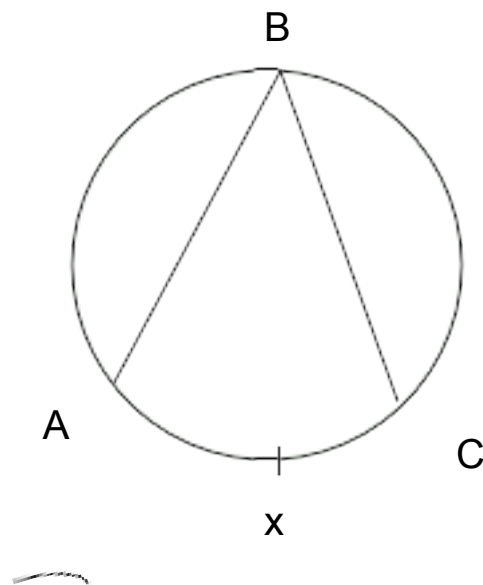


6.3 More on circles

6.31 Angles and Arcs determined by lines intersecting inside and on a circle

Thm 6.9

The measure of an angle inscribed in a circle is half the measure of the arc subtending it.



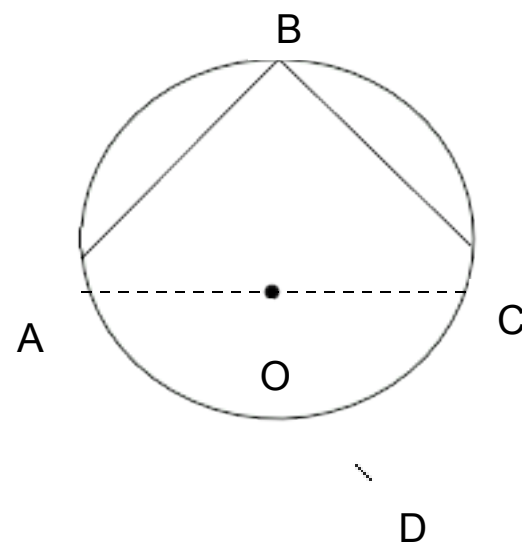
Corollary 6.9.1

An angle inscribed in a sem-circle is a right angle.

The measure of arc ADC is 180° or π radians

$$m(\angle ABC) = \frac{1}{2} m(\widehat{ADC})$$

$$= \frac{1}{2} (180^\circ) = 90^\circ \text{ or } \frac{\pi}{2} \text{ radian}$$



$\angle ABC$ is subtended by a sem-circle

Corollary 6.9.2

An angle inscribed in an arc less than a semi-circle is obtuse

$$m(\angle ABC) = \frac{1}{2}m(\widehat{ADC})$$

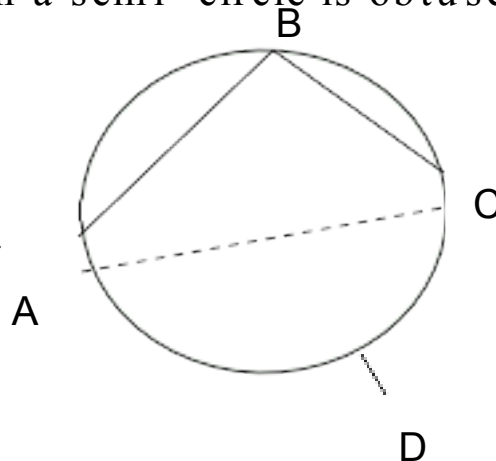
but $m(\angle ABC) < \text{length of a semi circle}$.

$$m(\widehat{ADC}) < 180^\circ$$

$$\therefore m(\widehat{ADC}) > 180^\circ$$

$$m(\angle ABC) = \frac{1}{2}m(\widehat{ADC}) > \frac{1}{2}(180^\circ)$$

$m(\angle ABC) > 90^\circ$, so, $\angle ABC$ is an obtuse angle.



Corollary 6.9.3

An angle inscribed in an arc greater than a semi-circle is acute.

Thm 6.10

Two parallel lines intercept congruent arcs on the same circle see the proof on your text book on page 250 & 251

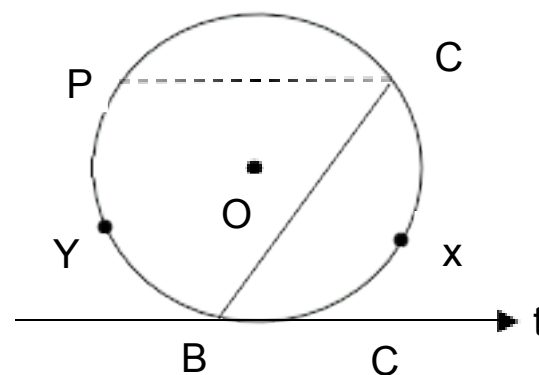
Thm 6.11

An angle formed by a tangent and a chord drawn from the point of tangency is measured by half the arc it intercepts

Given: circle O with $\angle ABC$ formed by

Tangent & chord $\overline{AB}$ at B, the point of contact.

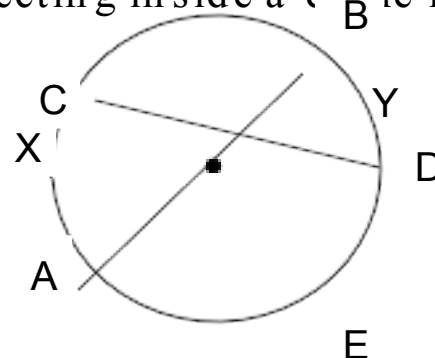
$$m(\angle ABC) = \frac{1}{2}m(\widehat{AC})$$



Thm 6.12

the measure of an angle formed by two chords intersecting inside a circle is half the sum of the measures of the arcs

subtending the angle & its vertically opposite angle



$$m(\angle BPD) = \frac{1}{2} m(\angle AXC) + \frac{1}{2} m(\angle BYD)$$

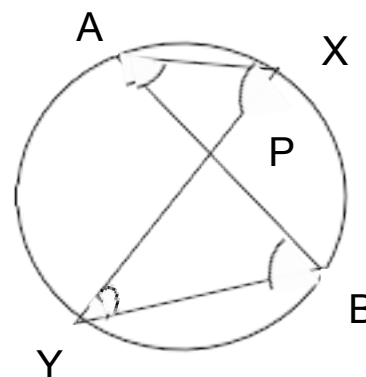


Remark:- the following result is sometimes called the product or rectangle property of a circle.

If two chords intersect in a circle

as shown in figure 6.47

Then $(AP)(PB) = (XP)(PY)$.



Eg. in fig. calculate the radius of the circle

Solution:- let the radius of the circle be r units long.

Then, $OD = r$ & $PD = 2r - 3$

Since $(AP)(PB) = (CP)(PD)$

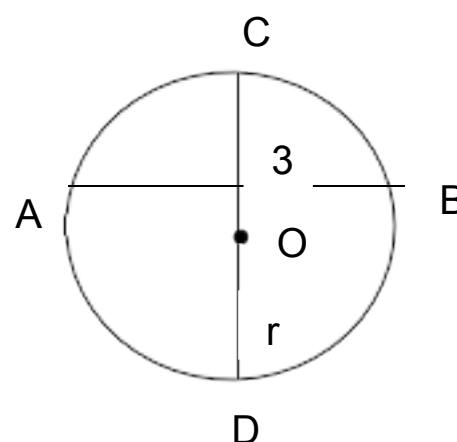
$$6 \times 6 = 3(2r - 3)$$

$$36 = 6r - 9$$

$$36 + 9 = 6r$$

$$\frac{45}{6} = \frac{6r}{6}$$

$$r = \frac{15}{2} = 7.5$$

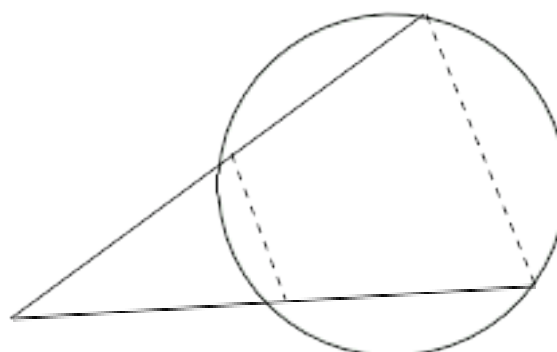


6.3.2 Angles and Arcs determined by lines intersecting outside a circle

Thm 6.13

The measure of the angle formed by the lines of two chords intersecting outside a circle is half the difference of the measure of the arcs they intercept.

$$(PA)(PB) = (PX)(PY)$$



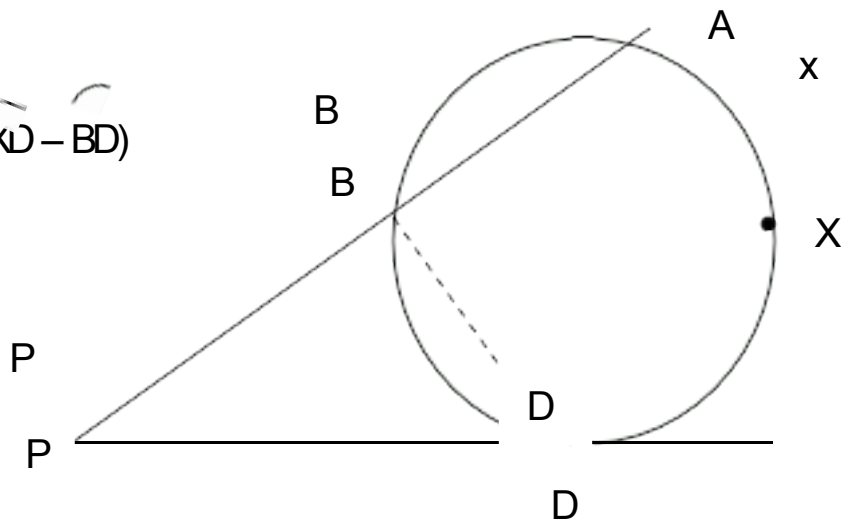
y

X

Thm 6.14

The measure of an angle formed by a tangent and secant drawn to a circle from a point outside the circle is equal to one-half the difference of the measures of the intercepted arcs.

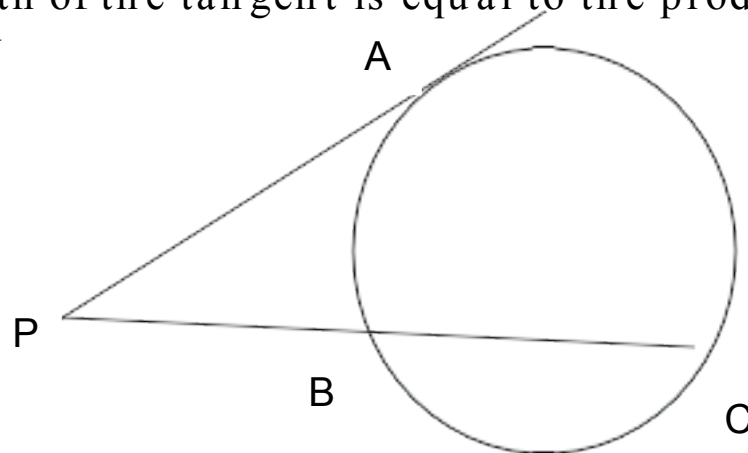
$$m(\angle P) = \frac{1}{2}m(\widehat{AXD} - \widehat{BD})$$



Th m 6.15

If a secant and a tangent are drawn from a point outside a circle, then the square of the length of the tangent is equal to the product of the lengths of line segments given by

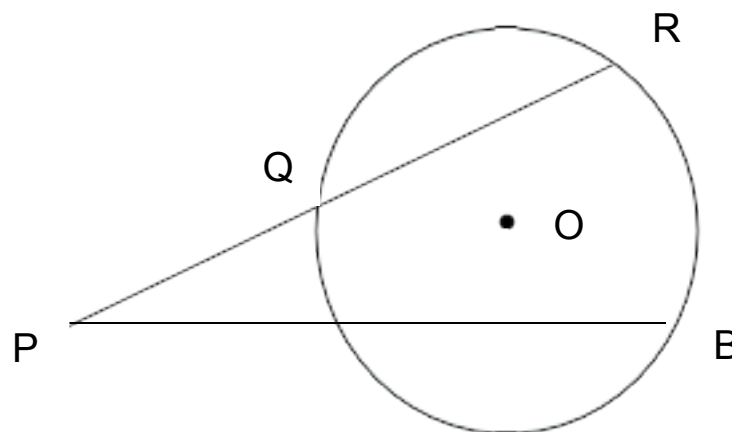
$$(PA)^2 = (PB)(PC)$$



Eg if the measure of arc AQ is 30° and the measure of arc BR is 60° what is the measure of $\angle P$?

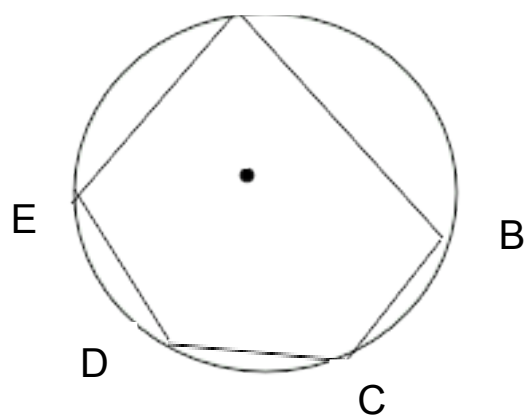
$$m(\angle P) = \frac{1}{2}m(\widehat{RB} - \widehat{QA})$$

$$= \frac{1}{2}(60^\circ - 30^\circ) = \frac{1}{2}(30^\circ)$$

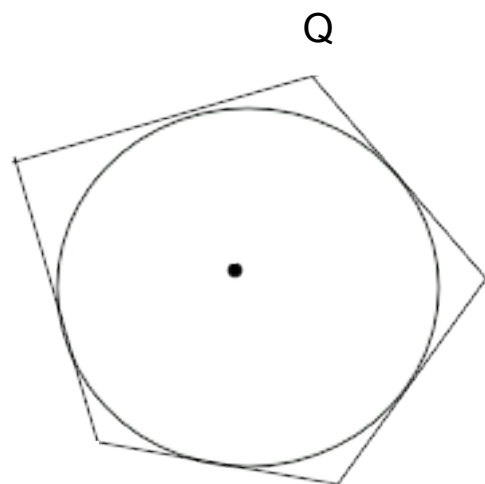


6.4 Regular polygons

A polygon whose vertices are on a circle is said to be inscribed in the circle. The circle is circumscribed about the polygon.



ABCDE is inscribed in the circle or the circle is circumscribed about the polygon.



the pentagon PQRST is circumscribed about the circle. the circle is inscribed in the pentagon.

6.4.1. perimeter of a regular polygon

The length of a side (s) of a regular polygon

$$S = 2r \sin \frac{180^\circ}{n}$$

The perimeter (P) of a regular polygon

$$P = 2nr \sin \frac{180^\circ}{n} \text{ or } P = ns$$

The apothem (a) of a regular polygon

$$a = r \cos \frac{180^\circ}{n}$$

6.4.2 area of a regular polygon

Thm 6.16

The area A of a regular polygon with n sides and radius r is

$$A = \frac{1}{2}nr^2 \sin \frac{360^\circ}{n} \text{ or } A = \frac{1}{2}ap$$

Eg. Find the area of a regular twelve-sided polygon with radius 3cm.

$$A = \frac{1}{2}nr^2 \sin \frac{360^\circ}{n} = \frac{1}{2}(12)(3\text{cm})^2 \sin \frac{360^\circ}{12} = 6 \times 9\text{cm}^2 \sin 30^\circ$$

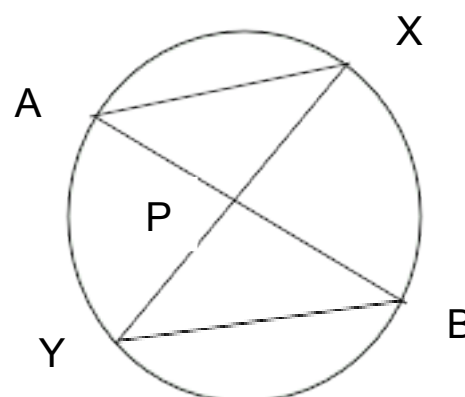
$$= 54\text{cm}^2 \times \sin 60^\circ$$

$$= 54\text{cm}^2 \times \frac{\sqrt{3}}{2} = 27\sqrt{3}\text{cm}^2$$

Work sheet

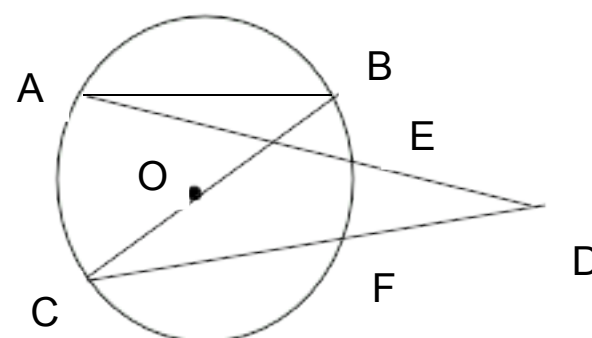
1. If two chord intersect in a circle at P and $\overline{AP}=4\text{cm}$, $\overline{PB}=6\text{cm}$ and $\overline{XP}=3\text{cm}$ then length of YP is

- A. $\frac{18}{4}\text{cm}$
- B. $\frac{9}{4}\text{cm}$
- C. 2cm

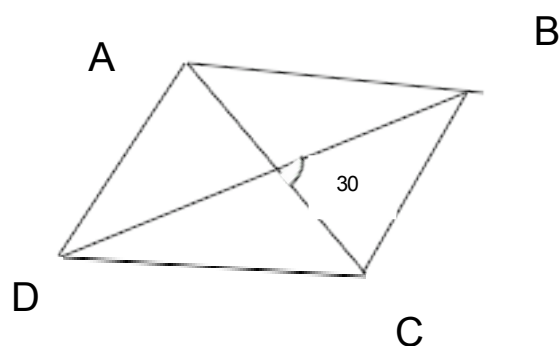


2. In the circle O, $\overline{CB}$ is a diameter of the circle. AD and CD intersect outside the circle. If $m(\angle BCF) = 60^\circ$, $m(\angle BAE) = 10^\circ$ and $m(\angle ABC) = 80^\circ$, then $m(\angle ADC)$ is equal to?

- A. 60°
- B. 40°
- C. 100°
- D. 30°

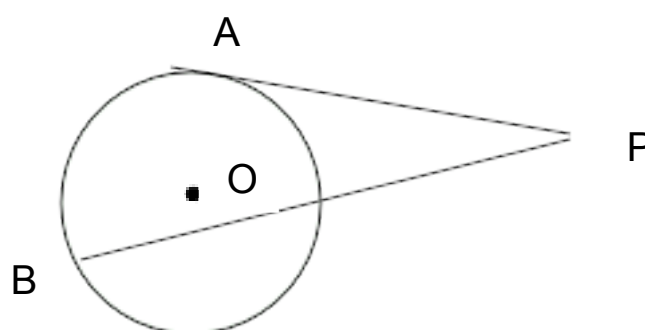


3. If the diagonal of a parallelogram are 10cm and 6cm and angle of 30° , then what is the area of the parallelogram?



- A. 20cm^2
- B. 15cm^2
- C. 40cm^2
- D. 30cm^2

4. On the circle shown below "O" is the centre and A,B and m are on the circle while PA is a tangent line and a secant line BP intersect outside the circle at:

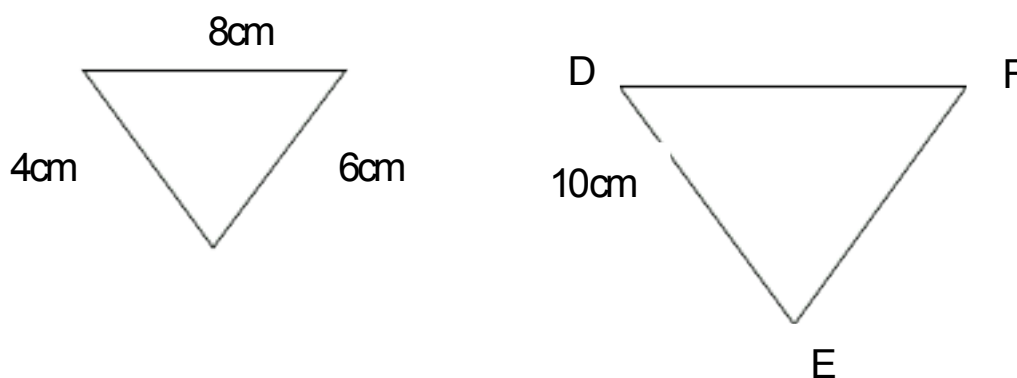


m

If the length of AP = 6cm and Pm = 4m, the Bm = _____

- A. 14cm B. 9cm C. 5cm D. 16cm

5. In the figure shown below, the two triangles are similar the shortest side of $\triangle DEF$ is 10cm. what is the area of $\triangle DEF$?



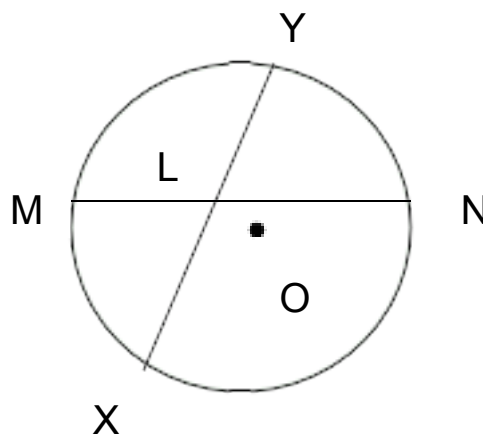
- A. $a(\triangle DEF) = \frac{\sqrt{15}}{4} \text{cm}^2$ C. $a(\triangle DEF) = 40 \text{cm}^2$
 B. $a(\triangle DEF) = \frac{3\sqrt{15}}{4} \text{cm}^2$ D. $a(\triangle DEF) = \frac{75\sqrt{15}}{4} \text{cm}^2$

6. What is the area of a regular six-sides polygon with radius 3cm

- A. $27\sqrt{3} \text{cm}^2$ B. $25\sqrt{3} \text{cm}^2$ C. $\frac{27}{2}\sqrt{3} \text{cm}^2$ D. $\frac{29}{2}\sqrt{3} \text{cm}^2$ E. none

7. In the following MX is an arc of 30° , and YN is an arc of 60° . What is the degree measure of $\angle YLN$?

- A. 35° C. 45°
 B. 90° D. 60°



8. An angle inscribed in a semi-circle is

- A. an obtuse angle C. a central angle
 B. a right angle D. an inscribed angle

9. The point of intersection of the altitudes of a triangle is called _____ of the triangle

- A. circum centre C. orthocenter

B. centroid

D. in centre