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UNIT 1

THE NUMBER SYSTEM

1.1. REVISION ON THE SET OF RATIONAL NUMBERS

1.1.1. NATURAT NUMBERS, INTEGERS, RIME NUMBERS AND COMPOSITE NUMBERS

- The set of natural numbers, denoted by N , is described by $N = \{1, 2, 3, \dots\}$
- The set of whole numbers, denoted by W , is described by $W = 0, 1, 2, 3, \dots\}$
- The set of integers, denoted by Z , is described by $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- Given two natural numbers m and p , m is called a multiple of p if there is a natural number q such that

$$m = p \times q$$

In this case, p is called a **factor** or **divisor** of m . We also say m is divisible by p . Similarly, q is also a factor or divisor of m , and m is divisible by q .

Definition 1.1 Prime numbers and composite numbers

- ♦ A natural number that has exactly two distinct factors, namely 1 and itself, is called a **prime number**.
- ♦ A natural number that has more than two factors is called a **composite number**.

Note- 1 is neither prime nor composite.

Prime numbers- $\{2, 3, 5, 7, 11, 13, 17, 19, 23, \dots\}$

Composite numbers - $\{4, 6, 8, 9, 10, 12, 14, 15, 16, \dots\}$

Divisibility Test

A number is divisible by

- 2, if its unit's digit is divisible by 2.
- 3, if the sum of its digits is divisible by 3.
- 4, if the number formed by its last two digits is divisible by 4.
- 5, if its unit's digit is either 0 or 5.
- 6, if it is divisible by both 2 and 3.
- 8, if the number formed by its last three digits is divisible by 8.
- 9, if the sum of its digits is divisible by 9.
- 10, if its unit's digit is 0.

Example 1 Use the divisibility test to determine whether 2,416 is divisible by 2, 3, 4, 5, 6, 8, 9 and 10.

Solution: ♦ 2, 416 is divisible by 2 because the unit's digit 6 is divisible by 2.

- ♦ 2,416 is divisible by 4 because 16 (the number formed by the last two digits) is divisible by 4.
- ♦ 2,416 is divisible by 8 because the number formed by the last three digits (416) is divisible by 8.
- ♦ 2,416 is not divisible by 5 because the unit's digit is not 0 or 5.
- ♦ Similarly you can check that 2,416 is not divisible by 3, 6, 9, and 10.

A factor of a composite number is called a **prime factor**, if it is a prime number. For example 2, 3 and 5 are the prime factors of 30.

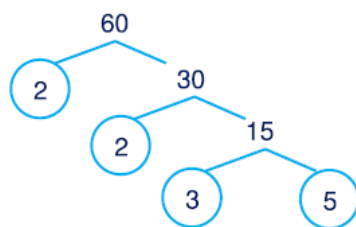
Every composite number can be written as a product of prime numbers. To find the prime factors of any composite number, begin by expressing the number as a product of two factors where at least one of the factors is prime. Then, continue to factorize each resulting composite factor until all the factors are prime numbers.

When a number is expressed as a product of its prime factors, the expression is called the **prime factorization** of the number.

For example, the prime factorization of 60 is

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5.$$

The prime factorization of 60 is also found by using a factoring tree.



Theorem 1.1 Fundamental theorem of arithmetic

Every composite number can be expressed (factorized) as a product of primes. This factorization is unique, apart from the order in which the prime factors occur.

Exerscise 1

Find the rime factorization of the following numbers

1. 365
2. 450
3. 1160
4. 912

1.1.2. Common Factors and Common Multiple.

A. Common Factors and Greatest Common Factors

Given two or more natural numbers, a number which is a factor of all of them is called a **common factor**. Numbers may have more than one common factor. The greatest of the common factors is called the **greatest common factor (GCF)** or the **highest common factor (HCF)** of the numbers.

➤ The greatest common factor of two numbers a and b is denoted by $\text{GCF}(a, b)$.

Example 2 Find the common factors of

- a. 36 and 6
- b. 32 and 27

Solution

a First, make lists of the factors of 36 and 60, using sets.

Let F_{36} and F_{60} be the sets of factors of 36 and 60, respectively. Then,

$$F_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$$

$$F_{60} = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$$

You can use the diagram to summarize the information. Notice that the common factors are shaded in **green**. They are 1, 2, 3, 4, 6 and 12 and the greatest is **12**.

$$\text{i.e., } F_{36} \cap F_{60} = \{1, 2, 3, 4, 6, 12\}$$

$$\text{Therefore, } \text{GCF}(36, 60) = 12.$$

b Similarly,

$$F_{32} = \{1, 2, 4, 8, 16, 32\} \text{ and}$$

$$F_{27} = \{1, 3, 9, 27\}$$

$$\text{Therefore, } F_{32} \cap F_{27} = \{1\}$$

$$\text{Thus, } \text{GCF}(32, 27) = 1$$

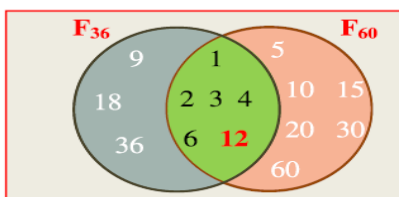


Figure 1.2

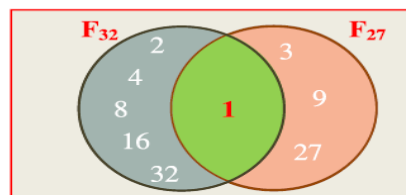


Figure 1.3

➤ Two or more natural numbers that have a GCF of 1 are called **relatively prime**.

Definition 1.2

The **greatest common factor (GCF)** of two or more natural numbers is the greatest natural number that is a factor of all of the given numbers.

Example 3 Using prime factorization find $\text{GCF}(180, 216, 54)$

Solution

Step 1 Express the numbers 180, 216 and 540 in their prime factorization.

$$180 = 2^2 \times 3^2 \times 5; \quad 216 = 2^3 \times 3^3; \quad 540 = 2^2 \times 3^3 \times 5$$

Step 2 As you see from the prime factorizations of 180, 216 and 540, the numbers 2 and 3 are common prime factors.

So, $\text{GCF}(180, 216, 540)$ is the product of these common prime factors with the smallest respective exponents in any of the numbers.

$$\therefore \text{GCF}(180, 216, 540) = 2^2 \times 3^2 = 36.$$

Exercise 2

Find GCF of the following numbers using prime factorization method.

1. 12 and 16
2. 10, 12 and 14
3. 320 and 560
4. 36, 48 and 64

B. Common Multiple and Least Common Multiple

Definition 1.3

For any two natural numbers a and b , the **least common multiple of a and b** denoted by $\text{LCM}(a, b)$, is the smallest multiple of both a and b .

Example 4 Find $\text{LCM}(8, 9)$.

Solution

3 Find $\text{LCM}(8, 9)$.

Let M_8 and M_9 be the sets of multiples of 8 and 9 r

$$M_8 = \{8, 16, 24, 32, 40, 48, 56, 64, \boxed{72}, 80, 88, \dots\}$$

$$M_9 = \{9, 18, 27, 36, 45, 54, 63, \boxed{72}, 81, 90, \dots\}$$

Therefore $\text{LCM}(8, 9) = 72$.

Prime factorization can also be used to find the LCM of a set of two or more than two numbers. A common multiple contains all the prime factors of each number in the set. The LCM is the product of each of these prime factors to the greatest number of times it appears in the prime factorization of the numbers.

Example 4 Find LCM (9, 21, 24).

Solution

$$9 = 3 \times 3 = 3^2$$

$$21 = 3 \times 7$$

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

The prime factors that appear in these factorizations are 2, 3 and 7.

Considering the greatest number of times each prime factor appears, we can get 2^3 , 3^2 and 7, respectively.

$$\text{Therefore, } LCM(9, 21, 24) = 2^3 \times 3^2 \times 7 = 504$$

Note For any natural numbers a and b , $GCF(a, b) \times LCM(a, b) = a \times b$.

Example 5

The GCF of two numbers is 12 and the LCM is 144. If one of the numbers is 48, what is the other number?

Solution

Given :- $GCF(a, b) = 12$, $LCM(a, b) = 144$ and $b = 48$

Required :- $a = ?$

$$GCF(a, b) \times LCM(a, b) = a \times b$$

$$12 \times 144 = a \times 48$$

$$1728 = 48a$$

$$36 = a$$

Exercise 3

1. Find the LCM of the following numbers.

- 72 and 80
- 65 and 105
- 30, 45 and 60
- 50, 70, and 100

- The GCF of two numbers is 3 and the LCM is 180. If one of the numbers is 45, what is the other number?
- Find the value of an odd natural number x if $LCM(x, 40) = 1400$.

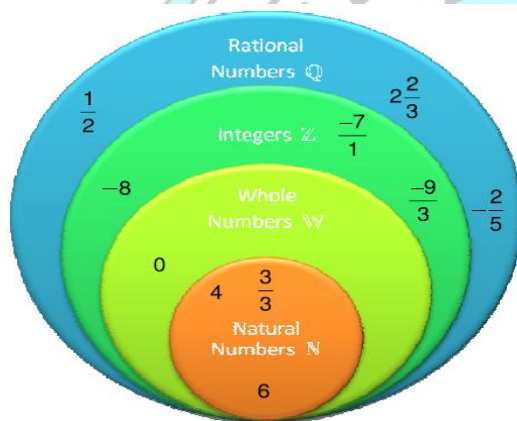
1.1.3. Rational numbers

Definition 1.4 Rational number

Any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$, is called a **rational number**. The set of rational numbers, denoted by $\mathbb{Q}$, is the set described by

$$\mathbb{Q} = \left\{ \frac{a}{b} : a \text{ and } b \text{ are integers and } b \neq 0 \right\}.$$

We can generalize the set of numbers by using the following diagram.



Example $3, \frac{5}{6}, \frac{-4}{9}, 0, \frac{102}{697}$ are examples of rational numbers.

Note:- In a rational number $\frac{a}{b}$, a is numerator and b is denominator.

Rational numbers (fractions) are of three type, these are

- Proper fractions:-** are fractions whose numerator is greater than its denominator.

Example $\frac{1}{2}, \frac{3}{5}, \frac{25}{89}, \frac{584}{968}, \frac{87415}{554884}, \dots$

- Improper fractions:-** are fractions whose denominator greater than its numerator.

Example $\frac{8}{2}, \frac{9}{5}, \frac{95}{89}, \frac{5784}{968}, \frac{8742515}{554884}, \dots$

- Mixed fractions:-** are the other way of writing Improper fractions.

Example $\frac{7}{2} = 3\frac{1}{2}, \frac{9}{5} = 1\frac{4}{5}, \frac{95}{89} = 1\frac{6}{89}, \frac{5784}{968} = 5\frac{944}{968}, \frac{8742515}{554884} = 15\frac{419255}{554884}, \dots$

When a rational number expressed as a fraction, it is often expressed in simplest form (lowest term). $\frac{a}{b}$ is in its lowest form when $GCF(a, b) = 1$.

Example 1 Write $\frac{30}{45}$ in simplest form.

Solution: $\frac{30}{45} = \frac{2 \times 3 \times 5}{3 \times 3 \times 5} = \frac{2}{3}$. (by factorization and cancellation)

Hence $\frac{30}{45}$ when expressed in lowest terms (simplest form) is $\frac{2}{3}$.

Exercise Write the following fractions in their lowest form.

1. $\frac{72}{63}$
2. $\frac{36}{40}$
3. $\frac{100}{800}$
4. $\frac{8}{12}$

1.2. THE REAL NUMBERS SYSTEM

1.2.1. Representation of Rational by Decimals

A rational number may also appear in the form of a decimal.

Remember that a fraction is another way of writing division of one quantity by another. Any fraction of natural numbers can be expressed as a decimal by dividing the numerator by the denominator.

Example 1 Show that $\frac{3}{8}$ and $\frac{7}{12}$ can each be expressed as a decimal.

Solution: $\frac{3}{8}$ means $3 \div 8$

$\frac{7}{12}$ means $7 \div 12$

$$\begin{array}{r} 0.375 \\ 8 \overline{) 3.000} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$$

$$\therefore \frac{3}{8} = 0.375$$

$$\begin{array}{r} 0.5833 \dots \\ 12 \overline{) 7.0000} \\ \underline{60} \\ 100 \\ \underline{96} \\ 40 \\ \underline{36} \\ 40 \\ \underline{36} \\ 4 \end{array}$$

$$\therefore \frac{7}{12} = 0.5833 \dots$$

The fraction (rational number) $\frac{3}{8}$ can be expressed as the decimal 0.375. A decimal like 0.375 is called a **terminating decimal** because the division ends or terminates, when the remainder is zero.

The fraction $\frac{7}{12}$ can be expressed as the decimal 0.58333... (Here, the digit 3 repeats and the division does not terminate.) A decimal like 0.58333... is called a **repeating decimal**. To show a repeating digit or a block of repeating digits in a repeating decimal number, we put a bar above the repeating digit (or block of digits). For example 0.58333... can be written as $0.58\overline{3}$, and 0.0818181... can be written as $0.08\overline{18}$. This method of writing a repeating decimal is known as **bar notation**.

The portion of a decimal that repeats is called the **repetend**. For example,

In $0.583333... = 0.58\overline{3}$, the repetend is 3.

In $1.777... = 1.\overline{7}$, the repetend is 7.

In $0.00454545... = 0.004\overline{5}$, the repetend is 45.

To generalize:

Any rational number $\frac{a}{b}$ can be expressed as a decimal by dividing the numerator a by the denominator b .

When you divide a by b , one of the following two cases will occur.

Case 1 The division process ends or terminates when a remainder of zero is obtained. In this case, the decimal is called a **terminating decimal**.

Case 2 The division process does not terminate as the remainder never becomes zero. Such a decimal is called a **repeating decimal**.

Expressing terminating and repeating decimals as fractions

➤ Every terminating decimal can be expressed as a fraction (a ratio of two integers) with a denominator of 10, 100, 1000 and so on.

Example 2 Express each of the following decimals as a fraction in its simplest form (lowest terms):

a 0.85

b 1.3456

Solution:

$$\mathbf{a} \quad 0.85 = 0.85 \times \frac{100}{100} = \frac{85}{100} = \frac{17}{20} \text{ (Why?)}$$

$$\mathbf{b} \quad 1.3456 = 1.3456 \times \frac{10000}{10000} = 1.3456 \times \frac{10^4}{10^4} = \frac{13456}{10000} = \frac{841}{625}$$

➤ If d is a terminating decimal number that has n digits after a decimal point, then we rewrite d as

$$d = \frac{10^n \times d}{10^n}$$

The right side of the equation gives the fractional form of d .

For example, if $d = 2.128$, then $n = 3$.

$$\therefore 2.128 = \frac{10^3 \times 2.128}{10^3} = \frac{2128}{1000} = \frac{266}{125}$$

✓ Repeating decimals can also be expressed as fractions (ratios of two integers).

Example 3 Express each of the following decimals as a fraction (ratio of two integers):

$$\mathbf{a} \quad 0.\overline{7}$$

$$\mathbf{b} \quad 0.\overline{25}$$

Solution: a Let $d = 0.\overline{7} = 0.777\ldots$ then,

$$10d = 7.777\ldots \quad (\text{multiplying } d \text{ by } 10 \text{ because } 1 \text{ digit repeats})$$

$$\text{Subtract } d = 0.777\ldots \quad (\text{to eliminate the repeating part } 0.777\ldots)$$

$$10d = 7.777\ldots \quad \mathbf{1}$$

$$\underline{1d = 0.777\ldots} \quad \mathbf{2} \quad (d = 1d)$$

$$9d = 7 \quad (\text{subtracting expression } \mathbf{2} \text{ from expression } \mathbf{1})$$

$$\therefore d = \frac{7}{9} \quad (\text{dividing both sides by } 9)$$

$$\text{Hence } 0.\overline{7} = \frac{7}{9}$$

$$\mathbf{b} \quad \text{Let } d = 0.\overline{25} = 0.252525\ldots$$

$$\text{Then, } 100d = 25.2525\ldots \quad (\text{multiplying } d \text{ by } 100 \text{ because } 2 \text{ digits repeat})$$

$$100d = 25.252525\ldots$$

$$\underline{1d = 0.252525\ldots} \quad (\text{subtracting } 1d \text{ from } 100d \text{ eliminates the repeating part } 0.2525\ldots)$$

$$99d = 25$$

$$\therefore d = \frac{25}{99}$$

$$\text{So, } 0.\overline{25} = \frac{25}{99}$$

The algebra used in the above example can be generalized as follows:

➤ In general, if d is a repeating decimal with k non-repeating and p repeating digits after the decimal point, then the formula

$$d = \frac{d(10^{k+p} - 10^k)}{10^{k+p} - 10^k}$$

is used to change the decimal to the fractional form of d .

Example 4 Express the decimal $0.3\overline{75}$ as a fraction.

Solution: Let $d = 0.3\overline{75}$, then,

$k = 1$ (number of non-repeating digits)

$p = 2$ (number of repeating digits) and

$k + p = 1 + 2 = 3$.

$$\begin{aligned} \Rightarrow d &= \frac{d(10^{k+p} - 10^k)}{(10^{k+p} - 10^k)} = \frac{d(10^3 - 10^1)}{(10^3 - 10^1)} = \frac{10^3 d - 10d}{10^3 - 10} \\ &= \frac{10^3 \times 0.3\overline{75} - 10 \times 0.3\overline{75}}{990} \\ &= \frac{375.\overline{75} - 3.\overline{75}}{990} = \frac{372}{990} \end{aligned}$$

Remark

- i Every rational number can be expressed as either a terminating decimal or a repeating decimal.
- ii Every terminating or repeating decimal represents a rational number.

Exercise

- Express each of the following decimals as a fraction
 - 0.88
 - $6.4\overline{57}$
 - $0.321\overline{56}$
 - $7.\overline{254}$
 - 123.567
 - $58.20\overline{05}$
 - 58.025
 - 0.2
- Express each of the following fractions as a decimal.
 - $\frac{89}{11}$
 - $\frac{8}{5}$
 - $\frac{11}{6}$
 - $\frac{20}{4}$

1.2.2. Irrational Numbers

Remember that terminating or repeating decimals are rational numbers, since they can be expressed as fractions. The square roots of perfect squares are also rational numbers.

For example, $\sqrt{4}$ is a rational number, since $\sqrt{4} = 2 = \frac{2}{1}$. Similarly, $\sqrt{0.09}$ is a rational number because, $\sqrt{0.09} = 0.3$ is a rational number.

If $x^2 = 4$, then what do you think is the value of x ?

$x = \pm \sqrt{4} = \pm 2$. Therefore x is a rational number. What if $x^2 = 3$?

If you first press the button 2 and then the square-root button, you will find $\sqrt{2}$ on the display.

i.e., $\sqrt{2} : 2 \sqrt{ } = 1.414213562 \dots$

$\sqrt{5} : 5 \sqrt{ } = 2.236067977 \dots$

Note that many scientific calculators, such as Casio ones, work the same as the written order, i.e., instead of pressing 2 and then the $\sqrt{ }$ button, you press the $\sqrt{ }$ button and then 2. Before using any calculator, it is always advisable to read the user's manual.

Note that the decimal numbers for $\sqrt{2}$ and $\sqrt{5}$ do not terminate, nor do they have a pattern of repeating digits. Therefore, these numbers are not rational numbers. Such numbers are called irrational numbers. In general, if a is a natural number that is not a perfect square, then $\sqrt{a}$ is an irrational number.

Note that:- A decimal number that is neither terminating nor repeating is an irrational number.

Definition:- Irrational number

An irrational number is a number that cannot be expressed as $\frac{a}{b}$, such that a and b are integers and $b \neq 0$.

Example 2 Show that $3 + \sqrt{2}$ is an irrational number.

Solution: To show that $3 + \sqrt{2}$ is not a rational number, let us begin by assuming that $3 + \sqrt{2}$ is rational. i.e., $3 + \sqrt{2} = \frac{a}{b}$ where a and b are integers, $b \neq 0$.

$$\text{Then } \sqrt{2} = \frac{a}{b} - 3 = \frac{a - 3b}{b}.$$

Since $a - 3b$ and b are integers (Why?), $\frac{a - 3b}{b}$ is a rational number, meaning

that $\sqrt{2} = \left(\frac{a - 3b}{b} \right)$ is rational, which is false. As the assumption that $3 + \sqrt{2}$ is

rational has led to a false conclusion, the assumption must be false.

Therefore, $3 + \sqrt{2}$ is an irrational number.

You can generalize the following facts:

- I. The sum of any rational number and an irrational number is an irrational number.
- II. The set of irrational numbers is not closed with respect to addition, subtraction, multiplication and division.

III. If p is a positive integer that is not a perfect square, then $a + b\sqrt{p}$ is irrational where a and b are integers and $b \neq 0$. For example, $3 + \sqrt{2}$ and $2 - 2\sqrt{3}$ are irrational numbers.

Exercise

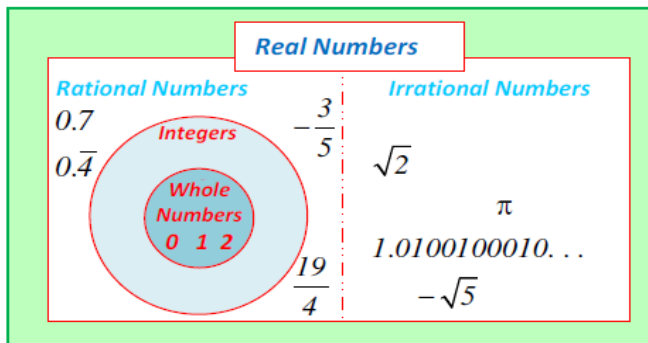
- 1) Identify each of the following numbers as rational or irrational:
 - a. $\frac{5}{6}$
 - b. $2.\overline{34}$
 - c. 3.121314151617 ...
 - d. 0.1011011101111 ...
 - e. $\sqrt{5} - \sqrt{2}$
 - f. $\sqrt[3]{72}$
 - g. $1 + \sqrt{7}$
- 2) For each of the following, decide whether the statement is 'true' or 'false'. If your answer is 'false', give a counter example to justify.
 - a. The sum of any two irrational numbers is an irrational number.
 - b. The sum of any two rational numbers is a rational number.
 - c. The sum of any two terminating decimals is a terminating decimal.
 - d. The product of a rational number and an irrational number is irrational.

1.2.3. Real Numbers

Definition Real numbers

A number is called a real number, if and only if it is either a rational number or an irrational number. The set of real numbers, denoted by R , can be described as the union of the sets of rational and irrational numbers. $R = \{x : x \text{ is a rational number or an irrational number.}\}$

The set of real numbers and its subsets are shown in the adjacent diagram.



1. Trichotomy property

For any two real numbers a and b , one and only one of the following is true

$$a < b \text{ or } a = b \text{ or } a > b.$$

2. Transitive property of order

For any three real numbers a , b and c , if $a < b$ and $b < c$, then $a < c$.

3. For any two non-negative real numbers a and b , if $a^2 < b^2$, then $a < b$.

You can use this property to compare two numbers without using a calculator.

For example, let us compare $\frac{5}{6}$ and $\frac{\sqrt{3}}{2}$.

$$\left(\frac{5}{6}\right)^2 = \frac{25}{36}, \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} = \frac{27}{36}$$

Since $\left(\frac{5}{6}\right)^2 < \left(\frac{\sqrt{3}}{2}\right)^2$, it follows that $\frac{5}{6} < \frac{\sqrt{3}}{2}$.

Exercise

1 Compare the numbers a and b using the symbol $<$ or $>$.

a $a = \frac{\sqrt{6}}{4}, b = 0.\bar{6}$

b $a = 0.432, b = 0.437$

c $a = -0.128, b = -0.123$

2 State whether each set (**a – e** given below) is closed under each of the following operations:

i addition **ii** subtraction **iii** multiplication **iv** division

a $\mathbb{N}$ the set of natural numbers. **b** $\mathbb{Z}$ the set of integers.

c $\mathbb{Q}$ the set of rational numbers. **d** The set of irrational numbers.

e $\mathbb{R}$ the set of real numbers.

1.2.4. Exponents and Radicals

A. Roots and radicals

Definition Square root

For any two real numbers a and b , if $a^2 = b$, then a is a square root of b .

Perfect squares also include decimals and fractions like 0.09 and $\frac{4}{9}$. Since $(0.3)^2 = 0.09$ and

$\left(\frac{2}{3}\right)^2 = \frac{4}{9}$, it is also true that $(-8)^2 = 64$ and $(-12)^2 = 144$.

So, you may say that -8 is also a square root of 64 and -12 is a square root of 144.

The positive square root of a number is called the **principal square root**.

The symbol $\sqrt{\quad}$, called a radical sign, is used to indicate the principal square root.

The symbol $\sqrt{25}$ is read as "the principal square root of 25" or just "the square root of 25" and $-\sqrt{25}$ is read as "the negative square root of 25". If b is a positive real number, $\sqrt{b}$ is a positive real number. Negative real numbers do not have square roots in the set of real numbers since $a^2 \geq 0$ for any number a . The square root of zero is zero.

Similarly, since $4^3 = 64$, you say that 64 is the cube of 4 and 4 is the cube root of 64. That is written as $\sqrt[3]{64} = 4$.

The symbol $\sqrt[3]{64} = 4$ is read as "the principal cube root of 64" or just "the cube root of 64".

Note that Each real number has exactly one cube root.

Definition Principal n^{th} root

If b is any real number and n is a positive integer greater than 1, then, the principal n^{th} root of b , denoted by $\sqrt[n]{b}$ is defined as:

$$\sqrt[n]{b} = \begin{cases} \text{the positive } n^{\text{th}} \text{ root of } b, & \text{if } b > 0. \\ \text{the negative } n^{\text{th}} \text{ root of } b, & \text{if } b < 0 \text{ and } n \text{ is odd.} \\ 0, & \text{if } b = 0. \end{cases}$$

Remark

- If $b < 0$ and n is even, there is no real n^{th} root of b , because an even power of any real number is a non-negative number.
- The symbol $\sqrt[n]{}$ is called a radical sign, the expression $\sqrt[n]{b}$ is called a **radical**, n is called the **index** and b is called the **radicand**. When no index is written, the radical sign indicates square root.

Example

- $\sqrt[4]{16} = 2$ because $2^4 = 16$
- $\sqrt{0.04} = 0.2$ because $(0.2)^2 = 0.04$
- $\sqrt[3]{-1000} = -10$ because $(-10)^3 = -1000$

Note that :- For any three positive real numbers a , b and c and a positive integer n if $a^n < b < c^n$, then $n \sqrt[n]{a} < \sqrt[n]{b} < c$.

Example Find a rational approximation of $\sqrt{43}$ to the nearest hundredth.

Solution: Use the above property and divide-and-average on a calculator.

Since $6^2 = 36 < 43 < 49 = 7^2$

$$6 < \sqrt{43} < 7$$

Estimate $\sqrt{43}$ to tenths, $\sqrt{43} \approx 6.5$

Divide 43 by 6.5

$$\begin{array}{r} 6.615 \\ 6.5 \overline{) 43.000} \end{array}$$

Average the divisor and the quotient $\frac{6.5 + 6.615}{2} = 6.558$

Divide 43 by 6.558

$$\begin{array}{r} 6.557 \\ 6.558 \overline{) 43.000} \end{array}$$

Now you can check that $(6.557)^2 < 43 < (6.558)^2$. Therefore $\sqrt{43}$ is between 6.557 and 6.558. It is 6.56 to the nearest hundredth.

Example Through successive trials on a calculator, compute $\sqrt[3]{53}$ to the nearest tenth.

Solution:

$3^3 = 27 < 53 < 64 = 4^3$. That is, $3^3 < 53 < 4^3$. So $3 < \sqrt[3]{53} < 4$

Try 3.5: $3.5^3 = 42.875$ So $3.5 < \sqrt[3]{53} < 4$

Try 3.7: $3.7^3 = 50.653$ So $3.7 < \sqrt[3]{53} < 4$

Try 3.8: $3.8^3 = 54.872$ So $3.7 < \sqrt[3]{53} < 3.8$

Try 3.75: $3.75^3 = 52.734375$ So $3.75 < \sqrt[3]{53} < 3.8$

Therefore, $\sqrt[3]{53}$ is 3.8 to the nearest tenth.

B. Meaning of fractional exponents

Definition The n^{th} power

If $b \in R$ and n is a positive integer greater than 1, then

$$b^{\frac{1}{n}} = \sqrt[n]{b}$$

Example Write the following in exponential form:

a. $\sqrt{7}$

c. $\sqrt[3]{16}$

b. $\frac{1}{\sqrt[3]{15}}$

d. $\sqrt[6]{25}$

Solution:

a. $\sqrt{7} = 7^{\frac{1}{2}}$

c. $\sqrt[3]{16} = 16^{\frac{1}{3}}$

b. $\frac{1}{\sqrt[3]{15}} = \frac{1}{15^{\frac{1}{3}}} = 15^{-\frac{1}{3}}$

d. $\sqrt[6]{25} = 25^{\frac{1}{6}}$

Theorem 1.2

For any two real numbers a and b and for all integers $n \geq 2$, $a^{\frac{1}{n}}b^{\frac{1}{n}} = (ab)^{\frac{1}{n}}$

Example Simplify each of the following.

a. $16^{\frac{1}{3}} \times 4^{\frac{1}{3}} = (16 \times 4)^{\frac{1}{3}} = 64^{\frac{1}{3}} = (4^3)^{\frac{1}{3}} = (4)^{3 \times \frac{1}{3}} = 4$

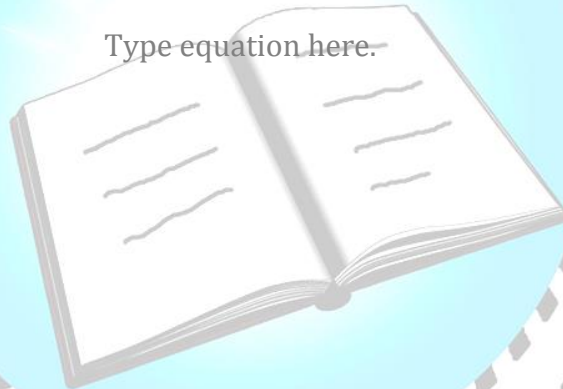
b. $100^{\frac{1}{3}} \times 10^{\frac{1}{3}} = (100 \times 10)^{\frac{1}{3}} = 1000^{\frac{1}{3}} = (1000^3)^{\frac{1}{3}} = (10)^{3 \times \frac{1}{3}} = 10$

c. $\sqrt[5]{8} \times \sqrt[5]{4} = 8^{\frac{1}{5}} \times 4^{\frac{1}{5}} = (8 \times 4)^{\frac{1}{5}} = 32^{\frac{1}{5}} = (2^5)^{\frac{1}{5}} = (2)^{5 \times \frac{1}{5}} = 2$

Theorem 1.3

For any two real numbers a and b where $b \neq 0$ and for all integers $n \geq 2$,

Type equation here.



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