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"To Achieve Big Goals, Start with Small Habits!"

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Mathematics for Natural Sciences (Math 1011)
Worksheet I - July 2021

Part I: Propositional Logic.

- (1) Which of the following sentences are propositions? Determine the truth value when-ever possible?
- (a) $5 < 9$ and there are infinitely many prime numbers.
 - (b) Nairobi is the capital of Kenya.
 - (c) Thank you for your consideration.
 - (d) Answer this question.
 - (e) $x^2 = y^2$ provided that $x = y$.
 - (f) How many courses are you taking this semester?
 - (g) $x + 5 = 11$.
 - (h) The first human kind lived in Ethiopia.
- (2) What is the negation of each of the following propositions?
- (a) Today is Friday.
 - (b) There is no pollution in Europe.
 - (c) $6 + 4 = 10$.
 - (d) The summer in Dire Dawa is hot and sunny.
- (3) Let p and q be propositions:
 p : It is below freezing.
 q : It is snowing.
- Write the following propositions using p , q and logical connectives.
- (a) It is below freezing and snowing.
 - (b) It is below freezing but not snowing.
 - (c) It is not below freezing and it is not snowing.
 - (d) It is either snowing or below freezing (or both).
 - (e) If it is below freezing, it is also snowing.
 - (f) It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
 - (g) That it is below freezing is necessary and sufficient for it to be snowing.
- (4) Given the following propositions
 p : You have the flu.
 r : You pass the course.
 q : You miss the final examination.
- Express each of the following propositions as an ordinary English sentence.
- a) $\neg q \Leftrightarrow r$
 - b) $\neg(p \wedge q)$
 - c) $(p \Rightarrow \neg r) \vee (q \Rightarrow \neg r)$.
- (5) Determine the truth value of p if
- (a) $(q \Rightarrow \neg p) \vee r$ is False
 - (b) $\neg q$ and $p \Rightarrow q$ are True
 - (c) $(p \vee \neg q) \Rightarrow (\neg r \vee p)$ is False.
- (6) If $\neg[\neg r \Rightarrow \neg(p \wedge q)]$ is true, then find the truth value of $[(p \Leftrightarrow r) \vee q] \Leftrightarrow (\neg p \wedge r)$.
- (7) Prove that

- (a) $(p \wedge q) \Rightarrow r \equiv p \Rightarrow (q \Rightarrow r)$
 (b) $(q \vee r) \Rightarrow p \equiv (q \Rightarrow p) \wedge (r \Rightarrow p)$.
- (8) For the following propositions, indicate whether it is a tautology, a contradiction, or neither. Use a truth table to decide.
 (a) $[p \wedge (p \Rightarrow q)] \Rightarrow q$
 (b) $(p \Rightarrow q) \wedge \neg(\neg q \Rightarrow \neg p)$.
 (c) $(\neg q \Rightarrow \neg p) \Rightarrow ((\neg q \Rightarrow p) \Rightarrow q)$.
- (9) Determine whether the following statements are valid or invalid.
 (a) If I do not wake up, then I cannot go to work.
 If I cannot go to work, then I will not get paid.
 Therefore, if I do not wake up, then I will not get paid.
 (b) If I study, then I will not fail Math 1011.
 If I do not play cards too often, then I will study.
 I failed Math 1011.
 Therefore, I played cards too often.
 (c) Today is either Thursday or Friday.
 On Thursdays I have to go to a lecture.
 Today is not Friday.
 Therefore, I have to go to a lecture today.
- (10) Let $P(x) : x$ is an integer greater than 5. $Q(x) : x$ is a natural number.
 Determine the truth value of the following propositions.
 (a) $P(3) \wedge Q(-2)$
 (b) $P(2) \Rightarrow Q(2)$
 (c) $[P(3) \Rightarrow Q(4)] \vee Q(9)$.
- (11) Let $P(x) : x$ is a prime number $Q(x) : x$ is an even number.
 $R(x) : x$ is an odd number. $S(x) : x$ is an integer.
 Write a sentence, which corresponds to each of the following:
 (a) $R(x) \vee Q(x) \Rightarrow P(x)$
 (b) $P(x) \Leftrightarrow Q(x) \vee R(x)$.
- (12) If $U = \mathbb{R}$, then find the truth value of $(\forall x)(\exists y)(\sqrt{x^2} = 16 \Rightarrow y + x = 10)$.
- (13) Let $P(x) : x$ is a composite number. $Q(x) : x$ is a prime number.
 Find the truth values of (i) $(\exists x)[P(x) \Leftrightarrow Q(x)]$ (ii) $(\forall x)[P(x) \Leftrightarrow Q(x)]$.
- (14) Find the truth values of the following where $U = \mathbb{R}$
 (a) $(\exists x)(\forall y)(x^2 < y^2)$
 (b) $(\forall x)[x \neq 0 \Rightarrow (\exists y)(xy = 4)]$.

Part II: Set Theory.

- (1) Write the following sets in complete listing or partial listing method.
 (a) $A = \{x | x \text{ is an integer and } 3 < x \leq 10\} \cap \{x | x \text{ is even integer}\}$.
 (b) $B = \{x \in \mathbb{N} | (x-1)(x-3) \neq 0 \Leftrightarrow x+1 = x\}$.
 (c) $C = \{x \in \mathbb{N} | x \geq 3 \Leftrightarrow x < 0\}$.
 (d) $D = \{x | x \in \mathbb{N} \wedge (x-1)(x-3) \neq 0 \Leftrightarrow x+1 = x\}$.
- (2) If $n(A \setminus B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.
- (3) For all subsets A, B and C of some Universal set U , prove or disprove the following statements.

- (a) $(A \subseteq B \wedge A \subseteq C) \Rightarrow A \subseteq B \cap C$
 (b) $A \subseteq B \Leftrightarrow A \cap B = A$
 (c) $A^c \setminus B^c = B \setminus A$
 (d) $A \subseteq B \Rightarrow A \cup (B \setminus A) = B$.
- (4) Let $U = \{x \in \mathbb{Z} \mid -12 \leq x \leq 6\}$.
 $A = \{x \in \mathbb{Z} \mid x = 2n \wedge x = 3m, \text{ for } -4 \leq n \leq 8 \text{ and } -2 \leq m \leq 3\}$.
 $B = \{0, 1, 3\}$ $C = \{x \in \mathbb{Z} \mid x = \frac{n}{3}, n \in A\}$.
 Then find
 (a) $A \setminus [B \cup C]^c$
 (b) $(A \setminus B) \setminus (A \cup C)^c$
 (c) $(A \cap B) \setminus C$.
- (5) (a) For every natural number n , define $A_n = \{x \in \mathbb{Z} \mid \frac{-24}{n} \leq 2x + 1 \leq \frac{24}{n}\}$.
 Then find
 (i) $\bigcup_{n=1}^6 A_n$
 (ii) $[(\bigcup_{n=1}^4 A_n) \setminus (\bigcap_{n=1}^3 A_n)]^c$
 (iii) $\bigcap A_n, n \in \mathbb{N}$.
- (b) For every natural number n , define $A_n = \{x \in \mathbb{N} \mid x = n \Rightarrow x < n\}$.
 Then find
 (i) $\bigcup_{n=1}^{10} A_n$
 (ii) $\bigcup A_n, n \in \mathbb{N}$.

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Worksheet II - July 2021

Part I: The real number system.

- (1) Two incandescent lamps are turned on at the same time. One blinks every 4 seconds and the other blinks every 6 seconds. In 60 seconds, how many times will they blink at the same time?
- (2) Write the following sets as an interval or union of intervals. In each case $x \in \mathbb{R}$.
 - (a) $A = \{x | x \neq 5 \wedge x \neq 8\}$.
 - (b) $B = \{x | x^2(x^3 - 1) \neq 0\}$.
 - (c) $C = \{x | x^3 - 2x \neq 0\}$.
 - (d) $D = \{x | (x - 2)(x + 3)(x + 4) = 0 \Rightarrow x \geq 2\}$.
- (3) Simplify each of the following sets and give the final result as an interval or union of intervals
 - (a) $([-3, 2] \cap (0, 5))^c$.
 - (b) $((-3, 5.6] \cap (-5, 2.43)) \cup (0.3, 11)$.
 - (c) $(2, 4]^c \cap ((-\infty, 5)) \cup [0, \infty))$.
 - (d) $(-0.3, 0.03) \cup ((-\infty, \frac{1}{30}) \cap [5, \infty))$.
- (4) Find the supremum and infimum of each of the following sets of real numbers:
 - (a) All numbers of the form $2^{-p} + 3^{-q} + 5^{-r}$, where p, q , and r take on all positive integer values.
 - (b) $S = \{3x^2 - 10x + 3 < 0\}$.
- (5) Find the smallest and the largest elements of the given set. Also the lub and glb whenever they exist.
 - (a) $A = \{x | x = 5 - \frac{1}{n}; n \in \mathbb{N}\}$.
 - (b) $B = \{x | x \in (-1, 1) \cup \{\frac{1}{n} | n \in \mathbb{N}\}\}$.
 - (c) $C = \{x | x = \frac{n}{n+1}; n \in \mathbb{N}\}$.
 - (d) $D = \{x | x = \frac{n+1}{n}; n \in \mathbb{N}\}$.
 - (e) $E = \{x | x = \frac{n^3}{2n^2+1}; n \in \mathbb{N}\}$.
- (6) Using the Principle of Mathematical Induction, prove that
 - (a) $1 + 4 + 7 + \dots + (3n - 2) = \frac{n(3n-1)}{2}$ for $n \geq 1$.
 - (b) $1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$ for $n \geq 1$.
 - (c) $2^{2n+1} + 7$ is a multiple of 3 for $n \geq 1$.
 - (d) $n^2 - n$ is even for $n \geq 1$.
 - (e) $6^n - 1$ is divisible by 5 for $n \geq 1$.
 - (f) $n! > 3^n$ for $n \geq 7$.
- (7) Consider the sequence of real numbers defined by the relations

$$x_1 = 1 \text{ and } x_{n+1} = \sqrt{1 + 2x_n} \text{ for } n \geq 1.$$

Use the Principle of Mathematical Induction to show that $x_n < 4$ for all $n \geq 1$.

- (8) Prove that $\sqrt{2} + \sqrt{3}$ is irrational number.
- (9) If $\frac{a}{b} < \frac{c}{d}$ with $b > 0, d > 0$, prove that $\frac{a+c}{b+d}$ lies between the two fractions $\frac{a}{b}$ and $\frac{c}{d}$.

Part II: The Complex number system.

- (1) Change the following complex numbers to polar form
 - (a) $-2 - 4i$.
 - (b) $1 - \sqrt{2}i$.
 - (c) -27 .
 - (d) $5i$.
- (2) Find the modulus and argument of
 - (a) $(1 - \sqrt{2}i)^2$.
 - (b) $(1 + i)^{11}$.
 - (c) $(\frac{1}{2} - \frac{\sqrt{3}}{2}i)^{15}$.
- (3) Find $z \in \mathbb{C}$ such that
 - (a) $\operatorname{Re}(z(1 + i)) + z\bar{z} = 0$;
 - (b) $\operatorname{Re}(z^2) + i\operatorname{Im}(\bar{z}(1 + 2i)) = -3$;
 - (c) $\operatorname{Im}(2 - i)z = 1$.
- (4) Express the following complex numbers in the form $a + bi$.
 - (a) $(1 + i)^3$.
 - (c) $i^5 + i^{16}$.
 - (b) $\frac{2+3i}{3-4i}$.
 - (d) $\frac{1}{2}(1 + i)(1 + i^{-8})$.
- (5) Compute the absolute value and the conjugate of
 - (a) $z = (1 + i)^6$.
 - (b) $w = i^{17}$.
- (6) In each case, determine all real x and y which satisfy the given relation.
 - (a) $x + iy = |x - iy|$.
 - (b) $x + iy = (x - iy)^2$.
 - (c) $\sum_{k=0}^{100} i^k = x + iy$.
- (7) Solve the following equations.
 - (a) $z^2 = 1 + i$.
 - (b) $z^2 - 4z + 6 = 0$.
 - (c) $z^4 + 4z^2 + 16 = 0$.
 - (d) $(\bar{z} - i)^3 = 1$.
 - (e) $z^5 = 32$.
- (8) Compute
 - (a) the square roots of $z = -1 - i$.
 - (b) the three cube roots of $z = -8$.
 - (c) the four fourth roots of $z = -1 + \sqrt{3}i$.
- (9) Write $\sin 4\theta$ and $\cos 4\theta$ in terms of $\sin \theta$ and $\cos \theta$. (Hint: use $(\cos \theta + i \sin \theta)^4$)
- (10) Find the locus of points $z = x + iy$ that satisfy
 - (a) $|z + 1| = |z - i|$.
 - (b) $3|z| = |z + 3i|$.
 - (c) $|z - 1| = 2$.
 - (d) $z^2 \bar{z} = z$.
- (11) If $\alpha = \cos \theta + i \sin \theta$ for $0 < \theta < 2\pi$, show that $\frac{1+\alpha}{1-\alpha} = i \cot \frac{\theta}{2}$.
- (12) If $z \neq -1$ and $\operatorname{Im}(z) \neq 0$, show that $\frac{z}{(1+z)^2}$ is real iff $z\bar{z} = 1$.
- (13) If for $z = x + iy$, $f(z) = e^x(\cos y + i \sin y)$, then show that
 - (a) $f(z_1 + z_2) = f(z_1)f(z_2)$.
 - (b) $\overline{f(z)} = f(\bar{z})$.
- (14) Find $z \in \mathbb{C}$ such that $z^2 \in \mathbb{R}$.