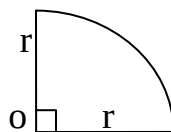


AMHARA NATIONAL REGIONAL STATE EDUCATION BUREAU
MATHEMATICS MODEL EXAMINATION FOR GRADE – 12
NATURAL SCIENCE STUDENTS / 2015 E.C.

TIME ALLOWED – 3:00 HOURS

I. CHOOSE THE CORRECT ANSWER FROM THE GIVEN
ALTERNATIVES.

1. The simplified form of the expression $\frac{8\sqrt{40} + 14\sqrt{160}}{16\sqrt{90}}$ is:
- A. $\frac{4}{3}$ B. $\frac{3}{2}$ C. $\frac{3\sqrt{10}}{2}$ D. $3\sqrt{5}$
2. Which of the following is the solution set of $1 + 2|x + 2| = 6$?
- A. $\left\{\frac{1}{2}, \frac{-9}{2}\right\}$ B. $\left\{\frac{3}{2}, \frac{-11}{2}\right\}$ C. $\{-5, -9\}$ D. $\emptyset$
3. Let $A \cap B' = \{1, 2, 3\}$ and $A' \cap B = \{0, 4\}$. Then which of the following is equal to $A \Delta B$?
- A. $\{0, 4\}$ B. $\{1, 2, 3\}$ C. $\{0, 1, 2, 3, 4\}$ D. $\emptyset$
4. Let $f(x) = \frac{x}{x-3}$ and $g(x) = \frac{x-1}{3x}$. What is the domain of $\frac{f}{g}(x)$?
- A. $\mathbb{R} \setminus \{0, 3\}$ B. $\mathbb{R} \setminus \{0, 1\}$ C. $\mathbb{R} \setminus \{1, 3\}$ D. $\mathbb{R} \setminus \{0, 1, 3\}$
5. A 300 cm long piece of wire is bent into a quarter of a circle center at O and radius 'r' as shown in the figure below:



What is the radius of the circle?

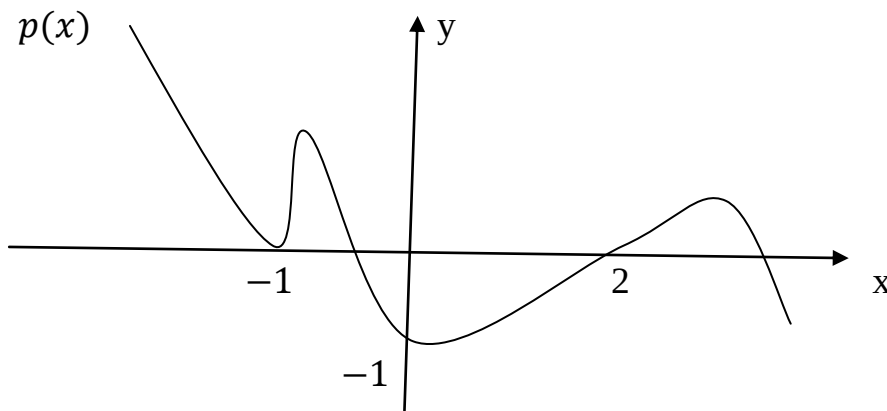
- A. $\frac{300}{\pi+4} \text{ cm}$ B. $\frac{300}{\pi+2} \text{ cm}$ C. $\frac{600}{\pi+4} \text{ cm}$ D. $\frac{600}{\pi+2} \text{ cm}$
6. The solution set of the equation $\left(\frac{4}{9}\right)^x \left(\frac{27}{8}\right)^{x-1} = \log_8 4$ is:
- A. $\{2\}$ B. $\{4\}$ C. $\{7\}$ D. $\{9\}$

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7. Which one of the following properties is **NOT** true about measures of location?

- A. The median is affected by extreme values.
- B. The mean is affected by extreme values.
- C. The median is unique for a given data set.
- D. The mode is not affected by extreme values.

8. Suppose $p(x)$ is a polynomial function whose graph is given below.



Which one of the following is **NOT** true about $p(x)$?

- A. $x^2 + 3x + 2$ is a factor of $p(x)$.
- B. The constant term of $p(x)$ is -1 .
- C. The possible degree of $p(x)$ is 5.
- D. The leading coefficient of $p(x)$ is positive.

9. The solution set of the quadratic inequality $4x^2 - x \leq 3x^2 + 2$ is equal to:

- A. $(-\infty, -1] \cup [2, \infty)$
- B. $[-1, 2]$
- C. $[2, \infty)$
- D. $(-\infty, -2) \cup (1, \infty)$

10. Which of the following is **NOT** true about the line $l : 2x - 2y = 10$?

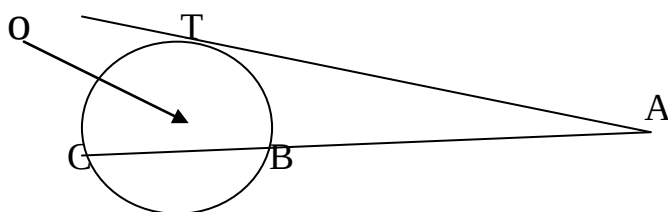
- A. It is perpendicular to the line $y = -x + 1$.
- B. It is parallel to the line $y = x + 1$.
- C. Its slope is -1 .
- D. Its y-intercept is -5 .

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11. If $\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta > 0$, then what is $\sec\theta$?

- A. $\frac{1}{4}$ B. 2 C. $\frac{5}{2}$ D. 4

12. $\overline{AT}$ and $\overline{AC}$ are tangent and secant lines to a circle O respectively in the figure below.



If $AT = 6\text{cm}$ and $BC = 9\text{cm}$, then what is the length of $\overline{AC}$?

- A. 3cm B. 6cm C. 7cm D. 12cm

13. What is the volume (in cm^3) of a hemisphere of radius 6 cm?

- A. 72π B. 108π C. 144π D. 288π

14. Let $f(x) = ax + b$ and $f(f(x)) = 4x - 1$. Then what are the possible values of b ?

- A. $-\frac{1}{3}$ and 1 C. -2 and 1
 B. 2 and -2 D. 2 and $-\frac{1}{3}$

15. Which one of the following functions is a one to one correspondence?

- A. $f: [0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = |x|$
 B. $f: \mathbb{R} \rightarrow [0, \infty)$ defined by $f(x) = x^2$
 C. $f: \mathbb{R} \rightarrow [0, \infty)$ defined by $f(x) = 3^x$
 D. $f: (0, \infty) \rightarrow \mathbb{R}$ defined by $f(x) = \log_2 x$

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16. If $p(x) = 3x^2$ and $q(x) = x^2 + x$, then what is the solution set of the equation

$$\frac{p(x)}{3q(x)} - \frac{1}{x} = \frac{1}{q(x)} ?$$

- A. $\{-1, 2\}$ B. $\{2\}$ C. $\{1, -2\}$ D. $\{0, -1\}$

17. Which one of the following is **NOT** true about the graph of the rational function $f(x) = \frac{x}{x^2+x-2}$?

- A. The x-axis is its horizontal asymptote.
B. The lines $x = -2$ and $x = 1$ are its vertical asymptotes.
C. It passes through the point $(0, 0)$.
D. It is symmetric with respect to the y- axis.

18. If the equation $x^2 + y^2 - 6y + k = 0$ represents a circle of radius 2 units, then the value of k is:

- A. -5 B. -4 C. 4 D. 5

19. What is the length of the minor axis of the ellipse with equation

$$\frac{(x+2)^2}{9} + \frac{(y-1)^2}{4} = 16 ?$$

- A. 8 B. 12 C. 16 D. 24

20. Which of the following is **NOT** true about the parabola whose equation is given by $2y^2 + 3x + 8y - 4 = 0$?

- A. The line $y = 2$ is its line of symmetry.
B. The length of its latus rectum is $\frac{3}{2}$ units.
C. It has a focus at the point $\left(\frac{29}{8}, -2\right)$.
D. The line $x = \frac{35}{8}$ is its directrix.

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27. How many three digit even natural numbers can be formed using the digits 1, 2, 3, 4, 5 if the digits are used at most once?
- A. 30 B. 40 C. 50 D. 60
28. The probability that Aster goes to France is $\frac{3}{5}$ and the probability that Marta goes to France is $\frac{3}{4}$. What is probability that at least one of them goes to France ?
- A. $\frac{9}{20}$ B. $\frac{17}{20}$ C. $\frac{9}{10}$ D. $\frac{19}{20}$
29. Let $A = \begin{pmatrix} 2 & 1 & 2 \\ 4 & 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} a & 2 \\ 1 & 3 \\ 2 & 1 \end{pmatrix}$ be two given matrices such that $C = AB$ and $C_{21} = 4$. Then what is the value of a ?
- A. -2 B. -1 C. 1 D. 2
30. If the system $\begin{cases} x + 2y - 3z = 5 \\ 2x - y - z = 8 \\ kx + y + 2z = 14 \end{cases}$ has a unique solution, then the value of k is:
- A. $k \neq -3$ B. $k \neq 3$ C. $k = -3$ D. $k = 3$
31. What is the value of $24A^{-1} + 2B^T$ if $A = \begin{pmatrix} 4 & -6 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & -2 \\ 1 & -3 \end{pmatrix}$?
- A. $\begin{pmatrix} 9 & 8 \\ -6 & -2 \end{pmatrix}$ C. $\begin{pmatrix} 9 & 2 \\ 0 & -2 \end{pmatrix}$
- B. $\begin{pmatrix} 2 & 4 \\ -10 & -9 \end{pmatrix}$ D. $\begin{pmatrix} 9 & -6 \\ 8 & -2 \end{pmatrix}$
32. What is the principal argument of z, if $z = -2 + 2\sqrt{3}i$?
- A. $\pi/4$ B. $\pi/3$ C. $2\pi/3$ D. $3\pi/4$

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33. Suppose $A = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$ and X is a 2×2 matrix such that $AX - A^T = 2A$.

Then, matrix X is represented by:

A. $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

C. $\begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$

B. $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

D. $\begin{pmatrix} 3 & 3 \\ 6 & 9 \end{pmatrix}$

34. If $|x + xi| = \sqrt{6}$, then the value of x is equal to:

A. $\pm \sqrt{3}$

B. $\pm \sqrt{6}$

C. ± 3

D. ± 6

35. $\sum_{n=2}^{30} \frac{-1}{n(n-1)}$ is equal to:

A. $\frac{-31}{30}$

B. $\frac{-29}{30}$

C. $\frac{29}{30}$

D. $\frac{31}{30}$

36. What is the n^{th} term of an arithmetic progression whose 2^{nd} term is twice of the 6^{th} term and the sum of these two terms is 96?

A. $A_n = 80 - 8n$

C. $A_n = 72 - 6n$

B. $A_n = 96 - 8n$

D. $A_n = 102 - 9n$

37. Which one is **true** about the geometric series $1 - x + x^2 - x^3 + x^5 + \dots$ that converges to $\frac{3}{4}$?

A. The common ratio, $r = \frac{1}{3}$.

C. The sum of the first two terms is $\frac{2}{3}$.

B. The fourth term, $G_4 = -\frac{1}{9}$.

D. The fifth term, $G_5 = -\frac{1}{27}$.

38. Let $f(x) = \frac{\sqrt{9x+81} + \sqrt{x+9}}{\sqrt{\frac{1}{9}x+1}}$. Then $\lim_{x \rightarrow -9} f(x)$ is equal to:

A. -18

B. -12

C. $3\sqrt{2}$

D. 12

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46. Suppose that x is differentiable with respect to y and y is differentiable with respect to x , and $x^3y + xy^3 = 10$. What is the value of $\frac{dy}{dx}$ at $x = 1$?
- A. $-\frac{13}{14}$ B. $-\frac{14}{13}$ C. $\frac{13}{14}$ D. $\frac{14}{13}$
47. Air is being pumped into a spherical balloon so that its volume increases at a rate of $100\pi \text{ cm}^3/\text{sec}$. How fast is the radius (in cm/sec) of the balloon increasing when the radius is 5 cm?
- A. 1 B. 2 C. 3 D. 4
48. What is the absolute maximum value of the function $f(x) = x^3 - 3x^2$ on the interval $[-1, 3]$?
- A. -4 B. 0 C. 2 D. 3
49. Which of the following is correct about the graph of the function $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x$?
- A. Its local maximum value is 0. C. It is decreasing on $[3, \infty)$.
B. It is concave upward in $(-\infty, -2)$. D. Its inflection point is $(2, \frac{2}{3})$.
50. What is the area A (in sq. units) of the region enclosed by the graph of $y = x$ and $y = x^2 - 3x + 3$?
- A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. 1 D. $\frac{4}{3}$
51. $\int_0^{\pi/2} \sin^2 x \cos x \, dx$ is equal to:
- A. 0 B. $\frac{1}{3}$ C. 1 D. Does n't exist
52. What is the value of k for which $\int_{\frac{1}{2}}^2 (x^2 + k) dx = \int_{\frac{1}{2}}^2 \left(\frac{2}{x^2}\right) dx$?
- A. $\frac{1}{4}$ B. $\frac{3}{8}$ C. $\frac{1}{2}$ D. 4

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53. $\int \left(\frac{x^2+1}{x} \right) dx$ is equal to:

A. $\frac{x^2}{2} + \ln|x| + c$

C. $x^2 + 2\ln|x| + c$

B. $x^2 - \ln|x| + c$

D. $x + 2\ln|x| + c$

54. Which one of the following is an antiderivative of $f(x) = \frac{1}{x^2 + 3x + 2}$?

A. $\ln|x + 1| + c$

C. $\ln \left| \frac{x+2}{x+1} \right| + c$

B. $\ln|x^2 + 3x + 2| + c$

D. $\ln \left| \frac{x+1}{x+2} \right| + c$

55. Let $\vec{A} = (2, 2)$ and $\vec{B} = (-2, -2)$ be two vectors. Then which of the following is **NOT** true?

A. $\vec{A}$ and $\vec{B}$ are orthogonal vectors.

B. The magnitude of $2\vec{A} + 3\vec{B}$ is $2\sqrt{2}$.

C. The cosine of the angle between $\vec{A}$ and $\vec{B}$ is -1 .

D. $\vec{A}$ and $\vec{B}$ are parallel vectors in the opposite direction.

56. Let $\vec{u} = (4, 1)$ and $\vec{v} = (2, 6)$. Then $(2\vec{u}) \cdot (3\vec{v})$ is equal to:

A. 42

B. 48

C. 84

D. 96

57. If a translation T takes the origin to $(-1, 1)$, then what is the equation of the image circle for the circle with equation $x^2 + y^2 = 4$ under this translation ?

A. $(x + 1)^2 + (y + 1)^2 = 4$

C. $(x - 1)^2 + (y + 1)^2 = 4$

B. $(x + 1)^2 + (y - 1)^2 = 4$

D. $(x - 1)^2 + (y - 1)^2 = 4$

58. The simplified form of $\frac{\tan^3 x + \frac{1}{\cot x}}{\sec^2 x}$ is:

A. $\sec x$

B. $\csc x$

C. $\tan x$

D. $\cot x$

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59. Which one of the following is correct about $f(x) = -2\sin\left(\frac{2x+\pi}{3}\right) + 4$?

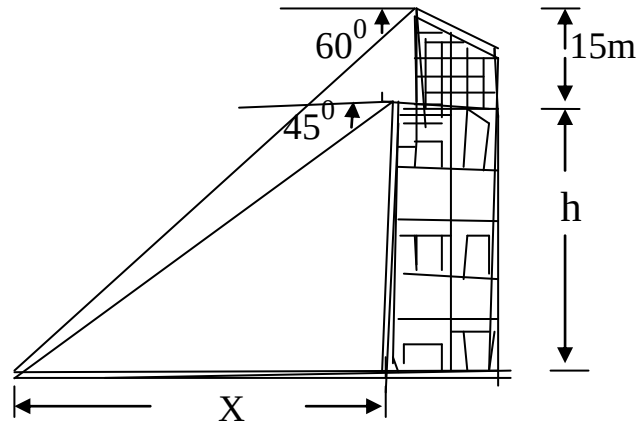
A. The amplitude is 4.

C. The phase shift is $\pi/2$.

B. The period is 3π .

D. The range is $[3, 5]$.

60. A tower 15meter high is on the bank of a river. It is observed that the angle of depression from the top of the tower to a point on the opposite shore is 60° and the angle of depression from the base of the tower to the same point on the opposite shore is observed to be 45° as shown in the figure below:



What is the width of the river (in meters)?

A. $\frac{15}{\sqrt{3}+1}$

B. $\frac{30}{\sqrt{3}+1}$

C. $\frac{15}{\sqrt{3}-1}$

D. $\frac{30}{\sqrt{3}-1}$

61. Which one of the following points is a point of intersection of a sphere with equation $(x - 2)^2 + (y - 1)^2 + (z - 1)^2 = 9$ and the y – axis?

A. $(0, -3, 0)$

B. $(0, 3, 0)$

C. $(0, -5, 0)$

D. $(0, 5, 0)$

62. What is the angle between the vectors $\vec{a} = (2, 0, 1)$ and $\vec{b} = (0, -1, 0)$?

A. $\theta = 45^\circ$

B. $\theta = 60^\circ$

C. $\theta = 90^\circ$

D. $\theta = 135^\circ$

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63. How far is the point p (3, 0, 2) from the sphere whose equation is

$$(x - 1)^2 + (y + 1)^2 + z^2 = 1?$$

- A. 2 B. 3 C. 4 D. 5

64. Which one of the following argument forms is invalid?

- A. $p \Rightarrow \neg q, p, r \Rightarrow q \vdash \neg r$ C. $p \Rightarrow (q \vee r), \neg r, p \vdash q$
B. $p \Rightarrow q, \neg q, \vdash \neg p$ D. $\neg p \Rightarrow q, q, \vdash p$

65. A student tried to show that for every integer n, $2n+7$ is odd. He took n to be even for the first time and n to be odd for the second time. In both cases he got that $2n+7$ is odd. The method of proof he used is said to be:

- A. Proof by contradiction. C. Direct proof.
B. Proof by exhaustion. D. Proof by Mathematical Induction.