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# Unit 1

## RATIONAL NUMBERS

### Learning Outcomes:

At the end of this unit, learners will be able to:

- ▶ Define and represent rational numbers as fractions
- ▶ Show the relationship among  $\mathbb{W}$ ,  $\mathbb{Z}$  and  $\mathbb{Q}$
- ▶ Order rational numbers.
- ▶ Solve problems involving addition, Subtraction, Multiplication and division of rational numbers
- ▶ Apply Rational Numbers to solve practical problems.
- ▶ Aware the four operations as they relate to Rational Numbers

### Main content

- 1.1 The Concept of Rational Numbers
- 1.2 Comparing and Ordering Rational Numbers
- 1.3 Operation and Properties of Rational Numbers
- 1.4 Applications of Rational Numbers
  - ※ Key terms
  - ※ Summary
  - ※ Review Exercise

## Introduction

You can begin your study of numbers by counting objects around you. The numbers used for this purpose are called counting numbers or natural numbers. They are 1,2,3,4,...by including 0 to natural numbers, we get the whole numbers, i.e., 0,1,2,3,...the negatives of natural numbers were then put together with whole numbers to make up integers. Integers are ...,−3,−2,−1,0,1,2,3,... We, thus, extend the number system, from natural numbers to whole numbers and from whole numbers to integers. You will be also introduced to fractions. These are numbers of the form  $\frac{\text{numerator}}{\text{denominator}}$  where the numerator is an integer and the denominator is non-zero integer. In this unit you will compare two fractions, find their equivalent forms and study all the four basic operations addition, subtraction, multiplication and division. In this Chapter, we shall extend the number system further. We shall introduce the concept of rational numbers along with addition, subtraction, multiplication and division of operations.

In this unit you will learn about rational numbers and their basic properties. You will also learn how to perform the four fundamental operations on rational number.

### 1.1. The Concept of Rational Numbers

The word ‘rational’ arises from the term ‘ratio’. You know that a ratio like 3:2 can also be written as  $\frac{3}{2}$ . Here, 3 and 2 are natural numbers.

## Group work 1.1

- Using the Venn diagram given below discuss the relationships between the sets of Natural numbers, whole numbers, Integer and Rational numbers.

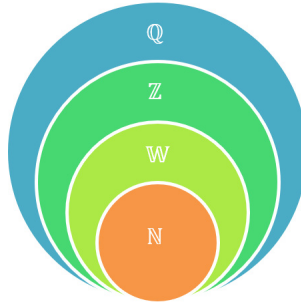


*Figure 1.1 the relation among  $N$ ,  $W$ ,  $Z$  and  $Q$*

- Define each of the following
  - Natural numbers
  - Whole numbers
  - Integers
- To which set(s) of numbers does each of the following numbers belong?
  - 22
  - $\frac{24}{6}$
  - 5.2
  - $\frac{3}{4}$
  - $-\frac{7}{9}$

## Activity 1.1

- Are the following statements true or false? Give reasons for your answers.

- a. Every whole number is a natural number.
  - b. Every integer is a rational number.
  - c. Every rational number is an integer.
2. What do you say about  $\frac{4}{0}$ ?
  3. What are rational numbers?
  4. What are the uses of rational numbers in our daily life?

## Fraction

A fraction is a number that represents a part of a whole. It consists of numerator and denominator. The numerator represents the number of parts of a whole, while the denominator is the total number of parts that make a whole, where a and b represents numerator and denominator respectively, and  $b \neq 0$ .

### Definition:

A fraction is a number of the form  $\frac{a}{b}$  where a and b are integers and  $b \neq 0$

### Example 1

Consider figure 1.2

1. What part of the table is painted red?
2. What part of the table is painted blue?
3. What part of the table is painted green?

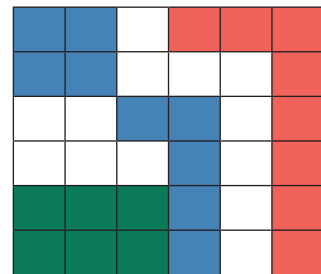


Figure 1.2: parts of the table

*Solution:*

The table is divided into 36 square regions.

1. 8 squares painted with red color. Therefore, the region colored with red is  $\frac{8}{36}$
2. 9 squares painted with blue color. Therefore, the region colored with blue is  $\frac{9}{36}$
3. 6 squares painted with green color. Therefore, the region colored with green is  $\frac{6}{36}$

*Types of fraction*

Fraction can be classified in to three. These are

1. Proper fraction: is a fraction where the numerator is less than the denominator. That is  $a < b$ , in  $\frac{a}{b}$ ,  $b \neq 0$

*Example : 2*

$\frac{3}{5}, \frac{9}{13}, \frac{7}{11}$  are examples of proper fractions.

2. Improper fraction: is a fraction where the numerator is greater than the denominator. That is  $\frac{a}{b}$  is said to be improper fraction if  $a \geq b$ ,  $b \neq 0$

*Example : 3*

$\frac{7}{5}, \frac{19}{13}, \frac{13}{11}$  are examples of improper fractions.

3. Mixed fraction:-is a fraction that is the combination of a natural number and proper fraction. It is basically an improper fraction.

*Example: 4*

$1\frac{3}{5}, 2\frac{4}{6}, 4\frac{7}{11}$  and are examples of mixed fractions.

Improper fractions can be converted to a mixed fraction.

### Example 1: 5

1. Convert the following improper fraction into mixed fraction.

a.  $\frac{3}{2}$

b.  $\frac{7}{3}$

### Solution:

a.  $\frac{3}{2} = 1 + \frac{1}{2} = 1\frac{1}{2}$

b.  $\frac{7}{3} = 2 + \frac{1}{3} = 2\frac{1}{3}$

### Example: 6

2. Convert the following mixed fraction into improper fraction

a.  $4\frac{3}{5}$

b.  $2\frac{3}{4}$

### Solution:

a.  $4\frac{3}{5} = \frac{(4 \times 5) + 3}{5} = \frac{23}{5}$

b.  $2\frac{3}{4} = \frac{(2 \times 4) + 3}{4} = \frac{11}{4}$

### Definition:

Any number that can be written in the form  $\frac{a}{b}$  where  $a$  and  $b$  are integers and  $b \neq 0$ , is called a rational number. The set of rational numbers, denoted by  $\mathbb{Q}$ , is defined by

$$\mathbb{Q} = \left\{ \frac{a}{b}, \text{ Where } a \text{ and } b \text{ are integers and } b \neq 0 \right\}$$

### Example : 7

$\frac{1}{2}, \frac{-5}{11}, \frac{-299}{100}, \frac{8}{7}, \frac{0}{21}, \dots$  are examples of rational numbers.

Any integer is a rational number. Because it can be written in the form  $\frac{a}{b}$  where  $b=1$ .

*Example 8:*

$$2 = \frac{2}{1}, 3 = \frac{3}{1} \text{ and so on}$$

**Equivalent rational numbers**

A rational number can be written with different numerators and denominators.

By multiplying the numerator and denominator of a rational number by the same non-zero integer, we obtain another rational number which is equivalent to the given rational number. This is exactly like (the technique of ) obtaining equivalent fractions.

*Example 9:*

$$\frac{-2}{3} = \frac{-2 \times 4}{3 \times 4} = \frac{-8}{12}$$

There fore  $\frac{-2}{3}$  and  $\frac{-8}{12}$  are equivalent rational numbers.

*Example 10:*

Which of the following pairs represent equivalent rational numbers?

- |                                        |                                         |                                      |
|----------------------------------------|-----------------------------------------|--------------------------------------|
| a. $\frac{-7}{21}$ and $\frac{3}{9}$   | d. $\frac{-8}{-5}$ and $\frac{-24}{15}$ | g. $\frac{7}{21}$ and $\frac{3}{9}$  |
| b. $\frac{-2}{-3}$ and $\frac{2}{3}$   | e. $\frac{-16}{20}$ and $\frac{20}{25}$ | h. $\frac{6}{-9}$ and $\frac{-4}{6}$ |
| c. $\frac{-3}{5}$ and $\frac{-12}{20}$ | f. $\frac{-7}{7}$ and $\frac{3}{-3}$    |                                      |

*Solution:*

Only b, c and g, are equivalent rational numbers.

**Positive and Negative rational numbers**

Consider the rational number  $\frac{2}{3}$ . Both the numerator and denomi-

nator of this number are positive integers. Such a rational number is called a positive rational number.

### *Example 11:*

$\frac{3}{8}, \frac{5}{7}, \frac{9}{2}, \dots$  are positive rational numbers.

The numerator is  $-\frac{5}{8}$  a negative integer, whereas the denominator is a positive integer. Such a rational number is called a **negative rational number**.

### *Example 12:*

$-\frac{5}{3}, -\frac{9}{11}, -\frac{3}{7}, \dots$  are negative rational numbers.

## *1.1.1 Representation of rational numbers on a number line*

### Activity 1.2

- Represent the numbers -3, -2, -1, 0, 1, 2, . . on the number line
- What are the numbers that are right of zero on the number line?
- What are the numbers that are left of zero on the number line?

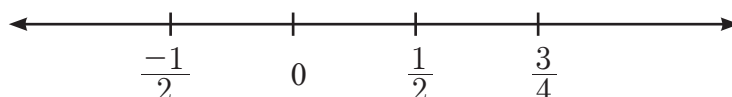
One of the most important properties of the rational number is that it can be represented graphically by points on a straight line. The point 0 is termed as the origin. Points right of 0 are called positive numbers and points left of 0 are called negative rational numbers. Each point on the number line correspond a unique rational number and vice-versa.

*Example 1:*

Represent the following numbers on the number line

a.  $-\frac{1}{2}, \frac{1}{2}$  and  $\frac{3}{4}$

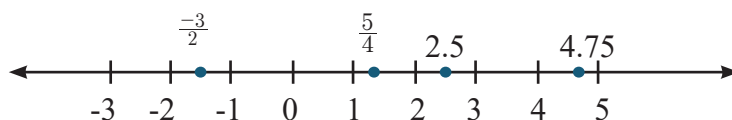
*Solution:*

*Example 2:*

Represent the following numbers on the number line

b.  $-\frac{3}{2}, \frac{5}{4}, 2.5, 4.75$

*Solution:*



Geometrically we say  $a$  is greater than  $b$  if  $a$  is located to the right of  $b$  on the number line.

## Rational Numbers in Standard form

Consider the rational numbers  $-\frac{2}{3}, \frac{4}{9}, \frac{7}{11}, -\frac{11}{15}$ , and  $\frac{-17}{19}$ . The denominators of these rational numbers are positive integers and 1 is the only common factor of the numerators and denominators. Further, the negative sign occurred only in the numerator. Such rational numbers are said to be in standard form.

**Note:** A rational number is said to be in the standard form if

- i. Its denominator is a positive integer and

- ii. The numerator and denominator have no common factor other than 1 are relatively prime.

If a rational number is not in the standard (lowest) form, then it can be reduced to the standard (lowest) form.

When a rational number is reduced in to its lowest forms, we divide the numerator and the denominator of the rational number by the same non- zero positive integers.

### Example 3:

Write the following rational numbers in standard (simplest) form

a.  $\frac{3}{4}$

b.  $\frac{6}{8}$

c.  $\frac{-12}{15}$

### Solution:

a. The standard (simplest) form of  $\frac{3}{4}$  is  $\frac{3}{4}$

b. In  $\frac{6}{8}$ , when both numerator and denominator is divide by 2, you have  $\frac{3}{4}$

Hence the standard (simplest) form of  $\frac{6}{8}$  is  $\frac{3}{4}$ .

c. In  $-\frac{12}{15}$ , when both numerator and denominator is divide by 3, you have  $-\frac{4}{5}$

Hence the standard (simplest) form of  $\frac{-12}{15}$  is  $-\frac{4}{5}$ .

## Exercise 1.1

1. Are the following pairs of rational numbers equivalent? Give the reasons to your answer.

a.  $\frac{3}{7}$  and  $\frac{12}{28}$

c.  $\frac{115}{15}$  and 15

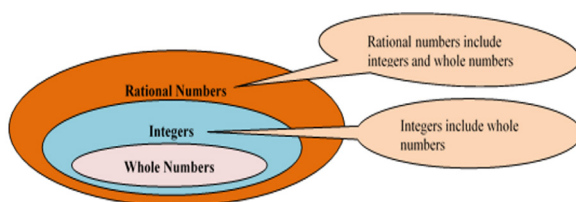
b.  $\frac{1}{9}$  and  $\frac{7}{63}$

d.  $\frac{85}{125}$  and  $\frac{17}{35}$

2. Find at least three equivalent rational numbers for each of them.
- a.  $\frac{6}{16}$  c.  $\frac{21}{36}$
- b.  $\frac{24}{54}$  d.  $\frac{65}{100}$
3. Rewrite each of the following rational number in the form of  $\frac{a}{b}$  where a and b are integers and  $b \neq 0$ .
- a.  $2\frac{4}{9}$  c.  $-86$  e.  $\frac{13}{23}$
- b.  $-6\frac{7}{8}$  d.  $345.658$  f.  $-6.83$
4. There are 36 students in the class. 21 are boys and 15 are girls. What fraction of the students are:
- a. Boys to girls b. Girls to boys

### 1.1.2. Relationship among $W$ , $\mathbb{Z}$ and $\mathbb{Q}$

- ◆ The set of whole numbers includes the natural numbers. Therefore, every natural number is also a whole number.
- ◆ The set of integers includes the set of whole numbers. Therefore, every whole number is also an integer.
- ◆ The set of rational numbers includes the set of integers. Therefore, every integer is also a rational number.
- ◆ The set of whole numbers is a subset of the set of integers and the set of integers is the subset of the set of rational numbers or.



## 1.1.3 Absolute value of Rational Number

### Activity 1.3

Write each of the following without the absolute value sign.

- a.  $|-3|$                       b.  $|3|$                       c.  $|0|$                       d.  $|\frac{-3}{2}|$

The absolute value of rational number is the distance from zero on the number line. The symbol for the absolute value of a number  $x$  is  $|x|$ . Since points corresponding to 3 and -3 are at the same distance from the points corresponding to zero we have  $|-3|=3$  and  $|3|=3$ .

From the above discussion we have the following properties

- i. If  $x$  is a positive number, then  $|x|=x$

### Example 1:

$|2|=2$  since the absolute value of positive rational number is the number itself.

- ii.  $|0|=0$

- iii. If  $x$  is negative number, then  $|-x| = -(-x) = x$

### Example 2:

$|-2| = -(-2) = 2$  since the absolute value of a negative rational number is the opposite of the number.

**Note:**

$|3| = 3$  is read as the absolute value of positive three is three.

**Definition:**

The absolute value of rational number 'x' is denoted by the symbol  $|x|$  and defined by

$$|x| = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -x & \text{if } x < 0 \end{cases}$$

**Example 3:**

Simplify each of the following absolute value expression.

a.  $|6 - 3|$

c.  $|0 - \pi|$

b.  $|2 - 7|$

**Solution :**

a. Since  $6 - 3 = 3$  and  $3 > 0$ , we have  $|6 - 3| = |3| = 3$

b. Since  $2 - 7 = -5$  and  $-5 < 0$ , We have  $|2 - 7| = -(-5) = 5$

c. Since  $0 - \pi = -\pi$  and  $-\pi < 0$ , We have  $|0 - \pi| = -(-\pi) = \pi$

**Equations Involving Absolute Value**

Geometrically,  $|x| = 5$  means the point with coordinate x is five unit from zero on the number line. Obviously the number line contains two points that are five units from the origin. That is, one is to the

right of the origin and the other to the left. Thus,  $|x| = 5$  has two solutions,  $x = 5$  and  $x = -5$ .

### Note:

The solution of the equation  $|x| = a$

For any rational number  $a$ , the equation  $|x| = a$  has

- i. Two solutions,  $x=a$  and  $x= -a$ , if  $a > 0$
- ii. One solution,  $x = 0$ , if  $a = 0$
- iii. No solution if  $a < 0$

### Example 4:

Solve the following absolute value equations

a.  $|x| = 6$

c.  $|x| + \frac{1}{3} = \frac{2}{3}$

b.  $|x| = -25$

### Solution:

a.  $|x| = 6$

Since  $6 > 0$ , we have two distinct solutions,  $x = 6$  and  $x = -6$ .

b.  $|x| = -25$

Since  $-25 < 0$ , we have no solutions, hence the solution set is empty set or  $\{ \}$  or  $\phi$ .

c.  $|x| + \frac{1}{3} = \frac{2}{3}$  Add  $-\frac{1}{3}$  to both sides

$$|x| + \frac{1}{3} - \frac{1}{3} = \frac{2}{3} - \frac{1}{3}$$

$$|x| = \frac{1}{3}, \text{ Since } \frac{1}{3} > 0, \text{ there is two distinct solutions, } x = \frac{1}{3} \text{ and } x = -\frac{1}{3}$$

$$x = -\frac{1}{3}$$

## Exercise 1.2

1. Complete the table below

|   |   |                |                |                 |    |   |     |               |     |      |
|---|---|----------------|----------------|-----------------|----|---|-----|---------------|-----|------|
| x | 5 | $-\frac{2}{3}$ | $3\frac{1}{4}$ | $-5\frac{6}{7}$ | -9 |   |     | $\frac{9}{2}$ | 3.4 | -3.7 |
| x |   |                |                |                 |    | 0 | 4.2 |               |     |      |

2. Find all rational numbers whose absolute values are given below.

a.  $4\frac{3}{4}$

b. 2.7

c.  $-6\frac{3}{8}$

3. Evaluate each of the following

a.  $|-7| - |31 - 11|$

d.  $|13 - 19|$

b.  $|\frac{-7}{15}| + |\frac{-7}{5} - \frac{2}{3}|$

e.  $|-4 + 12|$

c.  $|11 + (-11)|$

f.  $|4| + |-10| - |-3|$

4. Evaluate each expression

a.  $-6x + 2|x - 3|$ , when  $x = -3$

d.  $|y| - |x|$ , when  $x = 3$  and  $y = -7$

b.  $|m| - m + 3$ , when  $m = \frac{2}{3}$

e.  $-2|19 - y|$ , when  $y = -5$

c.  $|x| + |y|$ , when  $x = -3$  and  $y = -1$

5. Solve the following absolute value equations

a.  $|x| = 3\frac{2}{5}$

b.  $|x| + \frac{3}{5} = \frac{2}{5}$

c.  $3 - 2|x + 2| = 6$

e.  $|2x| = 16$

d.  $2|4x + 5| + 7 = 19$

f.  $|x - 4| = 7$

## 1.2 Comparing and ordering of rational numbers

### Group work 1.2

- Is there any integer between  $n$  and  $n+1$ , where  $n \in \mathbb{N}$
- Is there any whole number between  $n$  and  $n+1$ , where  $n \in \mathbb{W}$
- Arrange the following numbers in ascending order
  - $-15, 0, 65, -7, -31, 44$
  - $\frac{1}{2}, \frac{1}{3}, 1\frac{1}{4}, -2\frac{3}{5}, -3.5$
- Name all integers which lie between
  - $-2$  and  $3$
  - $0$  and  $1$
  - $-3$  and  $1$
  - $-4$  and  $1$
- Insert  $>$ ,  $<$  or  $=$  to express the corresponding relationship between the following pairs of rational numbers.
  - $-\frac{3}{5}$  \_\_\_\_\_  $-\frac{5}{3}$
  - $-\frac{9}{7}$  \_\_\_\_\_  $-2$
  - $\frac{150}{3}$  \_\_\_\_\_  $\frac{200}{5}$
  - $\frac{0}{2}$  \_\_\_\_\_  $-\frac{10}{2}$
  - $-2\frac{1}{2}$  \_\_\_\_\_  $-\frac{3}{2}$
  - $-3\frac{9}{8}$  \_\_\_\_\_  $-\frac{33}{8}$
- Alte compared two negative rational numbers  $-12$  and  $-15$  using number line. She wrote  $-12$  to the right of  $-15$ . Is she correct? Why?

## 1.2.1 Comparing Rational Numbers

You know how to compare two integers to tell which is smaller or which is greater.

Now let us see how we can compare two rational numbers.

In comparison of negative and positive integers, any positive or zero is larger than any negative number. Similarly, a negative rational number is to the left of zero whereas a positive rational number is to the right of zero on a number line. So, a negative rational number is always less than a positive rational number.

Here  $-511 < 0$  and  $37 > 0$

So that  $-511 < 37$ , and the same is true for  $-100 < 12$

To compare two negative rational numbers, we compare them ignoring their negative signs and then reverse the order.

For example, to compare  $-75$  and  $-53$ , we first compare  $75$  and  $53$ .

We get  $75 > 53$ , because  $7 \times 3 < 5 \times 5$  and we conclude that

$-75 < -53$  since  $-7 \times 3 > -5 \times 5$ .

Or by converting them to rational numbers with same denominators, it is possible to compare.

First find the LCM of  $5$  and  $3$ , which is  $\text{LCM}(5, 3) = 15$ . By changing  $-75$  and  $-53$  to its equivalent rational numbers  $-\frac{21}{15}$  and  $-\frac{25}{15}$ .

So we conclude that  $-75 < -53$ .

### Example 1:

Which of the following is correct?

a.  $\frac{100}{25} = 4$

b.  $2\frac{1}{3} > \frac{14}{9}$

c.  $-3\frac{4}{5} > -\frac{11}{5}$

d.  $\frac{n}{n+1} > \frac{n+1}{n+2}, n \in \mathbb{N}$

*Solution:*

a, and b are correct but c and d are not correct.

### Exercise 1.3

1. Which of the following is correct?

a.  $\frac{4}{11} < \frac{7}{11}$

c.  $\frac{5}{7} < \frac{5}{9}$

e.  $\frac{-3}{8} < \frac{0}{2}$

b.  $\frac{-5}{9} < \frac{3}{12}$

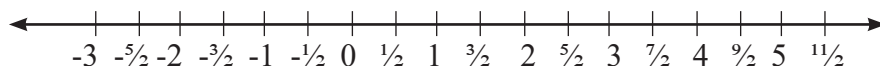
d.  $\frac{-11}{13} < \frac{-10}{11}$

## 1.2.2 Ordering Rational Numbers

You recall that the relation greater than  $>$  and less than  $<$  uses for ordering integers. Since the set of integer is the subset of the set of rational numbers, we define an order relation on the rational numbers which is consistent with the order relation in the set of integers.

The concept “less than” for rational numbers is similar to that of integers. Recall that for integers, the smaller of two numbers is to the left of the larger on the number line.

Consider the number line below



As shown in the above number line

a. -6 is to the left of -1,

d.  $-\frac{11}{2}$  is to the left of  $-\frac{9}{2}$ ,

c.  $-\frac{7}{2}$  is to the left of  $-\frac{5}{2}$ ,

e.  $-\frac{1}{2}$  is to the left of -1,

c.  $\frac{1}{2}$  to the left of 1,

f.  $\frac{5}{2}$  is to the left of 3

Therefore

$$-6 < -1, \quad -\frac{11}{2} < -\frac{9}{2}, \quad -\frac{7}{2} < -\frac{5}{2}, \quad -\frac{1}{2} < -1, \quad \frac{1}{2} < 1, \quad \frac{5}{2} < 3$$

In the same way, for two rational numbers the larger number is on the right of the smaller number on the number line.

### Example 1:

Compare the following pair of numbers using number line

a. 6.5 and -3.5

b.  $2\frac{3}{5}$  and  $3\frac{2}{3}$

c.  $5\frac{2}{7}$  and  $2\frac{3}{7}$

### Solution

a.  $-6.5 < -3.5$

c.  $5\frac{2}{7} > 2\frac{3}{7}$

b.  $2\frac{3}{5} < 3\frac{2}{3}$

d.  $-1.5 > -5.3$

Therefore from the above fact, it follows that:

- ✧ Every positive rational number is greater than Zero.
- ✧ Every negative rational number is less than Zero.
- ✧ Every positive rational number is greater than every negative rational number.
- ✧ Among two negative rational numbers, the one with the largest absolute value is smaller than the other.

**Note:**

In the system of rational numbers, there are many rational numbers between any two rational numbers.

**Example 2:**

$\frac{1}{2}, \frac{3}{2}, 2.7, 3.1, \frac{7}{2}$  are between 1 and 4

**Definition:**

Order of fractions with similar denominator

Let  $\frac{a}{c}$  and  $\frac{b}{c}$  be any fraction with  $c > 0$ , then  $\frac{a}{c} < \frac{b}{c}$  if and only if  $a < b$

**Example 3:**

a.  $-\frac{12}{5} < -\frac{8}{5}$  because  $-12 < -8$

b.  $\frac{41}{7} < \frac{55}{7}$  because  $41 < 55$

c.  $3\frac{4}{5} < 4\frac{6}{5}$  because  $19 < 26$

To test whether  $\frac{3}{4}$  is less than  $\frac{4}{5}$  we change  $\frac{3}{4}$  and  $\frac{4}{5}$  to equivalent fraction (fraction with the same denominator).

$$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20} \text{ and } \frac{4}{5} = \frac{4 \times 4}{5 \times 4} = \frac{16}{20} \text{ then by comparing } \frac{15}{20} \text{ and } \frac{16}{20},$$

i.e.  $\frac{15}{20} < \frac{16}{20}$

Therefore,  $\frac{3}{4} < \frac{4}{5}$  (because  $3 \times 5 = 15 < 16 = 4 \times 4$ )

**Definition:**

Order of fractions with different denominator

Let  $\frac{a}{b}$  and  $\frac{c}{d}$  be any fraction with  $b, d > 0$ , then  $\frac{a}{b} < \frac{c}{d}$  if and only if  $ad < bc$

**Example 4:**

- a.  $-\frac{7}{3} < \frac{3}{4}$ , because  $-7 \times 4 < 3 \times 3$  that is  $-28 < 9$
- b.  $\frac{2}{3} > -\frac{5}{4}$ , because  $2 \times 4 > -5 \times 4$  that is  $8 > -20$

**Example 5:**

Danchile grew  $\frac{3}{4}$  cm over the summer. Her friends also measured themselves and found that Samira grew  $\frac{2}{5}$  cm, Buwa grew  $\frac{5}{8}$  cm, and Lalego grew  $\frac{1}{2}$  cm. Which one is the correct order of their list that grew from least to most? Their height

- a. Danchile, Lalego, Buwa, Samira
- b. Samira, Buwa, Lalego, Danchile
- c. Lalego, Buwa, Danchile, Samira
- d. Samira, Lalego, Buwa, Danchile

Solution: d Answer or solution?

**Definition:**

For rational numbers  $\frac{a}{b}$  and  $\frac{c}{d}$  with  $b$  and  $d > 0$ . We say

- i.  $\frac{a}{b} > \frac{c}{d}$  if and only if  $ad > bc$  or
- ii.  $\frac{a}{b} < \frac{c}{d}$  if and only if  $ad < bc$

**Example 6:**

- a.  $\frac{5}{7} < \frac{8}{11}$  , because  $5 \times 11 < 7 \times 8$  or  $55 < 56$
- b.  $-\frac{5}{6} < \frac{12}{3}$  , because  $-5 \times 13 < 6 \times 12$  or  $-65 < 72$
- c.  $\frac{2}{-7} < \frac{8}{9}$  , because  $2 \times 9 < -7 \times -8$  or  $18 < 56$

The order properties of the set of integers  $Z$  remain to be true in the set of rational numbers  $Q$ .

The following are some of the properties of the set of rational numbers  $Q$  .

**Tracheotomy low:**

For any two rational numbers  $\frac{a}{b}$  and  $\frac{c}{d} \in Q$  , exactly one of the following is true

- a.  $\frac{a}{b} < \frac{c}{d}$                       b.  $\frac{a}{b} = \frac{c}{d}$                       c.  $\frac{a}{b} > \frac{c}{d}$

**Transitive low:**

For rational numbers  $\frac{a}{b}, \frac{c}{d}$  and  $\frac{e}{f} \in Q$  ,  $b \neq 0$  ,  $d \neq 0$  and  $f \neq 0$

If  $\frac{a}{b} < \frac{c}{d}$  and  $\frac{c}{d} < \frac{e}{f}$  , then  $\frac{a}{b} < \frac{e}{f}$

**Example 7:**

If  $\frac{2}{5} < \frac{4}{7}$  and  $\frac{4}{7} < \frac{3}{5}$  , then  $\frac{2}{5} < \frac{3}{5}$   $2/5 < 3/5$

That is  $2 \times 7 < 5 \times 4$  and  $4 \times 5 < 3 \times 7$ , then  $2 \times 5 < 3 \times 5$

**Addition low :**

For rational numbers  $\frac{a}{b}, \frac{c}{d}$  and  $\frac{e}{f} \in Q$  ,

$\frac{a}{b} < \frac{c}{d}$  , then  $\frac{a}{b} + \frac{e}{f} < \frac{c}{d} + \frac{e}{f}$

*Example 8:*

If  $\frac{2}{3} < \frac{5}{7}$ , then  $\frac{2}{3} + \frac{1}{4} < \frac{5}{7} + \frac{1}{4}$

**Multiplication law :**

For rational numbers  $\frac{a}{b}, \frac{c}{d}$  and  $\frac{e}{f} \in \mathbb{Q}$

- i. If  $\frac{a}{b} < \frac{c}{d}$  and  $\frac{e}{f} > 0$ , then  $\frac{a}{b} \times \frac{e}{f} > \frac{c}{d} \times \frac{e}{f}$
- ii. If  $\frac{a}{b} < \frac{c}{d}$  and  $\frac{e}{f} < 0$ , then  $\frac{a}{b} \times \frac{e}{f} > \frac{c}{d} \times \frac{e}{f}$

*Example 9:*

- a. If  $\frac{11}{12} > \frac{9}{10}$  and  $\frac{3}{4} > 0$ , then  $\frac{11}{12} \times \frac{3}{4} > \frac{9}{10} \times \frac{3}{4}$ , This is  $110 > 109$ ,
- b. and  $\frac{33}{48} > \frac{27}{40}$
- c. If  $\frac{2}{3} > \frac{1}{4}$  and  $\frac{-3}{5} < 0$ , then  $\frac{2}{3} \times \frac{-3}{5} < \frac{1}{4} \times \frac{-3}{5}$

## 1.3 Operations and properties of rational numbers

You know how to add, subtract, multiply and divide integers as well as fractions. Now let us study these basic operations on rational numbers.

### 1.3.1. Addition of Rational Numbers

#### 1. Addition of Rational Numbers with the same denominator

Let  $\frac{a}{b}$  and  $\frac{c}{b}$  are two rational numbers, and  $b \neq 0$  then

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$$

(add the numerators and take one of the denominators)

*Example 1:*

a.  $\frac{1}{4} + \frac{3}{4} = \frac{1+3}{4} = \frac{4}{4} = 1$

b.  $\frac{5}{7} + \frac{6}{7} = \frac{5+6}{7} = \frac{11}{7}$

## 2. Addition of Rational Numbers with different denominator

Let  $\frac{a}{b}$  and  $\frac{c}{d}$  are two rational numbers and  $b$  and  $d \neq 0$  then

$$\frac{a}{b} + \frac{c}{d} = \frac{a \times d + b \times c}{b \times d}$$

*Example 2:*

a.  $\frac{2}{9} + \frac{1}{3} = \frac{2 \times 3 + 9 \times 1}{9 \times 3} = \frac{6+9}{18} = \frac{15}{18} = \frac{5}{6}$

b.  $\frac{4}{7} + \frac{5}{2} = \frac{4 \times 2 + 7 \times 5}{7 \times 2} = \frac{8+35}{14} = \frac{43}{14}$

*1.3.2 Subtraction of Rational Number*

## 1. Subtraction of Rational Numbers with the same denominator

Let  $\frac{a}{b}$  and  $\frac{c}{d}$  are two rational numbers, and  $b \neq 0$  then

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b}$$

*Example 3:*

a.  $\frac{7}{8} - \frac{5}{8} = \frac{7-5}{8} = \frac{2}{8} = \frac{1}{4}$

b.  $\frac{15}{3} - \frac{11}{3} = \frac{15-11}{3} = \frac{4}{3}$

## 2. Subtraction of Rational Numbers with different denominators

Let  $\frac{a}{b}$  and  $\frac{c}{d}$  are two rational numbers, and  $b$ , and  $d \neq 0$  then

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

*Example 4:*

$$\text{a. } \frac{6}{5} - \frac{2}{3} = \frac{6 \times 3 - 5 \times 2}{5 \times 3} = \frac{18 - 10}{15} = \frac{8}{15}$$

$$\text{b. } \frac{23}{2} - \frac{9}{5} = \frac{23 \times 5 - 2 \times 9}{2 \times 5} = \frac{115 - 18}{10} = \frac{97}{10}$$

$$\text{c. } \frac{-7}{3} - \frac{14}{9} = \frac{-7 \times 9 - 3 \times (-14)}{3 \times 9} = \frac{-63 - 42}{27} = -\frac{105}{27} = -\frac{35}{9}$$

*1.3.3 Multiplication of Rational Numbers*

Let  $\frac{a}{b}$  and  $\frac{c}{d}$  are two rational numbers, and  $b, d \neq 0$  then

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

*Example 5:*

$$\text{a. } \frac{5}{3} \times \frac{7}{4} = \frac{5 \times 7}{3 \times 4} = \frac{35}{12}$$

$$\text{b. } \frac{15}{2} \times \frac{5}{3} = \frac{15 \times 5}{2 \times 3} = \frac{75}{6} = \frac{25}{2}$$

*1.3.4 Division of Rational Numbers*

The reciprocal of  $\frac{a}{b}$  is  $\frac{b}{a}$ , where  $a$  and  $b \neq 0$

*Example 6:*

$$\text{a. The reciprocal of } \frac{2}{3} \text{ is } \frac{3}{2}$$

$$\text{b. The reciprocal of } -6 \text{ is } -\frac{1}{6}$$

$$\text{c. The reciprocal of } 0 \text{ is undefined.}$$

For two rational numbers  $\frac{a}{b}$  and  $\frac{c}{d}$  and  $b, c, \text{ and } d \neq 0$  then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

*Example 7:*

$$a. \quad \frac{3}{7} \div \frac{2}{5} = \frac{3}{7} \times \frac{5}{2} = \frac{3 \times 5}{7 \times 2} = \frac{15}{14}$$

$$b. \quad \frac{8}{9} \div \frac{4}{5} = \frac{8}{9} \times \frac{5}{4} = \frac{8 \times 5}{9 \times 4} = \frac{40}{36} = \frac{10}{9}$$

$$c. \quad -3\frac{1}{5} \div 2\frac{1}{3} = -\frac{16}{5} \div \frac{7}{3} = -\frac{16}{5} \times \frac{3}{7} = \frac{-16 \times 3}{5 \times 7} = -\frac{48}{35}$$

**Properties of Rational Numbers****1. Closure properties**

The set of rational numbers is closed under the operation addition, multiplication and subtraction, but not under the operation division.

For all  $\frac{a}{b}$  and  $\frac{c}{d} \in \mathbb{Q}$ , where  $b \neq 0$  and  $d \neq 0$

$$a. \quad \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \in \mathbb{Q}$$

$$c. \quad \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \in \mathbb{Q}$$

$$b. \quad \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd} \in \mathbb{Q}$$

*Example 8:*

Consider  $\frac{3}{4}$  and  $\frac{5}{6} \in \mathbb{Q}$ , then

$$a. \quad \frac{3}{4} + \frac{5}{6} = \frac{3 \times 6 + 4 \times 5}{4 \times 6} = \frac{18 + 20}{24} = \frac{38}{24} \in \mathbb{Q}$$

$$b. \quad \frac{3}{4} - \frac{5}{6} = \frac{3 \times 6 - 4 \times 5}{4 \times 6} = \frac{18 - 20}{24} = -\frac{2}{24} \in \mathbb{Q}$$

$$c. \quad \frac{3}{4} \times \frac{5}{6} = \frac{3 \times 5}{4 \times 6} = \frac{15}{24} \in \mathbb{Q}$$

**2. Commutative property of (+) and (×) in  $\mathbb{Q}$** 

The operation addition and multiplication are commutative on the set of rational number.

For all  $\frac{a}{b}$  and  $\frac{c}{d} \in \mathbb{Q}$ ,  $b \neq 0$  and  $d \neq 0$

$$\text{i. } \frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$$

$$\text{ii. } \frac{a}{b} \times \frac{c}{d} = \frac{c}{d} \times \frac{a}{b}$$

### Example:9

Consider  $\frac{8}{9}$  and  $\frac{3}{7} \in \mathbb{Q}$

$$\text{a. } \frac{8}{9} + \frac{3}{7} = \frac{3}{7} + \frac{8}{9}$$

$$\text{b. } \frac{8}{9} \times \frac{3}{7} = \frac{3}{7} \times \frac{8}{9}$$

## 3. Associative property of (+) and (×) in $\mathbb{Q}$

The operation addition and multiplication are associative on the set of rational number.

For all  $\frac{a}{b}, \frac{c}{d}$  and  $\frac{e}{f} \in \mathbb{Q}$   $b \neq 0, d \neq 0$  and  $f \neq 0$

$$\text{i. } \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f} = \frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right)$$

$$\text{i. } \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f} = \frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right)$$

### Example:10

$$\text{i. } \frac{2}{3} + \left(\frac{4}{7} + \frac{9}{10}\right) = \left(\frac{2}{3} + \frac{4}{7}\right) + \frac{9}{10}$$

$$\text{ii. } \frac{-7}{8} \times \left(\frac{1}{9} \times \frac{11}{13}\right) = \left(\frac{-7}{8} \times \frac{1}{9}\right) \times \frac{11}{13}$$

## 4. Distributive property

Multiplication is distributive over addition

For all  $\frac{a}{b}, \frac{c}{d}$  and  $\frac{e}{f} \in \mathbb{Q}$ ,  $b \neq 0, d \neq 0$  and  $f \neq 0$

$$\frac{a}{b}(\frac{c}{d} + \frac{e}{f}) = \frac{a}{b} \times \frac{c}{d} + \frac{a}{b} \times \frac{e}{f}$$

*Example: 11*

$$\frac{-9}{10}(\frac{-3}{4} + \frac{2}{9}) = \frac{-9}{10} \times \frac{-3}{4} + \frac{-9}{10} \times \frac{2}{9}$$

## 5. Existence of identity

1. 0 is the additive identity element in the set of rational numbers

That is:  $\frac{a}{b} + 0 = 0 + \frac{a}{b} = \frac{a}{b}$ , for  $\frac{a}{b} \in \mathbb{Q}$

*Example 12:*

$$\frac{5}{6} + 0 = 0 + \frac{5}{6} = \frac{5}{6}$$

2. 1 is the multiplication identity element in the set of rational numbers ,

Since for all  $\frac{a}{b} \in \mathbb{Q}$ ,  $\frac{a}{b} \times 1 = 1 \times \frac{a}{b} = \frac{a}{b}$

## 6. Existence of Inverse

- i. For all  $\frac{a}{b} \in \mathbb{Q}$  there exists an additive inverse in rational numbers ,

denoted by  $-\frac{a}{b}$  , such that  $\frac{a}{b} + (-\frac{a}{b}) = -\frac{a}{b} + \frac{a}{b} = 0$

*Example 13:*

The additive inverse of rational numbers  $\frac{8}{9}$  is  $(-\frac{8}{9})$  .

Because  $\frac{8}{9} + (-\frac{8}{9}) = 0$

- ii. For every element  $\frac{a}{b} \neq 0 \in \mathbb{Q}$  ,there exists a multiplicative inverse ,namely  $\frac{b}{a}$  ,that is the reciprocal of  $\frac{b}{a}$  such that  $\frac{a}{b} \times \frac{b}{a} = \frac{b}{a} \times \frac{a}{b} = 1$

*Example 14:*

- a. The multiplicative inverse of  $\frac{3}{4}$  is  $\frac{4}{3}$ , because,  $\frac{3}{4} \times \frac{4}{3} = 1$
- b. The multiplicative inverse of  $\frac{-9}{11}$  is  $\frac{-11}{9}$ , because,  $\frac{-9}{11} \times \frac{-11}{9} = 1$

**Remark:**

0 is the only rational number that doesn't have a multiplicative inverse

For any two element  $\frac{a}{b}$  and  $\frac{c}{d} \in \mathbb{Q}$ ,

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

*Example 15:*

$$\frac{9}{5} \div \frac{7}{8} = \frac{9}{5} \times \frac{8}{7} = \frac{72}{35}$$

**Remark:**

- i.  $\mathbb{Q} \setminus \{0\}$  is closed under division.
- ii. Rational number  $\frac{a}{b}$  is said to be in lowest term, if and only if the numerator  $a$  and the denominator  $b$  are relatively prime. That is  $\text{GCF}(a,b)=1$

*Exercise 1.4*

1. Sketch a number line and mark the location of each fraction.

a.  $\frac{2}{3}$

b.  $-\frac{2}{3}$

c.  $-\frac{4}{5}$

d.  $\frac{4}{5}$

2. Compute the following problems in  $\mathbb{Q}$

a.  $\frac{2}{5} + \frac{3}{7}$

f.  $(\frac{1}{5} - \frac{7}{10}) \div \frac{9}{5}$

b.  $\frac{19}{18} - \frac{13}{9}$

g.  $(\frac{2}{9} \div \frac{1}{2}) \div \frac{3}{18}$

c.  $-\frac{3}{8} \times \frac{7}{13}$

h.  $-4\frac{2}{3} \times (-3\frac{1}{7}) \times \frac{14}{11}$

d.  $\frac{4}{11} \div \frac{7}{9}$

i.  $-2\frac{3}{5} - (1\frac{1}{2} + 4\frac{2}{3})$

e.  $(\frac{3}{4} + \frac{1}{2}) \div \frac{3}{8}$

3. Find an additive inverse of the following numbers in  $\mathbb{Q}$

a.  $\frac{6}{7}$

b.  $-3\frac{5}{8}$

c. 0

d.  $-\frac{17}{8}$

4. Find a multiplicative inverse of the following numbers in  $\mathbb{Q}$

a.  $4\frac{7}{17}$

b.  $-\frac{31}{65}$

c.  $-\frac{14}{19}$

d. 0

## Decimal Representations of the Rational Number

### Activity 1.4

1. Can every rational number be represented as a decimal?

2. Are all decimals rational numbers?

The decimal system of numbers is based on the digits 0 to 9. A number such as 43.27 is called a decimal fraction, a decimal point separating the integer part that is 43, from the fraction part, that is 0.27.

A number which can be expressed exactly as a decimal fraction is called a terminating decimals and those which cannot be expressed exactly as a decimal fraction are called non-terminating decimals .

Thus,  $\frac{7}{2} = 3.5$  and  $\frac{78}{100} = 7.8$  are called terminating decimals.

### Example 15

Find the decimal number for each of the following

a.  $\frac{11}{5}$

b.  $\frac{-7}{25}$

c.  $\frac{1}{3}$

### Solution

a. Dividing 11 by 5 , gives 2.2. Hence ,  $\frac{11}{5} = 2.2$

b. Dividing -7 by 25 , gives 0.28. Hence ,  $\frac{-7}{25} = 0.28$

c. Dividing 1 by 3, gives 0.888... Hence  $\frac{1}{3} = 0.3333.....$

Such a decimal is called non terminating repeating decimal.

Every rational number is a terminating decimal or a periodic decimal (repeating decimals).

### Example 16

Express 0.333... as in the form of  $\frac{b}{a}$  .

### Solution:

Let  $0.333... = \frac{p}{q}$  , where p and q are integers and  $q \neq 0$

Since we do not know what 0.333... is,

let us call it x and so  $x = 0.3333...$  (let's denote it by x )

Now, multiply both sides by 10 , it becomes

$$10x = 10 \times (0.333...) = 3.333...$$

Now,  $3.3333... = 3 + 0.333...$

$$= 3 + x, \text{ since } x = 0.3333...$$

$$10x = 3 + x$$

$$10x - x = 3 + x - x$$

$$9x = 3$$

Solving for x, we get  $x = \frac{3}{9} = \frac{1}{3}$ .

Hence,  $0.3333... = \frac{1}{3}$

*Example 17*

Change the following decimals to fraction form

- a. 10.105
- c.  $4.\overline{32}$
- b.  $3.\overline{2}$

*Solution*

a.  $10.105 = \frac{10105}{1000}$ , its simplest form is  $\frac{2021}{200}$

| b. $3.\overline{2}$                                                                                                                                                                                                                                                                       | c. $4.\overline{32}$                                                                                                                                                                                                                                  |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Let $x = 3.\overline{2}$ means $x = 3.222222...$<br>Then $10x = 32.\overline{2}$<br>Now, using subtraction<br>$10x - x = 32.\overline{2} - 3.\overline{2}$<br>$9x = 29$ , dividing both sides by 9<br>$x = \frac{29}{9}$<br>Therefore, $3.\overline{2} = \frac{29}{9}$ is rational number | Let $x = 4.\overline{32}$ which is $x = 4.3232....$<br>Then $100x = 432.3232$<br>Now, using subtract<br>$100x - x = 432.3232 - 4.3232....$<br>$99x = 428$<br>$x = \frac{428}{99}$<br>Therefore, $4.\overline{32} = \frac{428}{99}$ is rational number |

### Exercise 1.5

1. Change the following decimals to fraction form

- |                          |                        |                        |
|--------------------------|------------------------|------------------------|
| a. 40.202                | d. $0.\overline{01}$   | g. $0.23\dot{4}$       |
| b. $3.25\dot{}$          | e. $21.4\overline{06}$ | h. $1.11\overline{21}$ |
| c. $0.14285\overline{7}$ | f. $1.2\overline{101}$ | i. $4.0\overline{103}$ |

#### Note

Between two consecutive integers there is infinite rational numbers.

### Example 18

Find five rational numbers between 1 and 2.

#### Solution:

Check the sequence, which should come first? Giving a formula or writing an example? My suggestion is the reverse.

We can approach this question in at least two ways

#### Case 1:

Recall that to find a rational number between the numbers 1 and 2, you can add  $r$  and  $s$  and divide the sum by 2, that is  $\frac{r+s}{2}$  lies between  $r$  and  $s$ . So,  $\frac{1+2}{2} = \frac{3}{2}$  is a number between 1 and 2. That is  $1 < \frac{3}{2} < 2$ .

Next, find another rational numbers between 1 and  $\frac{3}{2}$  and  $\frac{5}{4}$  and  $\frac{3}{2}$

|   |                               |                                         |                                                    |                                                    |                                         |   |
|---|-------------------------------|-----------------------------------------|----------------------------------------------------|----------------------------------------------------|-----------------------------------------|---|
| 1 | $1 < \frac{3}{2} < 2$         | $1 < \frac{5}{4} < \frac{3}{2}$         | $\frac{5}{4} < \frac{11}{8} < \frac{3}{2}$         | $\frac{5}{4} < \frac{13}{8} < \frac{3}{2}$         | $\frac{3}{2} < \frac{7}{4} < 2$         | 2 |
|   | $\frac{1+3}{2} = \frac{3}{2}$ | $\frac{1+\frac{3}{2}}{2} = \frac{5}{4}$ | $\frac{\frac{5}{4}+\frac{3}{2}}{2} = \frac{11}{8}$ | $\frac{\frac{5}{4}+\frac{7}{4}}{2} = \frac{13}{8}$ | $\frac{\frac{3}{2}+2}{2} = \frac{7}{4}$ |   |

Therefore, the five rational numbers between 1 and 2 are  $\frac{3}{2}, \frac{5}{4}, \frac{11}{8}, \frac{13}{8}$  and  $\frac{7}{4}$ . are between 1 and 2.

Case 2:

The other option is to find all the five rational numbers in one steps. Since we want five numbers, we write 1 and 2 as rationales' with denominator 5+1 .

That is  $1 = \frac{6}{6}$  and  $2 = \frac{12}{6}$  . Then you can check that:  
 $\frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}$ , and  $\frac{11}{6}$  are all rational numbers between 1 and 2.  
So the five numbers are:  $\frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{6}$ , and  $\frac{11}{6}$ .  
or  $1 < \frac{7}{6} < \frac{4}{3} < \frac{3}{2} < \frac{5}{6} < \frac{11}{6} < 2$

Remark:

In general, there are infinitely many rational numbers between any two given rational numbers.

## 1.4. Real-life Applications of rational Numbers

In our daily life we frequently use some quantities which are not as whole number. For example, taxation..., prices which requires exchanges such as 50 cent, 25 cent of Ethiopian coins, and the likes require rational number representation.

Therefore there is a need of some other number such as rational number. In our daily life we use rational numbers for taxation, share some thing, interest and loans and so on.

### Activity1.5

1. How do you associate rational number with your daily life?  
(what is your answer for this question?)
2. If four friends divide an orange equally among themselves, then what fraction of an orange each friend will earn?
3. A house broker, Dancho receives a commission of  $1\frac{1}{2}\%$  of the selling price at a house .What commission he earns for selling a home for birr 350,000.

### 1.4.1 Applications in sharing among friends

Rational numbers are used to express a fraction of a whole in the sharing among individuals. Like a share of a whole for each member.

*Example 1:*

The cost /price/of lkg meat was 300 birr. Five men decided to eat and contribute money equally. What is the amount of each person will contribute?

*Solutions:*

Since each person will share the meat equally, each will get one-fifth of the meat. That is each person has to pay one-fifth of the cost

The amount of each person =  $\frac{1}{5} \times 300\text{birr} = 60\text{birr}$

*1.4.2. Application in calculating interest and loans*

**Simple interest**

In financial transactions, interest is an amount paid by a borrower to a lender or investor for the use of money over a period of time.

Interest that is paid as a percent of the amount borrowed or invested is called simple interest.

The formula for simple interest is the following:

| Simple Interest |                                                                                                                                  |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------|
| $I = prt$       | where I is interest earned (or owed)<br>p is principal invested (or borrowed)<br>r is annual interest rate<br>t is time in years |

*Example 2:*

Suppose 500 birr is invested for 2 years at 6 percent simple interest per year. What the simple interest earned at the end of 2 years?

*Solution:*

Given:  $p=500$  birr,  $t = 2$  years ,  $r = 6\% = 0.06$

$$I = prt, = 500 \times 2 \times 0.06 = 60$$

Hence, the simple interest earned at the end of 2 years is 60 birr.

*Example 3:*

Argiso borrowed 50,000 for 3 years at the rate of 3.5% per annual. Find the interest accumulated at the end of 3 years.

*Solution:*

$p = 50,000$  birr,  $r = 3.5\%$ , which is 0.035,  $t = 3$

Therefore since  $I = prt$

$$I = 50,000 \times 0.035 \times 3 = 5,250 \text{ birr}$$

*Example 4:*

Buwa plans to place his money in a deposit that matures in three months. The principal is 10,000 birr and 5% interest earned annually. Calculate how much he earns in three months

*Solution:*

$p = 10,000$  birr,  $r = 5\%$ , which is 0.05

$t = 3$  months , which is  $\frac{3}{12} = 0.25$

Therefore since  $I = prt$

$$I = 10,000 \times 0.05 \times 0.25 = 125$$

Therefore the interest after 3 months is 125 birr

### Simple Discount

If a lender deducts in advance the interest due on a loan, the amount deducted is called the discount. The formula for simple discount is the following

| Simple Discount |                                                                                                                                                                                                                            |
|-----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $D = Sdt$       | Where D is discount (amount deducted in advance by lender)<br><br>S is amount of the loan (amount paid back by the borrower at the end of the loan)<br><br>d is annual discount rate<br><br>t is time of the loan in years |

### Example 5:

The discount on a loan of 500 birr for 2 years at 6 percent simple discount is

### Solution:

$$S = 500 \text{ birr}, \quad d = 6\%, \quad t = 2 \text{ years}$$

$$D = Sdt$$

$$D = 500 \times 0.06 \times 2 = 60 \text{ birr}$$

Hence, the lender deducts 60 birr from the 500 birr loan.

The borrower receives 440 birr and must pay back 500 birr at the end of 2 years.

Hence, the lender deducts 60 birr from the 500 birr loan. The borrower receives 440 birr and must pay back 500 birr the end of 2 years.

### Example 6:

Dukamo borrows 1200 birr for 10 month at a simple interest rate of 15% per year . Determine the discount

### Solution:

The discount  $D$  is the interest on the loan that the bank deducts from the loan amount.

Given:  $S = 1200$ ,  $d = 15\%$  which is 0.15

$$t = 10/12$$

$$D = Sdt$$

$$D = 1200 \times 0.15 \times \frac{10}{12} = 150 \text{ birr}$$

Therefore the bank deducts 150 birr from the maturity value of 1200 and gives Dukamo 1050 birr. Dukamo is obligated to repay the bank 1200 birr.

## Exercise 1.6

1. A 10,000 face value discount note has a term of 4 months. The simple discount rate is 6%. Find the amount of the discount.
2. A person borrowed birr 40,000 at 5% simple interest per year. What is the interest owned in 10 years?
3. A woman deposited 50,000 birr in bank which charged an interest of 2%. Calculate the interest after 3 years.

4. Sara wants to borrow money from her mother and she is offered a 5 year simple interest loan of 7000 birr with 3% amount interest rate. What is Sara's interest

## Key terms

- |                     |                        |
|---------------------|------------------------|
| ◆ Fraction          | ◆ Mixed fraction       |
| ◆ Denominator       | ◆ Terminating decimals |
| ◆ Numerator         | ◆ Repeating decimals   |
| ◆ Proper fraction   | ◆ Rational numbers     |
| ◆ Improper fraction | ◆ Decimal number       |

## Summary

- Rational numbers are closed under the operations of addition, subtraction and multiplication.
- The operations addition and multiplication are
  - Commutative for rational numbers.
  - Associative for rational numbers.
- The rational number 0 is the additive identity for rational numbers.
- The rational number 1 is the multiplicative identity for rational numbers.
- The additive inverse of the rational number  $\frac{a}{b}$  is  $-\frac{a}{b}$  and vice-versa.

6. The reciprocal or multiplicative inverse of the rational number  $\frac{a}{b}$  is  $\frac{c}{d}$  if  $\frac{a}{b} \times \frac{c}{d} = 1$
7. Distributive of rational numbers: For all rational numbers a, b and c,  
 $a(b + c) = ab + ac$  and  $a(b - c) = ab - ac$
8. Rational numbers can be represented on a number line.
9. Between any two given rational numbers there are many rational numbers. The idea of mean helps us to find rational numbers between two rational numbers.
10. Properties of Zero

Let a and b be rational numbers:

- i.  $a + 0 = a$
- ii.  $a \times 0 = 0$
- iii.  $\frac{0}{a} = 0$ ,  $a \neq 0$
- iv.  $\frac{a}{0}$  is undefined
- v. Zero-factor property : if  $ab=0$ , then  $a=0$  or  $b=0$

## 11. Properties and Operations of Fractions

Let a, b, c, and d be rational numbers, variables, or algebraic expressions such that  $a \neq 0$  and  $d \neq 0$

- i. Equivalent Fractions:  $\frac{a}{b} = \frac{c}{d}$  if and only if  $ad = bc$
- ii. Rules of Signs:  $-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$  and  $\frac{-a}{-b} = \frac{a}{b}$
- iii. Generate Equivalent Fractions:  $\frac{a}{b} = \frac{ac}{bc}$

iv. Add or Subtract with Like Denominators:

$$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b} \quad \text{and} \quad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$$

v. Add or Subtract with Unlike Denominators:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \quad \text{and} \quad \frac{a}{b} - \frac{c}{d} = \frac{ad-bc}{bd}$$

vi. Multiply Fractions:  $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

## Review exercise

1. State whether the following statements are true or false:
  - a. The set of all even natural numbers is an infinite set.
  - b. Every non-empty subset of  $\mathbb{N}$  has the smallest element.
  - c. Every integer can be identified with a rational number.
  - d. For each rational number, one can find the next rational number.
  - e. There is no largest rational number.
  - f. Every integer is either even or odd.
  - g. Between any two rational numbers, there is an integer.
2. Find each of the following differences
  - a.  $18\frac{9}{10} - (-\frac{3}{4})$
  - b.  $-75.5 - |-75.5|$
  - c.  $\frac{3}{7} \div \frac{5}{8}$
  - d.  $-8 \div (-\frac{16}{21})$
  - e.  $\frac{-4}{7} \div \frac{3}{4}$

## 3. Simplify

a.  $-\frac{18}{5} \div \frac{9}{35} \times (-\frac{3}{7})$

e.  $\frac{1}{8} \div (\frac{1}{4} - \frac{1}{3})$

b.  $(1\frac{2}{3} \times 4\frac{2}{3}) \div \frac{-3}{7}$

f.  $\frac{15}{37} (\frac{4}{15} + \frac{23}{30} - \frac{3}{2})$

c.  $(5\frac{1}{6} \div 6\frac{3}{4}) \times 7\frac{5}{9}$

g.  $\frac{1}{7} (\frac{14}{3} \div \frac{1}{3})$

d.  $\frac{4}{3} + \frac{1}{2} (\frac{1}{2} \div \frac{3}{2})$

## 4. Solve each of the following absolute value equations

a.  $|2y - 4| = 12$

c.  $3|4x - 1| - 5 = 10$

b.  $|3x + 2| = 7$

d.  $|2x + 15| = -10$

5. if  $x = 6$  and  $y = 10$ , then find  $\frac{|x| - |3y|}{|xy|}$

# Unit 2

## SQUARES, SQUARE ROOTS, CUBES AND CUBE ROOTS

### Learning Outcomes:

At the end of this unit, learners will be able to:

- ▶ Understand the notion square and square roots and cubes and cube roots
- ▶ Determine the square of numbers
- ▶ Determine the square roots of the perfect square numbers
- ▶ Extract the approximate square roots of numbers by using the numerical table and scientific calculator.
- ▶ Determine the cube of numbers
- ▶ Extract the cube roots of perfect cubes.
- ▶ Apply squares, square roots, cubes and cube roots in the real-life situation

### Main content

- 2.1 Squares and Square Roots
- 3.2 Cubes and Cube Roots
- 2.3 Applications on squares, square roots, cubes and cube roots
  - ※ Key terms
  - ※ Summary
  - ※ Review Exercise

## Introduction

What you had learnt in the previous grade about multiplication will be used in this unit to describe special products known as squares and cubes of a given numbers. You will also learn what is meant by square roots and cube roots. Also you will learn how to compute them and their applications. What you will learn in this unit are basic and very important concepts in mathematics. So get ready and be attentive

## 2.1 Squares and Square Roots

Addition and subtraction are operations of the first kind while multiplication and division are operation of the second kind. Operations of the third kind are raising to a power and extracting roots. After a review of your knowledge about rational numbers, you will continue studying the square of a number and square roots in the present unit. Here, you will learn about the square of a number and square roots, their properties and basic operations upon them. Also, you will discuss some related concepts such as approximation, accuracy, and scientific notation.

### *2.1.1 Square of a Rational Number*

#### Group work 2.1

Discusses with your friends about the given numbers in the table below and Justify your answers for Questions 1, 2 and 3.

- 1. What do you observe about the numbers?
- 2. What do you observe about their sums including the missing numbers at the last row?
- 3. Can you express each sum as a product of two similar numbers?

|                 |     |     |     |     |     |     |     |  |
|-----------------|-----|-----|-----|-----|-----|-----|-----|--|
|                 | sum | sum | Sum | Sum | Sum | Sum | Sum |  |
| 1               |     |     |     |     |     |     |     |  |
| 1+3             |     | sum |     |     |     |     |     |  |
| 1+3+5           |     |     | sum |     |     |     |     |  |
| 1+3+5+7         |     |     |     | sum |     |     |     |  |
| 1+3+5+7+9       |     |     |     |     | sum |     |     |  |
| 1+3+5+7+9+11    |     |     |     |     |     | sum |     |  |
| 1+3+5+7+9+11+13 |     |     |     |     |     |     | sum |  |
| ?.....?         |     |     |     |     |     |     | sum |  |

Figure 2.1 the sum of natural numbers

- 4. Put three different numbers in the vertexes of the triangles so that when you add the numbers at the vertexes you always get a perfect square.

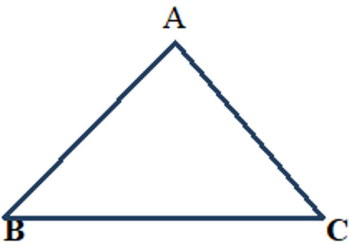


Figure 2.2

Let you have the numbers 1,4,9,16,25,36,49 and other such numbers,

Since, each numbers can be expressed as;

$$1 \times 1 = 1 ,$$

$$3 \times 3 = 9 ,$$

$$5 \times 5 = 25 , \text{etc.}$$

$$2 \times 2 = 4 ,$$

$$4 \times 4 = 16 \text{ and}$$

All such numbers can be expressed as the product of the number with itself.

Such numbers like 1,4,9,16,25,36,49, ... are known as square numbers. These numbers are also called perfect squares.

The above square numbers also can be express as,

$$1^2 = 1 ,$$

$$3^2 = 9 ,$$

$$5^2 = 25$$

$$2^2 = 4 ,$$

$$4^2 = 16 , \text{ and}$$

And you read it as  $1^2$  (one squared),  $2^2$  (two squared),  $3^2$  (three squared),  $4^2$  (four squared) and  $5^2$  (five squared), respectively etc.

Numbers such as 1,4,9,16,25, etc. are special numbers as these are the product of a number by itself.

If we express a number  $n$  in terms of the square of any natural number such as  $n^2$ , then  $n$  is a square number.

Whereas, the number 55 cannot be called a square number because it is the product of numbers 11 and 5. The number is not multiplied by itself.

What are square numbers?

### Definition:

When a rational number is multiplied by itself, its result is called a 'Square number'

If the number to be multiplied by itself is ‘n’, then the product (or the result  $(n \times n)$ ) is usually written as  $n^2$  and read as:

n Squared or the square of n or n to the power of 2.

When the same number is used as a factor for several times, you can use an exponent to show how many times this numbers is taken as a factor or base.

Power = Base<sup>ponent</sup>

$36 = 6^2$

In the square number:  $6^2 = 36$ , the numbers 2, 6,  $6^2$  and 36 are

|                     |                     |
|---------------------|---------------------|
| In $6^2 = 36$       | In $n^2 = p$        |
| 2 is exponent       | 2 is exponent       |
| 6 is base           | n is base           |
| $6^2$ is power form | $n^2$ is Power form |
| 36 is standard form | p is Standard form  |

How to Find Square Numbers

Finding square numbers are, just multiply a number by itself.

Note:

The following properties of squares are important:

- $(ab)^2 = a^2 \times b^2$
- $(a+b)^2 = a^2 + 2ab + b^2$
- $\left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2}$ , where  $b \neq 0$
- $(a-b)^2 = a^2 - 2ab + b^2$

### Example 1

Find the square of the following numbers without actual multiplication

a. 23, use  $(a+b)^2 = a^2 + 2ab + b^2$

b. 56, use  $(a-b)^2 = a^2 - 2ab + b^2$

### Solution

- a. Using  $(a+b)^2 = a^2 + 2ab + b^2$ , its square can be express by:

$$\begin{aligned} 23^2 &= (20+3)^2 \\ &= 20(20+3) + 3(20+3) \\ &= 20^2 + 20 \times 3 + 3 \times 20 + 3^2 \\ &= 400 + 60 + 60 + 9 \\ &= 400 + 2(60) + 9 \\ &= 400 + 120 + 9 \\ &= 529 \end{aligned}$$

- b. Using  $(a-b)^2 = a^2 - 2ab + b^2$ , its square can be express by :

$$\begin{aligned} 56^2 &= \\ (60-4)^2 &= 60(60-4) + 4(60-4) \\ &= 60^2 - 60 \times 4 - 4 \times 60 + 4^2 \\ &= 3600 - 240 - 240 + 16 \\ &= 3600 - 2(240) + 16 \\ &= 3600 - 480 + 16 \\ &= 3616 - 480 \\ &= 3,136 \end{aligned}$$

### Example 2

The first 10 square numbers are found by multiplying 1 by itself, then 2 by itself, as shown below.

$$1^2 = 1 \times 1 = 1,$$

$$2^2 = 2 \times 2 = 4,$$

$$3^2 = 3 \times 3 = 9$$

$$4^2 = 4 \times 4 = 16$$

$$5^2 = 5 \times 5 = 25$$

$$6^2 = 6 \times 6 = 36$$

$$7^2 = 7 \times 7 = 49$$

$$9^2 = 9 \times 9 = 81$$

$$8^2 = 8 \times 8 = 64$$

$$10^2 = 10 \times 10 = 100$$

### Note that:

- i. The squares of natural numbers are also natural numbers.
- ii.  $0 \times 0 = 0$  therefore  $0^2 = 0$
- iii. We give no meaning to the symbol  $0^0$
- iv. If  $a \in \mathbb{Q}$  and  $a \neq 0$ , then  $a^0 = 1$

For any rational number 'a',  $a \times a$  is denoted by  $a^2$  and read as "a squared" or "a to the power of 2" or "the square of a".

A square number is always a positive number. They can never be a negative number. Negative numbers can be multiplied by themselves to give a positive square number:

### Example 3

The number 5 multiplied by 5 is equal to 5 squared or  $5 \times 5 = 5^2$ .

The number -5 multiplied by -5 is equal to -5 squared or  $-5 \times -5 = (-5)^2$ .

$(-5)^2 = -5 \times -5 = 25$ . So, we can say here 25 is a positive square number

### Note:

Square numbers are those numbers that are produced by multiplying a rational number by itself.

If  $n$  is an integer, then  $n^2$  is the square of  $n$ .

### Example 4

The following are an example of square of rational numbers

- a.  $13^2 = 13 \times 13 = 169$
- b.  $16^2 = 16 \times 16 = 256$
- c.  $(20)^2 = 20 \times 20 = 400$
- d.  $(-30)^2 = -30 \times -30 = 900$
- e.  $\left(\frac{8}{9}\right)^2 = \frac{8}{9} \times \frac{8}{9} = \frac{64}{81}$
- f.  $\left(-\frac{11}{12}\right)^2 = -\frac{11}{12} \times -\frac{11}{12} = \frac{121}{144}$
- g.  $\left(\frac{p}{q}\right)^2 = \frac{p}{q} \times \frac{p}{q} = \frac{p^2}{q^2}$  where  $q \neq 0$
- h.  $(-m)^2 = -m \times -m = m^2$

## Perfect square

### Definition:

A perfect square is a number that is generated by multiplying two equal rational numbers by each other.

$$n \times n = n^2, \text{ for some } n \in \mathbb{Q}$$

Here,  $4 \times 4 = 4^2 = 16$ , hence ,

16 is a perfect square

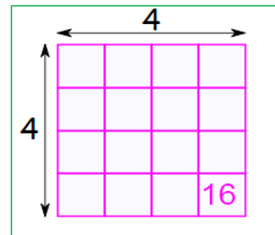


Figure 2.3 square

The number  $\frac{49}{81}$  is a perfect square because it can be expressed in the form of

$$\frac{7}{9} \times \frac{7}{9} = \left(\frac{7}{9}\right)^2 = \frac{49}{81}$$

Example 5

In the table below, if  $n$  is given in the row from  $n=1$  to  $n=7$ , then the value of  $n^2$  is

|       |   |   |   |   |   |   |   |   |
|-------|---|---|---|---|---|---|---|---|
| $n$   | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $n^2$ |   |   |   |   |   |   |   |   |

Solution

|                    |   |   |   |   |    |    |    |    |
|--------------------|---|---|---|---|----|----|----|----|
| $n$                | 0 | 1 | 2 | 3 | 4  | 5  | 6  | 7  |
| $n^2 = n \times n$ | 0 | 1 | 4 | 9 | 16 | 25 | 36 | 49 |

The unit digit of a perfect square can be only 0,1,4,5,6 and 9.  
There is no perfect square that ends with 2, 3, 7 or 8 at unit's place.  
The square of a number having:

- ◆ 1 or 9 at the units place ends in 1.
- ◆ 2 or 8 at the units place ends in 4
- ◆ 3 or 7 at the units place ends in 9.
- ◆ 4 or 6 at the units place ends in 6.
- ◆ 5 at the units place ends in 5

You have so far been able to recognize the squares of natural numbers; you also know that multiplication is closed in the set of rational numbers. Hence it is possible to multiply any rational number by itself.

Example 6

Which of the following is not a perfect square?

- a. 361                      b. 1156                      c. 1128                      d. 1681

Answer: c

### Example 8

A perfect square can never have the following digit at one's place.

- a. 1                      b. 6                      c. 5                      d. 3

Answer: d

### Example 8

Find the value of  $n^2$  if  $n$  is rational numbers for a given table below.

Where  $k \neq 0$

|       |     |                |                  |   |                |                |                 |    |
|-------|-----|----------------|------------------|---|----------------|----------------|-----------------|----|
| n     | -25 | $-\frac{2}{5}$ | $-\frac{11}{13}$ | 0 | $\frac{7}{30}$ | $\frac{3m}{k}$ | $2\frac{3}{10}$ | 70 |
| $n^2$ |     |                |                  |   |                |                |                 |    |

### Solution

Since the value of  $n$  is given, the value of  $n^2$  is given by:

a.  $(-25)^2 = -25 \times -25 = 625$

b.  $\left(-\frac{2}{5}\right)^2 = -\frac{2}{5} \times -\frac{2}{5} = \frac{4}{25}$

c.  $\left(-\frac{11}{13}\right)^2 = -\frac{11}{13} \times -\frac{11}{13} = \frac{121}{169}$

d.  $0^2 = 0 \times 0 = 0$

e.  $\left(\frac{7}{30}\right)^2 = \frac{7}{30} \times \frac{7}{30} = \frac{49}{900}$

f.  $\left(\frac{3m}{k}\right)^2 = \frac{3m}{k} \times \frac{3m}{k} = \frac{9m^2}{k^2}$

g.  $\left(2\frac{3}{10}\right)^2 = 2\frac{3}{10} \times 2\frac{3}{10} = \frac{529}{100}$

h.  $70^2 = 70 \times 70 = 4900$

### Example 9

a. Are  $2\frac{3}{10} \times 2\frac{3}{30}$  and  $4\frac{9}{100}$  equal?

b. Are  $-14^2$  and  $(-14)^2$  equal?

- c. Are  $100^2$  and  $2(100)$  equal?
- d. For what number  $n$  is  $n^2 = 2n$ ?

### Solution

a.  $2\frac{3}{10} \times 2\frac{3}{10} = \frac{23}{10} \times \frac{23}{10} = \frac{529}{100}$ , but  $4\frac{9}{100} = \frac{409}{100}$ ,

Therefore they are not equal.

b.  $-14^2 = -(14 \times 14) = -196$  and  $(-14)^2 = -14 \times -14 = 196$

Therefore, they are not equal.

c.  $100^2 = 100 \times 100 = 10,000$ , but  $2(100) = 2 \times 100 = 200$

Therefore, they are not equal.

d. The possible numbers are  $n=0$  and  $n=2$

if  $n=0$ ,  $0^2 = 2(0) = 0$

if  $n=2$ ,  $2^2 = 2(2) = 4$

### Exercise 2.1

1. Find the square of the following numbers without actual multiplication

a. 79, use  $(a+b)^2 = a^2 + 2ab + b^2$

b. 65, use  $(a-b)^2 = a^2 - 2ab + b^2$

2. How many perfect squares are there in the given table below?

| $\times$ | 1 | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|----------|---|---|---|----|----|----|----|----|----|----|
| 1        | 1 | 2 | 3 | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| 2        | 2 | 4 | 6 | 8  | 10 | 12 | 14 | 16 | 18 | 20 |
| 3        | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 |

|    |    |    |    |    |    |    |    |    |    |     |
|----|----|----|----|----|----|----|----|----|----|-----|
| 4  | 4  | 8  | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40  |
| 5  | 5  | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50  |
| 6  | 6  | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60  |
| 7  | 7  | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70  |
| 8  | 8  | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80  |
| 9  | 9  | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90  |
| 10 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

Fig 2.4

### 2.1.2 Use of table values and scientific calculator to find squares of rational numbers

#### Group work 2.2

Discusses with your friends about the numbers and Justify your answers

1. Is 2352 a perfect square? If not, what is the smallest multiple of 2352 which is a perfect square?
2. Find the exact value using rough calculation and scientific calculator and compare your answer with the value obtained from the table.
  - a. 1.59
  - b. 4.92
  - c. 7.15
  - d. 9.84

### Finding squares of rational numbers from the table

To find the square of a rational number when it is written in the form of a decimal is tedious and time consuming work. To avoid this te-

dious and time consuming work a table of squares is prepared and presented in the “Numerical tables” at the end of this book.

In this table the first column headed by x lists numbers starting from 1.0. The remaining columns are headed respectively by the digits 0 to 9.

Now if you want to determine the square of a number for example 2.26 proceed as follows.

- Step i.** Under the column headed by x, find the row with 2.2.
- Step ii.** Move to the right along the row until you get the column under 6, (or find the column headed by 6).
- Step iii.** Then read the number at the intersection of the row in (i) and the column ( ii), (see the illustration below). Hence  $(2.26)^2 = 5.108$

The function  $y=x^2$ ,  $1.00 \leq x \leq 3.00$

| x   | 0    | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-----|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1   | 1    | 1.02  | 1.04  | 1.061 | 1.082 | 1.102 | 1.124 | 1.145 | 1.166 | 1.188 |
| 1.1 | 1.21 | 1.232 | 1.254 | 1.277 | 1.277 | 1.322 | 1.346 | 1.369 | 1.392 | 1.416 |
| 1.2 | 1.44 | 1.464 | 1.488 | 1.513 | 1.513 | 1.562 | 1.588 | 1.613 | 1.638 | 1.644 |
| ⋮   | ⋮    | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     |
| 2.1 | 4.41 | 4.452 | 4.494 | 4.537 | 4.58  | 4.622 | 4.666 | 4.709 | 4.752 | 4.796 |
| 2.2 | 4.84 | 4.884 | 4.928 | 4.973 | 5.018 | 5.062 | 5.108 | 5.153 | 5.198 | 5.244 |
| ⋮   | ⋮    | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     |

Figure 2.5 sample Table of square roots

### Example 1

Find the exact value using scientific calculator and compare your answer with the value obtained from the table.

a.  $(2.26)^2$                       b.  $(1.98)^2$                       c.  $(275)^2$

### Solution

|      | The exact value<br>(scientific calculator) | Value obtained from the table                                                                     |
|------|--------------------------------------------|---------------------------------------------------------------------------------------------------|
| x    | $x^2$                                      |                                                                                                   |
| 2.26 | $(2.26)^2 = 5.1076$                        | $(2.26)^2 = 5.108$                                                                                |
| 1.98 | $(1.98)^2 = 3.924$                         | $(1.98)^2 = 3.920$                                                                                |
| 475  | $(475)^2 = 225625$                         | $(475)^2 = (4.75 \times 100)^2$<br>$= (4.75)^2 \times (100)^2$<br>$= 22.56 \times 10000 = 225600$ |

This example shows that the result obtained from the “Numerical table” is an approximation and closer to the exact value.

### Example 2

Find the value of the following rational numbers

a.  $(-3.5)^2$                       c.  $\left(-\frac{20}{15}\right)^2$                       e.  $- \left(-\frac{1.1}{11}\right)^2$   
 b.  $\left(\frac{19}{100}\right)^2$                       d.  $\left(\frac{3a}{4b}\right)^2$                       f.  $\left(8\frac{4}{7}\right)^2$

### Solution

The solution is as follows

a.  $(-3.5)^2 = -3.5 \times -3.5 = 12.25$

$$\text{b. } \left(\frac{19}{100}\right)^2 = \frac{19}{100} \times \frac{19}{100} = \frac{361}{10000}$$

$$\text{c. } \left(-\frac{20}{15}\right)^2 = -\frac{20}{15} \times -\frac{20}{15} = \frac{400}{225} = \frac{16}{9}$$

$$\text{d. } \left(\frac{3a}{4b}\right)^2 = \frac{3a}{4b} \times \frac{3a}{4b} = \frac{9a^2}{16b^2}, b \neq 0$$

$$\text{e. } -\left(-\frac{1.1}{11}\right)^2 = -\left(-\frac{1.1}{11} \times -\frac{1.1}{11}\right) = -\frac{1.21}{121} = -0.01$$

$$\text{f. } \left(8\frac{4}{7}\right)^2 = 8\frac{4}{7} \times 8\frac{4}{7} = \frac{60}{7} \times \frac{60}{7} = \frac{3600}{49}$$

## Exercise 2.2

1. Simplify the following and identify a perfect square.
  - a. Let  $x=12$  and  $y=5$ . Then what is the value of  $x^2+y^2$  ?
  - b. What is the value of  $6^2+8^2$ ? Is the result a perfect square?
  - c. Let  $x=\frac{1}{3}$  and  $y=\frac{1}{4}$ . Then what is the value of  $x^2+y^2$  ?
  - d. What is the value of  $\left(\frac{1+3+5+7}{1+(4 \times 6)}\right)$ ? Is the value a perfect square?
2. Find the square of the following rational numbers from the given table on the back page of your text book and check your values using scientific calculator.
 

|               |                 |
|---------------|-----------------|
| a. $(84.7)^2$ | c. $(0.0123)^2$ |
| b. $(333)^2$  | d. $(0.746)^2$  |
3. Find the square of the rational numbers from the numerical table and using scientific calculator.
 

|               |               |              |
|---------------|---------------|--------------|
| a. $(2.26)^2$ | b. $(1.98)^2$ | c. $(275)^2$ |
|---------------|---------------|--------------|

## 2.1.3 Square Roots of a Rational Number

### Group work 2.3

1. Discuss with your friends and justify your answers

- $(-7)^2 = 49$ , is -7 square root of 49?
- $-\left(-\frac{5}{19}\right)^2 = -\frac{25}{361}$ . Is  $-\frac{5}{19}$  square root of  $\frac{25}{361}$ ?
- Is  $-\frac{5}{19}$  the square root of  $\left(-\frac{5}{19}\right)^2$ ?

2. What is the simplest form of

- $\sqrt{22 + \sqrt{6 + \sqrt{9}}}$
- $\sqrt{5 + \sqrt{121}} - \sqrt{52 - \sqrt{1 + \sqrt{64}}}$
- If  $\frac{p}{q} = \sqrt{2}$ , then what is  $\sqrt{\left(\frac{p}{q}\right)^2 + \sqrt{8}\left(\frac{p}{q}\right) + 75}$
- $\sqrt{(\sqrt{2} + \sqrt{3})^2 - 2\sqrt{6} + 116}$

### Square Root of perfect squares

The inverse (opposite) operation of addition is subtraction and the inverse operation of multiplication is division. Similarly, finding the square root is the inverse operation of squaring.

Now, to get back the original number, we have to find the square root of the square number.

#### Example 1

What is the square root of 100 ?

## Solution

Finding the Square root is the inverse operation of squaring.

So, if  $100 = 10 \times 10$ , then  $10^2 = 100$

Inverse operation of squaring: the square root of 100 is 10

So the square root of any number gives us the value that can be squared to get the original number.

### Definition:

For a rational number  $x$ , if  $p = x^2$ , then  $x$  is called the square root of  $p$

The square root of a number  $x$  is the number whose square is  $x$ . Positive square root of a number  $x$  is denoted by  $\sqrt{x}$ .

## Example 2

What is the square root of?

a. 1,296

b. 3969

c. 300

## Solution

a. Finding the Square root is the inverse operation of squaring.

Inverse operation of squaring: the square root of 1296 is 36.

Using prime factorization the square root of 1296 is

$$1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$$

$$1296 = 2^2 \times 2^2 \times 3^2 \times 3^2, \text{ pairing numbers}$$

$$1296 = \underbrace{2^2 \times 3^2} \times \underbrace{2^2 \times 3^2}$$

$$1296 = 36 \times 36 = (36)^2$$

Hence, if  $p = 1296$  and  $x = 36$  it follows that

Using, if  $p = x^2$  and  $x = \sqrt{p}$ , then  $x = 36 = \sqrt{1296}$

Inverse operation of squaring: the square root of 1296 is 36.

b. Finding the Square root is the inverse operation of squaring.

Using prime factorization the square root of 3969

$$3969 = 3 \times 3 \times 3 \times 3 \times 7 \times 7$$

$$= \underbrace{3 \times 3 \times 3 \times 3}_{\text{pairing numbers}} \times \underbrace{7 \times 7}_{\text{pairing numbers}}$$

$$= \underbrace{3 \times 3}_{\text{pairing numbers}} \times \underbrace{3 \times 3}_{\text{pairing numbers}} \times \underbrace{7 \times 7}_{\text{pairing numbers}}$$

$$= 9 \times 9 \times 7 \times 7$$

$$= \underbrace{9 \times 7}_{\text{pairing numbers}} \times \underbrace{9 \times 7}_{\text{pairing numbers}}$$

$$= \underbrace{63 \times 63}_{\text{pairing numbers}}$$

$$= (63)^2$$

Hence, if  $p = 3969$  and  $x = 63$  it follows that

Using, if  $p = x^2$  and  $x = \sqrt{p}$ , then  $x = 63 = \sqrt{3969}$

Inverse operation of squaring: the square root of 3969 is 63.

c. Finding the Square root is the inverse operation of squaring.

$$300 = 2 \times 2 \times 3 \times 5 \times 5 \quad \text{Pairing numbers}$$

$$= \underbrace{2 \times 2}_{\text{pairing numbers}} \times 3 \times \underbrace{5 \times 5}_{\text{pairing numbers}}$$

$$= 3 \times \underbrace{2 \times 5}_{\text{pairing numbers}} \times \underbrace{2 \times 5}_{\text{pairing numbers}}$$

$$= 3 \times \underbrace{10 \times 10}_{\text{pairing numbers}}$$

Here the number 3 is not paired, so 300 is not a perfect square'

Example 3

2025 Plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Solution

Let the numbers of rows be  $x$ , then the area  $x^2 = 2025$

$x = \sqrt{2025}$

Inverse operation of squaring: the square root of 2025 is  $x=45$ .

The number of plants in each row is 45

Properties of Square roots

For all non-negative rational numbers  $a$  and  $b$

- a.  $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- b. For any  $a \geq 0$  and  $b > 0$ ,  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- c. For every rational number  $a$ ,  $\sqrt{a^2} = |a|$

Example 4

a.  $\sqrt{(-3)^2} = |-3| = 3$

b.  $\sqrt{\left(-\frac{3}{2}\right)^2} = \left|-\frac{3}{2}\right| = \frac{3}{2}$

Example 5

What do you observe about the square of two opposite numbers in the table below?

|          |   |    |    |   |     |    |     |    |
|----------|---|----|----|---|-----|----|-----|----|
| x        | 2 | -2 | -5 | 5 | -10 | 10 | -12 | 12 |
| p= $x^2$ |   |    | 25 |   | 100 |    | 144 |    |

### Solution

From the above table we have, the square of two opposite numbers is always a positive and equal. For example;

$$(2)^2 = (-2)^2 = 4$$

$$(10)^2 = (-10)^2 = 100$$

$$(5)^2 = (-5)^2 = 25$$

$$(12)^2 = (-12)^2 = 144$$

What do you observe about the square root of the numbers in the following table ?

| $x = \sqrt{p}$ | $x = \sqrt{4}$ | $x = \sqrt{25}$ | $x = \sqrt{100}$ | $x = \sqrt{144}$ |
|----------------|----------------|-----------------|------------------|------------------|
|                | 2              | 5               | 10               | 12               |

### Solution

From the above table we have, the square root of positive numbers is always a positive

#### For example

$$a. \sqrt{(2)^2} = \sqrt{(-2)^2} = \sqrt{4} = 2$$

$$b. \sqrt{(5)^2} = \sqrt{(-5)^2} = \sqrt{25} = 5$$

$$c. \sqrt{(10)^2} = \sqrt{(-10)^2} = \sqrt{100} = 10$$

$$d. \sqrt{(12)^2} = \sqrt{(-12)^2} = \sqrt{144} = 12$$

### Example 7

Find the square root of  $n$  if  $n$  is given below:

$$a. \frac{1}{100}$$

$$d. -\frac{441}{225}$$

$$b. \left(-\frac{25}{7}\right)^2$$

$$e. \frac{9a^2}{400}$$

$$c. (-0.17)^2$$

$$f. 10\frac{9}{16}$$

### Solution

a.  $n = \frac{1}{100} = \frac{1}{10} \times \frac{1}{10}$ , this implies that,  $n = \left(\frac{1}{10}\right)^2$ , thus  $\sqrt{\frac{1}{100}} = \frac{1}{10}$

b.  $n = \frac{625}{49} = \frac{25}{7} \times \frac{25}{7}$ , this implies that,  $n = \left(\frac{25}{7}\right)^2$ , thus  $\sqrt{\frac{625}{49}} = \frac{25}{7}$

c.  $n=0.0289=0.17 \times 0.17$ , this implies that,  $n=(0.17)^2$ , thus  $\sqrt{0.0289} = 0.17$

d.  $n = -\frac{441}{225}$ , this number cannot express in the form of perfect number. So, it has no square root.

e.  $n = \frac{9a^2}{400} = \frac{3a}{20} \times \frac{3a}{20}$ , this implies that,  $n = \left(\frac{3a}{20}\right)^2$ , thus  $\sqrt{\frac{9a^2}{400}} = \frac{3a}{20}$

f.  $n = 10\frac{9}{16}$ , is a mixed fraction.

First change to proper fraction  $n = \frac{169}{16} = \frac{13}{4} \times \frac{13}{4}$ , this implies that,  $n = \left(\frac{13}{4}\right)^2$ , thus  $\sqrt{\frac{169}{16}} = \frac{13}{4}$

### Example 8

Couple wants to install a square glass window that has an area of 500 square cm. Calculate the length of each side and the length of trim needed to the nearest tenth of cm.

### Solution

Given the area of =  $500\text{m}^2$

Let the length of side of the window is  $l$

Therefore, the area of square window is  $l^2=500\text{cm}^2$

$$l = \sqrt{500\text{m}^2} = (\sqrt{5 \times 100}) \text{ cm} = (\sqrt{5} \times \sqrt{100})\text{cm}$$

$$= (\sqrt{5} \times 10)\text{cm} \approx 2.23 \times 10 \approx 22.3\text{cm}$$

(from square root table  $\sqrt{5} = 2.23$ )

### Example 9

Simplify the following

a.  $\sqrt{8100}$

c.  $-\sqrt{1.69}$

b.  $\sqrt{\frac{400}{625}}$

d.  $-\sqrt{\frac{25p^2}{10000}}$

### Solution

a.  $\sqrt{8100} = \sqrt{90 \times 90} = 90$ , because  $(90)^2 = 8100$

b.  $\sqrt{\frac{400}{625}} = \frac{\sqrt{400}}{\sqrt{625}} = \frac{\sqrt{20 \times 20}}{\sqrt{25 \times 25}} = \frac{20}{25} = \frac{4}{5}$

c.  $-\sqrt{1.69} = -\sqrt{\frac{169}{100}} = -\frac{\sqrt{13 \times 13}}{\sqrt{10 \times 10}} = -\frac{13}{10} = -1.3$

d.  $-\sqrt{\frac{25p^2}{10000}} = -\frac{\sqrt{25p^2}}{\sqrt{10000}} = -\frac{\sqrt{25} \times \sqrt{p^2}}{\sqrt{100 \times 100}} = -\frac{\sqrt{5 \times 5 \times p \times p}}{100} = -\frac{5p}{100}$

### Exercise 2.3

1. Determine whether each of the following statements is true or false.

a.  $\sqrt{\frac{422}{22}} = 2$

d.  $\sqrt{-10000} = -100$

b.  $\sqrt{99} = 9$

e.  $\sqrt{-18 \times -18} = -18$

c.  $-\sqrt{\frac{121}{169}} = -\frac{11}{13}$

f.  $\sqrt{\frac{9}{100}} = \pm \frac{3}{10}$

2. Find the square root of each the following rational numbers

a. 640,000

d. 4.9284

g. 0.0001

b. 0.0144

e. 1.21

h. 961

c. 1,089

f. 7,744

3. A farmer wants to plough his square field of side 150m. How much areas will he have to plough?
4. What will be the number of unit squares on each side of a square graph paper if the total number of unit squares is 256?
5. Find the side of a square whose area is equal to the area of a rectangle with sides 6.4m and 2.5m
6. Find the square root of the following by prime factorization method.
 

|        |          |          |
|--------|----------|----------|
| a. 729 | c. 1,764 | e. 7,744 |
| b. 400 | d. 4,096 | f. 9,60  |
7. For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also find the square root of the square number so obtained.
 

|        |         |        |         |
|--------|---------|--------|---------|
| a. 720 | b. 1008 | c. 768 | d. 1100 |
|--------|---------|--------|---------|

### *2.1.4 Table values and scientific calculator to find square roots of rational number*

#### Group work 2.4

Discuss with your friends and justify your answers.

1. What is the approximate value of the following?
 

|                    |                   |                          |
|--------------------|-------------------|--------------------------|
| a. $\sqrt{0.0004}$ | c. $\sqrt{23.44}$ | e. $\sqrt{5.210}$        |
| b. $\sqrt{10.92}$  | d. $\sqrt{67.89}$ | f. $\sqrt{5} + \sqrt{3}$ |

2. Is  $\sqrt{12^2 + 5^2}$  a perfect square
3. What is the value of  $\sqrt{11.2345 + 13.7655}$  ?

## Approximating Square Roots

How to calculate the square root without a calculator?

Finding square roots by guess & check method. To find a decimal approximation, say  $\sqrt{2}$ , first make an initial guess, then square the guess, and depending how close you got, improve your guess. Since this method involves squaring the guess (multiplying the number times itself), it uses the actual definition of square root, and so can be very helpful in teaching the concept of square root.

### Example 1

What is the decimal approximation square root of  $\sqrt{20}$ ?

### Solution

To find the decimal approximation square root of  $\sqrt{20}$ . You can start out by noting that since  $\sqrt{16} = 4$  and  $\sqrt{25} = 5$ , then  $\sqrt{20}$  must be between 4 and 5.

Then make a guess for  $\sqrt{20}$ ; let's say for example that it is 4.5. Square that and see if the result is over or under 20, and improve your guess based on that. Repeat this process until you have the desired accuracy (amount of decimals). It's simple and can be a nice experiment for students!

### Example 2

Find  $\sqrt{6}$  to 4 decimal places

*Solution*

Since  $2^2 = 4$  and  $3^2 = 9$ , we know that  $\sqrt{6}$  is between 2 and 3. Let's guess (or estimate) that it is 2.5. Squaring that we get  $(2.5)^2 = 6.25$ . That's too high, so we reduce our estimate a little. Let's try 2.4 next. To find the square root of 6 to four decimal places we need to repeat this process until we have five decimals, and then we will round the result

| Estimate | Square of estimate | High/low                                                             |
|----------|--------------------|----------------------------------------------------------------------|
| 2.4      | 5.76               | Too low                                                              |
| 2.45     | 6.0025             | Too high but real close                                              |
| 2.449    | 5.997601           | Too low                                                              |
| 2.4495   | 6.00005025         | Too high so the square root of 6 must be between 2.449 and 2.4495.   |
| 2.4493   | 5.99907049         | Too low                                                              |
| 2.4494   | 5.99956036         | Too low, so the square root of 6 must be between 2.4494 and 2.4495   |
| 2.44945  | 5.9998053025       | Too low, so the square root of 6 must be between 2.44945 and 2.4495. |

This is enough iteration since we know now that  $\sqrt{6}$  would be rounded to 2.4495 (and not to 2.4494).

*Example 3*

What is the approximate value of  $\sqrt{10}$ ?

- a. to 1 decimal places
- b. to 2 decimal places

## Solution

a. to 1 decimal places

It is easy to work out the square root of a perfect square, but it is really hard to work out other square roots.  $\sqrt{10}$  ?

To guess the approximating value of  $\sqrt{10}$  , so we guess the answer is between 3 and 4. i.e. If  $3 < \sqrt{10} < 4$

Let us start the approximate value of  $\sqrt{10}$

Let's try by taking (guessing) 3.5,  $3.5 \times 3.5 = 12.25$

Let's try by taking (guessing) 3.2,  $3.2 \times 3.2 = 10.24$

Let's try by taking (guessing) 3.1,  $3.1 \times 3.1 = 9.61$

Getting closer to 10, but it will take a long time to get a good answer

At this point, I get out my calculator and it says

3.16227766016837933199889

b. to 2 decimal places

Let's try by taking (guessing) 3.14:  $3.14 \times 3.14 = 9.89$

Let's try by taking (guessing) 3.15:  $3.15 \times 3.15 = 9.92$

Let's try by taking (guessing) 3.16:  $3.16 \times 3.16 = 9.98$

Let's try by taking (guessing) 3.17:  $3.17 \times 3.17 = 10.04$

Getting closer to 10, but it will take a long time to get a good answer so, after 3 times around the answer is 3.16, which is pretty good, because:

$$3.16 \times 3.16 = 9.98$$

Most people in today’s world feel that since calculators can find square roots that students don’t need to learn how to find square roots using any pencil-and-paper method. However, learning at least the “guess and check” method for finding the square root will actually help the students understand and remember the square root concept itself.

Use table of square root

The square root of natural number

To find the square root of the natural number directly you can obtain the value from the given table below.

| x  | $\sqrt{x}$ | x  | $\sqrt{x}$ | x  | $\sqrt{x}$ | x  | $\sqrt{x}$ | x  | $\sqrt{x}$ |
|----|------------|----|------------|----|------------|----|------------|----|------------|
| 1  | 1.000      | 21 | 4.583      | 41 | 6.403      | 61 | 7.810      | 81 | 9.000      |
| 2  | 1.414      | 22 | 4.690      | 42 | 6.481      | 62 | 7.874      | 82 | 9.055      |
| 3  | 1.732      | 23 | 4.796      | 43 | 6.557      | 63 | 7.937      | 83 | 9.110      |
| 4  | 2.000      | 24 | 4.899      | 44 | 6.633      | 64 | 8.000      | 84 | 9.165      |
| 5  | 2.236      | 25 | 5.000      | 45 | 6.708      | 65 | 8.062      | 85 | 9.220      |
| 6  | 2.449      | 26 | 5.099      | 46 | 6.782      | 66 | 8.124      | 86 | 9.274      |
| 7  | 2.646      | 27 | 5.196      | 47 | 6.856      | 67 | 8.185      | 87 | 9.327      |
| 8  | 2.828      | 28 | 5.292      | 48 | 6.928      | 68 | 8.246      | 88 | 9.381      |
| 9  | 3.000      | 29 | 5.385      | 49 | 7.000      | 69 | 8.307      | 89 | 9.434      |
| 10 | 3.162      | 30 | 5.477      | 50 | 7.071      | 70 | 8.367      | 90 | 9.487      |
| 11 | 3.317      | 31 | 5.568      | 51 | 7.141      | 71 | 8.426      | 91 | 9.539      |
| 12 | 3.464      | 32 | 5.657      | 52 | 7.211      | 72 | 8.485      | 92 | 9.592      |
| 13 | 3.606      | 33 | 5.745      | 53 | 7.280      | 73 | 8.544      | 93 | 9.644      |
| 14 | 3.742      | 34 | 5.831      | 54 | 7.348      | 74 | 8.602      | 94 | 9.695      |
| 15 | 3.873      | 35 | 5.916      | 55 | 7.416      | 75 | 8.660      | 95 | 9.747      |
| 16 | 4.000      | 36 | 6.000      | 56 | 7.483      | 76 | 8.718      | 96 | 9.798      |

|    |       |    |       |    |       |    |       |     |       |
|----|-------|----|-------|----|-------|----|-------|-----|-------|
| 17 | 4.123 | 37 | 6.083 | 57 | 7.550 | 77 | 8.775 | 97  | 9.849 |
| 18 | 4.243 | 38 | 6.164 | 58 | 7.616 | 78 | 8.832 | 98  | 9.899 |
| 19 | 4.359 | 39 | 6.245 | 59 | 7.681 | 79 | 8.888 | 99  | 9.950 |
| 20 | 4.472 | 40 | 6.325 | 60 | 7.746 | 80 | 8.944 | 100 | 10.00 |

Figure 2.6 samples Table of square roots

### Example 4

From the above given table, find the value of the following

a.  $\sqrt{78}$

c.  $\sqrt{86}$

e.  $\sqrt{94}$

b.  $\sqrt{55}$

d.  $\sqrt{37}$

### Solution

a.  $\sqrt{78} = 8.832$

c.  $\sqrt{86} = 9.274$

b.  $\sqrt{55} = 7.416$

d.  $\sqrt{37} = 6.083$

e.  $\sqrt{94} = 9.695$

## Square root of rational numbers

To find the square root of a rational number when it is written in the form of a decimal is tedious and time consuming work. To avoid this tedious and time consuming work a table of square root is prepared and presented in the “Numerical tables” at the end of this book, in each of the following. In this table the first column headed by x lists numbers starting from 1.0. The remaining columns are headed respectively by the digits 0 to 9. The same table which you can use to determine squares of numbers can be used to find the approximate square roots, of numbers.

Example 5

Find  $\sqrt{2.993}$  from the numerical table

Solution

Step i. Find the number 2.993 in the body of the table for the function  $y = x^2$

Step ii. On the row containing this number move to the left and read 1.7 Under x. These are the first two digits of the square root of 2.993

Step iii. To get the third digit start from 2.993 move vertically upward and read 3.

Therefore  $\sqrt{2.993} \approx 1.73$ , that means.  $(1.73)^2 = 2.9929 \approx 2.993$

The function  $y = x^2$ ,  $1.00 \leq 3.00$

| x   | 0    | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-----|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1   | 1    | 1.02  | 1.04  | 1.061 | 1.082 | 1.102 | 1.124 | 1.145 | 1.166 | 1.188 |
| 1.1 | 1.21 | 1.232 | 1.254 | 1.277 | 1.277 | 1.322 | 1.346 | 1.369 | 1.392 | 1.416 |
| 1.2 | 1.44 | 1.464 | 1.488 | 1.513 | 1.513 | 1.562 | 1.588 | 1.613 | 1.638 | 1.644 |
| 1.3 | 1.69 | 1.716 | 1.742 | 1.769 | 1.769 | 1.822 | 1850  | 1.877 | 1.904 | 1.932 |
| 1.4 | 1.96 | 1.988 | 2.016 | 2.045 | 2.045 | 2.102 | 2.132 | 2.161 | 2.19  | 2.22  |
| 1.5 | 2.25 | 2.28  | 2.316 | 2.341 | 2.341 | 2.102 | 2.434 | 2.465 | 2.496 | 2.528 |
| 1.6 | 2.56 | 2.592 | 2.31  | 2.657 | 2.657 | 2.722 | 2.756 | 2.789 | 2.822 | 2.856 |
| 1.7 | 2.89 | 2.924 | 2.624 | 2.993 | 2.993 | 3.062 | 3.098 | 3.133 | 3.168 | 3.204 |
| ⋮   | ⋮    | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     |

Figure 2.3 Sample Table of square roots

**Note:**

If the radicand is not found in the body of the table, you can consider the number which is closer to it.

*Example 6*

From the given table above, find the value of the following

a.  $\sqrt{5.108}$

c.  $\sqrt{3.92}$

b.  $\sqrt{8.066}$

d.  $\sqrt{1.644}$

*Solution*

a.  $\sqrt{5.108} = 2.26$

c.  $\sqrt{3.92} = 1.98$

b.  $\sqrt{8.066} = 2.84$

d.  $\sqrt{1.644} = 1.29$

*Example 7*

Approximate the following value based on the given table

a.  $\sqrt{876}$

c.  $\sqrt{0.5417}$

b.  $\sqrt{9742}$

d.  $\sqrt{0.7755}$

*Solution*

a.  $\sqrt{876} = \sqrt{8.76 \times 100}$

$$= \sqrt{8.76} \times \sqrt{100}, (\text{from numerical table } \sqrt{8.76} = 2.96)$$

$$= 2.96 \times 10 = 29.6$$

b.  $\sqrt{9742} = \sqrt{97.42 \times 100}$

$$= \sqrt{97.42} \times \sqrt{100}, (\text{from numerical table } \sqrt{97.42} = 9.87)$$

$$= 9.87 \times 10 = 98.7$$

c.  $\sqrt{0.5417} = \sqrt{54.17 \times \frac{1}{100}} = \sqrt{54.17} \times \sqrt{\frac{1}{100}}$

$$= \sqrt{54.17} \times \sqrt{\frac{1}{100}} = \sqrt{54.17} \times \frac{1}{10},$$

$$= 7.36 \times \frac{1}{10} = 0.736 \quad (\text{from numerical table } \sqrt{54.17} = 7.36)$$

$$\text{d. } \sqrt{0.7755} = \sqrt{77.55 \times \frac{1}{100}} = \sqrt{77.55} \times \sqrt{\frac{1}{100}}$$

$$= \sqrt{77.55} \times \sqrt{\frac{1}{100}}$$

$$= \sqrt{77.55} \times \frac{1}{10}$$

$\sqrt{77.55}$  is between  $\sqrt{77.44}$  and  $\sqrt{77.62}$ , therefore,

$$\sqrt{77.55} = \sqrt{77.44} + \frac{1}{2}(\sqrt{77.62} - \sqrt{77.44})$$

$$\left(8.80 + \frac{1}{2}(8.81 - 8.80)\right) \times \frac{1}{10}$$

$$= \frac{1}{10}(8.80 + 0.005) \approx 0.8805$$

### Exercise 2.4

- Find the square root of the following rational numbers based on the given table and check your values using scientific calculator.

a. 19.18

c. 4998

e. 0.7673

b. 950

d. 4747

- Use the table of squares to find approximate value of the following rational numbers.

a.  $\sqrt{7.344}$

d.  $\sqrt{15.44}$

g.  $\sqrt{3.960}$

b.  $\sqrt{37.33}$

e.  $\sqrt{47.47}$

h.  $\sqrt{2927}$

c.  $\sqrt{1.988}$

f.  $\sqrt{0.7779}$

i.  $\sqrt{60.06}$

## 2.2 Cubes and Cube Roots of a Rational Number

### 2.2.1 Cube of a Rational Number

If the number to be cubed is 'c', then the product  $c \times c \times c$  which is usually written as  $c^3$  read as 'c' cubed. For example 2 cubed gives 8 because  $2 \times 2 \times 2 = 8$ . The product  $2 \times 2 \times 2$  can be written as  $2^3$  and which read as 2 cubed. This shows that the cube of 2 is obtained by multiplying 2 three times by itself.

### Activity 2.5

- Which of the following numbers are cubic?
 

|         |        |          |
|---------|--------|----------|
| a. 64   | c. 125 | e. 3150  |
| b. 1000 | d. 216 | f. -1000 |
- Which of the following numbers are perfect cube?
 

|        |         |         |
|--------|---------|---------|
| a. 128 | c. 432  | e. 2187 |
| b. 162 | d. 1024 |         |
- Find  $c^3$  in each of the following.
 

|            |              |                      |
|------------|--------------|----------------------|
| a. $c = 3$ | c. $c = 0.5$ | e. $c = \frac{3}{2}$ |
| b. $c = 7$ | d. $c = -5$  | f. $c = 0.2$         |

**Definition:**

The process of multiplying a rational number three times by itself is called cubing the number.

The product  $c \times c \times c = c^3$  read as 'c' cubed is written as

$b = c^3$

Where b is standard numeral form

3 represents exponent and

c is the base, and

$c^3$  is power form

*Example 1*

- a.  $3 \times 3 \times 3 = 3^3 = 27$

b.  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

d.  $-\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} = \left(-\frac{3}{2}\right)^3 = -\frac{27}{8}$
- c.  $-6 \times -6 \times -6 = -216$

*Example 2*

Identify the base, exponent, power form and standard numeral form:

- a.  $7^3$

b.  $13^3$

c.  $29^3$

*Solution*

| Cubic number | Base | Exponent | power form | standard form |
|--------------|------|----------|------------|---------------|
| a. $7^3$     | 7    | 3        | $7^3$      | 343           |
| b. $13^3$    | 13   | 3        | $13^3$     | 2197          |
| c. $29^3$    | 29   | 3        | $29^3$     | 24389         |

### Example 3

Rewrite the following numbers in cubic form.

a. 3375

c. 12167

e.  $\frac{4913}{12167}$

b. 4913

d. 35937

f.  $\frac{3375}{35937}$

### Solution

a.  $3375 = 15 \times 15 \times 15 = 15^3$ , therefore the number is 15

b.  $4913 = 17 \times 17 \times 17 = 17^3$ , therefore the number is 17

c.  $12167 = 23 \times 23 \times 23 = 23^3$ , therefore the number is 23

d.  $35937 = 33 \times 33 \times 33 = 33^3$ , therefore the number is 33

e.  $\frac{4913}{12167} = \frac{17}{23} \times \frac{17}{23} \times \frac{17}{23} = \left(\frac{17}{23}\right)^3$ , therefore the number is  $17/23$

f.  $\frac{3375}{35937} = \frac{15}{33} \times \frac{15}{33} \times \frac{15}{33} = \left(\frac{15}{33}\right)^3$  therefore the number is  $15/33$

### Definition:

A rational number  $x$  is called a perfect cube if and only if  $x = n^3$  for some  $n \in \mathbb{Q}$ .

### Example 4

a.  $1=1^3$

c.  $27=3^3$

e.  $125=5^3$

b.  $8=2^3$

d.  $64=4^3$

f.  $343=7^3$

Thus 1, 8, 27, 64, 125 and 343 are perfect cubes

### Example 5

Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

a. 81

c. 135

b. 128

d. 192

*Solution:*

a.  $81 = 3 \times 3 \times 3 \times 3$

Here, one 3 is left which is not in a triplet. If we divide 81 by 3, then it becomes a perfect cube. Thus,  $81 \div 3 = 27 = 3 \times 3 \times 3$  is a perfect cube. Hence, the smallest number by which 81 should be divided to make it a perfect cube is 3.

b.  $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Here, one 2 is left which is not in a triplet. If we divide 128 by 2, then it becomes a perfect cube. Thus,  $128 \div 2 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$  is a perfect cube. Hence, the smallest number by which 128 should be divided to make it a perfect cube is 2.

c.  $135 = 3 \times 3 \times 3 \times 5$

Here, one 5 is left which is not in a triplet. If we divide 135 by 5, then it becomes a perfect cube. Thus,  $135 \div 5 = 27 = 3 \times 3 \times 3$  is a perfect cube.

Hence, the smallest number by which 135 should be divided to make it a perfect cube is 5.

d.  $192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Here, one 3 is left which is not in a triplet. If we divide 192 by 3, then it becomes a perfect cube. Thus,  $192 \div 3 = 64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$  is a perfect cube. Hence, the smallest number by which 192 should be divided to make it a perfect cube is 3.

### Example 6

Complete the table given below

|       |                 |    |    |                |   |               |   |    |                |
|-------|-----------------|----|----|----------------|---|---------------|---|----|----------------|
| x     | $-\frac{40}{3}$ | -9 | -3 | $-\frac{3}{5}$ | 0 | $\frac{3}{5}$ | 5 | 11 | $\frac{40}{3}$ |
| $x^3$ |                 |    |    |                |   |               |   |    |                |

### Solution

$$\text{When } x = -\frac{40}{3}, x^3 = \left(-\frac{40}{3}\right)^3 = -\frac{64000}{27}$$

$$\text{When } x = -9, x^3 = (-9)^3 = -729$$

$$\text{When } x = -3, x^3 = (-3)^3 = -27$$

$$\text{When } x = -\frac{3}{5}, x^3 = \left(-\frac{3}{5}\right)^3 = -\frac{27}{125}$$

$$\text{When } x = 0, x^3 = (0)^3 = 0$$

$$\text{When } x = \frac{3}{5}, x^3 = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

$$\text{When } x = 5, x^3 = (5)^3 = 125$$

$$\text{When } x = 11, x^3 = (11)^3 = 1331 \quad \text{When } x = \frac{40}{3}, x^3 = \left(\frac{40}{3}\right)^3 = \frac{64000}{27}$$

Therefore, we have

|       |                     |      |     |                   |   |                  |     |      |                    |
|-------|---------------------|------|-----|-------------------|---|------------------|-----|------|--------------------|
| x     | $-\frac{40}{3}$     | -9   | -3  | $-\frac{3}{5}$    | 0 | $\frac{3}{5}$    | 5   | 11   | $\frac{40}{3}$     |
| $x^3$ | $-\frac{64000}{27}$ | -729 | -27 | $-\frac{27}{125}$ | 0 | $\frac{27}{125}$ | 125 | 1331 | $\frac{64000}{27}$ |

### Existence theorem

Theorem: For each rational number  $x$ , there is a rational number  $y$  such that  $y = x^3$

Approximation of calculation can be used for checking the results in cubing rational numbers.

### Example 7

Find the approximate values of  $x^3$  in each of the following.

a.  $x = 3.2$

b.  $x = 11.34$

c.  $x = 0.076$

- $3.2 \approx 3$  thus  $(3.2)^3 \approx 3^3 = 27$
- $11.34 \approx 11$  thus  $(11.34)^3 \approx 11^3 = 1331$
- $0.076 \approx 0.08$  thus  $(0.076)^3 \approx (0.08)^3 = 0.000512$

### Exercise 2.5

- a. 512000                      c.  $\frac{512}{729}$   
b. 16000                        d. -6859

2. Determine whether each of the following statement is true or false.

- $5^3 = 25 \times 5$
- $6^3 = 216$
- $(-7)^3 = 343$
- $\left(\frac{5}{7}\right)^3 = \left(-\frac{5}{7}\right)^3$
- $\left(\frac{3}{13}\right)^3 = \left(\frac{13}{3}\right)^3$
- $\left(\frac{8}{9}\right)^3 = -\left(\frac{8}{9}\right)^3$
- $-\left(\frac{2}{11}\right)^3 = \left(-\frac{2}{11}\right)^3$

3. Find  $x^3$  in each of the following.

- a.  $x = -8$                       d.  $x = 0.4$                       g.  $x = 4\frac{3}{5}$   
b.  $x = -4$                       e.  $x = -2\frac{4}{7}$                       h.  $x = 0.7$   
c.  $x = -\frac{4}{5}$                       f.  $x = -0.2$

4. Find the approximate values of  $x^3$  in two decimal places for each of the following.

- a.  $x = -3.39$                       c.  $x = 5.29$                       e.  $x = 7.88$   
b.  $x = 4.97$                       d.  $x = -1.015$                       f.  $x = 0.027$

5. Re write each of the following rational numbers as  $x^3$ .
- |                      |           |                       |
|----------------------|-----------|-----------------------|
| a. -64               | c. 1000   | e. $-\frac{125}{512}$ |
| b. $\frac{343}{216}$ | d. -0.008 | f. 125                |
6. Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.
- |        |        |         |
|--------|--------|---------|
| a. 192 | c. 500 | e. 1536 |
| b. 405 | d. 704 | f. 4374 |

### Properties of Cube of Number:

- i. Cubes of the numbers ending with the digits 0,1,4,5,6 and 9 end with digits 0,1,4,5,6 and 9 respectively. But cubes of the numbers ending with digits 2, 3, 7, and 8 do not end with digits 2, 3, 7, and 8 respectively.

### Examples 8

- |                                                              |
|--------------------------------------------------------------|
| a. $10^3=1000$ ; $20^3=8000$ ; $30^3=27000$ ; $40^3=64000$   |
| b. $11^3=1331$ ; $21^3=9261$ ; $31^3=29791$ ; $41^3=68921$   |
| c. $12^3=1728$ (the unit digit does not end with 2)          |
| d. $13^3=2197$ (the unit digit does not end with 3)          |
| e. $14^3=2744$ ; $24^3=13824$ ; $34^3=39304$ ; $44^3=85184$  |
| f. $15^3=3375$ ; $25^3=15625$ ; $35^3=42875$ ; $45^3=91125$  |
| g. $16^3=4096$ ; $26^3=17576$ ; $36^3=46656$ ; $46^3=97336$  |
| h. $17^3=4913$ (the unit digit does not end with 7)          |
| i. $18^3=5832$ (the unit digit does not end with 8)          |
| j. $19^3=6859$ ; $29^3=24389$ ; $39^3=59319$ ; $49^3=117649$ |

ii. The cube of a negative number is always negative.

### Example 1

a.  $(-12)^3 = -12 \times -12 \times -12 = -1728$

b.  $(-13)^3 = -13 \times -13 \times -13 = -2197$

c.  $(-17)^3 = -17 \times -17 \times -17 = -4913$

d.  $\left(-\frac{23}{11}\right)^3 = -\frac{23}{11} \times -\frac{23}{11} \times -\frac{23}{11} = -\frac{12167}{1331}$

e.  $\left(-\frac{32}{15}\right)^3 = -\frac{32}{15} \times -\frac{32}{15} \times -\frac{32}{15} = -\frac{32768}{3375}$

f.  $\left(-\frac{52}{33}\right)^3 = -\frac{52}{33} \times -\frac{52}{33} \times -\frac{52}{33} = -\frac{140608}{35937}$

iii. The cube of even number is even.

### Example 2

a.  $4^3 = 4 \times 4 \times 4 = 64$  is even

b.  $6^3 = 6 \times 6 \times 6 = 216$  is even

c.  $12^3 = 12 \times 12 \times 12 = 1728$  is even

d.  $36^3 = 36 \times 36 \times 36 = 46656$  is even

iv. The cube of odd number is odd.

### Example 3

a.  $7^3 = 7 \times 7 \times 7 = 343$  is an odd

b.  $9^3 = 9 \times 9 \times 9 = 729$  is an odd

c.  $27^3 = 27 \times 27 \times 27 = 19683$  is an odd

d.  $43^3 = 43 \times 43 \times 43 = 79507$  is an odd

- v. The sum of the cubes of first  $n$  natural numbers is equal to the square of their sum.

Let us consider the pattern below

$$1^3 + 2^3 = 3^2 = 9$$

$$1^3 + 2^3 + 3^3 = 6^2 = 36$$

$$1^3 + 2^3 + 3^3 + 4^3 = 10^2 = 100$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 15^2 = 225$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 = 21^2 = 441$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 = 28^2 = 784$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 = 36^2 = 1296$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 = 45^2 = 2025$$

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3 = 55^2 = 3025$$

- Vi. The sum of  $n$  consecutive odd numbers gives  $n^3$

Let us consider the following pattern below

$$1 = 1^3 = 1$$

$$3 + 5 = 2^3 = 8$$

$$7 + 9 + 11 = 3^3 = 27$$

$$13 + 15 + 17 + 19 = 4^3 = 64$$

$$21 + 23 + 25 + 27 + 29 = 5^3 = 125$$

$$31 + 33 + 35 + 37 + 39 + 41 = 6^3 = 216$$

$$43 + 45 + 47 + 49 + 51 + 53 + 55 = 7^3 = 343$$

$$57 + 59 + 61 + 63 + 65 + 67 + 69 + 71 = 8^3 = 512$$

$$73 + 75 + 77 + 79 + 81 + 83 + 85 + 87 + 89 = 9^3 = 729$$

The above pattern shows that the sum of  $n$  consecutive odd numbers is  $n^3$ .

*Example 4*

How many consecutive odd numbers are needed to obtain the following sum:

- |       |        |        |
|-------|--------|--------|
| a. 8  | c. 64  | e. 216 |
| b. 27 | d. 125 |        |

*Solution*

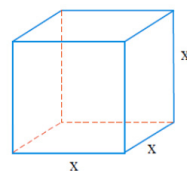
- Since  $8 = 2^3$ , there are two consecutive odd numbers whose sum is 8.
- Since  $27 = 3^3$ , there are three consecutive odd number whose sum is 27
- Since  $64 = 4^3$ , there are four consecutive odd number whose sum is 64
- Since  $125 = 2^3$ , there are five consecutive odd number whose sum is 125
- Since  $216 = 6^3$ , there are six consecutive odd number whose sum is 216

## Use of table values and scientific calculator to find cube of Rational number

### Activity 2.6

1. What is the side length of a cube whose volume is:

- |           |          |
|-----------|----------|
| a. 254.80 | c. 588.5 |
| b. 267.10 |          |



2. Using numerical table determine the cubes of the following
  - a. 2.26
  - b. 5.99
  - c. 8.86
  - d. 9.58
  - e. 9.99
  - f. 5.12
3. Using scientific calculator find the cube of the following in four decimal places.
  - a. 1.98
  - b. 8.55
  - c. 6.24
  - d. -2.273
  - e. -0.079
  - f.  $3\frac{2}{5}$

To find the cubes of a rational number when it is written in the form of a decimal is tedious and time consuming work. To avoid this tedious and time consuming work, a table of cubes is prepared and presented in the “Numerical Tables” at the end of this textbook. In this table the first column headed by ‘x’ lists numbers starting from 1.0. The remaining columns are headed respectively by the digit 0 to 9.0. Now if we want to determine the cube of a number, for example 1.95 proceed as follows.

- Step i. Find the row which starts with 1.9 (or under the column headed by x).
- Step ii. Move to the right until you get the number under column 5.
- Step iii. Then read the number at the intersection of the row in step (i) and the column step (ii)

Therefore, we find that  $(1.95)^3 = 7.415$ . See the illustration below.

$$y = x^3 \quad 1.00 \leq x \leq 9.99$$

| x   | 0     | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1.0 | 1.00  | 1.030 | 1.061 | 1.093 | 1.12  | 1.158 | 1.191 | 1.225 | 1.260 | 1.295 |
| 1.1 | 1.33  | 1.368 | 1.405 | 1.443 | 1.48  | 1.521 | 1.561 | 1.602 | 1.643 | 1.685 |
| ⋮   | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     | ⋮     |
| 1.8 | 5.832 | 8.930 | 6.029 | 6.128 | 6.230 | 6.332 | 6.435 | 6.539 | 6.645 | 6.751 |
| 1.9 | 6.859 | 6.968 | 7.078 | 7.189 | 7.301 | 7.415 | 7.530 | 7.645 | 7.762 | 7.881 |
| 2.0 | 8.00  | 8.121 | 8.242 | 8.365 | 8.49  | 8.615 | 8.742 | 9.870 | 8.999 | 9.129 |

Figure 2.3 sample Tables of cubes

Note that the steps (i) to (iii) are often shortened saying “1.9 under 5” Mostly the values obtained from the table of cubes are only an approximate value which of course serves almost for all practical purposes.

### Example 5

Using scientific calculator find the following

g.  $(1.1)^3$

h.  $(1.05)^3$

i.  $(0.7)^3$

j.  $\left(\frac{3}{2}\right)^3$

### Solution

Using the scientific calculator, the following are the solution

a.  $11^3 = 1.331$

c.  $(0.7)^3 = 0.343$

b.  $(1.05)^3 = 1.157625$

d.  $\left(\frac{3}{2}\right)^3 = \frac{27}{8} = 3.375$

### Example 6

Using numerical tables following cube

a.  $(3.19)^3$

b.  $(7.68)^3$

c.  $(9.99)^3$

## Solution

The cube of the decimal number 1.00 up to 9.99 is indicated in the cube table at back of your text book. Using across rows and column you can indicate the following decimal numbers in the table

a.  $(3.19)^3 = 32.46$

c.  $(9.99)^3 = 997.00$

b.  $(7.68)^3 = 453.00$

## Exercise 2.6

- Determine the cube of the following number from numerical table of cube

a. 3.55

d. 986

g.  $4\frac{1}{5}$

b. 65.8

e. 0.990

h. -6.95

c. 702

f. -9.97

i.  $-8\frac{1}{2}$

- Use scientific calculator and calculate the following numbers

a.  $3^3$

c.  $(-4.6)^3$

e.  $(5\frac{3}{5})^3$

b.  $(3.5)^3$

d.  $(\frac{5}{8})^3$

## 2.2.2 Cube Root of a Rational Number

### Group work 2.5

- Can you define a “cube root” of a number precisely by your own word?
- Find the cube root of  $12,167 \times 42,875$

**Definition:**

The cube root of a given number is one of the three identical factors whose product is the given number.

The cube-root of  $a$  is denoted by  $\sqrt[3]{a}$

where the number 3 is called the index and  $a$  is called the radicand.

**Example 1**

- a.  $\sqrt[3]{0} = 0$ , because  $0 \times 0 \times 0 = 0$
- b.  $\sqrt[3]{27} = 3$ , because  $3 \times 3 \times 3 = 27$
- c.  $\sqrt[3]{343} = 7$ , because  $7 \times 7 \times 7 = 343$
- d.  $\sqrt[3]{\frac{1}{27}} = \frac{1}{3}$ , because  $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$

**Note:**

- ✧ Every number has exactly one cube root.
- ✧ The cube root of a positive number is positive and
- ✧ the cube root of a negative number is negative

**Example 2**

- a.  $\sqrt[3]{8} = 2$ , because 2 is the only cube root of 8
- b.  $\sqrt[3]{27} = 3$ , because 3 is the only cube root of 27
- c.  $\sqrt[3]{-8} = -2$ , because  $-2$  the only cube root of  $-8$
- d.  $\sqrt[3]{-27} = -3$ , because  $-3$  the only cube root of  $-27$

e.  $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$ , because  $\frac{2}{3}$  the only cube root of  $\frac{8}{27}$

f.  $\sqrt[3]{-\frac{8}{27}} = -\frac{2}{3}$ , because  $-\frac{2}{3}$  the only cube root of  $-\frac{8}{27}$

## Cube Roots of Perfect Cubes

### Group work 2.6

1. Which of the following is perfect cube?

27, 42, 60, 64, 90, 111, 125, 133, 150, 216,  $y^3$ ,  $y^8$ ,  $y^9$ ,  $y^{12}$ ,  $y^{27}$

2. Compute the following

a.  $\sqrt[3]{125}$

c.  $\sqrt[3]{\frac{8}{125}}$

b.  $\sqrt[3]{1000}$

d.  $\sqrt[3]{-1000}$

### Note:

For any  $\frac{a}{b}, \frac{c}{d} \in \mathbb{Q}$ , the following properties are important

a.  $(a \times b)^3 = (a)^3 \times (b)^3$

b.  $\left(\frac{a}{b} \times \frac{c}{d}\right)^3 = \left(\frac{a}{b}\right)^3 \times \left(\frac{c}{d}\right)^3$

c.  $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

d.  $\left(\frac{a}{b}\right)^3 = \frac{c}{d}$  if and only if  $\frac{a}{b} = \sqrt[3]{\frac{c}{d}}$

A number is called a perfect cube, if it is the cube of a rational number.

*Example 1*

$$\text{a. } \sqrt[3]{\frac{1}{8}} = \frac{\sqrt[3]{1}}{\sqrt[3]{8}} = \frac{1}{2}$$

$$\text{c. } \sqrt[3]{-\frac{8}{125}} = -\frac{\sqrt[3]{8}}{\sqrt[3]{125}} = -\frac{2}{5}$$

$$\text{b. } \sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$$

**Definition:**

A perfect cube is a number that is a product of three identical factors of a rational number and its cube root is also a rational number.

*Example 2*

Find the cube root of each of the following

$$\text{a. } -343$$

$$\text{c. } -125$$

$$\text{e. } -\frac{8}{125}$$

$$\text{b. } \frac{1}{64}$$

$$\text{d. } 216$$

$$\text{f. } (-8)^3$$

*Solution*

$$\text{a. } \sqrt[3]{-343} = \sqrt[3]{-7 \times -7 \times -7} = -7$$

$$\text{b. } \sqrt[3]{\frac{1}{64}} = \sqrt[3]{\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}} = \frac{1}{4}$$

$$\text{c. } \sqrt[3]{-125} = \sqrt[3]{-5 \times -5 \times -5} = -5$$

$$\text{d. } \sqrt[3]{216} = \sqrt[3]{6 \times 6 \times 6} = 6$$

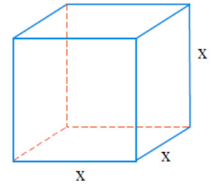
$$\text{e. } \sqrt[3]{-\frac{8}{125}} = \sqrt[3]{-\frac{2}{5} \times -\frac{2}{5} \times -\frac{2}{5}} = -\frac{2}{5}$$

$$\text{f. } \sqrt[3]{(-8)^3} = \sqrt[3]{-8 \times -8 \times -8} = -8$$

### Example 3

The length of the sides of a cube is related to the volume of the cube according to the formula,  $x = \sqrt[3]{v}$

- What is the side length of the cube if the volume is  $156,300 \text{ cm}^3$ .
- What is the side length of the cube if the volume is  $846,000 \text{ cm}^3$



### Solution

|                                                      |                                                      |
|------------------------------------------------------|------------------------------------------------------|
| a. $x = \sqrt[3]{v}$                                 | b. $x = \sqrt[3]{v}$                                 |
| $x = \sqrt[3]{156,300 \text{ cm}^3}$                 | $= \sqrt[3]{846,000 \text{ cm}^3}$                   |
| $= \sqrt[3]{15.63 \times 1000} \text{ cm}$           | $= \sqrt[3]{15.63 \times 1000} \text{ cm}$           |
| $= \sqrt[3]{15.63} \times \sqrt[3]{1000} \text{ cm}$ | $= \sqrt[3]{84.60} \times \sqrt[3]{1000} \text{ cm}$ |
| $= 2.5 \times 10 = 25 \text{ cm}$                    | $= 4.39 \times 10 \text{ cm} = 43.9 \text{ cm}$      |

### Exercise 2.7

- Determine whether each of the following statements is true or false.
  - $\sqrt[3]{-1331} = 11$
  - $\sqrt[3]{-\left(\frac{1}{8}\right)^3} = \frac{1}{8}$
  - $\sqrt[3]{-2197} = -13$
  - $\sqrt[3]{\frac{8}{512}} = \frac{1}{4}$
  - $\sqrt[3]{\frac{2}{3}} \times \sqrt[3]{\frac{4}{9}} = \frac{2}{3}$
  - $-(\sqrt[3]{-4943}) = -17$

2. Find the cube root of each of the following.

- a.  $\frac{0}{27}$

b. 729

c. 0.008
- d. 1728

e. 6859

f.  $\left(\frac{1}{2}\right)^3$
- g.  $-\frac{27}{2197}$

h.  $8x^3$

3. Evaluate each of the following

- a.  $\sqrt[3]{125}$

b.  $\sqrt[3]{\frac{8}{27}}$

c.  $\sqrt[3]{-125}$

j.  $\sqrt[3]{-\frac{343}{729}}$
- d.  $-\sqrt[3]{-\frac{1}{64}}$

e.  $\sqrt[3]{-1000}$

f.  $-\sqrt[3]{\frac{8}{27}}$
- g.  $\sqrt[3]{-\frac{64}{343}}$

h.  $-\sqrt[3]{-729}$

i.  $\sqrt[3]{-\frac{8}{27}}$

4. Simplify each of the following. Assume  $x, y \neq 0$

- a.  $\sqrt[3]{\frac{x^6}{64x^3}}$

b.  $\sqrt[3]{27b^3}$

c.  $\sqrt[3]{125a^3}$
- d.  $\sqrt[3]{\frac{x^4}{216x}}$

e.  $\sqrt[3]{\frac{x^2}{x^5}}$
- f.  $\sqrt[3]{8a^3b^{18}}$

g.  $\sqrt[3]{\frac{y^{10}}{y^4}}$

h.  $\sqrt[3]{64a^{15}b^{12}}$

### Table of Cube Roots

The same table which you can used to determine cubes of numbers can be used to find the approximate cube roots, of numbers.

#### Example 4

Find  $\sqrt[3]{7.53}$  from the numerical table

## Solution

Step i: Find the number 7.53 in the body of the table for the relation  $y=x^3$ .

Step ii: Move to the left on the row containing this number to get 4.0 under  $x$ . These are the first two digits of the required cube root of 7.53.

Step iii: To get the third digits start from 7.53 and move vertically upward and read 1 at the top. Therefore  $\sqrt[3]{7.53} \approx 1.96$

$$y=x^3, x = \sqrt[3]{y}, 1.00 \leq x \leq 9.99$$

| x   | 0     | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1.0 | 1.000 | 1.030 | 1.061 | 1.093 | 1.125 | 1.158 | 1.191 | 1.225 | 1.260 | 1.295 |
| 1.8 | 5.832 | 5.930 | 6.029 | 6.128 | 6.230 | 6.332 | 6.435 | 6.539 | 6.645 | 6.751 |
| 1.9 | 6.859 | 6.968 | 7.078 | 7.189 | 7.301 | 7.415 | 7.530 | 7.645 | 7.762 | 7.881 |
| 2.0 | 8.000 | 8.121 | 8.242 | 8.365 | 8.490 | 8.615 | 8.742 | 8.870 | 8.999 | 9.129 |

Figure 2.4 sample Tables of cube roots

## Exercise 2.8

- Use the numerical table and determine each of the following
  - $\sqrt[3]{32.77}$
  - $\sqrt[3]{991}$
  - $\sqrt[3]{42600}$
  - $\sqrt[3]{302500}$
  - $\sqrt[3]{329500}$
  - $\sqrt[3]{32770}$
  - $\sqrt[3]{991000}$
- Use scientific calculator and calculate each of the following

a.  $\sqrt[3]{3249}$

d.  $\sqrt[3]{7408}$

b.  $\sqrt[3]{21952}$

e.  $\sqrt[3]{185193}$

c.  $\sqrt[3]{27000}$

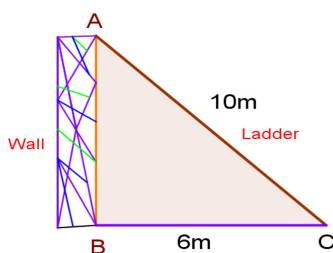
f.  $\sqrt[3]{274625}$

## 2.3. Real life application on squares, square roots, cube and cube roots

Square roots and cube roots are needed for many jobs in our daily life. For instance, we use it in Carpentry, Architecting, and Engineering designing buildings, flooring technology. Square roots used in the Pythagoras's theorem, which is used in many real life situations.

### Example 1

A ladder 10m long rests against a vertical wall. If the foot of the ladder is 6m away from the wall and the ladder just reaches the top of the wall, how high is the wall?



### Solution

Let AC be the ladder

Therefore  $AC = 10\text{m}$

Let BC be the distance between the foot of the ladder and the wall

Therefore  $BC = 6\text{cm}$

$\triangle ABC$  form a right angled triangle, right angled at B .

BY Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$(10\text{cm})^2 = AB^2 + (6\text{cm})^2 \text{ or}$$

$$AB^2 = 100\text{cm}^2 - 36\text{cm}^2$$

$$AB^2 = 64\text{cm}^2$$

$$AB = \sqrt{64\text{cm}^2} = 8\text{cm}$$

### Example 2

Burako has to paint a wall which is about 3m high. He has to put the base of the ladder 2m away from the wall to ensure it will not tip. What will be the length of the ladder required by Burako to complete his work?

### Solution:

Let the height of the wall is b

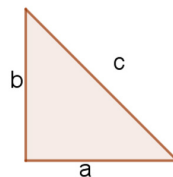
The distance from wall to the foot of the ladder is a

The length of ladder is c

$$c^2 = a^2 + b^2$$

$$c^2 = 3^2 + 2^2$$

$$c = \sqrt{13}$$



1. Couple wants to install a square glass window that has an area of 500 square cm. Calculate the length of each side and the length of trim needed to the nearest tenth of cm.

### *Solution*

Given the area of square =  $500\text{cm}^2$

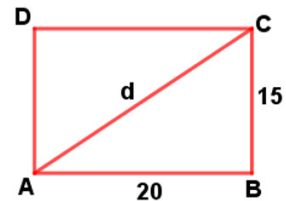
Let the length of side of the window is  $l$

Therefore, the area of square window is  $l^2 = 500\text{cm}^2$

$$\begin{aligned} l &= \sqrt{500\text{m}^2} = (\sqrt{5 \times 100})\text{cm} = (\sqrt{5} \times \sqrt{100})\text{cm} \\ &= (\sqrt{5} \times 10)\text{cm} \\ &\approx 2.23 \times 10 \approx 22.3\text{cm} \end{aligned}$$

(from square root table  $\sqrt{5} = 2.23$ )

2. Find the length of a diagonal of a rectangle with dimensions 20m by 15m.



### *Solution:*

Using Pythagoras theorem

We have length of diagonal of the rectangle

$$\begin{aligned} d^2 &= (l^2 + b^2) \\ d &= \sqrt{20^2 + 15^2}\text{ m} \\ &= \sqrt{400 + 225}\text{ m} \\ &= \sqrt{625}\text{ m} \\ &= 25\text{m} \end{aligned}$$

3. During a mass drill exercise, 6250 students of different schools are arranged in rows such that the number of students in each row is equal to the number of rows. In doing so, the instructor finds out that 9 children are left out. Find the number of children in each row of the square.

### *Solution*

Total number of students = 6250

Number of students forming a square =  $6250 - 9 = 6241$ , thus students form a big square which has number of rows equal to the number of students in each row.

Let the number of students in each row be  $x$ ,

then the number of rows =  $x$

therefore,  $x \times x = x^2 = 6241$

$$x = \sqrt{6241} = 79$$

Hence, there are 79 students in each row of the square formed

4. Butuna made a cuboid of plasticine. Length, breadth and height of the cuboid are 15 cm,

30 cm, 15 cm respectively. Lamite asks how many such cuboids will she need to make a perfect cube? Can you tell?

Butuna said, Volume of cuboid is

$$15 \times 30 \times 15$$

$$= 3 \times 5 \times 2 \times 3 \times 5 \times 3 \times 5$$

$$= 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$$

Since there is only one 2 in the prime factorization. So we need  $2 \times 2$ , i.e., 4 to make it a perfect cube. Therefore, we need 4 such cuboids to make a cube.

### Exercise 2.9

- Find the length of the side of a square whose area is  $144 \text{ m}^2$ . 
- A gardener has 1000 plants. He wants to plant these in such a way that the number of rows and the number of columns remain same. Find the minimum number of plants he needs more for this.
- The area of a rectangular field whose length is twice its breadth is  $2450 \text{ m}^2$ . Find the perimeter of the field.
- A hall has a capacity of 2704 seats. If the number of rows is equal to the number of seats in each row, then find the number of seats in each row.
- Sidama special force General wishes to draw up his 7500 soldiers in the form of a square. After arranging, he found out that some of them are left out. How many soldiers were left out?
- 8649 students were sitting in a lecture room in such a manner that there were as many students in the row as there were rows in the lecture room. How many students were there in each row of the lecture room?
- The area of a square plot is  $101\frac{1}{400} \text{ m}^2$ . Find the length of one side of the plot.

8. 2025 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.

## Summary

1. Process of multiplying a number by itself is called squaring the number
2. For each rational number  $x$  there is a rational number  $y$  ( $y \geq 0$ ) such that  $x^2 = y$
3. Square root of a number is one of its two equal factors.
4. A rational number  $x$  is called a perfect square, if and only if  $x = n^2$  for some  $n \in \mathbb{Q}$ .
5. The process of multiplying a number by itself three times is called cubing the number.
6. The cube root of a given number is one of the three identical factors whose product is the given number.
7. A rational number  $x$  is called a perfect cube, if and only  $x = n^3$  for some  $n \in \mathbb{Q}$ .

## Key words

- |                |                        |
|----------------|------------------------|
| ◆ Square       | ◆ Cubic perfect square |
| ◆ Square root  | ◆ Radical              |
| ◆ Prime factor | ◆ Index                |
| ◆ Cube         | ◆ Radicand             |

## Review exercise

1. Which of the following are perfect squares?  
 $\{0, 1, 4, 7, 12, 16, 25, 30, 36, 42, 49\}$
2. Which of the following are perfect squares?  
 $\{50, 64, 72, 81, 95, 100, 121, 140, 144, 169\}$ ?
3. Simplify :  $\sqrt{(-5)^2 + 5^2 - (-1)^2} + \sqrt{169 - 12^2}$
4. Simplify:  $\sqrt{10 - \left(\frac{8}{\sqrt{10}}\right)^2}$
5. Find the numbers
  - a. The sum of the square of three consecutive numbers is 50.
  - b. The sum of the square of three consecutive numbers is 77.
6. Prove that the difference of the square of an even number is multiple of 4.
7. Find the square root of
  - a.  $36x^2 - 60xy + 5y^2$
  - b.  $16x^2 + 24xy + 9y^2$
  - c.  $2x^2 + 2\sqrt{6}xy + 3y^2$
8. Find the cube root of  $27a^3 + 54a^2b + 36ab^2 + 8b^3$ .
9. Show that 64 can be written as either 26 or 43.
10. Look at this number pattern.  
 $7^2 = 49$   
 $67^2 = 4489$

$$667^2 = 444889$$

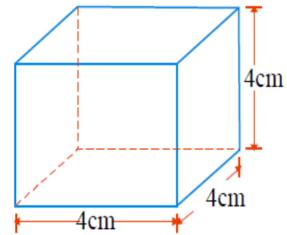
$$6667^2 = 44448889.$$

This pattern continues.

Write down the next line of the pattern.

11. In Figure below to the right find:

- a. The surface area of a cube.
- b. The volume of a cube



12. Find the exact perimeter of a square whose side length is  $5\sqrt[3]{16}$  cm
13. The length of the sides of a cube is related to the volume of the cube according to the formula:  $x=3\sqrt[3]{V}$ .
  - a. What is the volume of the cube if the side length is 25cm?
  - b. What is the volume of the cube if the side length is 40 cm?

# Unit 3

## LINEAR EQUATION AND INEQUALITIES

### Learning Outcomes:

At the end of this unit, learners will be able to:

- ▶ Graph linear equations of type  $y = mx + n$
- ▶ Solve linear inequalities
- ▶ Solve applications of linear inequalities

### Main content

- 3.1 Revision of Cartesian coordinate system
- 3.2 Graph of linear equations
- 3.3 Solving Linear Inequalities
- 3.4 Applications of Linear Equations and Inequalities
  - ※ Key terms
  - ※ Summary
  - ※ Review Exercise

## 3.1 Revision of Cartesian coordinate system

The unification of algebra and geometry into the branch of mathematics called coordinate geometry, or analytic geometry or Cartesian geometry. We use the rectangular (or Cartesian) coordinate system to get a visual image of order pairs of real numbers. The plane containing this number line is called coordinate plane. By using a Cartesian coordinate system, we can specify a point P in the plane with two rational number, also called coordinates. A Cartesian coordinate system consists of two perpendicular number lines, called coordinate axes, which meet at origin.

The coordinate of P is usually written as an ordered pairs  $(x,y)$  enclosed in parentheses, with abscissa first (x-axis) and the ordinate second (y-axis).

A coordinate plane contains the two number lines called the x-axis and y-axis which are perpendicular to each other.

- ◆ The horizontal line is called x-axis.
- ◆ The vertical line is called y-axis
- ◆ The point  $(0,0)$  at which the two axes intersect is called the origin.
- ◆ The part of y-axis that lies above x-axis is called positive y-axis and labeled with positive numbers.
- ◆ The part of y-axis that lies below x-axis is called negative y-axis and labeled with negative numbers.

- ◆ The part of x-axis that lies to the left of y-axis is called negative x-axis and labeled with negative numbers.
- ◆ The part of x-axis that lies to the right of y-axis is called positive x-axis and labeled with positive numbers.

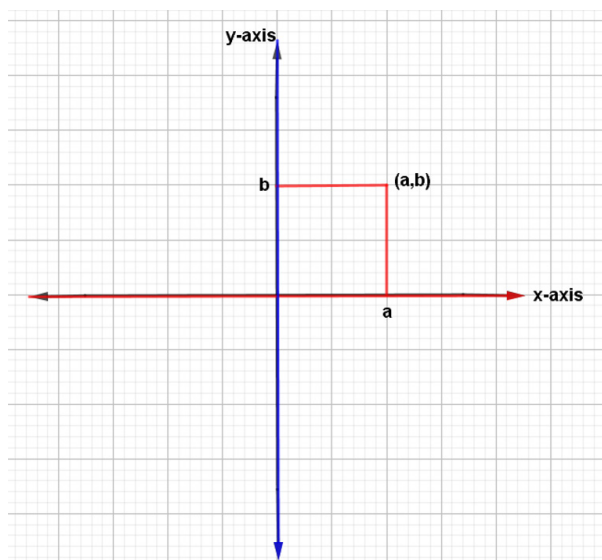


*Figure 3.1 Cartesian coordinate plane*

### Activity3.1

Plot each ordered pair in xy-plane and indicate at which quadrant points lie

- |           |            |           |
|-----------|------------|-----------|
| a. (3,1)  | c. (-1,-3) | e. (2,-2) |
| b. (-2,2) | d. (0,3)   |           |

### The four quadrants of the Cartesian coordinate plane

The two number lines, the x-axis and y-axis, divide the plane into four regions called quadrants. They are numbered as quadrant I, quadrant

II, quadrant III and quadrant IV.

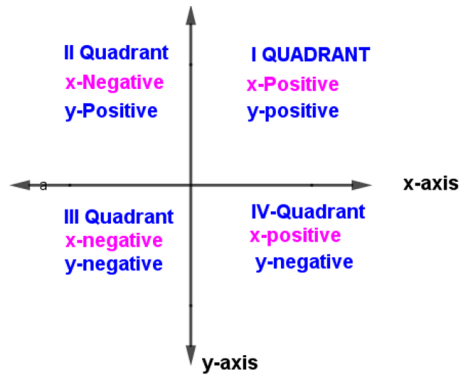


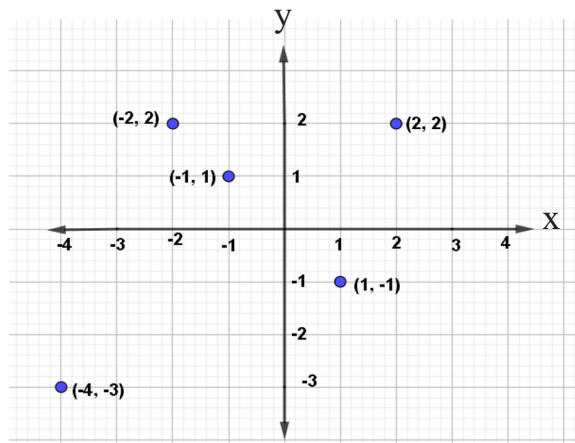
Figure 3.2 Quadrants

### Example 1

Plot each of the following pairs and indicate which quadrant a coordinate axis contains the ordered pair.

- |              |               |             |
|--------------|---------------|-------------|
| a. $(-1, 1)$ | c. $(-4, -3)$ | e. $(2, 2)$ |
| b. $(-2, 2)$ | d. $(1, -1)$  |             |

### Solution



## 3.2 Graphs of linear equations

### Linear equations in two variables

A linear equation in two variables,  $x$  and  $y$ , can be written in the standard form  $Ax + By = C$

where  $A$ ,  $B$ , and  $C$  are rational numbers., that is,  $A$  and  $B$  are not both zero. A solution of the equation consists of a pair of numbers

$(x_1, y_1)$  which satisfies the equation,  $Ax_1 + By_1 = C$  Solutions of the equation can be found by assigning arbitrary values to  $x$  and solving for  $y$  (or vice versa).

### Activity 3.2

1. State the slope and y-intercept of each line.

a.  $y = 2x + 4$

c.  $y = \frac{2}{3}x - 6$

b.  $y = -5x$

d.  $y = -\frac{1}{4}x + 2$

2. Translate each equation into slope-intercept form. Then, state the slope and y-intercept.

a.  $4y = 2x + 20$

d.  $5x = 1 - 10y$

b.  $6x - 3y = 12$

e.  $x = 1 - y$

c.  $3x + y = 4$

f.  $\frac{1}{2}y = 2x - 3$

### Horizontal and Vertical Lines

#### I. Vertical Lines ( $x = a$ )

Graph of an equation of the form  $x = a$ , where  $a \in \mathbb{Q}$

The graph of the equation  $x=a$ ,  $a \neq 0$  is a line parallel to the y-axis and at a distance of a unit from it.

The graph of  $x=a$ , for any constant  $a$ , is a vertical line with x intercept

**a. The graph, in figure 3.3**

- i. If  $a > 0$ , then the line lies to the right of the y- axis. Look at the line  $x=1$ , and  $x=2$
- ii. If  $a < 0$ , then the line lies to the left of the y-axis. Look at the line  $x=-2$ , and  $x=-1$
- iii. The graph of the equation  $x=0$  is the y-axis

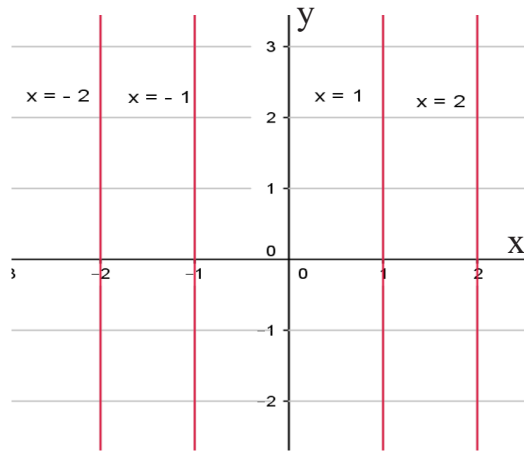


*Figure 3.3 graph of vertical lines*

## II. Horizontal Lines ( $y = b$ )

Graph of an equation of the form  $y = b$ , where  $b \in \mathbb{Q}$

The graph of the equation  $y=b$ ,  $b \neq 0$  is a line parallel to the x-axis and at a distance of  $b$  unit from it.

The graph of  $y = b$ , for any constant  $b$ , is a horizontal line with y intercept  $b$ .

**The graph, in figure 3.4**

- i. If  $b > 0$ , then the line lies above the x-axis. Look at the line,  $y = \frac{1}{2}$ , and  $y=2$

- ii. If  $b < 0$ , then the line lies below the x-axis. Look at the line  $y = -1$ , and  $y = -2$
- iii. The graph of the equation  $y = 0$  is the x-axis.

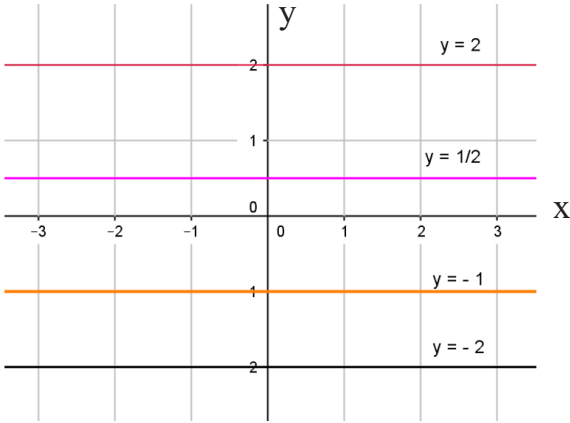


Figure 3.4 graph of horizontal lines

Example 1

Draw the graphs of each of the following on the same coordinate system

- a.  $y = 3$
- b.  $y = \frac{1}{2}$
- c.  $y = -2$

Solution

First by drawing table values for x, and y in which y is constant and following this you plot these points and realize that the points lie horizontal line.

- a.  $y = 3$ .....(Let this line denoted  $L_1$ )

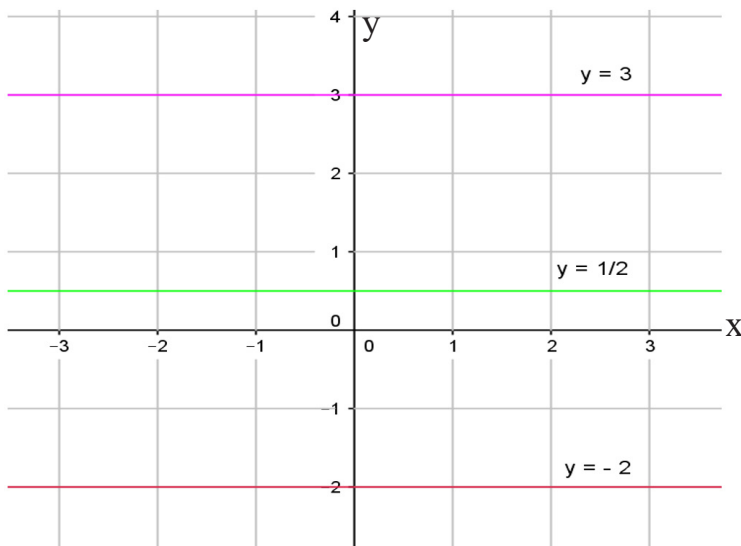
|                    |         |         |        |        |        |        |
|--------------------|---------|---------|--------|--------|--------|--------|
| x                  | -2      | -1      | 0      | 1      | 2      | 3      |
| y                  | 3       | 3       | 3      | 3      | 3      | 3      |
| Coordinates (x ,y) | (-2 ,3) | (-1 ,3) | (0 ,3) | (1 ,3) | (2 ,3) | (3 ,3) |

$y = \frac{1}{2}$  ..... (Let this line denoted  $L_2$ )

|                  |                     |                     |                    |                    |                    |                    |
|------------------|---------------------|---------------------|--------------------|--------------------|--------------------|--------------------|
| x                | -2                  | -1                  | 0                  | 1                  | 2                  | 3                  |
| y                | $\frac{1}{2}$       | $\frac{1}{2}$       | $\frac{1}{2}$      | $\frac{1}{2}$      | $\frac{1}{2}$      | $\frac{1}{2}$      |
| coordinates(x,y) | $(-2, \frac{1}{2})$ | $(-1, \frac{1}{2})$ | $(0, \frac{1}{2})$ | $(1, \frac{1}{2})$ | $(2, \frac{1}{2})$ | $(3, \frac{1}{2})$ |

b.  $y = -2$  ..... (Let this line denoted  $L_3$ )

|                  |            |            |           |           |           |           |
|------------------|------------|------------|-----------|-----------|-----------|-----------|
| x                | -2         | -1         | 0         | 1         | 2         | 3         |
| y                | -2         | -2         | -2        | -2        | -2        | -2        |
| coordinates(x,y) | $(-2, -2)$ | $(-1, -2)$ | $(0, -2)$ | $(1, -2)$ | $(2, -2)$ | $(3, -2)$ |



## The graphs of linear equations

$y = mx + b$ , Where  $m \neq 0$ ,  $b \neq 0$  and  $m, b \in \mathbb{Q}$

News magazines often show graphs comparing the rate of inflation, the federal deficit, or the unemployment rate to the time of year.

Businesses use graphs to report monthly sales statistics. Such graphs

provide geometric pictures of the way one quantity changes with respect to another. Frequently, the relationship between two quantities is expressed as an equation. This section introduces the basic procedure for determining the geometric picture associated with an equation. For an equation in the variables  $x$  and  $y$ , a point  $(a, b)$  is a solution point if the substitution of  $x=a$  and  $y=b$  satisfies the equation. Most equations have infinitely many solution points. For example, the equation  $3x+y=5$  has solution points  $(0, 5)$ ,  $(1, 2)$ ,  $(2, -1)$ ,  $(3, -4)$  and so on. The set of all solution points of an equation is the graph of the linear equation.

For example consider the equation  $2x + y = 4$ . If we substitute  $x = -2$  in the equation, we obtain  $2(-2) + y = 4$  or  $4 + y = 4$  or  $y = 8$

Therefore  $(-2, 8)$  is a solution.

If we substitute  $x = 3$  in the equation, we obtain  $2(3) + y = 4$  or  $6 + y = 4$  or  $y = -2$ . Hence,  $(3, -2)$  is a solution.

The following table lists possible values for  $x$  and the corresponding values for  $y = -2x + 4$ .

|                       |           |           |          |          |          |           |
|-----------------------|-----------|-----------|----------|----------|----------|-----------|
| $x$                   | -2        | -1        | 0        | 1        | 2        | 3         |
| $y$                   | 8         | 6         | 4        | 2        | 0        | -2        |
| coordinates( $x, y$ ) | $(-2, 8)$ | $(-1, 6)$ | $(0, 4)$ | $(1, 2)$ | $(2, 0)$ | $(3, -2)$ |

## Intercepts

### **x- intercept and y- intercept**

The graph of the linear equation  $Ax + By = C$  consists of all points  $(x, y)$  in the Cartesian coordinate system which satisfies the equation. This graph is a straight line when  $A$  and  $B$  are not both zero. This fact

is indicated by Fig. (a) and (b) below which shows a straight line containing the six solutions of  $2x + y = 4$ .

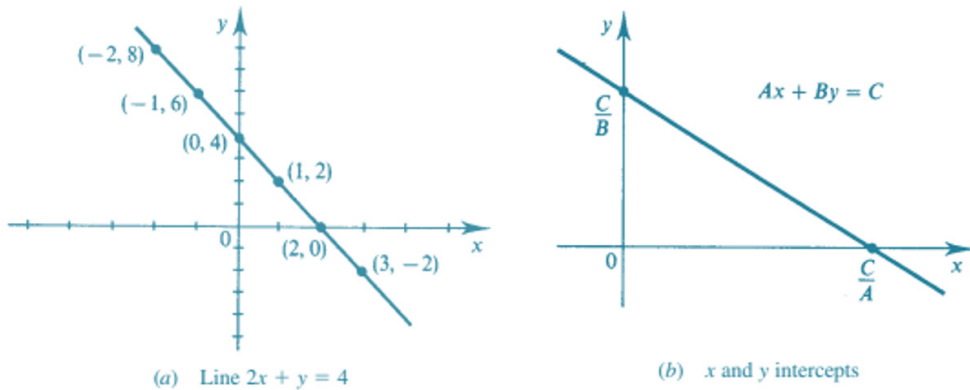


Figure 3.5 graph of  $Ax + By = C$

We plot the points that we found in Figure (a); they appear to lie on a line. So we complete the graph by joining the points by a line. The x intercept and the y intercept of the linear equation  $Ax + By = C$  are the points at which its graph crosses the x axis and y axis, respectively. as shown in (Figure 3.5 b)

Alternatively, the x intercept is the x value when,  $y=0$ , and y intercept is the y value when  $x=0$ .

Thus,  $x = \frac{C}{A}$  is the x intercept when  $A \neq 0$ , and

$y = \frac{C}{B}$  is its y-intercept when  $B \neq 0$ .

In particular, if  $A \neq 0$ , we usually plot the x-intercept, and if  $B \neq 0$ , we usually plot the y-intercept

### Example: 1

Find the x and y-intercepts for the following linear equations.

a.  $4x - 3y = 5$

b.  $\frac{1}{2}y + x = 1$

c.  $y = \frac{4}{5}x + 7$

d.  $y = x$

*Solution*a.  $4x - 3y = 5$  is in the form of  $Ax + By = C$ 

$A = 4, B = -3, C = 5$

 $x = \frac{C}{A}$  is the x intercept when  $y = 0, A \neq 0$ 

$$x = \frac{5}{4}$$

 $y = \frac{C}{B}$  is its y intercept when  $x = 0, B \neq 0$ .

$$y = \frac{5}{-3} = -\frac{5}{3}$$

b.  $\frac{1}{2}y + x = 1$ , first express in the form of  $Ax + By = C$ 

$y + 2x = 2, A = 2, B = 1, C = 2$

 $x = \frac{C}{A}$  is the x-intercept when,  $y = 0, A \neq 0$ 

$$x = \frac{2}{2} = 1$$

 $y = \frac{C}{B}$  is its y -intercept when  $x = 0, B \neq 0$ .

$$y = \frac{2}{1} = 2$$

c.  $y = \frac{4}{5}x + 7$ , first express in the form of  $Ax + By = C$ 

$y - \frac{4}{5}x = 7, A = -\frac{4}{5}, B = 1, C = 7$

 $x = \frac{C}{A}$  is the x-intercept when,  $y = 0, A \neq 0$ 

$$x = \frac{7}{-\frac{4}{5}} = -\frac{35}{4}$$

 $y = \frac{C}{B}$  is its y-intercept when,  $y = 0, B \neq 0$ .

$$y = \frac{7}{1} = 7$$

d.  $y = x$ ; express in the form of  $Ax + By = C$

$$y - x = 0, A = -1, B = 1, C = 0$$

$x = \frac{C}{A}$  is the x-intercept when  $y = 0, A \neq 0$

$$x = \frac{0}{-1} = 0$$

$y = \frac{C}{B}$  is its y-intercept when  $x = 0, B \neq 0. y = \frac{0}{1} = 0$

## The slope of the lines

$$\text{Slope} = \frac{\text{change in y value}}{\text{change in x value}}$$

In Figure to the right, consider a line drawn through the points

$P(x_1, y_1)$  and  $Q(x_2, y_2)$ .

From P to Q the change in the x coordinate is  $(x_2 - x_1)$

and the change in the y coordinate is  $(y_2 - y_1)$ .

By definition, the slope of the line AB is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

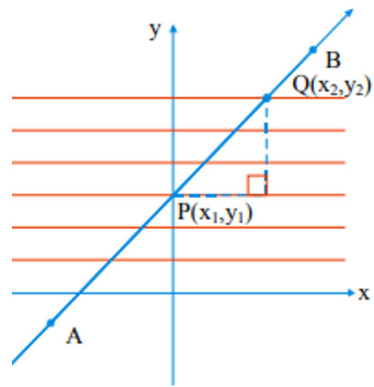


Figure 3.6 slope of line

### Definition:

**Definition** If  $x_1 \neq x_2$  the slope of the line through the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is the ratio:.

$$\text{Slope} = \frac{\text{change in y value}}{\text{change in x value}}$$

If we denote the slope of a line by the letter “m”.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = m(x - x_1)$$

### Example 2

Find the equation of the line if

- The line has slope 5 and passes through the points P(-1, 3)
- The line has slope  $\frac{1}{2}$  and passes through the points P(-4, -7)

### Solution:

- P(-1, 3) here  $x_1 = -1$  and  $y_1 = 3$  and  $m = 5$

From  $y - y_1 = m(x - x_1)$

$$y - 3 = 5[x - (-1)] = 5(x + 1) \dots \text{by substitution}$$

$$y = 5x + 8 \dots \text{by simplifying}$$

- P(-4, -7), here  $x_1 = -4$  and  $y_1 = -7$  and  $m = \frac{1}{2}$  ..... by substituting

From  $y - y_1 = m(x - x_1)$

$$y + 7 = \frac{1}{2}(x + 4)$$

$$y = \frac{1}{2}x - 5$$

### Example 3

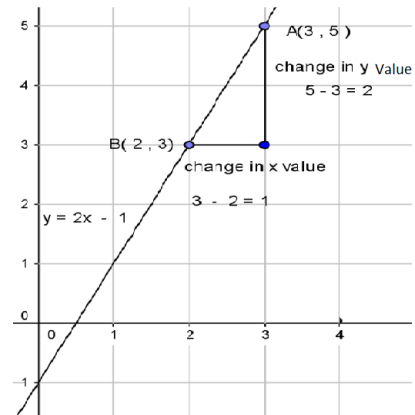
From the figure given below, the equation of the line is  $y = 2x - 1$ . The points A(3, 5) and B(2, 3) lie on the given graph.

If  $x_1 = 3$ ,  $x_2 = 2$ ,  $y_1 = 5$  and  $y_2 = 3$ , then the slope of the line is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{2 - 3} = \frac{-2}{-1} = 2$$

Therefore, the slope of the line is:

$$m = 2.$$



### Example 4

Find the slope of the line passing through the following points.

- P(1, 1) and Q(9, 6)
- A(5, -2) and B(-1, 8)
- M(6, 6) and N(-6, 6)
- T(9, -3) and R(9, 7)
- E(0, 0) and F(2, -2)

### Solution:

|    | Points                                                             | Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$                                                   |
|----|--------------------------------------------------------------------|--------------------------------------------------------------------------------------------|
| a. | (1, 1) and (9, 6)<br>$x_1 = 1, y_1 = 1$ and $x_2 = 9, y_2 = 6$     | $m = \frac{6 - 1}{9 - 1} = \frac{5}{8}$<br>The slope $m = \frac{5}{8}$                     |
| b. | (5, -2) and (-1, 8)<br>$x_1 = 5, y_1 = -2$ and $x_2 = -1, y_2 = 8$ | $m = \frac{8 + 2}{-1 - 5} = \frac{10}{-6} = -\frac{5}{3}$<br>The slope: $m = -\frac{5}{3}$ |

|    |                                                          |                                                                  |
|----|----------------------------------------------------------|------------------------------------------------------------------|
| c. | (6 ,6) and (-6 ,6)<br>$x_1=6, y_1=6$ and $x_2=-6, y_2=6$ | $m = \frac{6-6}{-6-6} = \frac{0}{-12} = 0$<br>The slope: $m = 0$ |
| d. | (9 ,-3) and (9 ,7)<br>$x_1=9, y_1=-3$ and $x_2=9, y_2=7$ | $m = \frac{7+3}{9-9} = \frac{10}{0} = ??$<br>It has no slope     |

**Note:**

- i. The horizontal line has a slope of 0.
- ii. The vertical line has no slope (undefined)

*Example 5*

Determine whether the solution points lie on the graph of  $y=10x-7$

- a. (2,13)
- b. (-1 ,-3 )
- c.  $(\frac{1}{2}, 2)$
- d.  $(\frac{6}{5}, -7)$

*Solution*

- a. Take the point (2 ,13) , and substitute the number  $x=2$  in the equation

$$y = 10(2)-7 = 20-7$$

$$y = 13$$

Therefore, the point (2 ,13) is the solution point of the equation  $y=10x-7$ , The point (2 ,13) lies on the graph of  $y=10x-7$

- b. Take the point (-1 ,-3) , and substitute the number  $x=-1$  in the equation

$$y = 10(-1)-7=-10-7$$

$$y = -17$$

$$-3 \neq -17$$

The point  $(-1, -3)$  is not the solution of the given equation  $y = 10x - 7$

Therefore, the point  $(-1, -3)$  does not lie on the graph of  $y = 10x - 7$

- c. Take the point  $(\frac{1}{2}, 2)$ , and substitute the number  $x = \frac{1}{2}$  in the equation

$$y = 10(\frac{1}{2}) - 7 = 5 - 7$$

$$y = -2$$

$$-2 \neq 2$$

The point  $(\frac{1}{2}, 2)$  is not the solution of the given equation  $y = 10x - 7$ .

Therefore, the point  $(\frac{1}{2}, 2)$  does not lie on the graph of  $y = 10x - 7$

- d. Take the point  $(\frac{6}{5}, 5)$ , and substitute the number  $x = \frac{6}{5}$  in the equation

$$y = 10(\frac{6}{5}) - 7 = 12 - 7$$

$$y = 5$$

$$5 = 5$$

The point  $(\frac{6}{5}, 5)$  is the solution of the given equation  $y = 10x - 7$

Therefore, the point  $(\frac{6}{5}, 5)$  lies on the graph of  $y = 10x - 7$ .

### Note:

It is always important to check your answer, even if you never make a mistake in your calculations.

## Graph of Linear Equation $y = mx + b$

The basic technique used for sketching the graph of an equation is the point plotting method.

- i. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
- ii. Make a table of values showing several solution points.
- iii. Plot these points on a rectangular coordinate system.
- iv. Connect the points with a straight line

### Example 6

Use point plotting and graph paper to sketch the graph of

- a.  $3x + y = 6$
- b.  $2y - x = 10$
- c. What is the slope of a line that rises from left to right?
- d. What is the slope of a line that falls from left to right?

### Solution:

a.  $3x + y = 6$

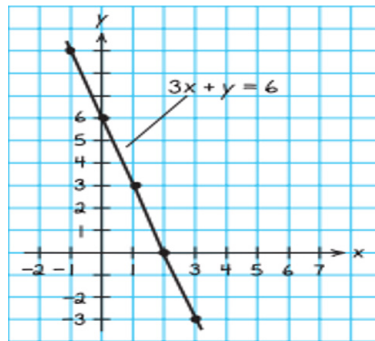
In this case you can isolate the variable  $y$

$$y = 6 - 3x, \text{ solve for } x$$

Using some negative, zero, and positive values you can obtain the following table of values (solution points)

|                        |         |       |        |        |          |
|------------------------|---------|-------|--------|--------|----------|
| x                      | -1      | 0     | 1      | 2      | 3        |
| $y = 6 - 3x$           | 9       | 6     | 3      | 0      | -3       |
| Solution point(x , y ) | (-1 ,9) | (0,6) | (1 ,3) | (2 ,0) | (3 ,-3 ) |

To locate the point  $(x, y)$ , start at the origin, go  $x$  directed units along the  $x$  axis and then  $y$  directed unit parallel to the  $y$  axis. The endpoint has the coordinates  $(x, y)$ ,

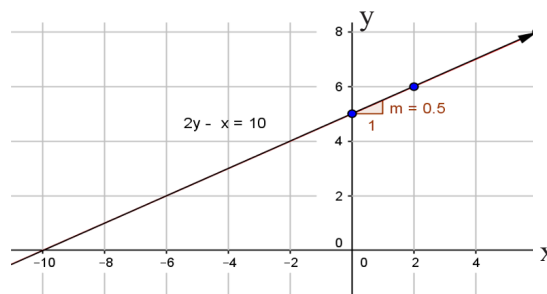


b.  $2y - x = 10$

In this case you can isolate the variable  $y$ ;  $y = \frac{1}{2}x + 5$ , then solve for  $x$  by taking negative, zero, and positive values for  $x$ , you can obtain the following table of values (solution points)

| $x$                    | -2      | -1                  | 0      | 1                   | 2      | 3                   |
|------------------------|---------|---------------------|--------|---------------------|--------|---------------------|
| $y = \frac{1}{2}x + 5$ | 4       | $\frac{9}{2}$       | 5      | $\frac{11}{2}$      | 6      | $\frac{13}{2}$      |
| Sol.n point( $x, y$ )  | (-2, 4) | $(-1, \frac{9}{2})$ | (0, 5) | $(1, \frac{11}{2})$ | (2, 6) | $(3, \frac{13}{2})$ |

To locate the point  $(x, y)$ , start at the origin, go  $x$  directed units along the  $x$  axis and then  $y$  directed units parallel to the  $y$  axis. The endpoint has the coordinates  $(x, y)$ ,

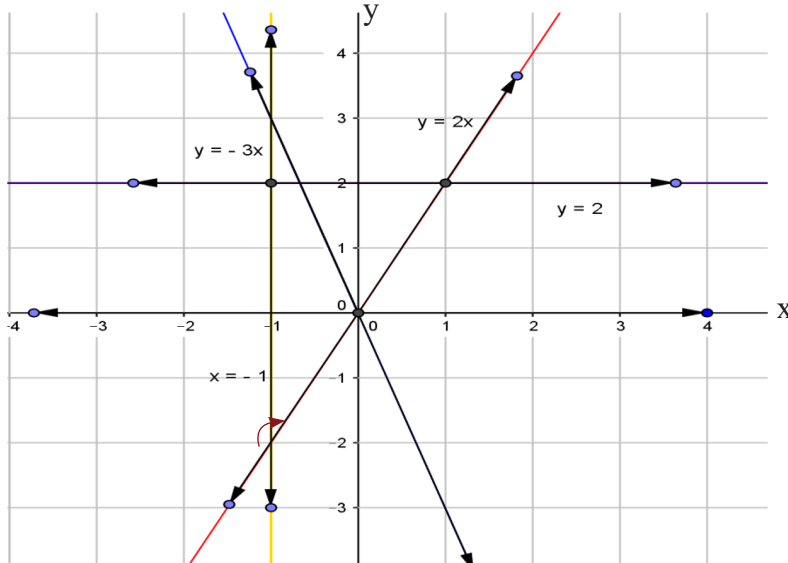


- c. The line that rises from left to right has a positive slope
- d. The slope of a line that falls from left to right has a negative slope

### Exercise 3.1

1. Plot the following lines
  - a.  $x = -3$ ,  $x = -1$ ,  $x = 3$ , and  $x = 2$
  - b.  $y = 4$ ,  $y = 1$ ,  $y = -2$ , and  $y = -3$
2. Express the following linear equations
  - i. In the form  $Ax + By + C = 0$  and indicate the
  - ii. Values of A, B and C in each case
  - iii. y- intercept and x- intercept in each case
  - iv. Draw the graph in each case
    - a.  $x - \frac{y}{4} - 5 = 0$
    - b.  $3y - \frac{1}{2}x = 1$
    - c.  $\frac{x + y}{2} = -4$
    - d.  $3x = 2y$
3. If the point (3,4) lies on the graph of the equation  $3y = ax - 9$ . Find
  - a. The value of a.
  - b. The slope of the line
  - c. The intercept
4. Show that the points A (1, 2), B (− 1, − 16) and C (0, − 7) lie on the graph of the linear equation  $y = 9x - 7$ .
5. Find m, if point (−7,3) lies on the graph of  $\left(y - \frac{3}{7}\right) = \frac{m(x-2)}{7}$
6. From the given lines below, identify
  - i. The line(s) which has a negative slope.

- ii. The line(s) which has a positive slope.
- iii. The line(s) which has undefined slope.
- iv. The line (s) which has a slope of zero.



7. Find the equation of the line if the line has slope as shown and passes through the given points
- a.  $P(-3, 2), m=2$
  - b.  $Q(7, -11), m=-5$
  - c.  $R(0, 8), m=-\frac{2}{3}$
  - d.  $M(0, 0), m=3$

### 3.3 Solving linear Inequality

In lesson 3.2 you have learnt about linear equation and the rules for transforming linear equality. Now in this sub topic you learn more about linear inequalities. The rules for transforming linear inequalities will be discussed in detail so as to find their solutions.

### Activity 3.3

1. Discuss the difference between the following pairs of inequality
  - i.  $x > 5$  and  $x \geq 5$
  - ii.  $x > 5$  and  $x < 5$
  - iii.  $x \geq 5$  and  $x \leq 5$
2. Discuss how to solve the inequalities of the form,
  - i.  $ax + b < c, a \neq 0$
  - ii.  $ax + b > c, a \neq 0$
3. Solve the following linear inequalities
  - i.  $3x + 6 < -24$
  - ii.  $5x + 12 < 9$
  - iii.  $\frac{3}{2}x + 6 > -9$
  - iv.  $-7x + \frac{2}{3} \leq 2$
  - v.  $\frac{13}{5}x - \frac{12}{7} < 9$
4. Determine whether the following values of  $x$  are the solution of the linear inequality or not.
  - i.  $7x - 8 > 0, x = 2, x = -1, x = 1, x = -2$
  - ii.  $3x + 12 < -4, x = 2, x = -1, x = 1, x = -2$
  - iii.  $0 \leq \frac{x+2}{5} < 2, x = 2, x = -1, x = 1, x = -2$
5. Identify, whether the following are linear inequality or not
  - a.  $x + 3 < 4$
  - b.  $8x - 6 > -21$
  - c.  $3x^2 - 2x \leq 11$
  - d.  $12x + 9 > 6x - 5$
  - e.  $-4x + \frac{2}{x} \leq 17$
  - f.  $8 - 2x^2 \geq 17x$
  - g.  $6x - 7^2 \leq 4^3$
  - h.  $-19x^3 + 21 > 23x - 18$
  - i.  $\frac{3}{2}x - \frac{11}{2} < 33$

**Definition:**

Mathematical sentence which contains one of the relations signs (symbols)  $<$ ,  $\leq$ ,  $>$ ,  $\geq$  or  $\neq$  are called inequalities.

**Definition:**

A linear inequality in one variable “x” is an inequality that can be written in the form of  $ax + b < 0$ ,  $ax + b \leq 0$  or  $ax + b > 0$ , or  $ax + b \geq 0$  where a and b are rational numbers and  $a \neq 0$ .

*Example 1*

Some examples of linear inequalities are:

a.  $-8x + \frac{2}{3} < 13$

c.  $-7x + \frac{2}{3} \leq 2$

b.  $\frac{3}{2}x + 6 > -\frac{19}{7}$

d.  $\frac{13}{5}x - \frac{12}{7} < \frac{9}{5}$

**Note:**

For a, b, c  $\in$  Q,  $a < b$ , then

a.  $a + c < b + c$

b.  $a \cdot c < b \cdot c$ , when  $c > 0$

c.  $ac > bc$  when  $c < 0$

Thus, to produce an equivalent inequality, we may add (subtract) the same quantity to (from) both sides of an inequality, or multiply (divide) both sides by the same positive quantity.

On the other hand, we must reverse the inequality symbol to produce an equivalent inequality if we multiply (divide) both sides by the same negative quantity.

## Rules of Transformation for Inequalities

The following rules are used to transform a given inequality to an equivalent inequality.

**Rule 1:** If the same number is added to or subtracted from both sides of an inequality, the direction of the inequality is unchanged. That is for any rational numbers  $a, b$  and  $c$ .

- i. If  $a < b$ , then  $a + c < b + c$
- ii. If  $a < b$ , then  $a - c < b - c$

**Rule 2:** If both sides of an inequality are multiplied or divide by the same positive number, the direction of the inequality is unchanged. That is for any rational numbers  $a, b$  and  $c$ .

- i. If  $a < b$  and  $c > 0$ , then  $ac < bc$ .
- ii. If  $a < b$  and  $c > 0$ , then  $\frac{a}{c} < \frac{b}{c}$  (provided  $c \neq 0$ ).

**Rule 3:** If both sides of an inequality are multiplied or divided by the same negative number, the direction of the inequality is reversed. That is for any rational numbers  $a, b$  and  $c$ .

- i. If  $a < b$  and  $c < 0$ , then  $ac > bc$ .
- ii. If  $a < b$  and  $c < 0$ , then,  $\frac{a}{c} > \frac{b}{c}$  (provided  $c \neq 0$ )

### Example 2

Solve the inequality:  $x + 5 < 12$

*Solution:*

$$x + 5 < 12 \dots\dots\dots \text{Given inequality}$$

$$x + 5 - 5 < 12 - 5 \dots\dots\dots \text{Subtracting 5 from both sides}$$

$$x < 7 \dots \dots \dots \text{Simplifying}$$

Thus, the solution is  $x < 7$

### Example 3

Solve the linear inequality  $5x+7 \geq 0, x \in \mathbb{Q}$

$$5x+7 \geq 0 \dots \dots \dots \text{Given inequalities}$$

$$5x+7-7 \geq 0-7 \dots \dots \dots \text{Subtracting 7 from both sides}$$

$$5x \geq -7 \dots \dots \dots \text{Simplifying}$$

$$\frac{5x}{5} \geq \frac{-7}{5} \dots \dots \dots \text{Dividing both sides by 5.}$$

$$x \geq \frac{-7}{5} \dots \dots \dots \text{Simplifying}$$

Therefore, the solution set is  $x \geq \frac{-7}{5}$

### Example 4

Solve the linear inequality  $5x+8(20-x) \geq 2(x-5), x \in \mathbb{N}$

#### Solution:

$$5x+8(20-x) \geq 2(x-5) \dots \dots \dots \text{Given inequalities}$$

$$5x+160-8x \geq 2x-10 \dots \dots \dots \text{Removing brackets}$$

$$160-3x-2x \geq 2x-10-2x \dots \dots \dots \text{Subtracting 2x from both sides}$$

$$160-5x \geq 10 \dots \dots \dots \text{Simplifying}$$

$$160-5x-160 \geq 10-160 \dots \dots \dots \text{Subtracting 160 from both sides}$$

$$-5x \geq -150 \dots \dots \dots \text{Simplifying}$$

$$\frac{-5x}{-5} \leq \frac{-150}{-5} \dots \dots \dots \text{Dividing both sides by -5.}$$

$$x \leq 30 \dots \dots \dots \text{Simplifying}$$

The solution of the inequality is  $x \leq 30$ .

Therefore, the solution set is  $\{x: x \leq 30\} = \{1, 2, 3, \dots, 30\}$

### Example 5

Find the solution set of the inequality  $3x - 5(x - 2) \geq 0$ ,  $x \in W$

### Solution

$$3x - 5(x - 2) \geq 0 \dots\dots\dots \text{The given inequality}$$

$$3x - 5x + 10 \geq 0 \dots\dots \text{Removing the brackets by distribution)}$$

$$-2x + 10 \geq 0 \dots\dots\dots \text{(Combining like terms)}$$

$$-2x + 10 - 10 \geq 0 - 10 \dots\dots\dots \text{(Subtracting 10 from both sides)}$$

$$-2x \geq -10 \dots\dots\dots \text{Simplifying}$$

$$\frac{-2x}{-2} \leq \frac{-10}{-2} \quad \text{(Dividing both sides by -2 reverse the inequality)}$$

$$x \leq 5 \dots\dots\dots \text{Simplifying}$$

Therefore, the solution set =  $\{x \in W: x \leq 5\} = \{0, 1, 2, 3, 4, 5\}$

The solution of an inequality is required to be only in a given domain. If so, a solution set should contain only those solutions that belong to the specified domain.

To find the solutions of a given inequality, you will use the rules of transformation for inequalities to get successive equivalent inequalities so that the least simplified form is either  $x > a$  or  $x < a$  or  $x \leq a$  or  $x \geq a$

In solving a linear inequality of the form  $ax + b > 0$ ,  $a \neq 0$ , you have to consider two cases. These are: When  $a > 0$  and when  $a < 0$

**Definition:**

The set of numbers from which value of the variable may be chosen should be meaningful and it is called the domain of the variable.

*Example 6*

Given the domain =  $\{1, 3, 5, 9, 11, 13, 15, 17, 19\}$

Find the solution set of the inequality  $x + 8 > 22$

*Solution*

$$x + 8 - 8 > 22 - 8$$

$$x > 14$$

Since 15, 17 and 19 are the solution of the given inequality  $x + 8 > 22$ , the set that contains these number is called the solution set of  $x + 8 > 22$ . You can now define the term solution set or truth set.

**Definition:**

The set containing all the solutions of an inequality is called the solution set or truth set of the inequality and denoted by S.S or T.S.

*Example 7*

Find the solution set of  $x - 4(x + 1) \geq -13 - (x - 2)$  in the set of natural numbers,  $\mathbb{N}$ .

*Solution:*

$x - 4(x + 1) \geq -13 - (x - 2)$ .....The given inequality

$$x-4x-4 \geq -13-x+2 \dots\dots\dots \text{(Removing brackets by distribution)}$$

$$-3x-4 \geq -11-x \dots\dots\dots \text{(Combining like terms)}$$

$$-3x+x \geq -11+4 \dots\dots\dots \text{(Collecting like terms)}$$

$$-2x \geq -7 \dots\dots\dots \text{(Divide of both sides by -2}$$

reverses the inequality)

$$x \leq \frac{7}{2}; \text{ i. e. } x \leq 3.5$$

$$\text{Thus, S.S} = \{1, 2, 3\}$$

Some inequalities may have no solution in the specified domain as in the following example.

### Example 8

Find the solution set of  $5x + 7 \leq 2x + 1$  in the set of whole numbers W

### Solution

$$5x + 7 \leq 2x + 1 \dots\dots\dots \text{(The given inequality)}$$

$$5x - 2x \leq 1 - 7 \dots\dots\dots \text{(Collecting like terms)}$$

$$3x \leq -6 \dots\dots\dots \text{(Combining like terms)}$$

$$\frac{3x}{3} \leq \frac{-6}{3} \dots\dots\dots \text{(Divide both sides by 3)}$$

$$x \leq -2$$

However, there is no negative whole number.

Therefore, S.S =  $\emptyset$  or  $\{ \}$

To rewrite mathematical sentences (phrases) the following linear inequalities expression are applicable.

| Using phrase                                                                    | Mathematical Inequality |
|---------------------------------------------------------------------------------|-------------------------|
| ◆ a is less than b                                                              | $a < b$                 |
| ◆ a is greater than b<br>◆ a exceeds b                                          | $a > b$                 |
| ◆ a is less than or equal to b<br>◆ a is at most b<br>◆ a is no more than b     | $a \leq b$              |
| ◆ a is greater than or equal to b<br>◆ a is at least b<br>◆ a is no less than b | $a \geq b$              |

### Example 9

Write the following sentence in linear inequalities

- The speed  $S$  of a car was at least 110 km/hr.
- Aster's average test score  $t$  exceeds 60.
- The height  $h$  of a cave was no more than 30m.
- The temperature  $t$  on the tennis court was no less than  $180^{\circ}\text{F}$ .
- The depth  $d$  of a certain pool was at most 12m.

### Solution:

- $S \geq 110$
- $t > 60$
- $h \leq 30$
- $t \geq 180^{\circ}\text{F}$
- $d \leq 12$

*Example 10:*

Six times a natural number is increased by 8 times the number is less than 28. What are the possible values of this number?

*Solution*

Let  $n$  be the number

$$6n + 8n < 28 \dots\dots\dots \text{Translated inequality.}$$

$$14n < 28 \dots\dots\dots \text{Collect like terms}$$

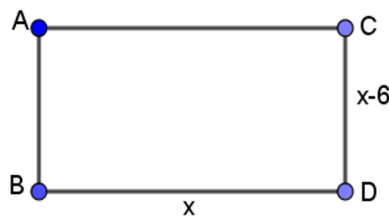
$$\frac{14n}{14} < \frac{28}{14} \dots\dots\dots \text{Dividing both sides by 14.}$$

$$n < 2 \dots\dots\dots \text{Simplifying.}$$

Therefore, the required natural number is less than 2. Thus the number is 1.

*Example 11:*

For the rectangle given below, find all values of  $x$  for which the perimeter is at most 24.



*Solution*

Let the perimeter =  $p$

$$AB + BC + CD + DA = p$$

$$x-6+x+x-6 + x = p \dots\dots\dots \text{substitution}$$

Thus,  $P = 4x - 12$ , but we need  $4x - 12 \leq 24$

$$4x - 12 \leq 24$$

$$4x - 12 + 12 \leq 24 + 12$$

$$4x \leq 36$$

$$\frac{4x}{4} \leq \frac{36}{4}$$

$x \leq 9$  and since  $x$  represents length,  $x - 6 > 0$ . Therefore, the values of  $x$  is  $x > 6$  or  $\{x \in \mathbb{N}: 6 < x \leq 9\} = \{7, 8, 9\}$

### Definition:

Two inequalities are said to be equivalent if they have exactly the same solution.

### Example 12

Some examples of equivalent inequalities are

- $-4x + 1 < 13$  and  $x > -3$
- $2x + 3 \geq 3$  and  $4x - 3 > -3$
- $\frac{13}{5}x - \frac{4}{5} \leq \frac{9}{5}$  and  $\frac{23}{6}x - \frac{13}{6} \leq \frac{10}{6}$
- $7x + \frac{2}{3} \leq 2$  and  $-42x + 6 \geq -2$

### Exercise 3.2

1. Solve the following inequality

a.  $2(-3x + 6) > -10$

b.  $\frac{3x}{7} + \frac{1}{4} < 1 + \frac{x}{2}$

c.  $\frac{3(4-2x)}{4} + 1 > \frac{-(x+1)}{3} - \frac{5}{6}$

d.  $\frac{-2(5-6x)}{4} + 3 < \frac{-7}{2} + 14x$

e.  $\frac{2(x+1)}{3} > 6 - \frac{1}{4} + 3x$

f.  $\frac{3x}{5} + \frac{1}{2} \geq \frac{x}{4} - \frac{2}{5}$

2. If six times a whole number increased by four is less than 34, then find the solution.
3. Find the solution set of the inequality in  $\mathbb{Q}$ 
  - a.  $\frac{1}{6}(2x - 24) < \frac{1}{3}x + 4$
  - b.  $\frac{1}{8}(4x - 4) > \frac{1}{3}x + 4$
  - c.  $7x + 6 \leq 3x + 2$
  - d.  $2x + 0.4 \geq 3.6$
  - e.  $5(x - 4) - 7 \leq -2(x - 3)$
  - f.  $4(2x - 17) - 7 > -5(3x - 8)$
4. Find the solution set of the following in the specified domain
  - a.  $5x + 2 > 2x - 10 : (x \in \mathbb{N})$
  - b.  $4x - 11 \geq 7x + 20 : (x \in \mathbb{Z})$
  - c.  $\frac{2}{x} \geq \frac{1}{x-1} : (x \in \mathbb{Z})$
  - d.  $\frac{3}{4}x - \frac{1}{2} < \frac{3}{2} : (x \in \mathbb{Z})$
  - e.  $\frac{2}{3}x - 5 \geq \frac{1}{2}x - 3 : (x \in \mathbb{W})$
5. Which of the following pairs of inequalities are equivalent?
  - a.  $-x + 1 < 11$  and  $x > -10$
  - b.  $5x + 9 \geq 9$  and  $3x - 7 > -7$
  - c.  $2x - \frac{7}{9} \leq \frac{11}{9}$  and  $\frac{19}{7}x - \frac{11}{7} \leq \frac{8}{7}$
  - d.  $5x - 3 \leq 4$  and  $-15x + 8 \geq -13$
  - e.  $-(x-3) - 4 < (x-3)$  and  $x < 2$
  - f.  $\frac{1}{3}(2x - 3) < 2 - x$  and  $\frac{2}{5}(x + 5) < \frac{3x-2}{5}$
6. Solve each of the following word problems.
  - a. Twice a whole number increased by four is not more than 8. Find all possible values of  $x$ .
  - b. A natural number is less than the sum of its opposite and 8. Find all such numbers.

- c. Find the two smallest consecutive even integers whose sum is at least 51.
- d. The perimeter of a rectangle field is 118m. If the length of the rectangle is 7m less than twice the width, what is the length of the field?

### 3.4 Application in linear Equation and Inequalities

We can apply linear equation and inequalities in many areas of our daily life such as, in Agriculture, Engineering, business, education, road transport, climate change, protection of environment, rising of temperature and so on. In this topic we will see some of their applications.

#### *Example 1*

A larger integer is 2 less than three times a small integer. The sum of the two integers is 18. Find the integers.

#### *Solution:*

Let the smaller  $x$

The larger number is  $3x-2$

The sum of the two integers is 18

$$x+(3x-2)=18$$

$$4x-2=18$$

$$4x=20; \quad x=5$$

The smaller number is 5 and the larger number is 13

### Example 2

The difference between two integers is 2. The larger is 6 less than twice the smaller. Find the two integers

### Solution

Let  $x$  represent the smaller integer

Let  $2x-6$  represents the larger integer then

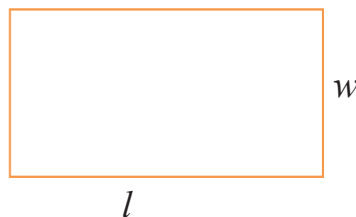
$$(2x-6)-x=2$$

$$x-6=2$$

$$x=8, \text{ Hence the numbers are } x=8 \text{ and } x=10$$

### Example 3

A farmer has a rectangular field whose perimeter measuring 64 feet. The length is 4 feet more than 3 times the widths find the dimensions of fields.



### Solution

Let  $w$  represents the width of the rectangle

Let  $3w+4$  represents the length

$$p = 2l + 2w$$

$$64 = 2(3w+4) + 2w = 6w + 8 + 2w$$

$$64 = 8w + 8$$

$$8w = 64 - 8 = 56$$

$w=7$ . Therefore, the width is 7 units and length is 25 units

### Example 4

Two sides of triangular field are 5 and 7 inches longer than the third side. If the perimeter of the field is 21 inch. Find the length of sides of the field.

### Solution

Let  $x$  represents the length of third side

Let  $x+5$  and  $x+7$  represents the length

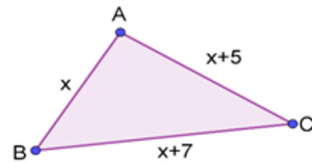
$$P=x+x+5+x+7$$

$$21=3x+12$$

$$21-12=3x$$

$$9=3x, \text{ i.e } x=3$$

The length of sides of the field is 3, 8 and 10



### Example 5

Majo has a land escape maintenance business. His monthly expenses are 1,100 birr. If he charges 60 birr per job how many jobs must he do earn a profit of at least 4000 a month?

### Solution

Let  $x$  the number jobs

$$60x-1100 \geq 4000$$

$$60x \geq 5100$$

$$x \geq 85 \text{ jobs}$$

### Example 6

Durete got a new job and will have to move. Her monthly income will be 5265 . To qualify to rent an apartment Durete monthly income must be at least three times as much as the rent. What is the highest rent Durete will qualify for?

### Solution

Let  $r$  = rent

Durete monthly income must be at least three times the rent

$$3r \leq 5265$$

$$r \leq 1755$$

A maximum rent is 1755 .

### Example 7

If a pair of shoes costs birr 52.50 including a  $7\frac{1}{4}\%$  tax. What is the original cost of the item before taxes are added?

### Solution

Begin by converting  $7\frac{1}{4}\%$  to a decimal  $\frac{29}{4}\% = 7.25\% = 0.0725$

Let  $C$  represents the cost of the item before taxes are added

$$\text{Amount of tax} = \text{tax rate} \times \text{cost of item} = 0.0725C$$

$$\text{Total cost} = \text{cost of item} + \text{amount of tax}$$

$$52.50 = C + 0.0725C$$

$$52.5 = 1.0725C$$

$$C = \frac{52.5}{1.0725} = 48.95$$

### Example 8

Between 1970 and 2010 average global temperature has increased by  $+1.1^{\circ}\text{F}(+0.6^{\circ}\text{C})$  or at a rate of  $0.028^{\circ}\text{F}/\text{year}$ . In 1970 the average global temperature was  $57.2^{\circ}\text{F}(+14.0^{\circ}\text{C})$ . Write a linear equation in the point slope form from what will be the temperature of population in the 2007?

### Solution

Point slope, coordinate of the point are (1970, 57.2)

The slope  $m = 0.28$

$$F - 57.2^{\circ}\text{F} = 0.028(y - 1970) \text{ in } 2020$$

$$F - 57.2 = 0.028(2020 - 1970)$$

$$F - 57.2 = 0.028(50)$$

$$F = 58.6^{\circ}\text{F}$$

### Example 9

The United States produced 1.2 billion tons of Carbon in 1985 at a growth rate of 0.02 billion tons per year. China produces 0.9 billion tons of Carbon in 2003 at a growth rate of 0.15 billion tons per year. In what year will China and US be producing the same amount of Carbon from fossil fuel consumption?

### Solution

First the two equations are:

$$\text{China: } y - 0.9 = 0.15(x - 2003)$$

$$\text{US: } y - 1.2 = 0.02(x - 1985) \text{ then simplify each}$$

$$\text{China: } y = 0.15x - 299.55 \text{ and}$$

$$\text{US: } y = 0.02x - 38.5$$

Now solve the two equations in intersection point  $(x, y)$

$$y = 0.15x - 299.5$$

$$y = 0.02x - 38.5, \text{ thus}$$

$$0.15x - 299.5 = 0.02x - 38.5$$

$$0.13x = 261$$

$$x = \frac{261}{0.13} = 2007.69 \approx 2008$$

$$y = 0.02x - 38.5 \text{ Substituting the value of } x \text{ here}$$

$$y = 0.02(2008) - 38.5 = 1.66 \text{ billion tone of carbon}$$

The intersection point is  $(2008, 1.66)$ . In the year 2008 China and US will each be producing 1.66 billion tons of carbon. After 2008 China will be producing more carbon than the Us.

## Unit Summary

1. An equation is a mathematical statement in which two quantities or two algebraic expressions are connected by the equality sign “=”.
2. A linear equation in one variable  $x$  is an equation which can be written in standard form  $ax + b = 0$ , where  $a$  and  $b$  are constant numbers with  $a \neq 0$ .

3. An inequality is a mathematical statement which contains the inequality symbols  $<$ ,  $>$ ,  $\leq$  or  $\geq$  to express that one quantity is greater than (or less than) another quantity.
4. Two linear inequalities are said to be equivalent, if and only if their solution sets are equal.
5. A linear inequality in one variable “x” is an inequality that can be written in the form of  $ax + b < 0$ ,  $ax + b \leq 0$  or  $ax + b > 0$ ,  $ax + b \geq 0$  where a and b are rational numbers and  $a \neq 0$ .
6. Rules of transformation for inequality:

Let a, b and c be any rational numbers

- a. If  $a < b$ , then  $a + c < b + c$ .....Addition property of inequality.
- b. If  $a < b$ , then  $a - c < b - c$ .....Subtraction property of inequality.
- c. If c is positive and  $a < b$ , then  $ac < bc$ .....Multiplication property of inequality.
- d. If c is positive and  $a < b$ , then .....Division property of inequality.
- e. If c is negative and  $a < b$ , then  $ac > bc$  .....Multiplication property of inequality.
- f. If c is negative and  $a < b$ , then .....Division property of inequality.

## Key terms

◆ Linear equation

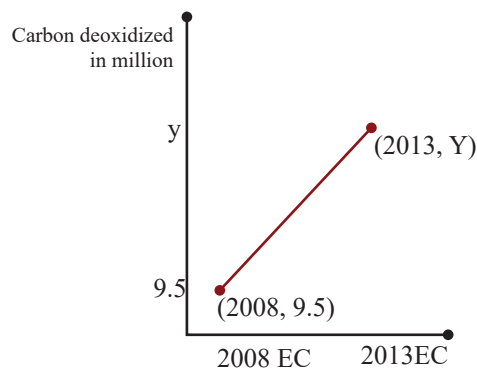
◆ Linear inequality

- ◆ Slope
- ◆ Abscissa
- ◆ Intercept
- ◆ Ordinate
- ◆ x-intercept
- ◆ Solution set
- ◆ y-intercept

## Review Exercise

1. Solve the following linear inequality under the domain of  $\mathbb{Q}$ 
  - a.  $\frac{2(x+1)}{3} > 6 - \frac{1}{4} + 3x$
  - b.  $\frac{3x}{5} + \frac{1}{2} \geq \frac{x}{4} - \frac{2}{5}$
  - c.  $\frac{-x-12}{4} \leq \frac{5x-7}{3} + 2$
  - d.  $2(x-3)-5 < 4(x-5)$
  - e.  $3(x+5)(x-1) < (3x+4)(x-2)$
  - f.  $3(x+4)(x-4) \geq (x-3)(3x+4)$
2. Find the solution set of the following inequalities under the given domain.
  - a.  $3x - 3 < -6, x \in \mathbb{W}$
  - b.  $7x + 6 \leq 3x + 2, x \in \mathbb{N}$
  - c.  $x - 4(x+1) \geq -13 - (x-2); x \in \mathbb{Z}^+$
  - d.  $14(x-4) < 8x - 16; x \in \mathbb{Q}^+$
3. The population of Bensa as a function of time  $t$  in years since 1995 follows:  $P(t) = 95,000 + 75t$ 
  - a. Find the population in 2004
  - b. Find the rate of change of the population.
  - c. Find the time and the year  $Y$  for the population to reach 3450 people
  - d. Solve for  $t$  in terms of  $P$ .

4. A computer costs birr 16000 and depreciates birr 1600 per year. Let  $V$  be the value (in birr) of the computer as a function of the age  $t$  (in years) of the computer. Find the equation relating  $V$  to  $t$
- Find  $V$  when  $t=2$ .
  - Find  $t$  when  $V=3200$ .
  - When does the computer lose all of its value?
5. Find the slope of the line passing through the following points.
- $A(3, 7)$  and  $B(5, -1)$
  - $C(1, -1)$  and  $D(3, 3)$
  - $E(6, 4)$  and  $F(-5, 0)$
  - $T(1, 8)$  and  $R(9, 8)$
  - $K(10, -3)$  and  $L(10, 1)$
  - $K(0, 0)$  and  $L(4, -4)$
- What will be the amount of carbon dioxide emitted in 2020 E.C?
  - Write equation of the graph using the formula  $y - y_1 = m(x - x_1)$
6. Ethiopia emits 9.5 million tons of carbon dioxide in 2008 E.C. If the rate of emission of every year is 0.3 million tone which is described in the following graph



# Unit 4

## SIMILARITY OF FIGURES

### Learning Outcomes:

At the end of this unit, learners will be able to:

- ▶ Know the concept of similar figures and related terminologies
- ▶ Understand the condition for triangles being similar.
- ▶ Apply tests to check whether two given triangles are similar or not.
- ▶ Apply real-life situations in solving geometric problems.

### Main content

#### 4.1 Similar Plane Figures

#### 4.2 Perimeter and Area of Similar Triangles

- ※ Key terms
- ※ Summary
- ※ Review Exercise

## 4.1 Similar Plane Figures

### Introduction

In terms of Mathematics, when two figures have the same shape but their sizes are different, then such figures are similar figures. For example, different sized photographs of a person i.e. stamp size; passport size, etc. depict the similar objects but are not congruent. In geometry, two similar shapes such as similar triangles, similar rectangles, similar square, are the shapes whose dimensions are in equal or common ratio but the size or length of their sides vary.

### Activity 4.1

Which of the following are similar figures?

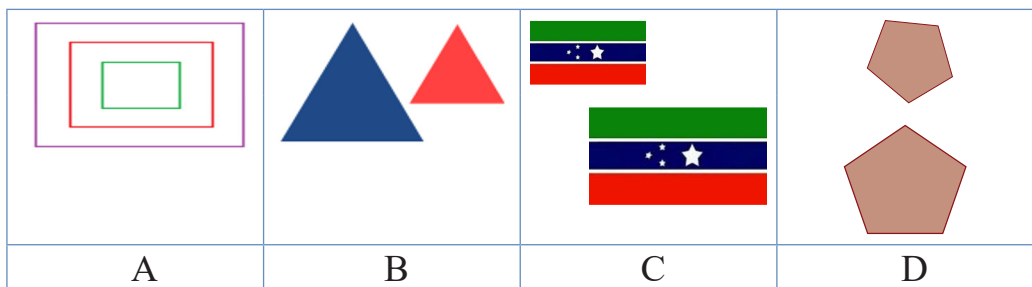


Figure -1

Similar figures are figures having the same shape. The objects which have exactly the same shape and size are known as congruent objects. For example, in real life you can see that, both the front wheels of a car, both hands of a person, etc. are examples of congruent figures or objects. But the similar shape objects have the same shape but their sizes may be different.

The following are an example of similar figures.

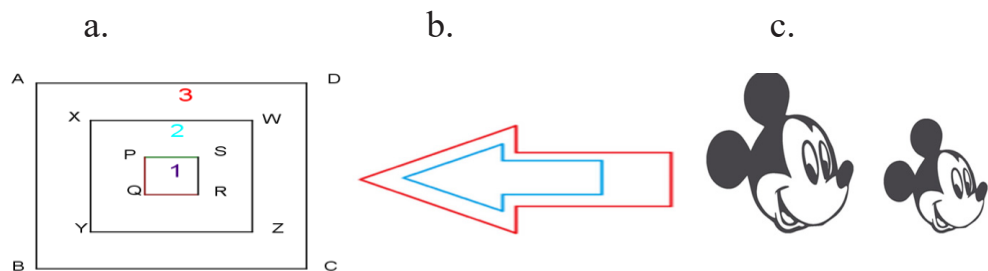


Figure 2

Because, Figure -2

- ◆ The three rectangles. PQRS, XYZW, and ABCD have the same shape but different sizes.
- ◆ The two arrows, the internal and the external have the same shape but different sizes.
- ◆ The two toys have the same shape but different sizes.

### 4.1.1 Definition and Illustration of Similar Figures

When an architect draws plans for a house or an engineer makes a drawing of a machine part, the result is drawn to scale showing the same objects in reduced sizes. This leads to the concept that two figures are similar if one of the figures can be uniformly enlarged so that it is identical with the other figure.

#### Group Work 4.1

1. Which of the following are similar figures?

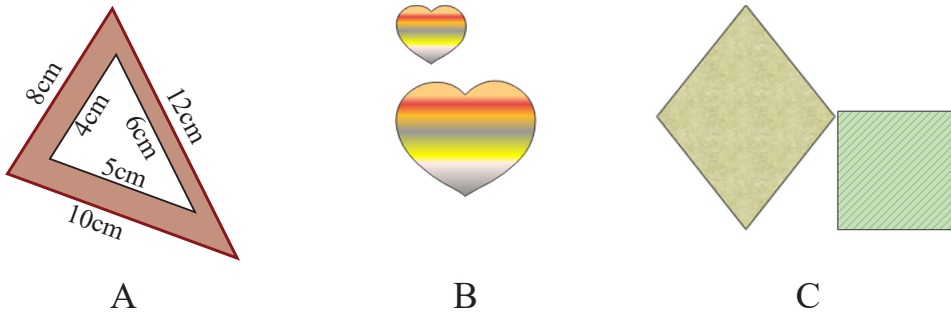


Figure -3

**Definition:**

Similar figures have exactly the same shape but not necessarily the same size. Corresponding sides of two figures are in the same relative position, and corresponding angles are in the same relative position.

**Example 2**

Observe this image of the butterflies and the squares

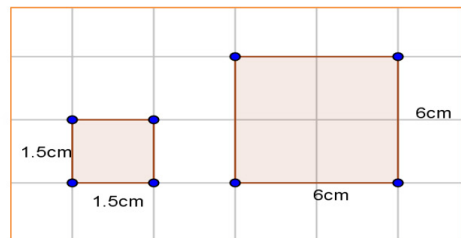


Figure - 4

The butterflies have the same shape but different sizes.

Therefore, the butterflies are similar.

The squares have the same shape but different in size.

Therefore, the squares are similar.

### Exercise 4.1

1. Are the following figures similar?

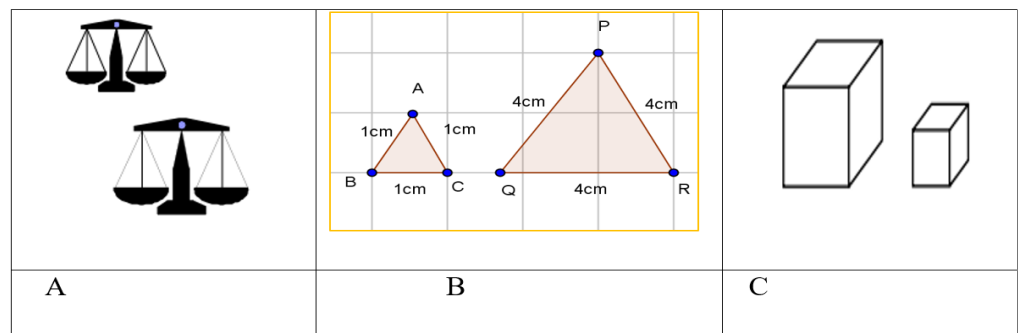


Figure -5

### 4.1.2 Similar Triangles

### Activity 4.2

1. Explain the following statements
  - a. How do you introduce similar triangles?
  - b. How do you know if two triangles are similar?
  - c. Can two triangles be similar and congruent?
  - d. Which type of triangles are always similar?
  - e. Can two isosceles triangles be similar?
2. Consider the figure 6 shown below:
  - a.  $\triangle XYZ$  is similar to  $\triangle PQR$ . Explain the reason.
  - b.  $\triangle XYZ$  is not similar to  $\triangle PRQ$ . Explain the reason.

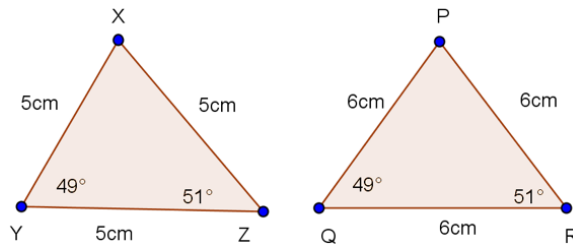


Figure 6

Similar triangles may have different individual lengths of the sides of triangles but their angles must be equal and their corresponding ratio of the length of the sides must be the same. We use the “ $\sim$ ” symbol to represent the similarity. If two triangles are similar, then we denote it as  $\triangle ABC \sim \triangle DEF$ .

The following figures are similar triangles with different size.

### Example 1

If  $\triangle ABC \sim \triangle DEF$ , then

- a. The lengths of corresponding sides are proportional

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

- b. All pairs of corresponding angles have equal measures.

$$m(\angle A) = m(\angle D), m(\angle B) = m(\angle E), m(\angle C) = m(\angle F).$$

So, the two triangles are similar.

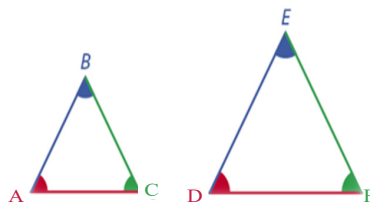


Figure 7

**Definition:**

Similar triangles are triangles that have corresponding sides are proportional and corresponding angles are congruent to each other. Similar triangles are the triangles that look the same but the sizes can be different.

There are various methods by which we can find if two triangles are similar or not. Let us learn more about similar triangles and their properties along with a few solved examples.

**Example 2**

Show that  $\triangle ABC$  and  $\triangle DEF$  are similar triangles.

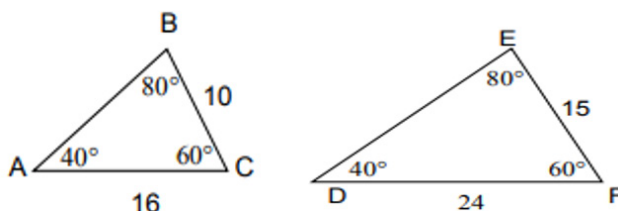


Figure 8

**Solution**

In similar triangles, the sides that are opposite to congruent angles are called corresponding sides. The lengths of each pair of corresponding sides can be written as a ratio. A proportion can then be written using the ratio of corresponding sides.

In  $\triangle ABC$  and  $\triangle DEF$  the corresponding angles are congruent. Also we can write a proportion as follows:

**Step 1.** Identify the pairs of corresponding sides. AC and DF are corresponding sides. (They are opposite to  $80^\circ$ ) BC and EF are corresponding sides. (They are opposite to  $40^\circ$ )

**Step 2.** Write a proportion using pairs of corresponding sides

$\frac{BC}{EF} = \frac{AC}{DF}$  or  $\frac{10}{15} = \frac{16}{24} = \frac{2}{3}$ , Therefore,  
 $\triangle ABC$  and  $\triangle DEF$  are similar triangles.

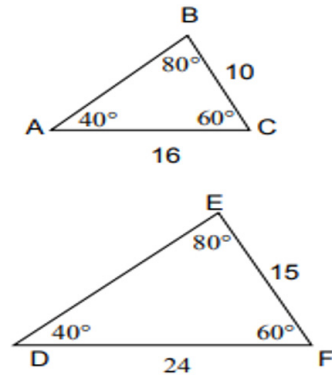


Figure 9

### Example 3

Let  $\triangle XYZ$  and  $\triangle RST$  are similar triangles. The length of side YZ is unknown and is represented as d. Find this length. (in cm)

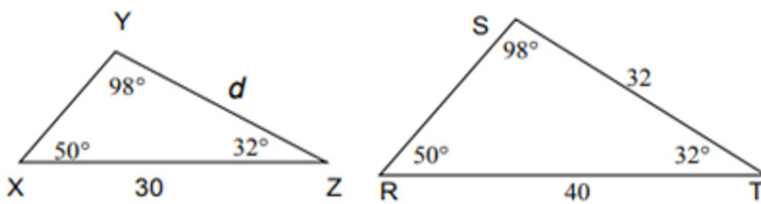
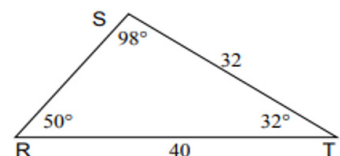
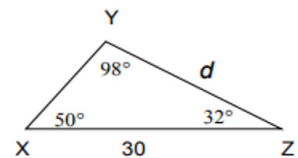


Figure 10

**Step 1.** Write a proportion using the corresponding sides. To simplify your work, write the proportion so that d (the unknown length) is the numerator of the left-side fraction.  $\frac{YZ}{ST} = \frac{XZ}{RT}$  or  $\frac{d}{32} = \frac{30}{40}$



**Step 2.** Cross multiply and solve for d

$$40d = 30(32) = 960, \quad d = 960/40 = 24$$

Therefore, The length of YZ, or d=24 cm

## Similarity of Polygons

We can use the fact that the sides of similar polygons are proportional. To determine whether if two polygons are similar or not we simply set up the ratios of corresponding sides and see if they are equal. If so, then the two polygons are similar. If not, then they are not similar.

For example, consider the two rectangles below:



*Figure 11*

As we've discussed, similar figures have the same shape. Therefore, when we have two similar figures, one is a larger or smaller version of the other. Because of this, when two polygons are similar, their sides are proportional. Being proportional means the ratios of corresponding sides on similar polygons are all equal. For example, consider these two similar rectangles.

$$\text{Ratio of the shorter sides} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Ratio of the longer sides} = \frac{3}{9} = \frac{1}{3}$$

Since  $\frac{1}{3} = \frac{1}{3}$ , the two sides of rectangles are proportional.

Therefore, the two rectangles are similar.

### Example 4

Is JKLM ~ WXYZ?

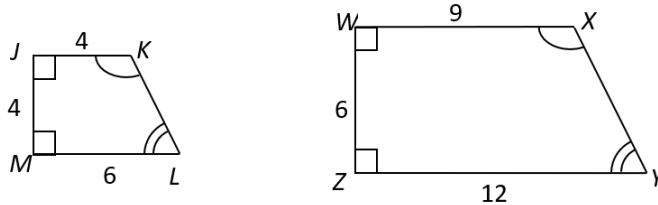


Figure 12

### Solution

The pairs of corresponding (matching) angles are congruent, but

The pairs of corresponding (matching) sides are

$$\frac{JM}{WZ} = \frac{ML}{ZY} = \frac{JK}{WX}$$

$$\frac{4}{6} \neq \frac{6}{12} \neq \frac{4}{9}$$

Corresponding sides are not in the same ratio.

Therefore, the figures are not similar.

### Example 5

In the figure 13,

$ABCDE \sim JKLMN$

- Find  $m(\angle K)$ .
- Find the length of MN

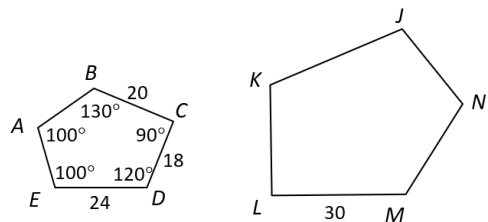


Figure 13

## Solution

a.  $m(\angle K) \equiv m(\angle B)$ , so  $m(\angle K) = 130^\circ$

b. We need to set up a proportion.

We know  $MN$  corresponds to  $DE$  and  $LM$  corresponds to  $CD$ .

So,  $\frac{MN}{DE} = \frac{LM}{CD}$ . This shows  $\frac{MN}{24} = \frac{30}{18}$  and  $18MN = 720$

$MN = 40$

## Exercise 4.2

- Two buildings stand side by side. The shorter building is 60 feet high and casts a shadow of 48 feet. At the same time of day, the taller building casts a shadow of 80 feet. What is the height ( $h$ ) of the taller building?

(Hint: Similar triangles are formed by the buildings, the ground, and the dotted lines that represent the sun's rays.)

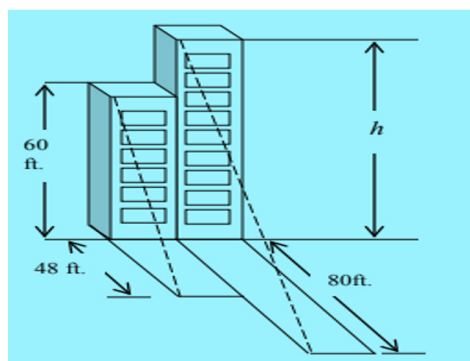


Figure 14

- Consider figures 15. Are the two polygons similar?

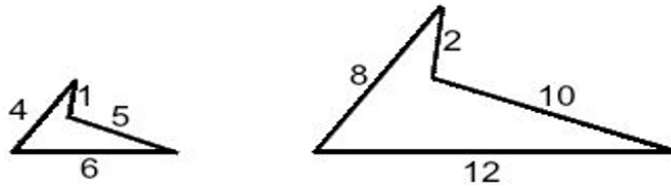


Figure 15

3. In the figure 16,  $\overline{PQ} \parallel \overline{AB}$
- Is  $\triangle ABC \sim \triangle PQC$  ?
  - Find the value of  $x$  and  $y$

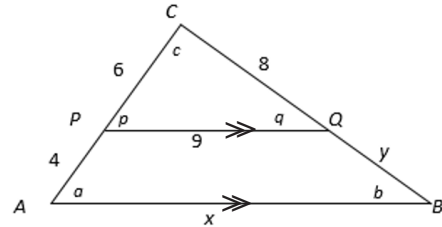


Figure 16

4. What are the preconditions that must be satisfied to say the two polygons  $ABCD \sim PQRS$  ?

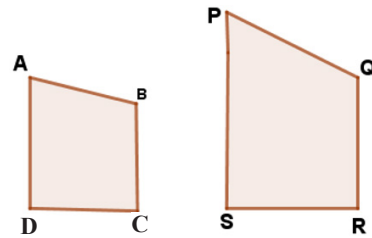


Figure 17

5. Are the two rectangles similar?

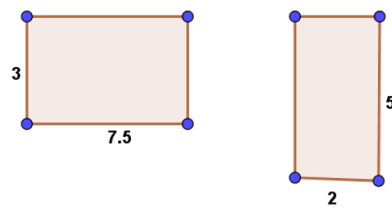


Figure 18

6. Are the two parallelograms similar?



Figure 19

7. A man is 6 feet tall is walking away from a street light. If the length of the man's shadow is 4 feet when he is 8 feet from the light, then how high is the light?

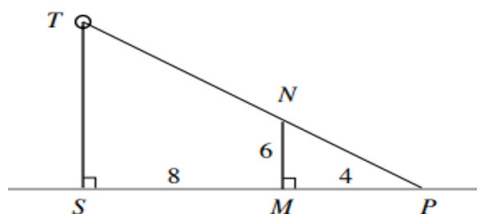


Figure 20

### 4.1.3 Tests for similarity of triangles (SSS, SAS, and AA)

#### Group work 4.3

Discusses with your friends and justify your answer

1. What are the theorems on similarity of triangles?
2. Is SSA a similarity theorem?
3. Can you apply AA, SAS and SSS similarity theorems to decide whether a given triangles are similar or not?

### Similar Triangles

The lengths of each pair of corresponding sides can be written as a ratio. A proportion can then be written using two of the ratios. We use rules when we do not have information about all sides and all angles of two triangles.

### Example 1

If two triangles have two pairs of sides in the same ratio and the included angles are also equal, then the triangles are similar.

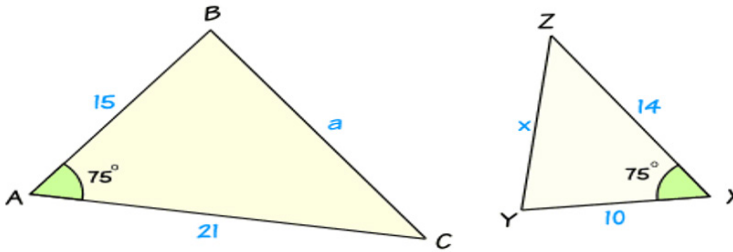


Figure 21

In this example we can see that:

- ◆ one pair of sides is in the ratio of  $21:14 = 3:2$
- ◆ another pair of sides is in the ratio of  $15:10 = 3:2$
- ◆ there is a matching angle of  $75^\circ$  in between them

So there is enough information to tell us that the two triangles are similar.

Let us see the theorems of similarity of triangles

### SSS similarity Theorem

#### Theorem 4.1

The SSS similarity Theorem states that if the three sides of one triangle are respectively proportional to the three sides of another, then the two triangles are similar. This essentially means that any such pair of triangles will be equiangular (All corresponding angle pairs are equal) also.

### Example 2

Consider the following figure 22:

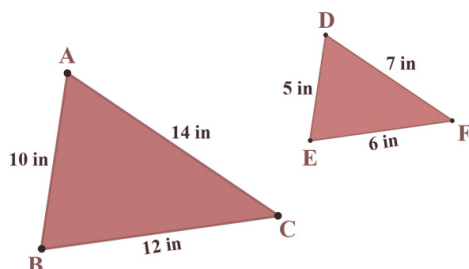


Figure 22

$$\frac{AB}{DE} = \frac{10}{5}, \frac{BC}{EF} = \frac{12}{6}, \frac{AC}{DF} = \frac{14}{7}$$

$$\text{That is, } \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = 2$$

The sides are proportional.

Therefore,  $\triangle ABC \sim \triangle DEF$ , by SSS similarity theorem.

#### Note:

If the three sides of a triangle are proportional to the corresponding sides of a second triangle, then the triangles are similar.

### Example 4

Justify whether the following triangles are similar

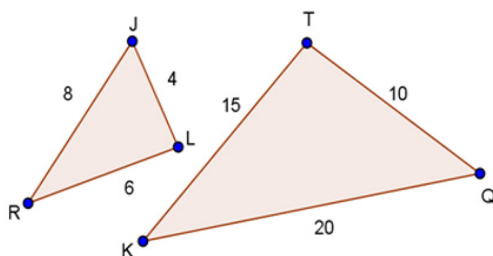


Figure 23

## Solution

By matching longest side to longest side, middle to middle, and shortest to shortest we have:

$$\frac{JL}{QT} = \frac{4}{10} = \frac{2}{5}, \frac{LR}{TK} = \frac{6}{15} = \frac{2}{5} \text{ and } \frac{JR}{QK} = \frac{8}{20} = \frac{2}{5}$$

$$\frac{JL}{QT} = \frac{LR}{TK} = \frac{JR}{QK} = \frac{2}{5}$$

Since the ratios of the corresponding sides are equal

Therefore,  $\triangle JLR \sim \triangle QTK$  by SSS Similarity theorem.

## SAS similarity Theorem

### Theorem 4.2

The SAS similarity Theorem states that If two sides of one triangle are respectively proportional to two corresponding sides of another, and if the included angles are equal, then the two triangles are similar.

### Note:

The emphasis on the word included. If the equal angle is a non-included angle, then the two triangles may not be similar.

## Example 5

Consider figure 24:

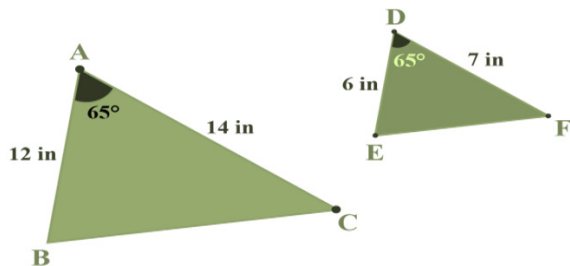


Figure 24

- ◆ The two corresponding sides are proportional,

$$\frac{AB}{DE} = \frac{AC}{DF} \text{ i.e. } \frac{12}{6} = \frac{14}{7} = 2$$

- ◆ The included angles are equal,  $m(\angle A) = m(\angle D)$

Therefore, by SAS similarity theorem,  $\triangle ABC \sim \triangle DEF$ .

### Example 6

In fig 25 if  $\triangle AGB \sim \triangle EDF$  are similar, then find the value of  $e$  if

$$\angle G = \angle D = 43^\circ, \quad b = 15, \quad f = 4, \quad a = 1$$

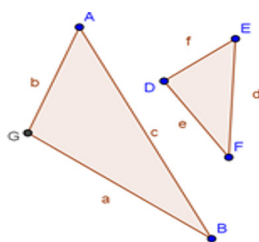


Figure 25

### Solution

Use the corresponding side lengths to write a proportion.

Since,  $\triangle AGB \sim \triangle EDF$  by SAS Similarity  $\angle G = \angle D = 43^\circ$

$$\frac{AG}{ED} = \frac{GB}{DF} = \frac{AB}{EF}, \text{ implies } \frac{15}{4} = \frac{1}{e}, \text{ solve for } e, 15e = 4 \text{ which is } e = \frac{4}{15}$$

### Example 7

Justify your answer that  $\triangle XYZ \sim \triangle ACB$  by SAS similarity theorem. In the given Fig 26  $\triangle XYZ$  and  $\triangle ACB$ , are given where,  $m(\angle Y) = m(\angle C) = 40^\circ$ ,  $XY = 6 \text{ cm}$ ,  $AC = 12 \text{ cm}$ ,  $YZ = 10 \text{ cm}$  and  $CB = 20 \text{ cm}$

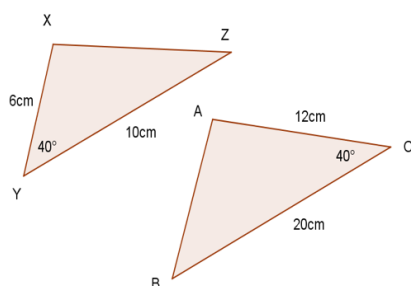


Figure 26

*Solution:*

Let  $\triangle XYZ$  and  $\triangle ACB$  be the triangles as shown. Then

| Statement s                                                                        | Reasons                                  |
|------------------------------------------------------------------------------------|------------------------------------------|
| $\angle Y \equiv \angle C$                                                         | Given                                    |
| $\frac{XY}{AC} = \frac{6\text{cm}}{12\text{cm}} = \frac{1}{2}$ , the ratio is 1:2  | two corresponding sides are proportiona. |
| $\frac{YZ}{CB} = \frac{10\text{cm}}{20\text{cm}} = \frac{1}{2}$ , the ratio is 1:2 |                                          |

Therefore  $\triangle XYZ \sim \triangle ACB$  by SAS similarity theorem

## AA (Angle – Angle) similarity Theorem

### Theorem 4.3

The AA similarity theorem states that if the two angles of one triangle are respectively equal to the corresponding two angles of the other, then the two triangles are similar. In short, equiangular triangles are similar.

### Example 1

Consider figure 27, in which  $\triangle ABC$  and  $\triangle DEF$  are equiangular

1.  $\angle A \equiv \angle D$ ,  $\angle B \equiv \angle E$  and  $\angle C \equiv \angle F$

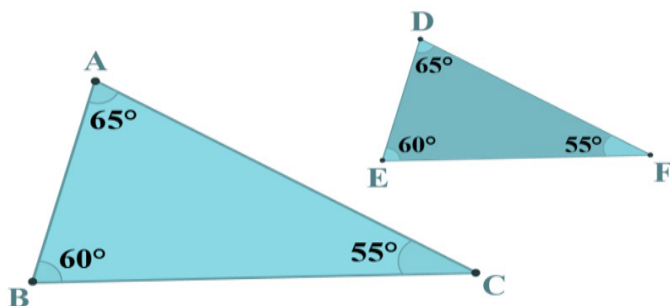


Figure 27

Using the AA similarity, we can say that these triangles are similar.  
 $\triangle ABC \sim \triangle DEF$

Example 2

In the figure 28 below,

- a. Are  $\triangle QPR$  and  $\triangle MRN$  similar?
- b. What is the value of  $x$ ,  $PR$  and  $PQ$  ?

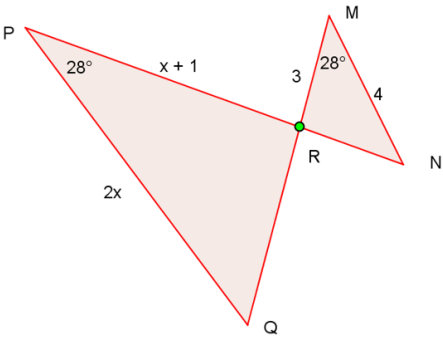


Figure 28

Solution:

| Statement s                        | Reasons                   |
|------------------------------------|---------------------------|
| $\angle P \equiv \angle M$         | Given                     |
| $\angle PRQ \equiv \angle MRN$     | Vertical opposite angle s |
| $\triangle PRQ \sim \triangle MRN$ | AA similarity             |

b. Hence,  $\triangle PRQ \sim \triangle MRN$  by AA similarity theorem .

Hence, The lengths of the corresponding sides are proportional:-

$\frac{PR}{MR} = \frac{RQ}{RN} = \frac{PQ}{MN}$  implies  $\frac{x+1}{3} = \frac{2x}{4}$  ..... solve for  $x$

$4(x+1) = 6x$

$4x+4=6x$  which is  $6x-4x=4$

$2x=4$

$x=2$  Hence, if  $x=2$  , then  $PR=3$  and  $PQ=4$

### Exercise 4.3

1. Consider Figure 29

$$AB = 2\sqrt{5}, BC = 2\sqrt{5} \text{ and } AC = 4$$

$$PQ = \sqrt{5}, QR = \sqrt{5} \text{ and } PR = 2$$

Show that the two triangles are similar.

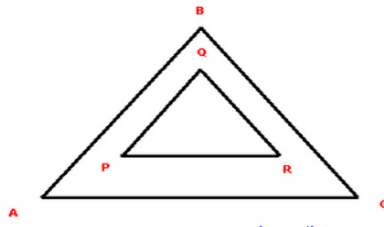


Figure 29

2. Are the following triangles are similar?

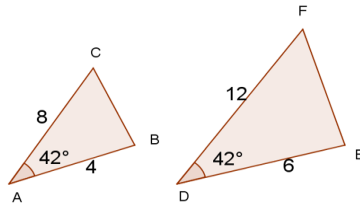


Figure 30

3. What is the value of  $x$

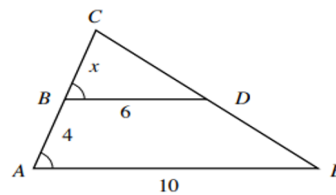


Figure 31

4. When a 6-feet student casts a 17-feet shadow, a flagpole casts a shadow that is 51 feet long. Find the height of the flagpole.
5. In fig 32 below determine similar triangles.

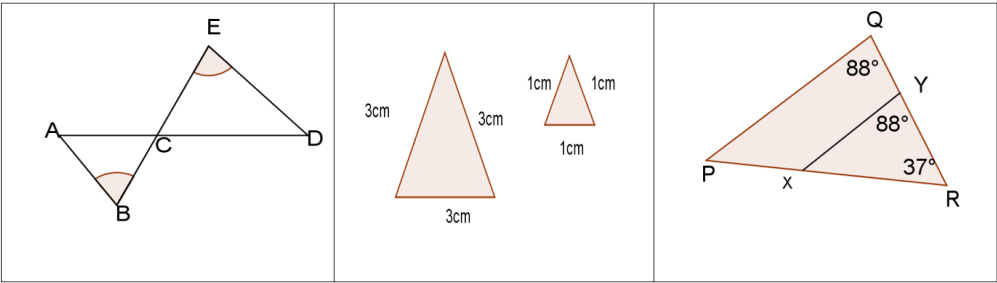
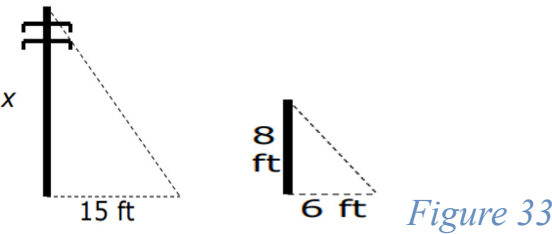
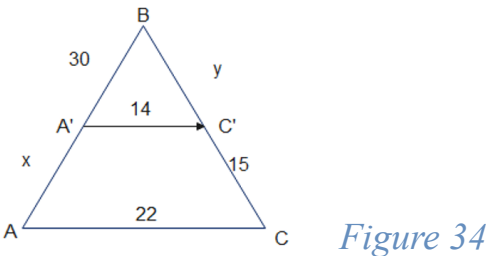


Figure 32

6. Find the height of the telephone pole.



7. Determine the length y of BC' and the length x of A'A are :



### 4.1.4 Perimeter and Area of Similar Triangles

Polygons are similar when the corresponding angles are equal, and the corresponding sides are in the same proportion. The scale factor for the sides of two similar polygons is the same as the ratio of the perimeters. In fact, the ratio of any part of two similar shapes (diagonals, medians, mid segments, altitudes, etc.) is the same as the scale factor. The ratio of the areas is the square of the scale factor. An

easy way to remember this is to think about the units of area, which are always squared. Therefore, you would always square the scale factor to get the ratio of the areas.

### Group work 4.4

1. Suppose figure 31a and figure b are given below:

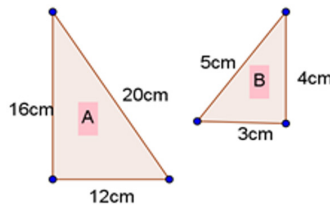


Figure 35a

Figure 35b

Based on the figures answer the following statements

- What is the scale factor of figure 35a to figure 35b?
- Determine the perimeters of figures 35a and 35b.
- What is the ratio of perimeters of figure 35a to figure 35b?
- Find the areas of figure 35a and figure b
- What is the ratio of areas of figure 35a to figure 35b?
- Express the relationship between scale factor and the ratio of perimeters.
- What is the relation between scale factor and the ratio of areas?

2. Let rectangle ABCD and EFGH in . Figure 36

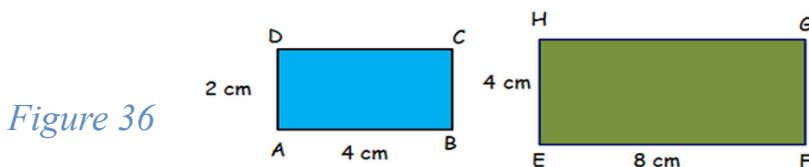


Figure 36

- a. Check whether the above rectangles are similar or not
  - b. Find the ratio of perimeters of rectangle ABCD to rectangle EFGH
  - c. Find the ratio of area of rectangle ABCD to rectangle EFGH
3. What is the ratio of the areas of two similar triangles if the ratio of
- a. their perimeters is  $\frac{5}{7}$
  - b. the corresponding sides is  $\frac{6}{11}$

### Perimeters of Similar Figures

If two figures are similar, then the ratio of their perimeters is equal to the ratio of their corresponding side lengths.

$$\frac{\text{perimetr of } \triangle ABC}{\text{perimetr of } \triangle DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

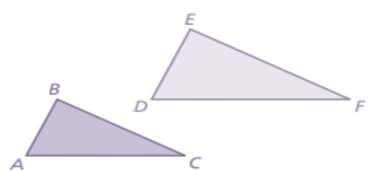


Figure 37

#### Theorem 4.4

If the ratios of the corresponding sides of two similar polygons is  $k$ , then the ratio of their perimeters is given by

$$\frac{P_1}{P_2} = \frac{S_1}{S_2} = k$$

#### Example 1

The length of two corresponding sides of two similar polygons are 4 and 7. If the perimeter of the smaller polygon is 20, find the perimeter of the larger polygon.

**Solution:**

Let  $s_1 = 4$  is the smaller side of the polygon

$s_2 = 7$  is the larger side of the polygon

Let  $p_1 = 20$  is perimeter of the smaller polygon

Let  $p_2 = x$  is perimeter of the larger polygon

Since, the polygons are similar, and then

$$\frac{p_1}{p_2} = \frac{s_1}{s_2} \text{ implies that, } \frac{20}{x} = \frac{4}{7} \text{ and this is equal to } 4x=140$$

$x=35$  , the perimeter of the larger polygon is 35.

**Note:**

If two similar figures have a scale factor of  $a : b$ , then the ratio of their perimeters is  $a : b$ .

**Areas of Similar Figures**

If two figures are similar, then the ratio of their areas is equal to the square of the ratio of their corresponding side length.

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

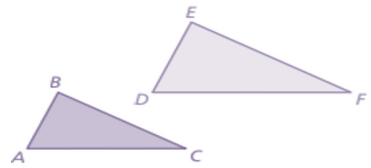


Figure 38

**Theorem 4.5**

If the ratios of the corresponding sides of two similar polygons is

$$\frac{s_1}{s_2} = k, \text{ then the ratio of their areas, is given by: } \frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2 = k^2$$

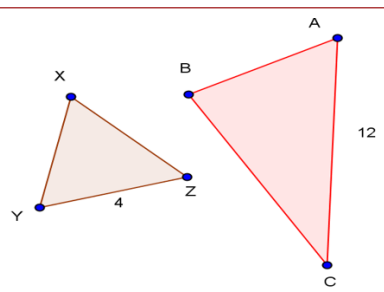
**Example 2**

Find the ratio of the areas of the similar triangles XYZ and ABC

*Solution*

$$\frac{\text{Area of } \triangle XYZ}{\text{Area of } \triangle ABC} = \left(\frac{4}{12}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

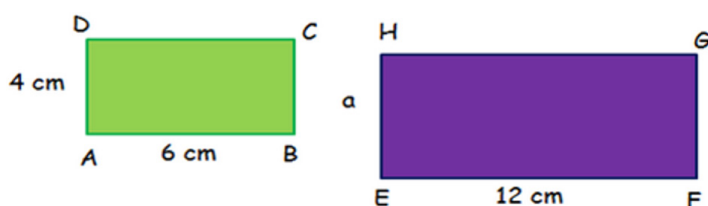
The ratio of the areas is  $\frac{1}{9}$  or 1:9

*Figure 39***Note:**

If two similar figures have a scale factor of  $a : b$ , then the ratio of their areas is  $a^2 : b^2$ .

*Example 3*

The two rectangles given below are similar. Find the perimeter of the rectangle EFGH.

*Figure 40**Solution:*

Because the above rectangles ABCD and EFGH are similar, the ratio of lengths of the corresponding sides will be proportional. That is,

$$\frac{AB}{EF} = \frac{AD}{EH}, \quad \frac{6}{12} = \frac{4}{a} \quad \dots \text{Simplify}$$

$$\frac{1}{2} = \frac{4}{a} \quad \text{Solve for } a;$$

$$8 = a$$

$$\begin{aligned}
 \text{Perimeter of rectangle EFGH is} &= 2(l + w) \\
 &= 2(12 + 8) \\
 &= 2(20) = 40\text{cm}
 \end{aligned}$$

So, the perimeter of the rectangle EFGH is 40cm.

Or directly,

Let  $p_s = 20$  is perimeter of the smaller rectangle is 20

Let  $p_l = x$  is perimeter of the larger rectangle

Since, the rectangles are similar, and

$$\frac{P_s}{P_l} = \frac{S_s}{S_l} \text{ implies that, } \frac{20}{x} = \frac{6}{12} = \frac{1}{2} \text{ and this is equal to } x=40$$

$x=40$  , the perimeter of the larger rectangle is 40cm.

### Exercise 4.4

- Two similar rectangles have a scale factor of 3:4
  - The perimeter of the larger rectangle is 68cm. What is the perimeter of the smaller rectangle?
  - The area of the smaller rectangle is  $72\text{cm}^2$ . What is the area of the larger rectangle?
- The perimeters of two similar triangles is in the ratio 3:4. The sum of their area is  $75\text{ cm}^2$ . Find the area of each triangle.
- Areas of two similar triangles are  $45\text{ cm}^2$  and  $80\text{ cm}^2$ . The sum of their Perimeter is 35cm. Find the perimeter of each triangle
- Two triangles are similar. The sides of the first triangle are 8, 5, and 12. The largest side of the second triangle is 24. Find the perimeter of the second triangle.

5. The ratio of the areas of two rectangles is 4:121. Find the perimeter of the larger rectangle if the perimeter of the smaller rectangle is 20 meters.
6. What is the value of  $a$  and the area of EFGH? If ABCD is similar to EFGH.

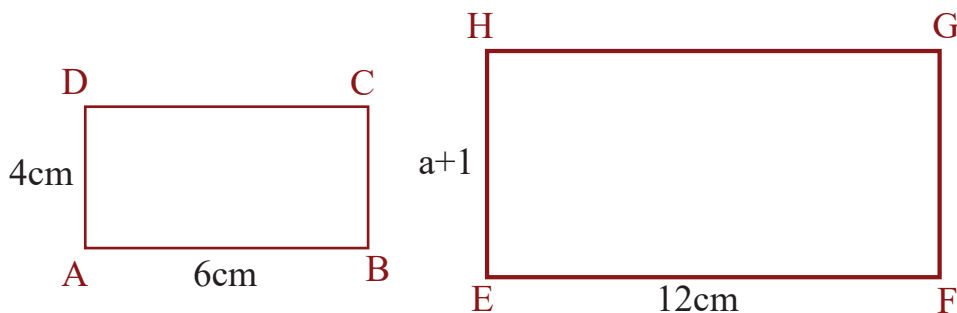


Figure 41

### Key words:

- ◆ Proportional
- ◆ Similar triangles
- ◆ AA similarity
- ◆ SAS similarity
- ◆ SSS similarity
- ◆ Corresponding
- ◆ Scale factor
- ◆ Similar figures

## Unit Summary

- ✧ In mathematics, saying that two figures are similar means that they share a common shape. They can be different sizes, but they must have the same shape
- ✧ Two triangles are similar if corresponding angles are congruent and corresponding sides are in the same ratio or proportion.

- ※ If two figures are similar then they are represented by the symbol ' $\sim$ '
- ※ SSS stands for “side- side,-side” , and means that if the three sides of one triangle are in proportion to the three sides of another triangle, then the two triangles are similar.

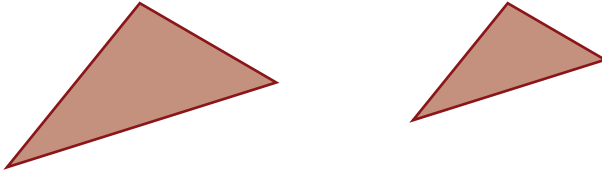


Figure 42

- ※ AA stands for “Angle -Angle” and means that the triangles have two of their angles equal.

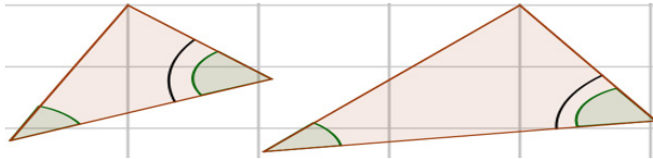


Figure 43

If two triangles have two of their angles equal, the triangles are similar.

- ※ SAS stands for “Side- Angle- Side” and means that if two triangles have two pairs of sides in the same ratio and the included angles are also equal, then the triangles are similar.

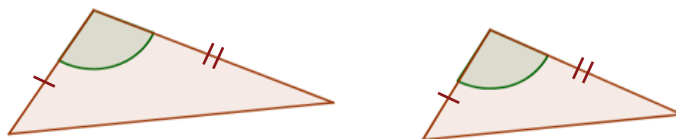


Figure 44

- ※ If the perimeters of two similar figures are in the ratio  $a : b$ , then their areas will be in the ratio  $a^2 : b^2$
- ※ If the areas of two similar figures are in the ratio  $a : b$ , then their perimeters will be in the ratio  $\sqrt{a} : \sqrt{b}$

## Review Exercise

1. If  $\triangle ABC \sim \triangle XYZ$ , then what is the value of  $k$ ?

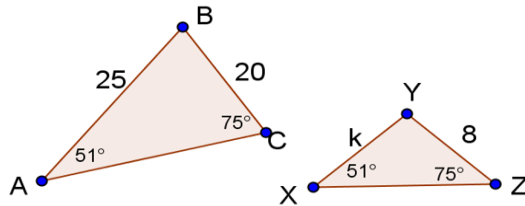


Figure 45

2. Determine whether the triangles are similar. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.

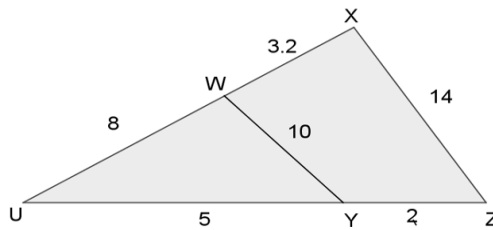


Figure 46

3. Find the value of  $x$  that makes  $\triangle ABC \sim \triangle DEF$ .

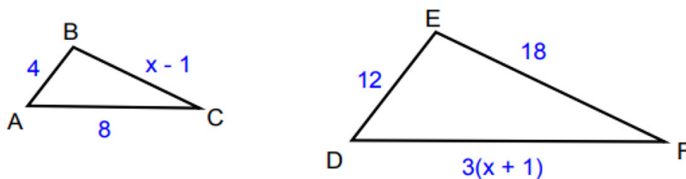


Figure 47

4. The two parallelograms shown below are similar. Find the perimeter of the parallelogram ABCD.

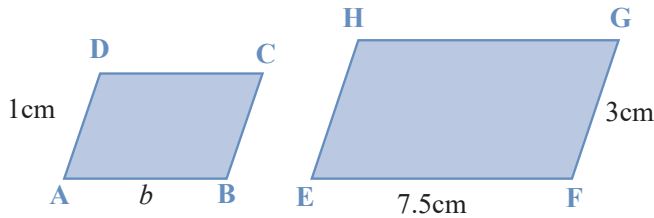


Figure 48

5. You place a picture on a page of a photo album. The page and the picture are similar rectangles.
- How many times greater is the area of the page than the area of the picture?
  - The area of the picture is 45 square centimeters. What is the area of the page?



Figure 49

6. The ratio of the corresponding side lengths of two similar computer screens is 14 : 21. The perimeter of the smaller screen is 42 inches. What is the perimeter of the larger screen?
7. What is the length of  $GD$  and  $DH$  from the given figure.

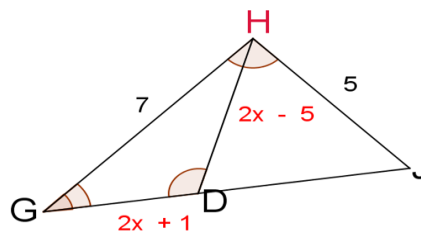


Figure 50

8. Rhombus A is similar to Rhombus B. What is the ratio (A to B) of the corresponding side lengths?

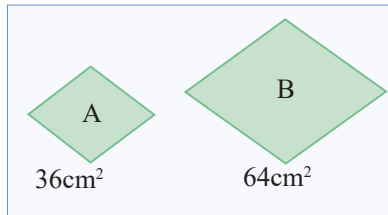


*Figure 51*

9. A research team wishes to determine the altitude of a mountain as follows (see figure below): They use a light source at L, mounted on a structure of height 2 meters, to shine a beam of light through the top of a pole P' through the top of the mountain M'. The height of the pole is 20 meters. The distance between the altitude of the mountain and the pole is 1000 meters. The distance between the pole and the laser is 10 meters. We assume that the light source mount, the pole and the altitude of the mountain are in the same plane. Find the altitude h of the mountain.

| The Function $y = x^2$ |       |       |       |       | $x \leq 1.00 \leq 9.99$ |       |       |       |       |       |
|------------------------|-------|-------|-------|-------|-------------------------|-------|-------|-------|-------|-------|
|                        | 0     | 1     | 2     | 3     | 4                       | 5     | 6     | 7     | 8     | 9     |
| 1.0                    | 1.000 | 1.020 | 1.040 | 1.061 | 1.082                   | 1.102 | 1.124 | 1.145 | 1.166 | 1.188 |
| 1.1                    | 1.210 | 1.232 | 1.254 | 1.277 | 1.277                   | 1.322 | 1.346 | 1.369 | 1.392 | 1.416 |
| 1.2                    | 1.440 | 1.464 | 1.488 | 1.513 | 1.513                   | 1.562 | 1.588 | 1.613 | 1.638 | 1.644 |
| 1.3                    | 1.690 | 1.716 | 1.742 | 1.769 | 1.769                   | 1.822 | 1850  | 1.877 | 1.904 | 1.932 |
| 1.4                    | 1.960 | 1.988 | 2.016 | 2.045 | 2.045                   | 2.102 | 2.132 | 2.161 | 2.190 | 2.220 |
| 1.5                    | 2.250 | 2.280 | 2316  | 2.341 | 2.341                   | 2.102 | 2.434 | 2.465 | 2.496 | 2.528 |
| 1.6                    | 2.560 | 2.592 | 2.310 | 2.657 | 2.657                   | 2.722 | 2.756 | 2.789 | 2.822 | 2.856 |
| 1.7                    | 2.890 | 2.924 | 2.624 | 2.993 | 2.993                   | 3.062 | 3.098 | 3.133 | 3.168 | 3.204 |
| 1.8                    | 3.240 | 3.276 | 2.958 | 3.349 | 3.349                   | 3.422 | 3.460 | 3.497 | 3.534 | 3.572 |
| 1.9                    | 3.610 | 3.648 | 3.686 | 3.725 | 3.764                   | 3.802 | 3.842 | 3.881 | 3.920 | 3.960 |
| 2.0                    | 4.000 | 4.040 | 4.080 | 4.121 | 4.162                   | 4.202 | 4.244 | 4.285 | 4.326 | 4.368 |
| 2.1                    | 4.410 | 4.452 | 4.494 | 4.537 | 4.580                   | 4.622 | 4.666 | 4.709 | 4.752 | 4.796 |
| 2.2                    | 4.840 | 4.884 | 4.928 | 4.973 | 5.018                   | 5.062 | 5.108 | 5.153 | 5.198 | 5.244 |
| 2.3                    | 5.290 | 5.336 | 5.382 | 5.429 | 5.476                   | 5.522 | 5.570 | 5.617 | 5.664 | 5.712 |
| 2.4                    | 5.760 | 5.808 | 5.856 | 5.905 | 5.954                   | 6.002 | 6.052 | 6.101 | 6.150 | 6.200 |
| 2.5                    | 6.250 | 6.300 | 6.350 | 6.401 | 6.452                   | 6.502 | 6.554 | 6.605 | 6.656 | 6.708 |
| 2.6                    | 6.760 | 6.812 | 6.864 | 6.917 | 6.970                   | 7.022 | 7.076 | 7.129 | 7.182 | 7.236 |
| 2.7                    | 7.290 | 7.344 | 7.398 | 7.453 | 7.508                   | 7.562 | 7.618 | 7.673 | 7.728 | 7.784 |
| 2.8                    | 7.840 | 7.896 | 7.952 | 8.009 | 8.066                   | 8.122 | 8.180 | 8.237 | 8.294 | 8.352 |
| 2.9                    | 8.410 | 8.468 | 8.526 | 8.585 | 8.644                   | 8.702 | 8.762 | 8.821 | 8.880 | 8.940 |
| 3.0                    | 9.000 | 9.060 | 9.120 | 9.181 | 9.242                   | 9.302 | 9.364 | 9.425 | 9.486 | 9.548 |
| 3.1                    | 9.610 | 9.672 | 9.734 | 9.797 | 9.860                   | 9.922 | 9.986 | 10.05 | 10.11 | 10.18 |
| 3.2                    | 10.24 | 10.30 | 10.37 | 10.43 | 10.50                   | 10.56 | 10.63 | 10.69 | 10.76 | 11.82 |
| 3.3                    | 10.89 | 10.96 | 11.02 | 11.09 | 11.16                   | 11.22 | 11.29 | 11.36 | 11.42 | 11.49 |
| 3.4                    | 11.56 | 11.63 | 11.70 | 11.76 | 11.83                   | 11.90 | 11.97 | 12.04 | 12.11 | 12.18 |
| 3.5                    | 14.25 | 12.32 | 12.39 | 12.46 | 12.53                   | 12.60 | 12.67 | 12.74 | 12.82 | 12.89 |
| 3.6                    | 12.96 | 13.03 | 13.10 | 13.18 | 13.25                   | 13.32 | 13.40 | 13.47 | 13.54 | 13.62 |
| 3.7                    | 13.69 | 13.76 | 13.84 | 13.91 | 13.99                   | 14.06 | 14.14 | 14.21 | 14.29 | 14.36 |
| 3.8                    | 14.44 | 14.52 | 14.59 | 14.67 | 14.75                   | 14.82 | 14.90 | 14.98 | 15.08 | 15.13 |
| 3.9                    | 15.21 | 15.29 | 15.37 | 15.44 | 15.52                   | 15.60 | 15.68 | 15.76 | 15.84 | 15.92 |
| 4.0                    | 16.00 | 16.08 | 16.16 | 16.24 | 16.32                   | 16.40 | 16.48 | 16.56 | 16.65 | 16.73 |
| 4.1                    | 16.81 | 16.89 | 16.97 | 17.06 | 17.14                   | 17.22 | 17.31 | 17.39 | 17.47 | 17.56 |
| 4.2                    | 17.64 | 17.72 | 17.81 | 17.89 | 17.98                   | 18.06 | 18.15 | 18.23 | 18.32 | 18.40 |
| 4.3                    | 18.49 | 18.58 | 18.66 | 18.75 | 18.84                   | 18.92 | 19.01 | 19.10 | 19.18 | 19.27 |

|     |       |       |       |       |       |       |       |       |       |       |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 4.4 | 19.96 | 19.45 | 19.54 | 19.62 | 19.71 | 19.80 | 19.89 | 19.98 | 20.98 | 20.16 |
| 4.5 | 20.25 | 20.34 | 20.43 | 20.52 | 20.61 | 20.70 | 20.79 | 20.88 | 21.90 | 21.07 |
| 4.6 | 21.16 | 21.25 | 21.34 | 21.44 | 21.53 | 21.62 | 21.72 | 21.81 | 22.85 | 22.00 |
| 4.7 | 22.09 | 22.18 | 22.28 | 22.37 | 22.47 | 22.56 | 22.66 | 22.75 | 23.81 | 22.94 |
| 4.8 | 23.04 | 23.14 | 23.33 | 23.33 | 23.43 | 23.52 | 23.62 | 23.72 | 24.80 | 23.91 |
| 4.9 | 24.01 | 24.11 | 24.24 | 24.30 | 24.40 | 24.50 | 24.60 | 24.70 | 25.81 | 24.90 |
| 5.0 | 25.00 | 25.10 | 25.20 | 25.30 | 25.40 | 25.50 | 25.60 | 25.70 | 26.83 | 25.91 |
| 5.1 | 26.01 | 26.11 | 26.21 | 26.32 | 26.42 | 26.52 | 26.63 | 26.73 | 27.88 | 26.94 |
| 5.2 | 27.04 | 27.14 | 27.25 | 27.35 | 27.46 | 27.56 | 27.67 | 27.77 | 28.94 | 27.98 |
| 5.3 | 28.09 | 28.20 | 28.30 | 28.41 | 28.52 | 28.62 | 28.73 | 28.84 | 28.94 | 29.05 |
| 5.4 | 29.16 | 29.27 | 29.38 | 29.48 | 29.59 | 29.70 | 29.81 | 29.92 | 30.03 | 30.14 |
| 5.5 | 30.25 | 30.36 | 30.47 | 30.58 | 30.69 | 30.80 | 30.91 | 31.02 | 31.14 | 31.25 |
| 5.6 | 31.36 | 31.47 | 31.58 | 31.70 | 31.81 | 31.92 | 32.04 | 32.15 | 32.26 | 32.38 |
| 5.7 | 32.46 | 32.60 | 32.72 | 32.83 | 32.95 | 33.06 | 33.18 | 33.29 | 33.41 | 33.52 |
| 5.8 | 33.64 | 33.76 | 33.87 | 33.99 | 34.11 | 34.22 | 34.34 | 34.46 | 34.57 | 34.69 |
| 5.9 | 34.81 | 34.93 | 35.05 | 35.16 | 35.28 | 35.40 | 35.52 | 35.64 | 35.76 | 35.88 |
| 6.0 | 36.00 | 36.12 | 36.24 | 36.36 | 36.48 | 36.60 | 36.72 | 36.84 | 36.97 | 37.09 |
| 6.1 | 37.21 | 37.33 | 37.45 | 37.58 | 37.70 | 37.82 | 37.95 | 38.07 | 38.19 | 38.32 |
| 6.2 | 38.44 | 38.56 | 38.69 | 38.81 | 38.94 | 39.06 | 39.19 | 39.31 | 39.44 | 39.56 |
| 6.3 | 39.69 | 39.82 | 39.94 | 40.07 | 40.20 | 40.32 | 40.45 | 40.58 | 40.70 | 40.83 |
| 6.4 | 40.96 | 41.09 | 41.22 | 41.34 | 41.47 | 41.60 | 41.73 | 41.86 | 41.99 | 42.12 |
| 6.5 | 42.25 | 42.38 | 42.51 | 42.64 | 42.77 | 42.90 | 43.03 | 43.16 | 43.30 | 43.43 |
| 6.6 | 43.56 | 43.69 | 43.82 | 43.96 | 44.09 | 44.22 | 44.36 | 44.49 | 44.62 | 44.77 |
| 6.7 | 44.89 | 45.02 | 45.16 | 45.29 | 45.43 | 45.56 | 45.70 | 45.83 | 45.96 | 46.10 |
| 6.8 | 46.24 | 46.38 | 46.51 | 46.65 | 46.79 | 46.92 | 47.06 | 47.20 | 47.33 | 47.47 |
| 6.9 | 47.61 | 47.75 | 47.89 | 48.02 | 48.16 | 48.30 | 48.44 | 48.58 | 48.71 | 48.86 |
| 7.0 | 49.00 | 49.14 | 49.28 | 49.42 | 49.56 | 49.70 | 49.84 | 49.98 | 50.11 | 50.27 |
| 7.1 | 50.41 | 50.55 | 50.69 | 50.84 | 50.98 | 51.12 | 51.27 | 51.41 | 51.55 | 51.70 |
| 7.2 | 51.84 | 51.98 | 52.13 | 52.27 | 52.42 | 52.56 | 52.71 | 52.85 | 52.99 | 53.14 |
| 7.3 | 53.29 | 53.44 | 53.58 | 53.73 | 53.88 | 54.02 | 54.17 | 54.32 | 54.46 | 54.61 |
| 7.4 | 54.76 | 54.91 | 55.06 | 55.20 | 55.35 | 55.50 | 55.65 | 55.80 | 55.94 | 56.10 |
| 7.5 | 56.25 | 56.40 | 56.55 | 56.70 | 56.85 | 57.00 | 57.15 | 57.30 | 57.45 | 57.61 |
| 7.6 | 57.76 | 57.91 | 58.06 | 58.22 | 58.37 | 58.52 | 58.68 | 58.83 | 58.98 | 59.14 |
| 7.7 | 59.29 | 59.44 | 59.60 | 59.75 | 59.91 | 60.06 | 60.22 | 60.37 | 60.52 | 60.68 |
| 7.8 | 60.84 | 61.00 | 61.15 | 61.31 | 61.47 | 61.62 | 61.78 | 61.94 | 62.09 | 62.25 |
| 7.9 | 62.41 | 62.57 | 62.73 | 62.88 | 63.04 | 63.20 | 63.36 | 63.52 | 63.67 | 63.84 |
| 8.0 | 64.00 | 64.16 | 64.32 | 64.48 | 64.64 | 64.80 | 64.96 | 65.12 | 65.29 | 65.45 |

|     |       |       |       |       |       |       |       |       |       |       |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 8.1 | 65.61 | 65.77 | 65.93 | 66.10 | 66.26 | 66.42 | 66.59 | 66.75 | 66.91 | 67.08 |
| 8.2 | 67.24 | 67.40 | 67.57 | 67.73 | 67.90 | 68.06 | 68.23 | 68.39 | 68.56 | 68.72 |
| 8.3 | 68.29 | 69.06 | 69.22 | 69.39 | 69.56 | 69.72 | 69.89 | 70.06 | 70.22 | 70.39 |
| 8.4 | 70.56 | 70.73 | 70.90 | 71.06 | 71.23 | 71.40 | 71.57 | 71.74 | 71.91 | 72.01 |
| 8.5 | 72.25 | 72.42 | 72.59 | 72.76 | 72.93 | 73.10 | 73.27 | 73.44 | 73.62 | 73.79 |
| 8.6 | 73.96 | 74.13 | 74.30 | 74.48 | 74.65 | 74.82 | 75.00 | 75.17 | 75.34 | 75.52 |
| 8.7 | 75.69 | 75.86 | 76.04 | 76.21 | 76.39 | 76.56 | 76.74 | 76.91 | 76.09 | 77.26 |
| 8.8 | 77.44 | 77.62 | 77.79 | 77.97 | 78.15 | 78.32 | 78.50 | 78.68 | 78.85 | 79.03 |
| 8.9 | 79.21 | 79.39 | 79.57 | 79.74 | 79.92 | 80.10 | 80.28 | 80.46 | 80.46 | 80.82 |
| 9.0 | 81.00 | 81.18 | 84.36 | 81.54 | 81.72 | 81.90 | 82.08 | 82.26 | 82.45 | 82.63 |
| 9.1 | 82.81 | 82.99 | 83.17 | 83.36 | 83.54 | 83.72 | 83.91 | 84.09 | 86.12 | 84.46 |
| 9.2 | 84.64 | 84.82 | 85.01 | 85.19 | 85.38 | 85.56 | 85.75 | 84.93 | 87.96 | 86.30 |
| 9.3 | 86.49 | 86.68 | 86.86 | 87.05 | 87.24 | 87.42 | 87.61 | 87.80 | 87.96 | 88.17 |
| 9.4 | 88.36 | 88.55 | 88.74 | 88.92 | 89.11 | 89.30 | 89.49 | 89.68 | 89.87 | 90.06 |
| 9.5 | 90.25 | 90.44 | 90.63 | 90.82 | 91.01 | 91.20 | 91.39 | 91.58 | 91.78 | 91.97 |
| 9.6 | 92.16 | 92.35 | 92.54 | 92.74 | 92.93 | 93.12 | 93.32 | 93.51 | 93.70 | 93.90 |
| 9.7 | 94.09 | 94.28 | 94.67 | 94.67 | 94.87 | 95.06 | 95.26 | 95.45 | 95.65 | 95.84 |
| 9.8 | 96.04 | 96.24 | 96.43 | 96.63 | 96.83 | 97.02 | 97.22 | 97.42 | 97.61 | 97.81 |
| 9.9 | 98.01 | 98.21 | 98.41 | 98.60 | 98.80 | 99.00 | 99.20 | 99.40 | 99.60 | 99.80 |

$(8.47)^2 = 71.74$

$(8.472)^2 = 71.77$

$(84.7)^2 = 7174$

$\sqrt{21.44} = 4.63$

$(0.847)^2 = 0.7174$

$\sqrt{0.2144} = 0.463$

If you move the comma in x one digit to the right (left), then the comma in x2 must be moved two digits to the right (left).

| The Function $y = x^3$ |       |       |       |       | $x \leq 1.00 \leq 9.99$ |       |       |       |       |       |
|------------------------|-------|-------|-------|-------|-------------------------|-------|-------|-------|-------|-------|
| x                      | 0     | 1     | 2     | 3     | 4                       | 5     | 6     | 7     | 8     | 9     |
| 1.0                    | 1.000 | 1.030 | 1.061 | 1.093 | 1.125                   | 1.158 | 1.191 | 1.225 | 1.260 | 1.295 |
| 1.1                    | 1.331 | 1.368 | 1.405 | 1.443 | 1.482                   | 1.521 | 1.561 | 1.602 | 1.643 | 1.685 |
| 1.2                    | 1.728 | 1.772 | 1.816 | 1.861 | 1.907                   | 1.953 | 2.000 | 2.048 | 2.097 | 2.147 |
| 1.3                    | 2.197 | 2.248 | 2.300 | 2.353 | 2.406                   | 2.460 | 2.515 | 2.571 | 2.628 | 2.686 |
| 1.4                    | 2.744 | 2.803 | 2.863 | 2.924 | 2.986                   | 3.049 | 3.112 | 3.177 | 3.242 | 3.308 |
| 1.5                    | 3.375 | 3.443 | 3.512 | 3.582 | 3.652                   | 3.724 | 3.796 | 3.870 | 3.944 | 4.020 |
| 1.6                    | 4.096 | 4.173 | 4.252 | 4.331 | 4.411                   | 4.492 | 4.574 | 4.657 | 4.742 | 4.827 |
| 1.7                    | 4.913 | 5.000 | 5.088 | 5.178 | 5.268                   | 5.359 | 5.452 | 5.545 | 5.640 | 5.735 |
| 1.8                    | 5.832 | 5.930 | 6.029 | 6.128 | 6.230                   | 6.332 | 6.435 | 6.539 | 6.645 | 6.751 |

|     |       |       |       |       |       |       |       |       |       |       |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1.9 | 6.859 | 6.968 | 7.078 | 7.189 | 7.301 | 7.415 | 7.530 | 7.645 | 7.762 | 7.881 |
| 2.0 | 8.000 | 8.121 | 8.242 | 8.365 | 8.490 | 8.615 | 8.742 | 9.870 | 8.999 | 9.129 |
| 2.1 | 9.261 | 9.339 | 9.528 | 9.664 | 9.800 | 9.938 | 10.08 | 10.22 | 10.36 | 10.50 |
| 2.2 | 10.65 | 10.79 | 11.94 | 11.09 | 11.24 | 11.39 | 11.54 | 11.70 | 11.85 | 12.01 |
| 2.3 | 12.17 | 12.33 | 12.49 | 12.65 | 12.81 | 12.98 | 13.14 | 13.31 | 31.48 | 13.65 |
| 2.4 | 13.82 | 14.00 | 14.17 | 14.35 | 14.53 | 14.71 | 14.89 | 15.07 | 15.25 | 15.44 |
| 2.5 | 15.63 | 15.81 | 16.00 | 16.19 | 16.39 | 16.58 | 16.78 | 16.97 | 17.17 | 17.37 |
| 2.6 | 17.58 | 17.78 | 17.98 | 18.19 | 18.40 | 18.61 | 18.82 | 19.03 | 19.25 | 19.47 |
| 2.7 | 19.68 | 19.90 | 20.12 | 20.35 | 20.57 | 20.08 | 21.02 | 21.25 | 21.48 | 21.72 |
| 2.8 | 21.95 | 22.19 | 22.43 | 22.67 | 22.91 | 23.15 | 23.39 | 23.64 | 23.89 | 24.14 |
| 2.9 | 24.39 | 24.64 | 24.90 | 25.15 | 25.41 | 25.67 | 25.93 | 26.20 | 25.46 | 26.73 |
| 3.0 | 27.00 | 27.27 | 27.54 | 27.82 | 28.09 | 28.37 | 28.65 | 28.93 | 29.22 | 29.50 |
| 3.1 | 29.79 | 30.08 | 30.37 | 30.66 | 30.96 | 26    | 31.55 | 31.86 | 32.16 | 32.46 |
| 3.2 | 32.77 | 33.08 | 33.39 | 33.70 | 34.01 | 34.33 | 34.65 | 34.97 | 35.29 | 35.61 |
| 3.3 | 35.94 | 36.26 | 36.59 | 36.93 | 37.26 | 37.60 | 37.93 | 38.27 | 38.61 | 38.96 |
| 3.4 | 39.30 | 39.65 | 40.00 | 40.35 | 40.71 | 41.06 | 41.42 | 41.78 | 42.14 | 42.51 |
| 3.5 | 42.88 | 43.24 | 43.61 | 43.99 | 44.36 | 44.47 | 45.12 | 45.50 | 45.88 | 46.27 |
| 3.6 | 46.66 | 47.05 | 47.44 | 47.83 | 48.23 | 48.63 | 49.03 | 49.43 | 49.84 | 50.24 |
| 3.7 | 50.65 | 51.06 | 51.48 | 51.90 | 52.31 | 52.73 | 53.16 | 53.58 | 54.01 | 54.44 |
| 3.8 | 54.87 | 55.31 | 55.74 | 56.18 | 56.62 | 57.07 | 57.51 | 57.96 | 58.41 | 58.86 |
| 3.9 | 59.32 | 59.78 | 60.24 | 60.70 | 61.16 | 61.63 | 62.10 | 62.57 | 63.04 | 63.52 |
| 4.0 | 64.00 | 64.48 | 64.96 | 65.45 | 65.94 | 66.43 | 66.92 | 67.42 | 67.92 | 68.42 |
| 4.1 | 68.92 | 69.43 | 69.91 | 70.44 | 70.96 | 71.47 | 71.99 | 72.51 | 73.03 | 73.56 |
| 4.2 | 74.09 | 74.62 | 75.15 | 75.69 | 76.23 | 76.77 | 77.31 | 77.85 | 78.40 | 79.95 |
| 4.3 | 79.51 | 80.06 | 80.62 | 81.18 | 81.75 | 82.31 | 82.88 | 83.45 | 84.03 | 84.60 |
| 4.4 | 85.18 | 85.77 | 86.35 | 86.94 | 87.53 | 88.12 | 88.72 | 89.31 | 89.92 | 90.52 |
| 4.5 | 91.13 | 91.73 | 92.35 | 92.96 | 93.58 | 94.20 | 94.82 | 95.44 | 96.07 | 96.70 |
| 4.6 | 97.34 | 97.97 | 98.61 | 99.25 | 99.90 | 100.5 | 101.2 | 101.8 | 102.5 | 103.2 |
| 4.7 | 103.8 | 104.5 | 105.2 | 105.8 | 106.5 | 107.2 | 107.9 | 108.5 | 109.2 | 109.9 |
| 4.8 | 110.6 | 111.3 | 112.0 | 112.7 | 113.4 | 114.1 | 114.8 | 115.5 | 116.2 | 116.9 |
| 4.9 | 117.6 | 118.4 | 119.1 | 119.8 | 120.6 | 121.3 | 122.0 | 122.8 | 123.5 | 124.3 |
| 5.0 | 125.0 | 125.8 | 126.5 | 127.3 | 128.0 | 128.8 | 129.6 | 130.3 | 131.1 | 131.9 |
| 5.1 | 132.7 | 133.4 | 134.2 | 135.0 | 135.8 | 136.6 | 137.4 | 138.2 | 139.0 | 139.8 |
| 5.2 | 140.6 | 141.4 | 142.2 | 143.1 | 143.9 | 144.7 | 145.5 | 146.4 | 147.2 | 148.0 |
| 5.3 | 148.9 | 149.7 | 150.6 | 151.4 | 152.3 | 153.1 | 154.0 | 154.5 | 155.7 | 156.6 |
| 5.4 | 157.5 | 158.3 | 159.2 | 160.1 | 161.0 | 161.9 | 162.8 | 163.7 | 164.6 | 165.5 |
| 5.5 | 166.4 | 167.3 | 168.2 | 169.1 | 170.0 | 171.0 | 171.9 | 172.8 | 173.7 | 174.7 |
| 5.6 | 175.6 | 176.6 | 177.5 | 178.5 | 179.4 | 180.4 | 181.3 | 182.3 | 183.3 | 184.2 |

|     |       |       |       |       |       |       |       |       |       |       |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 5.7 | 185.2 | 186.2 | 187.1 | 188.1 | 189.1 | 190.1 | 191.1 | 192.1 | 193.1 | 194.1 |
| 5.8 | 195.1 | 196.1 | 197.1 | 198.2 | 199.2 | 200.2 | 201.2 | 202.3 | 203.3 | 204.3 |
| 5.9 | 205.4 | 206.4 | 207.5 | 208.5 | 209.6 | 210.6 | 211.7 | 212.8 | 213.8 | 214.9 |
| 6.0 | 216.0 | 217.1 | 218.2 | 219.3 | 220.3 | 221.4 | 211.7 | 223.6 | 224.8 | 225.9 |
| 6.1 | 227.0 | 228.1 | 229.2 | 230.0 | 231.5 | 232.6 | 222.5 | 234.9 | 236.0 | 237.2 |
| 6.2 | 238.3 | 239.5 | 240.6 | 241.8 | 243.0 | 244.1 | 233.7 | 246.5 | 247.7 | 248.9 |
| 6.3 | 250.0 | 251.2 | 252.4 | 253.6 | 254.8 | 256.0 | 245.3 | 258.5 | 259.7 | 260.9 |
| 6.4 | 262.1 | 263.4 | 264.6 | 265.8 | 267.1 | 268.3 | 257.3 | 270.8 | 272.1 | 273.4 |
| 6.5 | 274.6 | 275.9 | 277.2 | 278.4 | 279.7 | 281.0 | 269.6 | 283.6 | 284.9 | 286.2 |
| 6.6 | 287.5 | 288.8 | 290.1 | 291.4 | 292.8 | 294.1 | 282.3 | 296.7 | 298.1 | 299.4 |
| 6.7 | 300.8 | 302.1 | 303.5 | 304.8 | 306.2 | 307.5 | 295.4 | 310.3 | 311.7 | 313.0 |
| 6.8 | 314.4 | 315.8 | 317.2 | 318.6 | 320.0 | 321.4 | 308.9 | 324.2 | 325.7 | 327.1 |
| 6.9 | 328.5 | 329.9 | 331.4 | 332.8 | 334.3 | 335.7 | 322.8 | 324.2 | 340.1 | 341.5 |
| 7.0 | 343.0 | 344.5 | 345.9 | 347.4 | 348.9 | 350.4 | 337.2 | 338.6 | 354.9 | 356.4 |
| 7.1 | 357.9 | 359.4 | 360.9 | 362.5 | 364.0 | 365.5 | 351.9 | 368.6 | 370.1 | 371.7 |
| 7.2 | 373.2 | 374.8 | 376.4 | 377.9 | 379.5 | 381.1 | 367.1 | 384.2 | 385.8 | 387.4 |
| 7.3 | 389.0 | 390.6 | 392.2 | 393.8 | 395.4 | 397.1 | 382.7 | 400.3 | 401.9 | 403.6 |
| 7.4 | 405.2 | 406.9 | 408.5 | 410.2 | 411.5 | 413.5 | 398.7 | 416.8 | 418.5 | 420.2 |
| 7.5 | 421.9 | 423.6 | 425.3 | 427.0 | 428.7 | 430.4 | 415.2 | 433.8 | 435.5 | 437.2 |
| 7.6 | 439.0 | 440.7 | 442.5 | 444.2 | 445.9 | 447.7 | 432.1 | 451.2 | 453.0 | 454.8 |
| 7.7 | 456.5 | 458.3 | 460.1 | 461.9 | 463.7 | 465.5 | 449.5 | 469.1 | 470.9 | 472.7 |
| 7.8 | 474.6 | 476.4 | 478.2 | 480.0 | 481.9 | 483.7 | 467.3 | 487.4 | 489.3 | 491.2 |
| 7.9 | 493.0 | 494.9 | 496.8 | 498.7 | 500.6 | 502.5 | 485.6 | 506.3 | 508.2 | 510.1 |
| 8.0 | 512.0 | 513.9 | 515.8 | 514.8 | 519.7 | 521.7 | 504.4 | 525.6 | 527.5 | 529.5 |
| 8.1 | 531.4 | 533.4 | 535.4 | 537.4 | 539.4 | 541.3 | 523.6 | 545.3 | 547.3 | 549.4 |
| 8.2 | 551.4 | 533.4 | 555.4 | 557.4 | 559.5 | 561.5 | 543.3 | 565.6 | 567.7 | 569.7 |
| 8.3 | 571.8 | 573.9 | 575.9 | 578.0 | 580.1 | 582.2 | 563.6 | 586.4 | 588.5 | 590.6 |
| 8.4 | 592.7 | 594.8 | 596.9 | 599.1 | 601.2 | 603.4 | 584.3 | 607.6 | 609.8 | 612.0 |
| 8.5 | 614.1 | 616.3 | 618.5 | 620.7 | 622.8 | 625.0 | 605.5 | 629.4 | 631.6 | 633.8 |
| 8.6 | 636.1 | 638.3 | 640.5 | 642.7 | 645.0 | 647.2 | 627.2 | 651.7 | 654.0 | 656.2 |
| 8.7 | 658.5 | 660.8 | 663.1 | 665.3 | 667.6 | 669.9 | 649.5 | 674.5 | 676.8 | 679.2 |
| 8.8 | 681.5 | 683.8 | 686.1 | 688.5 | 690.8 | 693.2 | 672.2 | 697.9 | 700.2 | 702.6 |
| 8.9 | 705.0 | 707.3 | 709.7 | 712.1 | 714.5 | 716.9 | 695.5 | 721.7 | 724.2 | 726.6 |
| 9.0 | 729.0 | 731.4 | 733.9 | 736.3 | 738.8 | 741.2 | 719.3 | 746.1 | 748.6 | 751.1 |
| 9.1 | 753.6 | 756.1 | 758.6 | 761.0 | 763.6 | 766.1 | 743.7 | 771.1 | 773.6 | 776.2 |
| 9.2 | 778.7 | 781.2 | 783.8 | 786.3 | 788.9 | 791.5 | 768.6 | 796.6 | 799.2 | 801.8 |
| 9.3 | 804.4 | 807.0 | 809.6 | 812.2 | 814.8 | 817.4 | 794.0 | 822.7 | 825.3 | 827.9 |

|     |       |       |       |       |       |       |       |       |       |       |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 9.4 | 830.6 | 833.2 | 835.9 | 838.6 | 814.2 | 843.9 | 820.0 | 849.3 | 852.0 | 854.7 |
| 9.5 | 857.4 | 860.1 | 862.8 | 865.5 | 868.3 | 871.0 | 846.6 | 876.5 | 879.2 | 882.0 |
| 9.6 | 884.7 | 887.5 | 889.0 | 893.1 | 895.8 | 898.6 | 873.7 | 904.2 | 907.0 | 909.9 |
| 9.7 | 912.7 | 915.5 | 918.3 | 921.2 | 924.0 | 926.9 | 901.4 | 932.6 | 935.4 | 938.3 |
| 9.8 | 941.2 | 944.1 | 947.0 | 449.0 | 952.8 | 955.7 | 929.7 | 961.5 | 964.4 | 967.4 |
| 9.9 | 970.3 | 973.2 | 976.2 | 979.  | 982.1 | 985.1 | 958.6 | 991.0 | 994.0 | 997.0 |

$(8.47)^3 = 607.6$   
 $(84.7)^3 = 607600$   
 $(0.847)^3 = 0.607$   
 $(8.472)^3 = 608.0$

$\sqrt[3]{123500} = 49.8$   
 $\sqrt[3]{0.1235} = 0.498$   
 $\sqrt[3]{123.5} = 4.98$

If you move the comm. In x one digit to the right (left), then the comma in  $x^3$  must be moved three digits to the right (left).