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UNIT 1

INTEGERS

Unit Outcomes

After completing this unit , you should be able to:

- ▶ Understand the concept of integers
- ▶ Represent integers on the number line
- ▶ Identify the commutative, associative and distributive properties of operations on integers
- ▶ Perform the operations addition and subtraction on integers
- ▶ Apply integers in real life situation

Main contents of the unit

- | | |
|------|---|
| 1.1. | Revision of whole and natural numbers |
| 1.2. | Introduction to integers |
| 1.3. | Comparing and ordering of integers |
| 1.4. | Addition and subtraction of integers |
| 1.5. | Multiplication and division of integers |
| 1.6. | Even and odd integers |

Key terms

Unit summary

Review exercises

Introduction

In early grades, you have learnt about Natural umbers, Whole number, ordering and comparing natural numbers and operations on whole numbers to perform mathematical calculations.

In this unit, you will learn comparing and ordering integers, representation of integers on the number line, basic operations on integers, and applying it in day-to-day activities.

Opening problem

The situations given below are what we face in our day-to-day activities. Write the integer that represents the opposite of each situation and explain what zero mean in each case.

- a. A deposit of 200 birr in a bank.
- b. 1000 feet above sea level
- c. A 5kg wieght increase

1.1 Revision of Natural and Whole numbers

Activity 1.1

- a. What is the difference between natural and whole numbers?
- b. Can you mention the smallest Natural number? the smallest whole number? If your answer is yes, write it in each case.
- c. What is the largest natural number?

From your lower grade mathematics, you can recall that

The set of all numbers are denoted by the symbol N and described as $N = \{1, 2, 3, \dots\}$

Other terms, that are used for natural numbers are ‘counting numbers’ or ‘cardinal numbers’.

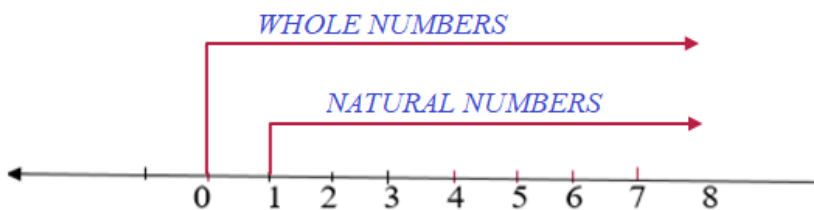
Note: Natural numbers starts from 1.

The set of Whole numbers are denoted by the symbol W and described as $W = \{0, 1, 2, \dots\}$

Whole numbers include all natural numbers and zero.

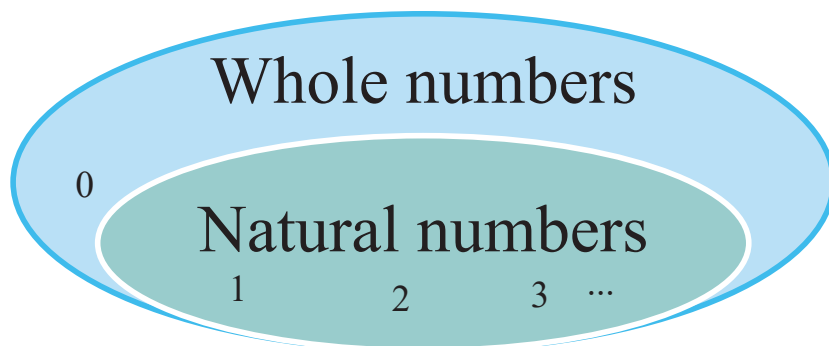
Whole numbers are positive numbers that do not have any fractions or decimals. Whole numbers start with zero.

Counting numbers and whole numbers can be visualized on a number line as follows



The diagram given below shows the relation between natural numbers and whole numbers.

Whole numbers contains all natural numbers or all natural numbers are included in whole numbers.



1.1.1 Classification of natural numbers

Natural numbers can be classified as Even and Odd or Composite and Prime.

Even natural numbers and Odd natural numbers.

Even natural numbers are natural numbers that are exactly divisible by 2.

The set of even numbers $\{2, 4, 6, \dots\}$

Odd natural numbers are numbers, which are not divisible by 2.

The set of odd numbers $\{1, 3, 5, \dots\}$

Exercise 1.1

- For each of the following numbers given below, classify as all even, all odd or collection of both.

$\{234, 568, 100, 0, 998\}$

$\{99, 1, 687, 2345, 277\}$

$\{678, 567, 456, 989\}$

2. Write all even numbers between 345 and 362.

Composite numbers and Prime numbers

Numbers that have more than two factors are said to be composite numbers.

For example {4, 6, 8, 9, ...} are composite numbers

Prime number is a number that is divisible by one(1) and itself only.

The numbers 2, 3, 5, 7, 11, 13, 17, ... are examples of prime numbers

Note:

- a. 1 is neither prime nor composite.
- b. 2 is the only even prime number.

1.1.2 Place value of a natural number

Activity 1.2

1. What is a place value?
2. Write the **place** and the **place value** of 3 in each of the following numbers. 234, 3564, 1023, 4532

In mathematics, every digit in a number has a place value. Place value can be defined as the value represented by a digit in a number on the basis of its position in the number.

For example, in the number 286, the numeral 8 is “in tens” place. So its place value is $8(10)=80$ and 2 is in hundreds place and its place value is $2(100)=200$.

Note: Units place, tens place, hundreds place, etc. indicates **positions** of a digit in a given number. However, place value indicates **the value** of the number in the indicated position.

Example 1:

In the number 2,345,697

Millions place	Hundred thousands place	Ten thousands place	Thousands place	Hundreds place	Tens place	units place
2	3	4	5	6	9	7

The number 2,345,679 can be written in expanded form as

2 million 3 hundred thousands 4 ten thousands 5 thousands 6 hundreds 7 tens and 9 ones.

$$\begin{aligned}
 2,345,679 &= 2(1,000,000) + 3(100,000) + 4(10,000) + 5(1000) + \\
 &\quad 6(100) + 7(10) + 9(1) \\
 &= 2,000,000 + 300,000 + 40,000 + 5000 + 600 + 70 + 9
 \end{aligned}$$

Exercise 1.2

- In the number, 40,382 describe the place values of each digit.
- Write the following numbers in numeral form
 - Seven thousand six hundred forty eight
 - Eleven thousand sixty four.
 - Two hundred eighty five thousand seven hundred fifty
 - Three million two hundred sixty six thousand seventy nine.
- Write the following numbers in words

- a) 2,345 b) 479,623 c) 98,235,016

1.1.3 Predecessor and successor of a whole number

Activity 1.3

- Write three natural numbers before and after 5
- What is the natural number immediately before 5?
- What is the number immediately after 5 in natural numbers?
- What is the number immediately before 1 in whole numbers?
- What is the number immediately before 1 in natural numbers?
- What is the number before 0 in whole numbers?

Note:

Predecessor of a whole number is a number, which is immediately before a given number.

Successor of a whole number is a whole number, which comes just after a given number.

Example2:

In the set of whole numbers, the predecessor of 3 is 2 and its successor is 4.

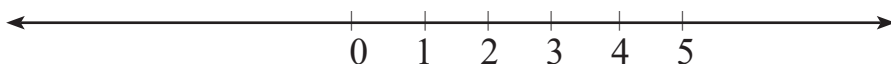
Note:

For a non - zero whole number a its predecessor is $a - 1$ and its successor is $a + 1$.

If you consider the number line given below, the numbers are spaced equally and for each number except 0, there are two numbers one to the right and the other to the left of the number.

The number, which to the left is the predecessor of the number and the number, which to the right of it is the successor.

For example, for 1 its predecessor is 0 and its successor is 2, for 2 its predecessor is 1 and its successor is 3 and so on.

**Example3:**

Find both the predecessor and successor of 97.

Solution:

The predecessor of 97 is $97 - 1 = 96$ and its successor is $97 + 1 = 98$. Predecessor of 99,999 is 99,998 and its successor is 100,000.

Note:

In the set of natural numbers, 1 has no predecessor.

In the set of whole numbers, 0 has no predecessor.

Dear students for each of the rules about predecessor and successor given above please give your own examples and check the validity of the rules. Enjoy it

Forward and back ward counting of numbers

In the set of whole numbers $W=\{0, 1, 2, 3, \dots\}$ the numbers are arranged by counting forward by ones starting from 0.

Example4:

Starting from 10 and counting by 2 forward we get 10, 12, 14, 16,

Starting from 20 back ward counting up to 0 by 5 gives 20,15,10,5, 0.

Activity 1.4

1. List the numbers from 934 to 854 by counting back by 10s.
How may 10s do you get?
2. List the numbers from 450 to 950 by counting forward in 100s.
How many 100s do you get?
3. For each of the following problems write the next number and describe whether it is counting forward or counting backward.

125, 150, 175, _____, _____, _____

359, 339, 319, _____, _____, _____

1.1.4 Basic operations on whole numbers

In lower grades, you have learnt about how to add or subtract two or more whole numbers that have equal number of digits.

Furthermore, you learned how to add or subtract whole numbers of different number of digits with idea of carrying in addition and with idea of borrowing in subtraction.

Now you can recall the following rules from your lower grade lessons.

- a. The sum of two whole number is a whole number.
- b. The difference of two whole numbers is not always a whole number.
- c. The product of two whole numbers is always a whole number.
- d. The quotient of two whole numbers is not always a whole number.

Note:
See and study the following pattern of first continued odd numbers. The sum of each set of odd numbers is the number of odd numbers multiplied by itself. Check the pattern taking next odd numbers.

		Sum	Pattern
First odd number	1	1	$1 \times 1 = 1$
First two odd numbers	1, 3	$1 + 3 = 4$	$2 \times 2 = 4$
First three odd numbers	1, 3, 5	$1 + 3 + 5 = 9$	$3 \times 3 = 9$
First four odd numbers	1, 3, 5, 7	$1 + 3 + 5 + 7 = 16$	$4 \times 4 = 16$
First five odd numbers	1, 3, 5, 7, 9	$1 + 3 + 5 + 7 + 9 = 25$	$5 \times 5 = 25$
First six odd numbers	1, 3, 5, 7, 9, 11	$1 + 3 + 5 + 7 + 9 + 11 = 36$	$6 \times 6 = 36$
...	...	...	...

Exercise 1.3

1. Perform the following operations

- a. $14,678 + 3,567$

b. $14,676 + 43,568$
- c. $14,678 - 3,569$

d. $54,678 - 25,567$

2. The population of a country in 2009 E.C was 110,000,000 and in two years it is increased by 712,560. What was the total number of population of the country in 2011?

1.2. Introduction to integers

Group work 1.1

A local bookstore has 30 books, which are most needed when it opens on Monday morning. On Monday, it sells six books. On Tuesday, it sells three books. On Wednesday, it receives or adds another 24 books to the store and sells eight books.

- How do you represent selling six books?
- How do you represent selling three books?
- How do you represent receiving 24 books?
- How do you represent selling eight books?
- How many books does the store have at the end of Wednesday?

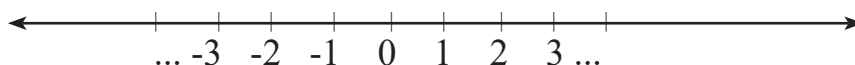
Understand that positive and negative numbers used together to describe quantities having opposite direction or values. For example, temperature above/below zero, elevation above/below sea level, credit /debit, profit/lose use positive or negative numbers to represent quantities in real-world context explaining the meaning of zero in each situation. Whole numbers are not enough to express such quantities. They are described using the new numbers called **integers**.

Definition:

The set of integer is denoted by either Z or I and described as

$$Z = \{..., -3, -2, -1, 0, 1, 2, 3, \dots\}$$

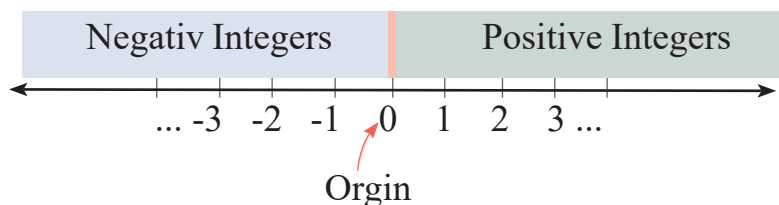
Again, we can show that the set of integers on the number line as follow:



The whole numbers, together with the new numbers $..., -5, -4, -3, -2, -1$ are called the integers.

The set of integers consist of:

- ◆ The numbers 1, 2, 3, 4, 5... are called the positive integers.
- ◆ The numbers $..., -5, -4, -3, -2, -1$ are called the negative integers.
- ◆ The number 0 is neither positive nor negative.



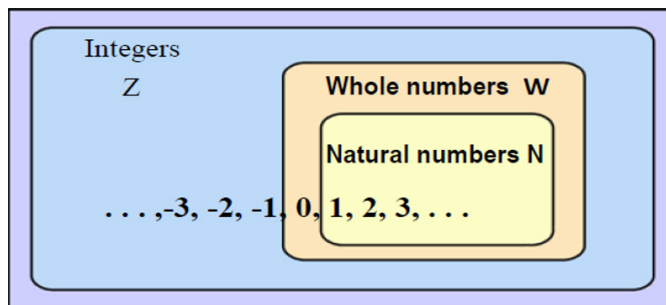
Note:

- ◆ For an integer a and b , $-a$ read as ‘negative of a ’ and $a-b$ read as ‘ a minus b ’.

For example -2 read as ‘negative 2’

- ◆ A number to the left of zero is a negative integer.
- ◆ A number to the right of zero is a positive integer.
- ◆ Zero is neither positive nor negative number. It is neutral.
- ◆ Positive integers are written without the sign (+). That is $+4$ and 4 are the same (similar).

The following diagram indicates the relations between N, W and Z



Note:

- ◆ All natural numbers are contained in the set of whole numbers.
- ◆ All whole numbers are contained in the set of integers.

Therefore, we can write the relation $N \subset W \subset Z$

Group work 1.2

1. Write an integer that represents each of the following concepts and explain what zero indicates in each case.
 - a. A credit of 100 birr.
 - b. A water bill charge of 50 birr on not paying in time.
 - c. Temperature of fifteen degree centigrade below zero.
 - d. An altitude of 2000feet below sea level.
2. A hole is drilled into the ground to the depth of 10m and a ball is dropped into it. What number would you use to represent the position of the ball compared to the ground level? Does this number belong to the set of whole numbers? Is it an integer?

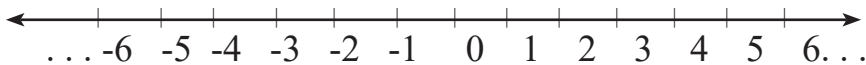
1.3 Comparing and ordering Integers

Activity 1.5

1. Order the integers -4,-5,2,6,4,-6,0,5 -1 from smaller to larger on the number line.
2. Write down the integers that come immediately before and immediately after the integers given below.
 - a. 1001
 - b. 0
 - c. - 999

1.3.1 The number line

The set of integers are represented as points on the number line. The numbers are spaced equally. 0 divides the number line in to two equal parts. Numbers to the right of 0 are positives and numbers to the left of zero are negatives.



An integer **a** is less than another integer **b** if **a** lies to the **left** of **b** on the number line.

The symbol “<” is used for ‘less than’.

For example,

$$-3 < 1, -5 < 2 \text{ and } -6 < 0.$$

An integer **b** is greater than another integer **a** if **b** lies to the **right** of **a** on the number line.

The symbol “>” is used for ‘greater than’.

For example: $1 > -5$

Note:

- ◆ Any negative integer is less than any positive integer.
- ◆ 0 is greater than any negative integer.

In general, comparing numbers **a** and **b** on the number line below,



HISTORICAL NOTE



Anders Celsius the Swedish astronomer who lived between 1701 and 1744 A.D. He devised a way of measuring temperature which was adjusted and improved after his death.

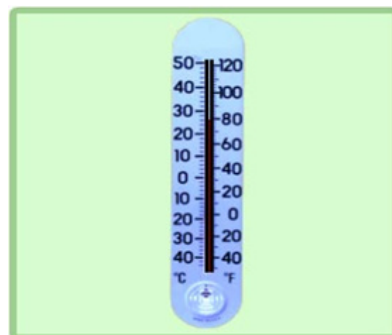
Integer numbers are used in telling the temperature in degree Celsius ($^{\circ}\text{C}$).

Thus if the temperature is 20 degree Celsius above zero, you can read as positive twenty degree Celsius ($+20^{\circ}\text{C}$) and the temperature is below zero you can read also negative twenty degree Celsius (-20°C).

Activity 1.6

Give the directed number describing each of the following temperatures.

- Seventy five above zero.
- Forty below zero.
- Twenty five below zero.
- Twenty one above zero.



Group work 1.3

- Draw a number line. Plot the following numbers on it and compare using $<$, $>$ or $=$

- a. $-3, -2, -1, 0, 1, 2, 3$
 - b. $-5, 2, 6, -6, 1, 4, 0, 5, -3$
 - c. Write your conclusion depending on a and b.
2. Assume that the table below shows the average temperatures of the five cities in Ethiopia in September. Compare the temperatures in Addis Ababa and Hawassa and compare the temperatures in Diredawa and Debrebrhan. Answer the questions below.
- i. Which city is the coldest?
 - ii. Which is the hottest?
 - iii. Write the temperatures in order from highest to lowest.

Cities	Temperature in $^{\circ}\text{C}$
Addis Ababa	10
Hawassa	19
Diredawa	20
Harar	14
Debrebrehan	-2

1.3.2 Predecessor and successor of an integer

Recall that the concept of Predecessor and successor from our discussion in whole numbers

- ◆ Predecessor of an integer n is the integer that is immediately before it. That is, if n is an integer then, its **predecessor is $n - 1$**
- ◆ Successor of an integer n is the integer that comes immediately after it. That is, if n is an integer then, its **successor is $n + 1$**

Example 5:

Predecessor of 5 is 4 and its successor is 6.

Predecessor of -5 is -6 and its successor is -4.

Predecessor of 0 in the set of integers is -1 and its successor is 1.

Example 6:

If $n = -99$, write predecessor of n and successor of n .

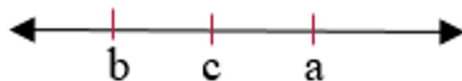
Solution:

Predecessor of n is $n - 1$.

Therefore, predecessor of $n = -99$ is $-99 - 1 = -100$

Successor of $n = -99$ is $-99 + 1 = -98$.

For any integer c , we can find its predecessor and successor on the number line.

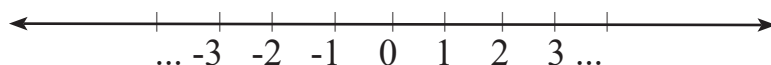


From the above number line, the successor of c is a and predecessor of c is b .

For example, on the number line given below, the predecessor of -1 is -2 and successor of -1 is 0.

Predecessor of -2 is -3 and successor of -2 is -1

Predecessor of 0 is -1 and successor of 0 is 1 and so on.



Note:

Predecessor and successor can be calculated only for integer numbers.

1.3.3 The opposite of an integer

The number -2 is in the opposite side of 2 on the number line, they are on the same distance from 0 , but they lie on the opposite side of 0 . Therefore, -2 is considered as the opposite of 2

◆ We write the opposite of the number a from 0 as $-a$.

The opposite of 2 is -2 and the opposite of -2 is 2 because they lie on opposite sides and at equal distance from 0 .

If a is a positive number, then $-a$ is a negative number. The negative of 10 is -10

Example 6

- a) -5 is the opposite of 5
- b) 12 is the opposite of -12
- c) The opposite of -2 is 2 . That is $-(-2) = 2$

◆ If a is a negative number, $-a$ is a positive number.

i.e. $-(-a) = a$

For example $-(-12) = 12$

◆ The opposite of 0 is 0 itself.

That is, $-0 = 0$.

Note:

The negative of a negative number is positive number.

Exercise 1.4

1. Mark all integers from -7 to 7 on the number line.
2. Using the number line compare the numbers. Which is the larger?
 - a. 0 or 5
 - b. -4 or -10
 - c. -1000 or 200
3. Insert the correct symbol either $>$, $<$ or $=$ between the following numbers and compare.
 - a. 234 _____ 567
 - b. -100 _____ -102
 - c. 0 _____ -1000
 - d. 1,000 _____ -100,000
4. Which of the following temperatures are colder?
 - a. 0°C or 5°C
 - b. -7°C or 2°C
 - c. -8°C or -5°C
 - d. -37°C or 37°C
5. Arrange the following numbers from the highest to the lowest (i.e. in decreasing order)
 - a. 3, -5, 7, -3, 9. _____
 - b. 8, -6, -2, -3, 5. _____
 - c. -9, -7, 6, -3, -2 _____
 - d. 4, -21, 0, -4, -5. _____
6. Determine the predecessor and successor of each of the following numbers

- a. 12 b. 0 c. -12 d. -101

Challenge problem

7. Rori and Dansita started saving each by 100Birr. Rori added 250Birr three times and Dansita added 200 Birr four times to their own savings. If Rori took 175Birr and Dasita took 190Birr from their savings to buy shoes, then
- Find the remaining amount of money in Rori's saving.
 - Find the remaining amount of money in Dansita's saving.
 - Compare the amounts of money the two women have in their savings now

1.4 Addition and subtraction of integers

Activity 1.7

1. Fill the following boxes using numbers 1 to 9 such that the sum in each row, column and diagonal is 15.

a	b	c
d	e	f
g	h	i

2. Aster has 4 blocks in one pile and 8 blocks in another pile. If she combines the piles, how many blocks will she have?

1.4.1 Addition of integers

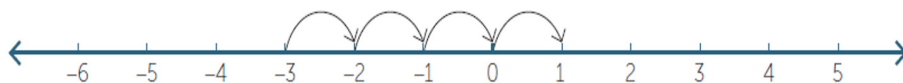
You have learnt how to add, subtract multiply and divide whole numbers by whole numbers for denominators different from zero. In this subsection, you will see how to add two integers using number line.

Example 7:

Find the sum of the following numbers using number line

a. $-3 + 4$

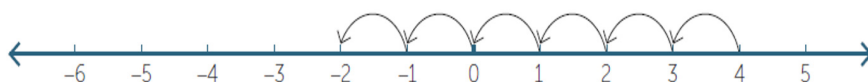
To add $-3 + 4$, start at -3 and move 4 steps to the right, so $-3 + 4 = 1$



When you add a positive integer, you move to the right along the number line.

b. $4 + (-6)$

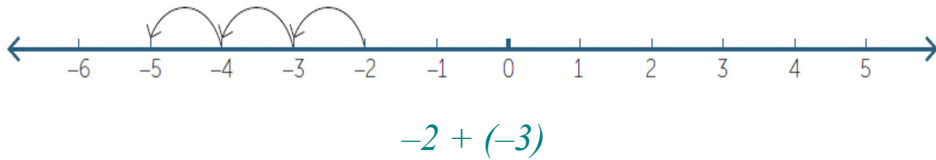
To add $4 + (-6)$ is the number you get by moving 6 steps to the left, beginning at 4.



Adding a negative integer to another integer means that you take a certain number of steps to the left on the number line.

c. $-2 + (-3)$

The result of the addition $-2 + (-3)$ is the number you get by moving 3 steps to the left, beginning at -2 .



1.4.2 Subtraction of integers.

Recall that the “-” sign can indicate the sign of a number if it is in front of a number and it indicates the operation of subtraction if it is in between two numbers.

For example -3 is to mean ‘negative 3’ and $9 - 3$ is to mean ‘nine minus three’.

Subtraction of integers can be written as the addition of the opposite number.

For integers a and b ,

$$a - b = a + (-b) \text{ and } -a + b = b - a$$

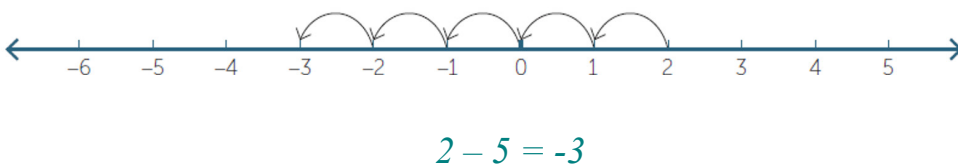
Similarly, as we did in addition of numbers using number line, we can perform subtractions, too.

Example 8:

Represent the difference of the following on the number line

a. $2 - 5$

When you subtract a positive integer start at 2 and move 5 steps to the left.



b. $5 - 2$

c. $-1 - 4$ (b & c are left as class work)

Therefore, you can represent the difference and the sum of numbers on the number line using similar way.

Group work 1.4

1. Find the following sums and differences using number line.

a. $7 + 5$

b. $-11 + 6$

c. $5 - 12$

d. $-5 - 9$

2. A man enters a lift at 10th floor. He goes up 5 floors then, down 7 floors. He then leaves the lift. In which floor is he on?

Note:

To subtract two integers, add the opposite of the second integer to the first integer. This can be written symbolically as

$$a - b = a + (-b).$$

For example:

$$(-15) - 75$$

Rewrite the subtraction operation as the sum of $(-15) - 75$

Add the first number and the opposite of the second number

$$(-15) + (-75).$$

The opposite of 75 is - 75.

$$-15 - 75 = -15 + (-75) = -90$$

Example 9:

Perform the following operations

a. $-25 + 120$

c. $-256 - 298$

b. $123 - 456$

d. $134 - (-282)$

Solution:

a. $-25 + 120 = 120 - 25$

$= 95$ (Remember that $-a + b = b - a$)

b. Subtracting 456 from 123 is the same as adding -456 and 123.

$$123 - 456 = 123 + (-456)$$

Since 456 is greater than 123 without considering the negative sign of 456, subtract 456 from 123 and make the result negative.

$456 - 123 = 333$. Then make the result negative.

Therefore, $123 - 456 = -333$

c. $-256 - 298 = -256 + (-298) = -554$

(add without considering the negative sign and make it the result negative).

d. Recall that $-(-a) = a$

$$134 - (-282) = 134 + 282 = 416$$

Example 10:

Deraro has read 4 chapters out of 6 chapters of grade 7 textbook. How many chapters does she have left to read?

Solution:

Number of chapters not read = Total number of chapters of the book
– number of chapters she read.

Number of chapters not read = $6 - 4 = 2$.

Example 11:

If Durete paid a tax of Birr12,000, 12,590, and 13,780 from her income in three years. How much money she paid in these three years?

Solution:

The total amount she paid is the sum of all the money 12,000, 12,590 and 13,780.

Total = $12,000 + 12,590 + 13,780 = 38,370$ Birr

Therefore, Durete paid a total tax of 38,370 birr

Exercise 1.5**1. Adding Integers**

a. $85 + (-96) =$ _____

g. $6 + (-33) =$ _____

b. $80 + 57 =$ _____

h. $6 + (-47) =$ _____

c. $86 + (-38) =$ _____

i. $(-78) + 69 =$ _____

d. $22 + (-41) =$ _____

j. $-72 + (-30) + 10 =$ _____

e. $-18 + (-45) =$ _____

k. $-83 + (-36) + 20 =$ _____

f. $-32 + 48 =$ _____

2. Subtracting Integers

- a. $1 - 3 =$ _____ f. $3 - (-2) =$ _____
- b. $2 - (-5) =$ _____ g. $-1 - 9 =$ _____
- c. $6 - (-9) =$ _____ h. $2 - 9 =$ _____
- d. $-7 - (-1) =$ _____ i. $-8 - (-1) =$ _____
- e. $-7 - 4 =$ _____

3. Evaluate $-x + y$ if $x = -3$ and $y = 4$

4. If kayamo paid a tax of 1,200, 985, and 1,100 Birr for the government from his income in three consecutive years. Calculate the total tax he paid in the three years.

5. Durete has Birr 1,234 in her account in a bank and she withdraw Birr 200, Birr150 and Birr 300. How much money remains in her account assuming no bank interest is calculated?

Properties of addition of integers

Group work 1.5

Copy and complete the following table below

a	b	c	$a + b$	$b + a$	$(a+b)+c$	$a+(b+c)$
2	3	5				
-3	-4	-6				
4	5	-7				

What do you understand from the above table?

The following properties of addition hold true for any integer numbers a , b and c

- i. Commutative property of addition

$$a+b = b+a$$

Example 12:

$$2+4 = 4+2$$

- ii. Associative property of addition

$$(a+b)+c = a+(b+c)$$

Example 13:

$$(5 + 3) + 4 = 5 + (3 + 4)$$

- iii. Property of zero

$$a + 0 = 0 + a = a$$

Example 14:

$$2 + 0 = 0 + 2 = 2$$

- iv. Property of opposite

$$a + (-a) = 0$$

Example 15:

$$200 + (-200) = 0$$

Note:

Subtraction of numbers is not commutative and not associative.

For integers a, b, and c,

i. Subtraction of numbers is not commutative

$$a - b \neq b - a$$

For example, $5 - 12 \neq 12 - 5$

$$5 - 12 = -7 \text{ but } 12 - 5 = 7$$

$$-7 \neq 7$$

$$5 - 12 \neq 12 - 5$$

This implies that subtraction is not commutative.

ii. Subtraction of numbers is not associative

$$a - (b - c) \neq (a - b) - c$$

For example, $4 - (3 - 2) \neq (4 - 3) - 2$

To check this

$$4 - (3 - 2) = 4 - 1 = 3$$

$$\text{But } (4 - 3) - 2 = 1 - 2 = -1$$

$$3 \neq -1$$

$$4 - (3 - 2) \neq (4 - 3) - 2$$

Therefore, subtraction of numbers is not associative.

Example 16:

Let temperatures of two places be 300°F and -250°F . What is the difference between the two temperatures?

Solution: The difference between the temperatures is highest minus the lowest temperature.

$$\begin{aligned}\text{That is, } 300^{\circ}\text{F} - (-250^{\circ}\text{F}) \\ = 300^{\circ}\text{F} + 250^{\circ}\text{F} = 550^{\circ}\text{F}\end{aligned}$$

Therefore, the difference between the two temperatures is 550°F .

Note:

When adding two integers of the same sign

- i. If the numbers are both positive, then the sum is positive.
- ii. If the numbers both are negative, then the sum is negative.
- iii. The sum of positive and negative integers is either positive, negative or zero. If the larger is positive, the sum is positive and if the larger is negative, the sum is negative. If they are opposite of each other the sum is zero

For example, $-25 + 17 = -8$, $-17 + 25 = 8$ and $-12 + 12 = 0$

Group work 1.6

1. Take any two numbers with different signs. Find the sum. Is the sum greater than or less than one of the numbers?
2. Take any two negative numbers. Find the sum. Is the sum greater than any of the numbers?

Exercise 1.6

1. Simplify the following.
 - a. $(-23+34) + 89$
 - b. $-101+((-345) - 345) - 567$
 - c. $99 - 23 - 34 - 67 + 88 - (-23) + 77$

d. $-18 - (-34)$

e. $(10 - (-41)) - 28$

2. Write pairs of integers whose sum gives
 - a. A positive integer
 - b. Zero
 - c. An integer smaller than both the integers.
 - d. An integer smaller than only one of the integers.

1.5 Multiplication and division of integers

1.5.1 Multiplication of integers

Activity 1.8

1. Multiply
 - a. 7×8
 - b.* -2×0
 - c.* $-2(3 + 12)$
2. A coach bought 3 soccer balls on sale. They normally sell for 260 Birr each. They were on sale for 70 Birr discount of the original price. The total tax on the balls was 40 Birr.
 - a. Write the expression to find the total amount the coach paid.
 - b. Find the total amount paid.

As you learnt in multiplication of whole numbers, multiplication of integers relate with addition. Multiplication is a **repeated addition**.

For example 3×2 is the same with adding three groups of twos together or two groups of threes together.

$$3 \times 2 = 2 + 2 + 2 = 6$$

OR

$$3 \times 2 = 3 + 3 = 6$$

We can use this idea for multiplication of integers also.

Note:

When multiplying integers, consider the signs in each case and follow the rules given below.

- i. Positive $\times$ Positive = Positive
- ii. Positive $\times$ Negative = Negative
- iii. Negative $\times$ Positive = Negative
- iv. Negative $\times$ Negative = Positive

Example 17:

Find the product of each of the following numbers

a. 4×-5

b. -3×5

c. -4×-6

Solution:

a. $4 \times -5 = (-5) + (-5) + (-5) + (-5) = -20$

b. $-3 \times 5 = (-3) + (-3) + (-3) + (-3) + (-3) = -15$

c. $-4 \times -6 = 4 \times 6$ (negative $\times$ negative = positive) = 24.

Properties of multiplication and division on integers

For any integer numbers a, b and c the following properties of multiplication are true

- i. Commutative property of multiplication

$$a \times b = b \times a$$

Example 18:

$$23 \times 21 = 21 \times 23$$

$$-12 \times 10 = 10 \times -12$$

- ii. Associative property of multiplication

$$(a \times b) \times c = a \times (b \times c)$$

Example 19:

Find the product of the questions below and check the associativity of multiplication.

a. $(25 \times 24) \times 4 = 25 \times (24 \times 4)$

Solution:

$$600 \times 4 = 25 \times 96$$

$$2400 = 2400$$

b. $(-2 \times 7) \times 9 = -2 \times (7 \times 9)$

Solution:

$$-14 \times 9 = -2 \times 63$$

$$-126 = -126$$

iii. Distributive property:

Multiplication is distributive over addition.

$$a \times (b+c) = (a \times b) + (a \times c) \quad (\text{Left distributive})$$

$$(a+b) \times c = (a \times c) + (b \times c) \quad (\text{Right distributive})$$

Example20:

a. $3 \times (4 + 12) = (3 \times 4) + (3 \times 12)$

$$3 \times (16) = (12) + (36)$$

$$48 = 48$$

b. $(23 + 15) \times 11 = (23 \times 11) + (15 \times 11)$

$$38 \times 11 = 253 + 165$$

$$418 = 418$$

Example 21:

Alemu and Dancho are playing a game. In the game, each player starts with zero points and the player with the most points at the end wins. Alemu gains 5 points twice, loses 12 points, and then gains 3 points. Dancho loses 3 points two times, loses 1 point, gains 6 points, and then gains 7 points.

- Write and evaluate an expression to find Alemu's score.
- Write and evaluate an expression to find Dancho's score.
- Who won the game?

Solution:

Alemu	
Gain	Lose
$2(5) + 3$	12

Dancho	
Gain	Lose
$6+7$	$3+3+1$

Alemu gain a total of 13 points and loses 12 points.

Dancho gain a total of 13 points and loses 7 points.

Remark:

Gain is positive and loss is negative

- Alemu's score is $5 + 5 + 3 - 12 = 13 - 12 = 1$
- Dancho's score is $6 + 7 - (3 + 3 + 1) = 13 - 7 = 6$.
- Dancho won the game because his net point is 6 but Alemu's net point is 1.

Note:

Addition is not distributive over multiplication.

For integers a, b and c we do not write as

$$a + (b \times c) = (a + b) \times (a + c)$$

1.5.2 Division of Integers

If multiplication is totaling of numbers, division is the distribution of numbers. Dividing integers is the reverse operation of multiplication. But the rules for division of integers are the same as multiplication rules. However, it is not necessary that the quotient will always be an integer.

When dividing integers, we use the following rules:

- i. Positive $\div$ Positive = Positive
- ii. Negative $\div$ Negative = Positive
- iii. Positive $\div$ Negative = Negative
- iv. Negative $\div$ Positive = Negative

Example 22:

Find the quotient in each case

a. $-12 \div 6$

Solution:

$$-12 = -2 \times 6$$

This indicates that the quotient is -2 .

Therefore, $-12 \div 6 = -2$

OR we can find the quotient performing the ordinary division as follows.

You can use the symbol “ $\overline{) }$ ” to perform division

$$\begin{array}{r} \text{divisor} \quad 6 \overline{) -12} \quad \text{dividend} \\ \underline{-12} \quad \text{quotient} \\ -2 \end{array}$$

Example 23:

Find the quotient $521 \div 17$

Solution:

$$\begin{array}{r}
 33.1764706 \\
 \underline{17 } \\
 564 \\
 \underline{-51 } \\
 54 \\
 \underline{-51 } \\
 30 \\
 \underline{-17 } \\
 130 \\
 \underline{-119 } \\
 11
 \end{array}$$

Dividing in this way until the end, we get the quotient 33.1764704.

$$564 \div 17 = 33.18 \text{ in 2 decimal places}$$

$$564 \div 17 = 33.176 \text{ in 3 decimal places.}$$

Example 24:

Dararo withdrew 50 Birr each day from her bank account and withdrew 350 Birr totally. Bolka withdrew 150 Birr at a time from his bank account and withdrew 900 Birr totally. Who made the greater number of withdrawals?

Solution:

Dararo withdrew 50 birr each day. She withdrew a total of 350 Birr.

Total withdrew = number of withdraw $\times$ amount of withdrew.

$$\text{Number of withdraw} = \frac{\text{Total with drew}}{\text{amout of withdrew}} = \frac{350}{50} = 7$$

Bolka withdrew 150 each time and a total withdrew is 900.

$$\text{Number of withdrew} = \frac{\text{Total with drew}}{\text{amout of withdrew}} = \frac{900}{150} = 6$$

Dararo made seven withdrawals whereas Bolka made six withdrawal.

Therefore, Dararo made more withdrawals.

Group work 1.7

1. Perform the following operations and compare answers in a and b and c and d
 - a. $12 \div 6$
 - b. $6 \div 12$
 - c. $12 \div (6 \div 3)$
 - d. $(12 \div 6) \div 3$
2. Write the quotient of $522 \div 17$ in
 - i. 2 decimal places and
 - ii. 3 decimal places.
3. Argiso found the quotient of two integers and got a positive integer. He added another integer to the quotient and got a positive integer. His sister, Kayime says that all the integers Argiso used to get this result must be positive. Do you agree? Explain.

Note:

◆ Division of integers is not commutative

For example $6 \div 3 = 2$ but $3 \div 6$ is 0.5. It gives different results

◆ Division of integers is not associative.

For example $16 \div (4 \div 2) = 16 \div 2 = 8$ but $(16 \div 4) \div 2 = 4 \div 2 = 2$

Exercise 1.7

1. Simplify each of the following using the properties of multiplication.

a. $-5 \times (3 + 9)$ b. $-4 \times (-30 + (-3))$ c. $2 \times (-13 \times -25)$

2. Multiplying Integers

a. $(-4)(-12) =$ _____	g. $-8 \times (-8) =$ _____
b. $(-8)(-10) =$ _____	h. $5 \times 1 =$ _____
c. $-10 \times 11 =$ _____	i. $(-3)(-8) =$ _____
d. $-2 \times 6 =$ _____	j. $7(-12) =$ _____
e. $4 \times (-10) =$ _____	k. $(-9)(-6)(2) =$ _____
f. $-5 \times 0 =$ _____	l. $(-10)(-7)(-4) =$ _____

3. Dividing Integers

a. $-48 \div 6 =$ _____	f. $-81 \div (-9) =$ _____
b. $-18 \div (-6) =$ _____	g. $25 \div (-5) =$ _____
c. $-10 \div 2 =$ _____	h. $-35 \div (-5) =$ _____
d. $-42 \div 6 =$ _____	i. $-70 \div (-7) =$ _____
e. $-16 \div (-8) =$ _____	j. $-78 \div -78 =$ _____

4. Find the quotient in each of the following and write it in

i. 2 decimal place

ii. 3 decimal place

a. $742 \div 11$	c. $3445 \div 21$
b. $675 \div 12$	

1.6 Even and odd integers.

Activity 1.9

49	24	45	-68	75	60
11	59	0	76	70	17
30	-15	69	80	19	71
22	18	90	79	-23	-89
-33	40	43	-22	89	66
20	74	34	13	-8	99

Using the above table, list

- a. Even numbers
- c. Odd numbers
- b. Negative even numbers
- d. Negative odd numbers.
- e. What do you say about zero? Is it even or odd?

Definition:

Even integer is an integer, which is exactly divisible by 2.

For examples $\{...,-6,-4,-2,0,2,4,6,...\}$ is the set of even integers.

Note:

If the last digit of a number is either of the numbers 0, 2, 4, 6, or 8, then the number is an even number.

Example 25:

The numbers 20, 234, 15780, 4536, 89042, 26958 are all examples of even numbers because their last digits 0, 4, 6, 2 and 8 are all even.

In general, if k is an integer, then $2k$ is an even integer.

Definition:

Odd integer is an integer, which is not divisible by 2.

For example $\{\dots, -5, -3, -1, 1, 3, 5, \dots\}$ is the set of odd integers..

Note:

If the last digit of a number is either of the numbers 1, 3, 5, 7, or 9, then the number is an odd number.

Example 26:

The numbers 21, 253, 1,235, 359, 21,487 are odd numbers because their last digits 1, 3, 5, 9, 7 are odd numbers in each case.

Note:

- ◆ Even integer plus 1 is an odd integer.
- ◆ If k is an integer, then $2k+1$ is an odd integer

For example, 2 is an even integer but $2+1 = 3$ is an odd integer.

10,234 is an even integer but $10,234 + 1 = 10,235$ is an odd integer.

Basic operation on even and odd integers

i. Addition and subtraction on even numbers.

- ◆ Sum of two even integers is an even integer

Example 27:

- a. $12 + 26 = 38$
- b. $-26 + 18 = -8$
- c. $-14 + (-12) = -26$

- ◆ Difference of two even integers is an even integer

Example 28:

- a. $26 - 12 = 14$
- b. $16 - (-8) = 16 + 8 = 24$
- c. $-32 - 18 = -32 + (-18) = -50$

As you can see from the above examples

$$\text{Even} + \text{Even} = \text{Even}$$

$$\text{Even} - \text{Even} = \text{Even}$$

- ◆ Sum of two odd integers is always an even integer.
- ◆ Sum of even and odd numbers is always an odd number.

Example 28:

$$13 + 25 = 38 \qquad -23 + 7 = -16 \qquad -17 + (-9) = -26$$

- ◆ The difference of two odd integers is always an even integer.

Example 29:

$$31 - 9 = 22$$

$$17 - 45 = -18$$

$$-11 - (-23) = -11 + 23 = 12$$

In all cases, you can see that

$$\text{Odd} + \text{Odd} = \text{Even}$$

$$\text{Odd} - \text{Odd} = \text{Even}$$

1.6.1 Multiplication on even and odd integers

◆ Product of two even numbers is an even number.

Example 30:

a. $4 \times 12 = 48$

b. $-6 \times 4 = -24$

c. $-8 \times -14 = 112$

◆ Product of even and odd is always an even number.

Example 31:

a. $2 \times 5 = 10$

b. $4 \times -11 = -44$

c. $-6 \times -3 = 18$

◆ Product of two odd numbers is always an odd number.

Example 32:

a. $5 \times 3 = 15$

b. $-9 \times 11 = -99$

c. $-13 \times -7 = 91$

In each case discussed above, you can see that product of even and odd is an even number.

$$\text{Even} \times \text{Even} = \text{Even}$$

$$\text{Even} \times \text{Odd} = \text{Even.}$$

$$\text{Odd} \times \text{Odd} = \text{Odd}$$

Group work 1.8

Why is 0 an even number? Discuss.

Exercise 1.8

1. Describe whether the following statements are true or false. Justify your answers.
 - a. The sum of two even integer numbers is an even number
 - b. The sum of two odd integer numbers is an odd integer.
 - c. The sum of even and odd numbers is neither even nor odd.
2. Using the formula $n = 2k$ for k is an integer, list five numbers. What do you say about these numbers? Are all they even?
3. For an integer k , what do you say about $2k, 4k, 6k, \dots$ are all even?
4. Using the formula $n = 2k + 1$, for k is an integer, list five numbers. What do you say about these numbers?

Key terms:

- | | |
|-------------------------------|----------------------------------|
| ♠ Natural number | ♠ Positive and negative integers |
| ♠ Whole number | |
| ♠ Even and odd numbers | ♠ Opposite |
| ♠ Prime and composite numbers | ♠ Predecessor |
| ♠ Integer | ♠ Successor |

Unit summary

Important points, which are discussed in this unit and you should know.

- ◆ The set of natural numbers denoted by the symbol N and given by $N = \{1, 2, 3, \dots\}$

- ◆ The set of whole numbers denoted by W and given by $W = \{0, 1, 2, \dots\}$

- ◆ The set of whole numbers contains 0 and all natural numbers.

- ◆ 0, 2, 4, 6, ... are even whole numbers

- ◆ 1, 3, 5, ... are odd whole numbers.

- ◆ 1, 2, 3, 5, 7, 11, 13, ... are prime numbers

- ◆ 4, 6, 8, 9, 12, ... are composite numbers

- ◆ 1 is neither prime nor composite number

- ◆ 2 is the only even prime number.

- ◆ The set of integers denoted by the symbol Z and given by $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$

- ◆ ..., -3, -2, -1 are negative integers

- ◆ 1, 2, 3, ... are positive integers

- ◆ 0 is neither positive nor negative

- ◆ The set of integers contains whole numbers and opposites of whole numbers.

i.e, $N \subset W \subset Z$

- ◆ For every integer a its opposite is $-a$.
- ◆ $-a$ and a lies to the left and right of 0 at equal distance from zero on the number line.

For example, -5 lies to the left of zero and its opposite 5 lies to the right of zero on the number line.

- ◆ For any integers a and b ;
 - ◆ $a = b$ if and only if $a - b = 0$.
 - ◆ $a > b$ if and only if $a - b$ is positive .
 - ◆ If a and b represent integer numbers then one of the following relation is true

$$a > b \text{ or } a = b \text{ or } a < b$$

- ◆ The following are rule of basic operations when working with integers

Adding Rules:

- ◆ Positive + Positive = Positive:
- ◆ Negative + Negative = Negative:
- ◆ The sum of a negative and a positive number or positive and negative is either negative, positive or zero.

Example:

$$-2 + 15 = 15 - 2 = 13 \text{ (Recall that } -a + b = b - a \text{)}$$

Subtraction Rules:

- ◆ Negative - Positive = Negative:
- $$(-5) - 3 = -5 + (-3) = -8$$

◆ Positive - Negative = Positive + Positive = Positive:

$$5 - (-3) = 5 + 3 = 8$$

◆ Negative - Negative = Negative + Positive

$$(-5) - (-3) = (-5) + 3 = -2 \quad (-3) - (-5) = (-3) + 5 = 2$$

Multiplication Rules:

◆ Positive x Positive = Positive: $3 \times 2 = 6$

◆ Negative x Negative = Positive: $(-2) \times (-8) = 16$

◆ Negative x Positive = Negative: $(-3) \times 4 = -12$

◆ Positive x Negative = Negative: $3 \times (-4) = -12$

Dividing Rules:

◆ Positive ÷ Positive = Positive: $12 \div 3 = 4$

◆ Negative ÷ Negative = Positive: $(-12) \div (-3) = 4$

◆ Negative ÷ Positive = Negative: $(-12) \div 3 = -4$

◆ Positive ÷ Negative = Negative: $12 \div (-3) = -4$

◆ Working with integers, the following properties are true

◆ Integer + integer = integer

◆ Integer – integer = integer

◆ Integer × integer = integer

◆ Even integer + even integer = even integer

◆ Odd integer + odd integer = even integer

◆ Even integer – even integer = even integer

◆ Odd integer – odd integer = even integer

◆ Even number – odd number = odd

◆ Odd – even = odd

◆ The quotient of two integers is not always an integer

For example $6 \div 2 = 3$, the quotient 3 is an integer but if you reverse and divide the result is not the integer, i.e, $2 \div 6 = 1/2$ which is not an integer.

◆ For integers a, b, and c

◆ $a+b = b+a$ addition of integers is commutative

◆ $a \times b = b \times a$ multiplication of integers is commutative.

◆ $a+(b+c) = (a+b)+c$ addition of integers are associative

◆ $a \times (b \times c) = (a \times b) \times c$ multiplication is associative

◆ Subtraction and division are not commutative and associative.

◆ Property of zero

◆ For any integer a, $0+a = a$.

◆ Property of an opposite

◆ For any integer a its opposite is $-a$ and $-a+a=0$

◆ Negative of a negative is positive

i.e, for an integer a, $-(-a) = a$

Review exercises on unit 1

1. Write an integer to describe each situation.
 - a. Earned 50 Birr interest.
 - b. 4 degrees below a normal.
 - c. 14 degrees below 0
 - d. Sea level
 - e. 100 feet above sea level
 - f. A decrease of 2 members.
 - g. A withdraw of 400 Birr
 - h. An increase in height of 3 inches
2. Write the opposite of each integer.
 - a. A loss of 18
 - b. A gain of 22
 - c. 30 feet below the surface of the ocean
 - d. A rise of 15 feet
 - e. 25 feet below sea level
 - f. 80 feet above the ground
 - g. A profit of ten Birr
 - h. Sea level
3. Draw a number line and write all integers between -10 and 10.

4. The average surface temperatures of Jupiter, Mars, Earth, and Moon are shown in the table below. Order the temperatures from least to greatest (in ascending order) and greatest to least (in descending order)

Name	Average surface temperature($^{\circ}\text{F}$)
Jupiter	-162
Moon	-10
Mars	-81
Earth	59

5. Use the number line to perform the following operations

a. $-2 + 8$ b. $11 - 23$ c. $-5 - 7$

6. Simplify each of the following

a. $-23 + 57 + (-27) - 32$ d. $(-5)(4)(-1)$
 b. $-14 - 29 - (25 - 13 + (-12))$ e. $(4)(-5)(-1)(-4)$
 c. $-24 - (-19) + 29$ f. $-64 \div (-8)$

7. Evaluate

a. $x + y$ for $x = 9$ and $y = -15$
 b. $x - y$ for $x = -2$ and $y = -3$
 c. $x - y - z$ for $x = -12$, $y = 16$ and $z = 34$
 d. $(x + y - z) \div (x - y)$ if $x = 12$, $y = 6$ and $z = 6$

8. Which expression results in a negative answer?

- a. A negative number divided by a negative number
 b. A positive number divided by a negative number

- c. A negative number multiplied by a negative number
 - d. A positive number multiplied by a positive number
 - e. A positive number multiplied by a negative number
9. Heri has a bank account balance of 3,579 Birr and then she write a check for 1,254 Birr. What is her new balance?
 10. The highest ever recorded temperature on earth was 136°F in the Libyan desert and the lowest was -126°F in Antarctica. What is the difference of these temperatures recorded on Earth?
 11. A scientist is measuring the temperature change in a chemical compound. The temperature dropped 11°F per hour from the original temperature. After 4 hours, the temperature was 90°F . Find the compound's original temperature.
 12. Dukamo and his family go on a trip to visit the Sheger Park. The bus tickets cost 250 Birr per a person. What is the total cost if there are 5 members in Dukamos' family?
 13. Masamo is reading a book of 4,029 pages. If Masamo wants to read the same number of pages every day. How many page would Masamo have to read each day to finish in 79 days?
 14. Three brothers each have their own savings. They borrow birr72 from their parents for concert tickets. Each brother must pay back an equal share of this amount. In addition, the youngest brother owes his parents birr15. By how much will the youngest brother's savings change after he pays his parents?

UNIT 2

RATIO, PROPORTION AND PERCENTAGE

Unit Outcomes

After completing this unit , you should be able to:

- ▶ Understand the notations of ratio and proportions.
- ▶ Solve problems involving ratio and proportions.
- ▶ Describe a percentage.
- ▶ Solve problems involving percentages.
- ▶ Relate fractions, decimals and percentages to real life situations.
- ▶ Apply the concept of percentage in solving real life problems.

Main contents of the unit

2.1. Ratio and proportion

2.1.1. Ratio

2.1.2. Proportion.

2.2. Revision on percentages

2.3. Application of Ratio, Proportion and percentage

2.3.1. Calculating Profit and loss as a percentage.

2.3.2. Simple interest

2.3.3. Compound interest

2.3.4. Ethiopian income Tax, Turn Over tax and VAT.

Key terms

Unit summary

Review exercises

Introduction

Ratio, rate and proportion are three interrelated concepts. They are used in many problem-solving opportunities. Ratio, proportion and percent are used extensively in shopping, profit or loss calculations, data analysis, banking, and other aspects of everyday life. In grade 5, you learnt about comparing two fractions, solving percentage applications, and so on. In this unit, you will study proportional relationships such as how to solve proportion, how to use the two special ratios: rates, which compare quantities with different units, and percent, which compare a number to 100.

Opening problem

Assume that there are 160 grade seven students in a school and there are only 80 mathematics textbooks. Does each grade seven student in a school get a text Book? If no, how do you determine the share of the books to the students?

2.1. Ratio and Proportion

2.1.1 Ratio

Activity 2.1

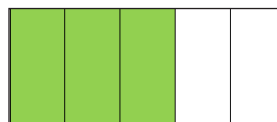
1. Write each of the following in simplest form

a. $\frac{2}{6}$

b. $\frac{3 \text{ Weeks}}{12 \text{ days}}$

2. Compare

- Shaded with unshaded
- Shaded with the total
- Unshaded with the total



Definition:

A ratio is a comparison of two or more quantities of the same kind by means of division.

The ratio of two quantities a to b is the quotient obtained by dividing a by b.

The ratio of a to b is written in any one of the following forms.

◆ a:b read as a to b

◆ $\frac{a}{b}$ read as a over b (In fraction form)

◆ $a \div b$ read as a divided by b (It is in quotient form)

Note:

In the ratio $a:b$;

- ◆ The quantities a and b are called terms of the ratio.
- ◆ The first term a is called antecedent.
- ◆ The second term b is called consequent.
- ◆ In ratio, $a : b \neq b : a$ unless otherwise $a = b$.

For example, the ratio of 2 to 3 is written as $2:3$ (using colon) or $\frac{2}{3}$ (as a fraction)

While 3 to 2 is written $3:2$ (using colon) or $\frac{3}{2}$ (as a fraction)

Note that 2 to 3 and 3 to 2 are different ratios.

The value of a ratio is usually expressed in its simplest form.

Example 1:

1. Write the ratio of the first number to the second number in the simplest form.
 - a. 20 to 40
 - b. 200 to 50

Solution:

- a. The ratio of 20 to 40 = $\frac{20}{40} = \frac{1}{2} = 1:2$
- b. The ratio of 200 to 50 = $\frac{200}{50} = \frac{4}{1} = 4:1$

Note:

- ◆ To find the ratios of two quantities, both the quantities must be expressed in the same unit of measure before their quotient is determined.
- ◆ Ratio has no unit

Example2:

Calculate the following ratios.

- a. Four hours to 75 minutes
- b. 75 cents to 2 birr
- c. 4 weeks to 12 days

Solution:

$$\text{a. Four hours to 75 minutes} = \frac{4\text{hrs}}{75\text{min}} = \frac{4(60)\text{min}}{75\text{min}} = \frac{16}{5}$$

$$\text{b. 75 cents to 2Birr} = \frac{75\text{cents}}{2(100)\text{cents}} = \frac{75}{200} = \frac{3}{8}$$

$$\text{c. 4weeks to 12days} = \frac{4\text{weeks}}{12\text{days}} = \frac{4(7\text{days})}{12\text{days}} = \frac{28}{12} = \frac{28}{12} = \frac{7}{3}$$

Classification of ratio

- i. Part-to-part comparison: Comparison of one part to another part.
- ii. Part – to – whole ratio comparison: Comparison of one part to the whole.
- iii. Equivalent ratio
- iv. Continued ratio

Example3:

In a class, there are 125 students 75 of them are girls and the rest are boys. Find

- The ratio of boys to girls
- Ratio of girls to boys
- Ratio of boys to total number of students
- Ratio of girls to total number of the students.

Solution:

Total number of the students = 125

Number of girls = 75

Number of boys = $125 - 75 = 50$

a. Ratio of boys to girls = $50 \div 75 = \frac{50}{75} = \frac{2}{3}$

Hence, the ratio is 2:3

b. Ratio of girls to boys = $\frac{75}{50} = \frac{3}{2}$

Hence ratio of girls to boys = 3:2

c. Ratio of boys to total number of the students is $\frac{50}{125} = \frac{2}{5}$

Therefore, ratio of boys to total = 2:5

d. Ratio of girls to total = $\frac{75}{125} = \frac{3}{5}$

Therefore, ratio of girls to boys is 3:5

In the above example, ratio of boys to girls or girls to boys is a part-to-part comparison whereas ratio of girls to total number of students or ratio of boys to total number is a part-to-whole comparison.

Equivalent ratio

Two ratios are equivalent (equal ratios) if one is obtained from the other by multiplying or dividing all the terms by the same non-zero number.

For example, $\frac{5}{3} = \frac{5}{3} \times \frac{2}{2} = \frac{10}{6}$ or, multiplying both the numerator and the denominator by 3 we get $\frac{15}{9}$. Using similar way equivalent ratios of $\frac{5}{3}$ are $\frac{10}{6}, \frac{15}{9}, \frac{20}{12}, \frac{25}{15}$ and so on.

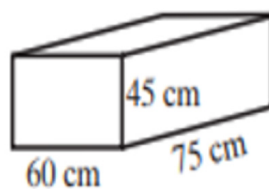
Continued ratio

Note:

- ◆ Continued ratio is a comparison of three or more quantities in a definite order
- ◆ The ratio of the numbers a, b, and c ($b, c \neq 0$) is a:b:c.
- ◆ The ratio a:b:c indicates the ratio a to b and the ratio b to c.

For example, the length of a rectangular solid is 75 centimeters, the width is 60 centimeters, and the height is 45 centimeters. The ratio of

- a. The length to the width is 75:60
- b. The ratio of width to the height is 60:45
- c. The continued ratio (ratio of length to width to height) is 75:60:45



Example 4:

A mixture contains six parts of oil, two parts of insecticide and ten parts of water by volume. Express the ratio of oil, insecticide, water in lowest terms.

Solution:

The ratio of Oil: Insecticide: Water = 6 : 2 : 10

$$= 3 : 1 : 5 \text{ (Dividing all the numbers by 2)}$$

One of the importance of ratio is to divide a whole into parts in any required size. Not only this, we can use ratio in scaling purpose also.

Example 5:

Divide 800 in the ratio of 3:5.

Solution:

The sum of the parts is $3 + 5 = 8$

Note that parts are terms of the ratio.

The first part is $\frac{3}{8}$ of 800 = 300. The second part is $\frac{5}{8}$ of 800 = 500

Example 6:

The scale on a map is 1:20,000. (1cm to 20,000 km). What actual distance does a length of 8cm on the map represent?

Solution:

1cm on the map represents 20,000 km on the ground

8cm on the map represents _____ km on the ground.

$1\text{cm} = 20,000\text{ km}$

$8\text{cm} = ?$

8cm on the map represents $8 \times 20,000\text{ km} = 160,000\text{ km}$ on the ground.

Group work 2.1

- 1. In your class if number of desks to number of students ratio is 1:3.What does this mean. Discuss in groups.
- 2. What do you say about the ratios $\frac{a}{b}$ and $\frac{b}{a}$? Can they be equal? Discuss.
- 3. On a set of architectural design for an office building, the scale is $1\frac{1}{2}\text{ inch} = 3\text{feet}$. Find the actual length of each room.

Room	Drawing Distances
Conference Room	7 inches
Lobby	2 inches
Mail Room	2.3 inches
Library	4.1 inches
Copy Room	2.2 inches
Storage	1.9 inches
Exercise Room	$3\frac{3}{4}$ inches
Cafeteria	$8\frac{1}{4}$ inches

Activity 2.2

A chemical engineer Dararo made 28 gallons of paint using white, black, yellow, and green in the ratio of 3:2:1:1 respectively. How many gallons of each material did she use?

Rate**Group work 2.2**

1. Alemu runs 300 meters in 40 seconds and Gatiso runs 200 meters in 25 seconds. Who is the faster runner?
2. Write the following in $\frac{a}{b}$ form.
 - a. 20km to 6hr
 - b. 100m to 25 sec
3. Dancho can make 6 glasses of orange juice from three oranges. At this rate, how many glasses of juice can he make from 8 oranges?

Definition:

Rate is a comparison of two quantities that have different units of measurements.

Rates describe a measure of how one-quantity changes for every unit of another quantity.

For example:

- ◆ A car moves 50km/hr means that a car travels 50km in one hour.
- ◆ Water flows 20 liters/minute means 20 liters of water flows in one minute.
- ◆ Price of goods increases five Birr per kilogram means that one kilogram of an object costs five Birr.
- ◆ It travels 30km/liter means that a vehicle travels 30km on one liter (of fuel).

Example 7:

Water is flowing into a dam at a constant rate of 600 liters per hour.

- a. How much water will flow into the dam in 2 hours?
- b. How long will it take for 12, 000 liters of water to flow into the dam?

Solution:

- a. 600 liters will flow into the dam in 1 hour.
 $600 \times 2 = 1200$ liters will flow into the dam in 2 hours.
- b. In 1 hr = 600×1 liters = 600liters
 In 10 hrs = 600×10 liters = 6000liters
 In 20 hrs = $600 \times 20 = 12,000$ liters

Therefore, it takes 20hrs for 12000 liters to flow into the dam.

Example 8:

If a car travels 120 km in 2 hours, its rate of speed is the ratio that compares the distance traveled in 2 hrs.

Solution:

$$\text{Rate} = \frac{\text{distance}}{\text{time}} = \frac{120\text{km}}{2h} = \frac{60\text{km}}{1h} = 60 \text{ km/h}$$

The abbreviation km/h read as 'kilometer per hour'

Note:

A rate that has a denominator of 1 is called a unit rate.

Group work 2.3

Write each rate in simplest form and give the unit rate

- 48 meters in 12 second
- 46 hours in 8 weeks
- 30,000 new job created in 6 months

Exercise 2.1

1. Write the following ratios in simplest form

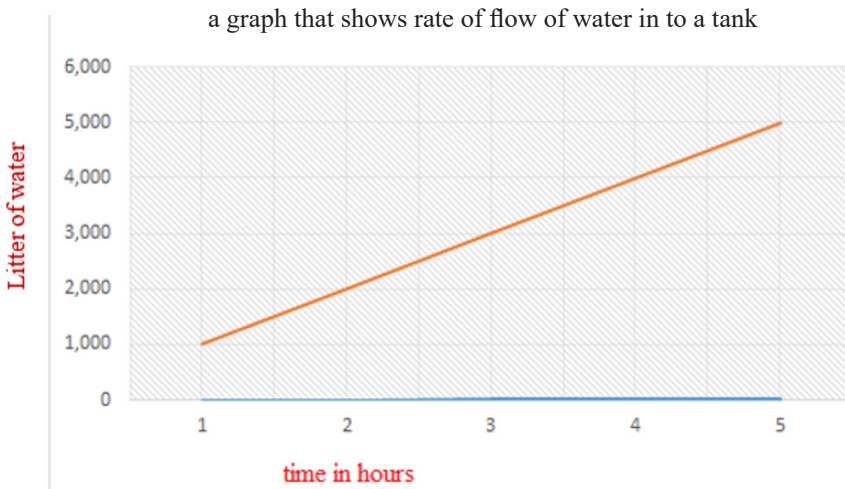
- | | | |
|-------------|------------|-------------|
| a. 24 to 72 | c. 40 to 5 | e. 0.2 to 8 |
| b. 8 to 32 | d. 5 to 4 | |

2. Write the following in unit rate.
 - a. 8 to 32
 - b. 72 to 1.2
3. If the ratio of two numbers is 5:1. Then, the larger number is how many times the smaller?
4. A student did six of ten problems correctly.
 - a. What is the ratio of the number of the right to the number of wrong?
 - b. For every two answers that were wrong, how many answers were right?
5. The perimeter of a rectangular garden is 30m, and the width is 6m. Find the ratio of
 - a. Length of the rectangle to its width
 - b. Length to the perimeter
 - c. Perimeter to width
 - d. Perimeter to length
6. Two numbers are in the ratio of 3:5 and their sum is 80. Find the numbers.
7. Ledamo, Ayele, and Bune are given 250 Birr. They decide to divide the money in the ratio of their ages, 10:8:7. Calculate the share of each.
8. Karissa scored 32 points in four tennis games. Express the average rate of the number of points Karissa scored per game in lowest term.

9. If $a : b = 3 : 2$, then find the value of $\frac{a+b}{a-b}$.
10. The graph below shows the amount of water in a tanker that flows in a similar rate each time.

Find

- The rate of flow and
- The amount of water at time $(t) = 10$ hours



2.1.2 Proportion

Activity 2.3

1. Determine whether the following fractions are equal.

a. $\frac{2}{3}$ and $\frac{12}{18}$

b. $\frac{11}{13}$ and $\frac{63}{78}$

c. $\frac{136}{85}$ and $\frac{8}{5}$

2. Assume seven people share 28 bananas and 12 people share 84 bananas. Is the share in each group the same? Discuss.

Definition:

A proportion is an equation that states the two ratios are equal.

Example 10:

a. $\frac{2}{3}$ and $\frac{12}{18}$ b. $\frac{11}{13}$ and $\frac{63}{78}$ c. $\frac{136}{85}$ and $\frac{8}{5}$

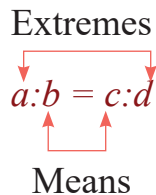
are examples of proportions

Note

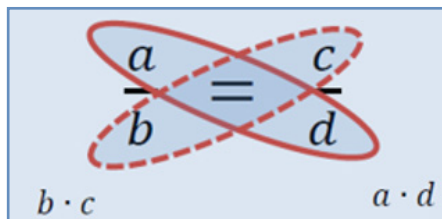
- i. Two ratios are said to be in proportion if they are equal
i.e., $\frac{a}{b} = \frac{c}{d}$ or $a:b = c:d$
- ii. In a proportion, the product of the means equals the product of the extremes.

That is, if $\frac{a}{b} = \frac{c}{d}$ or $a:b = c:d$ then, $a \times d = b \times c$.

This product is called the cross product of a proportion.



To check, whether $\frac{a}{b} = \frac{c}{d}$ is a true proportion, use cross multiplication.



Is $ad = bc$? If yes, then it is a true proportion.

Note:

In the proportion, $a:b = c:d$ or $\frac{a}{b} = \frac{c}{d}$;

- ◆ The four numbers a,b,c,and d are 1st, 2nd, 3rd, and 4th terms respectively.
- ◆ The 1st and 4th terms i.e. a & d are called the extremes and the 2nd and 3rd , i.e. b & c are called the means of the proportion.

Example 11:

1. Use the cross products to determine whether each pair of ratios forms a proportion or not.
 - a. $\frac{3}{4}$ and $\frac{18}{24}$
 - b. 4: 9 and 32:72
 - c. 3 to 7 and 15 to 36

Solution:

a. $\frac{3}{4}$ and $\frac{18}{24}$

The product of extremes is $3 \times 24 = 72$

The product of means is $4 \times 18 = 72$

Since the product of the extremes and the product of the means are equal, the ratio $\frac{3}{4}$ and $\frac{18}{24}$ forms proportion.

b. 4:9 and 32:72

The product of extremes is $4 \times 72 = 288$

The product of means is $9 \times 32 = 288$

Since the product of the extremes and the product of the means are equal, the ratio $\frac{4}{9}$ and $\frac{32}{72}$ forms proportion.

c. 3 to 7 and 15 to 36

The product of extremes is $3 \times 36 = 108$

The product of means is $7 \times 15 = 105$

The product of the extremes and the product of the means are not equal

Therefore, $\frac{4}{9}$ and $\frac{32}{72}$ are not forms a proportion.

Example 12:

The 1st, the 2nd and 4th terms of a proportion are 18, 24, and 48 respectively. Find the third term.

Solution:

Let the 3rd term be k,

Since 18, 24, k and 48 are in proportion, we can write as

$$\frac{18}{24} = \frac{k}{48}$$

$18 \times 48 = k \times 24$ by cross product of proportion

$$864 = 24k$$

$$k = \frac{864}{24} = 36 \text{ is the 3rd term}$$

Example 13:

Given the proportion $3:15 = 12:60$. Then, find the sum of the means.

Solution:

$3:15 = 12:60$ the given proportion

Then, $\frac{3}{15} = \frac{12}{60}$. The means are 12 and 15

Therefore, the sum of the mean of the proportion is $12+15 = 27$

Example 14:

The ages of a husband and his wife are in a ratio of 14:12 respectively.
How old is the husband if the wife is 36 years old.

Solution:

$H : W = 14:12$, where H = age of husband and W = age of the wife

$$\frac{H}{W} = \frac{14}{12} = \frac{7}{6}$$

$6H = 7W$, but age of the wife is given 36

$$H = \frac{7W}{6} = \frac{7(36)}{6} = 7(6) = 42$$

Therefore, age of the husband is 42 years.

Activity 2.4

1. The ratios $\frac{12}{21}$ and $\frac{36}{57}$ are not in proportion. If the same number from both denominators is subtracted, they become proportional. What is the number?
2. The ratio of two numbers is 4:5. After adding 20 to the smaller number and subtracting 20 from the greater number, the ratio becomes 14:13. Find the numbers.

Direct and inverse proportionality

Direct and inverse proportions are used to show how quantities and amount are related to each other. They are also described as directly proportional or inversely proportional.

The symbol used to denote the proportionality is ‘ $\propto$ ’.

Activity 2.5

1. The relation between a and b is given by

a. $b = 2a$ b. $b = \frac{a}{4}$ and c. $b = \frac{3}{a}$

Complete the table in each case and write what you understand from this.

a	1	2	3	4	5
b					

a	1	2	3	4	5
b					

a	1	2	3	4	5
b					

a. $b = 2a$ b. $b = \frac{a}{4}$ and c. $b = \frac{3}{a}$

2. The rate of decrease of temperature is 3.6°F for every 1000ft increase in altitude. What is the temperature if $h = 5,000\text{ft}$?

Note:

Given two variables x and y,

- ◆ y is said to be directly proportional to x ($y \propto x$) if there is a non-zero constant k such that $y = kx$
- ◆ y is inversely proportional to x ($\frac{1}{y} \propto x$) (varying inversely, in inverse variation, in inverse proportion, in reciprocal proportion) if there is a non - zero constant k such that $xy = k$

For example in activity 2.5 the relation between a & b in ‘question a’ is a direct proportionality whereas in ‘question c’ is an inverse proportionality.

Note:

- ◆ If x and y are directly proportional, then
 - a. when x increases y also increases
 - b. when x decreases y also decreases
- ◆ If x and y are inversely proportional, then
 - a. when x increases y decreases and
 - b. when x decreases y increases

For example, pressure and temperature have direct relationship. When pressure increases, temperature also increases and when pressure decreases, temperature decreases. $P \propto T$

However, pressure and altitude have inverse relation. When altitude increases, air pressure decreases and when altitude decreases air pressure increases. $P \propto \frac{1}{h}$

The rate of decrease of temperature is 6.5°C for each 1km altitude change. Or 3.6°F for every 1000ft increase in altitude.

Exercise 2.2

1. Determine whether each pair of ratios forms a proportion.

a. $\frac{1}{2}$ and $\frac{3}{9}$

b. $\frac{12}{14}$ and $\frac{2}{5}$

c. $\frac{25}{20}$ and $\frac{5}{4}$

2. Find the value of x in the following proportions.

a. $\frac{x}{25} = \frac{52}{100}$

c. $\frac{6.5}{1.3} = \frac{x}{2.5}$

e. $\frac{2}{3} = \frac{4x}{x+3}$

b. $\frac{12.5}{x} = \frac{15}{7.5}$

d. $\frac{16}{x+5} = \frac{5}{4}$

3. Write the corresponding equation of proportion for each of the following and solve for x.

a. 8 pencils in 2 boxes

20 Pencils in x boxes

b. x Birr for 5 Quna of wheat

14 Birr for 3 Quna of wheat

c. 6 Books for 10 students

x Books for 20 students

4. Find the sum of the means and product of the extremes in each of the following proportions.

a. 2:3 and 4:6

b. $\frac{5}{12} = \frac{15}{36}$

c. $\frac{a}{b} = \frac{c}{d}$, for b and d are non-zero numbers.

5. Find the mean proportion between the numbers

a. 9 and 16

b. 12 and 3

6. In making a cake, for each 2kg flour $\frac{1}{4}$ kg of sugar is required. How many kilogram of sugar is required for 3kg flour to make the mixture proportional?

7. A train travelled for 3 hours and covered a total of 180 km. Each hour it traveled the same distance. How many hours it takes to cover 840 km?

8. If water flows from a pipe at a constant rate and it takes 5 minutes to fill up a container with capacity of 35 liter, then how long will it take to fill a capacity of 77 liter?
9. The ages of two people add up to 25 and their ratio is 2:3. What is the age of the younger person?
10. A picture $3\frac{1}{4}$ inches long and $2\frac{1}{8}$ inches width is to be enlarged so that its length become $6\frac{1}{2}$ inches. What will be the width of the enlarged picture?
11. A student who is 5 feet tall casts an 8 foot shadow. At the same time, a tree casts a 40 foot shadow. How long is the tree?
12. If two cardboard boxes occupy 500 cubic centimeters space, then how much space is required to keep 200 such boxes?
13. In a scout camp, there is food provision for 300 cadets for 42 days. If 50 more persons join the camp, for how many days will the provision last?

2.2 Revision on percentage

The word percent is abbreviated from Latin word per centum, which means ‘per hundreds’, or ‘for every hundreds’ or ‘hundredth’.

Note:

- a. The symbol % used for the term percent.
- b. 1percent means $\frac{1}{100}$ ($1\% = \frac{1}{100}$)
- c. People use percent to give a quick comparison on a scale from 1 to 100.

- d. Percentages are comparisons of a given quantity (or part) with the whole amount (which we call 100).
- e. Percentages are commonly used to describe interest, sales prices, test results, inflation, and change in profit or loss
- f. Percentages can range from zero to higher than 100.

For example, 12% means $12 \times \frac{1}{100}$

12% of a quantity means $12 \times \frac{1}{100} \times \text{total amount}$.

Here 12% indicates a percent whereas, 12% of a quantity indicates the percentage.

For example 40 is 20% of 200

The phrase “20% of 200” indicates the percentage but 20% indicates the percent.

Percentage is more general whereas percent is specific. Most of the time percentage is expressed in words and percent is by specific number.

Group work 2.4

A number n is 25% of some number m and 35% of another number k . Describe the relationship between them. Is $m < k$, $m > k$, $m = k$ or is it impossible to determine the relationship? Discuss.

The concept of percent (%) is widely used in variety of mathematical applications, such as

- A price is decreased by 12%
- VAT in Ethiopia is 15%
- Number of patients is decreased by 48%

Example 15:

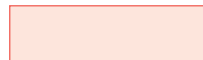
- 56 percent can be written as 56% and this means the ratio of 56 to 100 or $56\% = \frac{56}{100}$
- 120 percent can be written as 120% and this means the ratio of 120 to 100
i.e. $120\% = \frac{120}{100}$
 $100\% = \frac{100}{100} = 1.$

Note:

- ◆ Hundred percent means the whole. It indicates total of an object, or total of something.

For example, 100% of 20 Birr means completely 20 Birr.

Hundred percent shaded



Note:

- ◆ 0 % means zero or no value.

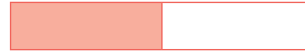
For example, 0% of 100 Birr means 0

Zero percent shaded or
100% unshaded



Example 16:

In the figure to the right,
half of it is shaded.



The whole table is considered as 100%

Half of the table = $\frac{1}{2}$ (the whole)

$$= \frac{1}{2} (100\%)$$

= 50% of the table is shaded

The percent is 50 and the percentage for this is the expression “shaded part is 50% of the whole”. The whole is considered as one. Hence 50% of 1 is $50 \times \frac{1}{100} \times 1 = \frac{1}{2}$.

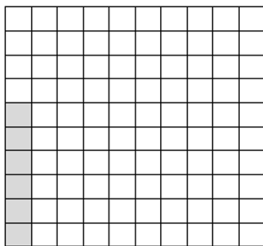
Example 17:

In the figure, $\frac{1}{4}$ of it is shaded.



i.e. 25% is shaded

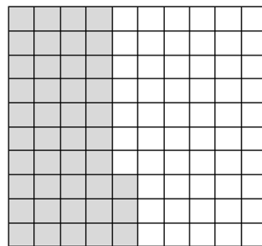
The place value models below are hundredths flats. You can also use a percent to tell how much is shaded.



6 hundredths

6 out of 100

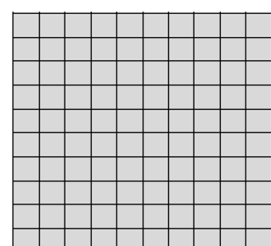
6% is shaded



43 hundredths

43 out of 100

43% is shaded



100 hundredths

100 out of 100

100% is shaded

Activity 2.6

1. Count total number of pages of your mathematics exercise book. How many pages are used? How many pages are free? Express both in percent.
2. If you covered only two units of your grade 7 mathematics lesson, express the remaining units in percent.
3. If you have 200 Birr and you give 75 Birr to your brother, what percent of the money remains with you?
4. What percent of 235 is 141?
5. Shade 25% of a square.

Relation between RATE(R), PERCENTAGE (P) and BASE (B) is given as follows:

$$\frac{R}{100} = \frac{P}{B} \quad \text{Or,} \quad \frac{B}{100} = \frac{P}{R}$$

$$A. \text{ Percentage} = \text{Percent (Rate)} \times \text{Base (Whole)}$$

Example 18:

What is 45% percent of 3,245?

Solution:

Percent (Rate) = 45%

Base = 3,245.

Required: Percentage

Percentage = percent (Rate) $\times$ Base (whole)

$$\text{Percentage} = \frac{45}{100} \times 3,245 = 1460.25$$

Example 19:

Deraro spends 5% of her salary for transportation in a month. If her salary is Birr 6000. What is her percentage expenditure for transport?

Solution:

$$\text{Base (B)} = 6000\text{Birr}$$

$$\text{Rate(R)} = 5\%$$

$$\text{Percentage (P)} = ?$$

$$P = \text{Percent (Rate)} \times \text{Base (whole)}$$

$$= 5\% \times 6000\text{Birr}$$

$$= \frac{5}{100} \times 6000\text{Birr}$$

$$= 5 \times 60\text{Birr}$$

$$= 300\text{Birr}$$

B. Percent (Rate) = Percentage divided by Base

$$P (\text{Rate}) = \frac{\text{percentage}}{\text{Base}} \times 100\%$$

Example 20:

What percent of 125 is 50?

Solution:

Given: Percentage = 50 and the whole (total) = 125

Required: Percent (Rate)

$$R = \frac{\text{percentage}}{\text{Base}} \times 100\% = \frac{50}{125} \times 100\% = 0.4 \times 100\% = 40\%$$

Example 21:

In a class there are 45 students, 18 of them come to school by car.

- What percent of the students come to school by car?
- How many students come to school on foot?
- What percent of the total students come to the school on foot?

Solution:

- a. Given: Base = 45, Percentage = 18. Rate = ?

$$\text{Rate} = \frac{\text{percentage}}{\text{Base}} \times 100\%$$

$$= \frac{18}{45} \times 100\%$$

$$= 2 \times 20\% = 40\%$$

- b. 40% of the students come to school by car. Number of students who come to school on foot = Base – who come to school by car

$$= 45 - 18 = 27$$

- c. Percent = $\frac{\text{percentage}}{\text{Base}} \times 100\%$

$$= \frac{27}{45} \times 100\%$$

$$= 0.6 \times 100\% = 60\%$$

Alternatively, you can use the following technique.

Total = Number of the students who come to school by car plus Number of the students who come to school on foot.

$100\% = 40\% + \text{Number of the students who come to school on foot}$

$100\% - 40\% = \text{Number of the students who come to school on foot}$

Therefore, total number of students who come to school on foot = 60%

C. Base (B) = Percentage divided by Rate

Example 22:

25% of what is 300 Birr?

Solution:

Percentage = 300

Rate = $25\% = \frac{25}{100} = 0.25$

Required: Base = ?

$$\begin{aligned} B &= \frac{P}{R} \\ &= \frac{300}{0.25} \\ &= \frac{30000}{25} = 1,200 \text{ Birr.} \end{aligned}$$

Example 23:

If 25% of Mohamed's salary is 1,500 Birr. What is his salary?

Solution:

Rate = 25%, Percentage = 1500

Required: Base = ?

Base = Percentage divided by rate

$$B = \frac{1500}{0.25} = 6000$$

Therefore, Mohamed's salary is Birr 6000.

Note:

The main difference between ratio and rate is

- ◆ We can represent a ratio as a percent
- ◆ We can't represent a rate as a percent

Writing a percentage in decimal and in fraction form.

Example 24:

Write the following percentage in decimal form and in fraction form.

- a. 3% b. 2.3% c. 32% d. 200%

Solution:

a. $3\% = \frac{3}{100} = 0.03$ in decimal notation form and

$3\% = \frac{3}{100}$ in fraction form.

b. $2.3\% = \frac{2.3}{100} = 0.023$ in decimal form

$2.3\% = \frac{2.3}{100} = \frac{23}{1000}$ in fraction form

c. $32\% = \frac{32}{100} = 0.32$ in decimal form

$32\% = \frac{32}{100} = \frac{8}{25}$ in decimal form.

d. $200\% = \frac{200}{100} = 2$ in fraction form.

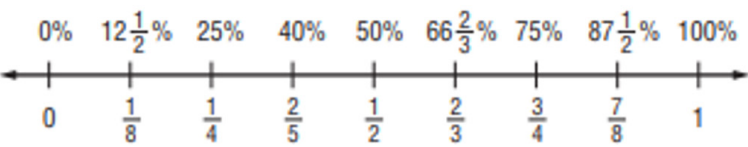
Group work 2.5

Copy and complete the table to the right

percentage	decimal	fraction
25 %	0.25	$\frac{1}{4}$
$\frac{7}{20}$ %		
26.5%		
	6.5	
	0.65	
		$\frac{3}{4}$
		$\frac{1}{25}$

Note:

When working with common percent like, 0%, 10%, 40%, 50%, it may be helpful to calculate the fraction form mentally.



For example to calculate 25% of 300 multiply it by $\frac{1}{4}$.
i.e. 25% of 300 = $300(\frac{1}{4}) = 75$.

Exercise 2.3

1. Express each of the following percent as a fraction in lowest form.
- a. 45%

b. $83\frac{1}{3}\%$

c. 12.5%

2. Express each fraction as a percent

a. $\frac{4}{5}$

b. $\frac{4}{3}$

c. 0.23

d. 2.66666...(round off to two decimal places)

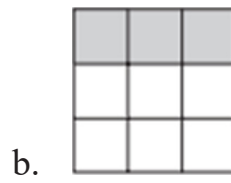
3. Express each of the following percentage as a decimal.

a. 25%

b. 125%

c. 0.23

4. Assuming that the regions in each figure are equal, write the shaded part in percent in each case. Which figure has the greatest shaded part?



5. Find

a. Increase of 50Birr by 70%

b. Decrease of 50Birr by 25%

6. Find

a. The percentage increase of 120 Birr to 150 Birr

b. The percentage decrease of 200 Birr to 120 Birr

c. If 15% of Jemal's salary is Birr 135.75, what is the amount of his full salary?

2.3 Application of ratio, proportion and percentage

2.3.1 Calculating profit and loss as a percentage

Activity 2.7

1. What does 100% of something mean? 120% of a quantity mean? 200% of a quantity mean. Give examples of such possible cases in real life situations.
2. Abebe had bought a sheep at Birr 2350 and sold it at Birr 2550. Had he got a profit? His loss? Discuss with friends.

The profit and loss statements is a report aimed at providing information on the costs and income of a certain business within respective period of time, such as a year, a quarter, month or even a day. They can also be expressed in percent.

Group work 2.6

Suppose a store advertises a sale, in which all merchandises at a 20% discount. How much you will save if the original price is Birr500.

Definitions, explanations and some examples of important terms in marketing system.

- a. **Discount:** is the amount by which the regular price of an item is reduced.

Note: **n% off** means that at a discount of n % of the selling price.

b. Cost Price:

The price, at which an article is purchased, is called the cost price, abbreviated as C.P.

c. Selling Price:

The price, at which an article is sold, is called its selling prices, abbreviated as S.P.

d. Profit/ Gain:

Profit is made when the selling price is greater than the cost price. The amount of profit is the difference between the selling price and the cost price.

$$\begin{aligned}\text{Profit/ Gain} &= \text{Selling price(S.p)} - \text{Cost price(C.p)} \\ &= \text{S.p} - \text{C.p}\end{aligned}$$

e. Loss:

Loss is made when the cost price is greater than the selling price. It is the difference between the cost price and the selling price.

$$\begin{aligned}\text{Loss} &= \text{Cost price(C.p)} - \text{Selling price(S.p)} \\ &= \text{C.p} - \text{S.p}\end{aligned}$$

Profit and Loss Based on Selling Price

To find the percent gain or loss, divide the amount gained or lost by the cost and multiply it by 100.

Percentage of Profit/gain: divide the amount of profit by the cost price and multiply by 100%

$$\begin{aligned}
 \text{Percent of profit/gain} &= \frac{\text{Selling price}(S.p) - \text{cost price}(C.P)}{\text{cost price}(CP)} \times 100\% \\
 &= \frac{s.p - c.s}{c.s} \times 100\% \\
 &= \frac{\text{profit}}{\text{cost price}} \times 100\%
 \end{aligned}$$

Percent of loss = divide the amount of loss by the cost price and multiply by 100%

$$\begin{aligned}
 \text{Percent of loss} &= \frac{\text{Cost price}(C.p) - \text{Selling price}(S.p)}{\text{cost price}(CP)} \times 100\% \\
 &= \frac{c.p - s.p}{c.s} \times 100 \\
 &= \frac{\text{loss}}{\text{cost price}} \times 100\%
 \end{aligned}$$

Example25:

A toy that cost 80 birr is sold at a profit of 20 birr. Find the percent or rate of profit.

Solution:

$$\begin{aligned}
 \text{Percentage of profit (\% profit)} &= \frac{\text{gain}}{\text{cost}} \times 100 \\
 &= \frac{20}{80} \times 100 \\
 &= \frac{2000}{80} \\
 &= 25\%
 \end{aligned}$$

Example26:

Shegena sells his hen for 130 at a profit of 65. What percent of profit on selling price does this represent?

$$\text{Percent of profit (\% profit)} = \frac{\text{gain}}{\text{selling price}} \times 100$$

$$= \frac{65}{130} \times 100$$

$$= \frac{650}{13} = 50$$

Therefore, percent of profit = 50%

Example 27:

A shop Keeper buys shoes for birr 125 and sold it for birr 160. Find

- i. The Profit
- ii. Percent of profit

Solution:

Cost price = 125 birr,

Selling price = 160 birr

- i. Profit/ Gain = Selling price(S.p)- Cost price(C.p)

$$= 160 \text{ birr} - 125 \text{ birr}$$

$$= 35 \text{ birr}$$
- ii. Percent of profit (% profit) = $\frac{\text{profit}}{\text{cost price}} \times 100\%$

$$= \frac{35}{125} \times 100\%$$

$$= 28\%$$

Therefore, the percentage of profit is 28%

Example 28:

A bicycle was purchased for birr 6000 and was sold two months later for birr 4500. Find

- i. The loss

ii. Loss in percent?

Solution:

Cost price = 6000Birr,

Selling price = 4500Birr

i. Loss = Cost price(C.p)-Selling price(S.p)

$$= C.p - S.p$$

$$= 6000 \text{ birr} - 4500 \text{ birr}$$

$$= 1500 \text{ birr}$$

ii. Loss in percent (%loss) = $\frac{\text{loss}}{\text{cost price}} \times 100\%$

$$= \frac{1500 \text{ birr}}{6000 \text{ birr}} \times 100\%$$

$$= 25\%$$

Therefore, percent of loss is 25%

Example29:

A toy is sold for 6 birr at a profit 25%. Find the profit and cost price.

Solution:

Given: Percentage of profit = 25%,

Selling price = 6 birr

i. S.P = C.p + 25% C.P

$$6 = C.P + 0.25 CP$$

$$6 = 1.25C.P$$

$$C.P = \frac{6}{1.25} = 4.8 \text{ Birr}$$

$$\begin{aligned}\text{ii. profit} &= \text{selling price} - \text{cost price} \\ &= 6 \text{ birr} - 4.8 \text{ birr} = 1.2 \text{ birr}\end{aligned}$$

Example 30:

A 2000 Birr bicycle is on sale with a 30% discount. How much is the sale price?

Solution:

Percentage of discount (loss) = 30%,

Cost price = 2000 Birr

$$\begin{aligned}\text{i. Loss} &= \text{cost price} \times \% \text{ discount (loss)} \\ &= 2000 \times 30\% \\ &= \frac{2000 \times 30}{100} = 600 \text{ birr.}\end{aligned}$$

$$\begin{aligned}\text{ii. Selling price} &= \text{cost price} - \text{Discount (loss)} \\ &= 2000 \text{ birr} - 600 \text{ birr} = 1400 \text{ birr}\end{aligned}$$

Exercise 2.4

1. A laptop is bought and then sold one year later for 8500 Birr, 15% less than what was originally paid for it. How much was paid for the computer originally?
2. Three friends started a part-time business. They plan, each month, to share the profits in the ratio of the number of hours that each worked. During the first month, Duguna worked 18 hours, Rori worked 30 hours, and Durete worked 12 hours.

- a. Express, in lowest terms, the ratio of the time Duguna: Rori: Durete that they worked during the first month.
 - b. Find the amount each of them should receive if the profit for this first month was 540 birr.
 - c. Find the amount each should receive if the profit for this first month was 1270 birr.
3. A student scores $\frac{18}{24}$ (18 marks out of 24 marks) in first test and $\frac{24}{48}$ (24 marks out of 48 marks) in a second test. Express each score as a percentage and hence state the better score.

2.3.2 Simple interest

Group work 2.7

There are 520,000 people in a country, if only one percent of this number increases each year, what will be total number of population of the country after 5 years?

When money is deposited in a bank or building society account, it commonly attracts interest; in a similar way, a borrower must normally pay interest on money borrowed. The rate of interest is usually (but not always) quoted as a rate percent per year. Up-to-date rates of interests are available from finance institute. There are two basic ways of calculating the amount of interest paid on money deposited. These are simple interest and compound interest.

Simple interest

Simple interest is a type of fee that is charged (or paid) only on the amount borrowed (or invested), and not on past interest.

Terms related to interests:

The principal amount (P)

The original amount of money borrowed or invested.

Interest (I)

The amount of money added to a loan or to a deposit based on an initial loan or investment and an interest rate.

Time (T)

This is the period at which money is borrowed or deposited. Time is normally expressed in months or years. It is denoted by a capital letter “T.”

Amount (A):

The amount is the sum of the total interest and the principal over a given period.

$$\text{Simple interest (I)} = \text{principal} \times \text{rate} \times \text{time}$$

$$I = P \times r \times t = Prt,$$

where I = simple interest
 P = principal
 r = interest rate (expressed percentage)
 t = time duration (in years)

Total amount (A) = Principal (P) + Interest (I)

$$= P + Prt$$

$$= P(1+rt)$$

$$A = P(1 + r t)$$

Example 31:

A woman deposited 5000 birr in a bank which charged simple interest of 5 percent per year. Calculate the interest and the amount she earned after 2 years.

Solution:

Principal (P) = 5000birr

Interest rate = 5%

Time duration = 2 years

Interest(I) = $P \times r \times t$

$$= 5000 \text{ birr} \times \frac{5}{100} \times 2$$

$$= 500 \text{ birr} \times 5 \times 2$$

= 500 birr is the interest of 5000 birr after 2 years.

Total amount (A) = Principal (P) + Interest (I)

$$= 5000 \text{ birr} + 500 \text{ birr}$$

= 5500 birr is the total amount

Note:

The time must in years to apply the simple interest formula. If you are given months, convert it to years.

Example32:

Find the total amount due on a loan of 600 birr at 16% interest at the end of 15 months.

Solution:

$$\text{Principal (p)} = 600 \text{ birr}$$

$$\text{Interest rate} = 16\%$$

$$\text{Time duration} = 15 \text{ months} = \frac{15}{12} = 1.25 \text{ year}$$

$$\text{Interest (I)} = P \times r \times t$$

$$= 600 \text{ birr} \times \frac{16}{100} \times 1.25$$

$$= 120 \text{ birr is the interest of 600 birr after 1.25 years.}$$

$$\text{Total amount (A)} = \text{Principal (P)} + \text{Interest (I)}$$

$$= 600 \text{ birr} + 120 \text{ birr} = 720 \text{ birr}$$

Example33:

Dukamo deposited 140 birr in a commercial bank of Ethiopia. At the end of the first year, he received 9.8Birr interest. What is the rate of interest?

Solution:

$$\text{Principal (p)} = 140 \text{ birr}$$

$$\text{Interest} = 9.8$$

$$\text{Time duration} = 1 \text{ years}$$

$$\text{Interest(I)} = P \times R \times T$$

$$9.8 \text{ birr} = 140 \text{ birr} \times \frac{r}{100} \times 1 \text{ year}$$

After 1 year, the amount of interest is given by

$$\text{Interest rate}(r) = \frac{9.8 \text{ birr} \times 100}{140 \text{ birr}} = 7$$

The interest rate (r) is 7 %

Example 34:

A boy borrowed 5000 birr from Sidama micro finance institution and paid back 5700 Birr a year later. What is the annual rate of interest?

Solution:

Principal (p) = 5000 birr

Duration = 1 year

Total amount (A) = 5700 birr

Interest = Total amount – principal amount

$$= 5700 \text{ birr} - 5000 \text{ birr} = 700 \text{ birr}$$

Interest (I) = $P \times R \times T$

$$700 \text{ birr} = 5000 \text{ birr} \times \frac{r}{100} \times 1 \text{ year}$$

$$\text{Interest rate (r)} = \frac{700 \text{ birr} \times 100}{5000 \text{ birr}} = 14$$

The interest rate is 14%

Exercise 2.5

1. Calculate (i) the interest payable and (ii) the total amount on the following deposits at simple interest.
 - a. Birr 300 borrowed for 2 years at 3% per year.

- b. Birr1000 invested for 4 years at 12% per year.
 - c. Birr 3600 invested for 6 months at 7.5 % per year.
2. Duguna added 400 birr into his saving account which attracting a simple interest at the rate of 5% per year. What interest will he receive if he leaves the money in the account for 8 years. What is the total amount?
 3. For how long would 500 birr have to be left in an account paying 4% simple interest per year to give a balance of 600 birr?
 4. Find the interest on a boat loan of 5,400 Birr at 16% annually at simple interest for four months.
 5. How much must Dancho deposit in a simple interest at a rate of 6% annually, if he wants to gain an interest of 120 Birr after two years?
 6. A total of 1,200 birr is invested at annual simple interest rate of 7% for 18 months. How much interest is earned on this investment?

2.3.3 Compound Interest

Group work 2.8

1. What does an interest of the interest mean? Discuss in groups.
2. Assume that there are wild goats in a jungle, some of them are males and the others are females. If each female give a birth at the age of 2 years and the jungle is suitable for their life, what do you think about increment of their number after 5 years?

Simple interest, which you studied in the previous lessons, is paid only on the initial principal of a saving account or loan. Compound interest is paid on the initial principal and on the interest earned in the past.

Unlike simple interest, compound interest on an amount accumulates at a faster rate than simple interest. The basic idea is that after the first interest period, the amount of interest is added to the principal amount and then the interest is computed on this higher principal. The latest computed interest is then added to the increased principal and then interest is calculated again. This process is completed over a certain number of compounding periods. We earn not only interest, but interest on the interest already paid. This is called compound interest.

In general, calculating compound interest depends on several factors. These factors include the amount of money deposited (p), the annual interest rate (r), the number of times the money is compounded per year (n), and the number of years the money is left in the bank (t). These factors lead to the formula

Total amount (S) = $p(1 + \frac{r}{n})^{nt}$ where;

P = principal or amount of money deposited

r = annual interest rate (in decimal form)

n = number of times compounded per year

t = time in years.

For example, the following chart shows how institutions such as banks calculate the interest and amount in each interest period. Let the initial (Principal) amount be 1000 and interest rate 0.03 as indicated in B column.

	A	B	C	D
1	RATE	0.03		
2				
3	PRINCIPAL	INTEREST	NEW PRINCIPAL	TIME (YR)
4	1000.00	30.00	1030.00	0.5
5	1030.00	30.90	1060.90	1.0
6	1060.90	31.83	1092.73	1.5
7	1092.73	32.78	1125.51	2.0
8	1125.51	33.77	1159.27	2.5
9	1159.27	34.78	1194.05	3.0
10	1194.05	35.82	1229.87	3.5
11	1229.87	36.90	1266.77	4.0
12	1266.77	38.00	1304.77	4.5
13	1304.77	39.14	1343.92	5.0

Example35:

If Birr20,000 is deposited in a bank with interest rate of 12% which compounded annually, what will be the total amount of money at the end 3rd year.

$$P = 20,000$$

$$T = 3$$

$$R = 12\%$$

$$n = 1$$

Required: A = ?

Solution:

$$\begin{aligned}
 \text{Total amount } (S) &= p \left(1 + \frac{r}{n}\right)^{nt} \\
 &= 20,000 \left(1 + \frac{12}{(100)(1)}\right)^{1 \times 3} \text{ Birr} \\
 &= 20,000(1+0.12)^3 \text{ Birr} \\
 &= 20,000 (1.12)^3 \text{ Birr} \\
 &= 20,000(1.12)(1.12)(1.12)\text{Birr} \\
 &= 28,098.56 \text{ Birr}
 \end{aligned}$$

Example 36:

If you deposit 6000 birr into an account paying 6% annual interest compounded quarterly, how much money will be in the account after 2 years?

Solution:

Principal = 6000 birr,

Interest rate = 6% = $\frac{6}{100} = 0.06$

Number of times compounded per year (n) = 4 times (since quarterly)

Time duration = 2 years.

$$\begin{aligned}\text{Total amount } (S) &= p\left(1 + \frac{r}{n}\right)^{nt} \\ &= 6000 \left(1 + \frac{0.06}{4}\right)^{4 \times 2} \\ &= 6000(1 + 0.015)^8 = 6000(1.015)^8 \text{ Birr}\end{aligned}$$

Exercise 2.6

1. A principal of 10,000 birr is deposited in a bank at the following interest rates. Then calculate the interest in each case after 2 years.
 - a. 2.75% interest, compounded annually.
 - b. at 2.7% interest, compounded semi-annually
 - c. at 2.65% interest, compounded quarterly

2. If you deposit 6500 birr into an account paying 7% interest compounded semiannually in a bank. How much money will be in your account after 7 years?

2.3.4 Ethiopian income Tax, Turn over Tax, VAT

Activity 2.8

1. Define the following terms in your own words
 - a. Tax
 - b. VAT
 - c. TOT
2. Who is VAT payer? Seller or buyer?

Business Income tax

Every year a great number of foreign and domestic companies join the various attractive and productive industries of Ethiopia. In addition to creating jobs, generating exports, and/or substituting imports. These companies broaden the tax base and contribute to generating income for the country. In addition, if you have a business company or intending to open a new business here in Ethiopia, here is the list of taxes imposed on business companies operating in Ethiopia.

Table 2.1 Business income tax rates for unincorporated or individual businesses

Taxable business income(I) per year (ETB)	Business income tax rate (BIT)	Deduction	Tax computation
0-7200	0	0	Exempted
7201-19800	10%	720	$BIT=(I \times 10\%) - 720$
19,801–38,400	15%	1,710	$BIT=(I \times 15\%) - 1,710$
38,401–63,000	20%	3,630	$BIT=(I \times 20\%) - 3,630$
63,001–93,600	25%	6,780	$BIT=(I \times 25\%) - 6,780$
93,601–130,800	30%	11,460	$BIT=(I \times 30\%) - 11,460$
Over 130,800	35%	18,000	$BIT=(I \times 35\%) - 18,000$

(Source: Federal Income Tax Proclamation (No. 979/2016))

BIT = Business Income Tax

Example37:

Assume, yearly income of a shop owner is 115,000 birr. What is the tax she should pay for the government?

Solution:

Business income (I) per year = 115,000 birr

Income tax rate(R) = 30%

Deduction (D) = 11,460 birr

Business income tax (BIT) = $IR - D$

$$= (I \times 30\%) - 11,460$$

$$= (115,000 \text{ birr} \times 30\%) - 11,460 \text{ birr}$$

$$= 34500 \text{ birr} - 11,460 \text{ birr} = 23040 \text{ birr}$$

Employment income tax

Employment income tax (EIT) is leveled each calendar month on the employment income that a resident employees receive during each month. An employee's income tax liability is calculated according to the marginal tax rate schedule shown in Table below.

Table 2.2 Employment income tax (EIT) rate of Ethiopia

Employment income per month (I)(ETB)	Employment income tax rate (EIT)	Deduction	Tax computation
0-600	0%	0	Exempted
601-1650	10%	60.00	$EIT=(I \times 10\%)-60$
1651-3200	15%	142.50	$EIT=(I \times 15\%)-143.50$
3201-5250	20%	302.50	$EIT=(I \times 20\%)-302.50$
5251-7800	25%	565.00	$EIT=(I \times 25\%)-565.00$
7801-10900	30%	955.00	$EIT=(I \times 30\%)-955.00$
More than 10900	35%	1500.00	$EIT=(I \times 35\%)-1500.00$

(Source: Federal Income Tax Proclamation (No. 979/2016))

Example38:

If the monthly Salary of Dukamo is 15,500Birr, what is the total tax he pay in one year?

Solution:

Dukamo's salary per month = 15,500

Income tax rate = 35%

Deduction = 1500 birr

Salary income tax per month (EIT) = $(I \times 35\%) - 1500.00$

$$= (15500 \times 35\%) - 1500.00$$

$$= 5425 \text{ birr} - 1500 \text{ birr}$$

$$= 3925 \text{ birr}$$

Dukamo pays 3925 Birr each month.

Therefore $3925 \text{ birr} \times 12 \text{ month} = 47,100 \text{ birr}$

The total tax he pay is 47,100 Birr in a year.

Turn over Tax (TOT)

Turn over tax is payable on goods and services supplied by a person who are not registered for VAT. Turn over tax proclamation states that ToT is at a rate of 2% on the supply of goods and services.

Example39:

The price of 1 kg Coffee is 120 birr including Turn over tax (TOT). How much is the turn over tax (TOT) on the coffee.

Solution:

$$\begin{aligned} \text{Turn over Tax (TOT)} &= 120 \text{ birr} \times \frac{2}{100} \\ &= \frac{120 \text{ birr} \times 2}{100} \\ &= 2.4 \text{ birr} \end{aligned}$$

The Value Added Tax (VAT)

Value added tax (VAT) is a tax imposed by the government on sales of goods and services other than those VAT exempted items.

VAT rate is 15% of the value added on to the original cost of every taxable transaction by a registered person.

Example40:

In a shop, the selling price of a Television including VAT is 15,000 Birr.

- What is the mount of VAT (Value added Tax) on the TV?
- What price must the TV be sold for?

Solution

Selling price of TV = 15000 birr

VAT rate = 15%

VAT = Selling price $\times$ VAT rate

$$= 15000 \text{ birr} \times \frac{15}{100}$$

$$= 150 \text{ birr} \times 15$$

$$= 2250 \text{ birr}$$

The VAT is 2250 birr.

Price of TV excluding VAT = total cost - VAT

$$= 15000 \text{ birr} - 2250 \text{ birr}$$

$$= 12750 \text{ birr}$$

Example41:

If the sales tax on a purchase of Birr800 is Birr12. What will be the sales tax on the purchase of Birr3000?

Solution:

$$\frac{800}{12} = \frac{3000}{n}$$

$$800n = 36,000$$

$$n = \frac{36000}{800} = \frac{360}{8} = 45 \text{Birr}$$

Exercise 2.7

1. Find the tax, a merchant should pay if her total income per year is 72,500 Birr.
2. Calculate the total tax an employee whose salary is Birr 8800 should pay?
(Refer table 2.1 and 2.2 for question number 1 and 2)
3. If you purchased goods for Birr2400, what is the total VAT you should pay?

Key terms

Ratio

Proportion

Percentage

Interest

Simple interest

Compound interest

Tax

Income tax

Turn over tax (TOT)

Value added tax(VAT)

Unit summary

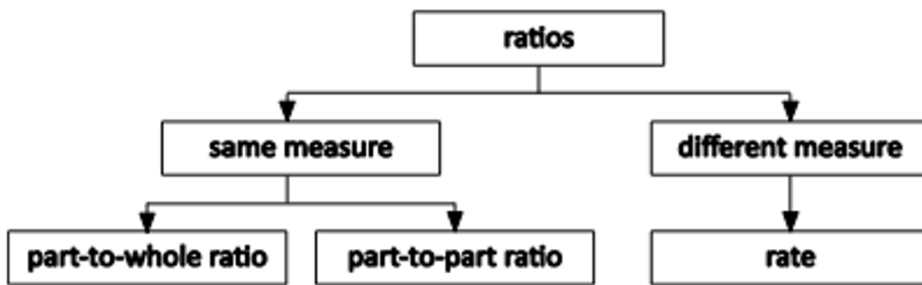
- ◆ A ratio is a comparison of two quantities of the same kind by means of division. Ratio has no unit. The ratio of a to b is written as any one of the following terms.

- ◆ $a : b$ read as a to b

- ◆ $\frac{a}{b}$ read as a over b (In fraction form)

- ◆ $a \div b$ read as a divided by b (It is in quotient form)

The diagram below illustrate the relation between rate and ratio.



- ◆ Ratios are classified as part-to-part comparison or part-to-whole comparison.

- ◆ Part-to-part comparison: Comparison of one part to another part of the whole.

- ◆ Part – to – whole ratio comparison: Comparison of one part to the whole.

◆ Equivalent ratio

- ◆ Two ratios are equivalent (equal ratios) if one is obtained from the other by multiplying or dividing all of the quantities by the same non-zero number.

Equivalent ratios behave like equivalent fractions.

Examples

$1 : 3 = 2 : 6 = 3 : 9 = 4 : 12$ and so on.

- ◆ **Continued ratio:** is a comparison of three or more quantities in a definite order.

For example, the ratio $a:b:c$ indicates the ratio a to b and the ratio b to c .

A rate is a comparison of two different units, such as miles per hour, or two different things measured with the same unit, such as cups of concentrate per cups of water.

- ◆ Rates are called unit rates when they are expressed denominators in terms of 1 unit.

Examples of unit rates; 60 miles per hour, 12 words per second etc.

◆ A proportion is the equality of two ratios.

◆ Two ratios are said to be in proportion if they are equal i.e.,

$$\frac{a}{b} = \frac{c}{d} \text{ or } a:b = c:d$$

- ◆ x and y are directly proportional if x increase y also increase or x decrease y also decrease.
- ◆ x and y are inversely proportional if x increase y decrease or x decrease y increase.
- ◆ To write a percent as decimal, divide it by 100 and remove the percent symbol.
- ◆ To write a decimal as a percent, multiply by 100 and add the percent symbol.

- ◆ Percentage (%) means per hundred or hundredths.

$$\text{Percentage} = \text{Percent(Rate)} \times \text{Base (whole)}$$

$$\text{Base} = \frac{\text{Percentage}}{\text{rate}}$$

$$\text{Base} = \text{Percentage}/\text{rate}$$

- ◆ The relation among Percent(P), base(B) and amount(A) can be summarized as follow:

$$\frac{\text{Percentage}}{100} = \frac{A}{B}$$

$$\text{Percent of increase} = \frac{\text{actual change}}{\text{Original amount}} \times 100\%$$

$$\text{Percent of Decrease} = \frac{\text{actual change}}{\text{Original amount}} \times 100\%$$

- ◆ Cost price: how much the item costs the seller
- ◆ Selling price: how much the item is resold for
- ◆ Profit: the money made from the resale
- ◆ Loss: the money lost from the resale
- ◆ Profit/ Gain = Selling price(S.p)- Cost price(C.p)
- ◆ Loss = Cost price(C.p)-Selling price(S.p)
- ◆ Percentage of profit/gain =profit/(cost price)×100%
- ◆ Percentage of loss =loss/(cost price)×100%
- ◆ Simple interest: is an interest paid on the original principal only overall interest periods.

- ◆ The formula for Simple interest(I) is given by:

$I = \text{principal} \times \text{rate} \times \text{time}$

$P = \text{principal}$

$I = Prt$, where ;

$r = \text{interest rate (per annum)}$

$I = \text{simple interest}$

$t = \text{time duration (in years)}$

Total amount (A) = Principal (P) + Interest (I)

$$= P + Prt = P(1+rt)$$

- ◆ In compound interest, the formula for the total amount of money(S) after 'n' years is given by:

Total amount (S) = $p(1 + \frac{r}{n})^{nt}$, where;

$P = \text{principal or amount of money deposited}$

$r = \text{annual interest rate}$

$n = \text{number of times compounded per year}$

$t = \text{time in years.}$

- ◆ Interest rate: the percent that is being given for an investment or for a loan. It depends on the amount of time the money is invested or borrowed.
- ◆ Turn over tax (TOT) is payable on goods and services supplied by a person who are not registered for VAT.
- ◆ To apply TOT, you add 2% extra on the original cost.
- ◆ Value added tax (VAT) is a tax imposed by the government on sales of goods and services in Ethiopia.
- ◆ To apply VAT, you add 15% extra on the original cost.

Review exercise

1. Express each ratio in simplest form
 - a. 12 to 8
 - b. 1.2 to 2.4
 - c. 6 to 0.5
 - d. $1\frac{1}{8}$ to $\frac{3}{8}$
 - e. 80meter to 16meter
 - f. 75gm to 500gm
 - g. 75min. to 3hrs
 - h. 2km to 150meter
2. Which of the following are equal ratios?
 - a. 3:5 and 6:15
 - b. 2:4 and 6:12
 - c. 3:4.5 and 5:7.5
 - d. 2:3 and 4:5
3. Complete the following spaces using appropriate word.
 - a. If $x = 5y$, then x and y vary _____ with each other.
 - b. If $xy = 10$, then x and y vary _____ with each other.
 - c. Both x and y are said to vary _____ with each other if for some positive number k , $xy = k$
 - d. Two quantities are said to vary _____ with each other if they increase (decrease) together in such a manner that the ratio of their corresponding values remains constant.

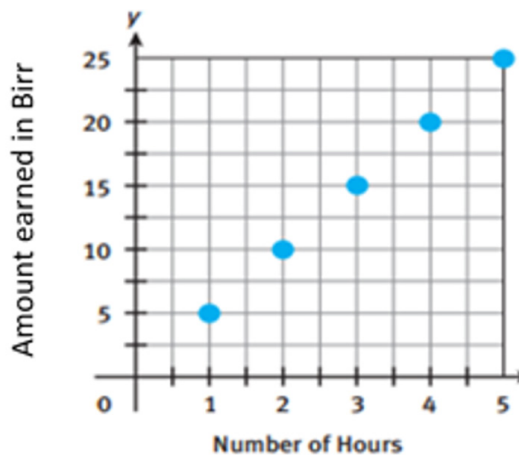
4. 100 persons had food provision for 24 days. If 20 persons left the place, the provision will last for
- a. 30 days
 - b. 120 days
 - c. $\frac{96}{5}$ days
 - d. 40 days
5. Express each percentage as a fraction in simplest form
- a. 50%
 - b. 85%
 - c. 75%
 - d. 18%
 - e. 66%
 - f. 345%
 - g. $15\frac{1}{4}\%$
 - h. 0.23%
6. Express each percentage as a decimal
- a. 2%
 - b. 77%
 - c. $2\frac{1}{5}\%$
 - d. 1.25%
7. Arrange each set in ascending order
- a. 72% , $\frac{1}{4}$, 0.92
 - b. $\frac{1}{5}$, 86%, 0.09
 - c. $\frac{5}{6}$, 70%, 0.44
8. Express the first quantity as a percent of the second quantity
- a. 38cm, 40cm
 - b. 42cm, 1.2meter
 - c. 3Birr , 75cents
 - d. 2years, 76weeks
9. Calculate the following
- a. Increase Birr 360 by 35%
 - b. Decrease Birr 200 by 12%

10. Solve each of the following problems using the percent equation

- 9 is what percent of 25?
- 48 is 64% of what number?
- 38 is what percent of 40?
- What is 43.6% of 20?
- 17.6 is $133\frac{1}{3}\%$ of what number?

11. Find 35% of a quantity if 20% of the quantity is 110meter.

12. Use the graph below to write a ratio showing the relationship between number of hours Chule spend on work and the money he earned.



13. If $A:B = 3:6$ and $B:C = 6:7$, then find $A:C$

14. In each of the following tables below, describe whether one variable varies directly or inversely as another. If it does, express the relation between the variables by means of a formula.

P	3	6	9
S	1	2	3

a

t	2	4	8
d	20	10	5

c

N	3	4	5
C	6	8	10

b

p	4	5	6
s	6	8	10

 $\dot{d}$

15. In each case below, one variable varies directly as the other. Write the formula that relates the variables and find the missing numbers.

h	1	2	y
A	5	x	25

a

h	4	8	y
s	6	x	15

b

I	2	8	y
W	1	x	7

C

16. State whether the relation between the variables in each question below is a direct proportion. Justify your answer.
 - a. $R + T = 20$
 - b. $15T = D$
 - c. $\frac{l}{m} = 3$
 - d. $bh = 12$
17. The variable x varies directly as t . when $x = 20$, $t = 5$, find t when $x = 48$.
18. Find two numbers whose sum is 160 and have the ratio 5 to 3.
19. A number n is 25% of some number x and 35% of another number y . describe the relationship between x and y . Is $x > y$, $x < y$, $x = y$ or is it not possible to determine the relationship?
20. On a field trip, there are 5 teachers, 117 students, and 16 parents. What does the ratio 21:138 represent?

21. The ages of Heri and Dancho are in the ratio 5:7. If age of Dancho is 28, what is age of Heri?
22. A football team played 36 games and won 24 games.
 - a. What is the ratio of number of games won to number of games played?
 - b. For every 3 games played how many games were won?
23. Dararo scored 81% for a test. If her actual score was 12.15. What was the maximum possible mark for the test?
24. The original price of a TV Birr 8500 increased by 30%. Calculate the selling price.
25. A television purchased for Birr10,000 and later it was sold for Birr 6450.
 - a. Calculate the loss
 - b. Express the loss as a percentage of C.P.
26. A company's quality control department found an average of 5 defective models for every 1000 models that were checked. If the company produced 70,000 models in a year, how many of them would be expected to be defective?
27. The weight of a 20 meter copper wire is 0.9 kg. Find the weight of 170 meter of the same wire.
28. There are 80 calories in a slice of cake that weighs 200gm. How many calories are there if the slice is 600 gm ?

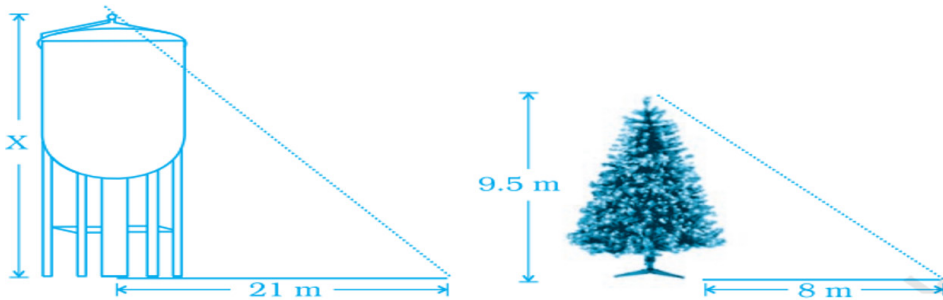
29. An alloy of gold and silver contains 2 parts of gold to 5 parts of silver by mass.
 - a. What fraction of the alloy is gold?
 - b. What fraction of the alloy is silver?
 - c. How much of each metal does 700g of the alloy contain?
30. If a family consumes p litters of milk in k days, represent the amount of milk consumed in m days.
31. If 30% of the 60 eggs that Gatiso found were broken. About how many of the eggs were broken?
32. Milla borrowed Birr3,700 at a simple interest rate of 6.9% to buy a used laptop. If the loan is for 2 years, how much money must she repay?
33. Merry deposits Birr1,200 in a bank account and does not add to it or make any withdrawals. After 3 years her amount increased to Birr1,430.40. What percent simple interest did the bank gave? Show your work.
34. Muna deposits Birr500 in a simple interest account offering 5.9% semiannually. After 6 months, she deposits another 350Birr. How much interest she will earn at the end of first year?
35. Which is the better option for a customer, to have 25% discount of each item purchased, or to have 25% discount of the total cost? Explain.

36. Every Student in a school choose one out of three sports options: Tennis(T), basketball(B) or swimming(S). Given that the pupils choose the options in the ratio $T:B:S = 4 : 2 : 3$ and 120 students choose tennis, find:

- the number of pupils in the school
- the number of students who choose swimming

37. A water tank casts a shadow of 21 m long. A tree of height 9.5 m casts a shadow 8 m long at the same time. The lengths of the shadows are

directly proportional to their heights. Find the height of the tank.



UNIT 3

LINEAR EQUATIONS

Unit Outcomes

After completing this unit , you should be able to:

- ▶ Identify terms and variables in algebraic expressions.
- ▶ Simplify algebraic expressions.
- ▶ Convert word problems in to mathematical expressions
- ▶ Develop skills on rearranging and solving linear equations.
- ▶ Apply the rules of transformation of linear equations for solving problems.
- ▶ Describe the Cartesian coordinate system
- ▶ Draw a line through the origin whose equation is given.
- ▶ Apply real life situations in solving linear equations.

Main contents of the unit

3.1. Algebraic terms and expressions

3.1.1. Use of variables in formula

3.1.2. Variables, terms and expressions

3.2.	Solving linear equations.
3.2.1.	Linear equations involving brackets
3.2.2.	Linear equations involving fractions
3.3	Cartesian coordinate
3.3.1.	The four quadrants of the Cartesian coordinate plane
3.3.2.	Coordinates and straight lines
3.4	Applications
Key terms	
Unit summary	
Review exercises	

Introduction

In your grade 5 and 6 lessons you have learned about algebraic terms and expressions such as variables, algebraic terms and values of simple algebraic expressions, solving linear equations by substitution and also solving one step linear equations such as $x+3=7$, then subtracting 3 from both sides which gives $x=4$. Some real-world application problems were also discussed. In this unit you will learn how to identify terms and variables in a given algebraic expression, how to rearrange and solve linear equations in one variable, apply rules of transformation of linear equations for solving problems in real-life situations, how to represent numbers on coordinate plane and also sketch simple linear equation graphs.

3.1. Algebraic terms and expressions

3.1.1. Use of variables in formula

As you can recall from your lower grade lessons, variable is a symbol, which represents some specific number or a particular object, whereas a formula is a rule that states a relationship between given quantities and unknowns.

For example, the formula that relates the area (A), base (b) and height (h) of a rectangle is:

$$A = bh$$

When the length of base and height of the rectangle varies, area of the rectangle also varies.

Example 1

Find area of the rectangle using the formula $A = bh$ given that

- height (h) = 6cm and its base (b) = 10cm.
- height(h) = 8cm and its base (b) = 15cm.

Solution:

- $A = bh$, $b=10\text{cm}$ and $h = 6\text{cm}$

$A = 10\text{cm} \times 6\text{cm} = 60\text{cm}^2$, cm^2 read as “square centimeter or centimeter square”

- $A = bh$, $b=8\text{cm}$ and $h = 15\text{cm}$

$A = 8\text{cm} \times 15\text{cm} = 90\text{cm}^2$

Group work 3.1

- Find the area of the triangle in each case. Use the formula $A = \frac{1}{2}bh$, where b is the base and h is the height of the triangle.
 - $b = 3\text{cm}$, $h = 8\text{cm}$
 - $b = 12\text{cm}$, $h = 5\text{cm}$
- Using the formula $P = 2(b+h)$. Calculate the perimeter of the rectangle whose sides are 6cm and 9cm.

3.1.2. Variables, terms and expressions

Definition:

Variable is a letter or a symbol that represents unknown value.

For example in the mathematical expression $3x+1$; the variable is x .

In the expression $2xy-3z+5$, the variables are x , y , and z .

Activity 3.1

Explain the concepts of

- Algebraic expressions.
- Terms
- Like terms, unlike terms and Constant term.

Definition:

Any expressions involving variables with mathematical operation is called an algebraic expression.

For example $21x$, $\frac{3}{4}x - 5$, $\frac{2x+7}{3}$, $8x + 7y + 1$ are some examples of algebraic expressions.

Note:

In mathematical expression $ax + b$, x is the variable, a is coefficient of x and b is the constant term.

Example 2

In each of the following algebraic expressions, determine the numerical coefficient and the constant terms.

a. $-2x+4$

c. $-x^3$

b. $\frac{3xy}{4} - 12$

Solution:

- The numerical coefficient of $-2x$ is -2 and the constant term is 4 .
- The numerical coefficient of $\frac{3xy}{4}$ is $\frac{3}{4}$ and the constant term is -12
- The numerical coefficient of $-x^3$ is -1 and the constant term is 0

Definition

Term is a constant number, a variable or product of a number and variable.

For example 2, $3x$, $4xy$, $2x^2$, etc are examples of terms.

Definition:

Like terms are terms, whose variables are the same and whose exponents of the variables are equal.

Terms, which are not like terms, are called unlike (not like) terms.

For example, $2xy$ and $-4xy$, x^2y and $5x^2y$, abc and $6abc$, $2xy^3z$ and $5xy^3z$ etc are examples of like terms but $2xy$ and $4xyz$, x^2y and xy , abc and a^2bc , etc are not like terms.

Example 3

Identify whether the following algebraic expressions are like terms or unlike terms.

- $3x$ and $20x$
- $12x^2$ and $12x$
- $\frac{3}{2}xy$, $-5xy$ and yx
- ab and xy

Solution:

- $3x$ and $20x$ are like terms. Because they have the same variables with equal exponents.

- b. $12x^2$ and $12x$ are unlike terms. Because they have the same variables but unequal exponents.
- c. $\frac{3}{2}xy$, $-5xy$ and yx are like terms. Because they have the same variables with equal exponents.
- d. ab and xy are unlike terms. Because they have different variables.

Definition:

An algebraic expression containing;

- Only one term is called monomial. ('mono' means one)
- Two terms is called binomial. ('bi' means two)
- Three terms is called trinomial ('tri' means three)
- More than three terms is called multinomial

Example 4

Classify each of the following expressions as monomial, binomial or trinomial

- $3x-4xy+6$
- 5
- $3x-2$
- $5xyz$

Expression	Terms	Number of terms	Its classification
$3x-4xy+6$	$3x, -4xy, 6$	3	trinomial
5	5	1	monomial
$3x-2$	$3x, -2$	2	binomial
$5xyz$	$5xyz$	1	monomial

Group work:

Using internet explain the concepts of variable, algebraic expression, like terms, and unlike terms.

Computing values of an algebraic expression

Computing an expression means that replacing or substituting the variable by specific value and simplifying it.

Example 5

Evaluate each of the following expressions below using the values

$x = 2$ and $y = 3$

- a. $3x - 2y + 3xy$
- b. $4xy + 2(x - y) - 6$
- c. $\frac{2xy(x - 3y)}{2x} + 4x - 5y + 5$

Solution:

- a. $3x - 2y + 3xy = 3(2) - 2(3) + 3(2)(3) = 6 - 6 + 18 = 18$
- b. $4xy + 2(x - y) - 6 = 4(2)(3) + 2(2 - 3) - 6 = 24 + 2(-1) - 6$
 $= 24 - 2 - 6 = 24 - 8 = 16$
- c. $\frac{2xy(x - 3y)}{2x} + 4x - 5y + 5 = \frac{2(2)(3)(2 - 3(3))}{2(2)} + 4(2) - 5(3) + 5$
 $= \frac{12(2 - 9)}{4} + 8 - 15 + 5$
 $= \frac{12(-7)}{4} - 7 + 5$
 $= 3(-7) - 2$
 $= -21 - 2 = -23$

Operations on an algebraic expression:

- ◆ Adding or subtracting an algebraic expression can be performed only between two or more like terms.
- ◆ Multiplying and dividing an algebraic expression can be performed between like and unlike terms.

During performing addition and subtraction of algebraic expressions;

- ◆ Add or subtract like terms.
- ◆ While adding or subtracting like terms, only the numerical coefficients are added or subtracted.
- ◆ Unlike terms cannot be simplified further. It will remain the same.

For example, $2x$ and $3x$ are like terms. So that $2x + 3x = 5x$, adding the coefficients and taking one of the variables. But, $2x$ and $3x^2$ are not like terms. Therefore, we cannot add or subtract one from the other.

Simplification of algebraic expressions.

To simplify algebraic expressions we follow some rules. Such as

- i. Remove the bracket if any
- ii. Collect like terms
- iii. Perform any of the operation
- iv. Write in simplest form

Example 6

Simplify the given expressions by collecting like terms

a. $2y + 3x - 10y + 8x$

c. $3a - (2b + 3) + 4(b + 2a) + 7$

b. $4a + 4b + 6 - 12a + 9b - 8 + 21a$

Solution:

a. First collect like terms

$3x$ and $8x$ are like terms , $2y$ and $10y$ are like terms.

So , $3x+8x+2y - 10y = 3x+8x+2y - 10y$

$$= (3+8)x+(2 - 10)y$$

$$= 11x+(- 8y) = 11x - 8y$$

b. $4a+4b+6 - 12a+9b - 8 + 21a$

$$= (4a - 12a+21a)+(4b + 9b) + (6 - 8)----- \text{ collect like terms}$$

$$= (4 - 12+21)a+(4+9)b+(- 2)$$

$$= 13a+13b - 2$$

c. $3a - (2b+3)+4(b+2a)+7$

$$= 3a - 2b - 3+ 4b+ 8a +7..... \text{ First, remove the brackets}$$

$$= 3a+8a + 4b - 2b - 3+7 \text{Collecting like terms together}$$

$$= 11a+2b+4..... \text{Performing the required operations.}$$

Group work 3.2

Simplify each of the following expressions and write them in simplest form

a. $12x + 3(y - 12x) + 3y + 5$

b. $3x(2 - 3y + 4) - 2y(x + 5) + 11xy + 4$

c. $6xy + 12y - 6x(y + 2) + 12x$

Transforming word problems to variable (mathematical) expression

Variables are very important in solving word problems. In this sub-topic, you will learn how to translate between variable expressions and verbal expressions.

Note:

Words or phrases that are used to express problems in words and their meaning:

Addition: Sum, add to, more than, increased by, total

Subtraction: Difference of, minus, subtracted from, less than, decreased by, less

Multiplication: Product, times, multiplies

Division: Quotient, divide in to, ratio

Example 7

Transform each of the following in to mathematical expression/symbolic expression.

- A number increased by four
- Four plus a number
- Four more than a number
- A number plus four

All a,b,c, and d have the same meaning and can be expressed as $x+4$, assuming that the unknown number is x .

Example 8

The following are the corresponding mathematical expressions of the given word statements.

- a. Five fewer than a number. $\text{-----} \rightarrow x-5$
- b. A number decreased by 10 $\text{-----} \rightarrow x-10$
- c. Five times a number is increased by 3 $\text{-----} \rightarrow 5x+3$
- d. Two divided by a number $\text{-----} \rightarrow \frac{2}{x}$
- e. Half a number is decreased by eight $\text{-----} \rightarrow \frac{1}{2}x - 8$
- f. The quotient of x by y $\text{-----} \rightarrow \frac{x}{y}$

Example 9

Ledamo is 3 years older than twice of age of his younger brother Kuma. Write the statement in mathematical expression.

Let age of Ledamo be L and age of Kuma be k, then $L = 2k + 3$

Group work 3.3

1. Change the following problem into mathematical expressions.
 - a. The cost of coffee is increased by 5
 - b. The difference of a number and 10
 - c. 2 times the sum of a number and 5
 - d. Five times a number decreased by 21

2. Write each of the following statements as an equation:

- a. 5 times a number equals 40.
- b. A number increased by 8 equals 15.
- c. 25 exceed a number by 7.
- d. A number exceeds 5 by 3.
- e. 5 subtracted from three times a number is 16.
- f. If 12 is subtracted from a number, the result is 24.
- g. Twice a number subtracted from 19 is 11.
- h. A number divided by 8 gives 7.
- i. 3 less than 4 times a number is 17.
- j. 6 times a number is 5 more than the number.

Exercise 3.1

1. Determine the terms and their numerical coefficients in each of the following algebraic expressions.

a. $\frac{x}{2} + 5$

c. $-2\frac{1}{4}y^2$

b. $0.5x + 5y + 1$

d. $-0.23x^2 + 2 - x + y$

2. Classify each algebraic expression in question one above as monomial, binomial or trinomial

3. Which of the following algebraic expressions are like terms and which are unlike terms.

a. $2xy$ and xyz

b. $\frac{5}{9}x^2$ and $\frac{9}{5}x$

- c. x and 1
- d. $x^2y^2z^2$ and $0.5(z^2x^2y^2)$
- e. $\frac{3}{5}a^9c^{10}$ and $\frac{20}{3}a^{10}c^9$
4. Compute each expression when $x = 1$ and $y = 2$
- a. $3y(4 - 2xy + 6x) + 5$
- b. $\frac{2x-3y}{2x} + xy - 11$
5. Write the following in mathematical expression
- a. Product of three times a number and half of another number
- b. Girma's age is 5 more than six times his son's age.
- c. Five less than four times a number.
- d. The sum of a number and five times another number.
6. Write a statement for each of the equations, given below:
- a. $x - 7 = 14$
- b. $2y = 18$
- c. $11 + 3x = 17$
- d. $2x - 3 = 13$
- e. $12y - 30 = 6$
- f. $\frac{2z}{3} = 8$
7. If base of a rectangle is twice its height. Write the formula which relates
- a. Perimeter of the rectangle
- b. Area of the rectangle
8. There are 5 ten Birr notes, 7 two hundred birr notes and 6 fifty birr notes in Abebe's pocket. What is the total amount of money in his pocket?

3.2. Solving linear equations

3.2.1 Linear equations involving brackets

Activity 3.2

1. What is the difference between an algebraic expression and an equation?
2. Fertilizer is applied at the rate of 2 kg per square meters. Find the area which 50 Kg of fertilizer can cover?
3. The sum of three consecutive natural numbers is 30. Find the numbers.

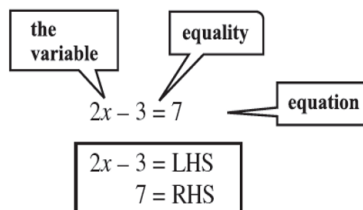
Definition:

Equation is an algebraic expression in which the left and right hand sides are joined by the 'equal to' sign.

For example, $-4 = 0$, $\frac{1}{2}x + 5 = 3 - 10x$, $x = 1$, $x^2 - 25 = 0$, etc are examples of equations.

The expression on the left of the equality sign is the Left Hand Side (LHS) and

The expression on the right of the equality sign is the Right Hand Side (RHS).



Definition:

The value of the variable that makes the left and right hand side expressions equal is said to be **solution** of the equation or **truth-value** or **root** of the equation.

For example, the equation $x+4=5$ is true for $x=1$,

i.e. for $x=1$, $x+4 = 5$

$$\begin{array}{ccc} 1+4 & = & 5 \\ \downarrow & & \downarrow \\ 5 & = & 5 \end{array}$$

LHS RHS (LHS is equal to RHS)

Therefore $x=1$ is the truth-value of the equation

Properties of Equation

For a, b and c are numbers:

a. Addition property of equation:

If $a=b$, then $a+c=b+c$

b. Subtraction property of equation

If $a=b$, then $a-c=b-c$

c. Multiplication property of equality

If $a=b$, then $ac=bc$

d. Division property of equation

If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ provided that $c \neq 0$

Note:

If a, b and c are numbers then;

i. $a + (b + c) = a + b + c$

ii. $a - (b + c) = a - b - c$

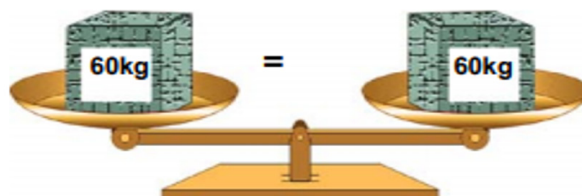
iii. $a \times (b + c) = (a \times b) + (a \times c)$

Methods of solving an equation

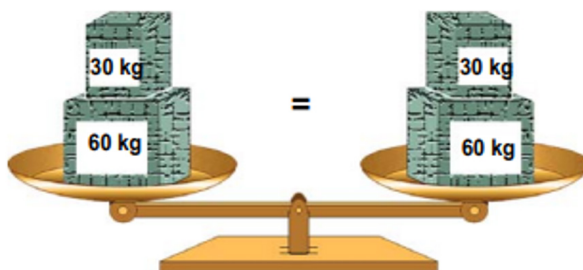
There are two methods by which an equation can be solved.

i. Balancing Method

This is true if two quantities are increased or decreased by the same amount, then the resulting quantities will also be equal. See figures below



Amount in both sides are equal



The same amount is add to both sides. Still both side are equal.

If we add the same amount to both sides or subtract the same amount from both sides, the equal sign remains the same.

In this method, both the LHS and RHS of the equation should be balance.

Example 10

Find the solution of the following equations.

a. $x - 5 = 10$

Solution:

To balance both sides, first we add 5 on both sides of the equation.

That is, $x - 5 + 5 = 10 + 5$ (add 5 to both sides)

We get, $x = 15$, this is the solution of the equation.

To check this, substitute the value of the variable obtained in to the original equation. If LHS equals to the RHS then, the value obtained is the solution.

From the above example, we obtained $x = 15$

Now, let us substitute this value into the original equation

$$x - 5 = 10.$$

$$15 - 5 = 10$$

$$10 = 10$$

$$\text{LHS} = \text{RHS}$$

Here, we observe that $\text{LHS} = \text{RHS}$, thus the solution obtained is correct.

b. $3x + 2 = 14$

Solution:

To balance both sides, first we subtract 2 from both sides of the equation.

$$\text{We have, } 3x + 2 - 2 = 14 - 2$$

$$3x = 12$$

Further, to balance the equation we divide both the sides by 3

$$\frac{3x}{3} = \frac{12}{3}$$

Solving this we get $x = 4$.

Now, substituting $x = 4$ into the equation $3x+2=14$, we get $3 \times 4 + 2 = 14$

So, the LHS = 14 and the RHS = 14

Here, we observe that LHS = RHS, thus the solution obtained is correct.

Therefore, $x = 4$ is the solution of the equation.

iii. Transposing Method

In this method, constants or variables are transposed from one side of the equation to other until the solution is obtained. In other words, it means collecting like terms to one side by adding or subtracting the same expression on both sides.

Example 11

Solve the following equations.

d. $x - 1 = 4$

First, we transpose the integer -1 from LHS to RHS, thus we get

$$x = 4 + 1$$

$x = 5$. Therefore, $x = 5$ is the solution of the equation.

e. $6x + 10 = 22$

First, transpose the integer 10 from LHS to RHS, thus the equation becomes

$$6x = 22 - 10$$

$$6x = 12$$

$$x = \frac{12}{6} = 2$$

Therefore, $x = 2$ is the truth-value of the equation.

Definition:

An equation of the form $ax + b = 0$ where, a and b are any numbers and $a \neq 0$ is called a linear equation with one variable.

For examples, $3x + 4 = 2$, $10x - 8 = 0.5$, $14 - 7x = 2$, $x = 1$, etc are linear equations in one variable, because the power of the variable in all cases is 1.

Note:

The highest power of the variable in a linear equation is 1.

Examples of some non-linear equations are:

$2x^2 + 1 = 0$, $x^2 - 1 = 4$, $x + x^2 = 8$, $\frac{3}{4}x^2 = 4$, etc. In these equations, the exponent of a variable is greater than one. Therefore, they are non-linear equations.

For linear equation $ax + b = 0$ where a, b are real numbers and $a \neq 0$

$$ax + b - b = 0 - b \quad (\text{Adding } -b \text{ on both sides})$$

$$ax = -b \quad (\text{Dividing both sides by } a \text{ for } a \neq 0)$$

$$\frac{ax}{a} = \frac{-b}{a}; \quad x = \frac{-b}{a}$$

Therefore, the truth-value or the solution of linear equation

$$ax + b = 0 \text{ is } x = \frac{-b}{a}.$$

Example 12

Find the truth-value of the following linear equations.

- $3x - 8 = 10$
- $9x - (2x - 3) = 17$
- $-2(5x - 4) = 1 - x + 12$
- $x - 2 = -8(x + 4) - 9$

Solution:

$$\text{a. } 3x - 8 = 10$$

$$3x - 8 + 8 = 10 + 8 \quad (\text{Adding } 8 \text{ on both sides})$$

$$3x = 18$$

$$\frac{3x}{3} = \frac{18}{3} \quad (\text{Dividing both sides by } 3)$$

$$x = 6.$$

Check: When $x=6$

$$3x - 8 = 10$$

$$3 \times 6 - 8 = 10$$

$$18 - 8 = 10$$

$$10 = 10 \text{ (Checked !!)}$$

Therefore, $x = 6$ is the truth value of the equation.

b. $9x - (2x - 3) = 17$

$$9x - 2x + 3 = 17 \quad (\text{Removing bracket})$$

$$7x + 3 = 17$$

$$7x + 3 - 3 = 17 - 3 \quad (\text{Subtract 3 from both sides})$$

$$7x = 14$$

$$\frac{7x}{7} = \frac{14}{7} \quad (\text{dividing both sides by 7})$$

$$x = 2$$

Check: When $x=2$

$$9x - (2x - 3) = 17$$

$$(9 \times 2) - (2 \times 2) + 3 = 17$$

$$18 - 4 + 3 = 17$$

$$17 = 17$$

$$17 = 17 \text{ is true. (Checked !!)}$$

Therefore, $x=2$ is the truth value.

c. $-2(5x-4)=1-x+12$

$$-2(5x)-2(-4) = 1-x+12$$

$$-10x+8=1-x+12 \quad (\text{Removing brackets})$$

$$-10x+8-8=-x+13-8 \quad (\text{Subtract 8 from both sides})$$

$$-10x=-x+5 \quad (\text{Combine like terms})$$

$$-10x+x=-x+x+5 \quad (\text{Add x to both sides})$$

$$-9x=5 \quad (\text{Combine like terms})$$

$$\frac{-9x}{-9} = \frac{5}{-9} \quad (\text{Dividing both sides by -9})$$

$$x = -\frac{5}{9}$$

Check: When $x = -\frac{5}{9}$

$$-2(5x - 4) = 1 - x + 12$$

$$-2\left(\left(5 \times -\frac{5}{9}\right) - 4\right) = 1 - \left(-\frac{5}{9}\right) + 12$$

$$-2\left(-\frac{25}{9} - 4\right) = \frac{5}{9} + 13$$

$$-2\left(\frac{-25-36}{9}\right) = \frac{5+117}{9}$$

$$-2\left(\frac{-61}{9}\right) = \frac{122}{9}$$

$$\frac{122}{9} = \frac{122}{9} \quad (\text{checked !!})$$

Therefore, $x = -\frac{5}{9}$ is the truth-value of the equation.

d. $x-2=-8(x+4)-9$

$$x-2 = -8x-32-9 \quad \dots\dots\dots (\text{Removing brackets})$$

$$x-2+2 = -8x-41+2 \quad \dots\dots\dots (\text{Add 2 to both sides})$$

$$x = -8x-39 \quad \dots\dots\dots (\text{Performing operation})$$

$$x+8x=-8x+8x-39 \quad \dots\dots\dots (\text{Add 8x to both sides})$$

$$9x = -39 \dots\dots\dots \text{(Performing operation)}$$

$$\frac{9x}{9} = -\frac{39}{9}$$

(Dividing both sides by 9)

$$x = -\frac{13}{3}$$

Check: When $x = -\frac{13}{3}$

$$x - 2 = -8(x + 4) - 9$$

$$x - 2 = -8x - 32 - 9$$

$$x - 2 = -8x - 41$$

$$\frac{-13}{3} - 2 = -8\left(\frac{-13}{3}\right) - 41$$

$$\frac{-13 - 6}{3} = \frac{104}{3} - 41$$

$$-\frac{19}{3} = \frac{104 - 121}{3}$$

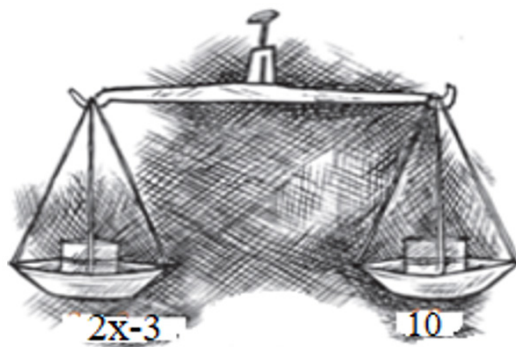
$$-\frac{19}{3} = \frac{-19}{3} \quad \text{(Checked !!)}$$

Therefore, $x = -\frac{13}{3}$ is the truth-value of the equation.

Exercise 3.2

1. Verify by substitution that
 - a. the root of $3x - 5 = 7$ is $x = 4$
 - b. the root of $3 + 2x = 9$ is $x = 3$
 - c. the root of $5x - 8 = 2x - 2$ is $x = 2$

- d. the root of $8 - 7y = 1$ is $y = 1$
 - e. the root of $\frac{z}{7} = 8$ is $z = 56$
2. Solve each of the following equations by the trial-and-error method:
- a. $y + 9 = 13$
 - b. $x - 7 = 10$
 - c. $4x = 28$
3. Solve the following linear equations and verify the answer.
- a. $7x - 12 = 9$
 - b. $5(10 - 4x) = 8 - x + 4$
 - c. $6x - 2(4x - 9) = 2(3 + 5x)$
 - d. $1 - 2(x - 1) = 4(3 - x) + 2x$
 - e. $19x + 8 - 1(x + 4) = 4x + 12 - 2(4 - 7x)$
 - f. $6(1 - 4x) + 7(2 + 5x) = 53$
 - g. $16(3x - 5) - 10(4x - 8) = 40$
4. Find the value of x if both LHS and RHS are balanced.



5. The Sum of two numbers is 49. One of the numbers is 11 more than the other. What are the numbers?
6. A carpenter cuts a 12-meter wood so that one piece is two times bigger than the other. What are the lengths of the two pieces?

3.2.2 Linear equations involving fractions

Group work 3.4

1. If a man buys 12 bananas and share half of the bananas to his sister and one-fourth of the bananas for his brother. How many bananas will each get? How many bananas remain with him?
2. Deraro thinks of a number. She subtracts $\frac{5}{2}$ from the number and multiples the result by 8. The result obtained is 3 times the same number she thought. What is the number?

Revision on Fractions

In number system, the number which written in the form of $\frac{a}{b}$, for $b \neq 0$ is a fraction. Although people today are making greater use of decimals and fractions as they work with calculators, computers, and the metric system; common fractions still surround us. We use fractions in everyday measures. For example in buying objects such as $\frac{1}{2}$ kg of coffee, $\frac{1}{2}$ kg of sugar, $\frac{1}{4}$ liter of milk, etc. Fractions are also used when sharing something.

Definition:

A fraction is a quotient of a number divided by any non-zero number.

For example, the arithmetic fraction $\frac{2}{5}$ indicates the quotient of 2 divided by 5.

Method of solving linear equation involving fractions

To solve linear equations involving fractions;

1. Remove fractions by multiplying both sides by least common multiple (LCM).
2. Solve the resulting equation by using one of the methods.
3. Check by substituting in the original equation.

Example 13

Solve the following linear equations.

a. $\frac{x}{5} - \frac{1}{3} = \frac{1}{15}$

Solution:

The LCM of 3, 5 and 15 is 15

$$15\left(\frac{x}{5} - \frac{1}{3}\right) = 15 \times \frac{1}{15}$$

(Multiply both sides by 15)

$$\frac{15x}{5} - \frac{15}{3} = \frac{15}{15}$$

(Removing brackets by using distributive property)

$$3x - 5 = 1$$

(Add 5 on both sides)

$$3x - 5 + 5 = 1 + 5$$

$$3x = 6$$

$$\frac{3x}{3} = \frac{6}{3} \quad (\text{Dividing}$$

both sides by 3)

$$x=2$$

Check: when $x=2$,

$$\frac{x}{5} - \frac{1}{3} = \frac{1}{15}$$

$$\frac{2}{5} - \frac{1}{3} = \frac{1}{15}$$

$$\frac{6-5}{15} = \frac{1}{15}$$

$$\frac{1}{15} = \frac{1}{15} \quad (\text{Checked!!})$$

Therefore, $x=2$ is the solution of the equation

$$\text{b. } \frac{x+1}{2} + \frac{x}{3} = \frac{x-1}{4}$$

Solution

$$\frac{x+1}{2} + \frac{x}{3} = \frac{x-1}{4}$$

$$\frac{12(x+1)}{2} + \frac{12(x)}{3} = \frac{12(x-1)}{4}$$

The LCM (2, 3, 4) = $2 \times 3 \times 4 = 12$

$$6(x+1) + 4x = 3(x-1)$$

$$6x+6+4x = 3x-3 \quad (\text{Removing brackets by distributive property})$$

$$10x+6 = 3x-3$$

$$10x+6 - 6 = 3x - 3 - 6 \quad (\text{Subtract 6 from both sides})$$

$$10x = 3x - 9$$

$$10x - 3x = 3x + 3x - 9 \quad (\text{Subtract 3x from both sides})$$

$$7x = -9$$

$$\frac{7x}{7} = \frac{-9}{7}$$

(Divide both sides by 7)

$$x = \frac{-9}{7}$$

Check: (Left as exercise)

Note:

Linear equation may have one solution, no solution or infinitely many solutions.

Example 14

a. $2x-1=11$

$$2x-1+1=11+1$$

$$2x = 12$$

$x = 6$. The equation has one solution.

b. $5x-3(x+4)=2x+17$

$$5x-3x-12=2x+17$$

$$2x-12=2x+17$$

$$2x-2x=17-12$$

$$0=5$$

This is always false. Therefore, the equation has no solution.

c. $18x+6(3-3x-4)=-6$

$$18x+6(-3x-1)=-6$$

$$18x-18x-6=-6$$

$-6 = -6$ is always true.

This implies that, there are many values of x , which makes the equation true. Therefore, the equation has many solutions.

Exercise 3.3

1. Solve the following linear equations.

a. $\frac{n}{4} - 5 = \frac{n}{6} + 12$

e. $-10 = \frac{40}{2-y}$

b. $\frac{3}{4}(x - 1) = -\frac{1}{2}$

f. $1.2x - 0.3 = 1.2(x + 1.2)$

c. $\frac{x+1}{3} - \frac{x}{4} = \frac{5x}{2}$

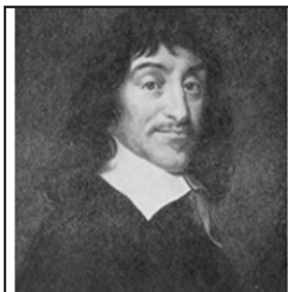
g. $\frac{2(c-3)}{5} - \frac{3(c-2)}{2} = \frac{7}{10}$

d. $\frac{2(a+9)}{9} = -1 + 2a$

2. What should be added to twice the rational number $\frac{-7}{3}$ to get $\frac{3}{7}$?
3. If one side of a triangle is one-third the perimeter, the second side is one-fifth the perimeter and length of the third side is 7 centimeters, what is the perimeter of the triangle?
4. Solve for F in terms of C, if $C = \frac{5}{9}(F - 32)$, where F- is temperature in degree Fahrenheit and C- is temperature in degrees Celsius.

3.3 Cartesian coordinate system

HISTORICAL NOTE

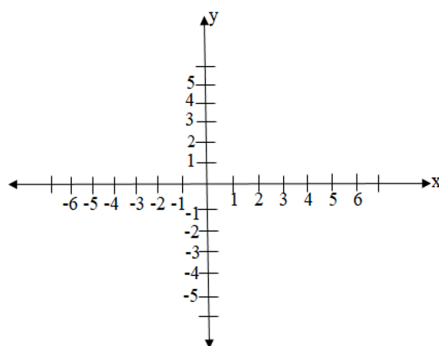


Rene Descartes (1596-1650): French scientific philosopher who developed a theory known as the mechanical philosophy. This philosophy was highly influential until superseded by Newton's methodology. Descartes was the first to make a graph, allowing a geometric interpretation of a mathematical function and giving his name to **Cartesian Coordinate**.

The coordinate plane is a two dimensional surface formed by two number lines. One number line is the horizontal and is called the x-axis. The other number line is the vertical and is called the y-axis. The two axes meet at the point called the origin, which is denoted by O. at the origin the values of both x and y are zero. we can use the coordinate plane to graph point, lines and more.

Note:

- ◆ On the number line, the values of x to the right of zero are positive and to the left of zero are negative and
- ◆ The values of y above the x-axis are positive and below the x - axis are negative

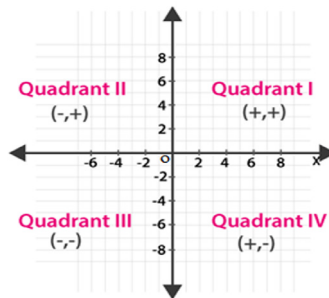


3.3.1 The four quadrants of the Cartesian coordinate plane

The Cartesian plane is sometimes called the xy- plane or the coordinate plane. The x-axis and the y –axis divide the coordinate plane in to four parts. They are called the quadrants. First quadrant (quadrant I), the second quadrant (Quadrant II), the third quadrant (Quadrant III) and the fourth quadrant (quadrant IV).

Quadrants and signs of value in each quadrant.

- ◆ O is the origin.
- ◆ In quadrant I both x and y values are positive
- ◆ In quadrant II value of x is negative and value of y is positive
- ◆ In quadrant III values of both x and y are negative
- ◆ In quadrant IV the value of x is positive and the value of y is negative

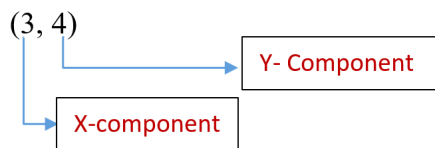


Example 15

The point (3, 4) lie in quadrant I because both x and y are positive.

In the coordinate (3, 4) the value of x is the first component, which is

3, and the value of y is the second component which is 4.



Example 16

The point $(-4, 3)$ lie in second quadrant

$(-5, -2)$ lie in third quadrant and

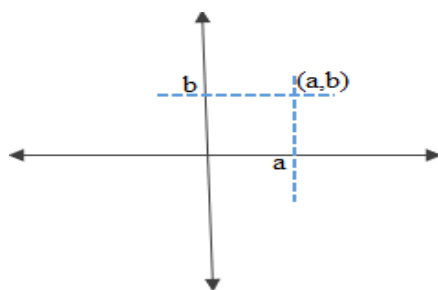
$(3, -6)$ lie in fourth quadrant

Plotting points on the coordinate plane.

Any point $P(a, b)$ has a specific place on coordinate axes. To plot it in a coordinate axes we follow the following steps.

- i. Draw the xy - plane
- ii. Write the corresponding values of x and y on the axes
- iii. Draw vertical line which pass through the point a
- iv. Draw horizontal line which passes through b
- v. Determine the intersection point of the two lines.

Therefore, the intersection point is the place of the point $P(a, b)$ on the coordinate axes.



Remark:

- ◆ The vertical or horizontal lines you draw for such case should be broken (dot lines)
- ◆ After you adapt how to plot numbers on a number line, no need of drawing such vertical or horizontal lines to plot on the number line.

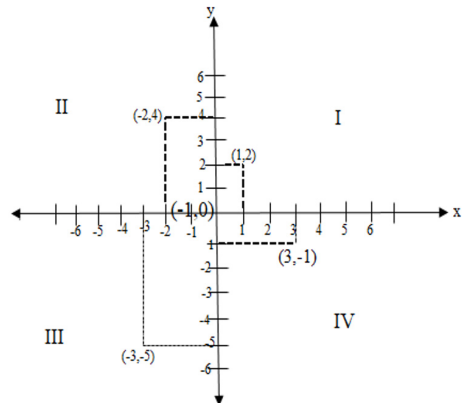
Example 17

Determine the quadrant of each of the following points and plot the points on the coordinate plane.

$(-2, 4)$, $(3, -1)$, $(-1, 0)$, $(1, 2)$ and $(-3, 5)$

Solution:

- i. $(-2, 4)$ lie in quadrant II
- ii. $(3, -1)$ lie in quadrant IV
- iii. $(-1, 0)$ lie on x-axis.
- iv. $(1, 2)$ lie in quadrant I
- v. $(-3, -5)$ lie in quadrant III



Group work 3.5

The coordinates of the corners of a shape are

A(2,4), B(4,1), C(2,-2), D(-2,-2), E(-4,1) and F(-2,4).

- a. Draw the shape
- b. What is name the shape formed?

3.3.2 Coordinates and straight line

Activity 3.3

1. Which of the following ordered pairs not satisfy the equation $2x+y = 0$?

- a. $(-1,2)$ b. $(2,1)$ c. $(1,3)$ d. $(0,0)$

2. Complete the table given below for $y = 2x$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y											

3. Plot the points in question 2 using coordinate axis and join them.

The expression $y = mx$ is called a function where x and y are variables and m is a constant. The constant m is the coefficient of x and it is called the slope of the line $y = mx$.

Note:

- ◆ Slope of a line is a measure of its steepness.
- ◆ Mathematically, slope is calculated as “rise over run” (change in y divided by change in x).

Let $p_1(x_1, y_1)$ and $p_2(x_2, y_2)$ be two points on a line, then the slope of the line is given by

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 18

Find the slope of a line, which passes through the following points.

- a. A (4,5) and B (2,3)
- b. A (-1,2) and B (-6,2)

Solution:

- a. The slope of a line which passes through point A and B is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 5}{2 - 4} = \frac{-2}{-2} = 1$$

- b. The slope of a line which passes through pint A and B is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{-6 - (-2)} = \frac{0}{-4} = 0$$

Example 19

Slope of the line $y = 2x$ is 2 and slope of the line $y = -4x$ is -4.

Sketching the graph of a line $y = mx$

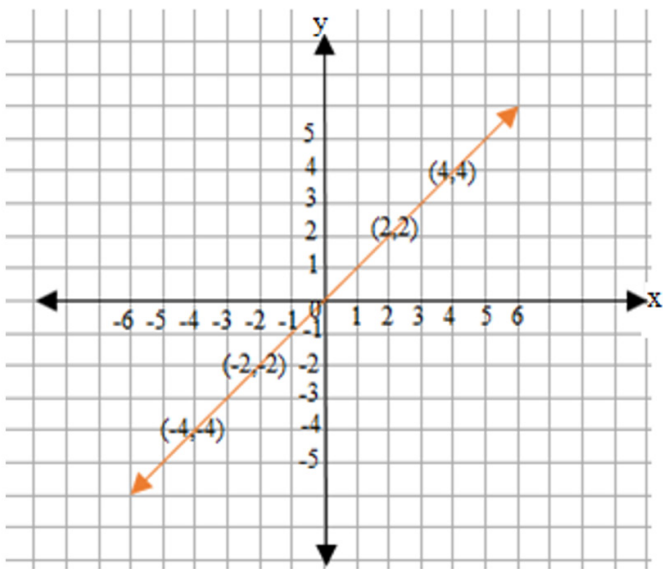
To sketch graph of the line $y = mx$ follow the steps given below.

- i. Take some values of x and calculate for y
- ii. Plot the points on the coordinate plane
- iii. Join the points, and then make the straight line.

Example 20

Draw the graph of the line $y = x$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	-5	-4	-3	-2	-1	0	1	2	3	4	5

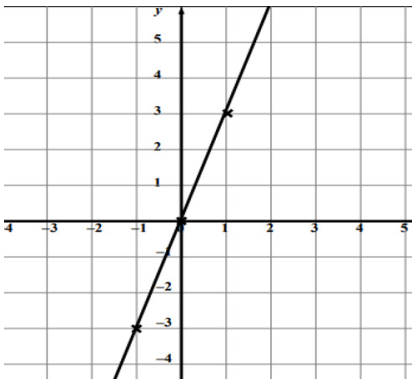


Example 21

Draw the graph of $y = 3x$

Solution:

x	-4	-3	-2	-1	0	1	2	3	4
y	-12	-9	-6	-3	0	3	6	9	12



Note:

The line $y = b$ is a horizontal line which passes through $(0, b)$

The line $x = a$ is a vertical line which passes through $(a, 0)$

In $y = b$, for every value of x the value of y is b .

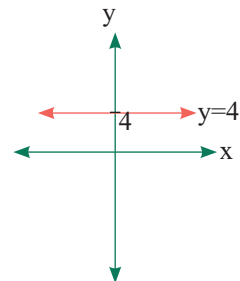
The function $y = b$ is called constant function.

Example 22

Sketch the graph of $y = 4$

x	-3	-2	-1	0	1	2	3
y	4	4	4	4	4	4	4

$y = 4$ is a horizontal line which crosses y-axis at 4



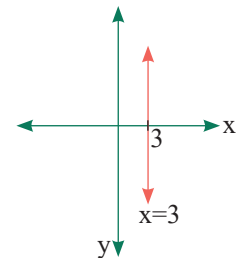
Example 23

Sketch the graph of the line $x = 3$

Solution

x	3	3	3	3	3	3	3
y	-3	-2	-1	0	1	2	3

The line $x = 3$ is a vertical line which crosses the x-axis at 3



In general, the line $y = b$ is a horizontal line and the line $x = b$ is a vertical line.

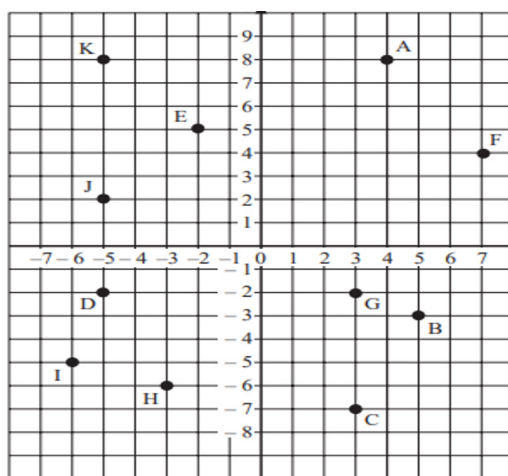
Group work 3.6

- Sketch the graphs of $y = 0$ and $x = 0$ using different coordinates.
Which axis is $x = 0$? Which axis is $y = 0$?

2. What is the slope of $y = mx$ when $m = 0$? What is slope of a horizontal line? What is slope of a vertical line?

Exercise 3.4

- Determine the quadrants of each of the following points
 - $(5, -6)$
 - $(-2, 8)$
 - $(-9, -1)$
 - $(0, 0)$
 - $(3, 0)$
 - $(0, 7)$
- Looking at the signs describe the quadrants
 - $(-, -)$
 - $(-, +)$
 - $(+, -)$
 - $(+, +)$
- Write down the coordinates of each of the points marked on the figure below.



- Determine the slope of each of the following functions:
 - $y = 4x$
 - $y = -11x$
 - $y = 3$
 - $y = 0$
 - $x = 2$

5. What is the slope of the line passing through the points P(2,4) and Q(-3,2)
6. Each set of points listed on a and b below lie on straight line.
 - a. (1, 4), (3,12), (-2,8)
 - b. (-6, 12), (-4, 8), (-2, 4), (2,-4). Then
 - i. Plot the points on coordinate plane
 - ii. Draw the line
 - iii. Compute the relationship between the x and y-components
 - iv. In each case, does the line pass through the origin? Give conclusion.

3.4 Application of linear equations.

Application of linear equation simplifies the process of solving real-world problems. This is done by using letters to represent unknowns, restating problems in the form of equations, and offering systematic techniques for solving those equations. In our day to day activities we use the idea of linear equation however we do not consider whether it is linear equation or not. For example, in selling or buying items, we apply it. In this sub-topic, you will learn working with one of the most important tools of mathematics, the equation. The ability to recognize and solve for variables types of equations is probably the most useful algebraic skill you will learn

In this sub topic, mostly we focus on solving word problems of linear equations.

To solve word problems we should consider the following strategies.

- i. Read the problem carefully several times if necessary until you understand the problem; know what is required and what is given.
- ii. Represent one of the unknown quantity by variable, say x , and write the other unknowns in terms of x . This is translation from words to mathematical expression.
- iii. Form an equation relating the known quantities to the unknown quantities.
- iv. Solve the equation and write answers to all unknowns.
- v. Check the answers by substituting to the original problem.

Activity 3.4

If Furra go to shop and buy 20 eggs and 3kg of sugar for Birr 180. Price of a kilogram of sugar is Birr 20. What is a price of an egg?

Example24:

The sum of two numbers is 60. One number is twice another. Find the numbers.

Solution:

Let one of the numbers be x . The other is twice of x , that is $2x$

The sum the numbers is 60

$$x + 2x = 60$$

$$3x = 60$$

$$x=20 \quad (\text{solving for } x)$$

The other number is $2(20) = 40$

Check: The sum of the numbers is $20+40 = 60$ -----correct
 40 is twice of 20 -----again correct.

Therefore, the numbers are 20 and 40

Example25:

Aman cuts 65cm wood so that one piece is 5cm more than four times the other. What are the lengths of the two pieces of wood?

Solution:

Let length of the smaller piece is x . Then, length of the larger piece is $4x + 5$

Sum of the two pieces = 65cm

$$x + 4x + 5 = 65\text{cm}$$

$$5x + 5 = 65\text{cm}$$

$$5x = 60\text{cm}$$

$$x = 12\text{cm}$$

The larger piece = $4(12) + 5 = 48 + 5 = 53\text{cm}$

Check: $53\text{cm} + 12\text{cm} = 65\text{cm}$.

Therefore, length of the smaller piece is 12cm and length of the larger piece is 53cm.

Example26:

Find the four consecutive even integers such that the sum of the first three exceeds the fourth by 8

Solution:

Let the first even integer is x , then the second is $x+2$, the third is $x+4$, and the fourth is $x+6$.

The sum of the first three is $x+x+2+x+4 = 3x + 6$

The sum of the first three exceeds the fourth by 8.

This implies $3x + 6 = x+6 + 8$

$$3x + 6 = x + 14$$

$$3x - x = 14 - 6 \text{ -----rearranging like terms}$$

$$2x = 8$$

$$x = 4 \text{ ----- solving for } x$$

The numbers are 4, 6, 8 and 10

Check:

Sum of the first three is $4 + 6 + 8 = 18$ but the fourth is 10, 18 is 8 more than 10.

Therefore, the numbers are 4, 6, 8, and 10

Example27:

The daughter is one-fourth her father's age at present. If after five years, the daughter becomes one-third of her father's age. Then calculate their present ages.

Solution:

Let age of the father is F and age of the daughter is D .

$$\frac{D}{F} = \frac{1}{4}. \text{ From this we get } F = 4D$$

After five years, age of the daughter will be $D + 5$ and age of her father will be $F + 5$.

After five years, $\frac{D+5}{F+5} = \frac{1}{3}$. From this we get $3(D+5) = F+5$

$$3(D + 5) = F + 5$$

$$3D + 15 = F + 5, \text{ but in the above equation } F = 4D$$

$$\text{Then } 3D + 15 = 4D + 5$$

$$15 - 5 = 4D - 3D$$

$$D = 10$$

$$F = 4D, \text{ substituting } D = 10$$

$$F = 4(10)$$

$$F = 40$$

Therefore, age of the daughter is 10 years and age of her Father is 40 years.

Group work 3.7

Find a number such that 10 less than two-thirds the number is one-fourth of the number.

Exercise 3.5

1. The sales price on a camera after a 20% discount is Birr 2000. What was the price of the camera before the discount?

2. The present age of a mother is 15 more than four times her daughter's age. The sum of their ages 65. Find their ages.
3. Fertilizer is applied at the rate of 30 grams per square meter. Find the area cover by 270 grams of fertilizer.
4. The number of boys and girls in a class are in the ratio 7:5. The number of boys is 8 more than the number of girls. What is the total number of students?

Challenge problem

5. The digits of a two-digit number differ by 3. If the digits are interchanged, and the resulting number is added to the original number, we get 143. What can be the original number?

Key terms

- | | |
|-------------------------|------------------------|
| ♠ Algebraic expressions | ♠ Multinomial |
| ♠ Variables | ♠ Linear equation |
| ♠ Like terms | ♠ Cartesian coordinate |
| ♠ Unlike terms | ♠ The slope of a line |
| ♠ Constant terms | ♠ Quadrants |
| ♠ Monomial | |
| ♠ Binomial | |
| ♠ Trinomial | |

Unit summary

- ◆ An algebraic expression is one or more mathematical terms containing variables and constants connected by mathematical operations.
- ◆ Variables are letters or symbols that represent unknown value in mathematical expressions.
- ◆ Coefficients are the number part of the terms that multiply a variable or power of the variables.
- ◆ Constants are the terms or elements that are represented only by numbers without a variable
- ◆ Term is a constant number, a variable or product of a number and a variable.
- ◆ Like terms are terms, whose variables are the same and whose exponents of the variables are equal.
- ◆ Unlike terms are terms which are not like terms.
- ◆ An algebraic expression containing:
 - ◆ One term is called monomial.
 - ◆ Two terms are called binomial.
 - ◆ Three terms are called trinomial.
 - ◆ Many terms are called multinomial.
- ◆ Steps should be followed in simplifying algebraic expressions:
 - Step 1: Remove the bracket if any
 - Step 2: Collect like terms

Step3: Perform any of the operation

Step4: Write in simplest form

- ◆ An algebraic expression to the left and right side which are joined by the 'equal' sign is called an equation.
- ◆ Any equation that can be written in the form $ax+b=0$, ($a \neq 0$) where a and b are numbers and x is a variable is called a linear or first degree equation in one variable.
- ◆ The highest power of the variable in linear equation is 1.
- ◆ For a, b and c are numbers;
 - ◆ If $a = b$ then, $a+c = b + c$ (addition property of equation)
 - ◆ If $a = b$ then, $a-c = b-c$ (subtraction property of equation)
 - ◆ If $a = b$ then, $ac = bc$ (multiplication property of equation)
 - ◆ If $a = b$ then, $\frac{a}{c} = \frac{b}{c}$ (for $c \neq 0$) (division property of equation)
- ◆ Linear equation may have one solution, no solution or infinitely many solutions.
- ◆ Key words used to translate word problems into algebraic expressions

Key words	Translation
Sum, increased by, more than, plus, added to, total	+
Difference ,decreased by, subtracted from, less, minus	-
Product, multiplied by, of, times, twice	×
Quotient, divided by, ratio, per	÷ <i>or</i> /
Is, total, result	=

- ◆ The Cartesian plane is called the xy- plane or the coordinate plane.
- ◆ In ordered pairs (a,b) the first component indicates the values of x and the second component indicates the values of y.

For example, in (-3, 4), -3 is the value of x and 4 is the value of y

- ◆ The x-axis and the y-axis divide the coordinate plane in to four quadrants.
 - ◆ In quadrant I both x and y values are positive.
 - ◆ In quadrant II x value is negative and y value is positive.
 - ◆ In quadrant III both x and y values are negative
 - ◆ In quadrant IV, x is positive and y is negative.
- ◆ In the linear equation $y=mx$, m is the slope of the line and its graph passes through the origin.
- ◆ The slop of the line is given by change in y divided by change in x. That is if $p_1 (x_1, y_1)$ and $p_2 (x_2, y_2)$ be two points on a line, then the slop of the line is given by; $m = \frac{y_2-y_1}{x_2-x_1}$ or $m = \frac{y_2-y_1}{x_2-x_1}$
- ◆ For b is any constant number, the line $y = b$ is a horizontal line and the line $x = b$ is a vertical line.
- ◆ The graph of $y = 0$ is the x-axis and $x = 0$ is the y -axis

Review Exercise:

1. In the algebraic expression $-2x^2y + 3xyz + \frac{x}{2} - y$ find;
 - a. The coefficient of x^2y
 - b. The coefficient of xyz
 - c. The coefficient of x
 - d. The coefficient of y
 - e. The constant term
 - f. Number of terms in the given algebraic expression
2. Simplify each of the following algebraic expressions.
 - a. $2xy + 5y - (8xy - 2y) + 3xy$
 - b. $0.5ab - 3.4ab + 4 - 0.4ab - \frac{1}{2}ab$
 - c. $x^2y^3z + y^3zx^2$
 - d. $xy - 3yz + z$
 - e. $-10z + 3(x + y + z) - 3y + 5z - (x + y)$
 - f. $b^2c - 5ac + 4ab - (b^2c - (b^2c - 2ac) - b^2c - 2ab)$
3. Translate the following into an algebraic expression.
 - a. Ten more than a number.
 - b. Three less than 4 times a number.
 - c. The quotient of a number and 12.
 - d. 12 times the sum of a number and 3.
 - e. The sum of a number and 3 is subtracted from 10 the result is 5.
 - f. 9 is subtracted from 13 times the sum of a number and 4, the result is -13.

4. Solve the following linear equations.

a. $3x+17=7-2x$

b. $5(1-2x)+4(1-x)=2x-6$

c. $\frac{2}{3}x - \frac{2}{5} = 4(x + 1)$

d. $\frac{6x+1}{3} + 1 = \frac{x-3}{6}$

e. $\frac{1}{3}(2x + 3) + \frac{1}{3}x = \frac{3}{2}(x - 2) - x$

5. The points with coordinates $(-6, -3), (2, 1), (4, 2),$ and $(6, 3)$ lie on a straight line.

a. Compute the missing numbers in the coordinates of other points that lie on the line

$(7, \underline{\quad}), (\underline{\quad}, -1), (3, \underline{\quad}), (\underline{\quad}, 4), (100, \underline{\quad})$

b. Describe the relationship between the x-and y-coordinates.

c. Will the points with coordinates $(24, 12)$ lie on the line? Give a reason for your answer.

d. Sketch the graph based on the given ordered pairs.

6. Draw the vertical or the horizontal line on the same coordinate plane.

a. $x = 3$

b. $x = 0$

c. $y = -3$

d. $y = 0$

7. Draw the graph of the function $y = 7x$ on coordinate plane.

8. A shape has corners at the points with coordinates $(3, -2), (4, 2), (-2, 2),$ and $(-3, -2).$

a. Draw the shape

b. What is the name of the shape?

9. Two numbers add up 589 and the first is 199 bigger than the second is. What are the two numbers?
10. What should be added twice to the number 35 in order to get 100?
11. There is a narrow rectangular plot reserved for school in Bushulo village in Hawassa. The length and width of the plot are in the ratio 11: 4. At the rate 100 birr perimeter, it will cost 7500 birr to fence the plot. What are the dimensions of the plot?
12. Rahel's age is 3 times her son's age. Ten years ago, she was five times her son's age. Find their present age.
13. According to working plan of a certain development agency 1800 different type of trees are planted in three consecutive years. The number of trees planted in the second year is twice of the first and number of trees planted in the third year is $\frac{3}{2}$ of the second year. How many trees are planted in each year?
14. A man created a triangular botanic garden such that the largest side is 6meter less than twice the smallest and the medium side is 5meter larger than the smallest side. If the total perimeter of the botanic garden is 47meter, what are the lengths of the three sides?
15. If the average daily temperature in Hawassa city is 27°C . What is th city's average daily temperature in degrees Fahrenheit?