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**NATIONAL EDUCATIONAL ASSESSMENT AND EXAMINATIONS AGENCY
(NEAEA)
ETHIOPIAN SECONDARY SCHOOL LEAVING CERTIFICATE EXAMINATION
(ESSLCE)
MATHEMATICS FOR NATURAL SCIENCE STREAM
TIKIMT 2013/14 NOVEMBER 2021**

BOOKLET CODE: 29
Number of Items: 65

SUBJECT CODE: 02
Time Allowed: 3 hours

Given two vectors $u = (1, 3)$ and $v = (-3, 5)$ in the plane. Then $6u + 2v$ is equal to

- A. (0, 18) C. (2, 28)
- B. (3, 3) D. (0, 28)

Which one of the following is the standard form of the equation of the circle centered at $(2, -3)$ and radius 5?

- A. $(x - 2)^2 + (y - 3)^2 = 25$
- B. $(x - 2)^2 + (y + 3)^2 = 25$
- C. $(x - 2)^2 + (y + 3)^2 = 5$
- D. $(x + 2)^2 + (y - 3)^2 = 25$

3. If f is a continuous function on $[a, b]$ and F is its anti-derivative, then which of the following is true?

- A. $\int_a^b F'(x) dx = f(b) - f(a)$
- B. $\int_a^b F(x) dx = f(b) - f(a)$
- C. $\int_a^b f(x) dx = F(b) - F(a)$
- D. $\int_a^b f'(x) dx = F(b) - F(a)$

4. Which of the following defines the equation of a sphere whose center is at $(0, -1, 2)$ and radius 6 units?

- A. $x^2 + (y+1)^2 + (z-2)^2 = 36$
- B. $x^2 + (y-1)^2 + (z-2)^2 = 36$
- C. $x^2 + (y+1)^2 + (z-2)^2 = 12$
- D. $x^2 + (y+1)^2 + (z+2)^2 = 36$

ANSWER (by Qube Academy, Qube)

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Booklet code 29.

1. Solution

Given two vectors $u = (1, 3)$ and $v = (3, 5)$

then $6u + 2v$

$$\begin{aligned} \text{let } w &= 6u + 2v = 6(1, 3) + 2(3, 5) \\ &= (6, 18) + (6, 10) \\ &= (6+6, 18+10) \\ &= (12, 28) \end{aligned}$$

ANSWER: D

2. Solution

Given center = $C(2, -3)$ and radius = $r = 5$

We know that the general formula of center is given by $(x-h)^2 + (y-k)^2 = r^2$, where (h, k) is center and r is radius

Thus $(x-2)^2 + (y+3)^2 = 25$ is the standard eqn of the circle

ANSWER: B

3. Solution

If f is continuous function on $[a, b]$ and F is its antiderivative, then ~~which~~ the following is true:-

$\int_a^b f(x) dx = F(b) - F(a)$, if $F'(x) = f(x)$ for all $x \in [a, b]$

ANSWER: C

BOOKLET CODE: 29

5. Which of the following represents the vector $(3, -5, 7)$ in the space using the standard unit vectors i, j and k ?
- A. $3i - 5j - 7k$ C. $3i - 5j + 7k$
 B. $3i + 5j + 7k$ D. $-3i - 5j - 7k$
6. Which one of the following is the scalar (dot) product of the vectors $(2, -4, 6)$ and $(-2, -1, 0)$?
- A. 0 C. 8
 B. -8 D. 6
7. What are the domain and range of the function $f(x) = 2x^{\frac{3}{4}}$ respectively?
- A. $\mathbb{R}$ and $[0, \infty)$ C. $[0, \infty)$ and $\mathbb{R}$
 B. $[0, \infty)$ and $(0, \infty)$ D. $[0, \infty)$ and $[0, \infty)$
8. Which one of the following defines a one-to-one function?
- A. $f = \{(x, y): x \text{ is a sister of } y\}$ C. $f = \{(x, y): y = x^2 + 1\}$
 B. $f = \{(x, y): y = 3x - 1\}$ D. $f = \{(x, y): y \text{ is a father of } x\}$

4. Solution

Given center = $C(0, -1, 2)$ and radius = $r = 6$
the standard eqn of sphere with center $C(a, b, c)$ and radius r is given by: $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$

$$\Rightarrow (x-0)^2 + (y+1)^2 + (z-2)^2 = (6)^2$$

$$\Rightarrow x^2 + (y+1)^2 + (z-2)^2 = 36$$

ANSWER: A

5. Solution

Given vector $(3, -5, 7)$
the standard vector of the given vector in terms of unit vectors is written as:

ANSWER: C $3i - 5j + 7k$

6. Solution

Given vector $(2, -4, 6)$ and $(-2, -1, 0)$

Let $\vec{a} = (2, -4, 6)$ and $\vec{b} = (-2, -1, 0)$

$$\text{thus } \vec{a} \cdot \vec{b} = (2, -4, 6) \cdot (-2, -1, 0) = -4 + 4 + 0 = 0$$

ANSWER: A

7. Solution

Given $f(x) = 2x^{3/4}$

Domain = $[0, \infty)$

since, for x less than 0 the function is undefined.

Range = $[0, \infty)$

since, for every value of x , y is greater than 0

ANSWER: D

8. Solution

Note any linear function is one-to-one function.

thus $f = \{(x, y) : y = 3x - 1\}$ is one-to-one function

ANSWER: B

9. The equation of a circle is given by $(x-3)^2 + (y+2)^2 = 9$. What are the center and the radius of the circle respectively?
- A. $(3, -2)$ and 3
B. $(3, 2)$ and 3
C. $(3, -2)$ and 9
D. $(3, 2)$ and 9
10. Which one of the following sentences has a truth value?
- A. Man is selfish.
B. We shall win!
C. Man is mortal!
D. Tomorrow is a beautiful day.
11. What is the determinant of the matrix $\begin{bmatrix} 1 & -7 \\ 6 & 5 \end{bmatrix}$?
- A. -37
B. -47
C. 47
D. 37
12. The additive inverse of the complex number $3 - 4i$ is
- A. $3 + 4i$
B. $-3 - 4i$
C. $3 - 4i$
D. $-3 + 4i$
13. What is the modulus of the complex number $-6 + 8i$?
- A. 5
B. 2
C. 10
D. 8

9. Solution

Given eqn of circle $(x-3)^2 + (y+2)^2 = 9$

but the standard eqn of circle is given by:

$$(x-h)^2 + (y-k)^2 = r^2, \text{ with } C(h,k) \text{ centre and } r, \text{ radius}$$

thus compare the two equations.

$$h=3, k=-2, r=\sqrt{9}=3 \text{ thus center} = C(h,k) = (3, -2)$$

ANSWER: A

10. Solution

- * A sentence which has a truth value (TRUE or FALSE) is said to be proposition statement.
- * from the given choices "Man is mortal" is a sentence which has a truth value

Solution: C

11. Solution
Given matrix $\begin{bmatrix} 1 & -7 \\ 6 & 5 \end{bmatrix}$

$$\begin{vmatrix} 1 & -7 \\ 6 & 5 \end{vmatrix} = (1 \times 5) - (-7 \times 6) = 5 + 42 = 47$$

ANSWER: C

12. Solution

Given complex number $3-4i$

and its additive inverse is given as:

ANSWER:

13. Solution

Let $z = -6+8i$

The modulus of z is given by

$$\begin{aligned} |z| &= \sqrt{(-6)^2 + (8)^2} \\ &= \sqrt{36+64} = \sqrt{100} = 10 \end{aligned}$$

$$|z| = 10$$

ANSWER: C

BOOKLET CODE: 29

14. What are the modulus and argument of the complex number $2 + 2i$ respectively?

- A. $2\sqrt{2}$ and $\frac{\pi}{4}$ C. $2\sqrt{2}$ and $-\frac{\pi}{4}$
B. 2 and $\frac{\pi}{4}$ D. 4 and $\frac{\pi}{4}$

15. Consider the following frequency distribution table.

Value	5	3	7	6	8
Frequency	3	4	5	3	5

Table 1

What is the sixth decile?

- A. 6.5 C. 7.5
B. 5 D. 7
16. In a certain health center there are 3 doctors, 8 nurses and 2 physicians. In how many ways one can form a group of 5 members consisting of 1 doctor, 3 nurses and 1 physician?
- A. 168 C. 336
B. 112 D. 330

14. Solution

Given let $z = 2 + 2i$

then, Modulus $= |z| = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$

$\arg(z) = \tan^{-1}\left(\frac{2}{2}\right) = \tan^{-1}(1) = \pi/4$

ANSWER: A

15. Solution

Given frequency distribution table:

Value	5	3	7	6	8
Frequency	3	4	5	3	5

* The decile of ungrouped data with even number of observation is given by:

$$D_i = \left(\frac{\frac{in}{10} + \frac{\frac{in}{10} + 1}{2} \right)^{th} \text{ item.}$$

So, to find the sixth decile first increasing order of magnitude

Arrange the data (Value) in

Value	3	5	6	7	8
Frequency	4	3	3	5	5

then, N = total frequency = 20
 $i = 6$ (sixth decile)

$$\text{then } D_6 = \left(\frac{\frac{6 \times 20}{10} + \left(\frac{6 \times 20}{10} + 1 \right)}{2} \right)^{th} \text{ item} = \left(\frac{12 + (12 + 1)}{2} \right)^{th} \text{ item}$$

$$= \left(\frac{12 + 13}{2} \right)^{th} \text{ item} = \frac{25^{th} \text{ item}}{2} = 12.5^{th} \text{ item} = 7$$

ANSWER: D

16. Solution

Soln: 1 doctor, 3 nurses and 1 physician can be selected in

$$C(3,1) \times C(8,3) \times C(2,1) = 3 \times 56 \times 2 = 336$$

Remember combination rule.

ANSWER: C

17. Which one of the following forms represents an exhaustive set of events if a die is thrown once?
- A. $\{1, 2\}, \{4\}, \{4, 5, 6\}$ C. $\{1, 2\}, \{2\}, \{4, 5\}, \{5, 6\}$
B. $\{1, 2, 3\}, \{4, 5\}, \{6\}$ D. $\{1, 2, 3\}, \{3, 4\}, \{6\}$
18. A bag contains 2 red, 3 yellow and 5 green balls. One ball is drawn at random. What is the probability that the drawn ball is NOT yellow?
- A. $\frac{3}{10}$ C. $\frac{4}{5}$
B. $\frac{1}{5}$ D. $\frac{7}{10}$
19. Which of the following is the product AB of the two matrices $A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 3 & 0 \end{bmatrix}$?
- A. $\begin{bmatrix} -11 & -10 \\ 11 & 5 \end{bmatrix}$ C. $\begin{bmatrix} 11 & -10 \\ 11 & 5 \end{bmatrix}$
B. $\begin{bmatrix} -11 & 10 \\ 11 & 5 \end{bmatrix}$ D. $\begin{bmatrix} -11 & 10 \\ 11 & -5 \end{bmatrix}$

17. Solution

* Exhaustive set of events: are events where at least one of them must necessarily occur every time the experiment is performed.

eg. If a die is thrown once

* the sample space is $S = \{1, 2, 3, 4, 5, 6\}$

From this, the events $\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$ are exhaustive events. The events $\{1, 2\}, \{3, 4\}, \{5, 6\}$ are also exhaustive events for this experiment.

Generally, events $E_1, E_2, \dots, E_n$ form a set of exhaustive events of a sample space S where $E_1, E_2, \dots, E_n$ are subsets of S and $E_1 \cup E_2 \cup \dots \cup E_n = S$

ANSWER: B

18. Solution

$$n(S) = 2 + 3 + 5 = 10$$

$$\rightarrow P(\text{yellow}) = \frac{3}{10}$$

$$\rightarrow P(\text{Not yellow}) = 1 - P(\text{yellow}) = 1 - \frac{3}{10} = \frac{10-3}{10} = \frac{7}{10}$$

ANSWER: D

19. Solution

Given $A = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 5 \\ 3 & 0 \end{bmatrix}$

then $AB = \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} -1 & 5 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} (-2 \ 3) \begin{pmatrix} -1 \\ 3 \end{pmatrix} & (-2 \ 3) \begin{pmatrix} 5 \\ 0 \end{pmatrix} \\ (1 \ 4) \begin{pmatrix} -1 \\ 3 \end{pmatrix} & (1 \ 4) \begin{pmatrix} 5 \\ 0 \end{pmatrix} \end{bmatrix}$

$$AB = \begin{bmatrix} 11 & -10 \\ 11 & 5 \end{bmatrix}$$

ANSWER: C

20. Which one of the following is **NOT** true about the determinant of a given square matrix A ?
- A. If A contains a row of zeros, then its determinant is zero.
 - B. If A has identical rows, then its determinant is zero.
 - C. The determinant of A is equal to the determinant of its transpose.
 - D. Interchanging two rows of A gives the same determinant.
21. Which of the following is the inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$?
- A. $\begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$
 - B. $\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$
 - C. $\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$
 - D. $\begin{bmatrix} -4 & -3 \\ -3 & -2 \end{bmatrix}$
22. Consider the system of linear equation $\begin{cases} 3x - ay = 1 \\ 6x + 4y = 5 \end{cases}$. Then which of the following is true?
- A. If $a > -2$, then the system has a unique solution.
 - B. If $a = -2$, then the system has a unique solution.
 - C. If $a = 2$, then the system has no solution.
 - D. If $a > 2$, then the system has infinitely many solutions.

20. Solution

⊗ Properties of a given square matrix A and B

- The determinant of A is equal to the determinant of its transpose: $|A| = |A^T|$
- If B is found by interchanging two rows (columns) of A, then $|B| = -|A|$
- If B is found by multiplying one row (one column) of A by scalar r, then $|B| = r|A|$
- If A has a row (or a column) of zeros, then the determinant of A is zero
- If A has two identical rows (or columns), then the determinant of A is zero.

ANSWER: D

21. Solution

let $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$, $A^{-1} = \frac{\text{adj} A}{|A|}$

thus, $|A| = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = 8 - 9 = -1$ and $\text{adj} A = (\text{cofactor } A)^T = \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$

therefore $A^{-1} = \frac{\text{adj} A}{|A|} = \frac{\begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}}{-1} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix}$

ANSWER: A

22. let first find the solution in terms of 'a'

$$-2/3x - ay = 1$$

$$+ \quad 6x + 4y = 5$$

$$\hline 2ay + 4y = 3$$

$$y(2a + 4) = 3$$

$$y = \frac{3}{2a + 4} \quad \text{--- (1)}$$

then solve for 'x'

$$6x + 4y = 5$$

$$x = \frac{5 - 4y}{6} = \frac{5 - 4\left(\frac{3}{2a + 4}\right)}{6}$$

$$x = \frac{5 - \frac{12}{2a + 4}}{6} = -\frac{1}{a + 2} + \frac{5}{6}$$

$$\text{now, } y = \frac{3}{2a + 4} \text{ and } x = \frac{5}{6} - \frac{1}{a + 2}$$

When $2a + 4 = 0$ or $a + 2 = 0$

$\Rightarrow a = -2$ the system has no solution because y and x is undefined at $a = -2$

ANSWER: C

23. What is the magnitude of the vector whose initial point is at $(1, -3)$ and terminal point is at $(4, 1)$?
- A. 5
B. 6
C. 1
D. 4
24. Let T be a translation that takes the origin into $(2, 3)$. Then which of the following is true about the translation T ?
- A. $(x - 4)^2 + (y - 3)^2 = 9$ is the translation of $(x - 2)^2 + (y + 6)^2 = 9$.
B. $(x - 5)^2 + (y - 6)^2 = 9$ is the translation of $(x - 3)^2 + (y - 3)^2 = 9$.
C. $(x - 4)^2 + (y + 3)^2 = 12$ is the translation of $(x - 2)^2 + (y + 6)^2 = 12$.
D. $(x + 4)^2 + (y - 3)^2 = 12$ is the translation of $(x - 2)^2 + (y + 6)^2 = 12$.

23. **Solution**

let point $A = (1, -3)$ & initial point
point $B = (4, 1)$ & terminal point

$$\text{the } \|AB\| = \sqrt{(4-1)^2 + (1-(-3))^2} = \sqrt{9+16} = \sqrt{25} = 5$$

Solution: A

24. **Solution**

given $T((0,0)) = (2,3) \Rightarrow W = (2,3)$ & translation vector

thus, (A) $(x-2)^2 + (y+6)^2 = 8$ is translated to:

$$\Rightarrow (x-2-2)^2 + (y+6-3)^2 = 8 \Rightarrow (x-4)^2 + (y+3)^2 = 8$$

(B) $(x-3)^2 + (y-3)^2 = 9$ is translated to:

$$\Rightarrow (x-3-2)^2 + (y-3-3)^2 = 9 \Rightarrow (x-5)^2 + (y-6)^2 = 9$$

(C) $(x-2)^2 + (y+6)^2 = 12$ is translated to:

$$\Rightarrow (x-2-2)^2 + (y+6-3)^2 = 12 \Rightarrow (x-4)^2 + (y+3)^2 = 12$$

(D) $(x-2)^2 + (y+6)^2 = 12$ is translated to

$$\Rightarrow (x-2-2)^2 + (y+6-3)^2 = 12 \Rightarrow (x-4)^2 + (y+3)^2 = 12$$

BOOKLET CODE: 29

25. Which of the following functions represents, the graph shown in Figure 2?

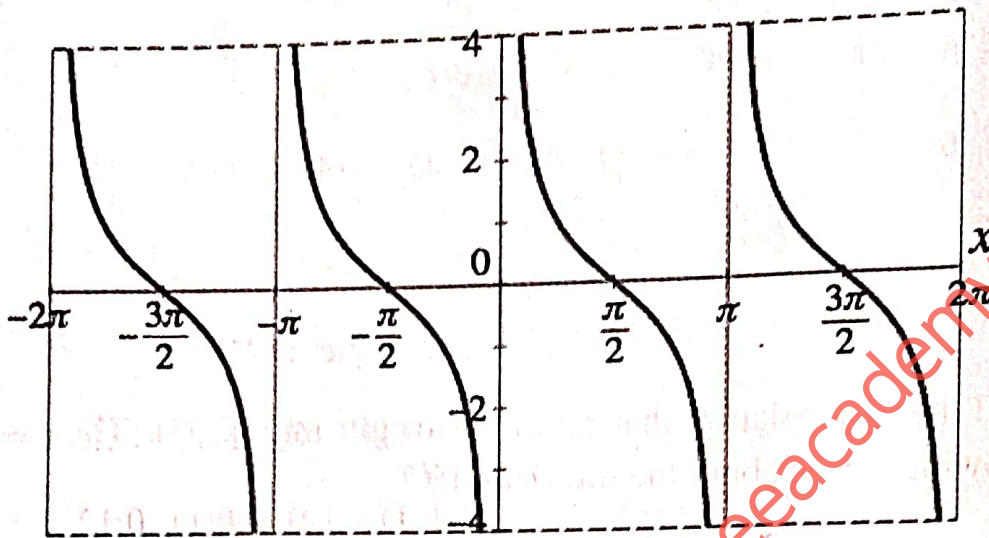


Figure 2

- A. $f(x) = \tan x$ C. $f(x) = \csc x$
 B. $f(x) = \sec x$ D. $f(x) = \cot x$

26. Which of the following is true?

- A. The domain of $y = \tan^{-1}x$ is $(-\infty, \infty)$.
 B. The domain of $y = \cos^{-1}x$ is $(-1, 1)$.
 C. $\sin^{-1}(0) = \frac{\pi}{2}$.
 D. $\tan^{-1}(0) = 1$.

25. Solution

the given graph is the graph of $f(x) = \cot x$ function

~~ANSWER: D~~

26. Solution

<u>function</u>	<u>Domain</u>	<u>Range</u>
* $y = \tan^{-1}(x)$	$(-\infty, \infty)$	$(-\pi/2, \pi/2)$
* $y = \cos^{-1}(x)$	$[-1, 1]$	$[0, \pi]$
* $y = \sin^{-1}(x)$	$[-1, 1]$	$(-\pi/2, \pi/2)$

⊗ $\sin^{-1}(0) = 0$, $\tan^{-1}(0) = 0$

Solution: A

~~27.~~

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27. Which one of the following is the fourth term of the sequence $\left\{\cos\left(\frac{n\pi}{2}\right)\right\}$?
- A. 0
B. -1
C. 1
D. 0.5
28. Which of the following is a geometric sequence?
- A. 1, 1.5, 2.5, 3.5, ...
B. 0.1, 0.01, 0.0011, 0.000111, ...
C. 8, -4, 2, -1 ...
D. 0.1, 0.11, 0.12, 0.13, ...
29. A factory that produces electric cable had sales Birr 1,000,000 the first day. During each successive days the sales increased by Birr 10,000 every day. What is the total sales of the factory in the first 10 days?
- A. Birr 10,000,000
B. Birr 1,090,000
C. Birr 20,900,000
D. Birr 10,450,000
30. What is the least upper bound of the sequence $\left\{2 + \frac{n}{n+1}\right\}_{n=1}^{\infty}$?
- A. 1
B. 0
C. 3
D. 2

27. SOLUTION

given sequence $A_n = \cos\left(\frac{n\pi}{2}\right)$

then $A_4 = \cos\left(\frac{4\pi}{2}\right)$
 $= \cos(2\pi) = 1$

ANSWER: C

28. SOLUTION

* A Geometric sequence is characterized by a common ratio (r) between successive terms

* let $\{G_n\}$ be a geometric sequence if and only if
 $\frac{G_2}{G_1} = \frac{G_3}{G_2} = \frac{G_4}{G_3} = \dots = \frac{G_{n+1}}{G_n} = r, r \in \mathbb{R} \setminus \{0\}$

thus from the given choice only choice 'C' fulfills this condition
 $r = \frac{-4}{8} = \frac{2}{-4} = \frac{-1}{2} \dots = \frac{-1}{2}$

ANSWER: C

29. SOLUTION

Given

$A_1 = 1,000,000$ & first day

$A_2 = A_1 + 10,000 = 1,000,000 + 10,000 = 1,010,000$

$A_3 = A_2 + 10,000 = 1,010,000 + 10,000 = 1,020,000$

...

$A_{10} - A_9 = A_2 - A_1 = 10,000 = d$

and $A_3 - A_2 = A_2 - A_1 = 10,000 = d$

thus it's Arithmetic sequence.

thus $S_{10} = \frac{n}{2}(2A_1 + (n-1)d), \quad n=10$

$S_{10} = \frac{10}{2}(2 \times 1,000,000 + (10-1) \times 10,000)$

$S_{10} = 5(2,000,000 + 90,000)$

$S_{10} = 5(2,090,000)$
 $= 10,450,000$

ANSWER: D

BOOKLET CODE: 29

31. Which one of the following sequences is a null sequence?

A. $\left\{ n \sin\left(\frac{1}{n}\right) \right\}$

C. $\left\{ \frac{n}{n+1} \right\}$

B. $\left\{ 1 - \frac{1}{n} \right\}$

D. $\left\{ \frac{\cos n}{n} \right\}$

32. Suppose $\{a_n\}$ and $\{b_n\}$ are sequences with $\lim_{n \rightarrow \infty} a_n = L$ and $\lim_{n \rightarrow \infty} b_n = M$. Then which of the following is **NOT** necessarily true?

A. $\lim_{n \rightarrow \infty} (3a_n + 2b_n) = 3L + 2M$

C. $\lim_{n \rightarrow \infty} \left(\frac{a_n}{b_n} \right) = \frac{L}{M}$

B. $\lim_{n \rightarrow \infty} (a_n - b_n) = L - M$

D. $\lim_{n \rightarrow \infty} a_n b_n = LM$

33. $\lim_{x \rightarrow \infty} \frac{3x^2 + 2x}{5x - x^2}$ is equal to

A. -1

C. 1

B. -3

D. 0

31. Solution

* Null sequence means, a sequence which its limit is approaches to 0 as $n \rightarrow \infty$
 i.e. if a_n is a sequence, then a_n is null sequence if

$$\lim_{n \rightarrow \infty} a_n = 0$$

$$\text{Thus (D) } \lim_{n \rightarrow \infty} \left\{ \frac{\cos n}{n} \right\} = 0$$

ANSWER: D

32. Solution

Suppose $\{a_n\}$ and $\{b_n\}$ are sequences with $\lim_{n \rightarrow \infty} a_n = L$ and $\lim_{n \rightarrow \infty} b_n = M$

$$\text{Then: } * \lim_{n \rightarrow \infty} (3a_n + 2b_n) = \lim_{n \rightarrow \infty} 3a_n + \lim_{n \rightarrow \infty} 2b_n = L + M$$

$$* \lim_{n \rightarrow \infty} (a_n - b_n) = \lim_{n \rightarrow \infty} a_n - \lim_{n \rightarrow \infty} b_n = L - M$$

$$* \lim_{n \rightarrow \infty} a_n b_n = \lim_{n \rightarrow \infty} a_n \times \lim_{n \rightarrow \infty} b_n = L \times M = LM$$

$$* \lim_{n \rightarrow \infty} \left(\frac{a_n}{b_n} \right) = \frac{\lim_{n \rightarrow \infty} a_n}{\lim_{n \rightarrow \infty} b_n} = \frac{L}{M}, \text{ but } M \neq 0$$

but in choice it doesn't say about M

ANSWER: C

33. Solution

$$\Rightarrow \lim_{x \rightarrow \infty} \frac{3x^2 + 2x}{5x - x^2} = \lim_{x \rightarrow \infty} \left(\frac{3 + \frac{2}{x}}{\frac{5}{x} - 1} \right) \text{ by dividing } x^2$$

$$\text{as } x \rightarrow \infty, \frac{2}{x} \rightarrow 0 \text{ and } \frac{5}{x} \rightarrow 0$$

$$\text{thus, } \Rightarrow \lim_{x \rightarrow \infty} \left(\frac{3 + \frac{2}{x}}{\frac{5}{x} - 1} \right) = \frac{(3 + 0)}{-1} = -3 //$$

ANSWER: B

34. Which of the following is the same as the derivative of a function f at a point P , where P is on the graph of f ?
- A. The continuity of the function at P .
 - B. The slope of a secant line through P .
 - C. The slope of the tangent line through P .
 - D. The graph of f has no jump at P .
35. What is the derivative of the function $f(x) = 3x^2 + 2\sqrt{x} - 4x$ at $x = 1$?
- A. -1
 - B. -3
 - C. 3
 - D. 1
36. What are the 2^{nd} and the n^{th} derivative of $f(x) = e^{3x}$ respectively?
- A. $4f(x)$ and $2^n f(x)$
 - B. $2f(x)$ and $nf(x)$
 - C. $9f(x)$ and $3^n f(x)$
 - D. $9f(x)$ and $n^2 f(x)$
37. Which of the following is true about the zero of a function?
- A. It is the y-intercept of the function.
 - B. It is the x-intercept of the function.
 - C. It is the concavity point of the function.
 - D. It is the critical point of the function.

34. Solution

① The derivative of a function f at a point P , is the same as the slope of the tangent line through point P , where point P is on the graph of f .

ANSWER: C

35. Solution

Given $f(x) = 3x^2 + 25x - 4x$

$$\Rightarrow f'(x) = 6x + \frac{2}{25x} - 4 = 6x + \frac{1}{x} - 4$$

$$\text{then } f'(1) = 6(1) + \frac{1}{1} - 4 = 6 + 1 - 4 = 3$$

ANSWER: C

36. Solution

Given $f(x) = e^{3x}$

then $f'(x) = 3e^{3x}$

$$f''(x) = 9e^{3x}$$

$$f'''(x) = 27e^{3x}$$

$$\vdots$$

$$f^{(n)}(x) = 3^n e^{3x}$$

$\therefore$ 2nd derivative of $f(x) = 9e^{3x}$ and n^{th} derivative of $f(x) = 3^n e^{3x} = 3^n f(x)$

ANSWER: C

37. Solution

* When we find the zero of a given fun in other way we find the x-intercept of the function

Eg $f(x) = 2x + 3$

its zero is found by making $f(x) = y = 0$

thus $2x + 3 = 0 \Rightarrow x = -3/2$ is the zero and x-intercept of the function f .

ANSWER: B.

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38. Which of the following is true about the function defined by $f(x) = x^3 - 3x^2$ on $[-1, 3]$?
- A. Its critical numbers are 0 and 2 and its maximum value is 15.
 - B. Its critical numbers are 0, 2, 3 and its maximum value is 0.
 - C. It has critical numbers at 0 and 2 and its maximum value is 0.
 - D. Its critical numbers are 0 and 2 and its maximum value is -4.
39. What is the number that satisfies Rolle's Theorem for $f(x) = x^2 - 3x + 1$ on $[0, 3]$?
- A. 1
 - B. $-\frac{3}{2}$
 - C. 2
 - D. $\frac{3}{2}$
40. What are the length L and width W of a rectangle with perimeter 10,000m that maximizes the area?
- A. $L = 2,500$ m and $W = 3,000$ m
 - B. $L = 2,500$ m and $W = 2,500$ m
 - C. $L = 5,000$ m and $W = 5,000$ m.
 - D. $L = 3,000$ m and $W = 2,500$ m.
41. If the radius r of a sphere is increasing at the rate of 2 cm/min, then the rate of change of the volume when $r = 1$ cm is
- A. $6\pi \text{ cm}^3/\text{min}$
 - B. $4\pi \text{ cm}^3/\text{min}$
 - C. $12\pi \text{ cm}^3/\text{min}$
 - D. $8\pi \text{ cm}^3/\text{min}$

38. Solution

Given $f(x) = x^3 - 3x^2$ on $[-1, 3]$

first let find the critical numbers

$$f'(x) = 3x^2 - 6x = 0$$

$$\Rightarrow 3x(x-2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

thus $x=0$ and $x=2$ are critical numbers

$$f(-1) = (-1)^3 - 3(-1)^2 = -1 - 3 = -4 \quad \leftarrow \text{left end point}$$

$$f(0) = 0^3 - 3(0)^2 = 0 \quad \leftarrow \text{at critical numbers}$$

$$f(2) = (2)^3 - 3(2)^2 = 8 - 12 = -4 \quad \leftarrow$$

$$f(3) = 3^3 - 3(3)^2 = 27 - 27 = 0 \quad \leftarrow \text{at right end point}$$

* thus, its maximum value is '0'

ANSWER: C

39. In order to satisfy Rolle's theorem

$$f(0) = f(3) \Rightarrow f(0) = 0^2 - 3(0) + 1 = 1$$

$$\Rightarrow f(3) = 3^2 - 3(3) + 1 = 1$$

since, the function satisfies the Rolle's theorem on the given interval $[0, 3]$ then there exist a number 'c' in $(0, 3)$ such $f'(c) = 0$

$$\text{but } f'(x) = 2x - 3 \Rightarrow f'(c) = 2c - 3$$

$$\Rightarrow 2c - 3 = 0 \Rightarrow c = \underline{\underline{3/2}}$$

ANSWER: D

40. Solution

$$2L + 2W = 10,000 \text{m} \quad \leftarrow \text{perimeter}$$

$$A(L) = LW \quad \leftarrow \text{function to be maximized (Area)}$$

$$* \text{ from the perimeter: } 2L + 2W = 10,000$$

$$\Rightarrow W = \frac{10,000 - 2L}{2} = 5,000 - L$$

then substitute it into $A(L)$

$$\Rightarrow A(L) = LW = (L)(5000 - L) \\ = -L^2 + 5000L = 5,000L - L^2$$

then find $A'(L) = 0$

$$A'(L) = 5000 - 2L = 0$$

$$\Rightarrow 2L = 5000$$

$$\Rightarrow L = 2500$$

$$\text{Now, solve for } W, W = 5000 - L \Rightarrow W = 5000 - 2500 \\ W = \underline{\underline{2500}}$$

$$\text{Thus } L = 2500, \text{ and } W = 2500$$

ANSWER: B

41. Solution

$$\text{given } \frac{dr}{dt} = 2 \text{ cm/min}, r = 1 \text{ cm}$$

$$\frac{dV}{dt} = ?$$

You know that Volume of sphere is given by

$$V = \frac{4}{3} \pi r^3$$

$$\text{then, } \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\Rightarrow \frac{dV}{dt} = 4\pi \times (1)^2 \times (2) = 4\pi \times 2 = \underline{\underline{8\pi \text{ cm}^3/\text{min}}}$$

ANSWER: D

42. Which of the following is equal to $\int (\cos x - 2x) dx$ where c is a constant?
- A. $\sin x + x^2 + c$.
B. $-\sin x - x^2 + c$.
C. $\sin x - x^2 + c$.
D. $-\sin x + x^2 + c$.
43. What is the area of the region bounded by $y = 0$ and the graph of $f(x) = 4 - x^2$ on the interval $[-1, 1]$?
- A. $\frac{22}{3}$
B. $\frac{2}{3}$
C. $\frac{26}{3}$
D. 8
44. Suppose $\int_1^3 f(x) dx = 6$ and $\int_1^2 f(x) dx = 10$, what is the value of $\int_2^3 f(x) dx$?
- A. 2
B. -4
C. 16
D. 4
45. Which of the following statements has a truth value T?
- A. $(p \wedge q) \Rightarrow (p \vee q)$
B. $(p \wedge q) \Rightarrow \neg(p \vee q)$
C. $(p \vee q) \Rightarrow (p \wedge q)$
D. $(p \Rightarrow q) \Rightarrow (p \vee q)$

42. Solution

Let $f(x) = \int (\cos x - 2x) dx$

$$F(x) = \int \cos x dx - \int 2x dx$$

$$= \sin x - x^2 + C$$

ANSWER: C

43. Solution

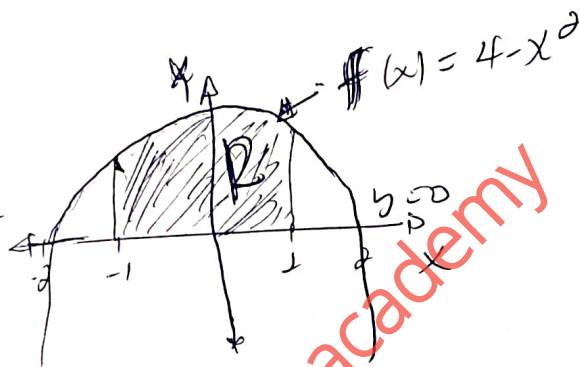
Since $f(x) > 4$ on $[-1, 1]$
then Area of the region is given by

$$A = \int_{-1}^1 (4 - x^2 - 0) dx$$

$$= \int_{-1}^1 (4 - x^2) dx$$

$$= \left(4x - \frac{x^3}{3} \right) \Big|_{-1}^1 = \left(4 - \frac{1}{3} \right) - \left(-4 + \frac{1}{3} \right)$$

$$= 4 - \frac{1}{3} + 4 = 2\frac{2}{3}$$



ANSWER: A

44. Solution

Given $\int_1^3 f(x) dx = 6$ and $\int_1^2 f(x) dx = 10$, $\int_2^3 f(x) dx = ?$

You know that if $C \in [a, b]$, then $\int_a^b f(x) dx = \int_a^C f(x) dx + \int_C^b f(x) dx$

thus $\int_1^3 f(x) dx = \int_1^2 f(x) dx + \int_2^3 f(x) dx$

$$\Rightarrow \int_2^3 f(x) dx = \int_1^3 f(x) dx - \int_1^2 f(x) dx = 6 - 10 = -4$$

ANSWER: B

45. Solution

(A) $(P \wedge Q) \Rightarrow (P \vee Q)$

* In all case these ~~statement~~ statement has a truth value T.

(B) $(P \wedge Q) \Rightarrow \neg(P \vee Q)$, in case of these, when $P \equiv T, Q \equiv T$ the statements has a truth value of F

(C) $(P \vee Q) \Rightarrow (P \wedge Q)$, has truth value F in case of $P \equiv T \& Q \equiv F$

(D) $(P \Rightarrow Q) \Rightarrow (Q \vee P)$, has truth value F in case of $P \equiv F \& Q \equiv F$

ANSWER: A

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46. The inverse of the function $y = 3x - 5$ is equal to
- A. $y = \frac{x+5}{3}$ C. $y = \frac{x-5}{3}$
- B. $y = \frac{x+3}{5}$ D. $y = -5x + 3$
47. Which of the following is the universal set of the expression $\frac{1}{x^2} + \frac{x+1}{x^2+x}$?
- A. $\mathbb{R} \setminus \{0, 1\}$ C. $\mathbb{R} \setminus \{0, 1, -1\}$
- B. $\mathbb{R} \setminus \{0\}$ D. $\mathbb{R} \setminus \{0, -1\}$
48. The graph of a rational function is given as in the Figure 1 below.

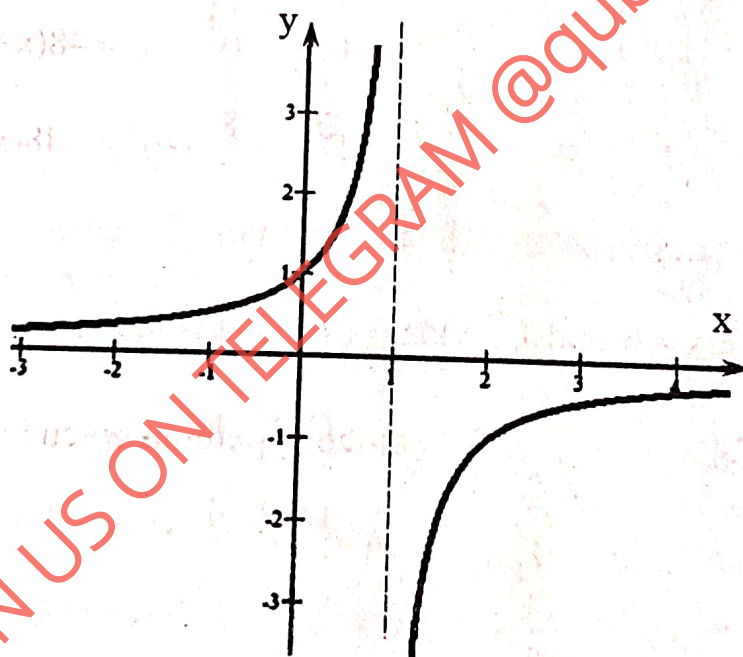


Figure 1

Which of the following is **NOT** true about the graph?

- A. $x = 1$ is its vertical asymptote.
- B. $y = 0$ is its horizontal asymptote.
- C. It is decreasing on $(1, \infty)$.
- D. It is increasing on $(-\infty, 1)$.

46. Solution.

Given $y = 3x - 5$

* In order to find the inverse of the given function replace x by y and y by x . then solve for y .

Now, $x = 3y - 5$

$$3y = x + 5$$

$$y = \frac{x+5}{3}$$

ANSWER: A

47. Solution

* In order to find the universal set of the given function set the denominator of the function different from zero.

thus, $x^2 \neq 0 \Rightarrow x \neq 0$

and, $x^2 + x \neq 0 \Rightarrow x(x+1) \neq 0 \Rightarrow x \neq 0 \text{ and } x \neq -1$

thus the universal set is $\mathbb{R} \setminus \{0, -1\}$

ANSWER: D

48. Solution

for the given graph

- $x=1$ is its vertical asymptote.
- $y=0$ is its horizontal "
- It is increasing on $(1, \infty) \cup (-\infty, 1)$

ANSWER: C

49. The equation of a line is given by $3x - 2y + 6 = 0$. Then which of the following is true about the line?
- A. Its y-intercept is -3 .
B. Its slope is $\frac{3}{2}$.
C. $(2, 5)$ lies on the line.
D. Its x-intercept is 2 .
50. Which one of the following is the standard form of the equation of the parabola with vertex $(4, 6)$ and focus $(-8, 6)$?
- A. $(x - 4)^2 = -12(y - 6)$
B. $(x - 4)^2 = 12(y - 6)$
C. $(y - 6)^2 = 48(x - 4)$
D. $(y - 6)^2 = -48(x - 4)$
51. Which of the following is true about the hyperbola given by $\frac{x^2}{4} - y^2 = 1$?
- A. Its foci are at $(-5, 0)$ and $(5, 0)$.
B. Its vertices are at $(-2, 0)$ and $(2, 0)$.
C. Its center is at $(2, 1)$.
D. An asymptote to the hyperbola is $y = 2x$.

49. Solution

Given equation $3x - 2y + 6 = 0$

• To find its y-intercept make $x = 0$

thus $-2y + 6 = 0 \Rightarrow 2y = 6 \Rightarrow y = 3$ is its y-intercept.

• To find its x-intercept set $y = 0$

$3x + 6 = 0 \Rightarrow x = -2$ is its x-intercept.

• to find its slope write in standard form

$$2y = 3x + 6$$

$$y = \frac{3}{2}x + 3, \quad m = \text{slope} = \frac{3}{2}$$

ANSWER: B

50. Solution

Since the parabola has vertex $(4, 6)$ and focus $(-8, 6)$ the second entry (k) of the vertex and focus is the same. It is a parabola whose axis is parallel to x-axis. Thus the standard eqn of the parabola is given by:

$$(y - k)^2 = -4p(x - h) \quad \text{since } -8 < 4$$

thus FOCUS = $F = (h - p, k) = (-8, 6)$, but $(h, k) = V(4, 6)$
 $\underline{h = 4}$ & $\underline{k = 6}$

$$\Rightarrow h - p = -8$$

$$\Rightarrow 4 - p = -8 \Rightarrow \underline{p = 12}$$

$$\text{then } (y - 6)^2 = -12 \times 4(x - 4) \Rightarrow \underline{(y - 6)^2 = -48(x - 4)}$$

ANSWER: D

51. Solution

given $\frac{x^2}{4} - \frac{y^2}{1} = 1$

From the eqn $a = \sqrt{4} = 2$ and $b = \sqrt{1} = 1$ and whose transverse axis is horizontal. Thus, and their center is $C(0, 0) = C(h, k)$, $h = 0$ & $k = 0$ and their vertices on the hyperbola are $V'(-2, 0)$ and $(2, 0)$.

Hence foci are $F'(h - c, k)$ and $F(h + c, k)$ which implies $F'(-\sqrt{5}, 0)$ & $F(\sqrt{5}, 0)$, since, $c^2 = a^2 + b^2 \Rightarrow c = \sqrt{5}$

• An asymptote to the hyperbola is $y = \pm \frac{b}{a}x = \pm \frac{1}{2}x$

ANSWER: B

BOOKLET CODE: 29

52. Two statements p and q are given by:

p: $\frac{22}{7}$ is an irrational number.

q: The sum of two odd integers is odd.

Then which of the following has a truth value T?

A. $p \wedge \neg q$

C. $\neg p \wedge q$

B. $p \vee \neg q$

D. $p \wedge q$

53. Which one of the following is the contrapositive of the statement: If 21 is a prime number, then 4 is an odd number?

A. If 4 is an odd number, then 21 is not a prime number.

B. If 4 is an odd number, then 21 is a prime number.

C. If 4 is not an odd number, then 21 is not a prime number.

D. If 21 is not a prime number, then 4 is an even number.

54. Which of the following is true where c is a constant?

A. $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = -2 \cos \sqrt{x} + c$

C. $\int \ln x dx = \frac{1}{x} + c$

B. $\int (3x+5)^6 dx = \frac{(3x+5)^7}{7} + c$

D. $\int x e^x dx = \frac{x^2}{2} e^x + c$

52. Solution

Given $P: \frac{22}{7}$ is an irrational number

Q = The sum of two odd integers is odd

We know that $P \equiv T$ and $Q \equiv F$

$$(A) P \wedge \neg Q \equiv T \wedge T \equiv T \quad (B) P \vee \neg Q \equiv T \vee T \equiv T$$

$$(C) \neg P \vee Q \equiv F \vee F \equiv F \quad (D) P \wedge Q \equiv T \wedge F \equiv F$$

ANSWER: A & B.

53. Solution

Given statement:

* If 21 is a prime number, then 4 is an odd number.
Then, its contrapositive statement is

* If 4 is not an odd number, then 21 is not a prime number.

ANSWER: C

54. Solution

① $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$, let integrate using substitution

$$\text{let } u = \sqrt{x}, \text{ then } du = \frac{1}{2\sqrt{x}} dx \Rightarrow 2du = \frac{dx}{\sqrt{x}}$$

$$\text{then } \Rightarrow \int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = \int \frac{\sin u}{\sqrt{x}} dx = \int \sin u \cdot \frac{1}{\sqrt{x}} dx$$

$$\Rightarrow \int 2 \sin u du = -2 \cos u + C \\ = \underline{\underline{-2 \cos \sqrt{x} + C}}$$

ANSWER: A

55. The acceleration of a body at any time t is given by $a(t) = 4 - 2t + 3t^2$ with initial velocity $v(0) = 5$. Then what is the velocity v of the body as a function of time?

A. $v(t) = 1 + 4t - t^2 + t^3$

C. $v(t) = -2 + 6t$

B. $v(t) = 5 + 4t - t^2 + t^3$

D. $v(t) = 9 - t^2 + t^3$

56. Which of the following argument is valid?

A. $p, \neg q \vdash p \Rightarrow q$

C. $p, q \vdash p \Rightarrow \neg q$

B. $\neg p, q \vdash p \Rightarrow q$

D. $p, \neg q \vdash p \wedge q$

57. Which of the following proof is more suitable to prove the assertion:

$$1 + 2^3 + 3^3 + \dots + n^3 = (1 + 2 + 3 + \dots + n)^2$$

A. Proof by counter example.

B. Proof by the principle of mathematical induction.

C. Proof by contradiction.

D. Proof by indirect proof.

55. Solution

Given $a(t) = 4 - 2t + 3t^2$

$$v(0) = 5$$

The acceleration of a body is given by $a(t) = \frac{dv(t)}{dt}$

thus $v(t) = \int a(t) dt$

$$v(t) = \int (4 - 2t + 3t^2) dt = 4t - t^2 + t^3 + C$$

but we have $v(0) = 5$

$$\text{thus } v(0) = 5 = 4(0) - 0^2 + 0^3 + C \Rightarrow C = 5$$

$$\text{thus } v(t) = 4t - t^2 + t^3 + 5 = t^3 - t^2 + 4t + 5$$

ANSWER: B

56. Solution

(B) $\neg P, q \vdash P \Rightarrow q$

from premises q is true, thus $P \Rightarrow q$ is true for all case of P

ANSWER: B

57. Solution

* Proof by the principle of Mathematical induction is suitable to prove the given assertion.

ANSWER: B

Scores	1 - 5	6 - 10	11 - 15	16 - 20
Frequency	4	6	8	2

What is the standard deviation of the scores ?

- A. 5 C. $\sqrt{21}$
B. 4 D. $\sqrt{20}$

59. What is the coefficient of x^2 in the binomial expansion of $\left(2x + \frac{3}{x^3}\right)^8$?

- A. 6 C. 576
B. 0 D. 500

60. Which of the following is true about the graph of $y = -4 \cos\left(\frac{2x}{3}\right)$?

- A. Its period is 3π .
- B. Its amplitude is -4 .
- C. The graph completes one cycle on the interval $\left[0, \frac{3\pi}{2}\right]$.
- D. The graph completes one cycle on the interval $\left[0, \frac{2\pi}{3}\right]$.

58. Solution

standard deviation for grouped data is given by.

$$Sd = \sqrt{\frac{\sum_{i=1}^n f_i (x_i - \bar{x})^2}{\sum f_i}}$$

given

Scores	1-5	6-10	11-15	16-20
Freq.	4	6	8	2

Where x_i is the midpoint or each class (class mark)
 $\bar{x}$ is the ~~mean~~ mean of the grouped data.

SCORES	f	class mark (x_i)	$f \cdot x_i$	$x_i - \bar{x}$ ($x_i - 10$)	$(x_i - \bar{x})^2$ ($x_i - 10$) ²	$f_i (x_i - \bar{x})^2$ $f_i (x_i - 10)^2$
1-5	4	3	12	-7	49	196
6-10	6	8	48	-2	4	24
11-15	8	13	104	3	9	72
16-20	2	18	36	8	64	128
$\sum f_i = 20$			$\sum f_i x_i = 200$			$\sum f_i (x_i - 10)^2 = 420$

$$\bar{x} (\text{mean}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{200}{20} = 10$$

$$Sd = \sqrt{\frac{\sum_{i=1}^n f_i (x_i - \bar{x})^2}{\sum f_i}} = \sqrt{\frac{420}{20}} = \sqrt{21} = \underline{\underline{\sqrt{21}}}$$

ANSWER: C

59. Solution

* for a non-negative integer n , the binomial Expansion of $(x+y)^n$ is given by: $(x+y)^n = C(n,0)x^n + C(n,1)x^{n-1}y + C(n,2)x^{n-2}y^2 + \dots + C(n,r)x^{n-r}y^r + \dots + C(n,n)y^n$

$$\begin{aligned} \text{thus } (2x + \frac{3}{x^3})^8 &= (2x)^8 + \binom{8}{1}(2x)^7 \cdot \frac{3}{x^3} + \binom{8}{2}(2x)^6 \cdot \left(\frac{3}{x^3}\right)^2 + \binom{8}{3}(2x)^5 \cdot \left(\frac{3}{x^3}\right)^3 \\ &+ \binom{8}{4}(2x)^4 \cdot \left(\frac{3}{x^3}\right)^4 + \binom{8}{5}(2x)^3 \cdot \left(\frac{3}{x^3}\right)^5 + \binom{8}{6}(2x)^2 \cdot \left(\frac{3}{x^3}\right)^6 \\ &+ \binom{8}{7}(2x)^1 \cdot \left(\frac{3}{x^3}\right)^7 + \binom{8}{8}\left(\frac{3}{x^3}\right)^8 \end{aligned}$$

As we see from the eqn there is ~~term~~ term that is x^2 thus coefficient of x^2 is '0'

ANSWER: B

60. Solution

Given $y = -4 \cos\left(\frac{2x}{3}\right)$

Compare it with $y = a \cos(kx)$, Period (P) = $\frac{2\pi}{|k|}$, amplitude = $|a|$

then $a = -4$ and $k = \frac{2}{3}$

thus * amplitude = $|a| = |-4| = 4$

* period = $\frac{2\pi}{|k|} = \frac{2\pi}{|2/3|} = 2\pi \times \frac{3}{2} = \frac{6\pi}{2} = \underline{\underline{3\pi}}$

ANSWER: A

61. Solution

* $\tan 60^\circ = \frac{h}{x}$, $\odot h = 8m + y$

* $\tan 30^\circ = \frac{y}{x} = \frac{h-8}{x}$

then, solve for x from both eqn.

$x = \frac{h}{\tan 60^\circ}$ --- (1)

$x = \frac{h-8}{\tan 30^\circ}$ --- (2)

then equate eqn (1) with (2)

$$\frac{h}{\tan 60^\circ} = \frac{h-8}{\tan 30^\circ} \Rightarrow \frac{h}{\sqrt{3}} = \frac{h-8}{1/\sqrt{3}}$$

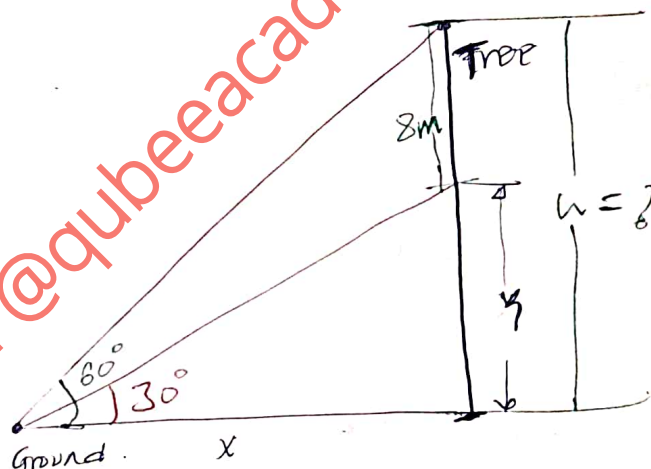
$$\Rightarrow \frac{h}{\sqrt{3}} = (h-8)\sqrt{3}$$

$$\Rightarrow h = (h-8)\sqrt{3} \cdot \sqrt{3} = (h-8) \cdot 3$$

$$\Rightarrow h = 3h - 24 \Rightarrow 2h = 24$$

$$\Rightarrow \underline{\underline{h = 12m}}$$

ANSWER: D



61. The angle of elevation of the top of a tree is found to be 60° as measured from a point on a level ground. What is the height of the tree if the angle of elevation of a point on the tree that is 8m below the top is 30° as measured from the same point on the ground?

A. 8m
B. 4m
C. 14m
D. 12m

62. Which of the following series is convergent?

A. $\sum_{n=1}^{\infty} 5^{11-2n}$
B. $\sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^{-n}$
C. $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$
D. $\sum_{n=1}^{\infty} 2^{2n-5}$

63. Let f be a function defined on a closed bounded interval $[a, b]$. Which of the following describes that f is continuous on $[a, b]$?

A. f is continuous on (a, b) , f is continuous from the right at a and from the left at b .
B. f is continuous at each point in (a, b) and at both a and b .
C. f is continuous on (a, b) , f is continuous from the left at a and from the right at b .
D. f is continuous on (a, b) , f is continuous from the left at a and from the left at b .

62. Solution

$$(A) \sum_{n=1}^{\infty} 5^{11-2n} = \sum_{n=1}^{\infty} 5^{11} \cdot 5^{-2n} = 5^{11} \sum_{n=1}^{\infty} 5^{-2n}$$
$$= 5^{11} \sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^{2n}$$

Since $r = \frac{1}{5} < 1$ it's convergent.

ANSWER: A

63. Solution

* Continuity on closed interval

A function f is continuous on the closed interval $[a, b]$ provided that

(i) f is continuous on (a, b)

(ii) f is continuous from the right, $\lim_{x \rightarrow a^+} f(x) = f(a)$

(iii) f is continuous from the left, $\lim_{x \rightarrow b^-} f(x) = f(b)$

ANSWER: A

64. A position of a particle is given by the equation $S(t) = 4t^4 - 4t + 1$ where t is measured in seconds and S in meters. Which of the following is **NOT** true about the motion of the particle?
- A. The velocity at 2 seconds is given by 124 m/sec.
 - B. The velocity v at any time t is given by $v(t) = 16t^3 - 4$.
 - C. The acceleration of the particle at 1 sec is 48 m/sec².
 - D. The particle is at rest at $t = 0.5$ seconds.
65. If $f'(1) = 4$, $f(1) = 3$ and $g'(3) = 5$, then $(g \circ f)'(1)$ is equal to
- A. 5
 - B. 3
 - C. 20
 - D. 15

THE END

64. Solution

* the position of a particle is given by eqn $s(t) = 4t^4 - 4t + 1$
then its velocity is given by

$$v(t) = \frac{d}{dt} s(t) = 16t^3 - 4$$

$$\text{thus at } t=2s, v(2) = 16(2^3) - 4 = 124 \text{ m/s}$$

* the acceleration of the particle is given by

$$a(t) = \frac{d}{dt} v(t) = 48t^2$$

$$\text{at } t=1 \text{ sec } a(1) = 48(1^2) = 48 \text{ m/sec}^2$$

* When $t=0.5$ seconds the velocity is given as

$$v(0.5) = 16(0.5)^3 - 4 = 2 - 4 = -2 \text{ m/s}$$

ANSWER: D.

65. Solution

$$\text{given } f'(1) = 4, f(1) = 3 \text{ \& } g'(3) = 5$$

$$\text{thus, } (g \circ f)'(1) = ?$$

$$\text{first we know that } (g \circ f)(x) = g(f(x))$$

$$\text{thus } (g \circ f)'(1) = \{g(f(1))\}' = g'(f(1)) \cdot f'(1), \text{ but } f(1) = 3$$

$$= g'(3) \cdot f'(1)$$

$$= 5 \times 4 = 20$$

ANSWER: C