

example

• find domain & Range of the following

a. $g(x) = \sqrt{x^2 - 6x + 8}$

b. $g(x) = \frac{1}{\sqrt{x^2 - 5x + 6}}$

Soln: $g(x) = \sqrt{x^2 - 6x + 8} \geq 0$

$$x^2 - 6x + 8 \geq 0$$

$$(x-4)(x-2) \geq 0$$

	2	-	4	+
$x < 2$	-	-	-	-
$2 < x < 4$	+	+	+	+
$x > 4$	-	-	-	-
$(x-4)(x-2)$	+	-	+	+

$$\therefore \text{Domain} = (-\infty, 2] \cup [4, \infty)$$

$$\text{Range} = \underline{\underline{[0, \infty)}}.$$

$$\textcircled{b} f(x) = \frac{1}{\sqrt{x^2 - 5x + 6}}$$

$$\text{Domain} : x^2 - 5x + 6 > 0$$

$$= (x-2)(x-3) > 0$$

$$\text{Domain} = (-\infty, 2) \cup \underline{\underline{(3, \infty)}}.$$

$$\text{Range} = \underline{\underline{(0, \infty)}}.$$

$$\textcircled{c} f(x) = 1 + 8x - 2x^2.$$

$$\text{Domain} = \mathbb{R} //$$

$$\text{Range} =$$

$$\text{Vertex} = \left(\frac{-b}{2a}, \frac{4ac - b^2}{4a} \right), \quad a = -2, b = 8, c = 1.$$

$$\text{Vertex} = \left(\frac{-8}{2(-2)}, \frac{4(-2)(1) - 64}{4(-2)} \right)$$

$$\text{Vertex} = \left(2, \frac{-8 - 64}{-8} \right) = \underline{\underline{(2, 9)}}.$$

$$\therefore \text{Range} = \underline{\underline{y \leq 9}}.$$

⑧ $f(x) = \frac{2x+3}{x-1}$ — M_1 : find f^{-1}
 — $\text{Dom } f^{-1} = \text{Range } f$.

Domain = $\mathbb{R} \setminus \{1\}$ M_2 : Find H.A.
 $R = \mathbb{R} \setminus \{HA\}$

Range = $\underline{\underline{\mathbb{R} \setminus \{2\}}}$.

Exercise

$$\textcircled{a} f(x) = \sqrt{3 - \sqrt{x^2 - 16}} \quad \textcircled{b} f(x) = \sqrt{3 + \sqrt{16 - x^2}}$$

Combination of function

$$\textcircled{1} (f+g)(x) = f(x) + g(x).$$

$$\textcircled{2} (f-g)(x) = f(x) - g(x).$$

$$\textcircled{3} (fg)(x) = f(x)g(x).$$

$$\textcircled{4} \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}.$$

$$\ast \text{Dom}(f+g) = \text{Dom}(f-g) = \text{Dom}(fg) = \text{Dom}(f) \cap \text{Dom}(g).$$

$$\ast \text{Dom}\left(\frac{f}{g}\right) = [\text{Dom}(f) \cap \text{Dom}(g)] \setminus \{g(x)=0\}$$

Composition of function.

$$① (f \circ g)(x) = f(g(x))$$

$$② (g \circ f)(x) = g(f(x))$$

└ Inside function
└ Outside function.

Domain of Composition of function.

Step 1: Find Domain of inside function.

Step 2: Find Domain of New function.

Step 3: Domain of Composition ^{forming the comp} function = Domain $S_1 \cap$ Domain S_2 .

Example:

Given $f(x) = \frac{x+1}{x^2-4x}$ and $g(x) = \sqrt{x+2} + \sqrt{2-x}$

Find

- a) $(f+g)(x)$ and its Domain.
- b) $(fg)(x)$ and its Domain.
- c) $(\frac{g}{f})(x)$ and its Domain.
- d) $(\frac{f}{g})(x)$ and its Domain.
- e) $(f+g)(0)$

$$\textcircled{c} (f+g)(x) = f(x) + g(x).$$

$$= \frac{x+1}{x^3-4x} + \sqrt{x+2} + \sqrt{2-x}$$

$$\cup \text{Domain}(f) = \mathbb{R} \setminus \{x^3-4x=0\}.$$

$$= \mathbb{R} \setminus \{-2, 0, 2\}.$$

$$\cup \text{Domain}(g) = \begin{matrix} x+2 \geq 0 \text{ and } 2-x \geq 0 \\ x \geq -2 \text{ and } 2 \geq x \end{matrix}$$



$$= [-2, 2]$$

$$\begin{aligned}
 \text{Dom}(f+g) &= \text{Dom}(f) \cap \text{Dom}(g) \\
 &= \mathbb{R} \setminus \{-2, 0, 2\} \cap [-2, 2] \\
 &= (-2, 2) \setminus \{0\}
 \end{aligned}$$

$$\begin{aligned}
 &= \underline{(-2, 0) \cup (0, 2)} = \text{Dom}(f-g)(x) = \text{Dom} \\
 &\quad (fg)(x)
 \end{aligned}$$

$$\checkmark \left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{x+2} + \sqrt{2-x}}{\frac{x+6}{x^3-4x}}$$

$$\left(\frac{g}{f}\right)(x) = \frac{(\sqrt{x+2} + \sqrt{2-x})(x^3-4x)}{x+6}$$

$$\begin{aligned}
 \text{Dom}\left(\frac{g}{f}\right)(x) &= [\text{Dom}(f) \cap \text{Dom}(g)] \setminus \{f(x)=0\} \\
 &= (-2, 2) \setminus \{-1, 0\} //
 \end{aligned}$$

$$(f+g)(0) = f(0) + g(0) = \underline{\underline{8}}$$

② Let $f(x) = \frac{x}{x^2-4}$ and $g(x) = \frac{x+1}{x-3}$. Find

① $(f \circ g)(x)$ and its Domain.

② $(g \circ f)(x)$ and its Domain.

Soln: $(g \circ f)(x) = g(f(x))$

$$= g\left(\frac{x+1}{x-3}\right)$$

$$= \frac{x+1}{x-3}$$

$$\frac{\left(\frac{x+1}{x-3}\right)^2 - 4}{\left(\frac{x+1}{x-3}\right)^2 - 4}$$

$$(f \circ g)(x) = \frac{x+1}{x-3} = \frac{\frac{x+1}{x-3}}{\frac{(x+1)^2}{(x-3)^2} - 4} = \frac{x+1}{x-3} \cdot \frac{(x-3)^2}{(x+1)^2 - 4(x-3)^2}$$

$$= \frac{x+1}{x-3} \cdot \frac{(x-3)^2}{(x+1)^2 - 4(x-3)^2}$$

$$= \frac{(x+1)(x-3)}{x^2 + 2x + 1 - 4(x^2 - 6x + 9)}$$

$$= \frac{(x+1)(x-3)}{x^2 + 2x + 1 - 4x^2 + 24x - 36}$$

$$= \frac{x^2 - 3x + x - 3}{-3x^2 + 26x - 35}$$

$$= \frac{x^2 - 2x - 3}{-3x^2 + 26x - 35}$$

Domain of $(f \circ g)(x)$.

Step 1: $\text{Dom}(g) = \mathbb{R} \setminus \{3\}$.

Step 2: $\text{Domain}(\text{New function}) = \mathbb{R} \setminus \{-3x^2 + 26x - 35 = 0\}$

$$\hookrightarrow -3x^2 + 26x - 35 = 0$$

$$\hookrightarrow -3x^2 + 21x + \underbrace{5x - 35}_{=0} = 0$$

$$\hookrightarrow -3x(x-7) + 5(x-7) = 0$$

$$\hookrightarrow (x-7)(-3x+5) = 0$$

$$\hookrightarrow x = \underline{7} \text{ or } x = 5/3 // = \mathbb{R} \setminus \{7, 5/3\}$$

Step 3: $\text{Dom}((f \circ g)(x)) = \mathbb{R} \setminus \{3\} \cap \mathbb{R} \setminus \{7, 5/3\}$
 $= \mathbb{R} \setminus \{3, 7, 5/3\}$

⊛ If $f(x) = 2x - 5$ and $g(x) = 3x + k$.

find the value of k so that

$$(f \circ g)(x) = (g \circ f)(x)$$

Solⁿ: $f(g(x)) = g(f(x))$

$$\hookrightarrow f(3x + k) = g(2x - 5)$$

$$\hookrightarrow 2(3x + k) - 5 = 3(2x - 5) + k$$

$$\hookrightarrow \frac{6x}{x} + 2k - 5 = \frac{6x}{x} - 15 + k$$

$$\hookrightarrow k = -15 + 5$$

$$\therefore k = -10 //$$

* Let $g(x) = 3x + 1$. Find $f(x)$ such that (a) $(g \circ f)(x) = 2x^2 - 6x - 9$.

(b) $(f \circ g)(x) = 2x^2 - 6x - 9$.

Soln — (a) $(g \circ f)(x) = g(f(x))$.

$$2x^2 - 6x - 9 = 3f(x) + 1$$

$$2x^2 - 6x - 10 = 3f(x)$$

$$\therefore f(x) = \frac{2}{3}x^2 - 2x - \frac{10}{3}$$

(b) $(f \circ g)(x) = f(g(x))$

$$(f \circ g)(x) = f(3x + 1)$$

$$2x^2 - 6x - 9 = f(3x + 1)$$

$$\text{Let } f(x) = ax^2 + bx + c$$

$$\hookrightarrow 2x^2 - 6x - 9 = a(3x+1)^2 + b(3x+1) + c$$

$$= a(9x^2 + 6x + 1) + b(3x+1) + c$$

$$= 9ax^2 + 6ax + a + 3bx + b + c$$

$$\hookrightarrow 2x^2 - 6x - 9 = 9ax^2 + (6a + 3b)x + a + b + c$$

Equating Coefficients of like terms

$$9a = 2 \Rightarrow a = \underline{\underline{2/9}}$$

$$6a + 3b = -6 \Rightarrow 6(2/9) + 3b = -6$$

$$4/3 + 3b = -6 \Rightarrow 3b = -6 - 4/3$$

$$3b = -22/3 \Rightarrow b = \underline{\underline{-22/9}}$$

$$a + b + c = -9$$

$$2/9 - 22/9 + c = -9$$

$$-20/9 + c = -9$$

$$C = -9 + \frac{20}{9} = \underline{\underline{-\frac{61}{9}}}$$

$$\therefore f(x) = \frac{2}{9}x^2 - \frac{22}{9}x - \frac{61}{9}$$

$$C = -9 + \frac{20}{9} = \underline{\underline{-\frac{61}{9}}}$$

$$\therefore f(x) = \frac{2}{9}x^2 - \frac{22}{9}x - \frac{61}{9}$$

Exercise.

① Given $f(x) = \frac{1-x}{3-\sqrt{x+1}}$ and $g(x) = \frac{2x-1}{\sqrt{4x^2-1}}$

find (a) Domain of $(fg)(x)$.

(b) Domain of $(\frac{f}{g})(x)$.

② Let $f(x) = \sqrt{x-4}$ and $g(x) = \sqrt{x^2-9}$
find Domain of $(f \circ g)(x)$.

③ Let $f(x) = \frac{x}{2x-1}$ and $g(x) = \frac{x+1}{3-x}$.

Find $(f \circ g)(x)$ and its domain.

④ Given $f(x) = \log \sqrt{4-x^2}$ and $g(x) = 3x+7$.

Find (a) Domain of $(f \circ g)(x)$.

(b) Domain of $(g \circ f)(x)$.

$g(x) = \frac{2x-1}{\sqrt{4x^2-1}}$

(x) .

$g(x) = \sqrt{x^2-9}$

(x) .