

2020/21 Academic Year 1st Quarter Mathematics Lesson Note One for Grade 8

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Unit One:

Squares, Square Roots, Cubes and Cube Roots

1.1. Squaring the number

The process of multiplying a rational number by itself is called squaring the number.

We can multiply any rational number by itself. For any rational number a/b : $\left(\frac{a}{b}\right)^2 = \frac{a}{b} \times \frac{a}{b} = \frac{a^2}{b^2}$

Examples some examples on squaring numbers

$$1^2 = 1 \times 1 = 1 \quad \longrightarrow 1^{\text{st}} \text{ square number}$$

$$2^2 = 2 \times 2 = 4 \quad \longrightarrow 2^{\text{nd}} \text{ square number}$$

$$3^2 = 3 \times 3 = 9 \quad \longrightarrow 3^{\text{rd}} \text{ square number}$$

$$4^2 = 4 \times 4 = 16 \quad \longrightarrow 4^{\text{th}} \text{ square number}$$

From the above example you can observe that the square of a rational number is also a rational number. In short squaring means self multiplying.

Note: i. if X is a rational number then X^2 is a rational number.

ii. $a^2 = (a) \cdot (a)$ and $2a = a + a$, hence a^2 **is not equal to $2a$**

For example: 1) $5^2 = (5) \times (5) = 25$ while $2(5) = 10$

2) $3^2 = (3) \times (3) = 9$ while $2(3) = 6$

Perfect square

A rational number X is called perfect square, if and only if $x = n^2$ for some $n \in \mathbb{Q}$

Example: the following rational numbers are perfect squares. 1, 4, 9, 16 etc because

$$1 = 1^2, \quad 4 = 2^2, \quad 9 = 3^2, \quad 16 = 4^2$$

Note: A perfect square is a number that is a product of a rational number times itself and its square root rational number.

Remember

- The squares of natural numbers are also natural numbers
- $0 \times 0 = 0$ therefore $0^2 = 0$
- We give no meaning to the symbol 0^0
- If $a \in \mathbb{Q}$ and $a \neq 0$, then $a^0 = 1$
- For any rational number 'a' $a \times a$ is denoted by a^2 and read as

“a squared” or “a to the power of 2” or “the square of a”

Existence theorem

For each rational number x , there is a rational number $y (y > 0)$ such that $x^2 = y$.

Consider the following examples on the existence theorem. If

a) $X = \frac{-3}{4}$, then $y = \frac{9}{16}$

b) $X = 0.5$, then $y = (0.5)^2 = 0.25$

1.2 The square root of a rational number

For any two rational numbers 'a' and 'b' if $a^2 = b$, then a is called the square root of 'b'. In short, square root is the reverse process of squaring.

Example: 1. The square root of 9 is 3.

2. The square root of 16 is 4.

The square root is denoted by the symbol " $\sqrt{}$ " and it is called **radical sign** and "a" is called **radicand**.

Hence

i. $\sqrt{9} = 3$ 3 is square root of 9.

ii. $\sqrt{16} = 4$ 4 is square root of 16.

From the above example the numbers 9 and 16 are called **radicand**.

Note: the value of square root is always non negative.

1.2.1. Square roots of perfect squares

Consider the following important properties of squares

$$(ab)^2 = a^2 \times b^2 \text{ and } \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2} \text{ (where } b \neq 0\text{)}$$

$$\text{Thus } (2 \times 3)^2 = 2^2 \times 3^2 = 36 \text{ and } \left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$$

- If a and b are non- negative rational numbers, then $\sqrt{axb} = \sqrt{a} \times \sqrt{b}$

$$\text{Thus } \sqrt{4 \times 9} = \sqrt{4} \times \sqrt{9} = \sqrt{2^2} \times \sqrt{3^2} = 2 \times 3 = 6$$

- If a and b are non – negative rational numbers, the $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, $b \neq 0$

$$\text{Thus } \sqrt{\frac{49}{36}} = \frac{\sqrt{49}}{\sqrt{36}} = \frac{\sqrt{7^2}}{\sqrt{6^2}} = \frac{7}{6}$$

- If 'a' and 'b' are non- negative, then $\sqrt{a^2b} = a\sqrt{b}$

$$\text{Thus } \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

Note: we can add or subtract like radicals

Example: simplify the following

$$\begin{aligned} \sqrt{8} + \sqrt{18} &= \sqrt{4 \times 2} + \sqrt{9 \times 2} \\ &= \sqrt{4} \times \sqrt{2} + \sqrt{9} \times \sqrt{2} \\ &= 2\sqrt{2} + 3\sqrt{2} \\ &= (2+3) \sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

1.2 cubes and cube roots

1.3.1. Cube of a number

Definition: A cube number is the result of multiplying a rational number by itself then multiplying by the number again.

For example, some few cube numbers are:

- ✓ $1 \times 1 \times 1 = 1$ ——— 1st cube number
- ✓ $2 \times 2 \times 2 = 8$ ——— 2nd cube number
- ✓ $3 \times 3 \times 3 = 27$ ——— 3rd cube number

Perfect cube

A rational number x is called perfect cube if and only if $x = n^3$ for some $n \in \mathbb{Q}$.

For example

8 ← perfect cube because $2^3 = 8$

Y^6 ← perfect cube because $(Y^2)^3 = Y^6$

Cube root of a number

Definition: The cube root of a given number is one of the three identical factors whose product is the given number.

Example

- $0 \times 0 \times 0 = 0$, So, 0 is the cube root of 0.
- $1/5 \times 1/5 \times 1/5 = 1/125$, So $1/5$ the cube root of $1/125$.

Note: if a and b are any numbers such that $b^3 = a$, then b is called the cube root of a and it can be written symbolically as $b = \sqrt[3]{a}$

Example = 1. $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$

$$2. \sqrt[3]{8/27} = \sqrt[3]{\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}} = \frac{2}{3} \text{ .So the answer is } 2/3.$$

The following properties of cubes are important:

- $(ab)^3 = a^3 \times b^3$ and $\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$ (when $b \neq 0$)

Thus $(2 \times 2)^3 = 2^3 \times 2^3 = 8 \times 8 = 64$ and $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$

A number is called a perfect cube, if it is the cube of a rational number.

- If a and b are numbers, then $\sqrt[3]{axb} = \sqrt[3]{a} \times \sqrt[3]{b}$

Example: Evaluation the following

a. $\sqrt[3]{8 \times 27} = \sqrt[3]{8} \times \sqrt[3]{27}$

$$= \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{3 \times 3 \times 3}$$

$$= 2 \times 3 = 6$$

b. $\sqrt[3]{-125 \times 27} = \sqrt[3]{-125} \times \sqrt[3]{27}$

$$= \sqrt[3]{-5 \times -5 \times -5} \times \sqrt[3]{3 \times 3 \times 3}$$

$$= -5 \times 3$$

$$= -15$$

- If a and b are rational numbers, then $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$, $b \neq 0$

Example:- evaluate the following

$$\text{a. } \sqrt[3]{\frac{216}{729}} = \frac{\sqrt[3]{216}}{\sqrt[3]{729}} = \frac{\sqrt[3]{6 \times 6 \times 6}}{\sqrt[3]{9 \times 9 \times 9}} = \frac{6}{9} = \frac{2}{3}$$

$$\text{b. } \sqrt[3]{0.0027} = \sqrt[3]{\frac{27}{1000}} = \frac{\sqrt[3]{27}}{\sqrt[3]{1000}} = \frac{3}{10} = 0.3$$

- If a and b are numbers, the $\sqrt[3]{a^3 b} = a \sqrt[3]{b}$

Example: simplify each of the following

$$\begin{aligned} \text{a. } \sqrt[3]{648} &= \sqrt[3]{216 \times 3} \\ &= \sqrt[3]{216} \times \sqrt[3]{3} \\ &= \sqrt[3]{6 \times 6 \times 6} \times \sqrt[3]{3} \\ &= 6\sqrt[3]{3} \end{aligned}$$

$$\begin{aligned} \text{b. } \sqrt[3]{27b^2} &= \sqrt[3]{27} \times \sqrt[3]{b^2} \\ &= \sqrt[3]{3 \times 3 \times 3} \times \sqrt[3]{b^2} \\ &= 3\sqrt[3]{b^2} \end{aligned}$$