



HARAMAYA IFA BORU SPECIAL
BOARDING SECONDARY SCHOOL
2016 E.C MATHEMATICS
ENTRANCE (2005-2010) WORK
BOOK FOR GRADE 12

CAAMSAA 2016/ MAY 2024
HARAMAYA SPECIAL BOARDING
2NDRY SCHOOL (MAYA CITY)
ADELE SUB CITY

Part one

CHOOSE THE BEST ANSWER FROM THE FOUR GIVEN ALTERNATIVES

1. If $\{a_n\}$ is a sequence such that $a_1 = 2$, and $a_{n+1} = a_n + 4$ for all $n \geq 1$, then $\sum_{n=1}^{35} a_n$ is equal to :

- ☐ A) 2460
 - ☐ B) 2458
 - ☐ C) 2450
 - ☐ D) 2442
-

2. If $f(x) = x^2 + 2\ln x$, then $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$?

- ☐ A) 5
 - ☐ B) 4
 - ☐ C) 2
 - ☐ D) 0
-

3. If $f(x) = e^{2x} + x - 3 \cos x$, then what is $f'(x)$?

- ☐ A) $e^{2x} + 1 - 3 \sin(x)$
 - ☐ B) $e^{2x} + 1 + 3 \sin(x)$
 - ☐ C) $4e^{2x} - 3 \cos(x)$
 - ☐ D) $4e^{2x} + 3 \cos(x)$
-

4. Which one of the following intervals does $f(x) = x^4 + 4x$ increase?

- ☐ A) $(-\infty, -1]$
 - ☐ B) $(-\infty, 0]$
 - ☐ C) $[-1, \infty)$
 - ☐ D) $(-\infty, \infty)$
-

7. What is the value of $|x| + 2x$ if $x \leq 0$?

- ☐ A) $-3x$
 - ☐ B) $3x$
 - ☐ C) $-x$
 - ☐ D) x
-

8. Suppose $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ if x is a 2×2 matrix such that $AX - A^T = 2A$, then what is the value of x ?

- ☐ A) $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$
- ☐ B) $\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$

- ☐ C) $\begin{pmatrix} 3 & 6 \\ 6 & 9 \end{pmatrix}$
- ☐ D) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

9. Suppose that A and B are 3 x 3 matrices, I is the identity matrix of order 3 such that $AB=2I$. If $\det B = |B| = 6$. What is $\det(A^T)$?

- ☐ A) 1/3
- ☐ B) 4/3
- ☐ C) 12
- ☐ D) 48

10. What is the 50th term of the sequence 3, 10, 17, 24, 31, ...?

- ☐ A) 310
- ☐ B) 346
- ☐ C) 510
- ☐ D) 531

12. Which one of the following functions has NO vertical asymptote?

- ☐ A) $f(x) = \ln(x+1)$
- ☐ B) $f(x) = \frac{x^2+1}{x^3+8}$
- ☐ C) $f(x) = \frac{x^2-9}{x-3}$
- ☐ D) $f(x) = \frac{x-1}{x^2-x}$

14. If $f(x) = \frac{1}{e^x+1}$, then which one of the following is equal to $f^{-1}(x)$ for $0 < x < 1$?

- ☐ A) $\ln(1-x) - \ln(x)$
- ☐ B) $e^{-x} + 1$
- ☐ C) $\ln(1/x+1)$
- ☐ D) $(1/e^{-x}) + 1$

15. If $x^2 - 6x + y^2 + k = 0$ is equation of a circle with radius 2, then what is the value of k?

- ☐ A) 13
- ☐ B) 5
- ☐ C) 4
- ☐ D) -4

16. If a line with angle of inclination of $3\pi/4$ passes through (0, 1), which one of the following is the equation of the line?

- ☐ A) $y = -x + 1$

- ☐ B) $y = x + 1$
- ☐ C) $y = -x - 1$
- ☐ D) $y = x - 1$

17. If $g(x) = x f(x) - \sqrt{f(x)}$ and $f(2) = f'(2) = 4$, then which of the following is equal to $g'(2)$?

- ☐ A) 11
- ☐ B) 8
- ☐ C) 2
- ☐ D) 0

18. What is the sum of the series $\sum_{n=1}^{\infty} \frac{2^{2n+1}}{5^{n-1}}$

- ☐ A) 40
- ☐ B) 20
- ☐ C) 10
- ☐ D) 8

19. What is the value of $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\frac{-x}{2}}$?

- ☐ A) $\frac{1}{\sqrt{e}}$
- ☐ B) $\sqrt{e}$
- ☐ C) e^{-2}
- ☐ D) ∞

20. Among students who took a quiz, 15 students scored 6, 20 students scored 7, 10 students scored 8 and 5 students scored 10. What is the average score of the students?

- ☐ A) 7.8
- ☐ B) 7.5
- ☐ C) 7.2
- ☐ D) 7.0

22. How many four digit even numbers can be formed from 1, 2, 3, 4 and 5 if the numbers start with 3?

- ☐ A) 40
- ☐ B) 50
- ☐ C) 100
- ☐ D) 120

25. At which value(s) of x does $f(x) = 0.25x^4 - 2x^2$ have a local maximum?

- ☐ A) $x = 4$
- ☐ B) $x = 0$

- ☐ C) $x = -2$ and $x = 2$
- ☐ D) $x = 0$ and $x = 2$

27. The population of certain country is currently 80 million with growth rate of 2% per year.

Given: $(0.02)^9 = 5.12 \times 10^{-16}$, $(1.02)^9 = 1.19$
 $(0.02)^{10} = 1.024 \times 10^{-17}$, $(1.02)^{10} = 1.22$

which one of the following is the best approximation of the population after 10 years?

- ☐ A) 81.9 million
- ☐ B) 86.8 million
- ☐ C) 95.2 million
- ☐ D) 97.6 million

28. The volume V of a melting ice cube after t seconds is $V = 2000 - 40t + 0.2t^2$ (in cm^3). How fast is the volume changing when $t=40$ seconds?

- ☐ A) $24 \text{ cm}^3/\text{sec}$
- ☐ B) $15 \text{ cm}^3/\text{sec}$
- ☐ C) $-15 \text{ cm}^3/\text{sec}$
- ☐ D) $-24 \text{ cm}^3/\text{sec}$

29. Which one of the following is equal to $\lim_{n \rightarrow \infty} \frac{1-n-3n^2}{6n^2+1}$?

- ☐ A) $1/6$
- ☐ B) $-1/2$
- ☐ C) $-1/6$
- ☐ D) $-\infty$

30. What is the solution set of $\frac{2}{x} - \frac{x-2}{x^2-2x} = 1 - \frac{2x-2}{3x-2}$?

- ☐ A) $\{1, -2\}$
- ☐ B) $\{1, 2\}$
- ☐ C) $\{-1\}$
- ☐ D) $\{1\}$

31. Let $f(x) = \begin{cases} 3^x + k, & \text{if } x \leq 0 \\ 3 \frac{\sin(2x)}{x}, & \text{if } x > 0 \end{cases}$ if f is continuous at $x=0$, then what is the value of k ?

- ☐ A) 6
- ☐ B) 5
- ☐ C) 2
- ☐ D) 0

32. Which one of the following is equal to $\int (1+x)^3 dx$?

- ☐ A) $(1+x)^3 - 3x + c$
☐ B) $(1+x)^3 + (\log_3 e)^3 + c$
☐ C) $(1+x)^3 \log_3 e - (\log_3 e)^2 3^x + c$
☐ D) $(1+x)^3 \log_3 e - 3^x (\log_3 e) + c$

33. A committee consisting of 3 students is to be selected from 10 candidates among which 4 are girls. What is the probability that at least one girl is selected?

- ☐ A) $5/6$
☐ B) $2/3$
☐ C) $1/3$
☐ D) $1/6$

34. A group of six students take their seats at random in a round table for a discussion. What is the probability that two specific students do NOT sit together?

- ☐ A) $3/5$
☐ B) $2/3$
☐ C) $2/5$
☐ D) $1/3$

35. Given $f(x) = \ln(x-1)$ and $g(x) = \sqrt{1-2x}$, which one of the following is the domain of $f \circ g$?

- ☐ A) $\{x \in \mathbb{R}: x > 1\}$
☐ B) $\{x \in \mathbb{R}: x \leq 1/2\}$
☐ C) $\{x \in \mathbb{R}: x < 0\}$
☐ D) $\{x \in \mathbb{R}: x > 1/2\}$

36. The mark that students scored in an examination is grouped in class intervals as shown in the following table.

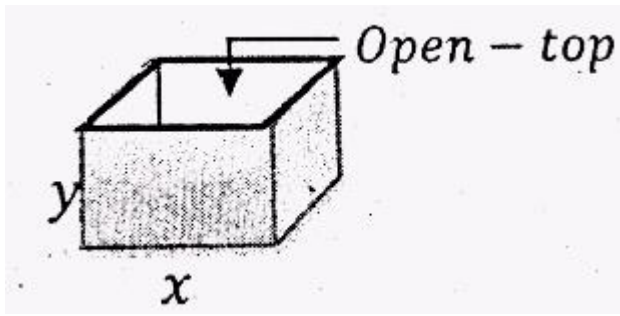
Class Interval (Mark)	Number of Students
55 - 64	8
65 - 74	12
75 - 84	20
85 - 94	6
95 - 100	4

What is the median of the mark?

- ☐ A) 25.0
☐ B) 75.5
☐ C) 77.0
☐ D) 79.5

37. A box seen below is to have a square base, an open top and volume of 32 cubic units. If x is the length

of each side of its base and y is its height, how many units should x and y be in order to make the box with the smallest amount of material?



- ☐ A) $x = 4, y = 2$
- ☐ B) $x = 2, y = 8$
- ☐ C) $x = \sqrt{8}, y = 4$
- ☐ D) $x = \sqrt{2}, y = 16$

38. Which one of the following is equal to $\int \frac{1}{x^2+x} dx$?

- ☐ A) $\ln|x^2 + x| + c$
- ☐ B) $2\ln|x + 1| + \ln|x| + c$
- ☐ C) $\ln|x| - \ln|x + 1| + c$
- ☐ D) $\ln|x| + \ln|x + 1| + c$

40 What is the value of $\int_0^{\frac{\pi}{2}} 2x \cos x \, dx$?

- ☐ A) $\pi - 2$
- ☐ B) $(\pi/2) + 1$
- ☐ C) $\pi + 2$
- ☐ D) $(\pi/2) - 1$

41. A box contains 5 white, 6 red and 4 black balls of all identical size. If 3 balls are randomly taken out of the box after the other, what is the probability that the first ball is white and both the second and third balls are red?

- ☐ A) $2/15$
- ☐ B) $3/15$
- ☐ C) $4/75$
- ☐ D) $5/91$

42. Which one of the following is the equation of the line tangent to the graph of $f(x) = 1/(x+1) + \cos x$ at $(0, f(0))$?

- ☐ A) $x + y = 1$
- ☐ B) $x - y = -2$
- ☐ C) $x + y = 2$
- ☐ D) $x + 4y = 2$

43. Which one of the following is equal to $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x^2-1}$?

- ☐ A) ∞
☐ B) 0
☐ C) -1/4
☐ D) 1/4

44. Which one of the following is equal to $\lim_{x \rightarrow 1} \frac{1-x}{1-\frac{1}{x^2}}$?

- ☐ A) 1
☐ B) 0
☐ C) -1/2
☐ D) Doesn't exist

45. Which one the following is equal to $\int \frac{\ln(xe^x)}{x} dx$

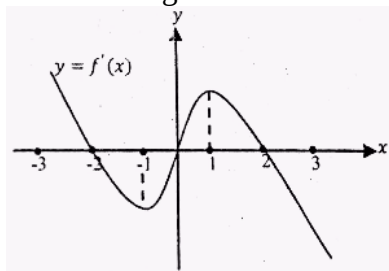
- ☐ A) $\ln |x| + \frac{1}{2}e^x + c$
☐ B) $\frac{1}{2}(\ln x)^2 + x + c$
☐ C) $\ln |x| + e^{2x} + c$
☐ D) $-\frac{1}{x^2} + (\ln x)^2 + C$

46. Consider the system $\begin{cases} \alpha x + y + z = 1 \\ x + 2y + 4z = 0 \\ 5x + y + z = 0 \end{cases}$ if the determinant of the coefficient matrix is 2, then what is the solution of the system of the equations?

- ☐ A) $(3\alpha, \frac{19\alpha}{2}, \frac{-11\alpha}{2})$
☐ B) $(3, \frac{19}{2}, \frac{-11\alpha}{2})$
☐ C) $(\frac{3}{\alpha}, \frac{-19\alpha}{2}, \frac{11}{2})$
☐ D) $(\frac{3}{2}, \frac{19}{2}, \frac{9}{2})$

47. Suppose f is differentiable on $(-\infty, \infty)$ and the graph of its derivative is as shown below. Which one of

the following is true about f ?



- ☐ A) f is decreasing on $(-\infty, -1) \cup [1, \infty)$
- ☐ B) f has a local minimum at $x = -2$
- ☐ C) f is concave down on $[0, \infty)$
- ☐ D) f is concave up on $(-1, 1)$

48. Which one of the following is true about the derivative of $f(x) = x|x|$?

- ☐ A) f is not differentiable at $x = 0$
- ☐ B) $f'(x) = 2|x|$, for every $x \in (-\infty, \infty)$
- ☐ C) $f'(x) = 2x$, for every $x \in (-\infty, \infty)$
- ☐ D) $f'(x) = |x| + x$, for every $x \in (-\infty, \infty)$

49. What is the area of the region between the graphs of $y = x^2$ and $y = -x + 2$, where $0 \leq x \leq 2$?

- ☐ A) 3
- ☐ B) 2
- ☐ C) $3/2$
- ☐ D) $2/3$

50. For what value of b does the parabola $p(x) = ax^2 + x + b$ pass through the points $(-1, 5)$ and $(2, -1)$?

- ☐ A) 9
- ☐ B) 3
- ☐ C) -3
- ☐ D) -15

51. If $f(x) = 2 - \frac{1}{2} \sin\left(\frac{\pi}{2}x\right)$, then which one of the following is the amplitude and period of f , respectively?

- ☐ A) $1/2$ and 4
- ☐ B) $-1/2$ and 4
- ☐ C) 2 and π
- ☐ D) $1/2$ and π

52. Which one of the following is equal to $\sec(\pi/2 - x) \sin^3 x + \cos 2x$?

- ☐ A) $2\cos x$
- ☐ B) $2\sin x$
- ☐ C) $\cos 2x$

☐ D) $\sin 2x$

53. Suppose $P(1, 2, 1)$ and $Q(1, 0, 2)$ are points in space and $\vec{A} = \overrightarrow{PQ}$. If $\vec{B}$ is parallel to $\overrightarrow{PQ}$ and $\vec{A} \cdot \vec{B} = -10$, then which one of the following is true?

- ☐ A) $\vec{A}$ and $\vec{B}$ has the same direction
- ☐ B) $\|\vec{B}\| = 10\|\vec{A}\|$
- ☐ C) $\|\vec{B}\| = \frac{1}{10}\|\vec{A}\|$
- ☐ D) $\|\vec{B}\| = 2\|\vec{A}\|$
-

54. What is the solution of $\cos^2 x + 0.5 \sin 2x = 1$ in the interval $[0, 2\pi)$?

- ☐ A) $\{0, \pi/4, \pi, 5\pi/4\}$
- ☐ B) $\{0, \pi/4, 3\pi/4, \pi\}$
- ☐ C) $\{0, \pi\}$
- ☐ D) $\{0, \pi/4, \pi\}$
-

55. If $\vec{u} = (-3, x)$ and $\vec{v} = (x, y - 2)$ are vectors, what is the value of y so that $\vec{u} + \vec{v} = 3\vec{u} - \frac{1}{2}\vec{v}$?

- ☐ A) $2/3$
- ☐ B) $-10/3$
- ☐ C) -4
- ☐ D) $-22/3$
-

56. Which one of the following points is closer to the sphere $x^2 + y^2 + z^2 - 2x + 6z + 9 = 0$?

- ☐ A) $(1, 0, 0)$
- ☐ B) $(0, 0, 0)$
- ☐ C) $(0, -1, 0)$
- ☐ D) $(0, 0, -1)$
-

57. Which one of the following describes the principle of Mathematical Induction on a set of natural numbers?

- ☐ A) if an assertion is true for a natural number n , then it is true for $n + 1$.
- ☐ B) if an assertion is true for 1 and it is true for $n = 1$, then it is true for some n .
- ☐ C) if an assertion holds for $n = 20$ and for an $n \geq 20$, then it is true for n implies true for $n + 1$.
- ☐ D) if an assertion is true for $n = 1$ and is true for $n = k$, whenever is true for $n = k + 1$.
-

58. What is the $\cot(\arcsin x)$ if $0 < x < 1$?

- ☐ A) $\frac{x}{\sqrt{1-x^2}}$
- ☐ B) $\frac{1}{x}\sqrt{1-x^2}$
- ☐ C) $\sqrt{1-x^2}$
- ☐ D) $\frac{1}{\sqrt{1-x^2}}$

59. Suppose $\vec{A} = 2\vec{j} - \vec{k}$ and $\vec{B} = 5\vec{i} + 15\vec{k}$, where $\vec{i}, \vec{j}$ and $\vec{k}$ are the standard unit vectors in the directions of positive x, y and z axis, respectively. Which one of the following is the unit vector in the direction of $\vec{A} + \frac{1}{5}\vec{B}$?

- ☐ A) $\frac{3}{5}\vec{i} + \frac{4}{5}\vec{k}$
- ☐ B) $\frac{1}{3}\vec{i} + \frac{2}{3}\vec{j} + \frac{2}{3}\vec{k}$
- ☐ C) $\frac{4}{5}\vec{j} - \frac{3}{5}\vec{k}$
- ☐ D) $\frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} + \frac{2}{3}\vec{k}$

60. A line given by a vector equation $\mathbf{r}(t) = (0, 3) + t(1, 1)$ is tangent to a circle at point (0,3). If the radius of the circle is 2. Which one of the following is the center of the circle?

- ☐ A) (1,4)
- ☐ B) (1,-4)
- ☐ C) (-1,2)
- ☐ D) (1,2)

63. Let $\vec{a} = 2\vec{i} + (x-1)\vec{j} + \vec{k}$ and $\vec{c} = \vec{i} - \vec{j} + y\vec{k}$ be vectors. If $\vec{a} \cdot \vec{c} = 0$ and $\|\vec{a}\| = 3$, which one of the following is a possible value of y?

- ☐ A) -4
- ☐ B) -1
- ☐ C) 3
- ☐ D) 4

64. If $\vec{A}$ is perpendicular to $\vec{B}$, what is the cosine of the angle between $\vec{A}$ and $\vec{A} - \vec{B}$?

- ☐ A) $\frac{\|\vec{A}-\vec{B}\|}{\|\vec{A}\|}$

- ☐ B) $\frac{\|\vec{A}\|}{\|\vec{A}-\vec{B}\|}$
- ☐ C) $\frac{\|\vec{A}-\vec{B}\|}{\|\vec{B}\|}$
- ☐ D) $\frac{\|\vec{B}\|}{\|\vec{A}-\vec{B}\|}$

65. Which one of the following is necessarily true?

- ☐ A) If $\|\vec{A}\| = \|\vec{B}\|$, then $\vec{A} = \vec{B}$
 - ☐ B) $\|k\vec{A}\| = K\|\vec{A}\|$, for any real number k
 - ☐ C) if $\vec{A}$ is parallel to $\vec{B}$, then $\vec{A} \cdot \vec{B} = 0$
 - ☐ D) if $\vec{u}$ is a unit vector in the direction of $\vec{A}$, then $\vec{A} \cdot \vec{u} = \|\vec{A}\|$
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PART TWO

1. A water tank is a circular cylinder with base radius 2m and height 3m. If the tank is empty and water is pumped into it at rate of $2\text{m}^3/\text{min}$, how long does it take for the tank to be full?

- ☐ A) 1.5 min
 - ☐ B) $3/2 \pi \text{min}$
 - ☐ C) 6π
 - ☐ D) 12 min
-

2. If $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ and $(2A + B)^T = A^T A$, then which one of the following is equal to B?

- ☐ A) $\begin{pmatrix} 1 & 0 & -2 \\ 2 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$
 - ☐ B) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
 - ☐ C) $\begin{pmatrix} 8 & 0 & -4 \\ 4 & 8 & 0 \\ 0 & 0 & -4 \end{pmatrix}$
 - ☐ D) $\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{pmatrix}$
-

3. Which one of the following represents a geometric sequence?

- ☐ A) 3, 1, 1/3, 1/9, 1/27,?
 - ☐ B) 1/2, -1/3, 1/4, -1/5, 1/6,?
 - ☐ C) 1,3, 6, 10, 15,?
 - ☐ D) -3, 6, -9, 12, -15,?
-

4. The sequence $\left\{ \frac{(n-1)(2n+1)}{1-n^2} \right\}_{n=1}^{\infty}$ converges to:

- ☐ A) $-\infty$
 - ☐ B) -2
 - ☐ C) 0
 - ☐ D) 1
-

5. If $F(x)$ is an ant derivative of $f(x) = 1 - 2/x^2$ and $F(1) = 0$, then $F(2)$ is equal to:

- ☐ A) 0
 - ☐ B) 1/2
 - ☐ C) -1/2
 - ☐ D) 3
-

6. Which one of the following expression is a polynomial expression?

- ☐ A) $x^2 - 3x + \sin x$
 - ☐ B) $\frac{4x^3 + 12x^2 - x}{\pi x^2}$
 - ☐ C) $\frac{2+\pi}{1+\pi^2}$
 - ☐ D) $2 - 3x^{\frac{2}{3}} + 7x^{\frac{5}{2}} + 3x^{-1}$
-

7. What is the distance from the origin to the line that passes through (1, 0) and (0, 1)?

- ☐ A) $1/2 \sqrt{2}$
 - ☐ B) 1
 - ☐ C) 1/2
 - ☐ D) $\sqrt{2}$
-

8. If the list of a measurement is 10, α , 5, α , 5, 10, 20, 15, 20, 5 with mean a then α in terms of a is equal to:

- ☐ A) $10a - 90$
 - ☐ B) $9a - 90$
 - ☐ C) $5a - 90$
 - ☐ D) $5a - 45$
-

9. The total cost (in Birr) of producing x iron sheets per day is $C(x) = 1,000 + 100x - 0.5x^2$, $0 \leq x \leq 100$. What is the marginal (rate of change of) cost at a production level of 80 iron sheets?

- ☐ A) 8.5

- ☐ B) 20
- ☐ C) 1,800
- ☐ D) 5,800

11. The following is the frequency distribution of a grouped data.

Class Intervals	Frequency (f)
3-7	2
8-12	2
13-17	10
18-22	6

What is the mean and standard deviation of the distribution, respectively?

- ☐ A) $15, 2\sqrt{5}$
- ☐ B) $15, \sqrt{7.5}$
- ☐ C) $12.5, 5\sqrt{2}$
- ☐ D) $12.5, \sqrt{15}$

$$\sum_{n=1}^{\infty} \left(\frac{2^n + 5^n}{10^n} \right)$$

13. What is the actual value of the sum

- ☐ A) 0.325
- ☐ B) 1
- ☐ C) $5/4$
- ☐ D) $37/9$

14. If $f(x) = \frac{x+1}{x-1}$ and $f(a) = 5$ then $f(2a)$ is equal to:

- ☐ A) 2
- ☐ B) 4
- ☐ C) 6
- ☐ D) 8

15. If $f(x) = \sqrt[3]{1 + e^{-x}}$, which of the following is equal to $f^{-1}(x)$?

- ☐ A) $\ln\left(\frac{1}{x^3-1}\right)$
- ☐ B) $\frac{1}{\ln(x^3-1)}$
- ☐ C) $\ln(1 - x^3)$
- ☐ D) $(1 + e^{-x})^3$

17. Given that $\lim_{x \rightarrow 3} f(x) = 5$ and $\lim_{x \rightarrow 3} g(x) = 11$, what is the value of $\lim_{x \rightarrow 3} \left(\frac{(f(x)-g(x))(g(x)^2-f(x))}{g(x)^2-f(x)} \right)$?

- ☐ A) -66/96
 - ☐ B) -1/16
 - ☐ C) 0
 - ☐ D) does not exist
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$$f(x) = \frac{x^2}{1+xg(x)}$$

18. If $g(2) = 1$ and $g'(2) = 10$, then which one of the following is equal to $f'(2)$ (the derivative of f at 2)?

- ☐ A) -8
 - ☐ B) -8/9
 - ☐ C) 4/3
 - ☐ D) 8/9
-

20. If $M = \begin{pmatrix} 0 & 1 & 2 \\ 3 & -1 & 0 \\ 5 & 2 & 4 \end{pmatrix}$ and $A^T M = 2I$, where A is a 3×3 matrix and I is the identity matrix of order 3, then what is $\det(A)$?

- ☐ A) 0.2
 - ☐ B) 4/17
 - ☐ C) 0.8
 - ☐ D) 1/17
-

21. If distinct codes (words of eight letters are formed by rearranging the letters in the word 'ABBEBAYE', how many of the codes begin with B or Y?

- ☐ A) 840
 - ☐ B) 630
 - ☐ C) 1680
 - ☐ D) 420
-

22. What should be the value of k so that the system of equation has a solution?

$$\begin{cases} x - y + z = 1 \\ -x + 5y - 4z = 1 \\ 2x + 2y - z = k \end{cases}$$

has a

- ☐ A) 0
 - ☐ B) 1
 - ☐ C) -4
 - ☐ D) 4
-

$$f(x) = \begin{cases} a \frac{\sin x}{x - |x|} & \text{if } x < 0 \\ e^{-x} + \cos x, & \text{if } x \geq 0 \end{cases}$$

23. Let if f is continuous at $x = 0$, then what is the value of a ?

- ☐ A) 4

- ☐ B) 2
- ☐ C) 1/2
- ☐ D) -4

24. What is the area of the region between the graphs of $y = -x^2 + 2$ and $y = |x|$, where $-1 \leq x \leq 2$?

- ☐ A) 11/6
- ☐ B) 25/6
- ☐ C) 7/3
- ☐ D) 11/3

25. What is the derivative of $\tilde{f}(x) = \int_0^{x^2+\pi} \frac{dt}{\sin t + 1}$?

- ☐ A) $\frac{\cos x}{\sin(x^2+\pi)+1}$
- ☐ B) $\frac{2x}{\sin(x^2+\pi)+1}$
- ☐ C) $\frac{2x \cos x}{\sin(x^2+\pi)+1}$
- ☐ D) $\int_0^{2x} \frac{dt}{\sin t + 1}$

26. If two lines $y=x$ and $y= x - 4$ are tangent to a circle at $(2, 2)$ and $(4, 0)$, respectively, then what is the equation of the circle?

- ☐ A) $(x - 2)^2 + y^2 = 4$
- ☐ B) $(x - 4)^2 + (y - 2)^2 = 4$
- ☐ C) $(x - 3)^2 + (y - 1)^2 = 2$
- ☐ D) $(x - 1)^2 + (y + 1)^2 = 10$

27. Which one of the following is the set of all critical numbers of $f(x) = \frac{1}{3}x^3 - |4x - 1|$?

- ☐ A) $\{1/4, 2\}$
- ☐ B) $\{-2, 1/4, 2\}$
- ☐ C) $\{-2, 2\}$
- ☐ D) $\{1/4\}$

28. Which one of the following is equal to $\int_0^{\frac{\pi}{2}} \frac{x - \sin x}{\sec x} dx$?

- ☐ A) $\frac{\pi-3}{2}$
- ☐ B) $\frac{\pi-1}{2}$

- ☐ C) $\frac{3-\pi}{2}$
- ☐ D) $\frac{\pi+3}{2}$

29. Which one of the following is true?

- ☐ A) a polynomial can have infinitely many vertical asymptotes
- ☐ B) the graph of a rational function can never cross its horizontal asymptote.
- ☐ C) the graph of $f(x) = \frac{3x-1}{x-1}$ has no horizontal asymptote.
- ☐ D) the graph of $f(x) = \frac{x^3-x}{x^2-x}$ has no vertical asymptote.

30. The simplified form of the derivative of $f(x) = \frac{1+\sin x}{\cos x}$ is

- ☐ A) $\sec x + \tan x$
- ☐ B) $\frac{1+\sin x}{\cos^2 x}$
- ☐ C) $\frac{1}{1+\tan x}$
- ☐ D) $\frac{\cos x}{\sin^2 x}$

31. If Q_i , D_i and P_i are respectively the i^{th} quartile, decile and percentile of a data arranged in an increasing order, then which one of the following is necessarily true?

- ☐ A) $Q_2 = \frac{Q_1+Q_3}{2}$
- ☐ B) $D_3 > P_{25}$
- ☐ C) $P_{25} > Q_1$
- ☐ D) $Q_2 = \text{mean of the data}$

32. If $f(x) = x^2\sqrt{2x+12}$, what is the slope of the tangent line to the graph of f at $x=2$?

- ☐ A) -4
- ☐ B) 2
- ☐ C) 18
- ☐ D) 17

33. If $f(x) = e^{2x} \sin x$, then $f'(x)$ is equal to

- ☐ A) $3e^{2x} \sin x - 4e^{2x} \cos x$
- ☐ B) $4e^{2x} \sin x + 2e^{2x} \cos x$
- ☐ C) $e^{2x} (3\sin x + 4\cos x)$
- ☐ D) $e^{2x} (4\sin x - 3\cos x)$

35. What is the sum of the series $\sum_{n=1}^{\infty} (-1)^n 3^{-2n}$?

- ☐ A) -1/8
- ☐ B) -0.13
- ☐ C) -0.1
- ☐ D) 1/8

37. A company produced 25,000 bulbs and randomly tested 2% of the product. Among the tested bulbs, if 40 have defect of type D_1 , 60 have defect of type D_2 and 25 have both types of defects, what is the probability that a bulb produced by the company had none of the defects?

- ☐ A) 0.95
- ☐ B) 0.80
- ☐ C) 0.85
- ☐ D) 0.20

39. Suppose $AX=b$, where A is a 3×3 matrix, $b=(b_1, b_2, b_3)^T$ and $X=(x, y, z)^T$. Which one of the following is necessarily true about this system of linear equations?

- ☐ A) The system has a solution only when $\det(A) \neq 0$.
- ☐ B) The Cramer's rule is suitable to solve the system if two rows of A are identical
- ☐ C) If $\det(A) \neq 0$ and the second column of A is a multiple of b , then $x = 0$
- ☐ D) If $b = 0$, then $x=(0, 0, 0)^T$ is the only solution of the system.

40. If $y = \sin(3x^2)$, then the simplified form of $\frac{d^2y}{dx^2}$ is :

- ☐ A) $-6\sin(3x^2)$
- ☐ B) $\cos(6x) - 6\sin(3x^2)$
- ☐ C) $6 \cos(3x^2) + 36x^2 \sin(3x^2)$
- ☐ D) $x^2 \cos(3x^2) + 6\sin(3x^2)$

41. If S is a set with 10 elements and $A \subseteq S$, what is the probability that A has 3 or more elements?

- ☐ A) 7/10
- ☐ B) 8/11
- ☐ C) 121/128
- ☐ D) 7/128

42. If $a_n = \left(\frac{n+3}{n+1}\right)^n$, then the limit of the sequence $\{a_n\}_{n=1}^{\infty}$ is equal to :

- ☐ A) 1
- ☐ B) $1/2 e$
- ☐ C) e^2
- ☐ D) $+\infty$

43. Which one of the following is equal to $\lim_{x \rightarrow \infty} \left(\frac{x}{x+2} \right)^{-3x}$?

- ☐ A) e^6
- ☐ B) e^{-3}
- ☐ C) $e^{-3/2}$
- ☐ D) e^{-6}

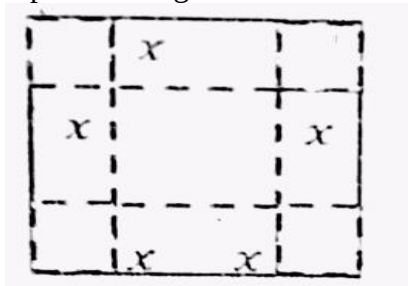
44. If a box with square base open top is made from $1,200\text{cm}^2$ material, what is the largest volume of the box in cm^3 ?

- ☐ A) 4000
- ☐ B) 8000
- ☐ C) 15,000
- ☐ D) 3000

45. Which one of the following is true about the graph of $f(x) = \frac{x^2-1}{x-x^2}$?

- ☐ A) $x = 0$ and $x = 1$ are its vertical asymptotes.
- ☐ B) $y = 1$ is its horizontal asymptote
- ☐ C) $y = x - 1$ is its oblique asymptote
- ☐ D) it is almost the same as the horizontal line $y = -1$ as $x \rightarrow \pm\infty$

47. Suppose that equal squares are cut from each of the four corners of a square cardboard whose sides are 72 centimetres long. [see the figure below.] The resulting flaps are then folded up to form a box without a top. How long should be each of the four squares that has to be cut off to maximize the volume of the box?



- ☐ A) 6cm
- ☐ B) 12cm
- ☐ C) 15cm
- ☐ D) 24cm

48. What is the area of the region between the graph of $f(x) = -x^2 + 4x - 3$ and the x axis from $x = 0$ to $x = 3$?

- ☐ A) $-2/3$
- ☐ B) $2/3$
- ☐ C) $4/3$
- ☐ D) $8/3$

49. Which one of the following is equal to $\int \frac{x + \ln(x+1)}{(x+1)^2} dx$?

- ☐ A) $\ln(x+1) + \frac{x}{x+1} + c$
- ☐ B) $(x+1)^2 + \frac{1}{x+1} + C$
- ☐ C) $(x+1)^2 - \frac{x}{x+1} + c$
- ☐ D) $\frac{x \ln(x+1)}{x+1} + c$

50. If $F(x) = f(2x+2)g(1-x^2)$, with $f(2) = -3$, $f'(2) = 4$, $g(1) = -5$, and $g'(1) = 1$, then what is the actual value of $F'(0)$?

- ☐ A) -40
- ☐ B) -20
- ☐ C) 0
- ☐ D) 19

51. Suppose that an airplane is descending at a speed of 50 miles per hour at an angle of 30° below the horizontal line. What is the x and y components, respectively of the velocity of the plane in terms of mile?

- ☐ A) $25\sqrt{3}, 25$
- ☐ B) $-25, 50\sqrt{3}$
- ☐ C) $25, -25\sqrt{3}$
- ☐ D) $-25\sqrt{3}, -25$

52. If a point (2, 5) is reflected under a line to the point(-3, 1), what is the line of reflection?

- ☐ A) $2x + 3y = 7$
- ☐ B) $x + 3y = 7$
- ☐ C) $8y + 10x = 19$
- ☐ D) $2x + 3y + 5 = 0$

53. Let the equation $x^2 + 2x + y^2 = 8$ represents a circle. Then which one of the following lines cut the circle at exactly two points?

- ☐ A) $4x + 3y + 19 = 0$
- ☐ B) $3x + 4y + 14 = 0$
- ☐ C) $2y = 5x + 43$
- ☐ D) $2x = y - 50$

54. Suppose $\vec{A} = 2\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{B}$ is a vector in space such that $|\vec{B}| = \vec{A} \cdot \vec{B}$ if $\vec{u}$ is a unit vector in the direction of $\vec{B}$, then $|\vec{A} + \vec{u}|^2$ is equal to

- ☐ A) 16
- ☐ B) 12
- ☐ C) 10
- ☐ D) 14

55. If $A = (-2, 3)$, $B = (3, 1)$ and C is any other point on the plane, then which one of the following is the coordinate form of $\overrightarrow{AC} - \overrightarrow{BC}$?

- ☐ A) $(-5, 2)$
- ☐ B) $(5, -2)$
- ☐ C) $(1, 4)$
- ☐ D) $(-1, -4)$

56. What is the equation of a line that passes through the point $(-1, -2)$ and parallel to the vector $(1, -1)$?

- ☐ A) $2x - y = 1$
- ☐ B) $x + y - 1 = 0$
- ☐ C) $x - 2y = 3$
- ☐ D) $y - 2x + 1 = 0$

57. Suppose P and Q are point in space such that the midpoint of PQ is on the negative z -axis and the distance between P and Q is 6. If $P = (2, -1, 0)$, then what is the coordinate of Q ?

- ☐ A) $(-2, 1, 4)$
- ☐ B) $(2, -1, 6)$
- ☐ C) $(2, -1, -6)$
- ☐ D) $(-2, 1, -4)$

59. An observer on level ground is at a distance $10\sqrt{3}$ m from a building. The angles of elevation to the bottom of the windows on the second and third floors are 30° and 60° , respectively. What is the distance h between the bottoms of the windows?

[you may use the values: $\sin 30^\circ = \cos 60^\circ = 1/2$ and $\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$]

- ☐ A) 15m
- ☐ B) 20m
- ☐ C) $15\sqrt{3}$ m
- ☐ D) 32m

61. What is the image of the line given by $(x, y) = (-1, 0) + t(3, 6)$, $t \in \mathbb{R}$, under the transition that takes $(1, 0)$ to $(0, 1)$ following by the reflection about the line $y = 2x$?

- ☐ A) $y = 2x + 3$
- ☐ B) $y = 2x - 3$
- ☐ C) $y = 2x + 6$
- ☐ D) $y = 2x - 5$

HSBSS, 2016

62. If $\theta = 2 \arctan (1/2)$, then which one of the following is equal to $\sec(\theta)$?

- ☐ A) $35/3$
 - ☐ B) $4/5$
 - ☐ C) $5/3$
 - ☐ D) $5/4$
-

63. If a translation T takes the circle $x^2 + y^2 - 2x + 6y + 3 = 0$ into the circle whose equation is $(x + 2)^2 + (y - 4)^2 = 7$, then what is the image of the origin under T ?

- ☐ A) $(-3, 7)$
 - ☐ B) $(1, 2)$
 - ☐ C) $(1, -3)$
 - ☐ D) $(-2, 4)$
-

65. Suppose $\vec{A} = 3\vec{i} - 4\vec{j}$ and $\vec{B}$ is a vector in the xy - plane such that the angle between $\vec{A}$ and $\vec{B}$ is $\frac{\pi}{3}$, if $\vec{u}$ is the unit vector in the direction of $\vec{B}$, then $\vec{A} \cdot (\vec{A} - 2\vec{u})$ is equal to:

- ☐ A) 20
 - ☐ B) 5
 - ☐ C) 15
 - ☐ D) 30
-

Part Three

1.. What is the maximum value of $f(x)=2x^2 - x^4 - 4$ on $[0, 2]$?

- ☐ A) -3
 - ☐ B) 3
 - ☐ C) -4
 - ☐ D) 12
-

3. Which of the following is convergent sequence?

- ☐ A) $1, 1/2, 1, 1/3, 1, 1/4, \dots$
 - ☐ B) $\{(-1)^n\}_{n=1}^{\infty}$
 - ☐ C) $\left\{10^{109} - \frac{1}{100^n}\right\}_{n=1}^{\infty}$
 - ☐ D) $\left\{n + \sin \frac{1}{n}\right\}_{n=1}^{\infty}$
-

4. Let $f(x)=2x(x^2 + 1)^4$, then which of the following is an anti- derivative of $f(x)$?

- ☐ A) $2x/5 (x^2 + 1)^5 + c$
- ☐ B) $2/5 (x^2 + 1)^5 + c$
- ☐ C) $x/5 (x^2 + 1)^5 + 1$
- ☐ D) $1/5 (x^2 + 1)^5 - 1$

5. Which one of the following is an arithmetic sequence?

- ☐ A) 3, 5, 7, 9, 11,...
- ☐ B) 3, 6, 12, 24, 28,...
- ☐ C) -3, 6, -9, 12, -15,...
- ☐ D) 1, 3, 6, 10, 15, 21,...

$$f(x) = \begin{cases} a \frac{\sin x}{x}, & x < 0 \\ e^{2x} - 2, & x \geq 0 \end{cases}$$

7. Let if f is continuous at $x=0$, then what is the value of a ?

- ☐ A) $1/2$
- ☐ B) 2
- ☐ C) $-1/2$
- ☐ D) -2

8. The Ozone level (in ppb- parts per billion) on a sunny day in metropolitan area is given by formula $p(t) = 80 + 12t - t^2$ where t is time in hours and $t = 0$ corresponds to 9A.M. what is the rate of increase of Ozone level after 3-hrs(i.e. at 12 A.M)?

- ☐ A) 6 ppb
- ☐ B) 12 ppb
- ☐ C) 107 ppb
- ☐ D) 113 ppb

9. Which of the following functions is one to one correspondence?

- ☐ A) $f: R \rightarrow R, f(x) = \tan x$, where R' is domain of f
- ☐ B) $g: R \rightarrow R, g(x) = 2^x$
- ☐ C) $h: [-\infty, \infty) \rightarrow [0, \infty) : h(x) = x^2$
- ☐ D) $r: [0, \infty) \rightarrow [0, \infty): r(x) = x + 5$

10. Different codes, each of which consisting of five characters, are to be generated in such a way that the first two characters are any of the English capital letters (A to Z) and the remaining three are any of the digits (0, 1, 2, ..., 9). How many distinct codes can be generated ?

- ☐ A) 468,000
- ☐ B) 260
- ☐ C) 676,000
- ☐ D) $26! \times 10!$

11. A certain meeting hall has 20 rows of seats. There are 20 seats in the first row, 22 seats in the second row, 24 seats in the third row, and so on. How many seats are there on the last (20th) row of the hall?

- ☐ A. 46

- ☐ B) 58
- ☐ C) 76
- ☐ D) 5240

12. Two perpendicular lines l_1 and l_2 are intersecting at $(-1, 2)$. If the angle of inclination of l_1 is 45° , then what is the equation of l_2 ?

- ☐ A) $y = -x + 3$
- ☐ B) $y = x + 3$
- ☐ C) $y = -x + 1$
- ☐ D) $y = x + 1$

13. What is the area of the region between the graph of $y = \sin x$ and x-axis where $0 \leq x \leq 2\pi$?

- ☐ A) 4
- ☐ B) 4π
- ☐ C) 2
- ☐ D) 2π

14. The following is the set of data representing the average mark of students: 91, 89, 93, 91, 87, 94, 92, 85, 91, 90, 96, 93, 89. Then which one of the following statements is true about the data?

- ☐ A) the median is 90.5
- ☐ B) the upper quartile is 92
- ☐ C) the range of marks is 11
- ☐ D) the mean is 91.5

15. If $f(x) = \ln(\sqrt{x^2 + 1})$, which of the following is equal to $f'(x)$?

- ☐ A) $\frac{x}{\sqrt{x^2 + 1}}$
- ☐ B) $\frac{x}{x^2 + 1}$
- ☐ C) $\frac{2x}{\sqrt{x^2 + 1}}$
- ☐ D) $\frac{2x}{x^2 + 1}$

17. What is the principal argument of $(5 + 5i)^{11}$?

- ☐ A) $\pi/2$
- ☐ B) $2\pi/3$
- ☐ C) $\pi/4$
- ☐ D) $3\pi/4$

21. If f and g are continuous on $\mathbb{R}$ and $a, b \in \mathbb{R}$, which one of the following is necessarily true?

- ☐ A) if $\int_a^b f(x)dx = \int_a^b g(x)dx$, then $f(x) = g(x)$ for all $x \in [a, b]$
- ☐ B) if $f'(x) = g'(x)$ for all $x \in [a, b]$, then $\int_a^b f(x)dx = \int_a^b g(x)dx$
- ☐ C) if $f(x) \geq 3$ for all $x \in [-2, 2]$, then $\int_{-2}^2 f(x)dx \geq 12$
- ☐ D) $\int_a^b f(x)dx = \int_b^a f(x)dx$

22. Consider the following system of equations:
$$\begin{cases} ax + by = 2 \\ x + 3y + 2z = 0 \\ 2x + y + z = 0 \end{cases}$$
 if the determinant of the coefficient matrix is 2, then what is the solution set of the system?

- ☐ A) $\{(1, 3, -5)\}$
- ☐ B) $\{(1/a, 1/b, 0)\}$
- ☐ C) $\{(-2, -6, 10)\}$
- ☐ D) $\emptyset$

23. For what values of a and b is the function $f(x) = \begin{cases} 1 - 3x^2, & \text{for } x \leq 1 \\ ax + b, & \text{for } x > 1 \end{cases}$ is differentiable at $x=1$?

- ☐ A) $a = 6, b = 0$
- ☐ B) $a = -3, b = 1$
- ☐ C) $a = 0, b = -2$
- ☐ D) $a = -6, b = 4$

24. Which one of the following is equal to $\int \frac{\ln x + x^2 e^x}{x} dx$?

- ☐ A) $\frac{1}{2} \ln^2 x + e^x(x^2 - 1) + c$
- ☐ B) $\frac{1}{2} \ln^2 x + e^x(x - 1) + c$
- ☐ C) $\frac{1}{x^2} \ln x + e^x(x - 1) + c$
- ☐ D) $\frac{-1}{x^2} \ln x + e^x(x^2 - 1) + c$

26. A city has two daily newspapers, X and Y. the following information was obtained from the survey of 100 residents of the city: 35 people subscribe to X, 60 people subscribe to Y and 20 subscribe to both newspapers. Then how many of the people in the survey do not subscribe to either of the newspapers?

- ☐ A) 5
- ☐ B) 25
- ☐ C) 40

☐ D) 55

27. Which one of the following is necessarily true?

- ☐ A) if $f'(x) = 0$ for all x in the interval I , then $f(x) = 0$ for all x in I .
- ☐ B) if $f(x) = x^2 \sin x + 5$, then there is $c \in (0, \pi)$ such that $f'(c) = 0$
- ☐ C) $f(x) = e^x$ is increasing on $(-\infty, \infty)$
- ☐ D) $f(c) = 0$, then f attains its maximum or minimum value at $x = c$

28. The derivative of the function $f(x) = \int_{-x}^x \frac{dt}{1+t}$ is :

- ☐ A) $\frac{2}{1-x^2}$
- ☐ B) $\ln |1 + x|$
- ☐ C) $\frac{1}{1+x}$
- ☐ D) $\ln \left| \frac{1+x}{1-x} \right|$

29. If $p(x) = 3x^2$ and $q(x) = x^2 + x$, then what is the solution set of $\frac{p(x)}{3q(x)} - \frac{1}{x} = \frac{1}{q(x)}$?

- ☐ A) $\{-1, 2\}$
- ☐ B) $\{2\}$
- ☐ C) $\{-3, 2\}$
- ☐ D) $\{-3\}$

31. What is the sum of all multiples of 3 between 20 and 200?

- ☐ A) 7,227
- ☐ B) 6,570
- ☐ C) 6,150
- ☐ D) 5,166

32. The inverse of the function defined by $g(x) = \frac{2x}{x+3}$ is equal to:

- ☐ A) $g^{-1}(x) = -\frac{2x}{x-3}$
- ☐ B) $g^{-1}(x) = -\frac{3x}{x-2}$
- ☐ C) $g^{-1}(x) = -\frac{x+3}{2x}$
- ☐ D) $g^{-1}(x) = -\frac{x+2}{3x}$

$$A = \begin{pmatrix} 0 & x & 0 \\ 1 & -1 & 1 \\ 0 & y & -1 \end{pmatrix} \quad \text{and} \quad A^{-1} = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 0 & 0 \\ 2 & 0 & -1 \end{pmatrix}$$

33. If , then what are the value of x and y?

- ☐ A) $x = 3, y = -2$
- ☐ B) $x = 2/3, y = 1/3$
- ☐ C) $x = -3, y = 2$
- ☐ D) $x = 1/3, y = 2/3$

34. If $f(2) = -e$, $f'(2) = 4$, $g(1) = -5$, $g'(1) = 1$ and $F(x) = f(2x + 2)(g(x - x^2))$, then what is the value of $F'(0)$?

- ☐ A) 19
- ☐ B) 0
- ☐ C) -20
- ☐ D) -40

35. A ladder 6m long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate (speed) of 0.5 m/s, how fast the angle between the top of ladder and the wall changing when the angle is $\pi/4$ rad?

- ☐ A) $\sqrt{2}/2$ rad/sec
- ☐ B) $\sqrt{2}/12$ rad/sec
- ☐ C) $\sqrt{2}/6$ rad/sec
- ☐ D) $\sqrt{2}/3$ rad/sec

36. The value(s) of x where the graph of the function $y = \frac{x^2 - 1}{x^3}$ crosses its horizontal asymptote is (are):

- ☐ A) $x = -2$
- ☐ B) $x = -1$ and $x = 1$
- ☐ C) $x = 0$
- ☐ D) $x = -\sqrt{2}$ and $x = +\sqrt{2}$

$$A = \begin{pmatrix} 3 & -2 & 8 \\ 0 & 6 & 7 \\ 0 & 4 & 5 \end{pmatrix},$$

37. If then $\det(A^T A)$ is equal to :

- ☐ A) 12
- ☐ B) 36
- ☐ C) 30
- ☐ D) 15

38. Suppose that the first 3 letters (A, B, and C) and number digits are to be used to form car plates in a small town. How many different plates can be formed in total that contains 1, 2 or 3 letters and then followed by 3 digits?

- ☐ A) 3,000

- ☐ B) 27,000
- ☐ C) 39,000
- ☐ D) 100,000

39. A ball is thrown vertically from ground up to a height of 16m. Each time it drops h meters, it rebounds $0.80h$ m. noting that the ball travels every height of h twice, what is the total vertical distance travelled by the ball before it comes to rest?

- ☐ A) 40m
- ☐ B) 80m
- ☐ C) 160m
- ☐ D) 320m

40. Suppose that a function f has a property that $f(x + y) = f(x)f(y)$ for all x and y and that $f(0) = 2$, $f'(0) = 1$. Then which one of the following represents the formula for the derivative $f'(x)$?

- ☐ A) $f(x) = 2f'(x) + 1$
- ☐ B) $f'(x) = f(x) + 2f'(x)$
- ☐ C) $f'(x) = f(x) + 2$
- ☐ D) no such function

$$\begin{cases} x + y - z = 1 \\ x + 2y - 3z = 1 \\ 2x + 3y - 4z = 2 \end{cases} ?$$

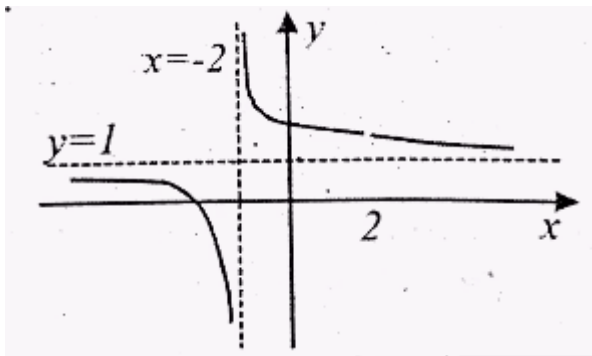
43. What is the solution of the system

- ☐ A) $\{(0, 2, 1)\}$
- ☐ B) $\{(1 - k, 2k, k) \mid k \in \mathbb{R}\}$
- ☐ C) $\{(2k + 1, -k, k) \mid k \in \mathbb{R}\}$
- ☐ D) $\emptyset$

44. Let $f(x) = \begin{cases} 3 - e^{2x}, & \text{if } x < 0.5 \\ \frac{2^x - 5}{x + 1}, & \text{if } x \geq 0.5 \end{cases}$, if c is a zero of f , that is, $f(c) = 0$, then which one of the following intervals must contain c ?

- ☐ A) $(-\infty, 0]$
- ☐ B) $[0, 1]$
- ☐ C) $[1, 2]$
- ☐ D) $[2, 3]$

45. Which of the following functions could most likely be drawn as in the Figure 1 below?



- ☐ A) $\frac{x+3}{x+2}$
- ☐ B) $\frac{x^2-2x}{x^2-4}$
- ☐ C) $\frac{-x^2-x+6}{x^2-4}$
- ☐ D) $\frac{x^2+x-6}{x^2-4}$

46. A measurement is grouped in to five class intervals with the following frequency distribution.

Class Intervals	5-15	15-25	25-35	35-45	45-55
Frequency	22	40	68	50	20

What are the first quartile Q_1 & the 75th percentile P_{75} of the measurement?

- ☐ A) $Q_1 = 20, P_{75} = 40$
- ☐ B) $Q_1 = 22, P_{75} = 40$
- ☐ C) $Q_1 = 20, P_{75} = 39$
- ☐ D) $Q_1 = 22, P_{75} = 39$

47. The number of shoes s that a factory can produce per day is a function of the number of hours t it operates:

$$s(t) = 40t \text{ for } 0 \leq t \leq 12$$

the daily cost c , in Birr, to manufacture s shoes is given by the function

$$c(s) = 0.1s^2 + 90s + 800$$

If the factory operates for 10 hours, what is the cost it incurs in producing as much shoes it can with in this time?

- ☐ A) Birr 400
- ☐ B) Birr 1,600
- ☐ C) Birr 52,800
- ☐ D) Birr 124,600

48. Three persons P_1 , P_2 and P_3 are firing at a target independently and have a probability 0.7, 0.5 and 0.4, respectively, of hitting the target. What is the probability that at least one of them hits the target?

- ☐ A) 0.95
- ☐ B) 0.85

- ☐ C) 0.91
 - ☐ D) 0.99
-

49. Which one of the following is equal to $\lim_{x \rightarrow \infty} \left(\frac{3x}{3x+2} \right)^{-3x}$?

- ☐ A) e^2
 - ☐ B) e^{-3}
 - ☐ C) e^{-2}
 - ☐ D) e^3
-

50. The total cost (in Birr) of producing x radio sets per day is given by the expression $0.25x^2 + 35x + 25$ and the price per set at which they may be sold is given by $50 - (0.5)x$. What should be the daily output to obtain a maximum total profit?

- ☐ A) 50 sets per day
 - ☐ B) 23 sets per day
 - ☐ C) 10 sets per day
 - ☐ D) no such case
-

51. A company manufactures x computer sets per month. The monthly marginal profit (in Birr) is given by: $p(x) = 165 - 0.1x$, for $0 \leq x \leq 400$. The company is currently manufacturing 10 sets of computer per month, but is planning to increase production. What is the total change in the monthly profit if the monthly production increased to 60 sets?

- ☐ A) Birr 50
 - ☐ B) Birr 1,865
 - ☐ C) Birr 8,075
 - ☐ D) Birr 18,635
-

52. If angle θ is an acute angle of a right triangle, what is the length of the side adjacent to θ , given the hypotenuse has 6 units length and $\sec \theta = 10/3$?

- ☐ A) 1.8 units
 - ☐ B) 2 units
 - ☐ C) 18 units
 - ☐ D) 20 units
-

53. If $A = (1, -2)$, $B = (-3, 2)$ and $\vec{v}$ is a position vector such that $2\vec{v} + \overrightarrow{AB} = \vec{0}$, then $\vec{v}$ is equal to

- ☐ A) $(2, 0)$
 - ☐ B) $(-1, 0)$
 - ☐ C) $(-2, 2)$
 - ☐ D) $(2, -2)$
-

54. If each of the compound propositions $P \vee Q$, $P \Rightarrow R$ and $\neg R$ is true, then which one of the following is True?

- ☐ A) P
- ☐ B) Q
- ☐ C) $Q \Rightarrow P$
- ☐ D) $P \wedge \neg R$

55. If $\vec{A} = 4\vec{i} - 3\vec{j}$ and $\vec{u}$ is a unit vector such that $|\vec{A} + \vec{u}|^2 = 27$, then the cosine of the angle between $\vec{A}$ and $\vec{u}$ is equal to

- ☐ A) 0.1
- ☐ B) 0.2
- ☐ C) 0.3
- ☐ D) 0.4

56. What is the possible value of x that solves the equation: $\sin^{-1} x + \cos^{-1}\left(\frac{3}{5}\right) = \pi$

- ☐ A) $\pi/3$
- ☐ B) $3/5$
- ☐ C) $5\pi/2$
- ☐ D) $4/5$

57. If $P = (3, \alpha - 1, \alpha + 2)$ and $Q = (2\alpha + 1, 3, 3\alpha)$ are points in space, what should be the value(s) of α so that the distance between the two points is 6?

- ☐ A) $\alpha = 1$, or $\alpha = 5$
- ☐ B) $\alpha = \frac{4+2\sqrt{7}}{3}$, or $\alpha = \frac{4-2\sqrt{7}}{3}$
- ☐ C) $\alpha = -1$, or $\alpha = 3$
- ☐ D) $\alpha = -3$, or $\alpha = 2$

58. If $(-1, 2, 2)$ and $(1, 0, -2)$ are end points of a diameter of a sphere, then which one of the following is true about the sphere?

- ☐ A) $(0, 1, 0)$ is a point on the sphere
- ☐ B) the equation of the sphere is $x^2 + (y-1)^2 + z^2 = 6$
- ☐ C) the equation of the sphere is $x^2 + (y-2)^2 + z^2 = 6$
- ☐ D) the radius of the sphere is 6

59. Suppose that the equation $x^2 + y^2 + z^2 + 2x + 8z = 6(y + 1)$, represents a sphere. Where is the point $(1, -1, 4)$ located relative to the sphere?

- ☐ A) inside the sphere
- ☐ B) on the sphere
- ☐ C) at the center of the sphere
- ☐ D) outside the sphere

HSBSS, 2016

60. Two ships, one with angle of depression 60° due east and the other with 30° due west are observed from a plane 1,000 metres above sea. If the two ships are on the same line, what is the distance between the two ships?

[you may use the values: $\sin 30^\circ = \cos 60^\circ = 1/2$ and $\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$]

- ☐ A) $\frac{1}{\sqrt{3}} 4,000m$
 - ☐ B) 2,000m
 - ☐ C) $\sqrt{3} 500m$
 - ☐ D) $\frac{1}{\sqrt{3}} 600m$
-

61. What is the image of the ellipse $(x - 1)^2 + 4y^2 = 1$ under the translation that takes $(1, 1)$ to $(0, 2)$ followed, by the reflection through the x-axis?

- ☐ A) $x^2 + 4(y - 1)^2 = 1$
 - ☐ B) $4x^2 + (y - 1)^2 = 1$
 - ☐ C) $x^2 + 4(y + 1)^2 = 1$
 - ☐ D) $4x^2 + (y + 1)^2 = 1$
-

62. What is the amplitude and period, respectively, of the graph of $f(x) = -6 \sin x \cos x$?

- ☐ A) 3, π
 - ☐ B) 6, π
 - ☐ C) 3, $\pi/2$
 - ☐ D) 6, 2π
-

63. What is the work done (in joule) when a force of 50 Newton is used to pull a cart 20 meters along path if the force is at an angle of 60° ?

[Remember that: Work=(Force) x (distance travelled)]

- ☐ A) 360
 - ☐ B) 500
 - ☐ C) 760
 - ☐ D) $1500\sqrt{2}$
-

64. Suppose L is the line through the center of the sphere $x^2 + y^2 + (z - 2)^2 = 9$ and the line intersects the sphere at $(1, 2, 4)$. What is the cosine of the angle between L and positive z-axis?

- ☐ A) $2/3$
 - ☐ B) $1/3$
 - ☐ C) $3/5$
 - ☐ D) $4/5$
-

65. Consider the formula for a natural number $n \in \mathbb{N}$:

$$2 + 4 + 8 + \dots + 2^n = 2^{n+1} + 1$$

To proof this formula a person has used the following argument.

“Assume the formula is true for $n=k$, for some $k \in \mathbb{N}$. Then the person has shown that the formula is also true for $n=k + 1$. And then, the person has concluded that, by the Principle of Mathematical induction, the formula is true for all natural numbers $n \in \mathbb{N}$.” which one of the following statements is true about the above argument?

- ☐ A) the formula holds true though it does not work for $n=1$
 - ☐ B) since the left-hand-side is an even number and the right hand side an odd number, the Principle of Mathematical Induction is False.
 - ☐ C) this is one example where the Principle of Mathematical Induction fails to work.
 - ☐ D) the above formula does not work for all natural numbers $n \in \mathbb{N}$
-

Part Four

1. Which one of the following is a one to one correspondence function from $A=[0, 1]$ to $B=[1, 2]$?

- ☐ A) $f(x) = x$
 - ☐ B) $f(x) = 1/3x^3 + 1$
 - ☐ C) $f(x) = 2x + 1$
 - ☐ D) $f(x) = x^2 + 1$
-

2. What is the solution set of $\frac{1-\frac{1}{x}}{1-\frac{1}{x^2}} = 3x^2 - \frac{x}{1+\frac{1}{x}} ?$

- ☐ A) $\{-1, 1/3\}$
 - ☐ B) $\{1/3\}$
 - ☐ C) $\{3, -1/3\}$
 - ☐ D) $\{-1/3\}$
-

3. Suppose $f(x) = \frac{Q(x)}{x(x^2-1)}$, where $Q(x)$ is a quadratic function. Which one of the following is necessarily true about the graph of f .

- ☐ A) $x = 0$, $x = 1$ and $x = -1$ are vertical asymptotes of graph of f .
 - ☐ B) the graph of f doesn't intersect with its horizontal asymptote.
 - ☐ C) the vertical asymptotes of the graph of f is only $x = -1$
 - ☐ D) the vertical asymptotes of graph of f are $x = 1$ and $x = -1$ if $Q(x) = 2x^2$.
-

4. Which one of the following is equation of a circle whose center is on y -axis and radius is 3?

- ☐ A) $x^2 + y^2 + 6y = 0$
- ☐ B) $(x-2)^2 + y^2 = 9$

- ☐ C) $x^2 + (y-2)^2 = 9$
☐ D) $x^2 - 2x + y^2 = 8$

5. The planet Mercury's orbit around the sun is an ellipse with eccentricity 0.206, length of the major axis 1.16×10^8 km and the sun at one focus. What is the max distance from Mercury to the sun?

- ☐ A) 6.99×10^8 km
☐ B) 6.99×10^7 km
☐ C) 9.66×10^7 km
☐ D) 9.66×10^8 km

8. What is the contrapositive of "If $x \in \mathbb{N}$, then x is integer and $x > 0$."?

- ☐ A) if x is not integer or $x < 0$, then $x \in \mathbb{N}$
☐ B) if x is integer or $x > 0$, then $x \in \mathbb{N}$
☐ C) if x is not integer or $x \leq 0$, then $x \notin \mathbb{N}$
☐ D) if $x \notin \mathbb{N}$, then x is not integer and $x \leq 0$

10. The following is sample frequency distribution of data with variable x .

X	3	5	6	7
Frequency	2	5	2	1

What are the mean ($\bar{x}$) and variance (δ^2) of the data?

- ☐ A) $\bar{x} = 5, \delta^2 = 0.7$
☐ B) $\bar{x} = 6, \delta^2 = 1.4$
☐ C) $\bar{x} = 6, \delta^2 = 0.7$
☐ D) $\bar{x} = 5, \delta^2 = 1.4$

11. A box contains 10 items of which 3 are defective. If 2 items are randomly taken out of the box, what is the probability that both items are not defective?

- ☐ A) $7/10$
☐ B) $4/7$
☐ C) $7/15$
☐ D) $49/100$

12. Items produced by accompany are subjected to two kinds of defects D1 and D2. Out of the total product, 5% have the defect D1, 10% have defect D2, and 2% have both defects. What is the probability that a randomly selected item has neither defect D1 nor defect D2?

- ☐ A) 0.13
☐ B) 0.5
☐ C) 0.98
☐ D) 0.87

13. If $A = \begin{pmatrix} 2 & 7 \\ 1 & 3 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$, then $(AB)^{-1}$ is equal to:

- ☐ A) $\begin{pmatrix} 4 & -3 \\ 4 & -5 \end{pmatrix}$
- ☐ B) $\begin{pmatrix} -2 & 5 \\ 2 & -4 \end{pmatrix}$
- ☐ C) $\begin{pmatrix} -3 & 11 \\ 1 & -3 \end{pmatrix}$
- ☐ D) $\begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$

14. Let $A = \begin{pmatrix} 0 & \alpha & \beta \\ 2 & 2 & 1 \\ 3 & 1 & 2 \end{pmatrix}$, $b = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$, and $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$. If $\det(A) = 3$, then what is the solution set of the system $AX = b$?

- ☐ A) $\{(6, -2, -8)^T\}$
- ☐ B) $\left\{\left(0, \frac{1}{\alpha}, \frac{5}{\beta}\right)^T\right\}$
- ☐ C) $\{(-3, 1, 4)^T\}$
- ☐ D) $\emptyset$

15. In set of complex numbers, what is the solution set of $x^2 + 4x + 5 = 0$?

- ☐ A) $\{\emptyset\}$
- ☐ B) $\{1 - 2i, 1 + 2i\}$
- ☐ C) $\{2 - i, 2 + i\}$
- ☐ D) $\{-2 - i, -2 + i\}$

17. If $\{A_n\}_{n=1}^{\infty}$ is an arithmetic sequence such that its 1st term $A_1 = -5$ and its 5th term $A_5 = 15$, then its 11th term A_{11} is equal to:

- ☐ A) 40
- ☐ B) 50
- ☐ C) 45
- ☐ D) 55

18. What is the sum of all multiples of 4 that are between 30 and 301?

- ☐ A) 12,882
- ☐ B) 11,288
- ☐ C) 6,288
- ☐ D) 6,882

19. The left hand side limit $\lim_{x \rightarrow 0^-} \frac{xe^x - |x|}{x}$ is equal to:

- ☐ A) 0

- ☐ B) 2
 - ☐ C) 1
 - ☐ D) does not exist
-

20. Which one of the following is equal to $\lim_{x \rightarrow 0} \frac{2x + \tan x}{x \sec x}$?
- ☐ A) 2
 - ☐ B) 0
 - ☐ C) 1
 - ☐ D) 3
-

21. Which one of the following is true about the horizontal asymptote(s) of the graph of $y = \frac{|x|+2}{x}$?
- ☐ A) $y = 2$ is the only horizontal asymptote of the graph.
 - ☐ B) $y = 1$ & $y = -1$ are the horizontal asymptote of the graph.
 - ☐ C) $y = 2$ & $y = -2$ are the horizontal asymptote of the graph.
 - ☐ D) $y = 1$ is the only horizontal asymptote of the graph.
-

22. If $f(x) = 2x^2 - 3x$, then $\lim_{x \rightarrow 1} \frac{f(x) - f(1)}{x - 1}$ is equal to:
- ☐ A) 1
 - ☐ B) -1
 - ☐ C) 7
 - ☐ D) ∞
-

23. If $f(x) = e^{3x} \cos x - \frac{x+\pi}{x^2+2}$, then $f'(0)$ is equal to:
- ☐ A) $3 - \pi/2$
 - ☐ B) $3/2$
 - ☐ C) $7/4$
 - ☐ D) $5/2$
-

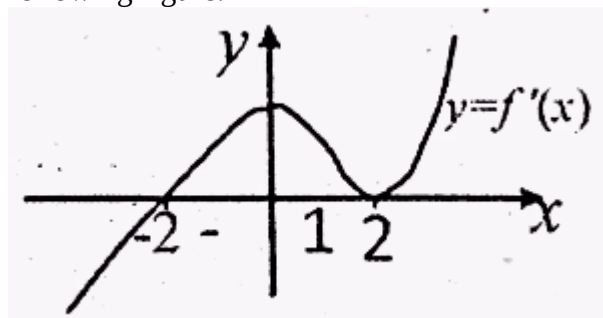
24. If $f(x) = \ln \sqrt{x^2 - 5}$, then which of the following is equal to $f'(x)$?
- ☐ A) $\frac{x}{x^2-5}$
 - ☐ B) $\frac{-x}{\sqrt{x^2-5}}$
 - ☐ C) $\frac{2x}{x^2-5}$

☐ D) $\frac{-x}{x^2-5}$

25. Let f be twice differentiable function on $\mathbb{R}$. which one of the following is necessarily true?

- ☐ A) if $f'(c) = 0$, at some $c \in \mathbb{R}$, then f has relative extreme at $x = c$.
☐ B) if $f'(x)$ is increasing, then graph of $y = f(x)$ is concave upward.
☐ C) if $f'(x) = 0$ for all $x \in \mathbb{R}$, then $f(x) = 0$ for all $x \in \mathbb{R}$
☐ D) if $f(x)$ is increasing, then $f''(x) = 0$ for all $x \in \mathbb{R}$

26. Suppose $f(x)$ is differentiable on $(-\infty, \infty)$ and the graph of its derivative, $y = f'(x)$ is as shown in the following figure.



- ☐ A) $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$
☐ B) $f(x)$ has a local extreme value at $x = 2$
☐ C) $f(x)$ has a local minimum value at $x = -2$
☐ D) $f(x)$ has a local maximum value at $x = 0$

27. A closed cylindrical can is to be made to hold 1000 cm^3 of oil. What are the dimensions (radius r and height h) that will minimize the total surface area of the can?

- ☐ A) $r = \frac{\sqrt[3]{\sqrt{50}}}{\pi}$, $h = 2 \frac{\sqrt[3]{\sqrt{50}}}{\pi}$
☐ B) $r = \frac{\sqrt[3]{\sqrt{500}}}{\pi}$, $h = 2 \frac{\sqrt[3]{\sqrt{500}}}{\pi}$
☐ C) $r = \sqrt[3]{\frac{50}{\pi}}$, $h = 2 \sqrt[3]{\frac{50}{\pi}}$
☐ D) $r = \sqrt[3]{\frac{500}{\pi}}$, $h = 2 \sqrt[3]{\frac{500}{\pi}}$

28. If $f(x) = 3x^2\sqrt{x^3 - 1}$, then which one of the following is anti-derivative of $f(x)$?

- ☐ A) $\frac{3x}{2}(x^3 - 1)^{3/2} + c$

- ☐ B) $\frac{2}{3}(x^3 - 1)^{3/2}$
- ☐ C) $\frac{3}{2}(x^3 - 1)^{3/2}$
- ☐ D) $\frac{2}{3}(x^3 - 1)^{3/2} + c$

29. Which one of the following is equal to $\int \frac{(\ln x)^2 + x^2 \cos x}{x} dx$?

- ☐ A) $1/x^2 \ln x + x \sin x - \cos x + c$
- ☐ B) $1/3 (\ln x)^3 + x \sin x - \cos x + c$
- ☐ C) $1/3 (\ln x)^3 + x \sin x + \cos x + c$
- ☐ D) $1/x^2 \ln x + x \sin x + \cos x + c$

30. The volume of the solid generated when the region bounded between graph

of $y = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 4, & 2 \leq x \leq 3 \end{cases}$ and x-axis is rotated about the x-axis:

- ☐ A) $32\pi/5$
- ☐ B) $112\pi/5$
- ☐ C) $112\pi/3$
- ☐ D) $64\pi/5$

31. If $f: A \rightarrow B$ and $g: B \rightarrow C$ are functions, then which one of the following is true about the composition function?

- ☐ A) domain of $g \circ f \subseteq$ domain of f
- ☐ B) range of $g \circ f \not\subseteq$ range of g
- ☐ C) domain of $g \circ f \not\subseteq$ domain of f
- ☐ D) range of $g \circ f \subseteq$ range of f

32. If the point (3, -2) is on the graph of $y = f(x)$, then which point is on the graph of $y = f^{-1}(x)$?

- ☐ A) (1/3, -2)
- ☐ B) (3, -1)
- ☐ C) (-2, 3)
- ☐ D) (3, -1/2)

33. The equation of the line that passes through (2, -1) and perpendicular to the line $3x + 4y = 6$ is:

- ☐ A) $-4x + 3y = 5$
- ☐ B) $4x - 3y = 5$
- ☐ C) $4x + 3y = 11$
- ☐ D) $-4x + 3y = -11$

35. The n^{th} term of the sequence: 1, -4, 9, -16,... is:

- ☐ A) $a_n = (-2)^n$
 - ☐ B) $a_n = (-1)^n n^2$
 - ☐ C) $a_n = (-1)2^n n^2$
 - ☐ D) $a_n = (-1)^{n-1} n^2$
-

36. Which one of the following is true about the horizontal asymptote(s) of the graph of $y = \frac{|x|+2}{x}$

- ☐ A) $y = 2$ is the only horizontal asymptote of the graph.
 - ☐ B) $y = 1$ and $y = -1$ are the horizontal asymptotes of the graph.
 - ☐ C) $y = 2$ and $y = -2$ are the horizontal asymptote of the graph.
 - ☐ D) $y = 1$ is the only horizontal asymptote of the graph.
-

37. For any $n \times n$ square matrix A , which of the following is true?

- ☐ A) $\text{Det}(A) = -\text{det}(A^T)$, where A^T is the transpose of A
 - ☐ B) If k is a scalar, then $\text{det}(kA) = k^n \text{det}(A)$
 - ☐ C) If B is a matrix obtained from A by interchanging of two rows of A , then $\text{det}(B) = \text{det}(A)$
 - ☐ D) If A is invertible, then $\text{det}(A) = \text{det}(A^{-1})$
-

38. The solution of the system of linear equation of
$$\begin{cases} x - 3y - 2z = 6 \\ 2x - 4y - 3z = 8 \\ -3x + 6y + 8z = -5 \end{cases}$$
 is:

- ☐ A) $x = -1, y = -3, z = -2$
 - ☐ B) $x = -1, y = -3, z = 2$
 - ☐ C) $x = 1, y = -3, z = 2$
 - ☐ D) $x = 1, y = 3, z = -2$
-

39. there are three children in a room, ages three, four, and five. If a four- year- old child enters the room then which one of the following is true?

- ☐ A) Mean age will stay the same but the standard deviation will increase.
 - ☐ B) Mean age will stay the same but the standard deviation will decrease.
 - ☐ C) Mean age and standard deviation will increase.
 - ☐ D) Mean age ad standard deviation will stay the same.
-

40. In how many more ways can 4 people be arranged in a row than if they were arranged in a circle?

- ☐ A) 1
 - ☐ B) 6
 - ☐ C) 18
 - ☐ D) 12
-

41. Two machines A and B work independently. The probability that both machines A and B work is 0.4. If the conditional probability that machine B works given that machine A works is 0.5, then the conditional

probability that machine A works given that machine B works is

- ☐ A) 0.8
 - ☐ B) 0.3
 - ☐ C) 0.5
 - ☐ D) 0.7
-

42. Which of the following is true about the function f defined by $f(x) = x^2 + e^{2x}$?

- ☐ A) f is decreasing for $x = 0$
 - ☐ B) f is increasing for $x = 0$
 - ☐ C) f has a relative minimum $x = 0$
 - ☐ D) f has a relative maximum at $x = 0$
-

43. The value of $\int_0^1 (x+1)e^{(x^2+2x)} dx$ is:

- ☐ A) $\frac{e^3}{2}$
 - ☐ B) $\frac{e^4 - e}{2}$
 - ☐ C) $\frac{e^3 - 1}{2}$
 - ☐ D) $e^3 - 1$
-

44. $\int_0^3 (x+1)^{\frac{1}{2}} dx =$ _____.

- ☐ A) $21/2$
 - ☐ B) $14/3$
 - ☐ C) 7
 - ☐ D) $16/3$
-

45. Given $f(x) = \begin{cases} x+1, & \text{for } x < 0 \\ \cos \pi x, & \text{for } x \geq 0 \end{cases}$, then $\int_{-1}^1 f(x) dx =$ _____

- ☐ A) $\frac{1}{2} + \frac{1}{\pi}$
 - ☐ B) $\frac{1}{2} - \frac{1}{\pi}$
 - ☐ C) $1/2$
 - ☐ D) $-1/2$
-

46. The sum of $\sum_{n=0}^{\infty} 5 \left(\frac{2}{3}\right)^n$ is _____.

- ☐ A) 0
 - ☐ B) 15
 - ☐ C) 10/3
 - ☐ D) 5
-

47. In which interval the sequence $\left\{ \frac{(-1)^n}{3n} \right\}_{n=1}^{\infty}$ is bounded?

- ☐ A) $\left[\frac{-1}{9}, \frac{1}{12} \right]$
 - ☐ B) $\left[\frac{-1}{3}, \frac{1}{6} \right]$
 - ☐ C) $\left[\frac{-1}{6}, \frac{1}{3} \right]$
 - ☐ D) $\left[\frac{-1}{12}, \frac{1}{9} \right]$
-

48. Which one of the following is true about the function given by the

$$f(x) = \begin{cases} \frac{x^2}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} ?$$

formula

- ☐ A. f is continuous except at $x = 0$
 - ☐ B. f has an infinite discontinuity at $x = 0$
 - ☐ C. f is continuous everywhere
 - ☐ D. f has $x = 0$ as a vertical asymptote
-

49. $\lim_{x \rightarrow \infty} x \sin \frac{1}{x}$ is equal to :?

- ☐ A) 0
 - ☐ B) 1
 - ☐ C) ∞
 - ☐ D) -1
-

50. $\frac{d}{dx} (\ln e^{2x})$ is equal to :?

- ☐ A. $\frac{1}{e^{2x}}$
- ☐ B. $\frac{2}{e^{2x}}$

- ☐ C) $2x$
- ☐ D) 2

51. If $f(x) = 2 + |x - 3|$ for all x , then the value of the derivative $f'(x)$ at $x = 3$

- ☐ A) -1
- ☐ B) Does not exist
- ☐ C) 1
- ☐ D) 2

52. The graph of $y = 5x^4 - x^5$ has a point of inflection at:

- ☐ A) $(3, 162)$ only
- ☐ B) $(4, 256)$ only
- ☐ C) $(0, 0)$ only
- ☐ D) $(0, 0)$ and $(3, 162)$

53. Let the angle between $\vec{V} = -2\vec{i} - \vec{j} + 2\vec{k}$ and $\overrightarrow{PQ}$ be 60° , where P and Q are space. If $\vec{V} \cdot \overrightarrow{PQ} = 2$, then what is the distance between P and Q ?

- ☐ A) $3/4$
- ☐ B) $4/5$
- ☐ C) $4/3$
- ☐ D) $5/4$

54. Consider the following assertion of a person and his proof "If x and y are equal positive integers, then $x + y = y$."

Proof: the following steps and reasons are used to prove the assertion.

Step	Reason
1. $x = y$	Given hypothesis
2. $x^2 = xy$	Multiply both sides of (1) by x
3. $x^2 - y^2 = xy - y^2$	Subtract y^2 from both sides of (2)
4. $(x - y)(x + y) = (x - y)y$	Factor both sides of (3)
5. $x + y = y$	Divide both sides of (4) by $x - y$

Step 5 completes the proof.

Which one of the following is true about this proof?

- ☐ A) It is a correct direct proof of the assertion.
- ☐ B) It follows the technique of a proof by contradiction because the steps lead to a contradiction.
- ☐ C) The proof is invalid because Step 4 does not lead to Step 5.
- ☐ D) The proof is invalid because Step 4 does not follow from Step 3.

55. Which one of the following is a valid assertion that can be proved by the principle of mathematical induction?

- ☐ A) $2^n > 10^n$ for every integer n such that $n \geq 6$.

- ☐ B) $r^2 > 0$ for every real number r such that $r \geq 1$.
- ☐ C) $n^2 + 10n > 2n^2$ for every natural number $n \geq 1$.
- ☐ D) $2^n > 8n$ for every integer n such that $n \geq 3$.

56. The image of a figure with vertices $A(1, 2)$, $B(3, 6)$, $C(-1, 2)$, and $D(-2, -2)$ after reflection across the x -axis is:

- ☐ A) $A'(1, -2)$, $B'(-3, -6)$, $C'(1, -2)$, and $D'(2, 2)$
- ☐ B) $A'(-1, 2)$, $B'(-3, 6)$, $C'(1, -2)$, and $D'(2, -2)$
- ☐ C) $A'(1, 2)$, $B'(3, -6)$, $C'(-1, -2)$, and $D'(-2, 2)$
- ☐ D) $A'(1, -2)$, $B'(3, 6)$, $C'(-1, 2)$, and $D'(-2, -2)$

57. If $\vec{V} = \vec{AB} + 3\vec{BA}$, where A and B are distinct points in the coordinate plane, then which one of the following is equal to $3\vec{V}$?

- ☐ A. $6\vec{AB}$
- ☐ B. $-6\vec{AB}$
- ☐ C. $12\vec{AB}$
- ☐ D. $-12\vec{AB}$

58. If $\vec{U}$ is a unit vector in the direction of $\vec{A}$ and $|\vec{A}| = 4$, then $\vec{A} \cdot \vec{U}$ is equal to _

- ☐ A) $1/4$
- ☐ B) 4
- ☐ C) $1/2$
- ☐ D) 2

59. If $\vec{A}$ and $\vec{B}$ are parallel vectors with opposite directions and $|\vec{B}| = |2\vec{A}|$, then equal to:

- ☐ A. $\vec{A}$
- ☐ B. $-\vec{A}$
- ☐ C. $-3\vec{A}$
- ☐ D. $3\vec{A}$

60. If $\cot(\theta) = 2$, then which of the following is equal to $\csc(\theta)$?

- ☐ A) $\sqrt{5}$
- ☐ B) $2/\sqrt{5}$
- ☐ C) $1/\sqrt{5}$

☐ D) 1/2

61. What is the amplitude and period, respectively, of the graph of $f(x) = 4\sin(x/3) \cos(x/3)$?

- ☐ A) 4, $\pi/3$
☐ B) 2, 3π
☐ C) 2, $2\pi/3$
☐ D) 4, 3π

62. A boat on a sea sailed from its station toward North with constant speed of 80 km/h. Another boat from the same station sailed 60° NE (North East) with constant speed of 100km/h. If the two boats started sailing at the same time, what is the straight distance between them after they have sailed for just 30 minutes?

- ☐ A) $10\sqrt{42}$ km
☐ B) 90 km
☐ C) $10\sqrt{41}$ km
☐ D) $10\sqrt{21}$ km

$$\vec{U} = \begin{pmatrix} 1 \\ k \\ -3 \end{pmatrix} \text{ and } \vec{V} = \begin{pmatrix} 2k \\ -5 \\ 4 \end{pmatrix}$$

63. What is the value k, for which the two vectors are perpendicular?

- ☐ A) 4
☐ B) -4
☐ C) 3
☐ D) -3

64. If one of the end point of the line segment is (3, 2, -4) and the mid-point is (4, 1, -2), then the coordinate of the other end point is:

- ☐ A) (5, 0, 0)
☐ B) (2, 0, 5)
☐ C) (5, 1, 2)
☐ D) (3, 1, 0)

65. What is the value of $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$.

- ☐ A) $\pi/4$
☐ B) $\pi/2$
☐ C) $-\pi/4$
☐ D) $-\pi/2$
-

$$A^{-1} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 0 & 4 & 5 \end{bmatrix}$$

1. If $A = (a_{ij})$ 3×3 is a square matrix with $A^{-1} = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 1 & 3 \\ 0 & 4 & 5 \end{bmatrix}$, then what is the cofactor of a_{23} ?

- ☐ A) -3/14
- ☐ B) -2/7
- ☐ C) 2/7
- ☐ D) -3/7

2. A team of 10 researchers consist 4 biologists and 6 chemists. If 3 persons are chosen randomly from the team, what is the probability that at least one is a biologist?

- ☐ A) 2/3
- ☐ B) 2/5
- ☐ C) 5/6
- ☐ D) 7/10

3. The probability that an electronic device produced by a company does not function properly is equal to 0.1. If 2 devices are bought, then what is the probability that at least one device function properly?

- ☐ A) 0.81
- ☐ B) 0.09
- ☐ C) 0.18
- ☐ D) 0.99

4. Two machines A and B produce respectively 60% and 40% of the total number of items of a factory. The percentages of defective output of these machines are 2% and 5%, respectively. If an item is selected at random, then what is the probability that the item is defective?

- ☐ A) 0.032
- ☐ B) 0.07
- ☐ C) 0.32
- ☐ D) 0.426

5. In how many ways can a committee of 3 members be formed from 7 candidates?

- ☐ A) 7
- ☐ B) 21
- ☐ C) 28
- ☐ D) 35

6. The following is a frequency distribution table of a grouped data with variable X.

X	3-7	8-12	13-17	18-22
Frequency	4	6	8	2

What is the mean ($\bar{x}$) and the variance (s^2) of the data respectively?

- ☐ A) $\bar{x} = 12, s^2 = 21$
- ☐ B) $\bar{x} = 12, s^2 = 25$
- ☐ C) $\bar{x} = 13, s^2 = 9$
- ☐ D) $\bar{x} = 13, s^2 = 16$

7. The expenditure of 100 families is given below.

Expenditure	0-9	10-19	20-29	30-39	40-49
No. of Families	14	23	f_1	21	f_2

The mode of the data is 23.5, what are the values of f_1 and f_2 .

- ☐ A) $f_1 = 27, f_2 = 15$
- ☐ B) $f_1 = 15, f_2 = 27$
- ☐ C) $f_1 = 25, f_2 = 17$
- ☐ D) $f_1 = 17, f_2 = 25$

8. The first group of 10 children has a mean weight of 15.6kg, the second group of another 10 children has a mean weight of 16kg, and the third group of children has a mean weight of 20kg. If the mean weight of all the children is 17kg, what is the total number of children in all of the three group?

- ☐ A) 28
- ☐ B) 29
- ☐ C) 30
- ☐ D) 32

11. Which of the following functions is a one to one correspondence?

- ☐ A) $f: [0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = |x|$
- ☐ B) $f: \mathbb{R} \rightarrow [0, \infty)$ defined by $f(x) = x^2$
- ☐ C) $f: \mathbb{R} \rightarrow [0, \infty)$ defined by $f(x) = 3^x$
- ☐ D) $f: (0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = \log_2 x$

12. If $f(x) = \sqrt{x^3}$ and $(f \circ g)(x) = \sqrt[4]{x}$, then what is the value of $g(8)$?

- ☐ A) $\sqrt[3]{2}$
- ☐ B) 2
- ☐ C) $\sqrt{2}$
- ☐ D) $2\sqrt{2}$

13. Which one of the following is the inverse of $f(x) = 8x^3 + 2$?

- ☐ A) $f^{-1}(x) = \frac{1}{8x^3+2}$
- ☐ B) $f^{-1}(x) = \frac{1}{2}\sqrt[3]{x-2}$
- ☐ C) $f^{-1}(x) = 8x^{-3} + 2$

☐ D) $f^{-1}(x) = \frac{1}{8}\sqrt[3]{x-2}$

$$\frac{1}{1+\frac{1}{x}} - \frac{1}{1-\frac{1}{x}} = \frac{x+\frac{1}{x}}{x-\frac{1}{x}}?$$

14. What is the solution set of

- ☐ A) { }
☐ B) {-1}
☐ C) {1}
☐ D) {-1, 1}

15. Which one of the following is true about the graph of $f(x) = \frac{x^3}{x^3(x-1)}$

- ☐ A) the vertical asymptotes of the graph are $x = 0$ and $x = 1$
☐ B) a horizontal asymptote of the graph is $y = 1$
☐ C) the graph intersects its horizontal asymptote at a point $(-1, 0)$
☐ D) the graph intersects the vertical line $x = 1$ at a point $(1, 2)$

16. What is the equation of a line that passes through point (a, a) in xy -plane if it is parallel to a line that passes through points (a, b) and (b, a) where $a \neq b$?

- ☐ A) $y = x$
☐ B) $y = -x$
☐ C) $y = -x + 2a$
☐ D) $y = 2x - a$

17. What are the values of the center (C) and radius (r) of a circle $x^2 + y^2 - 4x + 6y = 5$?

- ☐ A) $C = (-2, 3), r = 3\sqrt{2}$
☐ B) $C = (2, -3), r = 3\sqrt{2}$
☐ C) $C = (2, -3), r = 2\sqrt{3}$
☐ D) $C = (-2, 3), r = 2\sqrt{3}$

21. Let f be differentiable function with $f(1) = -1$ and $f'(1) = 1$.

If $g(x) = [f(2x + 1) + 2]^2$, then what is the value of $g'(0)$?

- ☐ A) 4
☐ B) 2
☐ C) -2
☐ D) -4

22. If $f(x) = \ln(x^2 + 2)$, then what is the value of $f''(1)$?

- ☐ A) $3/2$
☐ B) $5/9$
☐ C) $2/3$
☐ D) $2/9$

23. If $x^2 + xy = 10$, then what is the value of $\frac{dy}{dx}$ when $x = 2$?

- ☐ A) $-7/2$
- ☐ B) $2/7$
- ☐ C) $3/2$
- ☐ D) $7/2$

24. What is the equation of the tangent line to the graph of $f(x) = 3x^2 + 4x - 5$ at $(1, 2)$?

- ☐ A) $10x - y - 8 = 0$
- ☐ B) $-10x + y - 8 = 0$
- ☐ C) $-10x - y - 8 = 0$
- ☐ D) $10x + y - 8 = 0$

25. If $f(x) = \pi^2 + 1$, then what is the value of $f'(x)$?

- ☐ A) $2\pi + 1$
- ☐ B) 2π
- ☐ C) 2
- ☐ D) 0

26. Suppose f is continuous on $[2, 6]$ and the only solutions of the equation $f(x) = 7$ are $x = 2$ and $x = 5$. If $f(3) = 9$, then one of the following Can Not be the value of $f(4)$?

- ☐ A) 5
- ☐ B) 7.5
- ☐ C) 8
- ☐ D) 9

27. What is the value of k so that $f(x) = \begin{cases} \frac{\tan 2x}{x}, & \text{if } x > 0 \\ k - e^x, & \text{if } x \leq 0 \end{cases}$ is continuous at $x = 0$?

- ☐ A) 2
- ☐ B) 3
- ☐ C) 1
- ☐ D) 0

28. If f is continuous at $x = 0$ and $g(x) = \sqrt{x} (2f(x) + \frac{3}{\sqrt{x}})$ for all $x > 0$, then what is the value of $\lim_{x \rightarrow 0^+} g(x)$?

- ☐ A) 0
- ☐ B) 2

- ☐ C) 3
☐ D) 5

29. What is the value of $\lim_{x \rightarrow 0} \frac{\sin x \cos 2x}{x^2 + 3x}$?

- ☐ A) 1/3
☐ B) 2/3
☐ C) 1
☐ D) 2

30. If $a \neq 0$, then what is the value of $\lim_{x \rightarrow 0} \frac{\sin x \cos 2x}{x^2 + 3x}$?

- ☐ A) $1/a^2$
☐ B) $1/2a^2$
☐ C) $1/6a^2$
☐ D) 0

31. What are the values of λ and μ so that the system $\begin{cases} x + y + z = 6 \\ x + 2y + 3z = 10 \\ x + 2y + \lambda z = \mu \end{cases}$ has infinitely many solutions?

- ☐ A) $\lambda \neq 3$ and $\mu \in \mathbb{R}$
☐ B) $\lambda = 3$ and $\mu \neq 10$
☐ C) $\lambda = 3$ and $\mu = 10$
☐ D) $\lambda \in \mathbb{R}$ and $\mu = 10$

32. When $\begin{vmatrix} a & b & c \\ a & -a & a \\ a & a & -a \end{vmatrix} = a^3$, and $a \neq 0$, what is the solution

$$\begin{bmatrix} a & b & c \\ a & -a & a \\ a & a & -a \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}?$$

of

- ☐ A) $x = 0, y = 2a, z = 2a$
☐ B) $x = 1/a, y = -2a, z = 2a$
☐ C) $x = 1/a, y = 2/a, z = 2/a$
☐ D) $x = 0, y = 2/a, z = 2/a$

36. Which of the following relations holds for the sequence: -10, -3, 4, 11, ...?

- ☐ A) $a_n = a_{n-1} - 8$

- ☐ B) $a_n = a_{n-1} + 7$
 - ☐ C) $a_n = a_{n-1} - 7$
 - ☐ D) $a_n = a_{n-1} + 8$
-

37. What is the sum of $\sum_{n=1}^{30} (-1)^n \left(\frac{1}{n} + \frac{1}{n+1} \right)$?

- ☐ A) $-29/30$
 - ☐ B) $29/30$
 - ☐ C) $30/31$
 - ☐ D) $-30/31$
-

38. Which of the following is the sum of the series $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \dots$?

- ☐ A) -5
 - ☐ B) -3
 - ☐ C) 3
 - ☐ D) 5
-

39. Suppose a radioactive material loses one third of its mass per year. If its current mass is 81g, then how much will its mass be just after 7 years?

- ☐ A) 27g
 - ☐ B) $1/27$ g
 - ☐ C) $128/27$ g
 - ☐ D) $128/81$ g
-

40. Which one of the following is a convergent sequence?

- ☐ A) $\left\{ \left(\frac{5}{3} \right)^n \right\}$
 - ☐ B) $\left\{ \frac{2n}{n+1} \right\}$
 - ☐ C) $\left\{ \frac{n^2}{n+1} \right\}$
 - ☐ D) $\left\{ \frac{(-1)^n}{3} \right\}$
-

41. What is the value of the area of the region enclosed by the graph of $f(x) = e^x$ and $g(x) = x$ between the lines $x = -1$ and $x = 1$?

- ☐ A) $\frac{e^2 - 1}{e}$
- ☐ B) $e^2 - \frac{1}{e}$

HSBSS, 2016

- ☐ C) $e^2 - \frac{1}{e} + 2$
- ☐ D) $e - \frac{1}{e} + 2$

42. If $f(0) = -1$, $f(1) = 2$ and $f'(x)$ is continuous on $[0, 1]$, then which of the following is equal

to $\int_0^1 f'(x) \sqrt{2 + f(x)} dx$?

- ☐ A) $16/3$
- ☐ B) $14/3$
- ☐ C) $8/3$
- ☐ D) $4/3$

43. Which of the following is equal to the volume of a solid generated when the region bounded by the

graph of $y = 2\sqrt{x+1}$ and x-axis, when $0 \leq x \leq 2$, rotates about the x-axis?

- ☐ A) 8π
- ☐ B) $8\pi^2$
- ☐ C) 16π
- ☐ D) $16\pi^2$

44. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 + 6t$ for time $t \geq 0$. If the particle is at position $x = 2$ at a time $t = 0$, what is the position of the particle at $t = 1$?

- ☐ A) 6
- ☐ B) 9
- ☐ C) 11
- ☐ D) 12

45. Which of the following is equal to $\int x(e^x + \sin(x^2)) dx$?

- ☐ A) $e^x(x+1) + \frac{1}{2}\sin(x^2) + c$
- ☐ B) $e^x(x-1) - \frac{1}{2}\sin(x^2) + c$
- ☐ C) $e^x(x-1) - \frac{1}{2}\cos(x^2) + c$
- ☐ D) $e^x(x-1) + \frac{1}{2}\cos(x^2) + c$

46. What is the value of $\int_0^{\ln\sqrt{3}} \frac{e^x}{e^{-x} + e^x} dx$?

- ☐ A) $\frac{1}{2} \ln 2$
- ☐ B) $\ln 4$
- ☐ C) $\ln 2$
- ☐ D) 1

47. Suppose $f: (-\infty, \infty) \rightarrow \mathbb{R}$ is differentiable and the graph of its derivative $y = f'(x)$, is as shown in the

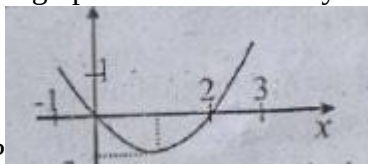


figure. Which one of the following is true about f ?

- ☐ A) f is increasing on $(1, \infty)$
- ☐ B) f is concave upward on $(0, \infty)$
- ☐ C) f has no relative maximum value
- ☐ D) f has a relative minimum value at $x = 2$

48. If $2 \leq f(x) \leq 4$ for all values of x , then the value of $f(8) - f(2)$ is between which of the following numbers?

- ☐ A) 14 and 24
- ☐ B) 12 and 24
- ☐ C) 12 and 18
- ☐ D) 8 and 10

49. Which one of the following is the set of critical numbers of $f(x) = \frac{4}{3}x^3 + |x|$?

- ☐ A) $\left\{\frac{1}{2}\right\}$
- ☐ B) $\left\{0, \frac{1}{2}\right\}$
- ☐ C) $\left\{0, -\frac{1}{2}\right\}$
- ☐ D) $\left\{-\frac{1}{2}, 0, \frac{1}{2}\right\}$

50. A tin can of volume $54\pi \text{ cm}^3$ is to be made in the form of a right circular cylinder that has both flat top and flat bottom. What is the base radius of the tin if it is to be made of the least amount of metal?

- ☐ A) 2 cm
- ☐ B) 3 cm
- ☐ C) 4 cm
- ☐ D) 6 cm

51. Air is being pumped into a spherical balloon so that its volume increase at a rate of $50 \text{ cm}^3/\text{s}$. How fast

is the radius of the balloon increasing when the diameter is 5 cm?

- ☐ A) $\frac{1}{50\pi}$ cm/s
- ☐ B) $\frac{1}{25\pi}$ cm/s
- ☐ C) $\frac{5}{\pi}$ cm/s
- ☐ D) $\frac{2}{\pi}$ cm/s

52. Which one of the following is a valid assertion that can be proved by principle of mathematical induction?

- ☐ A) the sum of any two positive rational numbers is positive.
- ☐ B) $r^2 \geq 1$, for every real number $r \geq 1$
- ☐ C) $n^2 \geq 4n$, for every integer $n \geq 4$
- ☐ D) $2^2 \leq 2^n$, for every integer $n \leq 100$

53. Consider the assertion: "The sum of positive irrational numbers is positive irrational number". Which one of the following is correct about the assertion?

- ☐ A) taking irrational numbers such as $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \dots$ and so on, if we add any two of them, the sum is irrational. Therefore, the assertion is true.
- ☐ B) the sum of $1 + \sqrt{2}$ and $1 - \sqrt{2}$ is 2, which is rational. This is a counter example that disproves the assertion.
- ☐ C) the sum of $\sqrt{7}$ and $\sqrt{2}$ is a counter example that shows the assertion is false.
- ☐ D) the assertion can be disproved by taking the sum of $1 + \sqrt{2}$ and $2 - \sqrt{2}$ as a counter example.

54. Suppose "If $x \in A$, then $y \in B$ " is a True statement. Then, which one of the following is necessarily true?

- ☐ A) $y \in B$
- ☐ B) if $x \notin A$, then $y \notin B$
- ☐ C) if $y \in B$, then $x \in A$
- ☐ D) if $y \notin B$, then $x \notin A$

55. Let $\vec{A}$ and $\vec{B}$ be vectors in space such that $\vec{A} \cdot \vec{B} = -2$ and $\vec{B} = 6\vec{i} - 7\vec{j} + \sqrt{1}$ the angle between $\vec{A}$ and $\vec{B}$, then what is the value of $|\vec{A}|$?

- ☐ A) $\frac{1}{5} \cos \theta$
- ☐ B) $\frac{1}{5 \cos \theta}$

HSBSS, 2016

- ☐ C) $-\frac{1}{5}\cos\theta$
- ☐ D) $\frac{-1}{5\cos\theta}$

56. If P(2, $\sqrt{5}$, 1) and Q(3, 0, 9) are points on a sphere whose center is on z-axis, then which one of the following point is outside of the sphere?

- ☐ A) (-4, 3, 5)
- ☐ B) (2, -2, 1)
- ☐ C) (3, 1, 1)
- ☐ D) (0, 0, 0)

57. If A(x, 0, 2), B(3, 0, 2) and C(2, $\sqrt{3}$, 2) are vertices of an equilateral triangle in space, then what is the value of x?

- ☐ A) 5
- ☐ B) 3
- ☐ C) 2
- ☐ D) 1

58. A patrol boat on a sea sailed from its station 7km to the North; and changed its course and sailed $4\sqrt{2}$ km in the direction of 45° South-East. What is the shortest (straight) distance the boat should travel in order to return to its station?

- ☐ A) 5km
- ☐ B) 7km
- ☐ C) $5\sqrt{2}$ km
- ☐ D) $5+\sqrt{2}$ km

59. Let $\overrightarrow{PQ}$ be a vector with initial point P = (1, 5) and terminal point Q = (4, 0). If $\vec{v} = x\vec{i} + 2\vec{j}$ is parallel to $\overrightarrow{PQ}$, then what is the value of x?

- ☐ A) -6/5
- ☐ B) -2/5
- ☐ C) -3
- ☐ D) 3

60. For two non-zero vectors $\vec{a}$ and $\vec{b}$ if $\|\vec{a} + \vec{b}\| = \|\vec{a}\|$ which of the following is true?

- ☐ A) $2\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{b}$
- ☐ B) $2\vec{a} + \vec{b}$ and $\vec{b}$ are perpendicular
- ☐ C) $2\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$ are not parallel

☐ D) $\vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{b} = 0$

61. A line given by the vector equation $(x, y) = (-t, 6+2t)$, $t \in \mathbb{R}$, is tangent to a circle at a point $(1, 4)$. What is the radius of the circle if its center is on the y-axis?

- ☐ A) $\sqrt{5}$
☐ B) $\frac{1}{2}\sqrt{5}$
☐ C) $2\sqrt{5}$
☐ D) $\sqrt{10}$

62. What is the translation vector $u = (h, k)$ so that the equation $x^2 + 2y^2 + 6x - 8y + 15 = 0$ is transformed to an equation of the form $x^2 + 2y^2 + d = 0$, where d is constant?

- ☐ A) $u = (-3, 2)$
☐ B) $u = (3, -2)$
☐ C) $u = (-2, 3)$
☐ D) $u = (2, -3)$

63. If $\cot \theta = \sqrt{8}$ and θ is first quadrant angle, then what is the value of $\csc \theta$?

- ☐ A) $\frac{1}{3}$
☐ B) 3
☐ C) $\sqrt{8}/3$
☐ D) $1/\sqrt{8}$

64. If $\theta = \arctan(2)$, then what is the value of $\sin(2\theta)$?

- ☐ A) $\frac{2}{5}$
☐ B) $\frac{4}{5}$
☐ C) $\frac{4}{\sqrt{5}}$
☐ D) $\frac{2}{\sqrt{5}}$

65. Which one of the following is true?

- ☐ A) the amplitude of $f(x) = \sin 3x$ is 3
☐ B) the period of $f(x) = 2\sin 4x$ is π
☐ C) the period of $f(x) = 3\cos(0.5x - \pi/3)$ is 4π
☐ D) the amplitude of $f(x) = -5\cos(3x + 2) - 2$ is 7
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