

Activity 1. 6

1. Does every positive number have a square root?
2. Can you find the square roots of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$, $\sqrt{13}$ etc

We have discussed about terminating decimals such as

$$\frac{3}{4} = 0.75$$

and repeating decimals such as

$$\frac{5}{7} = 0.\overline{714285}$$

3. Are there non-terminating and non-repeating decimals?

We have also discussed about perfect squares such as:

$$\sqrt{64} = 8, \text{ since } 8 \times 8 = 64$$

There are decimals which are not *terminating* and not *repeating*. Such decimals are called *irrational numbers*. Irrational numbers are not perfect squares.

The following numbers are irrational numbers.

0.1234567891011....

0.1010010001....

$\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, etc.

Discuss with your friends why $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, etc.

are irrational numbers, but $\sqrt{4}$ is a rational number

Irrational numbers are numbers which can not be expressed as ratio of two integers. They are neither repeating nor terminating decimals.

4. Can every number be written in the form of a rational number?
5. Which of the following are rational numbers? Underline them. 0 , $\sqrt{16}$, $\frac{3}{5}$, $\sqrt{12}$, -24 , 1 , $\sqrt{10}$,
6. Which of the following are irrational numbers? Underline them. $0.454545\dots$, 0 , -24 , 1 , 78 ,
7. Is the sum of two rational numbers rational or irrational? Give two examples.
8. Is the sum of two irrational numbers rational or irrational?

Real Numbers

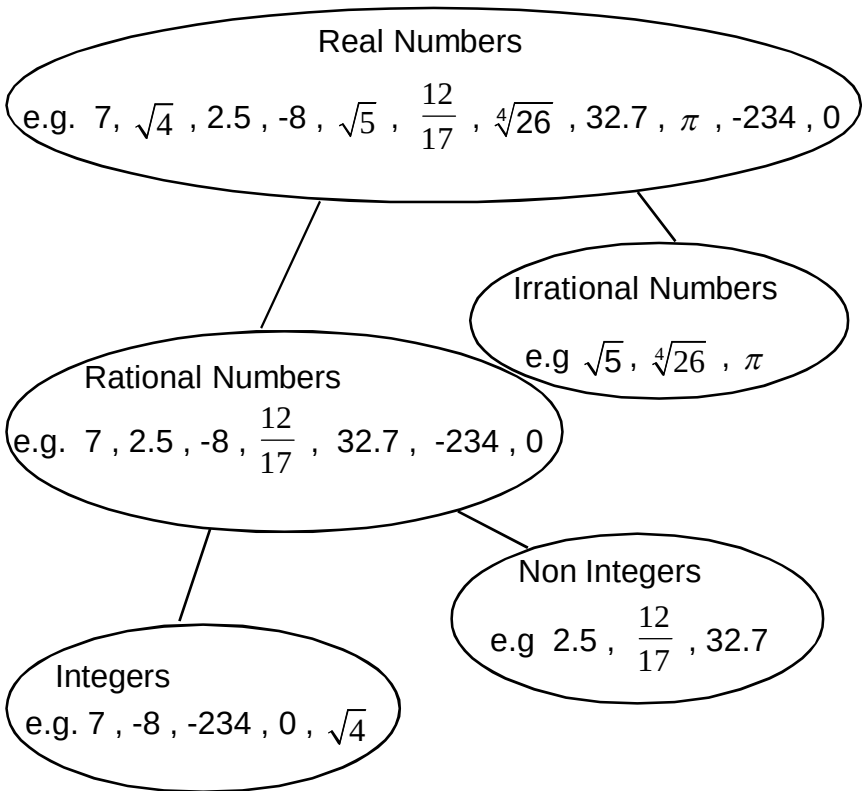
Activity 1.7

1. We discussed about different types of numbers.

- Are all rational numbers real numbers?
- Are all irrational numbers real numbers?
- What are real numbers?

The *rational* numbers and the *irrational* numbers are two distinct sets of numbers. Taking together rational and irrational numbers form the set of *real numbers* (R).

Recall the relationship between the different types of numbers you have learnt from the following diagram.



2. Classify each of the following by making $\checkmark$.

Number	Natural numbers	Whole numbers	Integers numbers	Rational	Irrational numbers	Real numbers
2.5						
− 0.6						
0						
24						
$7\frac{1}{4}$						
$\frac{-7}{11}$						
$\sqrt{9}$						
$\sqrt{5}$						
0.53535						
3...						
$\sqrt{11}$						
- 234						

Square Root of Positive Numbers by General Method .

Activity 1.8

1. A contractor built a square hall having an area of 4489 m^2 . How long is the hall?

Aster said, since the hall is a square its length and width are equal. So we must find the square root of the area to find the side of the hall.

$$l = \sqrt{4489}$$

But she could not able to find the square root by factorization. Ali said why do not we do by the general method? Can you do it? Yes, I can follow these steps.

- Mark off the digits in groups of 2 starting from the right.
- Estimate the highest root for 44.
- Add 6 and 6 and think of a number after 2 so that $12 \times x$, where the product is less or equal to 889.

$$\begin{array}{r|l}
 67 & \\
 \hline
 44,89 & \\
 \underline{36} & 6 \times 6 \\
 889 & 127 \times 7 \\
 \underline{889} &
 \end{array}$$

Hence, $\sqrt{4489} = 67$

The length of the hall is 67 m.

Zahra said I did not understand your steps Ali. Can you solve, if the area of the hall is 17424 m^2 by the same method ?

$$\begin{array}{r}
 132 \\
 \overline{) 1,74,24} \\
 \underline{1} \\
 = 74 \\
 \underline{69} \\
 = 524 \\
 \underline{524} \\
 = = =
 \end{array}
 \begin{array}{l}
 1 \times 1 \\
 23 \times 3 \\
 262 \times 2
 \end{array}$$

Hence $\sqrt{17424} = 132$

The length of the hall is 132 m.

2. Do you understand his steps?
3. Discuss about each step with your partners and explain to the class your suggestions.
4. Find the square root of each of the following numbers by the general method.
 - a) 324 b) 1225 c) 1296 d) 6561 e) 18 f) 20
5. 5329 square tiles are needed to cover a square floor. How many tiles are there in each row?
6. A man walked around a square field having an area of 1444 m^2 . How far did he walk on one side of the field?
7. The square of a number is 5184. Find the number?

Chapter 2

Equations and Inequalities

By the end of the chapter you will be able to:

- a) express statements (concepts) mathematically and formulate a mathematical equation from a situational problem;
- b) compare the values of the variables and express one in terms of the other;
- c) formulate and solve expressions of linear equations, linear inequalities, quadratic equations and word problems

2.1. Equations

Activity 2.1

1. The table below shows the age range of 6 students in grades 6 and 8 at your school.

Grade 6	12	13		15		17
Grade 8	14	15	16	17	18	

Based on the above table, answer the following questions.

- a) Complete the table.
- b) What relationship do you notice between the pair of ages of the two grades?
- c) Write a mathematical expression that represents the relationship?
- d) Do you observe that in each case the age of a grade 8 student is 2 years more than the age of a grade 6 student?

That is, age of a grade 8 = age of grade 6 + 2.

- e) What will be the age of a grade 8 student, If the age of a grade 6 student is:

i) 20 ii) 25 iii) x

Mathematically it can be represented by $y = x + 2$ where y is the age of a grade 8 student and x is the age of a grade 6 student.

2. The table below shows the number of fishes collected by Semere and Abraham in Massawa during the 1st week of July 2004.

Days	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Semere	10	15	20	30		38	
Abraham	19	24	29	39	40		63

- Complete the above table.
- What difference do you observe between the number of the fish collected by the two persons?
- Write an equation that shows the relationship of the number of fishes collected by the two boys? Ibrahim formulated an equation: $y = x + 9$, where y is the number of fish collected by Abraham and x is the number of fish collected by Semere.
- Do you agree with Ibrahim's equation? Is there any other description?

3. Belay and Kalid have some amount of money in Nakfa as indicated below.

Belay	4	5	6	7	8	9	10	
Kalid	11	13	15	17	19			43

- What relationships do you observe between the amounts of money of the two boys?
- Write an equation that shows the relationship of the situation.
- Complete the table.
- How much money will Belay own when Kalid owns 63 Nakfa?

4. A teacher gave the following question to the students.
 "Seid has a certain Nakfa in his pocket, while Haile has 3 Nakfa less than twice Seid's money".

- a) Write a mathematical equation for the relationship of the money they have?
- b) Tsehay derived the formula $y = 2x - 3$ for the relationship. Do you agree with Tsehay's equation? Complete the table using Tsehay's equation.

Said	Haile
5	
	11
8	
12	
	35
	41
30	

- c) Find the values of the ordered pairs that satisfy the equation for $x = 2, 3$ and 4 .

Activity 2.2

1. Find the values in each case if $x = -2, 0$, and 3 ,

a) $\frac{x+2}{3}$

b) $\frac{x}{2} - 1$

c) $7x - 9$

d) $-2x - 1$

e) $\frac{3(5x-1)}{4} - \frac{2x}{3}$

f) $\frac{7x}{5} + 2$

2. Solve for a if:

a) $2a + 3 = 5$

b) $5(a - 1) - 2a = 30$

c) $\frac{a}{3} - 4 = 10 + \frac{a}{5}$

d) $\frac{a-2}{4} - \frac{a+5}{6} = 2$

e) $6(a - 6) = 2(4 - a)$

f) $5(a + 4) = 3(-a - 2)$

3. Asmara soap factory packs 150 Gazella soaps in each box.
 - a) How many soaps are packed in 17 boxes?
 - b) How many boxes are needed to pack 75,000 soaps?
 - c) How many boxes are needed to pack 500 soaps?
 - d) How many soaps are packed in x boxes?
 - e) Write a mathematical sentence for the situation.

4. The age of the father is 2 years more than three times the age of his son.
 - a) Express the age of the father in terms of the age of his son
 - b) If the sum of their ages is 98 years, how old is the father?

5. Semere is three years less than twice the age of his sister. Five years ago the sum of their ages was 23 years. Find their present ages?

2.2 System of Linear Equations (Simultaneous)

Activity 2.3

1. Fatuma and Semira have 15 Nakfa all together.
 - a) How much do you think each may have?
 - b) Write some ordered pairs that satisfy the situation.

Adiam: made the following to derive the possible solutions for the relationship.

Fatuma	Semira
10	5
9	6
8	7
.	.
.	14

N.B. The above relation can be reversed such that Fatuma's share could be replaced by Semira's share.

- c) Can you express mathematically, if Fatuma's money is represented by "y" and Semira's money is represented by "x"?

Alem expressed the equation by $y + x = 15$.

- d) Do you agree with Alem?

- e) Do the ordered pairs: (1, 14), (2, 13), (3, 12), (4, 11), (5, 10), (6, 9) and (7, 8) belong to the equation?

2. The teacher gave additional information; that the difference of the amount of money which each of them have is 3 Nakfa.

- a) Could you suggest how to express mathematically the given situational problem?

- b) Alem does as follows:

let Fatuma's share be y, and Semira's share be x;
then $y - x = 3$.

Combining the two equations we get:

$$\begin{cases} y + x = 15 \\ y - x = 3 \end{cases}$$

- c) Can you solve the equations simultaneously?

Such system of equations can be solved by either elimination or substitution method.



A. Elimination Method:

3. Ali uses elimination method and he did as follows:

$$\begin{cases} y + x = 15 \\ y - x = 3 \end{cases}$$

Adding up the two equations

$$\begin{cases} 2y + 0 = 18 \\ y = 9 \end{cases} \quad (\text{See that } x + (-x) = 0. \text{ Here } x \text{ is eliminated})$$

To get the value of x , we replace $y = 9$ in either one of the equations.

$$9 + x = 15$$

$$x = 15 - 9$$

$$x = 6$$

Hence, $x = 6$ and $y = 9$.

4. Solve the equations below by elimination method:

$$2y + 3x = 4 \quad (1)$$

$$5y - 2x = 29 \quad (2)$$

Can we eliminate one of the variables simply by adding or subtracting the first equation from the second ?



To eliminate one of the variables we need to multiply by some numbers that make one of the variables in three equations equal but opposite in sign. If we multiply equation (1) by 2 and equation (2) by 3, the newly formed system looks like:

$$\begin{cases} 4y + 6x = 8 \\ 15y - 6x = 87 \end{cases}$$



Adding up the equations: we get $19y + 0 = 95$

$$19y = 95$$

$$y = 5$$

To get the value of x , We replace $y = 5$ in either one of the original equations.

$$5(5) - 2x = 29$$

$$25 - 2x = 29$$

$$-2x = 29 - 25$$

$$-2x = 4$$

$$x = -2$$

$$x = -2 \text{ and } y = 5$$

Activity 2.4

Solve each of the following pairs of equations by elimination method:

1. $x + y = 14$

$$x - y = 4$$

2. $x - y = -1$

$$2x - y = 0$$

3. $3x - y = 9$

$$4x - y = -14$$

4. $x - y = -1$

$$3x - y = 5$$

5. $5x + 4y = 4$

$$3x + 4y = 8$$

6. $3x + 5y = 5$

$$3x + 9y = -3$$

7. $3x + 2y = 0$

$$-x + y = 5$$

8. $3x - y = -2$

$$x - 3y = 10$$

9. $3x - 2y = 7$

$$4x + y = 13$$

10. $3x - y = 9$

$$2x + 2y = 14$$

11. $3x + 28y = 1$

$$6x - 7y = 25$$

12. $2x + 5y = 24$

$$4x + 3y = 20$$

13. $2x + 7y = 17$

$$5x + 3y = -1$$

14. $5x - 2y = 9$

$$7x - 2y = 14$$

B. Substitution Method:

Almaz used substitution method and she did it as follows:

$$\begin{cases} y + x = 15 \\ y - x = 3 \end{cases}$$

In the second equation, expressing y in terms of x ; we get

$$y = x + 3$$

Replacing $y = x + 3$ in $y + x = 15$;

$$(x + 3) + x = 15;$$

$$2x + 3 = 15 \text{ putting like terms together;}$$

$$2x = 12$$

$$x = 6$$

Replace $x = 6$ in either one of the equations;

$$y = x + 3$$

$$y = 6 + 3 = 9$$

$$\therefore x = 6 \text{ and } y = 9$$

Activity 2.5

Solve each of the following pairs of simultaneous equations by the substitution method:

$$\begin{aligned} 1. \quad x &= 5 - y \\ 2x - 2y &= 5 \end{aligned}$$

$$\begin{aligned} 2. \quad 2x - y &= 1 \\ y &= 13 - 5x \end{aligned}$$

$$\begin{aligned} 3. \quad 4x + 3y &= 25 \\ y &= 2x - 5 \end{aligned}$$

$$\begin{aligned} 4. \quad y &= -1 - 2x \\ 7x - 5y &= -1 \end{aligned}$$

$$\begin{aligned} 5. \quad 4x + y &= 14 \\ x &= 13 - 5y \end{aligned}$$

$$\begin{aligned} 6. \quad x &= -5 + 3y \\ 3x + 2y &= 7 \end{aligned}$$

$$\begin{aligned} 7. \quad x &= y - 3 \\ 4y &= 2x - 5 \end{aligned}$$

$$\begin{aligned} 8. \quad y &= x - 3 \\ x &= 2y + 21 \end{aligned}$$

$$\begin{aligned} 9. \quad 2x + y &= 4 \\ x - 5y &= 2 \end{aligned}$$

$$\begin{aligned} 10. \quad x - 5y &= 7 \\ 2x + y &= 3 \end{aligned}$$

$$\begin{aligned} 11. \quad 2x - y &= 4 \\ x + 3y &= 18 \end{aligned}$$

$$\begin{aligned} 12. \quad x + 5y &= 1 \\ 4x - y &= -2 \end{aligned}$$

$$\begin{aligned} 13. \quad x &= 11 - 2y \\ x &= 3y - 4 \end{aligned}$$

$$\begin{aligned} 14. \quad y &= 4 - 3x \\ 2y &= 4x - 3 \end{aligned}$$

Activity 2.6

1. Using both elimination and substitution methods solve the following systems of equations (simultaneous equation)

$$a) \begin{cases} y + 2x = 10 \\ y - 2x = 12 \end{cases}$$

$$b) \begin{cases} 2y + x = 10 \\ 2y - x = 12 \end{cases}$$

$$c) \begin{cases} 3y - 2x = 15 \\ 3y + 2x = 9 \end{cases}$$

$$d) \begin{cases} 2x + 2y = 12 \\ 2x - 2y = 12 \end{cases}$$

$$e) \begin{cases} 6x + 4y = 20 \\ 3x + y = 3 \end{cases}$$

$$f) \begin{cases} y + x = -10 \\ 2x - 3y = 0 \end{cases}$$

2. Solve the following simultaneous equations:

$$a) \begin{aligned} y - 3x &= 10 \\ 2y + 5x &= -12 \end{aligned}$$

$$b) \begin{aligned} 6x + 2y &= 34 \\ y &= x + 5 \end{aligned}$$

$$c) x + y + 2 = 20$$

$$d) x + y = 18$$

$$x = \frac{1}{2}y$$

$$2x + 2y = 30$$

$$e) \begin{aligned} x + y + 2 &= 23 \\ 3x &= y \end{aligned}$$

$$f) \begin{aligned} 3x + 4y &= 14 \\ 4x + 3y &= 7 \end{aligned}$$

3. The sum and difference of two distinct numbers is 30 and 6 respectively. What are the numbers?

4. Yemane paid 260 Nakfa for 8 kg of sugar and 3 kg of coffee at Semere's shop. Aster paid 400 Nakfa for 10 kg of sugar and 5kg of coffee from the same shop. Find the cost for 1 kg of sugar and coffee.

5. The cost of 7 hens and 10 eggs is 80 Nakfa and the cost of 10 hens and 7 eggs is 107 Nakfa. Find the cost of each hen and egg?
6. Abraha bought 10 sheep and 15 goats for 3500 Nakfa. If the cost of a sheep is twice that of a goat, what is the cost of one sheep and one goat?
7. Berhe bought one cow and one goat on Saturday. The cost of the cow plus the cost of the goat is 3500 Nakfa. The cost of the cow minus the cost of the goat is 2000 Nakfa. Find the cost of the cow as well as the cost of the goat.
8. In an electric shop the price of 5 televisions and 3 video decks is 37,000 Nakfa. The price of 4 televisions minus 2 video decks is 12,000 Nakfa. Find the price of one television, and the price of one Video deck.

2.3 Quadratic Equations:

Activity 2.7

1. Mr Ibrahim gives a task to his students to find the dimension of a rectangular shaped garden near the class in which its area is 32 m^2 . The length of the garden is twice its width.
 - a) Write a mathematical expression for the situation.
 - b) Find the length and width of the garden.

Haile did it as follows:

$$\text{a) Width (w) } \times \text{ Length (l) = Area of rectangle (A)}$$

$$w \times l = A$$

$$l = 2w$$

$$w(2w) = A$$

$$2w^2 = A$$

b) $2w^2 = 32 \text{ sqm}$

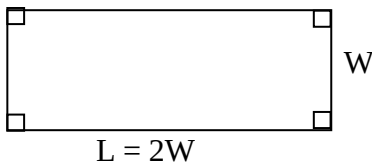
$w^2 = 16 \text{ sqm}$

$w = \sqrt{16} \text{ m}$

$w = 4 \text{ m}$

$l = 2w$

$= 2 \times 4 = 8 \text{ m}$



2. Find the products of the following:-

a) $x(2x - 1) = 2x^2 - x$

b) $3x(4 - 2x) = 12x - 6x^2$

$$\begin{aligned} \text{c) } (x - 1)(x + 2) &= x(x + 2) - 1(x + 2) \\ &= x^2 + 2x - x - 2 \\ &= x^2 + x - 2 \end{aligned}$$

Such products are known as quadratic equations and it is always written as $ax^2 + bx + c = 0$, where a,b,c are numbers.

3. Find the product of the following:

a) $y(y - 3) = y^2 - 3y$

b) $(x - 3)(x - 2)$

c) $x(5 - 6x)$

d) $(x - 3)(x + 3)$

e) $(y + 2)(2y - 1)$

f) $x(x - 9)$

4. Rewrite the following in the form $ax^2 + bx + c = 0$

a) $x^2 = x - 6$

b) $5y = y^2 - 6$

c) $4x^2 - 4x = 1$

d) $8 = 2x + 15x^2$

e) $3 - 12y = 12y^2$

f) $5z^2 = 6z + 1$

5. Factorize the following quadratic expressions:

a) $x^2 + 4x = x(x + 4)$

b) $3a^2 - 9a = 3a(a - 3)$

c) $x^2 - 3x$

d) $4x^2 - 4x$

e) $5x^2 - 25x$

f) $x^2 + 10x$

g) $6x - x^2$

h) $25x - x^2$

i) $(x + 3)^2$

j) $(x - 3)^2$

k) $x^2 - 4$

l) $x^2 - 9$

6. Factorize the following:

a) $10x - 15y$

c) $24xyz - 6xy$

e) $16x^2 - 8y^2$

g) $18y^2x - 9yx^2$

i) $x - x^2$

k) $x^2y - 4xyz$

b) $50xy - 120x$

d) $6abc + 12bcd$

f) $yx^2 + y^2x$

h) $10x - 15x^2$

j) $2a^2b^2 - 8a^2b$

l) $abc + a^2b + bc$

7. Simplify the following quadratic expressions:-

a) $4a^2 - a^2 - 1 = 3a^2 - 1$

b) $5a^2 - 2a^2 - 7a^2 + 11a^2 - 30$

$$= 5a^2 + 11a^2 + (-2a^2 - 7a^2) + (-30)$$

(grouping similar terms)

$$= 16a^2 - 9a^2 - 30$$

$$= 7a^2 - 30$$

c) $(3x^2 + 2x - 1) - (2x^2 + x + 1)$

d) $5(a - 7) - 3(4 - a)$

e) $(6y^2 - 5y) + 2(4y - y^2)$

f) $(2y^2 + x) + 2(3y - x^2)$

g) $(4x^2 + 5x) - 2(4x + x^2)$

h) $3(2y^2 - 5y) - 2(4y - 3y^2)$

Solving Quadratic Equations:**Activity 2.8**

1. Find the product of the following:

a) $2 \times 0 = 0$

b) $-2 \times 0 = 0$

c) $0 \times (-150) = 0$

d) $0 \times 25 = 0$

2. What is the product of $x \times 0$ 3. What is the difference between $x \times 0$ and $0 \times x$

If $A \times B = 0$, then either $A = 0$ or $B = 0$

4. Solve:

a) $x(x - 2) = 0$

b) $(x - 1)(x - 2) = 0$

c) $(x - 1)(x + 1) = 0$



Solution:

a) The two factors are x and $x - 2$. If the product is zero, then one of the factors must be zero. So, $x(x-2) = 0$,
then $x = 0$ or $x - 2 = 0$
 $x = 0$ or $x = 2$

b) $(x - 1)(x - 2) = 0$
 $x - 1 = 0$ or $x - 2 = 0$
 $x = 1$ or $x = 2$

c) $(x - 1)(x + 1) = 0$
 $x - 1 = 0$ or $x + 1 = 0$
 $x = 1$ or $x = -1$

Solve the following equations:

5. $7x(2x - 8) = 0$

6. $(x - 7)(x - 8) = 0$

7. $(y - 7)(y + 8) = 0$

8. $(x + 5)(x + 2) = 0$

9. $(z + 2)(z - 4) = 0$

10. $(8x + 7)(3x - 8) = 0$

11. $(3x + 6)(4 - 2x) = 0$

12. $(2a + 14)(3a - 9) = 0$

13. $(7x + 7)(4x - 8) = 0$

14. $(1 + 2y)(1 - 2y) = 0$

2.4 Inequalities:

Activity 2.9

1. Ali has 10 Nakfa while Abdu has 5 Nakfa in their pockets.
Express this in a mathematical expression?
2. One sack of sugar weighs 60kg, while one sack of salt weighs more than one sack of sugar. Express this in a mathematical equation.
3. The number of students in your class is less than 50.
Express the situation mathematically by representing the number of students by x .
4. Let your age be x . Express your age in relation to your grade. We represent less than by ($<$), greater than by ($>$), less or equal to ($\leq$) and greater or equal to ($\geq$) in inequalities.
5. Express the following mathematically:
 - a) m is greater than twice n .
 - b) Twice m minus five times n is less than 4.
 - c) m squared plus n squared is greater than 3.
 - d) y is less than $4x$ minus 2.
6. For what value of x will the following inequalities be true if " x " is the set of whole numbers.
 - a) $x+3 < 5$; b) $2x+1 \geq 7$

Solution:

$$\begin{aligned} \text{a) } x+3 &< 5 \\ x &< 5 - 3 \\ x &< 2 \\ \therefore x &= \{0, 1\} \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & 2x + 1 \geq 7 \\
 & 2x \geq 7 - 1 \\
 & 2x \geq 6 \\
 & x \geq 3, \\
 \therefore x &= \{3, 4, 5, 6, \dots\}
 \end{aligned}$$

7. Solve $3x - 1 \leq 8$, if x is an integer.

$$\begin{aligned}
 3x &\leq 9 \\
 x &\leq 3, \\
 x &= \{\dots, -1, 0, 1, 2, 3\}
 \end{aligned}$$

8. Express the following mathematical statements in the form of mathematical inequalities.

a) four minus twice a is greater or equal to 26.

b) thrice " c " plus " b " is less than 5.

c) five times x minus half of y is less than or equal to 6.

d) Seven times " r " is less than or equal to thrice " s " plus 8.

9. Express the following mathematical inequalities in the form of mathematical statements.

a) $y > x - 1$

b) $5x \geq 2y - 1$

c) $a < 3b + 5$

d) $3k \geq 4m - 2$

10. Solve the following inequalities under the indicated values.

a) $x + 5 < 6$, x is a whole number.

b) $2x - 3 \geq 5$, x is an Integer.

c) $\frac{5x - 3}{4} < 3$, If x is real number.

Between any two rational numbers on the number line there are infinite numbers of other rational or irrational numbers. In fact, every point on the number line corresponds to a real number and every real number corresponds to a point on the number line. Look at the following



11. Solve the following when “x” is in the set of real numbers.

- a) $4y + 3 > 11$
- b) $2y - 2 < 6$
- c) $2x + 4 \geq 6$
- d) $3x - 3 \leq 6$

Solution:

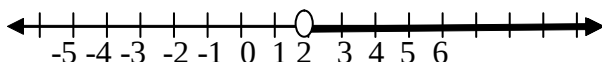
a) $4y + 3 > 11$

$$4y > 11 - 3$$

$$4y > 8$$

$$y > \frac{8}{4}$$

$$y > 2$$



N. B. In this case 2 is not included in the truth set.

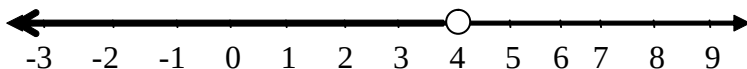
b) $2y - 2 < 6$

$$2y < 6 + 2$$

$$2y < 8$$

$$y < \frac{8}{2}$$

$$y < 4$$



N. B. In this case 4 is not included in the truth set.

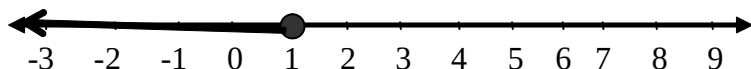
c) $2x + 4 \leq 6$

$2x \leq 6 - 4$

$2x \leq 2$

$x \leq \frac{2}{2}$

$x \leq 1$



N. B. In this case 1 is included in the truth set

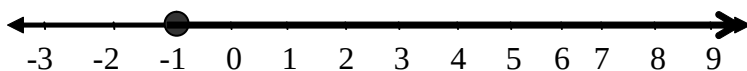
d) $3x - 3 \geq -6$

$3x \geq -6 + 3$

$3x \geq -3$

$x \geq \frac{-3}{3}$

$x \geq -1$



N. B. In this case -1 is included in the truth set

Activity 2. 10:

Show the values of x that satisfy each of the following using a number line for real numbers.

1. $3x < 0$

2. $2x - 1 > 7$

3. $4x + 3 \geq 5$

4. $-7x \geq 21$

5. $6 - x < 8$

6. $-5x < -25$

7. $-x - 5 > 4$

8. $3 - 4x \geq 12$

9. $4 > 4 - x$

10. $5 - \frac{x}{2} < -5$

11. $-\frac{1}{4}x \geq -4$

12. $\frac{x}{2} \leq -4$

Chapter 3

Graph

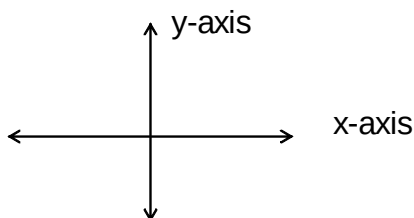
At the end of this chapter you will be able to:

- a) formulate and solve mathematical equation from a situational problem;
- b) simplify expressions by grouping similar terms.

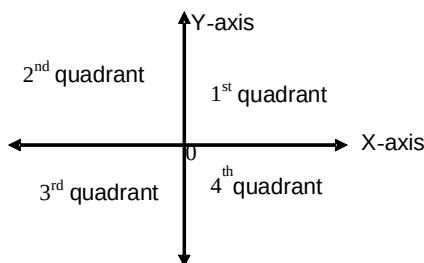
3.1 Plotting Points on a Number Plane

Activity 3.1

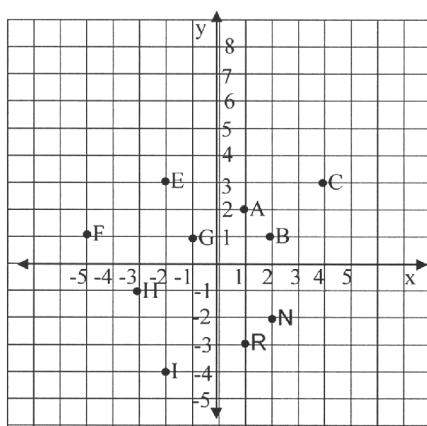
1. Rahel drew x- axis and y-axis on a plane. The two axes (singular axis) meet at a point (0,0) called the origin.



- a) What do you observe about the two lines?
 - b) Into how many parts do the two lines divide the plane?
 - c) What do we call each region? Discuss with your partner.
2. Look at the following figure.
 - a) The region to the right of the y-axis and above the x-axis is the first quadrant.
 - b) The region to the left of the y-axis and above the x-axis is the second quadrant.
 - c) The region to the left of the y-axis and below the x-axis is the third quadrant.
 - d) The region to the right of the y-axis and below the x-axis is the fourth quadrant.



3. Write the coordinates for each of the following.



A(____), B(____), C(____), E(____),
 F(____), G(____), H(____), I(____),
 M(____), N(____), R(____),

4. In which quadrant do the following points lie?

a) (3,5) b) (-4,-2) c) (2,-6) d) (-5,6)

5. Where does a point lie?

- a) If the 1st coordinate is negative and the 2nd coordinate is positive.
- b) If the 1st coordinate is positive and the 2nd coordinate is negative.
- c) If both coordinates are negative.
- d) If both coordinates are positive.

6. a) If the point (a, b) lies on the 3rd quadrant, where does the point (b, a) lie?

b) If the point (a, b) lies on the 1st quadrant, where does the point (b, a) lie?

c) If the point (a, b) lies on the 2nd quadrant, where does the point (b, a) lie?

d) If the point (a, b) lies on the 4th quadrant, where does the point (b, a) lie?

3.2 Graphs of Linear Equations

Activity 3.2

1. The price of a pen is 3 more than the price of a pencil.

a) How much will the price of the pen be, if the price of the pencil is

i) 1 Nakfa? ii) 2 Nakfa? iii) 3 Nakfa? iv) 4 Nakfa?

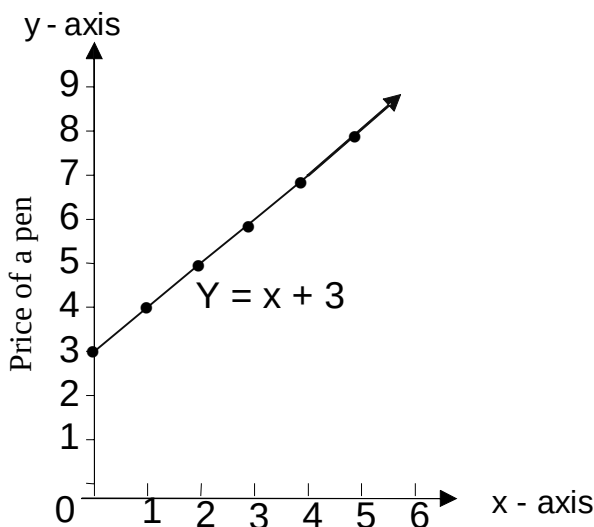
The relationship of their prices will be of the form
(1,4),(2,5),(3,6),(4,7)....

b) let y be the price of the pen and x be the price of the pencil, then the equation will be $y = x + 3$

c) The equation can also be described by the following table

X	1	2	3	4	5	etc
Y	4	5	6	7	8	etc

d) The equation $y = x + 3$ can be described using a graph



2. One number is 1 less than two times another number.

a) Write an equation that describes the relationship of the numbers.

b) Draw a graph that describes the relationship of the two numbers.

Solution:

- a) If we let one number to be x and the second number y , then the relationship of these numbers will be given by the equation $y = 2x - 1$

The equation can be described using the following table.

$$Y = 2x - 1$$

When $x = -3$, then $y = 2(-3) - 1 = -6 - 1 = -7$

When $x = -2$, then $y = 2(-2) - 1 = -4 - 1 = -5$

When $x = -1$, then $y = 2(-1) - 1 = -2 - 1 = -3$

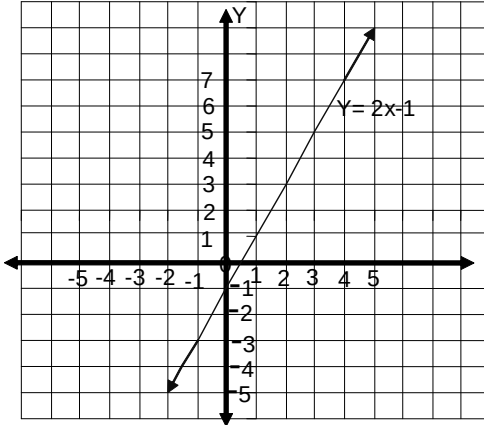
When $x = 0$, then $y = 2(0) - 1 = 0 - 1 = -1$

When $x = 1$, then $y = 2(1) - 1 = 2 - 1 = 1$

When $x = 2$, then $y = 2(2) - 1 = 4 - 1 = 3$

x	-3	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3	5

- b) The graph of the relationship is as follows:

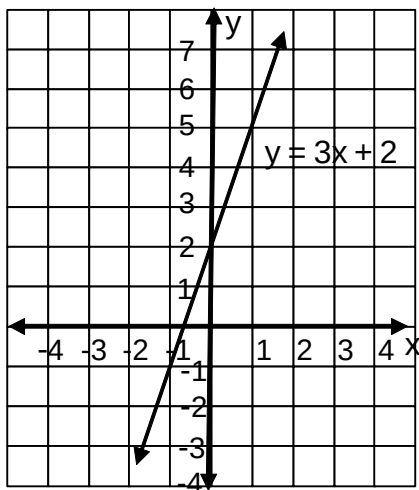


To draw the graph of a linear equation

- make a table of coordinates
- Draw the number plane that is the x -axis and the y -axis.
- Plot the points and draw the straight line joining them using a ruler.

3. Draw the graph of $y = 3x + 2$

x	-3	-2	-1	0	1	2	3
y	-7	-4	-1	2	5	8	11



Graph of a linear equation can also be drawn by taking any two points which are members of the equation.

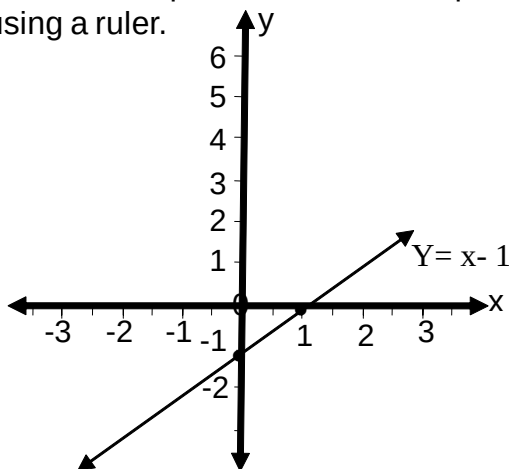
4. Draw the graph of $y = x - 1$:

Take any two ordered pairs of the equation $y = x - 1$.

a) If $x = -2$, then $y = -2 - 1 = -3$, the ordered pair will be $(-2, -3)$.

b) If $x = 2$, then $y = 2 - 1 = 1$, the ordered pair will be $(2, 1)$.

c) Plot these ordered pairs on the number plane and join them using a ruler.



5. a) Where does the line cross the x-axis?
- b) Where does the line cross the y-axis?
- c) What is the name given to the points where the line crosses the x-axis?
- d) What is the name given to the points where the line crosses the y-axis?

Intercepts:

The *x-intercept* of a line is the point where the line crosses the x-axis and is denoted by $(a, 0)$, where a is a real number.

The *y-intercept* of a line is the point where the line crosses the y-axis and is denoted by $(0, b)$, where b is a real number.

Find the intercepts of $y = 2x + 4$

- a) To find x-intercept, let $y=0$

$$2x + 4 = 0$$

$$2x = -4$$

$$x = -2 \text{ is the x intercept or } (-2, 0).$$

- b) To find y intercept, let $x=0$

$$y = 2x + 4$$

$$y = 2(0) + 4 = 4$$

$$y = 4 \text{ is the y- intercept or } (0, 4).$$

Activity 3.3

1. Find the x-intercept and the y-intercept for each of the following.
 - a) $y = x + 3$
 - b) $y = -x + 5$
 - c) $3x - y = 6$
 - d) $2x + 3y - 4 = 0$
2. Express y in terms of x in each of the following equations.
 - a) $y - 3x + 3 = 0$
 - b) $3x + 2y - 6 = 0$
 - c) $x - 5y + 4 = 0$
 - d) $2x + 5y - 1 = 0$
3. Express each of the equations in number 2, as x in terms of y .

4.

Can we draw linear equations by taking the intercepts?

Yes, let's draw the graph of
 $Y = 2x + 4$.

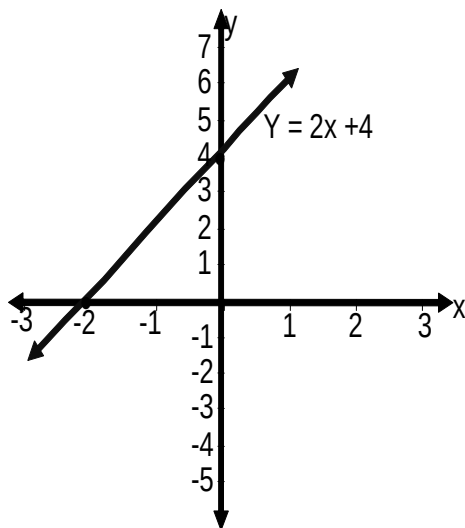


- a) Find the intercepts
- i) x- intercept is $(-2, 0)$, and
 - ii) y- intercept is $(0, 4)$

b) Plot the points on the plane,

c) Draw a line through the points.

The equation can be described by the graph below.



5. Aster has 3 more oranges than Kedija.

Let x be the number of oranges of Kedija and
 y be the number of oranges of Aster.

- a) Write an equation relating x and y .
 b) Describe the relationship by the table.

x	1	2	3	4
y				

c) Draw the graph of the equation

6. Two numbers x and y are related by the equation $y = 2x - 3$.

a) Complete the following table about the two numbers.

x	-2	-1	0	1	2	3
y						

b) Draw the graph of the equation

7. y is one more than twice x .

- a) Express y in terms of x .
 b) Describe the relationship on a table where the range of x is -3 to 3.
 c) Draw the graph of the equation.

8. In the equation $y = 3x - 3$

- a) Find the x -intercept and the y -intercept.
 b) Draw the graph of the equation.

9. Draw the graphs of the following by completing the following tables.

a) $y = 3$

b) $x = -4$

x	-2	-1	0	1	2
y					

x					
y	-2	-1	0	1	2

3.3 Solving Systems of Linear Equations Graphically

Activity 3.4

In chapter 2 we discussed how to solve systems of linear equations algebraically. Now, we will see how to solve system of linear equations using graphs.

1. Solve the system graphically

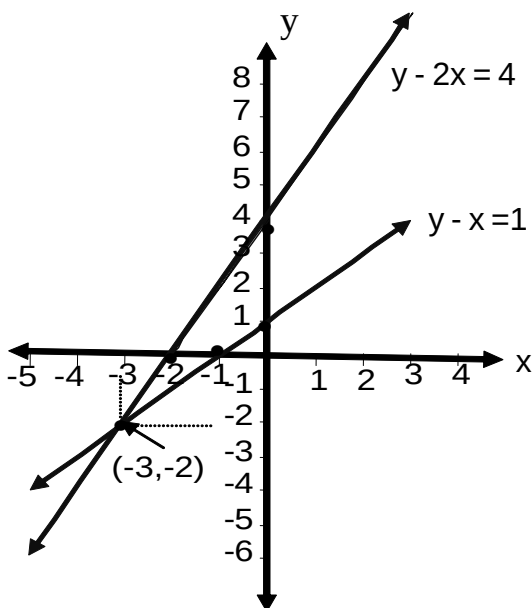
$$\begin{cases} y - x = 1 \\ y - 2x = 4 \end{cases}$$

- a) Draw the graph of $y - x = 1$.
Find the x and the y- intercepts.

x	0	-1
y	1	0

- b) On the same set of axis draw the graph of $y - 2x = 4$ by using the intercept.

x	0	-2
y	4	0



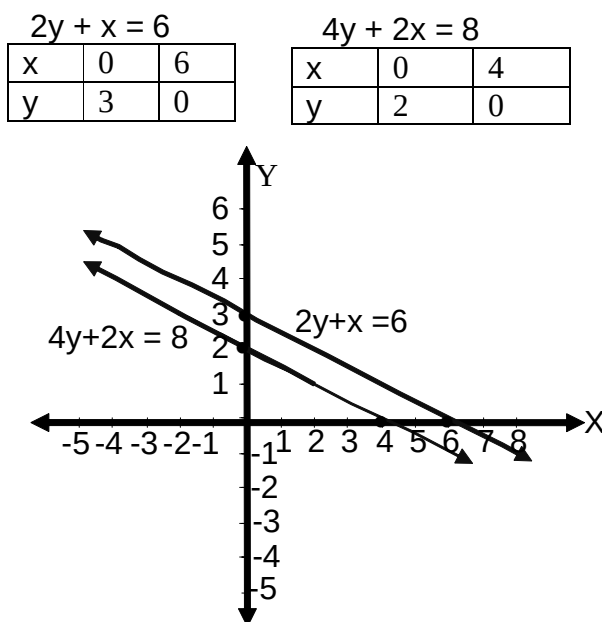
The solution of the system of the linear equation is the intersection point of the two lines that is $(-3, -2)$.

Hence $x = -3$ and $y = -2$ are the solutions.

2. Solve the system graphically

$$\begin{cases} 2y + x = 6 \\ 4y + 2x = 8 \end{cases}$$

Draw the graphs of $2y + x = 6$ and $4y + 2x = 8$ on the same set of axis.



- What do you observe?
- What is the name given to the lines that do not intersect?
- What is the solution set of the system?

3. Solve the following systems of linear equations graphically.

a) $\begin{cases} 2x - y = 1 \\ x + y = 5 \end{cases}$

b) $\begin{cases} 2x + y = 4 \\ x + 2y = 5 \end{cases}$

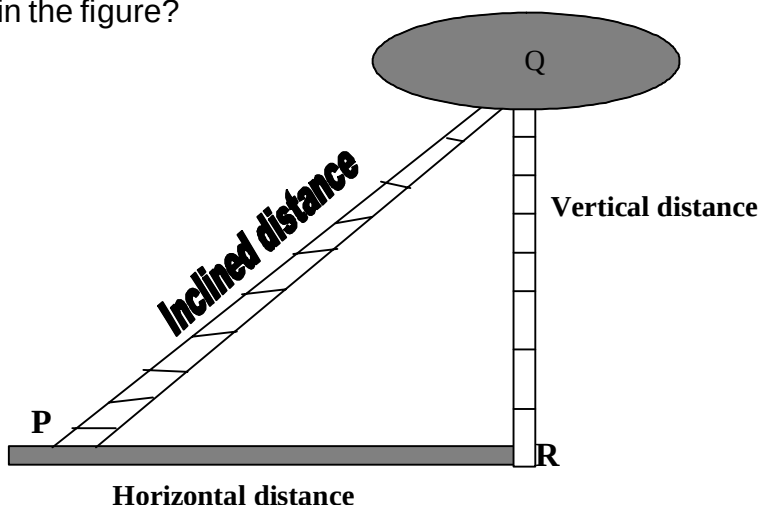
c) $\begin{cases} 3x - y = 1 \\ 6x - 2y = 4 \end{cases}$

d) $\begin{cases} 4x - y = 2 \\ 8x - 2y = 4 \end{cases}$

3.4 Slope of a Line

Activity 3.5

1. Have you ever climbed to the top of a building, a tower or a tree?
2. Have you observed people climbing a tree?
3. How do you climb to the top of the tower from the point **P** to **Q** in the figure?

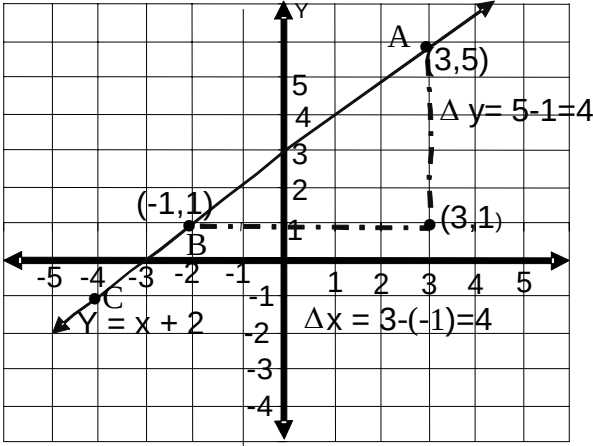


4. If Sirak climbed from **P** to **Q** through the inclined ladder as shown on the figure below;
 - a) Does he cover a horizontal distance (PR);
 - b) Does he cover a vertical distance (QP);
 - c) What is the name given to the ratio of the vertical change to horizontal change ($\frac{\text{Vertical distance}}{\text{Horizontal distance}}$)?

The ratio of the vertical change to the horizontal change is said to be slope of a line.

Slope of a line can also be defined as $\frac{\text{change of } y}{\text{change of } x} = \frac{\Delta y}{\Delta x}$

5. Consider a line passing through two distinct points A (3,5) and B (-1,1)



- What is the vertical change between the points A and B?
- What is the horizontal change between the points A and B?
- What is the ratio of the vertical change to the horizontal change?

$$\begin{aligned}\text{Slope} &= \frac{\Delta y}{\Delta x} = \frac{5-1}{3-(-1)} \\ &= \frac{4}{4} \\ &= 1\end{aligned}$$

The slope of a line through two distinct points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by:

$$\begin{aligned}m &= \frac{\text{Vertical distance}}{\text{Horizontal distance}} \\ &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}\end{aligned}$$

6. A line passes through the points P(3,1) and Q(8,11)

- What is the vertical change?
- What is the horizontal change?
- What is the slope of the line?

Solution:

a) Vertical change = $11-1 = 10$ or $1-11 = -10$

b) Horizontal change = $8-3 = 5$ or $3 - 8 = -5$

c) Slope = $\frac{10}{5} = 2$ or slope (m) = $\frac{-10}{-5} = 2$

7. Find the slope of the line:

- passing through the following pairs of points A (-2,4) and B(1,-5)
- whose equation is $y = 2x + 4$

Solution:

a) A(-2,4) and B(1,-5)

$x_1 = -2, y_1 = 4$ and $x_2 = 1, y_2 = -5$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 4}{1 - (-2)} = \frac{-9}{1 + 2} = \frac{-9}{3} = -3$$

b) take any two points on the line for instance,

x	0	-2
y	4	0

that is, (0,4) and (-2,0)

$$m = \frac{0 - 4}{-2 - 0} = \frac{-4}{-2} = \frac{4}{2} = 2$$

8. Find the slopes of the lines determined by the following pairs of points.

a) (-2,-3) and (2,5)

c) (8,-3) and (-2,-5)

b) (4,-3) and (-2,7)

d) (2,4) and -3,4)

9. Using the above method find the slope of a line whose equation is:

a) $y = 3x + 5$

b) $y = -3x + 2$

c) $y - 2x = 4$

d) $y + 2x - 3 = 0$

e) $2y - 6x = -12$

f) $3x - y + 4 = 0$

Chapter 4

Power Notation

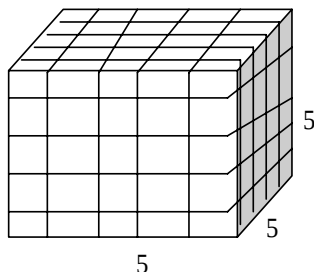
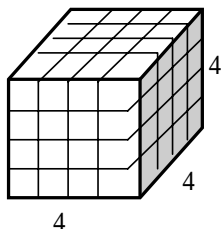
By the end of the chapter you will be able to:

- a) describe properties of power notations;
- b) simplify and solve equations and expressions involving power notations;
- c) relate power notations with other forms of equations.

4. 1 Positive Integral Exponents:

Activity 4.1

1. A teacher asked his class to find the total number of cubic units in each of the following figures.



Nejat applied a short method and calculated as follows

a) $4 \times 4 \times 4 = 64$

b) $5 \times 5 \times 5 = 125$

There are $4^3 = 64$ cubic units There are $5^3 = 125$ cubic units

2. A task was given to Nejat and Elsa to prime factorize which one is the short method?

a) 256

b) 432

c) 729

Nejat wrote the prime factorization as follows:

a) $256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8$

b) $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$

c) $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$

Elsa said to Nejat this way of writing 2^8 , $2^4 \times 3^6$ is a short form of the repeated multiplication.

For example $6 \times 6 \times 6 \times 6$ can be written as 6^4 and

$7 \times 7 \times 7 \times 7 \times 7$ can be written as 7^5 .

3. Write each of the following repeated multiplication in a short form.

a) $3 \times 3 \times 3 \times 3 \times 3$

b) $5 \times 5 \times 5 \times 5$

c) $12 \times 12 \times 12$

d) $m \times m \times m \times m \times m$

e) $13 \times 13 \times 13$

f) $15 \times 15 \times 15 \times 15$

g) $7 \times 7 \times 7 \times 11 \times 11$

h) $a \times a \times a \times b \times b$

i) $0.6 \times 0.6 \times 0.6 \times 0.6$

The system of writing repeated multiplication in a short form such as $7 \times 7 \times 7 \times 7 \times 7$ as 7^5 is called **exponential** form or **power notation**.

In 7^5 , 5 is called *exponent* or *index*,

7 is called a base and

7^5 is a *power*.

4. Write each of the following in power form.

a) $4 \times 4 \times 4 \times 4 \times 4$

b) $12 \times 12 \times 12 \times 12 \times 12$

c) $(-5) \times (-5) \times (-5)$

d) $m \times m \times m \times m \times m \times m$

e) $3 \times 3 \times 3 \times 7 \times 7 \times 7 \times 7$

f) $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$

g) $\frac{3}{5} \times \frac{3}{5} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7}$

5. Write each of the following using single exponents.

a) $2^3 \times 2^4$

b) $3^3 \times 3^4$

c) $7^2 \times 7^4$

d) $3^2 \times 3^4 \times 3^3$

e) $12^5 \times 12^3 \times 12^2$

f) $5^4 \times 5^5$

g) $a^4 \times a^5$

h) $6^2 \times 6^4 \times 6$

Power Rules

From the previous activities we noticed that a simple way of expressing expressions of the form:

$$2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2)$$

$$2^3 \times 2^4 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$2^3 \times 2^4 = 2^7$$

Hana formulated that:

$$2^3 \times 2^4 = 2^{3+4} = 2^7, \text{ And}$$

$$2^5 \times 2^3 \times 2^2 \text{ can be written as } 2^{5+3+2} = 2^{10}?$$

A. For a positive real number a , x and y any real number we have: $a^x \times a^y = a^{x+y}$.

If powers have the same base we add the exponents.

Activity 4.2

1. Rewrite the following as powers with a single exponent.

- | | |
|------------------------------|--------------------------------|
| a) $7^5 \times 7^4$ | b) $3^3 \times 3^2 \times 3^4$ |
| c) $5^6 \times 5^4$ | d) $3^5 \times 3^4 \times 3^3$ |
| e) $2^5 \times 2^3 \times 2$ | f) $9^4 \times 9^5$ |
| g) $k^4 \times k^7$ | h) $6^3 \times 6^4 \times 6$ |

2. A teacher asked his students to write $(2^4)^3$ as a power with a single exponent. One of her students Omer did like this:

$$\begin{aligned}(2^4)^3 &= 2^4 \times 2^4 \times 2^4 \\ &= 2^{4+4+4} \\ &= 2^{12}\end{aligned}$$

While another student Ibrahim did like this:

$$\begin{aligned}(2^4)^3 &= 2^{3 \times 4} \\ &= 2^{12}\end{aligned}$$

- Are all methods correct?
- Justify them by taking different examples.
- Which method do you think is shorter?

3. Re write using a single exponent.

- a) $(3^2)^3$ b) $(12^3)^4$ c) $(m^4)^5$ d) $(6^3)^7$ e) $(n^4)^4$.

4. Is there any difference between $(5^4)^3$ and $(5^3)^4$?

B. For a positive real number a and any real numbers x and y :

$$(a^x)^y = a^{x \cdot y}$$

5. Evaluate each of the following.

a) $(4^2)^3$ b) $(m^3)^4$

c) $(a^4)^5$ d) $(10^3)^7$

e) $(15^4)^1$

6. Indicate the difference in meaning and find a simple numeral.

a) $(2 \times 3)^2$ and 2×3^2

b) $(2x)^2$ and $2x^2$

c) 4×3^2 and $(4 \times 3)^2$

d) $3^2 \times 4^2$ and $(3 \times 4)^2$

7. A teacher gave to Fana and Zahra to evaluate the following types of activities.

a) $3^3 \times 4^3$ and b) $10^2 \times 6^2$

Fana did like this: a) $3^3 \times 4^3 = 3 \times 3 \times 3 \times 4 \times 4 \times 4$
 $= (3 \times 4) \times (3 \times 4) \times (3 \times 4)$
 $= 12 \times 12 \times 12$
 $= 12^3$, and

b) $10^2 \times 6^2 = 10 \times 10 \times 6 \times 6$
 $= (10 \times 6) \times (10 \times 6)$
 $= 60 \times 60$
 $= 60^2$

While Zahra did like this: a) $3^3 \times 4^3 = (3 \times 4)^3$
 $= 12^3$, and

b) $10^2 \times 6^2 = (10 \times 6)^2$
 $= 60^2$

- a) Did you understand both methods?
b) Are both methods correct all the time?
c) Which method do you think is easier? Why?

C. For positive real numbers a and b,

$$a^m \times b^m = (a \times b)^m$$

If powers have the same exponent then multiply the bases and take one.

8. Write each of the following using single exponents.

a) $3^4 \times 4^4$

b) $5^3 \times 2^3 \times 3^3$

c) $m^6 \times m^6$

d) $3^4 \times 3^3$

e) $(6^5)^2$

f) $7^4 \times 7^5$

g) $4m^3 \times n^3$

h) $6^3 \times 6^4$

i) $23^5 \times 23^4$

j) $12^3 \times 5^3$

Division with Integral Exponents

Activity 4.3

1. A teacher was discussing about simplification of integral exponents. He gave the following activities to his students.

Simplify $\frac{7^6}{7^4}$ and $\frac{8^3}{8^7}$.

Rahel simplified it like this:

$$\begin{aligned}\frac{7^6}{7^4} &= \frac{7 \times 7 \times 7 \times 7 \times 7 \times 7}{7 \times 7 \times 7 \times 7} \\ &= 7^2\end{aligned}$$

She also simplified $\frac{8^3}{8^7}$ in the same way as:

$$\begin{aligned}\frac{8^3}{8^7} &= \frac{8 \times 8 \times 8}{8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8} \\ &= \frac{1}{8 \times 8 \times 8 \times 8} \\ &= \frac{1}{8^4}\end{aligned}$$

Nejat did the above activities as follows:

$$\begin{aligned}\frac{7^6}{7^4} &= 7^{6-4} & \text{and} & \quad \frac{8^3}{8^7} = \frac{1}{8^{7-3}} \\ &= 7^2 & & \quad = \frac{1}{8^4}\end{aligned}$$

2. Whose method do you think is shorter?

3. Simplify each of the following using either method.

a) $\frac{6^7}{6^4}$ b) $\frac{4^6}{4^3}$ c) $\frac{m^7}{m^2}$ d) $\frac{a^{11}}{a^6}$

e) $\frac{9^8}{9^5}$ f) $\frac{13^4}{13^2}$ g) $\frac{m^3}{m^7}$ h) $\frac{x^7}{x^9}$

4. What did you understand from the above solutions?

For any real number a , $a > 0$, $a \neq 1$

a) $\frac{a^x}{a^y} = a^{x-y}$, when $x > y$, and

$$\frac{a^x}{a^y} = \frac{1}{a^{y-x}} \text{ , when } y > x$$

b) $\frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x$

5. Express each of the following in the form $\left(\frac{a}{b}\right)^x$

a) $\frac{5^4}{7^4}$ b) $\frac{3^5}{4^5}$ c) $\frac{2^7}{9^7}$ d) $\frac{m^m}{a^m}$ e) $\frac{20^5}{9^5}$

f) $\frac{13^4}{25^4}$ g) $\frac{x^6}{y^6}$ h) $\frac{8x^3}{27y^3}$

6. Simplify each of the following.

a) $\frac{5^4}{5^4}$ b) $\frac{6^7}{6^7}$ c) $\frac{r^5}{r^5}$ d) $\frac{x^3}{x^3}$

7. What did you conclude from the results of problem number 6?

8. What meaning can you suggest for 5^0 , 6^0 , ...?

9. Beshir simplified $\frac{3^5}{3^5}$ as follows:

$$\frac{3^5}{3^5} = \frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3} = \frac{243}{243} = 1$$

and arrived to
a conclusion that

$$\frac{3^5}{3^5} = 3^{5-5} = 3^0 = 1$$

10. Do you agree with him?

11. Verify the activities in question number 2 above by this method.

For a non zero real number m ,
 $m^0 = 1$.

Activity 4.4

Write each of the following in the simplest form.

1. $\frac{10^4}{10^4}$

2. $\frac{5^4 \times 5^3}{5^7}$

3. $\frac{2^4 \times 2^3 \times 2^5}{2^6 \times 2^7}$

4. $\frac{2^5 \times 3^2}{2^8 \times 9^2}$

5. $\frac{a^6 \times b^3}{a^4 \times b^5}$

6. $\frac{(2^2)^5}{2^6 \times 2^4}$

$$\frac{2^2}{2^5} = 2^{2-5} = 2^{-3}, \text{ Is the answer same?}$$

4. 2 Powers with Negative Exponents

Activity 4.5

1. A teacher gave a task to the class to simplify $\frac{2^2}{2^5}$

Jemal worked it as follows:

$$\begin{aligned}\frac{2^2}{2^5} &= \frac{2 \times 2}{2 \times 2 \times 2 \times 2 \times 2} \\ &= \frac{1}{2 \times 2 \times 2} \\ &= \frac{1}{2^3}\end{aligned}$$

2. Berhe worked as follows:

$$\frac{2^2}{2^5} = \frac{1}{2^{5-2}} = \frac{1}{2^3} \quad \text{subtracting the indices}$$

3. Which method do you think is easier?
4. Can you do it by any other method?
5. Dawud one of the clever students asked,
What about if we subtract 5 from 2?

$$\frac{2^2}{2^5} = 2^{2-5} = 2^{-3} \quad \text{Is the answer the same?}$$

Is the value of $2^{-3} = \frac{1}{2^3}$ why?

6. Ahferom said yes, and justified his answer as follows:

$$\begin{aligned}2^{-3} &= 2^{-3} \times \frac{2^3}{2^3} \quad \text{multiplying by } \frac{2^3}{2^3} = 1 \\ &= \frac{2^{-3+3}}{2^3} \quad \text{by multiplication rule} \\ &= \frac{2^0}{2^3} \quad \text{addition of opposites} \\ &= \frac{1}{2^3}.\end{aligned}$$

Hence $2^{-3} = \frac{1}{2^3}$.

7. Do you agree with the suggestion of Ahferom?

8. Can we conclude that $\frac{5^3}{5^7} = 5^{3-7} = 5^{-4} = \frac{1}{5^4}$?

Generally for $a > 0, a \neq 1$

$$a^{-x} = \frac{1}{a^x}$$

9. Simplify, the following and give your answers with non - negative exponents or zero.

a) $a^{-6} \times a^4 \times a^{-5}$

b) $m^{-4} \times m^{-3} \times m^9$

c) $k^{-4} \times k^{-3} \times k^{-7}$

d) $\frac{n^5 \times m^4}{n^7 \times m^{-3}}$

e) $\frac{r^{-5} \times s^4}{r^4 \times s^3}$

f) $\frac{d^3 \times c^6}{d^4 \times c^{-2}}$

Powers with Rational Exponent

Activity 4.6

Nebiyat and Fatna summarized what they learned about power notation as follows:

a) $a^x \cdot a^y = a^{x+y}$

b) $a^x \cdot a^x = (ab)^x$

c) $a^x \div a^y = a^{x-y}$

d) $a^x \div a^y = \frac{1}{a^{y-x}}$

1. Can you verify all the above rules by taking two examples for each?
2. Do the rules work for fractional exponents?
3. Discuss the following examples and give reasons for each step.

$$\begin{aligned} \text{a) } 5^{\frac{1}{2}} \times 5^{\frac{1}{2}} &= 5^{\frac{1}{2} + \frac{1}{2}} \\ &= 5^1 \text{ Why?} \\ &= 5 \text{ Why?} \end{aligned}$$

$$\left(5^{\frac{1}{2}}\right)^2 = 5^{\frac{1}{2}} \times 5^{\frac{1}{2}} = 5$$

$$\text{Is } 5^{\frac{1}{2}} = \sqrt{5} ?$$

$$\begin{aligned} \text{b) } 7^{\frac{1}{2}} \times 7^{\frac{1}{2}} &= 7^{\frac{1}{2} + \frac{1}{2}} \\ &= 7^1 \text{ Why?} \\ &= 7 \text{ Why?} \end{aligned}$$

$$\left(7^{\frac{1}{2}}\right)^2 = 7^{\frac{1}{2}} \times 7^{\frac{1}{2}} = 7$$

$$\text{Is } 7^{\frac{1}{2}} = \sqrt{7} ?$$

$$\text{c) } \left(\frac{3}{5}\right)^{\frac{1}{2}} \times \left(\frac{3}{5}\right)^{\frac{1}{2}} = \left(\frac{3}{5}\right)^{\frac{1}{2} + \frac{1}{2}}$$

$$\begin{aligned} &= \left(\frac{3}{5}\right)^1 \text{ Why?} \\ &= \frac{3}{5} \text{ Why?} \end{aligned}$$

$$\text{Is } 5^{\frac{1}{2}} = \sqrt{5}$$

$$\text{d) } 4^{\frac{1}{3}} \times 4^{\frac{1}{3}} \times 4^{\frac{1}{3}} = 4^{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}}$$

$$\begin{aligned} &= 4^{\frac{3}{3}} \text{ Why?} \\ &= 4 \text{ Why?} \end{aligned}$$

4. What can you say about the meaning of $8^{\frac{1}{5}}$ from the above examples?

For a > 0 and n a postive integer

$$a^{\frac{1}{n}} = \sqrt[n]{a}, n=2$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}, n \geq 3$$

5. Simplify the following

$$\text{a) } 49^{\frac{1}{2}} = \sqrt{49} = 7$$

$$\begin{aligned} \text{b) } 625^{\frac{1}{4}} &= \sqrt[4]{625} \\ &= \sqrt[4]{5 \times 5 \times 5 \times 5} \\ &= 5, \text{ since } 5^4 = 625 \end{aligned}$$

$$\begin{aligned} \text{c) } 8^{\frac{1}{3}} &= \sqrt[3]{8} \\ &= \sqrt[3]{2 \times 2 \times 2} \\ &= 2 \end{aligned}$$

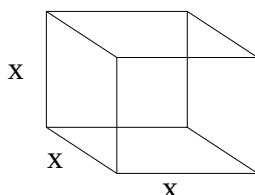
6. Simplify each of the following.

- a) $36^{\frac{1}{2}}$ b) $400^{\frac{1}{2}}$ c) $(\frac{144}{25})^{\frac{1}{2}}$ d) $16^{\frac{1}{2}}$
 e) $169^{\frac{1}{2}}$ f) $225^{\frac{1}{2}}$ g) $289^{\frac{1}{2}}$ h) $75^{\frac{1}{2}}$

7. Simplify each of the following.

- a) $64^{\frac{1}{3}}$ b) $81^{\frac{1}{4}}$ c) $27^{\frac{1}{3}}$ d) $125^{\frac{1}{3}}$
 e) $243^{\frac{1}{5}}$ f) $729^{\frac{1}{6}}$ g) $256^{\frac{1}{8}}$ h) $(\frac{27}{8})^{\frac{1}{3}}$

8. A teacher asked his class to calculate the length of the sides of this cube giving its volume to be 8 cubic unit.



Asmerom solved it as follows.

He let each edge to be x, then $x \cdot x \cdot x = 8$

$$x^3 = 8$$

$$x^3 = 2^3$$

$$x = 2$$

The length of each edge of the cube is 2 units.

a) Is Asmerom correct?

b) How do you justify it?

9. Rahma asked Asmerom, if he could also solve for x, in $32^x = 2$.

Asmerom worked it as follows.

First he expressed 32 as a power form.

$$(2^5)^x = 2, \text{ since } 32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

$$2^{5x} = 2^1 \quad \text{Why?}$$

$$5x = 1 \quad \text{Why?}$$

$$x = \frac{1}{5}$$

10. Did you understand the above steps?

11. In each of the following find the value of x.

a) $16^x = 4$

b) $27 = 3^x$

c) $625^x = 5$

d) $2^x = 32$

e) $x^3 = 27$

f) $2^x = 64$

g) $(\frac{1}{3})^x = \frac{1}{81}$

h) $(\frac{8}{27}) = (\frac{2}{3})^x$

Scientific Notation (Standard Notation)

Activity 4.7

1. Write the following big numbers in a simple form.

a) 93, 000, 000

b) 125,000

Abdu did as follows:

a) 93 million

b) 125 thousands

While Meriem did as follows:

a) 9.3×10^7

b) 1.25×10^5

b) Which way of writing do you think is convenient?

c) What do we call the Meriem's way of writing?

2. Write the following using the second method.

a) 546

b) 7200

c) 89.6

Solution:

a) $546 = 5.46 \times 10^2$

b) $7200 = 7.2 \times 10^3$

c) $89.6 = 8.96 \times 10^1$

3. Did you understand the above way of writing?

Expressing a number as a product of two numbers such that one is between 1 and 10, and the other is as a power of 10 is called *scientific notation or standard notation*.

4. Express each of the following in expanded form.

- | | |
|-------------|------------|
| a) 5600 | b) 70900 |
| c) 93.32 | d) 0.00432 |
| e) 0.0105 | f) 6.78 |
| g) 49000000 | h) 0.651. |

5. Write each of the following in decimal numeral.

- | | |
|--------------------------|-------------------------|
| a) 5.7×10^4 | b) 6.32×10^3 |
| c) 9.7×10^{-2} | d) 8.1×10^{-3} |
| e) 4.52×10^{-5} | f) 8.6×10^5 . |

6. Write the following in standard notation.

- | | |
|----------|-------------|
| a) 379 | b) 60800 |
| c) 0.379 | d) 0.00254. |

7. Write each of the following in scientific notation.

- | | |
|------------|-------------|
| a) 4679 | b) 937.4 |
| c) 9300000 | d) 0.000546 |
| e) 0.00709 | f) 45.9. |

Chapter 5

Logarithms

By the end of the chapter you will be able to:

- a) relate and identify relationship between logarithms and power notations (Exponential notations).
- b) verify properties of logarithms using numbers; operates numbers with logarithms; and apply properties of logarithms to solve elementary problems

5.1 Relationship between Exponential and Logarithmic Notations

Activity 5.1

1. We have discussed about power notations which are important for writing very big or very small numbers. Now we shall deal with logarithms.
 - a) Have you ever heard about logarithms?
 - b) What are logarithms?

Logarithms are expressing exponential notation in another form. They are used to simplify lengthy and time consuming computations that arise in solving problems. They are important in higher mathematics, science and technology.

2. We express the power form $2^4 = 16$ as $\log_2 16 = 4$.
It is read as 4 is equal to the logarithm of 16 to the base 2, and is called *logarithmic notation*.
Look at the following examples.
 - a) $5^3 = 125$, can be written as $\log_5 125 = 3$.
 - b) $3^4 = 81$, can be written as $\log_3 81 = 4$.
 - c) $7^2 = 49$, can be written as $\log_7 49 = 2$.

For positive real numbers a and x , such that $x = a^y$,
($a \neq 1$) can be written as $y = \log_a x$

3. A mathematics teacher asked her class to evaluate $\log_2 32$. Abdu said it is easy; you only need to know "To what power must 2 be raised to get 32?"

And he worked it as follows:

$$32 = 2^5, \text{ then } \log_2 32 = x$$

$$2^x = 2^5 \quad (\text{changing to power notation})$$

$$x = 5$$

4. Is Abdu correct?

5. Can you evaluate by the same method $\log_7 49$?

Solution:

Since $49 = 7^2$, then $\log_7 49 = 2$.

6. Rewrite the following exponential equations in logarithmic notation.

a) $125 = 5^3$

b) $49 = 7^2$

c) $81 = 3^4$

d) $2 = 32^{\frac{1}{5}}$

e) $2 = 16^{\frac{1}{4}}$

f) $128 = 2^7$

g) $625^{\frac{1}{4}} = 5$

h) $3 = 27^{\frac{1}{3}}$

7. Change the following to exponential notation.

a) $\log_{10} 100 = 2$

b) $\log_x 25 = 2$

c) $\log_4 97 = y$

d) $\log_4 \sqrt{32} = m$

e) $\log_{\left(\frac{1}{2}\right)} 64 = -6$

f) $\log_{27} 81 = k$

g) $\log_{\left(\frac{2}{3}\right)} \left(\frac{27}{8}\right) = -3$

h) $\log_{\sqrt{3}} 243 = x$

8. Evaluate the following:

a) $\log_5 125$

b) $\log_3 81$

c) $\log_2 128$

d) $\log_{13} 169$

Solving Logarithmic Equations

Can you solve the following two equations?

a) $\log_x 32 = 5$

b) $\log_3(x + 5) = 3$



Abdu: By converting to power notations, he solved for the unknowns:

a) $\log_x 32 = 5$

$$x^5 = 32$$

$$x^5 = 2^5$$

$$x = 2$$

b) $\log_3(x + 5) = 3$

$$x + 5 = 3^3$$

$$x + 5 = 27$$

$$x = 27 - 5$$

$$x = 22$$

Activity 5.2

1. Fill in the blank spaces.

a) $3^5 = 243$, then $\log_3 243 = \underline{\hspace{2cm}}$.

b) $2^{-5} = \frac{1}{32}$, then $\log_2 \left(\frac{1}{32} \right) = \underline{\hspace{2cm}}$.

c) $\left(\frac{1}{2} \right)^4 = \frac{1}{16}$, then $\log_{\frac{1}{2}} \left(\frac{1}{16} \right) = \underline{\hspace{2cm}}$.

d) $3^3 = 27$, then $\log_3 27 = \underline{\hspace{2cm}}$.

2. Solve the following logarithmic equations.

a) $\log_5 x = 3$ for x . b) $\log_x 81 = 4$ for x .

c) $\log_5 125 = r$ for r . d) $y = \log_4 \frac{1}{16}$ for y .

e) $\log_8 m = 2$

f) $\log_5 t = 5$

g) $\log_7 a = 3$

h) $\log_n \frac{1}{8} = 3$

i) $\log_9 3 = y$

j) $\log_k \frac{1}{243} = -5$

5.2 Basic Properties of Logarithms

We have discussed the relationship between power notation and logarithmic notation.

Activity 5.3

Consider the following:

1. We know that $64 = 16 \times 4$

$$\log_2 64 = x$$

$$2^x = 26$$

$$x = 6$$

$$\log_2 16 = h$$

$$2^h = 24$$

$$h = 4$$

$$\log_2 4 = k$$

$$2^k = 22$$

$$k = 2$$

Note that: $6 = 4 + 2$

$$x = h + k$$

$$\log_2 64 = \log_2 16 + \log_2 4$$

Property 1

The logarithm of the product of two positive numbers is equal to the sum of the logarithms of the numbers

$$\log_a (x \cdot y) = \log_a x + \log_a y, (x, y, a > 0 \text{ and } a \neq 1)$$

(Product law)

2. Evaluate: a) $\log_3(9 \times 27)$ b) $\log_2(32 \times 64)$

Solution:

$$\begin{aligned} \text{a) } \log_3(9 \times 27) &= \log_3 9 + \log_3 27 \quad (\text{Since } 3^2 = 9 \text{ and } 3^3 = 27) \\ &= \log_3 3^2 + \log_3 3^3 \\ &= 2 + 3 \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{b) } \log_2(32 \times 64) &= \log_2 32 + \log_2 64 \quad (\text{Since } 2^5 = 32 \text{ and } 2^6 = 64) \\ &= \log_2 2^5 + \log_2 2^6 \\ &= 5 + 6 \\ &= 11 \end{aligned}$$

3. Express as a sum of logarithms.

$$\begin{array}{ll} \text{a) } \log_3 5ab & \text{b) } \log_a(16 \times 64) \\ \text{c) } \log_7 xyz & \text{d) } \log_5 8x^2y^3 \end{array}$$

4. Express as a single logarithm

$$\begin{array}{ll} \text{a) } \log_a 5 + \log_a 14 & \text{b) } \log_m 65 + \log_m 2 \\ \text{c) } \log_n r + \log_n s & \text{d) } \log_2 9 + \log_2 14 \\ \text{e) } \log_3 5 + \log_3 14 \end{array}$$

5. Evaluate the following logarithms.

$$\begin{array}{ll} \text{a) } \log_3(81 \times 27) & \text{b) } \log_2(16 \times 64) \\ \text{c) } \log_7(49 \times 7) & \text{d) } \log_5(125 \times 25) \\ \text{e) } \log_2(128 \times 64) & \text{f) } \log_2 \left(\frac{1}{64} \times 16 \right) \\ \text{g) } \log_3(243 \times 81) & \text{h) } \log_6(36 \times 216) \end{array}$$

Activity 5.4

1. We have discussed how to simplify power notation. Look at the following examples

Exponential form

$$\frac{32}{8} = 4$$

$$\frac{2^5}{2^3} = 2^2$$

$$2^{5-3} = 2^2$$

Logarithmic form

$$\log_2 \left(\frac{32}{8} \right) = \log_2 4$$

$$\log_2 32 - \log_2 8 = \log_2 4$$

$$5 - 3 = 2$$

$$2 = 2$$

$$\log_2 32 = x$$

$$2^x = 2^5$$

$$x = 5$$

$$\log_2 8 = h$$

$$2^h = 2^3$$

$$h = 3, \text{ and}$$

$$\log_2 4 = k$$

$$2^k = 2^2$$

$$k = 2$$

$$\text{N. B} \quad 2 = 5 - 3$$

$$k = x - h$$

$$\therefore \log_2 \left(\frac{32}{8} \right) = \log_2 8 - \log_2 4$$

2. Evaluate: a) $\log_2 (16 \div 4)$
b) $\log_3 (27 \div 243)$

Solution:

$$\text{a) } \log_2 (16 \div 4) = \log_2 16 - \log_2 4$$

$$= 4 - 2$$

$$= 2$$

$$\text{b) } \log_3 (27 \div 243) = \log_3 27 - \log_3 243$$

$$= 3 - 5$$

$$= -2$$

Property - 2

For any positive numbers m , n and a , ($a \neq 1$)

$$\log_a \frac{m}{n} = \log_a m - \log_a n$$

The logarithm of the numerator minus the logarithm of the denominator.

(Quotient rule)

3. Express as a single logarithm

a) $\log_a 15 - \log_a 7$

b) $\log_b 42 - \log_b 7$

c) $\log_m 6 - \log_m 5$

d) $\log_8 m - \log_8 m$

e) $\log_5 2 - \log_5 3$

f) $\log_5 5 - \log_5 10$

4. Evaluate the following logarithms

a) $\log_2 \left(\frac{64}{16} \right)$

b) $\log_3 \left(\frac{9}{27} \right)$

c) $\log_5 \left(\frac{125}{25} \right)$

d) $\log_2 \left(\frac{1}{64} \right)$

e) $\log_2 \left(\frac{64}{128} \right)$

f) $\log_3 \left(\frac{27}{81} \right)$

g) $\log_5 \left(\frac{125}{625} \right)$

h) $\log_7 \left(\frac{1}{49} \right)$

For positive numbers m , n and a , ($a \neq 1$)
 $\log_a m^n = n \times \log_a m$ (Power rule)

5. Evaluate a) $\log_2 8$ b) $\log_3 27$

Solution:

$$\begin{aligned} \text{a) } \log_2 8 &= \log_2 (2)^3 \\ &= 3 \times \log_2 (2) \\ &= 3 \times 1 \\ &= 3 \end{aligned}$$

$$\begin{aligned} \text{b) } \log_3 27 &= \log_3 (3)^3 \\ &= 3 \times \log_3 (3) \\ &= 3 \times 1 \\ &= 3 \end{aligned}$$

6. Evaluate the following logarithms.

a) $\log_4 8 + \log_4 2$ b) $\log_4 8 + \log_4 16$

c) $\log_6 72 - \log_6 2$ d) $\log_2 40 - \log_2 5$

e) $\log_{10} 5 + \log_{10} 20$ f) $\log_5 50 - \log_5 2$

g) $\log_3 324 - \log_3 4$ h) $\log_8 4 + \log_8 16$

7. Find the value of x

a) $\log_{10} x = -2$ b) $\log_2 x = 5$

c) $\log_{10} 4x = 2$ d) $\log_{10} (x-5) = 3$

e) $\log_8 (x+3) = 2$ f) $\log_4 x + \log_4^5 = 2$

g) $\log_6 (x+8) + \log_6 3 = 2$ h) $\log_5 (x+3) - \log_5 x = 1$

i) $\log_3 (x-2) + \log_3 5 = 3$ j) $\log_{10} x + \log_{10} 4 = 0$

k) $3\log_{10} x = 3$

5.3 Common Logarithm

Activity 5.5

We have discussed how to evaluate logarithms. Look at the following examples.

1. $\log_2 6 = \log_2 2^4 = 4\log_2 2 = 4$, that is $2^4 = 16$
2. $\log_2 64 = \log_2 2^6 = 6\log_2 2 = 6$, that is $2^6 = 64$
3. $\log_3 27 = \log_3 3^3 = 3\log_3 3 = 3$, that is $3^3 = 27$
4. $\log_3 243 = \log_3 3^5 = 5\log_3 3 = 5$, that is $3^5 = 243$
5. $\log_5 625 = \log_5 5^4 = 4\log_5 5 = 4$, that is $5^4 = 625$
6. Did you understand each step?
7. Can you find the values of the following examples by the above methods?

- | | |
|-------------------|----------------|
| a) $\log_2 12$ | b) $\log_5 20$ |
| c) $\log_6 47$ | d) $\log_9 36$ |
| e) $\log_{10} 27$ | |

8. How can we solve logarithms of the above types?
9. Discuss about it with your classmates.

Logarithms of any numbers can be found, if the numbers are expressed as logarithms in base 10. There is also a ready table of Logarithms with a base 10 called **common logarithms**.

N.B.

Logarithms without a base are to be taken that the base is 10.

10. Re-write the following using logarithms to base 10.

a) $10^1 = 10$ b) $10^2 = 100$

c) $10^4 = 10000$ d) $10^3 = 1000$

e) $100^{\frac{1}{2}} = 10$ f) $10000^{\frac{1}{4}} = 10$

g) $1000^{\frac{1}{3}} = 10$

Solution:

a) $10^1 = 10 \Rightarrow \log_{10} 10 = 1$

b) $10^2 = 100 \Rightarrow \log_{10} 100 = 2$

c) $10^4 = 10000 \Rightarrow \log_{10} 10000 = 4$

11. Can you do the others?

12. What did you understand about the logarithm of a number?

Characteristic and Mantissa

Activity 5.6

1. Look at the following facts. Discuss about each fact with your partner.

For any one digit positive integer:

$$1 < 9 < 10, \text{ if } \log_{10} 1 < \log_{10} 9 < \log_{10} 10$$

$$0 < \log_{10} 9 < 1$$

For any two digit numeral say 58.

$\therefore \log_{10} 9$ is between 0 and 1. Hence it is a decimal number.

$$10 < 58 < 100, \text{ if } \log_{10} 10 < \log_{10} 58 < \log_{10} 100$$

$$1 < \log_{10} 58 < 2$$

$$\log_{10} 58 = 1 + \text{a decimal number}$$

2. Similarly for three digit numeral say 580 we can do as.

$$100 < 580 < 1000, \text{ if } \log_{10} 100 < \log_{10} 580 < \log_{10} 1000$$

$$2 < \log_{10} 580 < 3$$

$$\log_{10} 580 = 2 + \text{a decimal number}$$

3. It is also true that:

$$58 = 5.8 \times 10^1, \text{ then}$$

$$\log_{10} 58 = \log_{10} (5.8 \times 10)$$

$$= \log_{10} 5.8 + \log_{10} 10 \text{ why?}$$

$$= \text{a decimal number} + 1$$

$$= 1 + 0.7634 \leftarrow, \text{ (this is obtained from the table of logarithm.)}$$

$$= 1.7634$$

4. Similarly $\log_{10} 580$

Since $580 = 5.80 \times 10^2$. then

$$\log_{10} 580 = \log_{10} (580 \times 10^2) \text{ why?}$$

$$= \log_{10} 580 + \log_{10} 10^2 \text{ why?}$$

$$= 0.7634 + 2$$

$$= 2.7634$$

In the above example the values 1 and 2 are called characteristics and the decimal numeral 0.7634 called mantissa.

0.7634	+	2
↑		↑
mantissa		characteristics

5. Find the characteristic and the mantissa of $\log_{10} 436$

Solution:

$$\begin{aligned}
 \log_{10} 436 &= \log_{10} (4.36 \times 10^2) \\
 &= \log_{10} 4.36 + \log_{10} 10^2 \\
 &= 0.6405 + 2 \\
 &= 2.6405
 \end{aligned}$$

The characteristic is 2
The mantissa is 0.6405

- a) Did you understand each step?
- b) What do we call the system of writing the number 436 as 4.36×10^2 ?
- c) To find the logarithm of a number in base 10, we should write the number in scientific notation.
- d) Do you remember what a scientific notation is?

6. Find the value of $\log 0.00572$.

Solution:

$$\begin{aligned}
 \log_{10} 0.00572 &= \log_{10} (5.72 \times 10^{-3}) \\
 &= \log_{10} 5.72 + \log_{10} 10^{-3} \\
 &= 0.7574 + -3 \\
 &= \overline{3.7574}.
 \end{aligned}$$

The characteristic is -3

The mantissa is 0.75574.

We can also write

The line over the number 3 indicates only the number 3 is negative (the characteristic). The mantissa can not be negative.

7. Write the characteristics of each of the following by observation.
- | | |
|-----------|----------|
| a) 486 | b) 91 |
| c) 2348 | d) 9 |
| e) 0.0046 | f) 0.345 |
8. Find the mantissa of the following numbers using logarithmic table.
- | | |
|---------|------------|
| a) 563 | b) 23.7 |
| c) 6184 | d) 0.00436 |
| e) 5.98 | |
9. Find the logarithms in each of the following questions using table of logarithm.
- | | |
|----------------|------------------|
| a) $\log 32$ | b) $\log 128$ |
| b) $\log 500$ | d) $\log 789000$ |
| e) $\log 4200$ | f) $\log 9$ |
| g) $\log 256$ | h) $\log 150$ |
10. Evaluate the following logarithms using table of logarithm
- | | |
|-------------------------|-------------------------|
| a) $\log 9.8$ | b) $\log 0.0346$ |
| c) $\log 0.00049$ | d) $\log 56.4$ |
| e) $\log \sqrt{452}$ | f) $\log \sqrt[3]{54}$ |
| g) $\log \sqrt{0.0045}$ | h) $\log \sqrt[3]{253}$ |

LOGARITHMS OF NUMBERS

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

N	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7716	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9779
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9314
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

Chapter 6

Lines and Angles

By the end of the chapter you will be able to:

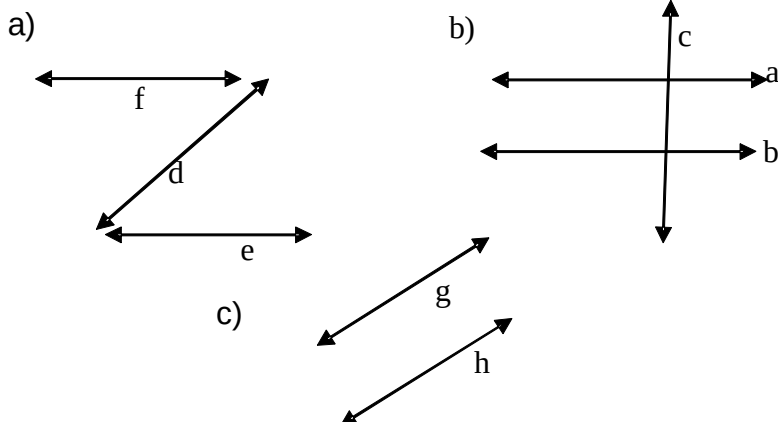
- a) identify parallel lines and perpendicular lines;
- b) identify corresponding, alternate interior, alternate exterior and vertically opposite angles;

6.1 Parallel and Perpendicular Lines

A. Parallel Lines

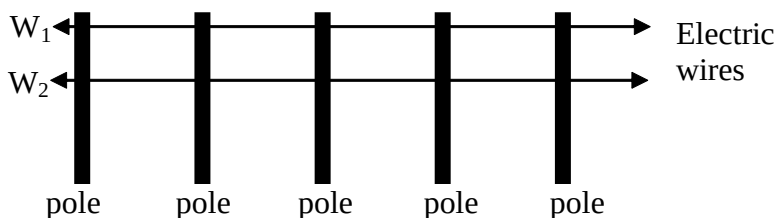
Activity 6.1

1. Look at the following lines:

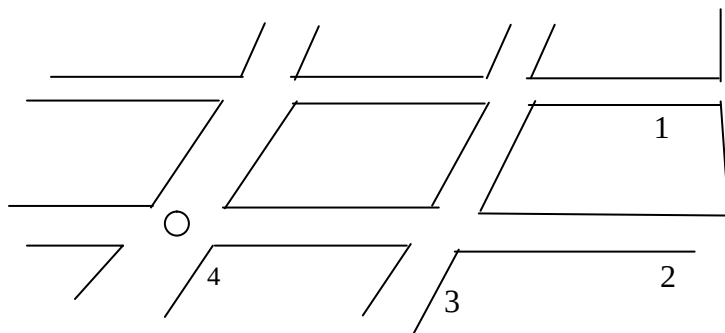


- a) Which of the above lines do you think will intersect?
- b) Which of the above lines do you think will not intersect?
- c) What do we call lines in the same plane that do not intersect?

2. Nejat called lines that do not intersect as *parallel lines*. And she gave some examples of parallel lines around her home such as electric wires on poles.



3. Lemlem also pointed out some more examples that are parallel streets that she has seen in Asmara.

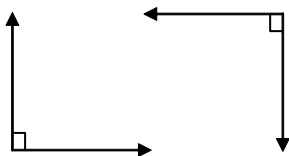


Lines in the same plane that do not intersect are called *parallel lines*.

4. Can you name some more examples of parallel lines?
5. Draw two parallel lines.
6. Draw two non-parallel lines.
7. What is the degree measure of the angle between the two non-parallel lines?
8. What is the degree measure of the angle between the two parallel lines?
9. Draw three lines in which two of the lines are parallel while the other intersects one of the two parallel lines.

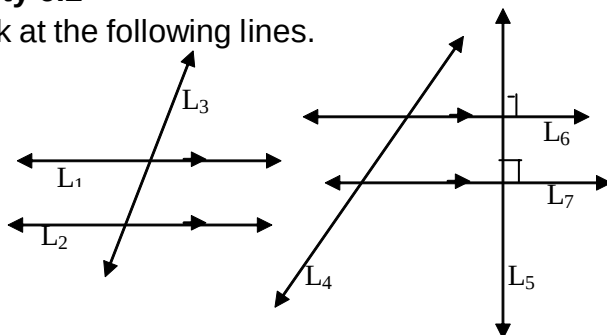
B. Perpendicular Lines

When two lines intersect, and if the angle between them is 90° , we call the two lines *perpendicular lines*. Look at the figure below.



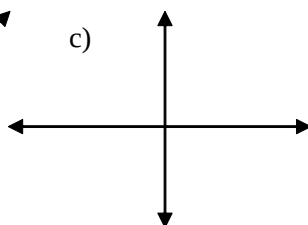
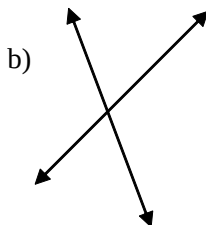
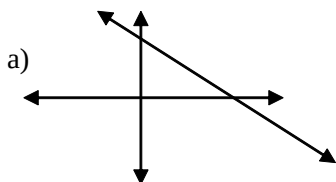
Activity 6.2

1. Look at the following lines.

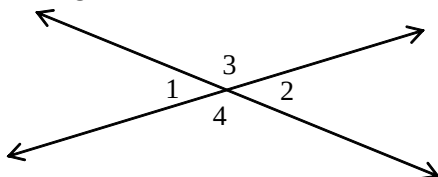


- Which of the above lines are parallel?
- Which of the above lines are perpendicular?
- Which of the above lines are intersecting but not perpendicular?
- Draw two perpendicular lines.
- Name some perpendicular lines around your class room.

2. Use a set square to check whether the lines are perpendicular or not.

**6.2 Vertically Opposite Angles.**

Angles whose sides form two pairs of opposite rays are called vertically opposite angles.



$\angle 1$ and $\angle 3$ are vertically opposite angles

$\angle 2$ and $\angle 4$ are vertically opposite angles

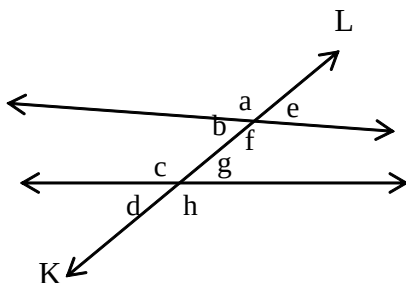
N.B Vertically opposite angles are equal.

That is: $m(\angle 1) = m(\angle 3)$

$m(\angle 2) = m(\angle 4)$

6.3 Corresponding, Alternate Interior and Alternate Exterior Angles :

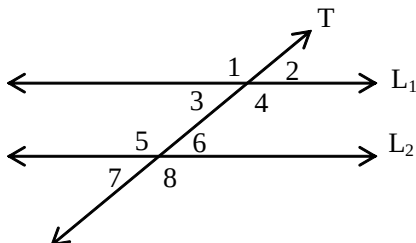
A straight line such as KL which cuts two or more other straight lines is called a **transversal**.



1. Certain pair of angles formed by a transversal has a special name. In the figure above:
 - a) The pairs of angles: 'a' and 'c', 'b' and 'd', 'e' and 'g', and 'f' and 'h' are **corresponding angles**.
 - b) The angles b, f, g and c are **interior angles**;
 - c) The pair of angles: b and g, and f and c are **alternate interior angles**, and
 - d) The pair of angles: a and h, and e and d are **alternate exterior angles**

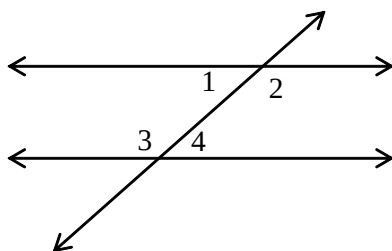
Look at the figure below

L_1 and L_2 are parallel lines; and T is a transversal



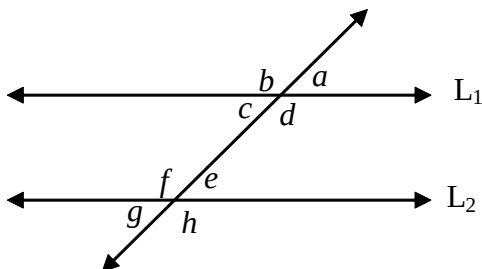
2. If a transversal crosses two parallel lines the corresponding angles are congruent.
 Angles **1** and **5** are congruent angles
 Angles **3** and **7** are congruent angles
 Angles **2** and **6** are congruent angles
 Angles **4** and **8** are congruent angles
3. If a transversal crosses two parallel lines the alternate interior angles are congruent
 Angles **3** and **6** are congruent
 Angles **4** and **5** are congruent

Two interior angles with different vertices on opposite sides of a transversal are called alternate interior angles.

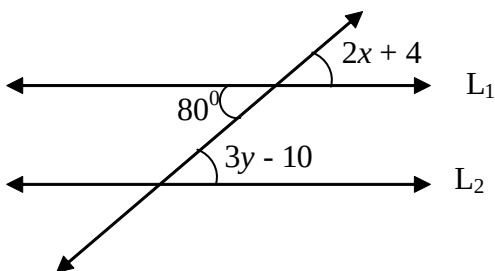


Activity 6.3

1. In the diagram below L_1 and L_2 are parallel
 - a) Name the pairs of interior angles.
 - b) Name the pairs of alternate exterior angles.
 - c) Name the pairs of corresponding angles.
 - d) Name the pairs of vertical opposite angles.



2. In the diagram L_1 and L_2 are parallel. Find x and y



3. Find the complementary and supplementary angles of the following.

a) 40°

b) 18°

c) 54°

d) 89°

4. Two angles are supplementary and the measure of the larger angle is three times the measure of the smaller angle. Find the measure of the angles.

Chapter 7

Right Angled Triangle and Trigonometric Ratio

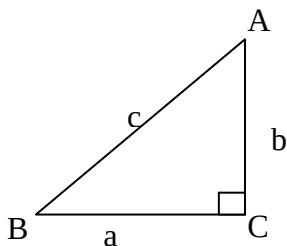
By the end of the chapter you will be able to:

- know and identify the angle relationship and side relationship of a right angled triangle;
- describe the sine, cosine and tangent ratio of right angled triangle;
- know the trigonometric value of sine, cosine and tangent of the common angles 30° , 45° , 60° and 90° .

7.1 Right-Angled Triangles

Activity 7.1

- Take a piece of a rectangular paper; draw a line from one corner to the other opposite corner. Then cut the paper diagonally.
 - How many pieces of paper do you have?
 - What type of triangles are formed?
- ABC is a right angled triangle in which angle $C = 90^\circ$



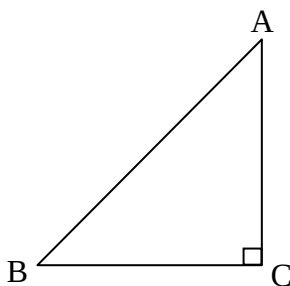
The longest side of a right- angled triangle is called *hypotenuse* and it is opposite to the right angle. The other two sides of a right angled triangle are called the legs of the triangle.

Note that: the sides are represented by small letters (a, b, c,) and the angles are represented by capital letters (A, B, C).

- Which side is the hypotenuse?
- What is the length of the hypotenuse?
- Name the two legs of the triangle and write their length.
- Name the angles of the triangle and the sides opposite to the angles

Labelling a Right Angled Triangle

In the figure below ABC is a right angled triangle in which angle C is 90° . The sides of the right angle are named as *opposite* or *adjacent* with respect to the angles.

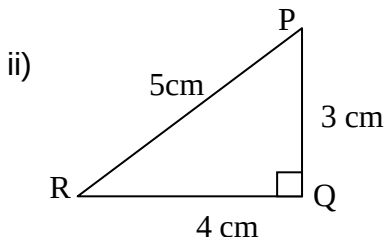
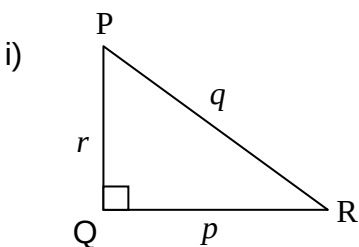


- AB is the hypotenuse
- With respect to angle B, AC is the opposite side and BC is the adjacent side
- With respect to angle A, BC is the opposite side and AC is the adjacent side.

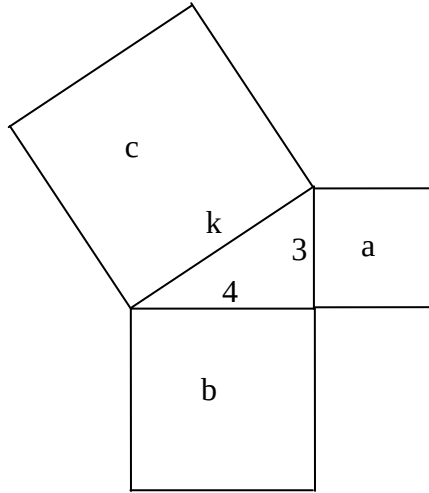
Activity 7.2

1. For each of the right angled triangles given below write:

- The side opposite to angle R.
- The side adjacent to angle R.
- The hypotenuse.



2. Construct a right-angled triangle about the middle of your page. Make the legs of the right angle of length 3 cm and 4cm. Construct squares based on the sides.



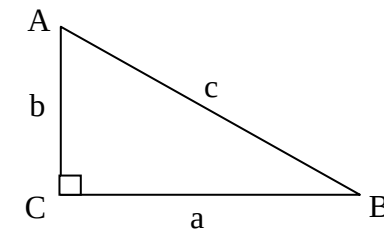
Use the figure above to answer each of the following questions:

- Measure the hypotenuse and write the value of k .
- What is the area of square a ?
- What is the area of square b ?
- What is the area of square c ?
- What is the total area of square a and b ?
- How does this total compare with the area of square c ?
- What do you conclude from the result of your work?

7. 2 Pythagoras Theorem:

In any right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two legs.

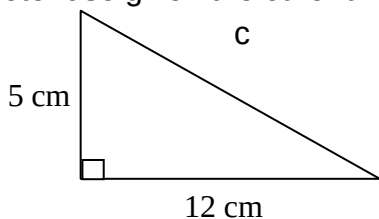
This statement is known as the **Pythagoras Theorem**.



That is, $c^2 = a^2 + b^2$

Activity 7.3

1. Find the hypotenuse given the other two sides.



Solution:

$$c^2 = a^2 + b^2$$

$$c^2 = (5 \text{ cm})^2 + (12 \text{ cm})^2$$

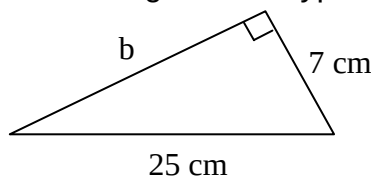
$$c^2 = 25 \text{ cm}^2 + 144 \text{ cm}^2$$

$$c^2 = 169 \text{ cm}^2$$

$$c = \sqrt{169 \text{ cm}^2}$$

$$c = 13 \text{ cm}$$

2. Find the unknown side given the hypotenuse and one side



Solution:

$$c^2 = a^2 + b^2$$

$$(25 \text{ cm})^2 = (7 \text{ cm})^2 + b^2$$

$$625 \text{ cm}^2 = 49 \text{ cm}^2 + b^2$$

$$625 \text{ cm}^2 - 49 \text{ cm}^2 = b^2$$

$$576 \text{ cm}^2 = b^2$$

$$b = \sqrt{576 \text{ cm}^2}$$

$$b = 24 \text{ cm}$$

3. In $\triangle ABC$, $m(\angle C)$ is 90° , AB is 5 cm and AC is 4 cm. Calculate the length of BC ?

Solution

$$c^2 = a^2 + b^2$$

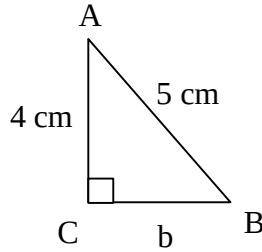
$$(5\text{cm})^2 = b^2 + (4\text{cm})^2$$

$$25\text{cm}^2 - 16\text{cm}^2 = b^2$$

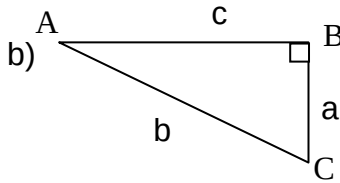
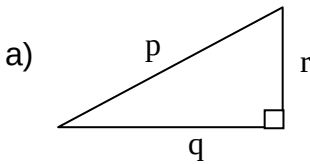
$$b^2 = 9\text{cm}^2$$

$$b = \sqrt{9\text{cm}^2}$$

$$b = 3\text{cm}$$



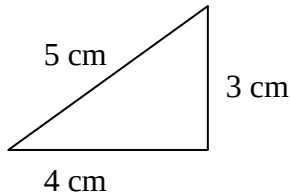
4. Write the Pythagorean equation for each of the following



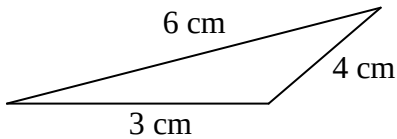
5. Construct a triangle whose sides are 3 cm, 4 cm and 5 cm long.

a) Measure the angles.

b) What is the measure of the angle opposite the longest side?



6. Construct a triangle whose sides are 3 cm, 4 cm and 6 cm



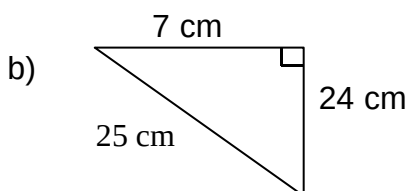
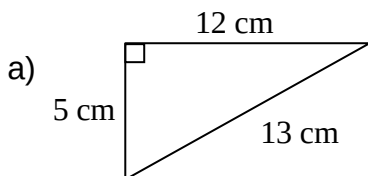
$$\text{Is } 3^2 + 4^2 = 6^2?$$

- Measure the angles.
- What is the measure of the angle opposite the longest side?
- What do you conclude from the result of your work?
- If the lengths of the sides of a triangle are a , b , & c and if $a^2 + b^2 = c^2$. Is the triangle a right angled?

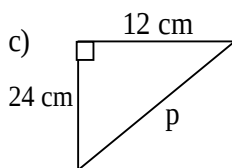
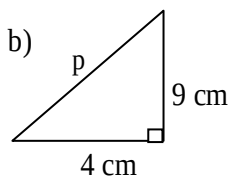
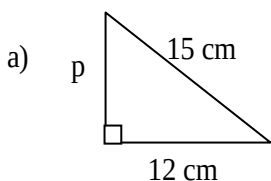
7. The lengths of the sides of a triangle are 8cm, 15cm, and 17cm. Show that the triangle is a right-angled triangle.

8. The lengths of the sides of a triangle are 8cm, 18cm, and 12cm. Is the triangle a right angled triangle?

9. State whether the measurements are correct or not, for the following right-angled triangles



10. Find the values of p in each of the following right angled triangles.

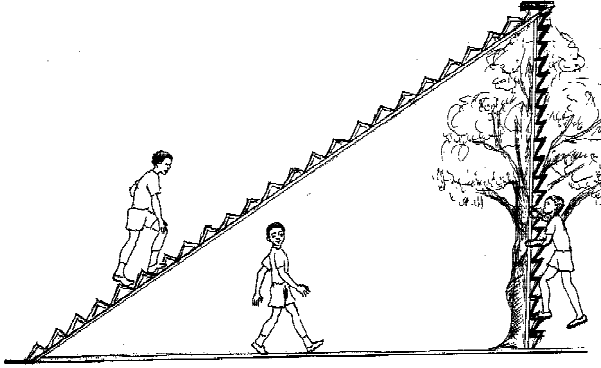


11. In $\triangle ABC$, angle B is 90° . $AB = 8$ cm, and $BC = 6$ cm. Find the length of AC .

12. In $\triangle DEF$, angle F is 90° , $EF = 10$ cm, and $DE = 26$ cm. Find the length of DF .

7.3 Trigonometric Ratios:

Consider the figure below where the three children made their activities. One climbed along the a vertical ladder, the second along the inclined ladder and the third moved along the ground.



- Do you agree that a right angled triangle is formed?
- What are your observations?
- Is there a relationship between the lengths of the vertical ladder, inclined ladder and the ground distance?

There is a relationship between the sides and the angle formed in the figure. Mathematicians call the relationship trigonometric it is a Greek word; and Trigon -means Three sided and Metric - means measurement.

Hence, trigonometric means measurement about the relationship of a triangle.

- What do we mean by trigonometric relationships?

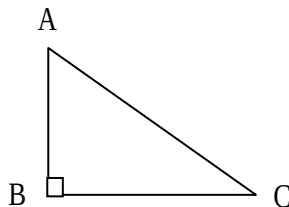
Activity 7.3

- Measure the length of the
 - inclined ladder
 - vertical ladder
 - ground distance
- Write 6 different ratios of the lengths of the sides of the figure.

3. The ratio of the length of
- Vertical ladder to inclined ladder is called sine
 - Horizontal distance to inclined ladder is called cosine
 - Vertical ladder to horizontal ladder is called tangent

A. The Sine of an Angle:

$$\text{Sine of an angle} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$



Triangle ABC is a right-angled triangle $m(\hat{B}) = 90^\circ$

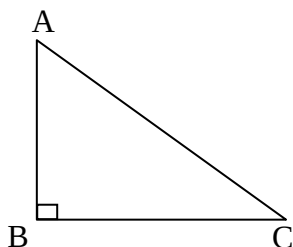
$$\sin \hat{A} = \frac{BC}{AC}, \text{ and } \sin \hat{C} = \frac{AB}{AC}$$

B. The Cosine of an Angle:

The cosine of an angle is given by:

$$\text{Cosine of an angle} = \frac{\text{Adjacent side}}{\text{Hypotenuse}}$$

Triangle ABC is a right angled triangle in which $m(\hat{B}) = 90^\circ$



$$\cos \hat{A} = \frac{AB}{AC} \text{ and } \cos \hat{C} = \frac{BC}{AC}$$

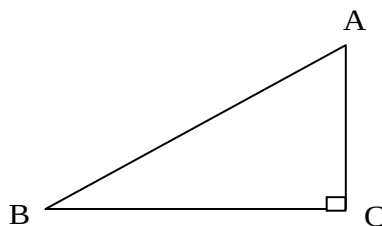
C. The Tangent of an Angle:

The tangent of an angle is given by

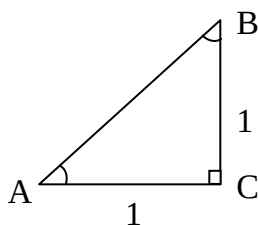
$$\text{Tangent of an angle} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

Triangle ABC is a right-angled triangle in which

$$m(\hat{C}) = 90^\circ.$$



$$\tan \hat{B} = \frac{AC}{BC} \quad \text{and} \quad \tan \hat{A} = \frac{BC}{AC}$$

Special Right - angled Triangles**A. The $45^\circ - 45^\circ - 90^\circ$ triangle Trigonometric Ratio of 45°** 

In an isosceles right angled triangle ABC with angle $C = 90^\circ$, let $a = 1$ and $b = 1$ unit. The acute angles A and B have equal measures each being 45° .

$$\begin{aligned} \text{Since } c^2 &= a^2 + b^2 \\ c^2 &= 1^2 + 1^2 \\ c^2 &= 2 \\ c &= \sqrt{2} \end{aligned}$$

The trigonometric ratios of 45° angle can be expressed as follows.

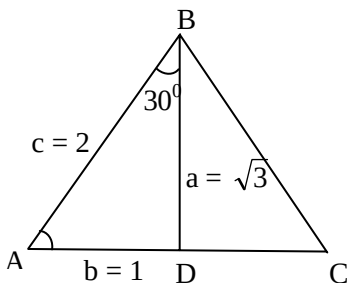
$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

B. The 30-60-90 Degrees Triangle:

The trigonometric ratios of 30° and 60° may be obtained by using an equilateral triangle. Draw an equilateral triangle, ABC whose side is 2 units.



BD is the **altitude** of $\triangle ABC$, $\triangle ABD$ and $\triangle CBD$, so BD bisects angle B and side AC. Why?

In the right angled triangle ABD, $c = 2$ units and $b = 1$ units.

$$a^2 = c^2 - b^2 \text{ why?}$$

$$a^2 = 2^2 - 1^2$$

$$a^2 = 3$$

$$a = \sqrt{3}$$

Using $\triangle ABD$

a) $\sin 30^\circ = \frac{1}{2}$

d) $\sin 60^\circ = \frac{\sqrt{3}}{2}$

b) $\cos 30^\circ = \frac{\sqrt{3}}{2}$

e) $\cos 60^\circ = \frac{1}{2}$

c) $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

f) $\tan 60^\circ = \sqrt{3}$

Summary of Values for Trigonometric Ratios of 30° , 45° and 60° .

Angle	30°	45°	60°
Sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cos	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{2}$
Tan	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

Activity 7.4

- Find the trigonometric ratios of 45° using a right isosceles triangle whose leg is 4 units.
- Find the trigonometric ratios of 30° and 60° using an equilateral triangle whose side is:
 - 6 units.
 - 4 units

3. Find the numerical values of each of the following

a) $\frac{\sin 60^\circ}{\sin 30^\circ}$

b) $\frac{\cos 45^\circ}{\cos 60^\circ}$

c) $\frac{\tan 45^\circ}{\sin 30^\circ}$

d) $\frac{\sin 60^\circ}{\cos 60^\circ}$

4. Find the degree measure of each angle x

a) $\cos x = \frac{1}{2}$

b) $\cos x = \frac{\sqrt{3}}{2}$

c) $\sin x = \frac{\sqrt{3}}{2}$

d) $\tan x = \sqrt{3}$

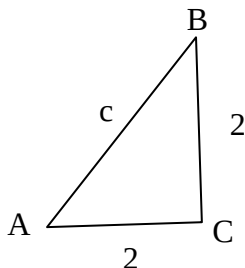
e) $\tan x - 1 = 0$

f) $2\sin x = 1$

5. In a 30° , 60° , 90° triangle the length of the leg opposite to the 30° angle is 4 cm . Find the length of the hypotenuse and adjacent sides.

6. In a 30° , 60° , 90° triangle if the side opposite the 30° angle is x units find the lengths of the remaining sides.

7. If a = 2 units and b= 2 units, find c and write the trigonometric ratios of 45°



8. Given an isosceles right-angled triangle with hypotenuse of length:

a) $10\sqrt{2}$ units

b) 5 units

c) a units

Find the length of the legs of the triangle.

9. In a 45° , 45° , 90° triangle if the length of the hypotenuse is x units what is the length of the legs?

Chapter 8

Congruency and Similarity

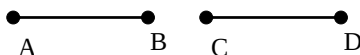
By the end of the chapter you will be able to:

- relate the corresponding angles and sides of one polygon with another polygon;
- identify whether a pair of polygons are congruent or not;
- identify the ratio of proportionality and the relationship of two similar polygons;
- do investigative reasoning and understanding in dealing with congruency and similarity proofs.

8.1 Congruent Figures

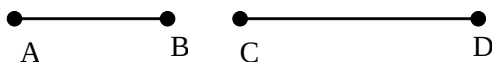
Activity 8.1

1. Halima and Azieb have sticks of the same kind and of the following lengths.



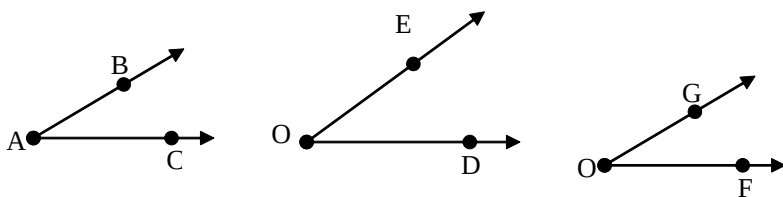
- Measure the length of the sticks.
- What can you say about the sticks?

2. Adem and Alem drew the following segments.



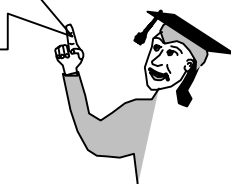
- Measure the length of the segments.
- Which segment is longer?
- What can you say about the segments?

3. Amar, Amanuel and Zebiba drew three angles as described below



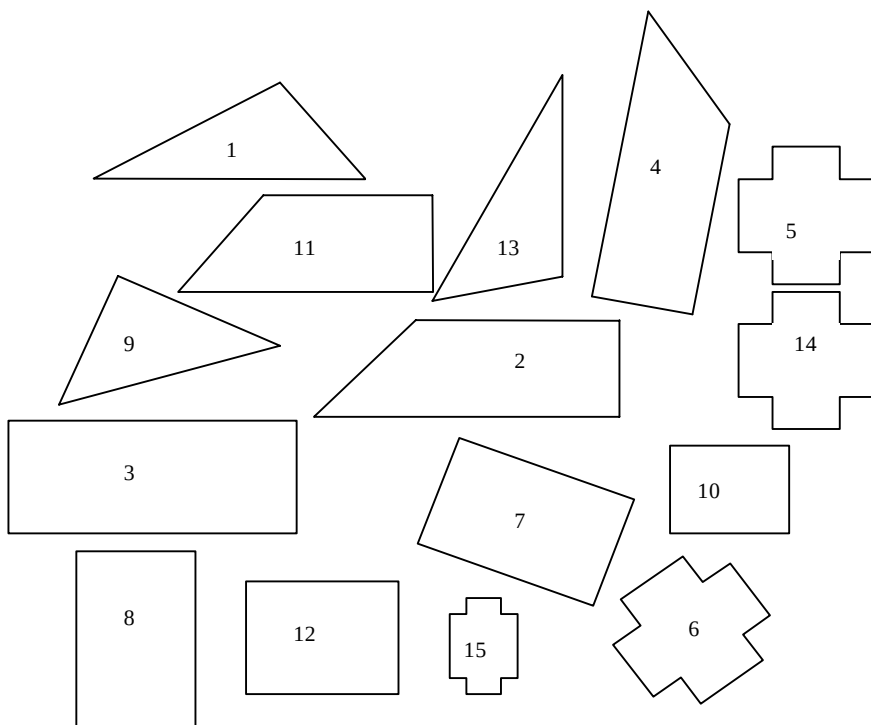
- a) Compare the degree measures of the angles.
- b) Do they have the same measure?
- c) How do you compare the angles?
- d) Which angles fit to one another?
- e) What do you say about the angles which fit to one another?

Angles which fit to one another are said to be equal, most commonly they are said to be congruent

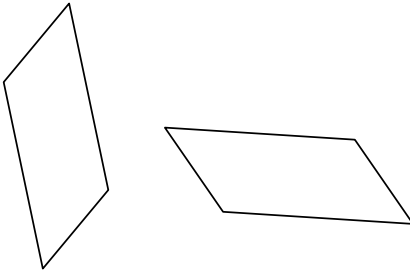


Activity 8.2

1. Look at the following figures:



- a) Which of the figures have the same shape?
 - b) Which of the figures have the same size?
 - c) Which of the above figures are congruent?
2. Draw a rhombus of side 3cm long and a square of side 3cm long.
 - a) Do they fit exactly?
 - b) Are the two figures congruent?
 3. Haile drew the following:

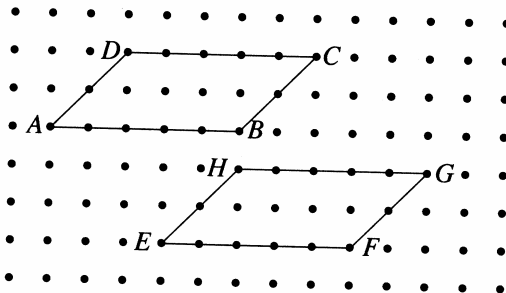


- a) Do they fit exactly?
- b) Are they congruent?

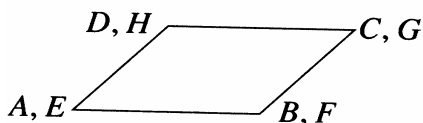
If two figures are of the same shape and the same size they are congruent.



4. Look at the following pair of parallelograms.



- Are they of the same shape?
- Are they of the same size?
- Are they congruent?
- If we place parallelogram ABCD over parallelogram EFGH, will they fit exactly?

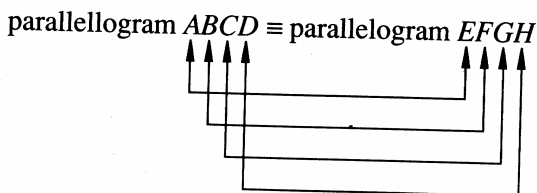


We notice that $\hat{A}$ is congruent to $\hat{E}$. We call $\hat{A}$ and $\hat{E}$ corresponding angles. Can you find other corresponding angles?

- Side AB is congruent to side EF. We call side AB and side EF corresponding sides. Can you find other corresponding sides?

To indicate the congruency of two parallelograms, we write:
 Parallelogram ABCD $\equiv$ parallelogram EFGH

The symbol $\equiv$ stands for 'is congruent to'. The letters are written such that their positions indicate the corresponding vertices of the two parallelograms.



- If two polygons are congruent, their
- corresponding angles are congruent.
 - corresponding sides are congruent

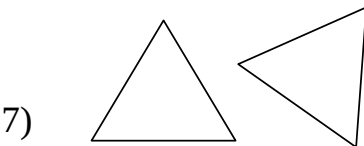
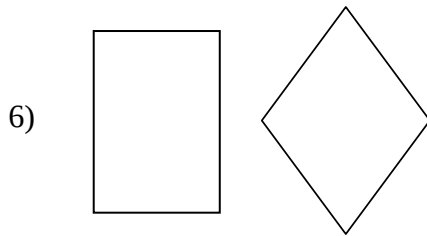
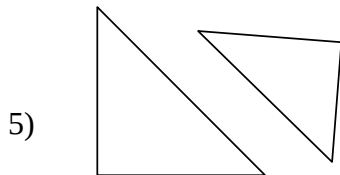
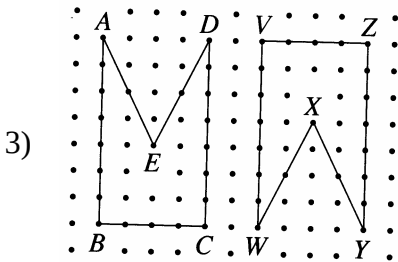
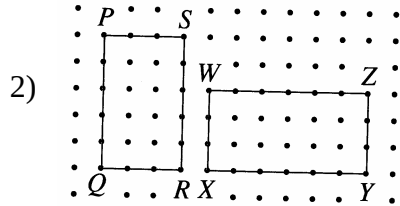
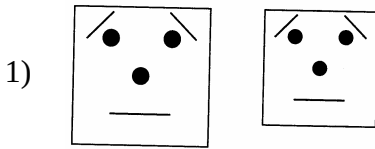


The converse is also true . If two polygons have congruent corresponding angles and congruent corresponding sides , we say that the two polygons are congruent .



Activity 8.3

Which of the following pair of figures are congruent?

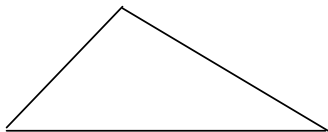


Congruent Triangles

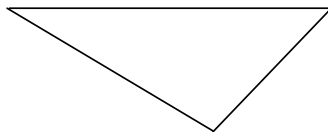
Activity 8. 4

1. Fana and Fatna drew two different triangles. They want to check whether the triangles are congruent or not?

Fana's drawing



Fatna's drawing

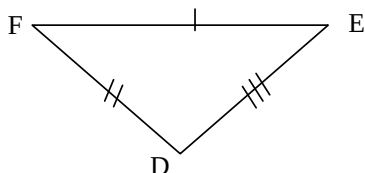
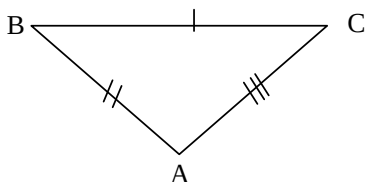


- a) Copy the above triangles in a tracing paper, put one triangle over the other. Do the two triangles coincide?
- b) Construct a triangle with sides 10 cm, 12 cm and 13 cm. Check whether your triangle is congruent to the one drawn by your classmate?
- c) Construct a triangle with sides 5 cm, 7 cm and 10 cm. And let your friend draw a triangle with sides 5 cm, 7 cm and 9 cm. Are the two triangles congruent?

A. Three Sides (SSS)

If all the three sides of one triangle are congruent to the corresponding sides of another triangle, the triangles are congruent.

2. Look at the triangles below



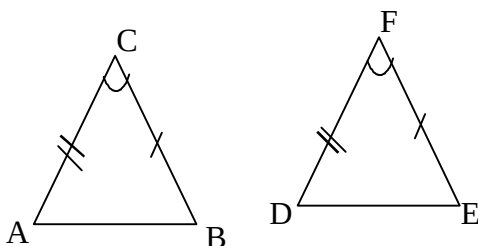
- a) Identify the corresponding sides and corresponding angles?
- b) Are the corresponding sides congruent?
- c) Are the two triangles congruent?
- d) Are the corresponding angles congruent? Why?

3. A teacher gave an assignment to Abraham and Ibrahim to construct a triangle with two sides 12cm and 14cm and an included angle of 50° .
- Draw a triangle as that of Abraham and Ibrahim?
 - Is your triangle congruent to the one drawn by your classmate?
 - Could you check the congruency of the triangles by Haile's approach? Why?

B. Two Sides and an Included Angle (SAS)

If two sides and the included angle of one triangle are equal to the corresponding sides and an included angle of another triangle, then the triangles are congruent

4. Look at the following figures.



- Identify the corresponding sides and corresponding angles?
- Are the corresponding sides congruent?
- Are the corresponding angles congruent? Why?
- Are the two triangles congruent? Why?

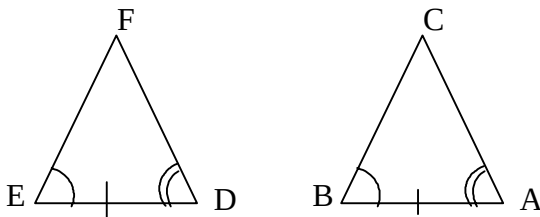
Is there any more tests that we can use to find out whether two triangles are congruent or not?



C. Two Angles and a Side (AAS or ASA or SAA)

If two angles and a side of one triangle are equal to the corresponding angles and a side of another triangle, then the triangles are congruent

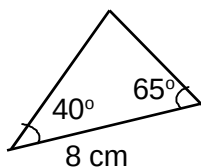
5. Look at the following figures.



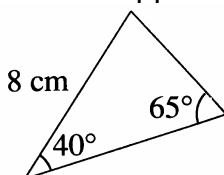
- Identify the corresponding sides and corresponding angles?
- Are the corresponding angles congruent?
- Are the corresponding sides congruent? Why?
- Are the two triangles congruent? Why?

6. Three students drew different triangles as shown below.

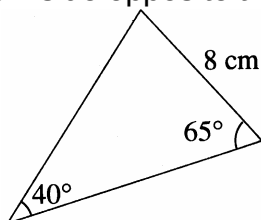
Haile: has the 8 cm side between the 65° and 40° angles,



Ibrahim: has the 8 cm side opposite the 65° angle,



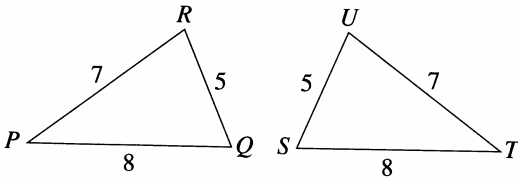
Fana: has the 8 cm side opposite the 40° angle,



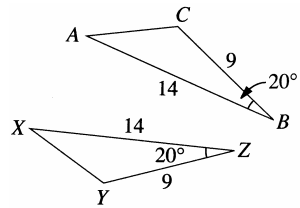
- Label each vertex of each triangle.
- Identify the corresponding sides and corresponding angles?
- Are the corresponding angles equal?
- Are the corresponding sides congruent?
- Are the two triangles congruent? Why?

7. Name the postulate by which the following pair of triangles are congruent.

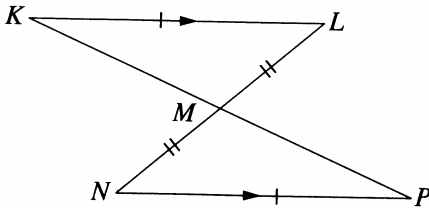
a)



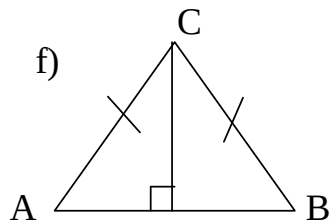
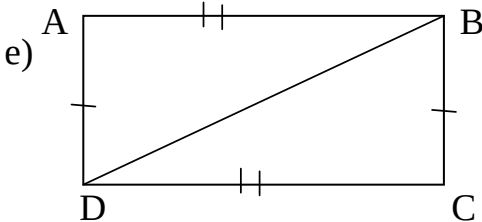
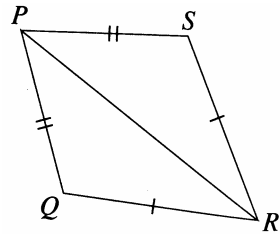
b)



c)



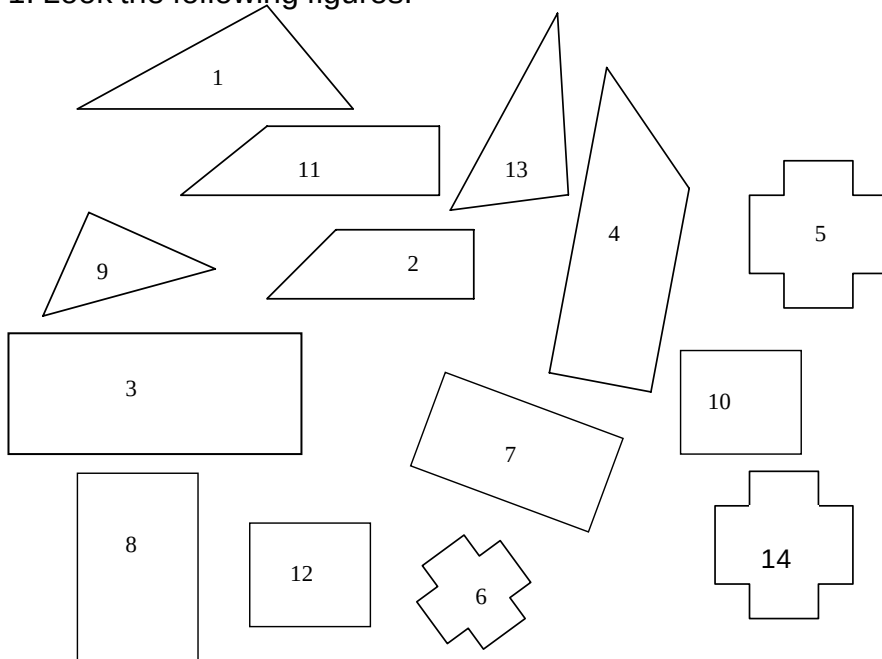
d)



8.2 Similarity:

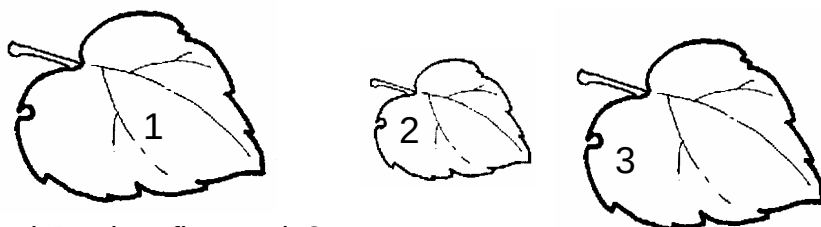
Activity 8.5

1. Look the following figures:



- Which of the figures have the same shape?
- Which of the figures have the same size?
- Which of the figures are similar?

2. Look at the following figures:

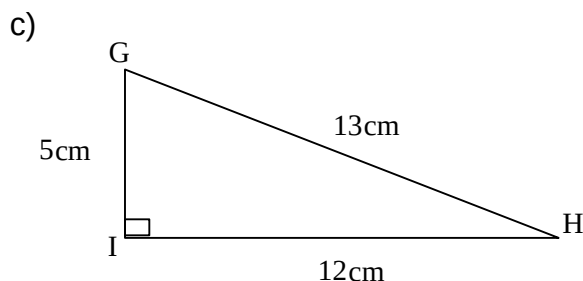
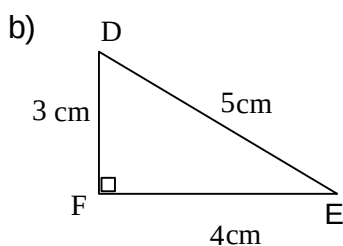
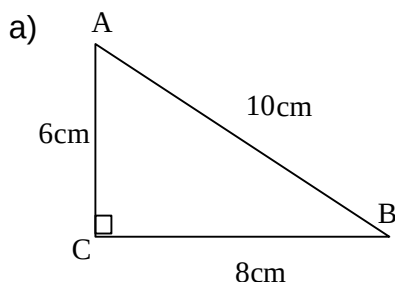


- Do they fit exactly?
- Do they match to one another?
- What do you think these figures are?

Two figures are similar if the:

- corresponding angles are equal; and
- corresponding sides are proportional (sides have the same ratio).

3. Which of the following triangles have proportional sides?



Solution:

a) If $\triangle ABC$ is similar to $\triangle DEF$, then the following is true.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{10}{5} = \frac{8}{4} = \frac{6}{3} = 2$$

The ratios of the corresponding sides are equal.

b) If $\triangle ABC$ is similar to $\triangle GHI$, then the following is true.

$$\frac{AB}{GH} = \frac{BC}{HI} = \frac{AC}{GI}, \text{ but}$$

$$\frac{10}{13} \neq \frac{8}{12} \neq \frac{6}{5}$$

The ratios of the corresponding sides are not equal. Hence, the two triangles are not similar.

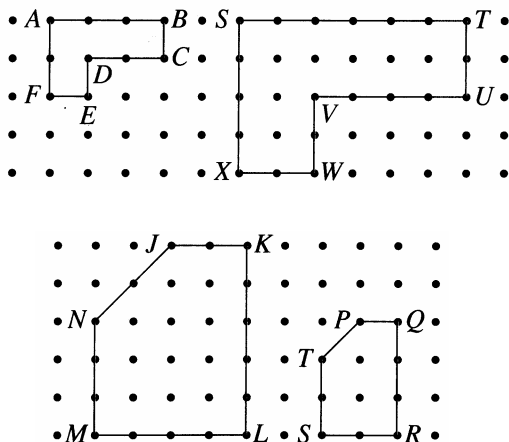
c) If $\triangle DEF$ is similar to $\triangle GHI$, then the following is true.

$$\frac{DE}{GH} = \frac{EF}{HI} = \frac{DF}{GI} \quad \text{but}$$

$$\frac{5}{13} \neq \frac{4}{12} \neq \frac{3}{5}$$

The ratios of the corresponding sides are not equal.
Therefore, the two triangles are not similar.

4. Which of the following pair of figures are similar? Why?



5. Which of the following are always similar?

- Two equilateral triangles
- Two isosceles triangles
- Two rectangles
- Two squares
- Two right - angled triangles
- Two congruent polygons

Chapter 9

Business Mathematics

By the end of the chapter you will be able to:

- a) describe different terms of business mathematics such as: Profit, loss, commission, discount and simple interest; and use in your daily activity.
- b) solve problems related to direct and indirect variations,

9.1 Profit and Loss

In business, when merchants make money they have to measure how much profit or loss they make.

How do they know their profit?

Activity 9.1

So let's look at two merchants Sara and Ali.

1. Sara and Ali are business men. They trade in a whole sale. Sara is egg seller. She buys eggs at 80 Nakfa for 100 and sells them at 110 Nakfa for 100.
2. Ali is a book seller. He buys books at 150 Nakfa for 10 and sells them at 180 Nakfa for 10.

Which transaction is better?

To find out which transaction is better, Beshir a friend of Ali worked as follows:

Profit = Selling price- Cost price

Sara's profit = $110 - 80 = 30$ Nakfa from 100 eggs.

Ali's profit = $180 - 150 = 30$ Nakfa from 10 books.

He concluded that both have the same profit and hence they are equally profitable business. Do you agree with Beshire's argument?

Elsa who is clever in mathematics said, " I do not agree".

To find out which transaction is better business, we have to express the **profit** as a percentage of the cost, and she calculated it as follows:

Sara's profit = $110 - 80 = 30$ Nakfa.

$$\text{Her profit \% is } \frac{30}{80} = \frac{x}{100}$$

$$x = \frac{30 \times 100}{80}$$

$$= \frac{3 \times 25}{2}$$

$$x = 37.5 \%$$

Ali's profit = $180 - 150 = 30$

$$\text{His profit in \% is } \frac{30}{150} = \frac{x}{100}$$

$$x = \frac{30 \times 100}{150}$$

$$x = 20\%$$

Hence, Sara's business has a better transaction.

- a) Do you agree with Elsa's calculation? Why or why not?
- b) Which way of comparison will give you enough information to determine a better business?
- c) Discuss about it with your partners.
- d) In a business transaction when do you have a profit?

3. A shop keeper bought goods for 250 Nakfa and sold them at 265 Nakfa.

- a) What is his profit?
- b) What is the profit %?

Solution

$$\text{a) Profit} = S. P - C. P$$

$$= 265 - 250$$

$$= 15 \text{ Nakfa}$$

$$\begin{aligned}
 \text{b) Profit \%} &= \frac{\text{profit} \times 100}{\text{C.P}} \\
 &= \frac{15 \times 100}{250} \\
 &= 6 \%
 \end{aligned}$$

4. Nura bought 12 trousers for 2200 Nakfa and sold them for 2090 Nakfa.

a) Does she have profit or loss?

b) Find her profit or loss %?

Solution:

If selling price is greater than cost price then, we have a profit. But if cost price is greater than selling price, we have a loss.

a) Hence Nura has a loss since 2200 is greater than 2090.

$$\begin{aligned}
 \text{Loss} &= \text{C.P.} - \text{S. P.} \\
 &= 2200 - 2090 \\
 &= 110 \text{ Nakfa}
 \end{aligned}$$

$$\begin{aligned}
 \text{Loss \%} &= \frac{\text{loss} \times 100}{\text{C.P}} \\
 &= \frac{110 \times 100}{2200} \\
 &= 5\%
 \end{aligned}$$

Profit or loss is usually expressed as a percentage of the cost to determine which business is more profitable or not.

5. A woman bought a hen for 50 Nakfa and sold it for 70 Nakfa. Find her profit %.

6. Articles that cost 25 Nakfa per dozen are sold 35 Nakfa per dozen. What is the profit Percent?

7. A man bought a television set for 4500 Nakfa and sold it at a profit of 5 %. Find the selling price.
8. By selling goods for 2544 Nakfa, a merchant made a profit of 6 %. How much did it cost him?
9. A radio that costs 500 Nakfa was sold at 370 Nakfa. Find the loss %.
10. Find the cost price of a refrigerator which was sold at 10,000 Nakfa at a profit of 25 %.
11. A cow was sold for 3500 Nakfa at a loss of 12.5 %. Find the cost price.
12. An article which costs 500 Nakfa was sold at a profit of 9 %. Find the amount of profit and the selling price.
13. Two qualities of coffee costing 25 and 30 Nakfa per kg are mixed in the ratio 3 : 2 , the mixture is sold at a profit of 20%. Find the selling price per kg.
14. By selling goods for 280 Nakfa, a loss of 28 Nakfa was made by a merchant. How much did the goods cost him?
15. A person sold 24 hens for 1152 Nakfa and lost 4 %. For what price should he sell each hen so as to gain 10 %.
16. Find the profit or loss percentage in the following transactions.

<u>Item</u>	<u>cost price</u>	<u>selling price</u>
Soap	3.25 Nakfa	3.75 Nakfa
Tea	15. 25 Nakfa	15 Nakfa
Sugar	7 Nakfa	7.50 Nakfa
Coffee	30 Nakfa	27 Nakfa

9.2 Discount and Commission

Activity 9.2

1. Expo Shoe Factory announces to the public that on the occasion of the New Year there is a 10 % discount on the prices of shoes in their Factory.

Azieb wanted to know how much she would pay for a pair of shoes that are marked 300 Nakfa and she calculated it as follows:

Solution:-

a) Discount % = 10 %

$$\begin{aligned}\frac{10}{100} &= \frac{x}{300} \\ 100x &= 300 \times 10 \\ x &= \frac{300 \times 10}{100} \\ x &= 30\end{aligned}$$

Discount = 30 Nakfa
cost price = marked price - discount
= 300 - 30 = 270
I have to pay 270 Nakfa.

b) But Nura calculated it like this:

$$\begin{aligned}\frac{90}{100} &= \frac{x}{300} \\ 100x &= 300 \times 90 \\ x &= \frac{300 \times 90}{100} \\ x &= 270 \text{ Nakfa}\end{aligned}$$

You have to pay 270 Nakfa.

2. a) Did you understand both methods?
b) Which method do you think is easy?
3. A furniture seller gives 12 % discount on goods paid in cash. How much cash would I have to pay for a cupboard marked 5200 Nakfa?
4. A sweater is marked 120 Nakfa. Find the net price after a discount of 20 %.
5. An article marked 80 Nakfa was sold for 64 Nakfa. Find the percentage of discount.
6. After getting a 15 % discount, I paid 170 Nakfa for a pair of shoes. What was the marked price?
7. How much cash must a man have to pay for a dress that is marked 240 Nakfa, if he got 12 % discount?
8. Fill the blank spaces.

Item	Original price	Discount	Paid in cash
TV	4800	15.5%	
Refrigerator	10000		8500
Radio		24%	480
Furniture	5000	12%	
Coat	1200		1000

Commission and Brokerage

Activity 9.3

1. Ato Beshir is a sales agent for the Red Sea Corporation. In addition to his monthly salary of 800 Nakfa, he gets a commission of 3 % from his sales, from the corporation. In a certain month he sold goods worth 6000 Nakfa.
 - a) How much commission did he receive?
 - b) How much income did he get in that month?

Solution:

a) Abraham calculated it as follows:

$$\begin{aligned}\frac{3}{100} &= \frac{x}{6000} \\ 100x &= 3 \times 6000 \\ x &= \frac{3 \times 6000}{100} \\ x &= 180 \text{ Nakfa}\end{aligned}$$

He received a commission of 180 Nakfa

b) $800 + 180 = 980$ Nakfa.

His income in that month was 980 Nakfa.

2. Berhe is a house agent (broker). He rents or sells houses. He charges 2% commission from what he sells and 20% of one month house rent from what he rents.

a) In a certain month he got 250 Nakfa from a house rent commission. How much did he rent the house?

b) If he got 4000 Nakfa commission from a house sale. Find the total sale of the house?

Solution:

a) Jimea calculated Berhes' house rent as follows: She let x to be the house rent. Then

$$\begin{aligned}\frac{20}{100} &= \frac{250}{x} \\ 20x &= 100 \times 250 \\ x &= \frac{100 \times 250}{20} \\ x &= 1250\end{aligned}$$

$$\begin{aligned} \text{b) } \frac{2}{100} &= \frac{4000}{x} \\ 2x &= 100 \times 4000 \\ x &= \frac{100 \times 4000}{2} \end{aligned}$$

$$x = 200,000 \text{ Nakfa}$$

a) Is Jimea's calculation correct?

b) Can you solve them by any other method?

3. a) Did you understand each step?

b) Can you explain to the class what a commission and a brokerage mean?

4. Elsa is a broker and charges 2% brokerage. In a certain month she sold a house worth 250000 Nakfa.

Calculate her brokerage income.

Solution:

$$\begin{aligned} \frac{2}{100} &= \frac{x}{250000} \\ 100x &= 2 \times 250000 \\ x &= \frac{2 \times 250000}{100} = 5000 \end{aligned}$$

She got an income of 5000 Nakfa.

5. A sales man has basic salary of 840 Nakfa. He receives 10 % commission for sales he made. In a certain month his total earring was 960 Nakfa, find his total sales.

6. By selling goods worth 6540 Nakfa, a sales agent received a commission of 163.50 Nakfa. Find the rate of commission.

7. A commission agent received 468 Nakfa as, a commission on his sale amounting to 7800 Nakfa. Find the rate of commission.

8. A man bought a house for 200000 Nakfa paying brokerage of $1\frac{1}{2}\%$. At what price will he sell it to obtain 10% more than what it cost him before?
9. I bought my bicycle for 1000 Nakfa through a broker paying him 5% brokerage. If I had sold it 850 Nakfa, find my loss and my loss %.
10. What expense decreased by 20% of it self gives 480?
11. Nebi got 15 % increase in his income. If his present income is 750Nakfa, find his income before the increase.

9.3 Banking: Simple Interest

Activity 9.4

1. Negash is grade 8 student in Bana middle school. One day his father took him to a shop to buy a new cloth for him. But he was so eager to save money in a Bank and told his father to deposit the 300 Nakfa in the Bank. At the end of the first year his father withdrew his money and received 318 Nakfa and showed to Negash.

Negash explained to his class mates about this case and asked them to show him how to calculate the interest.

His friend Ali said to him that, the 300 Nakfa is called *principal*. The 318 Nakfa is called *amount*, and the difference $318 - 300 = 18$ Nakfa is called *simple interest*.

Negash asked Ali if there is any rate. If there is a rate how do we calculate, asked? Oh ! easy:

$$\frac{18}{300} = \frac{x}{100}$$

$$300x = 18 \times 100$$

$$x = \frac{18 \times 100}{300}$$

$$x = 6\%$$

2. Negash asked another question to Ali. How much would have I received had my father deposited 450 Nakfa?

Ali calculated it as follows

$$\frac{6}{100} = \frac{x}{450}$$

$$100x = 450 \times 6$$

$$x = \frac{450 \times 6}{100}$$

$$x = 27$$

27 Nakfa is the interest

$$27 + 450 = 477 \text{ Nakfa}$$

Negash would have received 477 Nakfa

But Elsa said, we have the following formula to find the simple interest.

$$I = \frac{P \times R \times T}{100}$$

Where P = principal
 R = rate
 T = time
 I = simple interest
 A = amount, where
 $A = P + I$

3. Find the simple interest on a deposit of 6000 Nakfa at 4% per year for 3 years.
4. A woman borrowed 3000 for 2 years. If the rate of interest is 12% per year, find the simple interest that she will pay.

5. A man deposited 450 Nakfa in a bank. After 4 years he got back 558 Nakfa. Find the rate of simple interest.
6. If 350 Nakfa amounts to 420 Nakfa at 5% per annum. Find the time.
7. Find the principal which amounts at simple interest to 1200 Nakfa in 4 years at 5% per annum.
8. A man borrowed 500 Nakfa on the condition that he will repay 750 Nakfa at the end of 4 years. What is the rate of simple interest?

9.4 Variations (Direct and Indirect Variation)

A. Direct Variation:

Activity 9.5

1. Haile works in a company. The company pays his salary on the basis of working days. The rate of pay for one day is 45 Nakfa. How much will he get, if he works:
a) For 5 days? b) For 8 days?
c) For 10 days? d) For 20 days?
2. How many days did he work, if he got :
a) 135 Nakfa? b) 270 Nakfa?
c) 1350 Nakfa? d) 1800 Nakfa?
3. What relation did you observe between the number of working days and the amount of pay?
4. What happens to the amount of pay if the number of working days increase?

In these cases did you observe that when the number of working days increases the total amount of pay also increases and when the number of working days decreases the amount of pay decreases.

For any two quantities x and y , if x increases as y increases or x decreases as y decreases we call the variation a direct variation.

If there are two quantities x and y such that, if x_1 increases to x_2 as y_1 increases to y_2 or vice - versa then $\frac{x_1}{x_2} = \frac{y_1}{y_2}$

5. A hair stylist earns 60 Nakfa in 4 hrs.

a) How much does she earn in 10 hrs?

b) Draw a graph which shows the hair stylist income if she works for the given hours.

Solution

a) If she earns n_1 Nakfa in a time t_1 hrs , then she will earn n_2 Nakfa in a time t_2 .

$$\frac{t_1}{t_2} = \frac{n_1}{n_2}$$

$$\frac{4}{10} = \frac{60}{n_2}$$

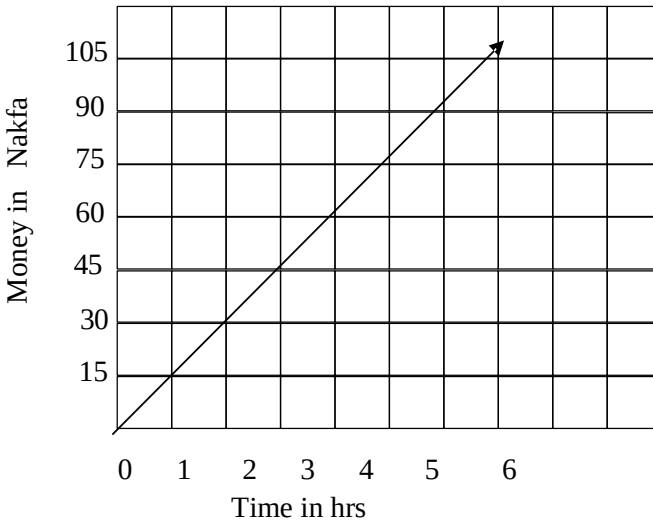
$$4 \times n_2 = 60 \times 10$$

$$n_2 = \frac{60 \times 10}{4}$$

$$= 150 \text{ Nakfa}$$

She would earn 150 Nakfa in 10 hrs.

b) To draw the graph for her income for the first five hours let us find the rate for 1hr. Since she earns 60 Nakfa in 4 hrs, then she earns $60 \div 4 = 15$ Nakfa in 1hr.



6. A car travelled 720 km at 50 km/hr. How far would it travel at 60 km/hr?

Solution:

Let it travel a distance of d_1 at a speed of v_1 , and then it would travel a distance of d_2 at a time of v_2

$$\frac{d_1}{d_2} = \frac{v_1}{v_2}$$

$$\frac{720}{d_2} = \frac{50}{60}$$

$$d_2 = \frac{720 \times 60}{50} = 864$$

The car would travel a distance of 870 Km.

Activity 9.6

- Calcium and chlorine combine in the weight ratio of 18:32. How much chlorine will combine with 36g of calcium?
- A plane flew 1800 km at 120 km/hr. During the same time
 - How many km would it travel at 200 km/hr?
 - How fast must it go to fly 900 km?

3. 135 kg of coffee was shared by three families in the proportion of 11, 9 and 7. Calculate the share of each family.
4. Silver combines with sulphur in the ratio of 27:4. How much sulphur will combine with 135 g of silver?
5. If x varies directly as y and $x = 4$ when $y = 12$, find y when $x = 8$.
6. A rope 36 m long was cut into three pieces whose lengths were in the Proportion 3, 4 and 5. Find the length of each piece.
7. If $x : y = 3 : 4$ and $y : z = 8 : 9$, find $x : z$.
8. The price of 7 kg of onion or 6 kg of potatoes is 42 Nakfa. Find the Price of 9kg of onions and 4 kg of potatoes.
9. A moving object moves 32 km in 2 hrs. How many km will it move in 12 hrs at the same rate? Draw a graph for the moving object for the first 5 hrs.
10. The weight of an object on Mars varies directly as its weight on Earth. An object weighs 95 kg on Earth and 40 kg on Mars. How much would a 190 kg object weigh on Mars?
11. If x varies directly as y and $x = 6$ when $y = 21$, find x when $y = 2$.
12. Two quantities are related by the equation $y = 3x$, find the values of y when the values of x are from -3 to 3? Draw a graph for the relationship.

B. Indirect Variation

Solomon and Mohamed are farmers. One day they discussed about the effect of price on their agricultural products.

If the prices of the commodities decrease will the number of buyers increase or decrease?

What effect does it have on the production of the items?

Discuss about them with your partners.

Activity 9.7

1. If prices of agricultural products such as onion, tomatoes, potatoes and others in a market become cheaper and cheaper:
 - a) Are people motivated to buy more and more or to buy less and less?
 - b) What about the farmers, are the farmers motivated to increase their productivity? Or decrease their productivity? Why?
2. If the production of goods increases will their price tend to increase or tend to decrease? Discuss about these points with your partner and explain your opinion to your class.
3. Are production of goods and prices of the goods directly proportional? Why?
4. Let us also discuss about the following concept. If we move from low lands to high lands (low altitude to high altitude) does the pressure increase or decrease? Discuss about it with your classmates?
5. If the speed of a car increases to complete a journey, does the time required increase or decrease?

In the above discussion you observed that the increase of one quantity may cause the decrease of the other quantity.

In mathematics when two quantities are related such that the *increase* of one *quantity* causes the *decrease* of the other *quantity* or vice-versa, the two quantities are said to be inversely related.

If two quantities x and y are related such that as x_1 increases to x_2 but y_2 decreases to y_1 and then we say the two quantities are inversely related. We express them

mathematically as: $\frac{x_1}{x_2} = \frac{y_2}{y_1}$

6. It takes 4 days for 12 people to build a house. How long would it take for 3 people to do the same work at the same speed ?

Solution:

Let the number of days needed to build the house for p_1 people be d_1 , then the number of days needed for p_2 people would be d_2

$$\begin{aligned}\frac{d_1}{d_2} &= \frac{p_2}{p_1} \\ \frac{4}{d_2} &= \frac{3}{12} \\ d_2 &= \frac{4 \times 12}{3} \\ &= 16 \text{ days}\end{aligned}$$

As time goes down number of people goes up.

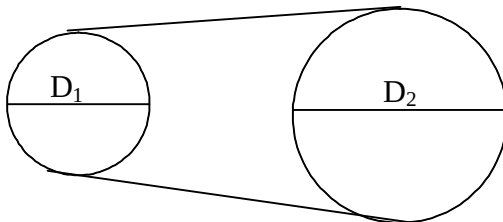
7. If y varies inversely as x , and $y = 6$ when $x = 20$, find x when $y = 24$.

Solution:

$$\begin{aligned}\frac{x_1}{x_2} &= \frac{y_2}{y_1} \\ \frac{20}{x_2} &= \frac{24}{6} \\ x_2 &= \frac{6 \times 20}{24} = 5, \quad \text{then } x_2 = 5.\end{aligned}$$

Activity 9.8

1. A bus travelling a distance of 20 km, at a speed of 20 km/hr takes 1 hr. At a speed of 40 km/hr it will take $\frac{1}{2}$ hr. How long will it take at a speed of 60 km/hr to cover the same distance?
2. Two weights of 40 kg and 25 kg are in balance on a lever bar. If the 40 kg weight is at 20 cm from the fulcrum, how far is the 25 kg weight from the fulcrum?
3. If m varies inversely as n , and $m = 24$ when $n = 8$ find n , when $m = 16$.
4. If R varies inversely as S , and $R = 8$ when $S = 30$ find S , when $R = 6$.
5. A cyclist completes a trip in 6 hrs at 20 km/hr. How long would the same trip take at a speed of 12 km/hr?
6. A triangle with height 24 m and base 6m have equal areas with another triangle with base 18 m. Find the height of the second triangle.
7. Two pulleys with diameters 7m and 28 m are connected by a belt. The smaller pulley is revolving at a speed of 360 revolutions per second. Find the speed of the larger pulley



Chapter 10

Volume and Surface Area of Solid Figures

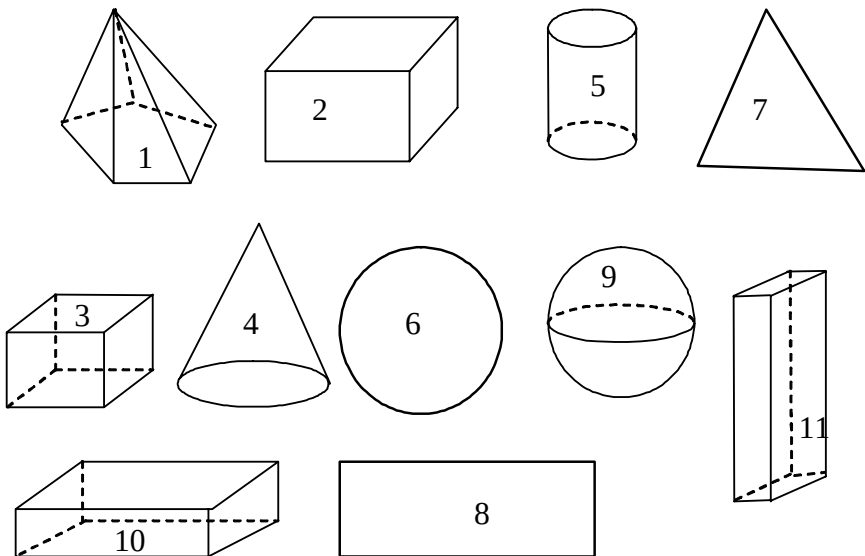
By the end of the chapter you will be able to:

- a) identify the shape and structure of solid figures such as prisms and pyramids
- b) explore the relationship between the length of their dimensions, base area and the volume of prisms and pyramids
- c) estimate, measure and calculate the base area, lateral surface area, total surface area and volume of a cylinder and a cone;

10.1. Solid Figures

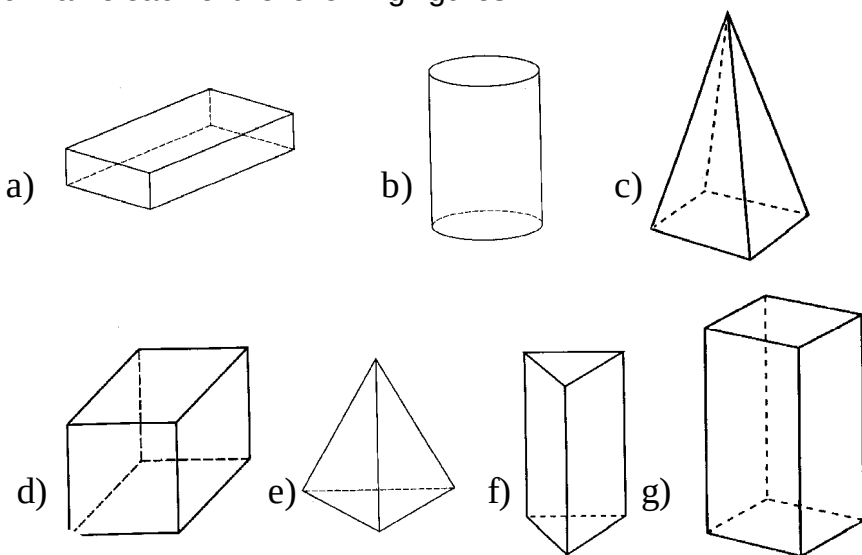
In grade 6 and grade 7 we have discussed about three dimensional objects. So far we have discussed about rectangular prisms and cylinders. In this topic we will extend our study of geometry including figures having three dimensions such as pyramids and cones.

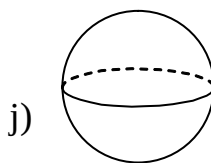
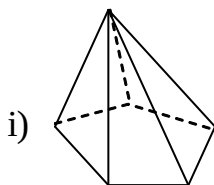
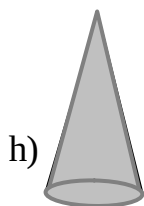
Look at the solid figures that are given below:



Activity 10.1

1. Which of the above figures are with three dimensions?
2. Prepare a model of:
 - a) A square prism
 - b) A pyramid
 - c) A cube
 - d) A cylinder
 - e) A rectangular prism
 - f) A rectangular pyramid
 - g) A triangular prism
 - h) A cone
 - i) A square pyramid
3. What is the difference between:
 - a) a square prism and a cube?
 - b) a rectangle and a rectangular prism?
 - c) a rectangular prism and a rectangular pyramid?
 - d) a triangular pyramid and triangular prism?
4. What is the difference between a pyramid and a prism?
5. What is the difference between a cone and a cylinder?
6. Name each of the following figures.





10.2 Volume of Solid Figures:

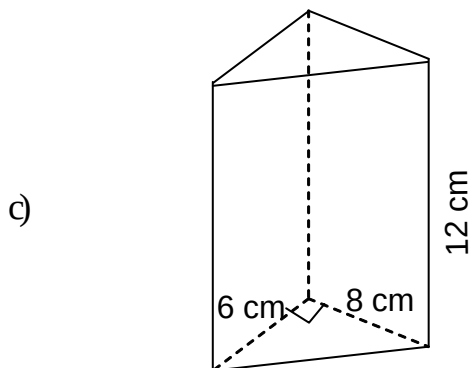
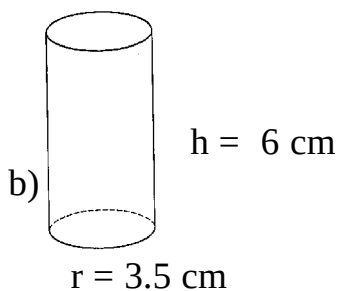
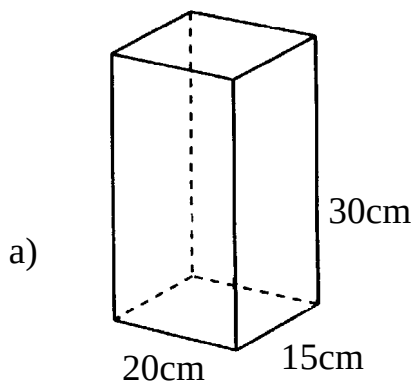
Volume of any prism is given by.

$$\text{Volume} = \text{base area} \times \text{height}$$

$$V = B \times h$$

Activity 10.2

1. Find the Volume of the following figures:



Solution:

a) $V = B \times h$

$$= (15\text{cm} \times 20\text{cm}) \times 30\text{cm}$$

$$= 300 \text{ sqcm} \times 30\text{cm}$$

$$= 9000 \text{ cm}^3$$

b) $V = B \times h$

$$= (\pi r^2)h \quad \text{assume } \left(\pi = \frac{22}{7}\right)$$

$$= \left(\frac{22}{7} \times \left(\frac{7}{2} \text{ m}\right)^2\right) \times 6 \text{ m}$$

$$= \left(\frac{77}{2} \text{ m}^2\right) \times 6 \text{ m}$$

$$= 231 \text{ m}^3$$

c) $V = B \times h$

$$= \frac{1}{2} (wl) h$$

$$= \frac{1}{2} (6 \text{ cm} \times 8 \text{ cm}) 12 \text{ cm}$$

$$= 24 \text{ cm}^2 \times 12 \text{ cm}$$

$$= 288 \text{ cm}^3.$$

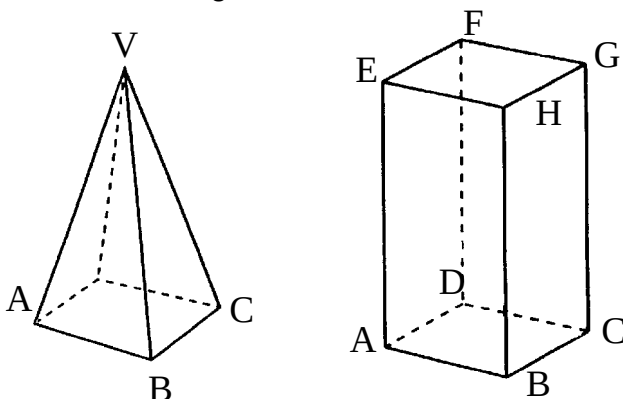
2. A water tank whose shape is a rectangular prism has a dimension of 3m by 4m by 6m. How many liters of water does it contain?
3. The circumference of the base of a cylinder is 44cm. if its height is 8cm long, find its volume.
4. The base radius of a cylinder is 3.5 cm and its volume is 308 cm^3 . Find its height?
5. Find the height of a rectangular prism with base area 80 cm^2 and volume 960 cm^3 .
6. The base of a prism is a right angled triangle of sides 3 cm, 4 cm and 5 cm. If its height is 10 cm, find its volume.

Volume of Pyramids and Cones:

Activity 10.3

1. A teacher gave a task to his class to calculate the volume of two containers whose shape are of a prism and pyramid. Each base is a square of side length 4cm; and height of each is 9cm as shown in the figures below.

Which container has a greater volume?



To know the volume of the pyramid Haile uses the following procedure:

- First he filled the pyramid with water and poured it into the prism. However, the prism never filled once.
- He poured three times, and then the prism becomes full.
- He calculates the volume of the prism. That is,

$$V = B \times h$$

$$V = (4 \text{ cm} \times 4 \text{ cm}) \times 9 \text{ cm}$$

$$V = 16 \text{ cm}^2 \times 9 \text{ cm}$$

$$V = 144 \text{ cm}^3$$
- He argues that the volume of the pyramid is $\frac{1}{3}$ the volume of the prism. Hence the Volume of the

$$\text{Pyramid} = \frac{144}{3} \text{ cm}^3 = 48 \text{ cm}^3.$$

e) He formulated that the volume of any pyramid is given by:

$$V = \frac{1}{3} B \times h$$

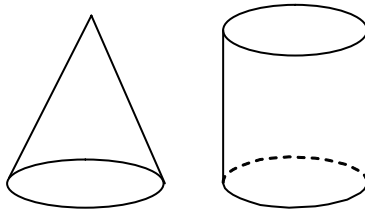
f) Prepare a model of the above solids and prove the argument of Haile.

What about the volume of a cone?



2. Sara formulated the volume of a cone using the experiment of Haile. She used the following procedures:

a) She prepared a cylinder and a cone with the same radius of their bases and heights, that is 7cm and 9 cm respectively, as shown in the figure below:



b) First she filled the cone with water and poured it into the cylinder. However, the cylinder never filled once.

c) She poured three times, and then the cylinder becomes full.

d) She calculates the volume of the cylinder That is,

$$V = B \times h$$

$$V = \pi r^2 h$$

$$V = \frac{22}{7} (7 \text{ cm})^2 \times 9 \text{ cm}$$

$$V = 154 \text{ cm}^2 \times 9 \text{ cm}$$

$$V = 1386 \text{ cm}^3$$

- e) She argues that the volume of the cone is $\frac{1}{3}$ the volume of the cylinder.

$$\begin{aligned} \text{Hence the volume of the cone} &= \frac{1386}{3} \text{ cm}^3 \\ &= 462 \text{ cm}^3. \end{aligned}$$

- f) Finally she formulated that the volume of any cone is given by:

$$V = \frac{1}{3} B \times h$$

$$V = \frac{1}{3} \pi r^2 h$$

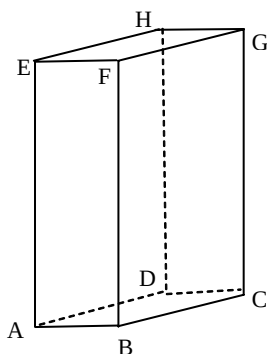
- g) Prepare models of the above cone and a cylinder and prove the argument.
3. The volume of a pyramid is 162 cm^3 . If the altitude of the pyramid is 9 cm find its base area.
 4. Find the volume of a pyramid with a rectangular base of length 4 cm and width 6 cm; and its altitude is 8cm.
 5. The diameter of the base of a cone is 12 cm and its altitude is 14 cm calculate the volume of the cone.
 6. Ahmed constructed a conical water tank whose volume is 1848 m^3 If the diameter of the base is 28 m, what will its height be?

10.3 Surface Areas of Solids:

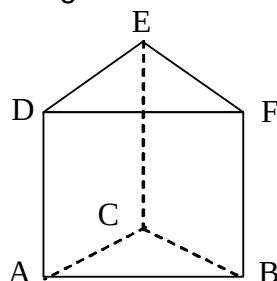
Surface Area of Prisms:

Activity 10.4

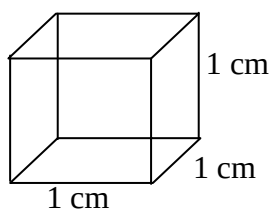
1. Tecle constructed a rectangular prism as shown below.



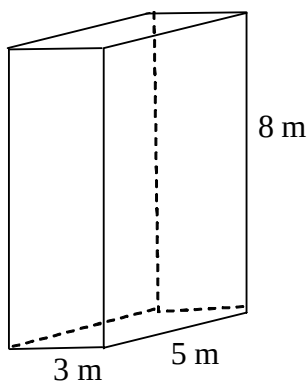
- How many faces does a rectangular prism have?
 - Name the base of the prism
 - What do we call the remaining faces?
 - List the lateral edges of the prism?
 - What is the total surface area of the prism?
 - Beshir said the total surface area of a prism is the sum total of the two base areas and the lateral surface areas. Do you agree with Beshir?
2. Look at the drawing below:



- What is the name of the above figure?
- How many faces does it have? Name them.
- What are the shape of the bases?
- What are the shapes of the lateral faces?



3. a) What is the name of the above figure? Why?
 - b) How many faces does it have?
 - c) What is the shape of the base?
 - d) What are the shapes of the lateral faces?
4. Amar constructed a rectangular prism of bases 3 m and 5 m long; and height of 8 m. The draft of his model is given below:



Calculate:

- a) Area of the bases:
- b) Lateral surface area
- c) Total surface area

Solution:

- a) There are two congruent bases; that are upper and lower bases.

$$\begin{aligned}
 \text{Area of bases} &= 2(l \times w) \\
 &= 2(5 \text{ m} \times 3 \text{ m}) \\
 &= 30 \text{ m}^2
 \end{aligned}$$

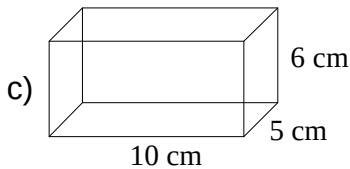
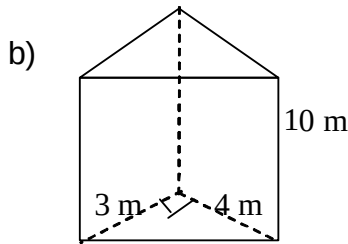
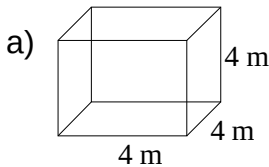
- b) There are four lateral faces in which two opposite faces are congruent.

$$\begin{aligned}\text{Lateral surface area} &= 2(lh) + 2(wh) \\ &= 2(3 \text{ m} \times 8 \text{ m}) + 2(5 \text{ m} \times 8 \text{ m}) \\ &= 48 \text{ m}^2 + 80 \text{ m}^2 \\ &= 128 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{c) Total surface area} &= \text{Area of bases} + \text{Lateral surface area} \\ &= 30 \text{ m}^2 + 128 \text{ m}^2 \\ &= 158 \text{ m}^2\end{aligned}$$

Activity 10.5

1. Find the total surface area of the following figures:



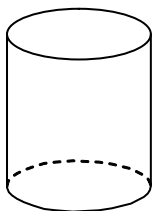
2. A rectangular prism has length of 12 cm width of 10 cm and height 16 cm. Find its:
 - a) Base area;
 - b) Lateral surface area; and
 - c) Total surface area.
3. Find the total surface area of a cube whose edge is 6 cm long.
4. The base of a prism is a right angled triangle whose sides are 6 cm, 8 cm and 10 cm. If its altitude is 14 cm, find its:
 - a) Base area;
 - b) Lateral surface area; and
 - c) Total surface area.

5. The total surface area of a cube is 600 cm^2 . Find the length of the side.
6. A rectangular prism has a base of 7 cm wide and 5 cm long. If the lateral surface area of the prism is 96 cm^2 , how high is the prism?

Surface Area of a Cylinder:

Activity 10.6

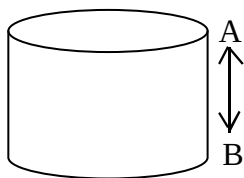
1. Look at the diagram below:



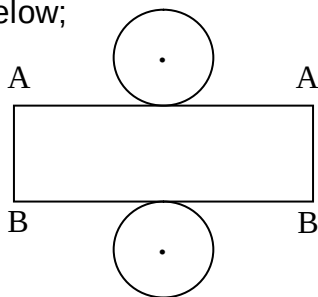
- a) What is the shape of the figure?
- b) How many faces are there?
- c) What is the shape of the base?
- d) How many bases are there?
- e) What is the total surface area of the figure?

2. Do the following activities:

- a) Prepare a cylindrical shape can as follows;

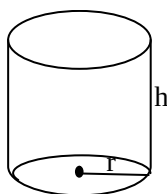


- b) Cut along the side up to the two bases; you will find as shown below;



- c) What do you observe?
 d) Berhe suggested that the length of the rectangle (AA or BB) is equal to the circumference of the circle, and that $L = 2 \pi r$
 e) Halima said that the width of the rectangle (AB) is equal to the height of the cylinder ($w = h$)
 f) Do you agree with them?
 g) Prove their experiment by the same procedure.

3. Given the figure below: (use $\pi = \frac{22}{7}$)



a) Lateral surface area = Area of a rectangle form
 $= lw$

$$= 2 \pi r h$$

b) Area of the base = $\pi r^2 + \pi r^2$

$$= 2 \pi r^2$$

c) Total surface area = Lateral surface area +
 Area of the base

$$= 2 \pi r h + 2 \pi r^2$$

$$= 2 \pi r (h + r)$$

4. The base radius of a cylinder is 7cm and its height is 10cm;

a) The area of the bases = $2 \pi r^2$

$$= 2 \times \frac{22}{7} \times (7\text{cm})^2$$

$$= 308 \text{ cm}^2$$

b) The lateral surface area $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \text{ cm} \times 10 \text{ cm}$$

$$= 440 \text{ cm}^2$$

c) Total surface area

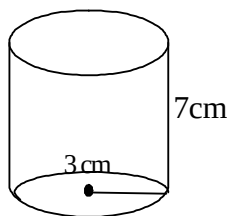
$= \text{Lateral surface area} + \text{Area of the bases}$

$$= 440 \text{ cm}^2 + 308 \text{ cm}^2$$

$$= 748 \text{ cm}^2$$

5. For the figure given below find:

- the area of the bases;
- the lateral surface area; and
- the total surface area.



6. A cylinder has a base radius of 3.5cm and altitude of 8cm, calculate;

- the area of the bases;
- the lateral surface area;
- the total surface area.

7. The lateral surface area of a cylinder is 220 cm^2 . If the height of the cylinder is 5 cm find:

- the area of the bases.
- the total surface area.

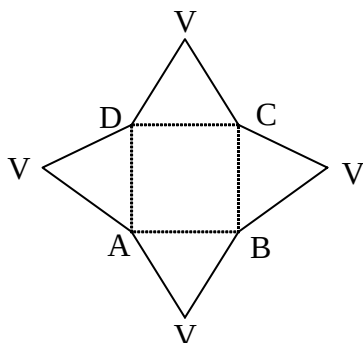
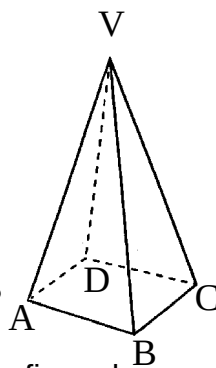
8. The total surface area of a cylinder is 1672 cm^2 and the base radius is 14 cm long. Find:

- the height of the cylinder
- the volume of the cylinder

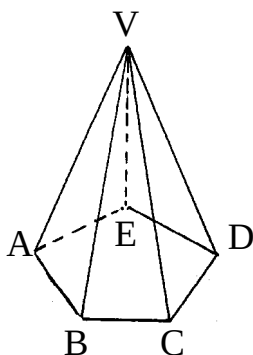
Surfaces of Pyramids and Cones:**Activity 10.7**

1. Look at the diagram below:

- What is the shape of the figure?
- How many faces are there?
- What is the shape of the base?
- How many bases are there?
- What are the shapes of the lateral faces?
- How many lateral faces are there?
- Name the lateral faces and the base of the figure by referring to the diagram below.

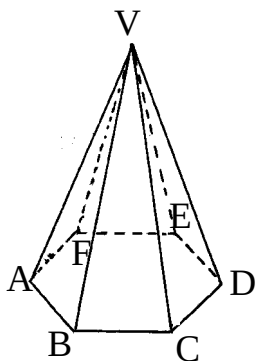


2. Look at the diagrams below:



- What is the name given to the figure?
- How many faces are there?
- What is the shape of the base?
- What are the shapes of the lateral faces?
- How many lateral faces are there?

3. Look at the diagram below:



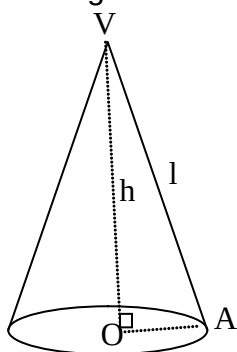
- a) What is the name given to the figure?
- b) How many faces are there?
- c) How many bases are there?
- d) What is the shape of each lateral face?
- e) How many lateral faces are there?

4. What do you observe from the above questions?

5. If the base of a pyramid is a 10 sided polygon, how many lateral faces does the pyramid have?

6. a) If the base of a pyramid is an n -sided polygon, how many lateral faces does the pyramid have?
- b) If the base of a pyramid is a regular polygon what can you say about the lateral faces?
- c) If the sides of the base of a pyramid are not congruent what can you say about the lateral faces?

7. Look at the diagram below:



- What is the shape of the figure?
 - What is the shape of the base?
 - How many faces are there?
 - What is the shape of the lateral surface of the figure?
 - How many lateral faces are there?
 - What is the name given for "l"?
 - What is the relationship between h, r and l?
 - Cut the cone along "AV". What do you think it will be?
8. If the radius and the altitude of a cone are 3.5 cm and 7cm respectively, find:
- the base area.
 - the slant height.

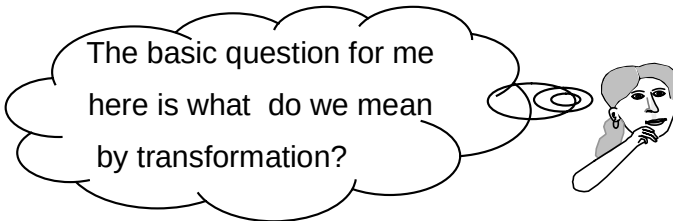
Chapter 11

Transformations

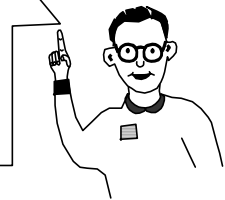
By the end of the chapter you will be able to:

- a) identify that reflection and translation are rigid motions.
- b) understand and explore the relationship between an object and its image during reflection or translation;

Mihret was wondering about the word transformation and she said:



In mathematics, Transformation means a 'a change' in *position*, *size* or *shape*. For example, when an object moves from one position to another, it undergoes a change in position. When an object is enlarged it changes in size.



In this chapter we will discuss two types of transformations:

- a) Reflection,
- b) Translation,

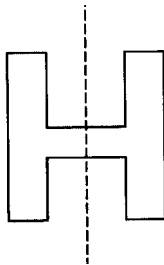
11.1 Reflections

Activity 11.1

1. Look at your self in a plane mirror.
 - a) What does the image of your reflection look like?
 - b) When you raise your right hand, does it become a left or a right hand?
 - c) Move one step backward. Does your reflection move one step backward or forward?
2. Use a mirror or imagine that you are looking at one point and state what your image or reflection does when you;
 - a) Smile
 - b) Touch an ear
 - c) Place a pencil behind the left ear
 - d) Put your hands together
 - e) Move a step to your right
3. Stand infront of a plane mirror. What will happen to your image, when you:
 - a) Walk backwards?
 - b) Raise your left arm?
 - c) Turn to the right?
 - d) Swings your right arm?
 - e) Touch the mirror.

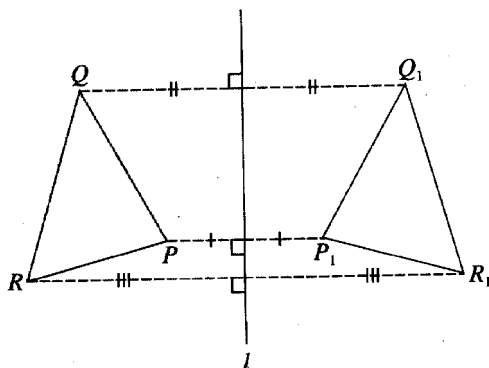
Mirror Line or Line of Reflection

4. Look at the figures below:



The dotted line is called a mirror of reflection or a line of reflection.

5. In the diagram below, line 'l' is the line of reflection.

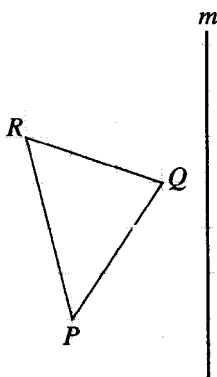


The reflection of $\triangle PQR$ in the line "l" is $\triangle P_1Q_1R_1$. We call

$\triangle P_1Q_1R_1$, the image of $\triangle PQR$. In addition to this P_1 is the image of P, Q_1 is the image of Q and R_1 is the image of R.

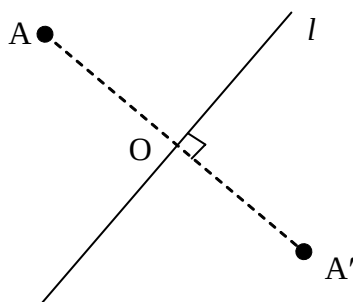
Activity 11. 2

1. Triangle PQR is reflected in the line 'm'. Draw its image.

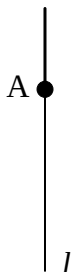


2. The image of A (-2, 4) under a reflection is A(2, 4).
Draw the line of reflection.

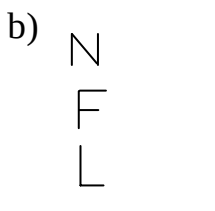
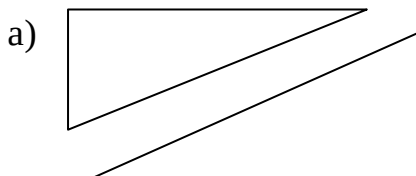
3. If point A is not on line l , the image of A is A^1 as shown in the figure below



- If you fold the plane through the line l , what do you think A and A^1 be?
 - What do you say about the line -segments AO and line A^1O ?
4. If point A is on a line of reflection as shown in the figure below:



- Where is the position of the image of A.
 - If you fold the plane through the line l , what do you think A and A^1 (the image of A).
 - What do you say about the distance between A and A^1 ?
5. Draw the reflection of the following objects in the line given.



c)

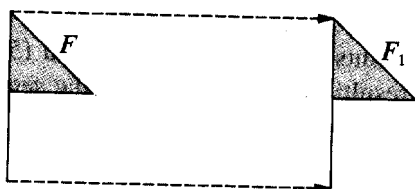
F

d)

p

11.2 Translation

Look at the flag below.



It is moved from one position to another position in a straight line in one direction. This kind of transformation is referred as a **translation**.

Under a translation, all points of an objects move in the same direction and over the same distance. Therefore, F and its image F_1 are congruent.

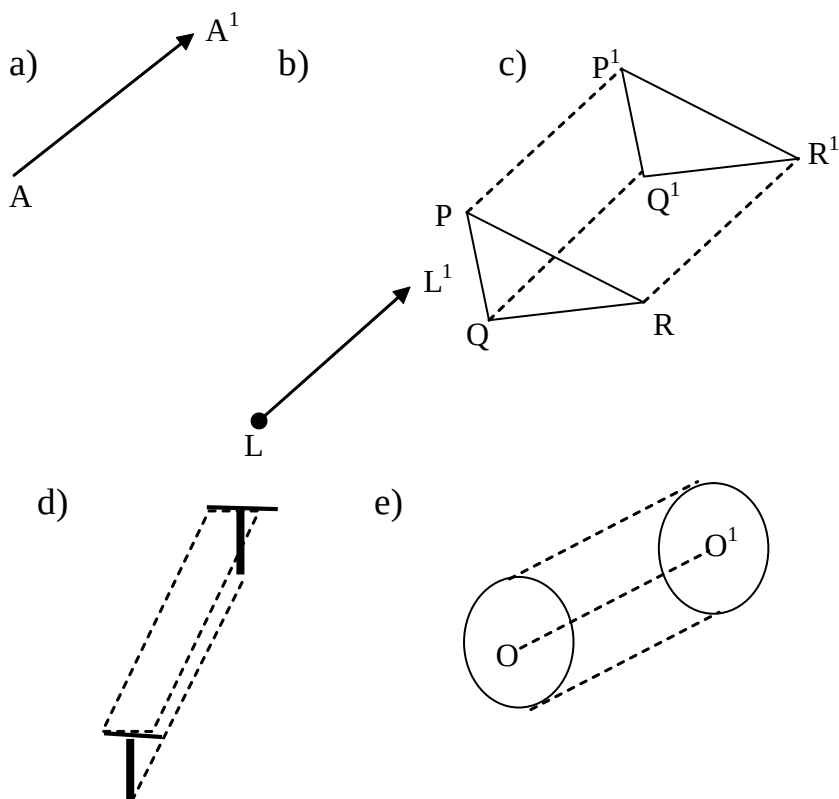
A translation can be represented by an arrow. The arrow shows the **direction** and the length of the arrow represented the **distance** of the translation.



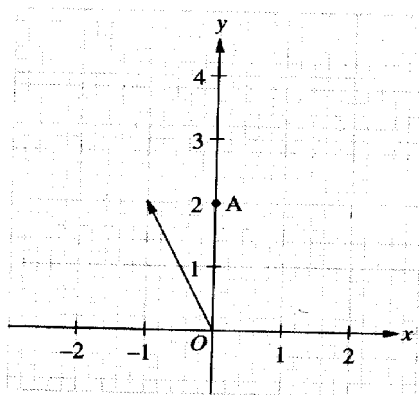
Under translation, a point A will move to A' in the direction as shown by an arrow with the length of the arrow represented the distance moved.

Activity 11.3

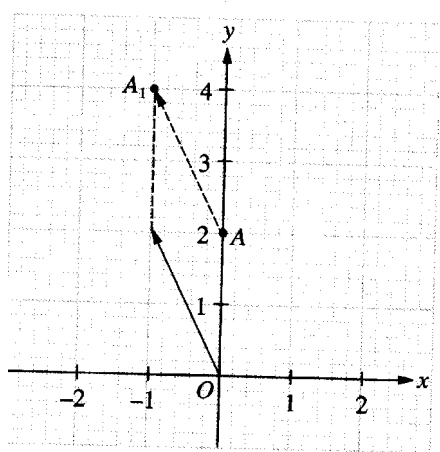
1. In the figure AA^1 is 3 cm long. Suppose now every figure in the plane moves 3 cm in the same direction as the arrow. The figure shows the same objects before and after the motion.



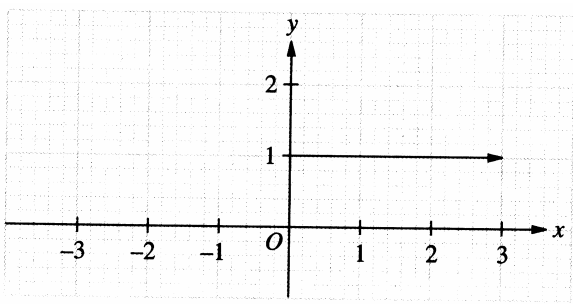
2. A translation takes $(0, 0)$, 2 units upward and 1 unit to the left as shown in the diagram. What will be the coordinates of the image of $A(0, 2)$ by the same translation?



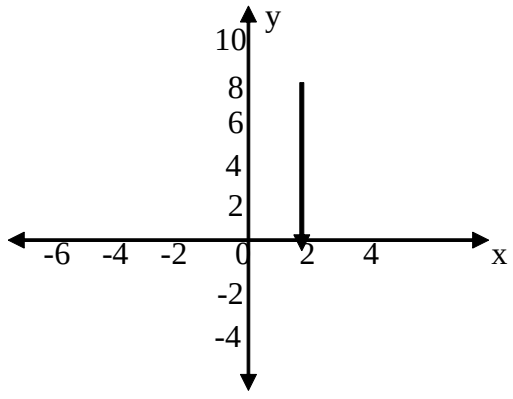
Since A moves in the direction shown by the arrow to a distance which is the same length as the arrow. To get the image of A, we construct a parallelogram with OA as one side of the parallelogram as shown below.



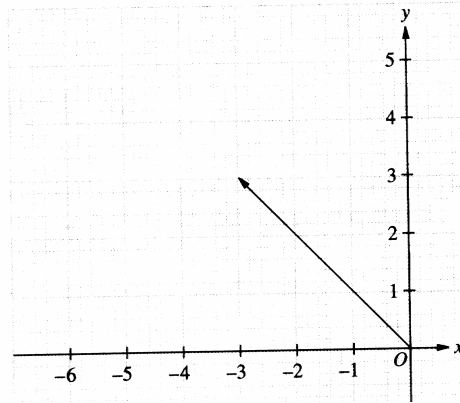
3. Translate the following figure 2 units to the left and 1 unit upward.



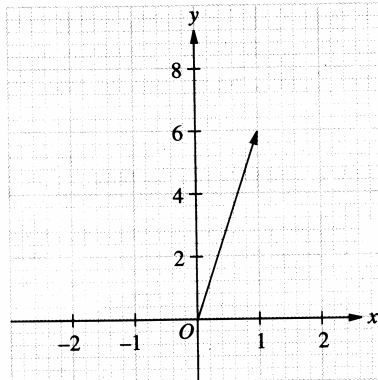
4. Translate the following figure 2 units to the right and 2 unit downward.



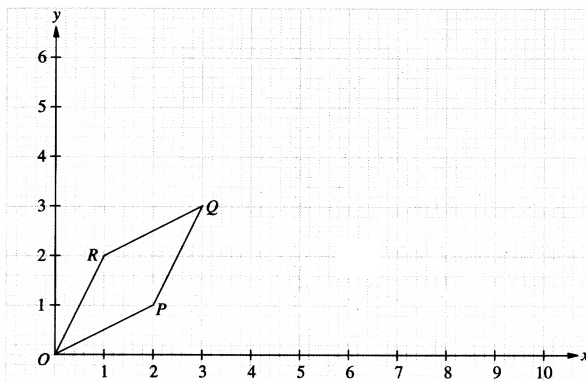
5. Translate the following figure 3 units to the left and 1 unit upward.



6. Translate the following figure 2 units to the left and 4 unit upward.



7. The vertices of a parallelogram $OPQR$ are $O(0,0)$, $P(2,1)$, $Q(3,3)$ and $R(1,2)$, as shown below. Translate the parallelogram 6 units to the right and 2 units upward.



8. Under a translation, the point $(0,0)$ is mapped onto the point $(-4,0)$. Draw the image of each of the following under this translation.
- a) Points $A(3,2)$, $B(-2,4)$, $C(5,-3)$ and $D(-2,-3)$
 - b) Triangle ABC with vertices $A(2,1)$, $B(2,3)$ and $C(4,1)$.
 - c) Rectangle $PQRS$ with vertices $P(1,1)$, $Q(2,1)$, $R(2,4)$ and $S(1,4)$.
 - d) Parallelogram $EFGH$ with vertices $E(1,1)$, $F(4,1)$, $G(7,6)$, and $H(4,6)$

Reflection and Translation are changes in position or direction, they never change the shape or size of the objects. These are called rigid motions or rigid transformation. Under rigid motion:

The image of a point is a point.

The image of a line segment is congruent line segment.

The image of a triangle is a congruent triangle.

Hence, rigid motion preserves sizes, shape and distance of an object.

Chapter 12

Statistics and Propbability

By the end of the chapter you will be able to:

- a) construct frequency distribution table from a given unorganized data;
- b) calculate the mean, the median, the mode, the range, the mean deviation and the standard deviation; from a given frequency distribution table;
- c) associate the playing game of coins and die with the mathematical implication;
- d) estimate the certainty of unpredicted futures.

12.1 Measures of Central Tendency

You have learned about mean, median and mode in grade 7. The word mean (arithmetic mean) is an average of the collected data. The word “Average” is common in every day life. For instance we speak of:

Average cost;
Average income;
Average speed;
Average temperature;
Average rainfall;
Average grade; and so on

Activity 12.1

1. What was your average score in the first semester?
2. What is the average income of your family members?
3. What is the average cost of the televisions in an electrical shop?
4. What is the average rainfall in Eritrea?

5. What is the average temperature in Massawa during summer?
6. How will you find the average age of the students in your class?
7. The score grades of Ali in the first semester is listed below.

Subject	Mark out of 100
English	85
Mathematics	90
Civics	87
Geography	83
History	82
Science	88
Social studies	85
Physical education	80
Total marks	680

Find his average mark.

Solution:

$$\text{Average mark} = \frac{\text{total marks}}{\text{number of subjects}}$$

$$= \frac{680}{8}$$

$$= 85$$

Activity 12.2

1. The scores of 10 students in a mathematics test are:
8, 9, 6, 7, 0, 8, 3, 4, 10, and 5
Find: a. The mean score
b. The median score, and
c. The mode score
2. The data given below is the amount of rainfall in millimeter that was recorded in one day in July 2005 in 9 different places of Eritrea .31,7,11,18,43,20,7,5,20
Find: a. the mean (average) rainfall of the places
b. the median rainfall of the places, and
c. the mode of the rainfall
3. In Asmara the following measures of temperature in $^{\circ}\text{C}$ was recorded in the last week of December in 1993
1, -3, 4, 0, -1, 5, 8
Find: a. the mean of the temperatures
b. the median of the temperatures
c. the mode of the temperatures

Activity 12.3

1. In grade 7 you have discussed how to organize data and analyze the organized data. For instance; The score of 20 grade 8 students of mathematics out of 100% is given below:
80, 70, 90, 85, 90, 80, 75, 80, 85, 90,
85, 75, 90, 85, 80, 85, 70, 90, 80, 85.
a) Organize the data in a frequency table.
b) Find the mean, median and mode for the students score.

Solution:

- a) Note that several of the numbers occur more than once. We will write our numbers in order and describe the frequency of each value.

Marks out of 100	Frequency (F)	F.M
70	2	$2 \times 70 = 140$
75	2	$2 \times 75 = 150$
80	5	$5 \times 80 = 400$
85	6	$6 \times 85 = 510$
90	5	$5 \times 90 = 450$
Total	20	$= 1650$

$$\text{b) Mean} = \frac{1650}{20} = 82.5$$

The mean weight of the total players is 82.

The numbers are arranged in order of magnitude. We have 20 people in our sample. The median number will be the average of the 10th and 11th numbers. The 10th number is 80, and the 11th number is 85.

$$\frac{80 + 85}{2} = 82.5$$

The **median** weight is 82.5.

For the **Mode**, look at the frequencies. We have a score that has a frequency of 6. The **mode** is 85.

2. The following are scores of 25 students in mathematics out of 6%

- Construct a frequency distribution table.
- Find the mean
- Find the median

d) Find the mode of the given data.

1	3	2	5	6
3	5	4	2	6
4	6	1	5	3
2	6	3	1	4
3	2	3	4	6

3. An electrical shop sells different types of television sets. The price per set and the number of sets sold in 6 days are shown below.

Type of television	Number of sets sold	Price per set
National T .V.	5	Nakfa 4500
Sony T .V.	7	Nakfa 4000
Toshiba T .V.	3	Nakfa 4000
Philips T .V.	6	Nakfa 6000
JVC T .V.	3	Nakfa 3500

- a) What is the average number of sets sold per day?
 b) What is the average price per set?
 c) What is the modal T. V. set?

4. The following frequency distribution table is the ages of kindergarten children

Age	3	4	5	6
Number of students	4	8	2	1

- Find: a) The mean
 b) The median
 c) The mode

5. The monthly rainfall and maximum temperature recorded in a certain town during 2005 are as follows:

Month	Rainfall in mm	Max. TemperatureIn $^{\circ}\text{C}$
January	5	18
February	6	20
March	20	27
April	40	25
May	35	26
June	45	24
July	75	23
August	60	22
September	20	23
October	10	21
November	15	19
December	5	17

The mean, the mode and the median are the measures of central tendency

- What is the average rainfall of the year?
- What is the average temperature of the year?
- What is the modal temperature?
- Which month is the hottest?
- Which month is coldest?
- What is the difference between the hottest and the coldest month?
- What is the range of rainfall in the year?

12.2 Measures of Variation (Dispersion)

In the last section we have learned methods of summarizing observations by computing the mean, median and mode. In this section we will learn different measures which indicate how dispersed a set of observation is.

Activity 12.4

1. A teacher describes the scores of Saron and Bahlbi as shown below:

students	English	Maths	Science	History	Geography
Saron	80	81	79	78	82
Bahlbi	63	94	80	93	70

- What is the mean score of Saron?
- What is the mean score of Bahlbi?
- Did you find the mean score?
- Which student is cleverer than the other?

You have observed that the two students have the same mean. This result (mean) can not tell us which student is cleverer. It simply summarizes the information.

Hence, the measure of central tendency is not enough to describe data. We have other types of measures called measure of dispersion (variation) Some measures of dispersion are:

- the range (R)
- the mean deviation
- the standard deviation

A. Range:

2. The score of 10 grade seven students in mathematics test is: 8, 6, 7, 4, 12, 8, 11, 5, 14, 13

What is the difference between the largest and smallest value of the given data?

Solution :-

largest value is 14 and smallest value is 4

$$14 - 4 = 10$$

10 is the range of the data.

B. Mean Deviation

3. In a specific year the average passing mark was 60%
 Adiam's average score was 68 %
 Biniam's average score was 54 %
 How far did they deviate from the mean?

Solution:

Adiam: $68 - 60 = 8$.

Adiam deviated 8 marks above the average

Biniam: $54 - 60 = -6$.

Biniam deviated 6 marks below the average.

4. The following results are obtained by ten students in Mathematics final exam out of 100.
 10,15,20,25,30,35,50,45,80,90

Find

- The mean of the scores
- The mean deviation of the scores.

Solution:

$$\begin{aligned} \text{a) mean} &= \bar{x} = \frac{10+15+20+25+30+35+50+45+80+90}{10} \\ \bar{x} &= \frac{400}{10} = 40. \end{aligned}$$

- The result of 6 students is below the mean and the result of 4 students is above the mean. That is, their results deviated as follow

Grade	Deviation from the mean
10	$10 - 40 = - 30$
15	$15 - 40 = - 25$
20	$20 - 40 = - 20$
25	$25 - 40 = - 15$
30	$30 - 40 = - 10$
35	$35 - 40 = - 5$
45	$45 - 40 = 5$
50	$50 - 40 = 10$
80	$80 - 40 = 40$
90	$90 - 40 = 50$

The negative sign (-) indicates that the value is below the mean and the positive sign (+) indicate that the value is above the mean. 10, 15, 20, 25, 30, 35 are below the mean and their results deviate by 30, 25, 20, 15, 10 and 45, 50, 80, and 90 are above the mean. That is their results deviate by 5, 10, 40 and 50 above the mean.

$$\begin{aligned}
 \text{The mean deviation (M.D)} &= \frac{30+25+20+15+10+5+5+10+40+50}{10} \\
 &= \frac{210}{10} \\
 &= 21
 \end{aligned}$$

The mean deviation of the data is 21. Hence there is a great difference within the students' achievements.

5. The results obtained by ten students in mathematics final exam out of 100 in another section are:

71, 72, 73, 73, 74, 74, 77, 78, 79, and 79

Find: a) the mean of the scores

b) the mean deviation of the scores.

Solution:

$$\begin{aligned} \text{a) Mean} = \bar{x} &= \frac{71 + 72 + 73 + 73 + 74 + 74 + 77 + 78 + 79 + 79}{10} \\ &= \frac{750}{10} = 75. \end{aligned}$$

The result of 6 students is below the mean and the result of 4 students is above the mean. That is, 71, 72, 73, 73, 74, 74 are below the mean and their results deviate by 4, 3, 2, 2, 1 and 1, and 77, 78, 79 and 79, are above the mean. That is their results deviate by 2, 3, 4 and 4 above the mean.

$$\begin{aligned} \text{The mean deviation (M.D)} &= \frac{4 + 3 + 2 + 2 + 1 + 1 + 2 + 3 + 4 + 4}{10} \\ &= \frac{26}{10} \\ &= 2.6. \end{aligned}$$

The mean deviation of the data is 2.6. Hence there is a small difference within the students' achievements.

C. Standard Deviation:-

Variance is a first step to know a standard deviation of a statistical data. It is nothing but it is the mean (average) of the deviation mean square. And standard deviation is a positive square root of variance.

6. Calculate the variance of questions number 3 and 4.

Grade	Value-Mean	(Value-Mean) ²
10	$10 - 40 = -30$	$(-30)^2 = 900$
15	$12 - 40 = -28$	$(-28)^2 = 784$
20	$20 - 40 = -20$	$(-20)^2 = 400$
25	$25 - 40 = -15$	$(-15)^2 = 225$
30	$30 - 40 = -10$	$(-10)^2 = 100$
35	$35 - 40 = -5$	$(-5)^2 = 25$
45	$45 - 40 = 5$	$(5)^2 = 25$
50	$50 - 40 = 10$	$(10)^2 = 100$
80	$80 - 40 = 40$	$(40)^2 = 1600$
90	$90 - 40 = 50$	$(50)^2 = 2500$
	Total	= 6500

$$\begin{aligned}\text{Variance} &= \frac{6500}{10} \\ &= 650\end{aligned}$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\text{Variance}} \\ &= \sqrt{650} \\ &= 25.5.\end{aligned}$$

Hence, there is high variation (dispersion) between the students achievement.

Grade	Value- Mean	(Value-Mean) ²
71	$71 - 75 = -4$	$(-4)^2 = 16$
72	$72 - 75 = -3$	$(-3)^2 = 9$
73	$73 - 75 = -2$	$(-2)^2 = 4$
73	$73 - 75 = -2$	$(-2)^2 = 4$
74	$74 - 75 = -1$	$(-1)^2 = 1$
74	$74 - 75 = -1$	$(-1)^2 = 1$
77	$77 - 75 = 2$	$(2)^2 = 4$
78	$78 - 75 = 3$	$(3)^2 = 9$
79	$79 - 75 = 4$	$(4)^2 = 16$
79	$79 - 75 = 4$	$(4) = 16$
	Total	= 80

$$\begin{aligned}\text{Variance} &= \frac{80}{10} \\ &= 8.\end{aligned}$$

$$\begin{aligned}\text{Standard deviation} &= \sqrt{\text{Variance}} \\ &= \sqrt{8} \\ &= 2.83.\end{aligned}$$

Hence, there is a small variation (dispersion) between the students' achievements.

7. find the range of the following data

- 5, 10, 12, 6, 3, 10
- 165, 178, 163, 170, 172
- 99, 85, 90, 100, 96

8. Find the mean deviation of the scores of 10 students in mathematics test 24,30,35,39,43,44,46,50,51.58
9. Find the variance and standard deviation of the following:
 - a) 1, 3, 4, 5 and 7
 - b) 2, 6, 8, 10, and 14

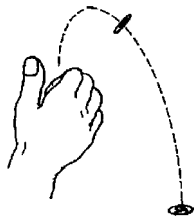
12.3 Probability:

Activity 12.5

1. A Meteorologist made a report on the weather forecast. He said within these three days there will be rain over the entire region. Is he certain that it will rain within these days?
2. Can you tell with certainty that it will be sunny or rainy tomorrow?
3. How many days of rain do you expect in the month of July in the southern region of Eritrea?
4. If a patient is hospitalized are you 100% sure that he will get better?
5. Which team will be the winner in the football match in the next world cup?
6. Who will be the star player in the next national football cup? What do you learn from the above questions? These are just to make you aware that there are situations in which the outcome (result) of a certain course of action cannot be known before hand with certainty.

Probability: - is a branch of mathematics that enables us to measure the degree of uncertainty.

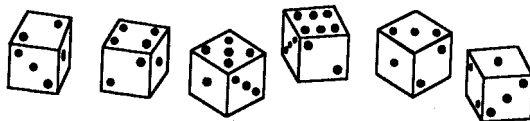
7. Take a coin. Have a look at both faces. Do you observe that the faces are different? In probability one face is called “head”(martyrs with flag) and the other face is called “a tail” (an animal).If you toss a coin, what would you expect to get? “Tail ” or “ Head ”?



Therefore, the probability of getting:

- a) “A head” is 1 out of 2, that is $\frac{1}{2}$
- b) A tail is 1 out of 2, that is $\frac{1}{2}$
- c) “A head” or A tail is 2 out of 2, that is $\frac{2}{2} = 1$
- d) “A head” and A tail at the same time is 0 out of 2,
that is $\frac{0}{2} = 0$

8. If you role a die, the top face of the die may show 1 or 2 or 3 or 4 or 5 or 6



Therefore the probability of showing:

- a) “1” is 1 out of 6 that is $\frac{1}{6}$
- b) “3” is 1 out of 6, that is;
- c) “2 or 5” is 2 out of 6, that is

d) "1, 2, 3, 4, 5 or 6 " is $\frac{6}{6} = 1$

e) "2 and 5" at the same time is 0 out of 6,
that is = 0

N.B. The probability of always possible is $100\% = 1$; and
The probability of impossible is $0\% = 0$

9. In a class there are 30 boys and 20 girls.

a) What is the probability of selecting a boy as a monitor?

b) What is the probability of selecting a girl as a monitor?

Solution:

the total number of the students in a class is 50

a) The probability of a boy being a monitor is:

$$30 \text{ out of } 50 \text{ or } \frac{30}{50} = \frac{3}{5} = 0.6$$

b) The probability of a girl being a monitor is:

$$20 \text{ out of } 50 \text{ or } \frac{20}{50} = \frac{2}{5} = 0.4$$

Activity 12.6

1. The rounded score of students in their mathematics examination is given below:

Grade	0	10	20	30	40	50	60	70	80	90	100
Number of students	0	0	0	5	10	15	20	25	20	5	0

a) What percent of the students score 80?

b) What percent of the students score below 50?

c) What percent of the students score above 70?

d) What percent of the students score from 40 to 60?

e) What percent of the students score the modal value?

f) What percent of the students score the grade near the mean?

- g) What percent of the students score 100?
- h) What is the probability of getting a student who scores 100?
- i) What is the probability of getting a student who scores 30?

2. Ghenet, Haile, Ali, Yosief, Neima and Mohamed were playing a game on throwing a die. Each member had his own number. 1, 2, 3, 4, 5 and 6 for Ghenet, Haile, Ali, Yosief, Neima and Mohamed respectively. They throw a die 6 times. Organise your self in a group of 6 members and do the same experiment.

Answer the following questions.

- a) Which number is shown up on the first throw?
 - b) Which number is shown up on the second throw?
 - c) Which number is shown up on the third throw?
 - d) Which number is shown up on the fourth throw?
 - e) Which number is shown up on the fifth throw?
 - f) Which number is shown up on the sixth throw?
 - g) What is the probability of showing 6 on the upper face of the die?
3. The height of some student in a certain class of grade 7 students is given below.

Height in cm	165	166	167	168	169	170	171	172	173
Number of students	10	5	2	3	16	3	6	4	1

- a) What is the probability of choosing a student of height 167 cm?
- b) What is the probability of choosing a student with a modal height as a monitor?
- c) What is the probability of getting a student of a height of 175 cm in the class?
- d) What is the probability of choosing a student of height above 160 cm and below 180 cm?