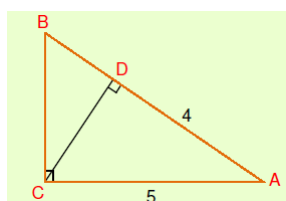


Bahir Dar University
Department of Mathematics
Pre-University Remedial Program for the 2014 EC ESSLCE Examinees

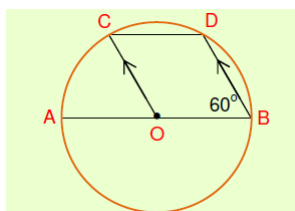
Mathematics for Natural Science and Mathematics for Social Science
Worksheet-II on Chapter 3-4, May 12/2015 E.C.

Chapter 3

1. Let $A(0, 0)$, $B(6, 0)$ and $C(0, 4)$ be vertices of $\triangle ABC$. Find the point of intersection of the medians of $\triangle ABC$.
2. In right angled triangle ABC shown in figure, $\overline{CD}$ is altitude to the hypotenuse $\overline{AB}$. If $\overline{AC} = 5$ units and $\overline{AD} = 4$ units, find the length of $\overline{BD}$ and $\overline{BC}$.

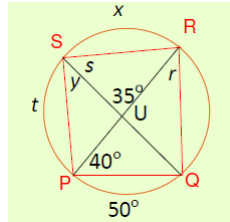


3. The mid-points of the sides of a rectangle are the vertices of a quadrilateral. What kind of quadrilateral is it? Prove your answer.
4. The mid-points of the sides of a parallelogram are the vertices of a quadrilateral. What kind of quadrilateral is it? Prove your answer.
5. Prove each of the following:
 - a. If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
 - b. If the diagonals of a quadrilateral bisect each other and one angle of the quadrilateral is a right angle, then the quadrilateral is a rectangle.
 - c. If all the four sides of a quadrilateral are congruent, then the quadrilateral is a rhombus.
 - d. The diagonals of a rhombus are perpendicular to each other.
6. In figure below, $\overline{AB}$ is a diameter. O is the centre of the circle. If $\overline{OC} \parallel \overline{BD}$ and $m(\angle ABD) = 60^\circ$, find $m(\angle OCD)$.

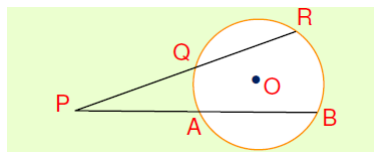


7. Find the area of a regular nine-sided polygon with radius 5 units.
8. Find the area of a regular twelve-sided polygon with radius 3 units.
9. Prove that the area A of an equilateral triangle inscribed in a circle with radius r is $A = \frac{3\sqrt{3}r^2}{4}$. Use this formula to find the area of an equilateral triangle inscribed in a circle with radius a. 2 cm b. 3 cm c. $\sqrt{3}$ cm .

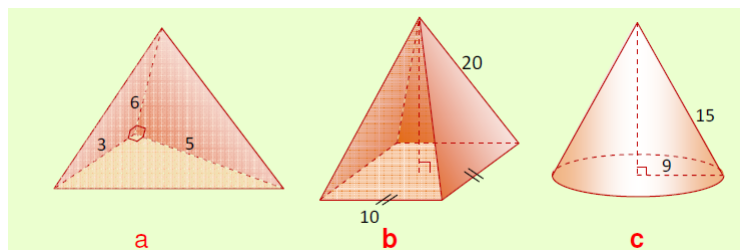
10. A chord of a circle of radius 6 cm is 8 cm long. Find the distance of the chord from the centre.
11. The altitude of a rectangular prism is 4 units and the width and length of its base are 3 units and 2 units respectively. Find: a. the lateral surface area b. the total surface area c. the volume
12. Find the values of the angle marked x, y, s, r and t as shown in figure.



13. A lateral edge of a right prism is 6 cm and the perimeter of its base is 20 cm. Find the area of its lateral surface.
14. Find the perimeter of the base of a right prism for which the area of the lateral surface is 180 square units and the altitude is 4 units.
15. The base of a right prism is an equilateral triangle of length 3 cm and its lateral surfaces are rectangular regions. If its altitude is 8 cm, then find: a. the total surface area of the prism b. the volume of the prism.
16. If the measure of arc AQ is 30° and the measure of arc BR is 60° , what is the measure of $\angle P$?

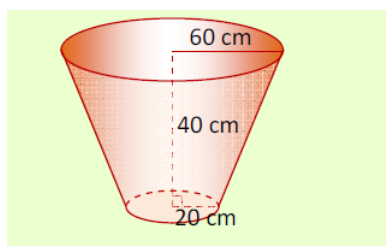


17. If the diagonal of a cube is $\sqrt{12}$ cm, find the area of its lateral surface.
18. Imagine a cylindrical container in which the height and the diameter are equal. Find expressions, in terms of its height, for its a. total surface area b. volume.
19. Calculate the volume of each of the following solid figures:



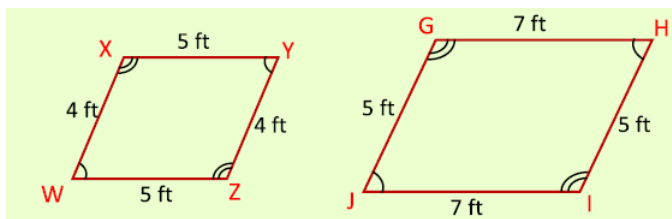
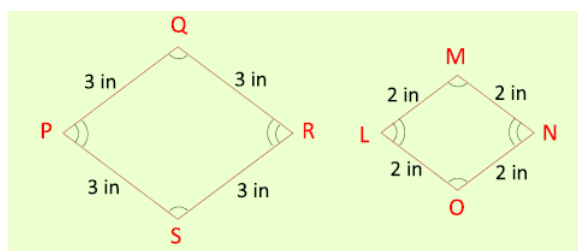
20. The volume of a pyramid is 240 cm^3 . The pyramid has a rectangular base with sides 6 cm by 4 cm. Find the altitude and lateral surface area of the pyramid if the pyramid has equal lateral edges.
21. The lateral edge of a regular tetrahedron is 8 cm. Find its altitude.
22. Find the volume of a cone of height 12 cm and slant height 13 cm.

23. Find the volume and surface area of a spherical football with a radius of 10 cm.
24. The lower base of a frustum of a regular pyramid is a square of side 6 cm, and the upper base has side length 3 cm. If the slant height is 8 cm, find: a. its lateral surface area b. its total surface area.
25. The areas of bases of a frustum of a pyramid are 25 cm^2 and 49 cm^2 . If its altitude is 3 cm, find its volume.
26. A frustum of a regular square pyramid whose lateral faces are equilateral triangles of side 10 cm has altitude 5 cm. Calculate the volume of the frustum.
27. The altitude of a pyramid is 10 cm. The base is a square whose sides are each 6 cm long. If a plane parallel to the base cuts the pyramid at a distance of 5 cm from the vertex, then find the volume of the frustum formed.
28. A frustum of height 5 cm is formed from a right circular cone of height 10 cm and base radius 4 cm. Calculate: a. the lateral surface area b. the volume of the frustum.
29. A frustum of a regular square pyramid has height 2 cm. The lateral faces of the pyramid are equilateral triangles of side 3 2 cm. Find the volume of the frustum.
30. A container is in the shape of an inverted frustum of a right circular cone as shown in Figure. It has a circular bottom of radius 20 cm, a circular top of radius 60 cm and height 40 cm. How many litres of oil could it contain?



31. Find the measure of an interior angle of a regular polygon with: a. 10 sides b. 20 sides c. 12 sides
32. Find the number of sides of a regular polygon, if the measure of each of its interior angles is: a. 150° b. 160° c. 190°
33. If the measure of a central angle of a regular polygon is 18° , find the measure of each of its interior angles.
34. Can a regular polygon be drawn such that the measure of each exterior angle is: a. 20° b. 16° c. 15°
35. Show that the area A of a square inscribed in a circle with radius r is $A = 2r^2$.
36. Find the length of a side and the perimeter of a regular nine-sided polygon with radius 5 units.
37. Find the radius of a regular hexagon with perimeter 48 units.
38. The radius of a circle is 12 units. Find the perimeter of a regular inscribed: a. triangle b. heptagon c. decagon

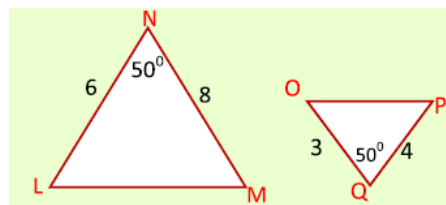
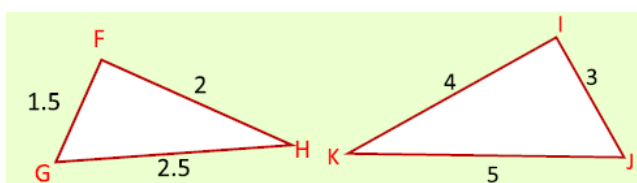
39. Decide whether or not each pair of polygons is similar. Explain your reasoning.



40. State whether each of the following statements is true or false.

- If two triangles are similar, then they are congruent.
- If two triangles are congruent, then they are similar.
- All equilateral triangles are congruent.
- All equilateral triangles are similar.

41. Which of the following pairs of triangles are similar? If they are similar, explain why.



- Two triangles are similar. A side of one is 2 units long. The corresponding side of the other is 5 units long. What is the ratio of:
 - their perimeters?
 - their areas?
- The areas of two similar triangles are 144 unit^2 and 81 unit^2 .
 - What is the ratio of their perimeters?
 - If a side of the first is 6 units long, what is the length of the corresponding side of the second?
- The sides of a polygon have lengths 5, 7, 8, 11, and 19 units. The perimeter of a similar polygon is 75 units. Find the lengths of the sides of the larger polygon.

Chapter 4

- Find the distance between the points $(\sqrt{2}, 1)$ and $(-\sqrt{6}, \sqrt{3})$.
- Let $A(3, -7)$ and $B(-1, 4)$ be two adjacent vertices of a square. Calculate the area of the square.
- Show that the plane figure with vertices:
 - $A(5, -1)$, $B(2, 3)$ and $C(1, 1)$ is a right angled triangle.
 - $A(-4, 3)$, $B(4, -3)$ and $C(3\sqrt{3}, 4\sqrt{3})$ is an equilateral triangle.
 - $A(2, 3)$, $B(6, 8)$ and $C(7, -1)$ is an isosceles triangle.
- Find the coordinates of the mid-point of the line segments joining the points $(1, 4)$ and $(-2, 2)$.
- Find the coordinates of the point R that divides the line segment joining the points $A(1, 3)$ and $B(-4, -3)$ in the ratio 2:3.

6. Find the gradients of the lines passing through the points P (3, 7) and Q (1, 9).
7. A (2, -3), B (7, 5) and C (-2, 9) are the vertices of triangle ABC. Find the gradient of each of the sides of the triangle.
8. Find the slope of the line whose angle of inclination is:
 - a. 30°
 - b. 75°
 - c. 150°
 - d. 90°
 - e. 0°
9. Find the angle of inclination of the line if its slope is:
 - a. $-\sqrt{3}$
 - b. 1
 - c. 0
 - d. $1/\sqrt{3}$
 - e. $-\sqrt{3}/3$
10. Find the equation of the line passing through the points A (-2, -4) and B (-1, 5).
11. Find the equation of the line with slope $m = -\pi$, passing through the point P(0, 0).
12. Find the equation of the line with slope $m = \pi$ and y-intercept $b = 2$.
13. Given $y + 2 = 2(x - 3y + 1)$, find the slope and y-intercept.
14. A line passes through the points A (5, -1) and B (-3, 3). Find:
 - a. the point-slope form of the equation of the line.
 - b. the slope-intercept form of the equation of the line.
 - c. the two-point form of the equation of the line. What is its general form?
15. Find the slope of the line that is perpendicular to the line joining P (2, -3) and Q (-3, -2).
16. Use slope to show that the quadrilateral ABCD with vertices A (-5, -2), B (-3, 1), C (3, 0) and D (1, -3) is a parallelogram.
17. Let L be the line with equation $2x - 3y = 6$. Find the slope-intercept form of the equation of the line that passes through the point P (2, -1) and is:
 - a. parallel to L
 - b. perpendicular to L .
18. Find the equation of a line passing through the point P and parallel to the line L for:
 - a. $L: 2x - 5y - 4 = 0$; $P(-1, 2)$
 - b. $L: 3x + 6 = 0$; $P(4, -6)$.
19. Find the equation of the line passing through the point P (2, 5) and:
 - a. parallel to the line passing through the points A (3, 1) and B (-1, 3)
 - b. parallel to the line $L: x + y = 2$
 - c. perpendicular to the line $L: y = x + 1$.
20. Determine k so that the line with equation $4x + ky = 12$ will be parallel to the line with equation $x = 3y$.
21. Find the distance from the point $P(-3, -5)$ to the line $2x - 3y + 11 = 0$.
22. Write the standard form of the equation of a circle with the given centre and radius.
 - a. $C(-2, 3), r = 5$
 - b. $C(8, 2), r = \sqrt{2}$
 - c. $C(-2, -1), r = 4$
23. Find the coordinates of the centre and the radius of the circle: $x^2 + y^2 - 8x + 12y - 12 = 0$

- 24.** Write the equation of the circle described below:
- It passes through the origin and has centre at (5,2).
 - It is tangent to the y-axis and has centre at (3, -4).
 - The end points of its diameter are (-2, -3) and (4, 5).
- 25.** A circle has centre at (5, 12) and is tangent to the line with equation $2x - y + 3 = 0$. Write the equation of the circle.
- 26.** Find the equation of the tangent line to the circle $x^2 + y^2 = 145$ at the point (9, -8).
- 27.** Write the equation of each parabola given below:
- Vertex(-2, 5); focus (-2, -8)
 - focus (-3, 12); directrix $y=3$
 - vertex (-2, 1); axis $y=1$; $p=1$
 - focus (5, 0); $p=4$; vertical axis
- 28.** Given the parabola $(y + 2)^2 = -16(x - 3)$. Find vertex, focus, directrix and sketch the graph.
- 29.** Write the equation of each ellipse given below:
- $C(0, 0)$; $a=6$, $b=4$; horizontal major axis
 - foci (-3, 0), (3, 0); $a=8$
- 30.** Given ellipse $\frac{(y + 2)^2}{25} + \frac{(x - 1)^2}{4} = 1$. Find centre, foci, vertices of ellipse and sketch the graph.
- 31.** Find the equation of the ellipse with
- centre at (1, 4) and vertices at (10, 4) and (1, 2).
 - foci at (-1, 0), (1, 0) and the length of the major axis 6 units.
 - vertex at (6, 0), focus at (-1, 0) and centre at (0, 0).
- 32.** Find the equation of the hyperbola with the information given below:
- centre at $C(0, 0)$; $a=8$, $b=5$, having horizontal transverse axis.
 - foci at $F(10, 0)$ and $F'(-10, 0)$; $2a=16$.
 - centre $C(-1, 4)$, $a=2$, $b=3$; vertical transverse axis.
- 33.** Name centre, foci, vertices and the equations of the asymptotes of each hyperbola below. Also sketch their graph.
- $\frac{x^2}{36} - \frac{y^2}{81} = 1$.
 - $\frac{y^2}{25} - \frac{x^2}{16} = 1$.
 - $\frac{(x + 3)^2}{9} - \frac{(y + 6)^2}{36} = 1$.
 - $\frac{(y - 3)^2}{25} - \frac{(x - 2)^2}{25} = 1$.
- 34.** Write the equation of each hyperbola satisfying the following conditions:
- Centre $C(4, -2)$; focus $F(7, -2)$; Vertex $V(6, -2)$
 - Centre $C(4, 2)$; Vertex $V(4, 5)$; equation of one asymptote is $4y - 3x = -4$
 - Vertices at $V(0, -4)$, $V'(0, 4)$; foci at $F(0, 5)$, $F'(0, -5)$