Assignment I

Department of Mathematics

Mathematics for Social Sciences

(30%)

1. Let p: 3 is an odd number.

q: 23 is a prime number.

r: 9 is an even number.

State each of the following in words, and determine the truth value of each.

a. $p \vee r$.

b. $p \wedge q$.

c. $q \Leftrightarrow r$.

d. $p \Longrightarrow q$.

e. $r \Longrightarrow \neg q$.

f. $\neg q \Longrightarrow \neg p$.

2. Determine the truth value of the following statements.

a. $(\forall x \in \mathbb{N})(x + 1 \geq 2)$.

b. $(\exists x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y + 3 = 8)$.

c. $(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(x + y = 5)$.

d. $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})(x + y = 5)$

3. Give formal proof to show that the following argument forms are valid.

a. $p \Longrightarrow \neg q, p, r \Longrightarrow q \vdash \neg r$.

b. $\neg p \wedge \neg q, (q \vee r) \Longrightarrow p \vdash \neg r$.

4. Let $f(x) = x^2$ and $g(x) = \sqrt{x}$.

a) Find $(fog)(x)$ and its domain.

b) Find $(gof)(x)$and its domain

c) Are $(f \circ g)(x)$ and $(g \circ f)(x)$ the same functions? Explain.

5. Sketch the graph of $f(x) = \frac{x^2+1}{x}$.

6. Find the inverse of $f(x) = e^{(3x-1)}$.

7. Solve the following linear systems

a. $x_1 + x_2 = -1$

$x_1 - x_2 = 0$

$-2x_1 + x_2 = 3$

$b.\ 4x_1 - x_2 - x_3 = 1$

$2x_1 + 2x_2 + 3x_3 = 10$

$5x_1 - 2x_2 - 2x_3 = -1$

8. Evaluate the following limits, if they exist

a. $\lim_{x \to 3^-} \frac{2}{x-3}$

b. $\lim_{t \to -\infty} \left(1 + \frac{4}{x+1}\right)^x$

c. $\lim_{x \to \infty} \frac{4}{x-1}$

9. Use the First or Second Derivative Test to determine relative extreme values of the function and Sketch the graph of the function $f(x) = (x^2 - 1)^2$

10. Find the area of the region between the graphs of the function

$f(x) = x^2 and\ g(x) = 2 - x$