The 2nd Ethiopian Mathematical Olympiad

Second Round

Date: December 30, 2023 **Time limit: 3 hours 15 min**

Please enter the following information in **PRINT**.

NAME: **GENDER:** M ☐ F ☐

SCHOOL: **CLASS:**

INSTRUCTIONS AND REGULATIONS (please read it carefully):

- **For each of the following 12 multiple-choice questions mark with cross (×) the alternative answer which you find is true. If you, at some point, want to change the marking add a clear note. Only one of the alternatives A, B, C, D and E is true.**
- **For following 5 questions full solutions must be provided.**
- **No calculators, cellphones or other electronic devices are allowed.**
- **For your own calculations use scratch papers.**
- **If you need additional paper or go to the toilet please raise your hand and wait until one of the invigilators will approach you.**
- **Students communicating with each other during the test will be removed from the classroom.**
- **When the invigilators announces "TIME'S UP", please collect your papers (with this page on top) and hand them over immediately. Do NOT include scratch papers.**
- **Note that the total time allowed is 3 hours and 15 minutes.**

Wishing you Good Luck

Tilahun Abebaw, Layla Sorkatti and Paul Vaderlind

DO NOT write in the boxes below please

Problems	3 points	4 points	5 points	Open Questions	**Total Score**
Score	/12	/16	/20	/35	/83

Each correct answer will be awarded **3 points**.

1. The product $(8) \cdot (888 \ldots 8)$, where the second factor has k digits, is an integer whose digits have a sum of 1000. What is k ?

☐ (A) 901 ☐ (B) 911 ☐ (C) 919 ☐ (D) 987 ☐ (E) 991

2. A box contains three bags. One bag contains one white stones and three stones stones. Another bag also contains one white stone and three black stones. The remaining bag contains one white stone and four black stones. You reach into the box and randomly pull out a bag and then reach into the bag and randomly pull out one stone. What is the probability that this stone is white?

☐ (A) $\frac{1}{9}$ ☐ (B) $\frac{3}{13}$, ☐ (C) $\frac{7}{30}$ ☐ (D) $\frac{1}{4}$ ☐ (E) $\frac{11}{36}$

3. For how many positive integers n is $\frac{n}{30-n}$ also a positive integer?

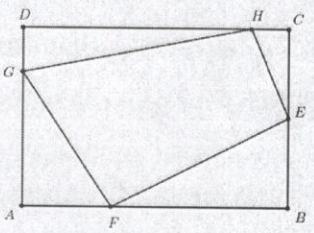
☐ (A) 4 ☐ (B) 5 ☐ (C) 6 ☐ (D) 7 ☐ (E) 8

4. The last non-zero digit of the number $2^{59} \cdot 3^4 \cdot 5^{53}$ is

☐ (A) 1 ☐ (B) 2 ☐ (C) 4 ☐ (D) 6 ☐ (E) 9

Each correct answer will be awarded **4 points**.

1. The point E divides the side BC of the rectangle $ABCD$ in two equal segments (in proportion $1:1$). The point F divides AB in proportion $1:2$. Point G divides DA in proportion $1:3$, and finally the point H divides CD in proportion $1:4$. Find the area of the quadrilateral $EFGH$ if the area of $ABCD$ equals 120.



☐ (A) 67 ☐ (B) 70 ☐ (C) 72 ☐ (D) 75 ☐ (E) 77

2. Dan and Sarah have each a measuring tape of length 1m. Dan's tape got stuck in a door and was extended by 4 cm. Sarah forgot her tape in a pocket and it shrank in laundry by 5 cm. However, on both tapes cm-marks remained evenly distributed.

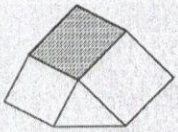
   Measuring the school yard, Dan noted the length of 23 m 75 cm. What length will Sarah get measuring the yard with her tape?

☐ (A) 23 m ☐ (B) 24 m ☐ (C) 25 m ☐ (D) 26 m ☐ (E) 27 m

3. Eric and Marina write poems. Each one writes two or three poems every day. During some period of time Eric wrote 43 poems while Marina wrote 61. How many days was this period of time?

☐ (A) 19 ☐ (B) 20 ☐ (C) 21 ☐ (D) 22 ☐ (E) 23

4. The figure is formed with two squares with sides 4 and 5 cm, a triangle with area 8 cm^2 and a shaded parallelogram. What is, in cm^2, the area of the parallelogram?



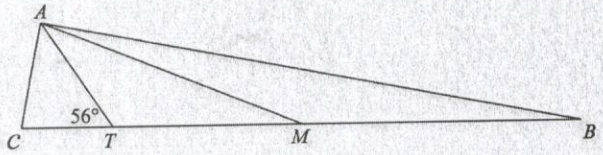
☐ (A) 15 ☐ (B) 16 ☐ (C) 18 ☐ (D) 20 ☐ (E) 21

Each correct answer will be awarded **5 points**.

1. Seven cards lie in a box. Numbers from 1 to 7 are written on these cards (exactly one number on each card). Tilahun takes, at random, 3 cards from the box and Layla takes 2 cards (two cards are left in the box). Then Tilahun tells Layla: "I know that the sum of the numbers on your cards is even". The sum of card's numbers that Tilahun has is equal to

☐ (A) 10 ☐ (B) 12 ☐ (C) 6 ☐ (D) 9 ☐ (E) 15

2. In the figure, M is a middle point of BC, $AM = MB$ and AT is a bisector of the angle BAC. What is a measure of the angle AMB if angle $ATC = 56°$?

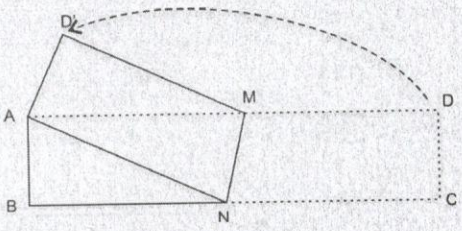


☐ (A) 120° ☐ (B) 135° ☐ (C) 142° ☐ (D) 150°, ☐ (E) 158°

3. How many different four-digit numbers are there with the sum of its digits being 9 and such that the digit 0 is not included?

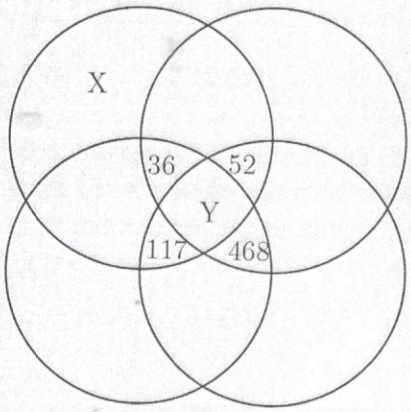
☐ (A) 32 ☐ (B) 36 ☐ (C) 40 ☐ (D) 48 ☐ (E) 56

4. A rectangular piece of paper $ABCD$ of size 4 cm $\times$16 cm is folded along the line MN such that vertex C coincides with vertex A, as shown in the picture. What is the area of the pentagon $ABNMD$?

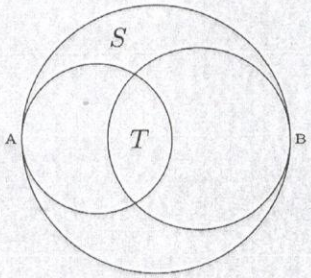


☐ (A) 17 ☐ (B) 27 ☐ (C) 37 ☐ (D) 47 ☐ (E) 57

1. Each of the four circles contains a number. To each intersection of two or more circles one assign the number which is a product of numbers in intersecting circles. Some of those products are shown in the figure below. What is the sum of numbers in regions X and Y?



2. We are given three circles with diameters 3, 4 and 5. The smaller circles are tangent to the larger circle from the inside, at opposite points A and B (see figure). The smaller circles divide the disk contained in the larger circle to several parts. We will take one part whose shape is that of a "curved triangle", i.e. has the arcs of three different circles as its boundary. Let its area be S, and let the area bounded by the intersection of the smaller circles be T.



Calculate $\frac{T}{S}$.

3. At least three people are sitting around a round table. T of them speak the truth and remaining L are liars. Those who speak the truth *always* do so, and liars *always* lie. Each one claims that both of his neighbours are of the same kind (either truth-tellers or liars). What can be the value of the ratio $\frac{L}{T}$? Find all possible answers.

4. For a rational number x the expression $3x^5 + 2x^2 + 1$ is an integer. Show that x is an integer.

The 3rd Ethiopian Mathematics Olympiad

Stage 2 - Regional Level

Date: May 10, 2025 **Time limit:** 4 hours

Please enter the following information in **PRINT**.

NAME:	**GENDER:** M ☐ F ☐
SCHOOL:	**CLASS:**

INSTRUCTIONS AND REGULATIONS (please read it carefully):

- **For each of the following 12 multiple-choice questions mark with cross (×) the alternative answer which you find is true. If you, at some point, want to change the marking add a clear note. Only one of the alternatives A, B, C, D and E is true. For the last five 7-points questions a full solution is required.**
- **No calculators, cellphones nor other electronic devices are allowed. Only paper, pen, eraser, straightedge and compass.**
- **Please do not make any calculation on the problem pages. Those are only for answers. For your own calculations use scratch papers.**
- **If you need additional paper or go to the toilet please raise your hand and wait until one of the invigilators will approach you.**
- **Students communicating with each other during the test will be removed from the classroom.**
- **When the invigilators announces "TIME'S UP", please collect your papers (with this page on top) and hand them over immediately. Do NOT include scratch papers.**
- **Note that the total time allowed is 4 hours.**

DO NOT write in the boxes below please

Problems	3 points	4 points	5 points	Open Questions	**Total Score**
Score	/12	/16	/20	/35	/83

Each correct answer will be awarded **3 points**.

1. At least how many people must be present in a cafe so that we can claim, that there must be at least two people of the same gender born in the same month among them?

   ☐ (A) 13 ☐ (B) 15 ☐ (C) 17 ☐ (D) 25 ☐ (E) 37

2. Including 1 and the number itself, how many distinct factors does 10^n have?

   ☐ (A) n^2+n ☐ (B) n^2+n+1 ☐ (C) n^2-2n+1 ☐ (D) n^2+2n+1 ☐ (E) n^2+2n

3. Consider a sphere of radius 3 with centre at the origin of a cartesian coordinate system. How many points on the surface of this sphere have integer coordinates??

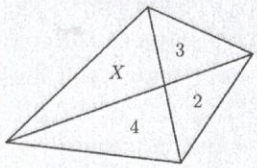
   ☐ (A) 30 ☐ (B) 24 ☐ (C) 12 ☐ (D) 6 ☐ (E) 3

4. Angelica is given an equilateral triangle ABC with side 3 cm. Her teacher asks her to draw a second triangle, whose sides are all parallel to the sides of $\triangle ABC$ with exactly 1 cm distance between the sides of $\triangle ABC$ and the sides of the new triangle. How many different triangles can Angelica draw for her teacher?

   ☐ (A) 1 ☐ (B) 2 ☐ (C) 3 ☐ (D) 4 ☐ (E) 8

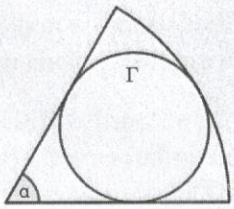
Each correct answer will be awarded **4 points**.

1. Two diagonals split the quadrilateral into four triangles. The areas of three of them are shown in the figure. Find the area of the fourth triangle.



   ☐ (A) 5 ☐ (B) 6 ☐ (C) $4\sqrt{2}$ ☐ (D) 7 ☐ (E) $5\sqrt{2}$

2. The circle Γ is inscribed in the sector with the radius 18 and the angle $\alpha = 60^o$. The area of the circle Γ is equal to



   ☐ (A) 12π ☐ (B) 30π ☐ (C) 36π ☐ (D) 24π ☐ (E) 18π

3. How many prime numbers p are there such that the number p^2+2 also is a prime number?

   ☐ (A) 0 ☐ (B) 1 ☐ (C) 2 ☐ (D) 3 ☐ (E) infinitely many

4. One of the solutions to the equation $x^3 - 5x^2 + 5x - 1 = 0$ is $x = 2 + \sqrt{3}$. What is the sum of the remaining two solutions?

☐ (A) 4 ☐ (B) $5 + \sqrt{3}$ ☐ (C) $3 + \sqrt{3}$ ☐ (D) $3 - \sqrt{3}$ ☐ (E) $2 + 2\sqrt{3}$

Each correct answer will be awarded **5 points**.

1. The sum of the first 4 numbers of an arithmetical progression is 12, and the sum of the first 12 numbers is 4. What is the sum of the first 16 elements?

☐ (A) -2 ☐ (B) -6 ☐ (C) -12 ☐ (D) -16 ☐ (E) -20

2. Given real numbers a, b and c, such that $a > b, b > \frac{1}{c}$ and $\frac{1}{a} > c$. Which statement below must necessary be true?

☐ (A) $c > 0$ ☐ (B) $c < 0$ ☐ (C) $c > a$ ☐ (D) $a > 0$ ☐ (E) none of previous must necessarily be true

3. There are 200 lightbulbs in a room, numbered from 1 to 200. Each lightbulb can be turned on and of with a normal switch. From the beginning all lightbulbs are in the "off" position.

   In the first move all the switches which are a multiples of 1 are flipped (i.e. all lightbulbs are turned on).

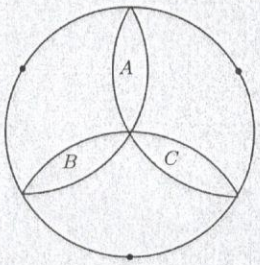
   Then, all the even-numbered (multiples of 2) switches are flipped.

   In the following steps all switches marked by multiples of 3 are flipped, then those marked by multiples of 4, and so on, until, in the last, 200-th step only the switch marked by 200 is flipped.

   Which of the following lightbulbs will be turned on at the end of the process?

☐ (A) 165 ☐ (B) 166 ☐ (C) 167 ☐ (D) 168 ☐ (E) 169

4. Within a circle with radius 1, three circular arcs are drawn, each with radius 1 and centred in the vertices of an equilateral triangle inscribed in the circle. The three regions A, B and C are then removed. What is the area of the remaining part of the circle?



☐ (A) $\frac{3\sqrt{3}}{2}$ ☐ (B) $\frac{\pi\sqrt{3}}{4}$ ☐ (C) $\pi - \frac{3\sqrt{3}}{4}$ ☐ (D) $\pi - \frac{3\sqrt{3}}{2}$ ☐ (E) $6 - \pi$

Each correct SOLUTION will be awarded **7 points**.

1. After school Paul goes to a busstop and takes the first bus which arrives (only two buses make stop there). The bus going to the right takes him to the library, while the bus going to the left takes him to the football field. The buses in each direction go regularly every 10 minutes. After a few months Paul discovers that he has been four times as often at the football field than in the library. How can we explain this asymmetry?

2. Let a and b be two positive integers such that a^2 is divisible by $a+b$. Show that b^2 also is divisible by $a+b$.

The Fourth Rwanda Mathematics Competition

Stage 2 - regional elimination

Date: April 29th, 2022 **Time limit: 3.5 hours**

Please enter the following information in **PRINT**.

NAME:	**GENDER:** M ☐ F ☐
SCHOOL:	**CLASS:**

INSTRUCTIONS AND REGULATIONS (please read it):

- **This stage of the RMC consists of two parts: 11 Multiple-choice questions and 5 Open questions.**
- **For each of the 11 multiple-choice questions mark with cross ($\times$) the answer which you find is true. If you, at some point, want to change the marking add a clear note. Only one of the alternatives A, B, C, D and E is true.**
- **For each of the 5 open questions full, well-justified solutions are requested.**
- **No calculators, cellphones nor other electronic devices are allowed. Only paper, pen, eraser, straightedge and compass.**
- **For multiple-choice questions, please do not make any calculation on the problem pages. Those are only for answers. For your own calculations use scratch papers.**
- **If you need additional paper or go to the toilet, please raise your hand and wait until one of the invigilators will approach you and address your request.**
- **Students communicating with each other during the test will be asked to leave the classroom.**
- **When the invigilators announces "TIME'S UP". please collect your papers (with this page on top) and hand them over immediately. Do NOT include scratch papers.**
- **Note that the total time allowed is 3.5 hours.**

Please, **DO NOT** write in the boxes below.

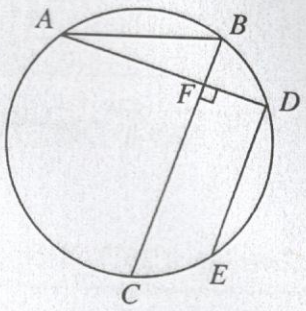
Problems	3 points	4 points	5 points	Open	Total Score
Score	/12	/12	/20	/35	/79

NAME:

PART 1

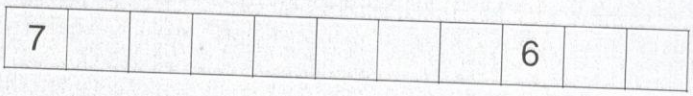
Each correct answer to the first set of four questions will be awarded **3 points**.

1. In the adjacent figure, AB and BC are chords of a circle with $AB < BC$. Let D be the point on the circle such that AD is perpendicular to BC and let E be the point on the circle such that DE is parallel to BC. What is the measure of $\angle ABE$?



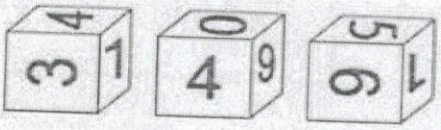
☐ (A) $45°$ ☐ (B) $75°$ ☐ (C) $105°$ ☐ (D) $60°$
☐ (E) $90°$

2. In the picture below we have 11 fields. In the first field we have number 7 and in the ninth field we have number 6. What number has to be placed in the second field if the following condition has to be true: sums of each three successive fields are equal to 21?



☐ (A) 7 ☐ (B) 8 ☐ (C) 6 ☐ (D) 10 ☐ (E) 5

3. There are three identical cubes in the picture below, with numbers 0, 1, 3, 4, 5, 9 written identically on each cube. What is the total sum of numbers on the bottom side of the cubes?



☐ (A) 6 ☐ (B) 8 ☐ (C) 9 ☐ (D) 10 ☐ (E) 12

4. The result of a school-test was as follows: the average mark for the class was 9.4, while the average mark for the boys was 8.6, and for the girls – 9.8. What ratio of the whole class are boys?

☐ (A) $\frac{3}{4}$ ☐ (B) $\frac{2}{3}$ ☐ (C) $\frac{1}{2}$ ☐ (D) $\frac{1}{3}$ ☐ (E) $\frac{1}{4}$

Each correct answer to the next set of four questions will be awarded **4 points**.

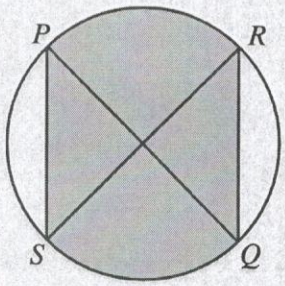
5. If a, b, c, d and e are different integers and $(4-a)(4-b)(4-c)(4-d)(4-e) = 12$, then $a+b+c+d+e$ equals

☐ (A) 17 ☐ (B) 20 ☐ (C) 24 ☐ (D) 32 ☐ (E) 40

6. Using the digits 1, 4 and 8 (each digit at most one time) it is possible to compose four perfect squares: $1 = 1^2, 4 = 2^2, 81 = 9^2, 841 = 29^2$. How many different perfect squares is possible to compose using the digits 1, 6 and 9?

☐ (A) 2 ☐ (B) 3 ☐ (C) 4 ☐ (D) 5 ☐ (E) 6

7. In the adjacent figure, PQ and RS are diameters of a circle with radius 4. If PQ and RS are perpendicular, what is the area of the shaded region?



☐ (A) $8 + 4\pi$ ☐ (B) $8 + 8\pi$ ☐ (C) $16+4\pi$ ☐ (D) $16+8\pi$
☐ (E) $16 + 16\pi$

Each correct answer to the last set of four questions will be awarded **5 points**.

9. A family drives from a city to the beach, at 60 km/h. On the return trip the speed is 20 km/h. What is the average speed for the whole trip?

☐ (A) 24 km/h ☐ (B) 30 km/h ☐ (C) 40 km/h ☐ (D) 45 km/h ☐ (E) 50 km/h

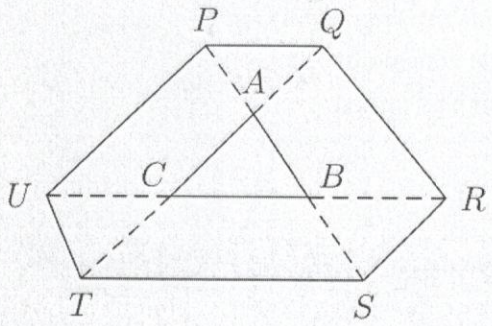
10. Sides of the triangle ABC are extended on both sides to points P, Q, R, S, T and U (see the figure below) so that

$PA = AB = BS$,

$TC = CA = AQ$ and

$UC = CB = BR$.

Moreover, the area of triangle ABC is 1. What is the area of the hexagon $PQRSTU$?



☐ (A) 9 cm^2 ☐ (B) 10 cm^2 ☐ (C) 12 cm^2 ☐ (D) 13 cm^2 ☐ (E) 15 cm^2

11. M and N are two natural numbers, such that $51 \leq N \leq 101$ and $19 \leq M \leq 49$. What is the greatest integer value (whole number) of the expression $\dfrac{N+M}{N-M}$?

☐ (A) 20 ☐ (B) 30 ☐ (C) 40 ☐ (D) 50 ☐ (E) 60

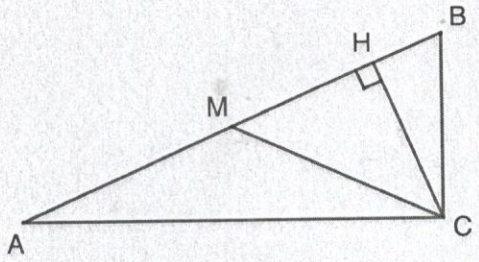
12. How many ten-digit numbers composed only of digits 1, 2 and 3 exist, in which any two neighboring digits differ by 1 (it is not necessary that all three digits must occur in such a number).

☐ (A) 16 ☐ (B) 32 ☐ (C) 64 ☐ (D) 80 ☐ (E) 100

NAME: **PART 2**

Each **solution** will be awarded up to **7 points**.

1. In the right triangle ABC, with the right angle at C, the length of the height $CH = 1$ cm and the length of the median $CM = 2$ cm. Find the angle BAC.



2. A positive integer n has 2022 digits, all of them being 9. What are the digits of n^2.

3. 100 participants of The Rwanda Mathematics Competition had to solve 4 problems. 90 students solved the first problem, 80 solved the second problem. 70 students solved the third problem, and 60 solved the fourth. None managed to solve all 4 problems. Only those who succeeded with the third and the fourth problems were announced the winners. How many winners were there?

4. Find all prime numbers p such that $p^3 + p^2$ is a perfect square.

5. For which real numbers a the equation $3^{2x} - 3^x + 1 = a$ has two different real positive roots.