



TUTORIAL COLLAGE

L E A R N I N G I S T H E K E Y
T O S U C C E S S .



Part III: Write the simplified answer on the space provided

23. Let $A = \{1, 2, 3, 4\}$ and $B = \{x \in \mathbb{N} : -1 \leq x \leq 7\}$. Then

a. $A \cup B$ is $\{1, 2, 3, 4, 5, 6, 7\}$

b. $A \cap B$ is $\{1, 2, 3, 4\}$

c. $A \Delta B$ is $\{5, 6, 7\}$

..... (3 points)

24. The truth value of statements p and q are F and T respectively, then the truth value of

$q \Rightarrow p$ is F

..... (1 points)

25. Let $z = 3 + i$. Then

a. $\text{Re}(z)$ is 3

b. $\bar{z}$ is $3 - i$

c. The additive inverse of z is $-3 - i$

..... (3 points)

26. Let $f(x) = \sqrt{x^2 - 2x}$. Then

a. $f(0)$ is 0

b. the domain of $f(x)$ is $(-\infty, 0] \cup [2, \infty)$

c. If $f(x) = 0$, then the value(s) of x is/ are 0 and 2

.... (3 points)



A

15. The greatest lower bound of a sequence $\left\{\frac{(-1)^n}{n}\right\}$ is

A. -1

B. 1

C. $\frac{1}{2}$

D. $-\frac{1}{2}$

D

16. Which one of the following statements is not true about any complex numbers Z_1 and Z_2 ?

A. $\overline{\overline{Z_1}} = Z_1$

B. $Z_1 + \overline{Z_1} = 2\text{Re}(Z_1)$

C. $\left|\frac{Z_1}{Z_2}\right| = \frac{|Z_1|}{|Z_2|}; Z_2 \neq 0$

D. $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$

B

17. Let order pairs $(x, 2)$ and $(2x - 9, 2)$ are equal, then the value of x is

A. -9

B. 9

C. 3

D. -3

18. The codomain of a relation $R: \mathbb{N} \rightarrow \mathbb{R}$ is

A. $\mathbb{N}$

B. $\emptyset$

C. $\mathbb{R}$

D. $\mathbb{N} \cap \mathbb{R}$

D

19. Which one of the following is a function?

A. $\{(x, y): x \text{ is a father of } y\}$

B. $\{(x, y): x = y^2\}$

C. $\{(x, y, x + y): \forall x, y \in \mathbb{Z}\}$

D. $\{(x, y): x + y = 1\}$

20. Which one of the following statements is true about sets A , B and C ?

A. $A \times B = B \times A$

B. $A \setminus B = B \setminus A$

C. $(A \Delta B) \Delta C = A \Delta (B \Delta C)$

D. $(A \times B) \times C = A \times (B \times C)$

21. Let $A = [-1, \infty)$, $B = (-\infty, 1]$ and $C = (1, \infty)$, then $C \cup (A \cap B)$ is

A. $(-\infty, 1]$

B. $[-1, \infty)$

C. $\mathbb{R}$

D. $(1, \infty)$

D

22. Which one of the following is not true about functions

$f(x) = x + 1$ and $g(x) = \frac{1}{x}$?

A. $(f + g)(x) = \frac{x^2 + x + 1}{x}$

B. $(f \cdot g)(x) = \frac{x + 1}{x}$

C. $(f \circ g)(x) = \frac{x + 1}{x}$

D. Domain of $(f \circ g)(x)$ is $\mathbb{R}$



Part I: Write "True" if the statement is correct or "False" if the statement is incorrect on the space provided (1 pt. each)

- False 1. For any function f and g , $(f \circ g)(x) = (g \circ f)(x)$.
False True 2. The domain of a relation R is equal to the range of the inverse relation of R .
True 3. Any relation is a function.
True 4. The set of real numbers is a subset of the set of complex numbers.
True 5. Any set is a subset of itself.
True 6. The principal argument of a complex number $Z = i$ is $\frac{\pi}{2}$.
False 7. The set of natural numbers is closed under division.
False 8. The sum of two irrational numbers is always irrational number.
False 9. The range and codomain of any function are the same.
True 10. The simplified form of $(1 - i)^{10}$ and $(1 - i)^{18}$ are equal.

Part II: Choose the correct answer from the given alternatives and write your answer on the space provided (1.5 pts. each)

- D 11. Let $A = \{1, 2\}$ and $B = \{3, 4\}$, then which one of the following relations is a relation from A to B ?
A. $\{(3, 1), (4, 2)\}$ B. $\{(1, 3), (2, 5)\}$
C. $\{(3, 1)\}$ D. $\{(1, 3), (2, 4)\}$
- B 12. Which one of the following statements is not true about integers a , b and c ?
A. $a + b = b + a$ B. $a - b = b - a$
C. $(a \times b) \times c = a \times (b \times c)$ D. $a \times (b + c) = a \times b + a \times c$
- A 13. The GCF of 100 and 150 is
A. 50 B. 300 C. 25 D. 150
- C 14. Which one of the following numbers is not rational number?
A. $\frac{1}{2}$ B. $3.\overline{14}$ C. $\sqrt{2}$ D. $\sqrt{25}$

16. Let the length of the transverse of a hyperbola is 16 and the end points of the conjugate axis are (5, -5) and (5, 3), then find:

- The equation of the hyperbola
- The vertex and the foci of the hyperbola.
- Sketch the graph

Solⁿ: (i) eqn of the hyperbola is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

1PT

Where the center $C(h, k)$ = mid point of conjugate axis

$$\Rightarrow C(h, k) = (5, -1)$$

$a = 8$ (half of length of transverse axis)

$b = 4$ (half of length of conjugate axis)

$$\Rightarrow \frac{(x-5)^2}{64} - \frac{(y+1)^2}{16} = 1 \quad \text{is eqn of the hyperbola.}$$

ii) the vertices are

$$V(h-a, k) \text{ and } V'(h+a, k)$$

$$= V(-3, -1) \text{ and } V'(13, -1)$$

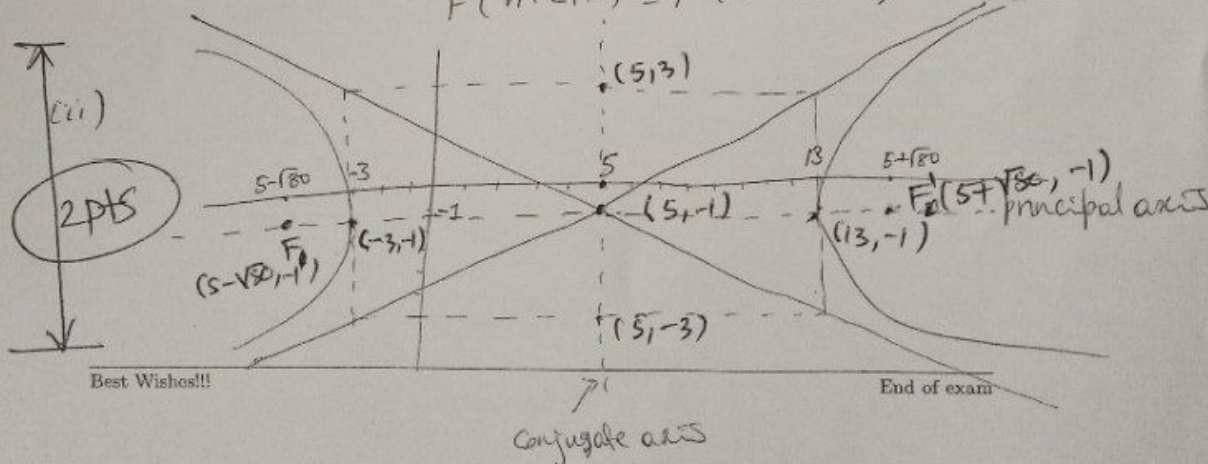
1PT

$$c = \sqrt{a^2 + b^2} = \sqrt{80}$$

$\Rightarrow$ the foci are $F(h-c, k) = F(5-\sqrt{80}, -1)$ and

$$F'(h+c, k) = F'(5+\sqrt{80}, -1)$$

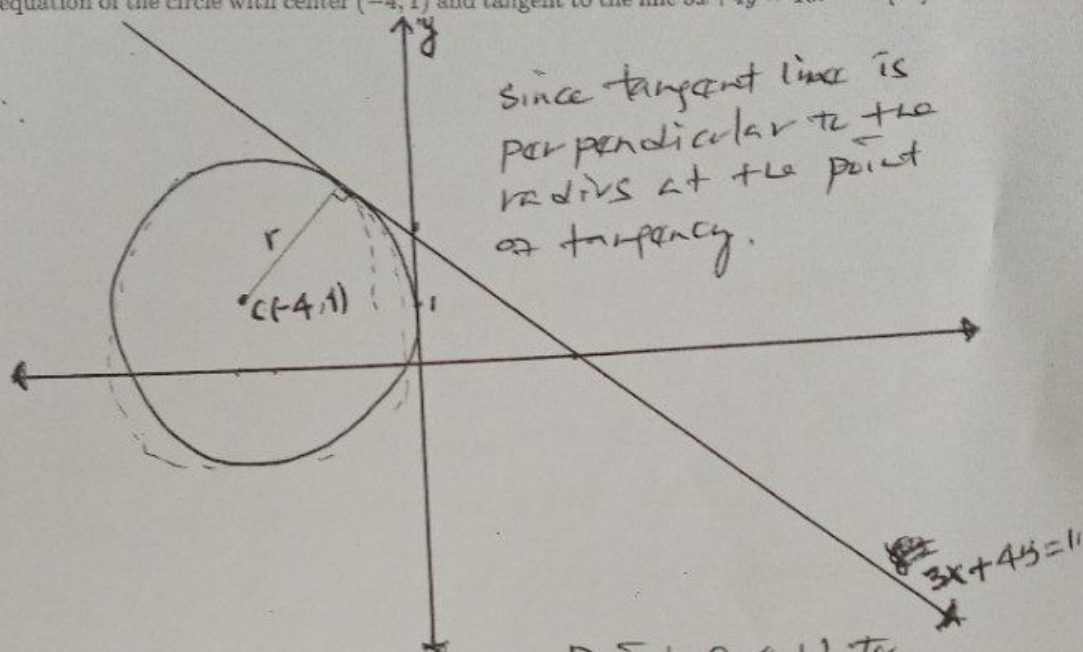
1PT



15. Find the equation of the circle with center $(-4, 1)$ and tangent to the line $3x + 4y = 10$.

[05]

Soln :



Since tangent line is perpendicular to the radius at the point of tangency.

By distance formula from a point $(-4, 1)$ to the line $3x + 4y - 10 = 0$ is the radius of the circle.

$$3\text{pt} \left\{ r = \frac{|3(-4) + 4(1) - 10|}{\sqrt{9 + 16}} = \frac{|-18|}{5} = 18/5 \right.$$

Hence, the equation of a circle will be

$$(x - h)^2 + (y - k)^2 = r^2 \quad \left\{ (1\text{pt}) \right.$$

$$1\text{pt} \left\{ \therefore (x + 4)^2 + (y - 1)^2 = (18/5)^2 \right.$$

14. Suppose the polynomial function $f(x) = 2x^3 + px^2 - 3x + k$. If f is divided by $x + 2$ and $x - 1$, then the remainders are -20 and -2 respectively.

(i) Find the values of p and k .

(ii) Find the zeros of f .

Soln : By remainder theorem, we have

$$(i) \quad f(-2) = -20 \Rightarrow k + 4p = -10 \quad \dots (1) \quad | \quad (1pt)^{(06)}$$

$$f(1) = -2 \Rightarrow k + p = -1 \quad \dots (2)$$

$$\Rightarrow k = 2 \text{ and } p = -3$$

(1pt)

$$\text{Hence, } f(x) = 2x^3 - 3x^2 - 3x + 2$$

(ii) By rational root theorem, we have a zero of polynomial may be one of $\pm 1, \pm \frac{1}{2}, \pm 2$

$$\text{Now, } f(1) = -2 \neq 0$$

$$f(-1) = 0$$

$\therefore x+1$ is factor of $p(x)$

— And using long division, we get that

$$f(x) = 2x^3 - 3x^2 - 3x + 2 = (x+1)(2x^2 - 5x + 2)$$

other zero of $f(x)$ will be

$$2x^2 - 5x + 2 = 0$$

$$x = \frac{+5 \pm \sqrt{25 - 16}}{4} = \frac{+5 \pm \sqrt{9}}{4}$$

$$x = \frac{+5 \pm 3}{4}, \quad x = 2 \text{ and } x = \frac{1}{2}$$

Hence, the zeros of $f(x)$ are $-1, \frac{1}{2}$ and 2 .

13. Find the value of x for which the graphs of $f(x) = 3^x$ and $g(x) = 4^{-x+2}$ intersect.
[Hint: Use common logarithm]

[05]

Soln $3^x = 4^{-x+2}$ gives us the point of intersection — (1pt)

$$\Rightarrow \log_{10} 3^x = \log_{10} 4^{-x+2} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{1 pt}$$

$$\Rightarrow x \log_{10} 3 = (-x+2) \log_{10} 4$$

$$\Rightarrow x \log_{10} 3 + x \log_{10} 4 = 2 \log_{10} 4$$

$$\Rightarrow x \log_{10} 12 = \log_{10} 16$$

$$\Rightarrow x = \frac{\log_{10} 16}{\log_{10} 12} = \log_{12} 16$$

3 pts

Or

$$3^x = 4^{-x+2} \Rightarrow 3^x = \frac{16}{4^x}$$

$$\Rightarrow 12^x = 16$$

$$\Rightarrow \log_{12} 16 = x$$

Part II Show all the necessary steps of all questions in this section using the answer space provided.

11. Show that $\sqrt{2}$ is an irrational number.

[03]

proof: suppose that $\sqrt{2}$ is rational number.

$\Rightarrow \sqrt{2} = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$ and $\text{G.C.F}(a, b) = 1$

$$\Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2$$

$$\Rightarrow a^2 \text{ is even}$$

$$\Rightarrow a \text{ is even}$$

$$\Rightarrow a = 2k, k \in \mathbb{Z}$$

$$\Rightarrow 4k^2 = 2b^2$$

$$\Rightarrow b^2 = 2k^2$$

$$\Rightarrow b^2 \text{ is even}$$

$$\Rightarrow b \text{ is even}$$

$$\Rightarrow \text{G.C.F}(a, b) \geq 2$$

i.e. a & b are not relatively prime or $\frac{a}{b}$ is not in simplest form.

$\Rightarrow$ Which is a contradiction

$\Rightarrow \sqrt{2}$ is not rational no.

Hence, $\sqrt{2}$ is an irrational no.

12. Find the fourth root of $-1 + \sqrt{3}i$.

[04]

$$\text{Let } Z = -1 + \sqrt{3}i$$

$$\Rightarrow r = |Z| = 2$$

$$\Rightarrow \theta = \text{Arg}(Z) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\therefore \theta = \frac{5\pi}{6}$$

$$\Rightarrow \sqrt[4]{Z} = r^{1/4} e^{i(\frac{\theta + 2k\pi}{n})}$$

$$= \sqrt[4]{2} e^{i(\frac{5\pi/6 + 2k\pi}{4})}$$

$$, n=4, k=0, 1, 2, 3$$

$$, k=0, 1, 2, 3$$

$$= \sqrt[4]{2} \left[\cos\left(\frac{5\pi/6 + 2k\pi}{4}\right) + i \sin\left(\frac{5\pi/6 + 2k\pi}{4}\right) \right]$$

$$Z_0 = \sqrt[4]{2} \left[\cos\left(\frac{5\pi}{24}\right) + i \sin\left(\frac{5\pi}{24}\right) \right]$$

$$Z_1 = \sqrt[4]{2} \left[\cos\left(\frac{17\pi}{24}\right) + i \sin\left(\frac{17\pi}{24}\right) \right]$$

$$Z_2 = \sqrt[4]{2} \left[\cos\left(\frac{29\pi}{24}\right) + i \sin\left(\frac{29\pi}{24}\right) \right]$$

$$Z_3 = \sqrt[4]{2} \left[\cos\left(\frac{41\pi}{24}\right) + i \sin\left(\frac{41\pi}{24}\right) \right]$$

Best Wish!!!

Please go on to the next page

A. $f^{-1}(x) = \frac{1}{x+3}$

C. $f^{-1}(x) = 3 - \frac{1}{x}$

B. $f^{-1}(x) = \frac{1}{x} - 3$

D. $f^{-1}(x) = x + 3$

9. Suppose that when the polynomial $p(x)$ is divided by $x - 5$, the quotient is $3x^4 - 5x^2 + 2x - 5$ with a remainder 4, then we may conclude that

A. $x - 4$ is a factor of p and 4 is the zero of p

B. $x + 5$ is not a factor of p and -5 is not a zero of p

C. $x - 5$ is a factor of p and 5 is a zero of p

D. $x - 5$ is not a factor of p and 5 is not a zero of p

10. Let $f(x) = x^3 + 5x + 1$, then which one of the following is true for any real number x

A. f is onto but not one to one

B. f is one to one correspondence

C. f is neither one to one nor onto

D. f is one to one but not onto

Part I Write the most simplified form of your answers only on the space provided for the following items. Each item will be awarded 2 marks for complete answers.

1. Let p and q be propositions which stands for

$$p(x) : |x| = -1 \quad q(x) : e^{2x} - 4 = 0$$

then the truth value of $(\exists x)(p(x) \Rightarrow q(x))$ is True (T) [02]

2. Let $A \cup B = \{a, b, d, e, x, y, z\}$ and $A \cap B = \{b, e, y\}$.

(a) If $B - A = \{x, z\}$, then $A = \{b, e, y, a, d\}$

(b) If $A - B = \emptyset$, then $B = A \cup B = \{a, b, d, e, x, y, z\}$

[02]

3. For propositions p, q and r , let the truth value of the compound proposition $\neg[\neg r \Rightarrow \neg(p \wedge q)]$ be True, then the truth value of

(a) $(p \wedge q) \Rightarrow (\neg q \vee p)$ is True (T)

(b) $\neg(p \Leftrightarrow (q \vee r))$ is False (F)

4. If $z = \sqrt{2}(\cos(\frac{\pi}{24}) + i\sin(\frac{\pi}{24}))$, then the value of z^4 is $2\sqrt{3} + i$ [02]
 $4[\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})] = 4e^{i\frac{\pi}{6}}$ [02]

5. The *lub* and *glb* of the set $S = \{3.01, 3.0101, 3.010101, \dots\}$ are $3.\overline{01}$ and 3.01 [02]

6. The inverse of the function $f(x) = \frac{x+1}{x-1}$, $x \neq 1$ is $f^{-1}(x) = \frac{x+1}{x-1}$ [02]

7. Let $f(x) = x^2 - 3x$ and $g(x) = 1 + x$, then the solution of $f(g(x)) = 0$ is $\{-1, 2\}$ [02]

8. For what value of k , $x^2 + y^2 - 6y + k = 0$ represents a circle? $k \leq 9$ [02]

9. The vertex, focus and directrix of the parabola $x^2 - 8y - 6x + 1 = 0$ are $(3, -1)$, $(3, 1)$ and $y = -3$ respectively.

10. The eccentricity of the ellipse $\frac{(x-2)^2}{4} + \frac{(y+1)^2}{9} = 1$ is $\frac{\sqrt{5}}{3}$ [02]

[02]

Wolaita Sodo University
College of Natural & Comp. Science
Department of Mathematics

Basic Mathematics Final Examination For 1st year Natural Science

Time allowed: $2\frac{1}{2}$ hrs.

Max.Load: 50% Date: 28/04/12 E.C

Name: _____ Section: _____

Part I(1.5pt) For each of the following statement write TRUE if the statement is correct and FALSE if the statement is incorrect in the shaded region only.

TRUE 1. The range of an odd degree polynomial function is the set of all real numbers.

FALSE 2. Vertical line is linear function.

TRUE 3. The amplitude of the function $f(x)=\sin(2x)$ is one.

TRUE 4. The function $f(x) = \pi^{0.5x}$ is an exponential growth function.

FALSE 5. Hyperbolic function is periodic

FALSE 6. The domain of the function $f(x)=\operatorname{csch} x$ is $\mathbb{R} \setminus \{1\}$.

FALSE 7. The graph of a function is a curve in the xy -plane.

FALSE 8. The range of $f(x)=\sec x$ is $\mathbb{R} \setminus (-1, 1)$.

TRUE 9. The locus of the equation $x^2 + y^2 - 2x + 4y + 5 = 0$ is a point circle.

FALSE 10. The function $f: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R}$, defined by $f(x) = \frac{2x-1}{x-1}$ has no an inverse.

Part II:(1.5pt) For the following question, choose the correct answer from the given options & write the letter of your choice in the shaded region only.

C 1) Which one of the following is the slope of a line whose equation is $2(y-x-4)=3(1-3x)+y$ is:

A) -11 B) -8 C). -7 D) -5 E) none of the above

E 2) Let $f(x) = (x^4 - 2x^2 - 8)(x^3 - 2x^2 - x + 2)$. Then the sum of multiplicities of roots 1 & 2 is;

A) 7 B) 5 C) 3 D) 2 E) 4

3) If the y-intercept and the x-intercept are 2 & -3 respectively, then which one is the equation of the line?

- A) $3x-2y-6=0$ B) $3x-2y+6=0$ C) $3y-2x-6=0$
D) $3y-2x+6=0$ E) none of the above

4) Which of the following is false about the function $f(x) = 2^x - 1$?

- A) The domain is $\mathbb{R}$ B) The line $y=0$ is the horizontal asymptote.
C) It is an increasing function D) The range is $(-1, \infty)$ E) none of the above

5) The distance b/n the line whose equation $3y-4x+13=0$ and the point $(-1, 1)$ is;

- A) $\frac{6}{5}$ B) 2 C) 4 D) 5 E) none of the above

6) Let $R = \{(x, y) : y \geq x - 3\}$ be a relation. Then which one is false about R^{-1} ?

- A) $R^{-1} = \{(y, x) : y \geq x - 3\}$ B) $R^{-1} = \{(y, x) : x \geq y + 3\}$
C) $R^{-1} = \{(x, y) : y \leq x + 3\}$ D) $R^{-1} = \{(x, y) : x \geq y - 3\}$ E) B & D

7) If the equation $x^2 + y^2 + 6y - 2(x + 3) + 2k = 0$ represent a real circle, then which one of the following cannot be possible value of k ?

- A) 4 B) 5 C) 7 D) 8 E) -5

8) Which of the following is false about the function $f(x) = \log_{(0.1)} x$?

- A) It is a decreasing function B) It approaches the negative y-axis.
C) Its inverse is $f^{-1}(x) = 10^{-x}$ D) Its x-intercept is at $(1, 0)$ E) none of the above

9) Which of the following is a polynomial function?

- A) $f(x) = x^3 + 2x^2 - 3\sqrt{x} + 2$ B) $f(x) = 3x^7 + 4|x|$ C) $f(x) = \sqrt[3]{5}$
D) $f(x) = 3x^4 - 2x^3 + 6x^{-2} + 7x - 2$ E) B & D

10. Which one is false about a rational function? $f(x) = \frac{x-3}{x^2+4x-12}$

- A) Domain of $f(x)$ is $\mathbb{R}/\{2, -6\}$ B) The horizontal asymptote is $y = 0$
C) The x-intercept is $(3, 0)$ D) The y-intercept is $(\frac{1}{4}, 0)$ E) none of the above



3) If the y-intercept and the x-intercept are 2 & -3 respectively, then which one is the equation of the line?

- A) $3x-2y-6=0$ B) $3x-2y+6=0$ C) $3y-2x-6=0$
D) $3y-2x+6=0$ E) none of the above

4) Which of the following is false about the function $f(x) = 2^x - 1$?

- A) The domain is $\mathbb{R}$ B) The line $y=0$ is the horizontal asymptote.
C) It is an increasing function D) The range is $(-1, \infty)$ E) none of the above

5) The distance b/n the line whose equation $3y-4x+13=0$ and the point $(-1, 1)$ is;

- A) $\frac{6}{5}$ B) 2 C) 4 D) 5 E) none of the above

6) Let $R = \{(x, y) : y \geq x - 3\}$ be a relation. Then which one is false about R^{-1} ?

- A) $R^{-1} = \{(y, x) : y \geq x - 3\}$ B) $R^{-1} = \{(y, x) : x \geq y + 3\}$
C) $R^{-1} = \{(x, y) : y \leq x + 3\}$ D) $R^{-1} = \{(x, y) : x \geq y - 3\}$ E) B & D

7) If the equation $x^2 + y^2 + 6y - 2(x + 3) + 2k = 0$ represent a real circle, then which one of the following cannot be possible value of k ?

- A) 4 B) 5 C) 7 D) 8 E) -5

8) Which of the following is false about the function $f(x) = \log_{(0.1)} x$?

- A) It is a decreasing function B) It approaches the negative y-axis.
C) Its inverse is $f^{-1}(x) = 10^{-x}$ D) Its x-intercept is at $(1, 0)$ E) none of the above

9) Which of the following is a polynomial function?

- A) $f(x) = x^3 + 2x^2 - 3\sqrt{x} + 2$ B) $f(x) = 3x^7 + 4|x|$ C) $f(x) = \sqrt[3]{5}$
D) $f(x) = 3x^4 - 2x^3 + 6x^{-2} + 7x - 2$ E) B & D

10. Which one is false about a rational function? $f(x) = \frac{x-3}{x^2+4x-12}$

- A) Domain of $f(x)$ is $\mathbb{R}/\{2, -6\}$ B) The horizontal asymptote is $y = 0$
C) The x-intercept is $(3, 0)$ D) The y-intercept is $(\frac{1}{4}, 0)$ E) none of the above



III: (1.5pts) Short Answers:

Write the most simplified answers in the given space only.

Suppose ℓ_1 and ℓ_2 are perpendicular lines intersecting at $(-1, 2)$. If the angle of inclination of ℓ_1 is 45° , then the equation of ℓ_2 is $y = x + 3$.
Suppose the endpoints of a line segment AB are $A(-5, 1)$ and $B(5, 10)$. Then the coordinates of point P if P divides segment AB in the ratio 2:3 is $(-1, \frac{23}{5})$.

$p(x) = x^5 - 2x^2 - 3$ is divided by $x + 1$ the quotient is $x^4 - x^3 - x^2 - 3x - 6$.

$f(x) = e^{3x-1}$. Then $f^{-1}(x) = \frac{\ln x + 1}{3}$.

General equation of a circle tangent to a line whose equation is $2x - y - 5 = 0$ and centered at point $(-1, 3)$ is: $x^2 + y^2 - 2x + 6y - 10 = 0$.

Slope-intercept equation of a line which passes through $(-1, \sqrt{3})$ and whose inclination angle is 105° is: $y - \sqrt{3} = \frac{1}{2}(x + 1)$.

General value of x for $\cos x + \sin x = 1$ is $n\pi$ or $\frac{\pi}{2} + 2n\pi$ where n is $0, 1, 2, 3, \dots$.

Amplitude and period of $f(x) = -7 \sin\left(\frac{\pi}{4}x - \frac{\pi}{4}\right)$ are 7 & $\frac{\pi}{4}$ respectively.

Workout(8pts): (Each contains 2, 3, 3pts respectively.)

necessarily steps Clearly in the attached paper.

Find the zero of a polynomial function $f(x) = 2x^3 - 5x^2 + 14x + 39$, then find

$(3 - x)$. Sketch the graph of the function f and identify the domain, range, and asymptotes of f .



WACHEMO UNIVERSITY

MATHS FINAL EXAM

Part I. Short Answer Questions (10pts)

Direction: For each of the following questions write your short and simplified answers only in the space provided. (2pts each)

1. Let $f(x) = x + 4$ and $g(x) = x^2 - 4$. Then the domain of $\left(\frac{f}{g}\right)(x) =$ _____
2. If $f(x) = \sqrt{x^3}$ and $(f \circ g)(x) = \sqrt[4]{x}$, then $g(64) =$ _____
3. If $f(x) = \frac{ax+b}{2}$ and $f^{-1}(x-1) = \frac{2x-5}{2}$, then the value of a & b respectively = _____
4. The simplified form of $\sin\left(\frac{\pi}{2} - x\right) \tan x =$ _____
5. Suppose $f(x) = \frac{2x^2 - ax + 4}{x-2}$ be a rational function. If the line $y = 2x - 3$ is its oblique asymptote, then the value of $a =$ _____

Part II. Workout Questions (40pts)

Direction: Attempt all questions by showing all the necessary steps neatly and clearly on the answer sheet provided. (5pts each)

1. Find the cube root of $z = 1 + i^5$.
2. When $2x^3 - 5x^2 + 4x - 8$ is divided by $ax^2 + bx + c$ the quotient is $x - 2$ and the remainder is $bx - a$. Then find the values of a, b and c .
3. Solve for x if $x = 12^{\log_6 8x}$.
4. Sketch the graph of $f(x) = 1 + \log_2(3 - x)$ and identify the domain, range, intercepts and asymptotes.
5. Find the inverse of $y = \cosh x$.
6. If the line $3x - 2y - m = 0$ divides the line segment joining $P(-1, -3)$ and $Q(4, 5)$ internally in the ratio $PR: RQ = 2:3$ at R . Find the value of m .
7. Consider a circle whose center is on the y -axis. If a line given by $y = -2x$ is tangent to the circle at $\left(\frac{-1}{2}, 1\right)$. Find the equation of the circle.
8. Show that the directrix of the parabola $y^2 = 4px$ is tangent to the circle whose diameter is the latus rectum of the parabola. Also find the length of the chord of the circle along the y -axis.

SZ

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Part I. Short Answer Questions (10pts)

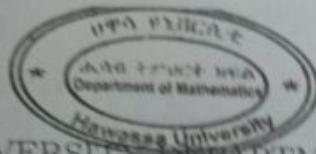
Direction: For each of the following questions write your short and simplified answers only on the space provided. (2pts each)

1. Let $f(x) = x + 4$ and $g(x) = x^2 - 4$. Then the domain of $\left(\frac{f}{g}\right)(x) =$ _____.
2. If $f(x) = \sqrt{x^3}$ and $(f \circ g)(x) = \sqrt[4]{x}$, then $g(64) =$ _____.
3. If $f(x) = \frac{ax+b}{2}$ and $f^{-1}(x-1) = \frac{2x-5}{2}$, then the value of a & b respectively = _____.
4. The simplified form of $\sin\left(\frac{\pi}{2} - x\right) \tan x =$ _____.
5. Suppose $f(x) = \frac{2x^2 - ax + 4}{x-2}$ be a rational function. If the line $y = 2x - 3$ is its oblique asymptote, then the value of $a =$ _____.

Part II. Workout Questions (40pts)

Direction: Attempt all questions by showing all the necessary steps neatly and clearly on the space provided. (5pts each)

1. Find the cube root of $z = 1 + i^5$.



HAWASSA UNIVERSITY DEPARTMENT OF MATHEMATICS

BASIC MATH FOR NATURAL SCIENCE FINAL EXAM -Jan. 27-2020 G.C.

Name: _____ ID.No. _____ Sec. _____ Time 2 : 30 hrs.

Part I. Choose the best answer to each of the following questions and write your answer on the space provided in the answer sheet [1.5 points each]

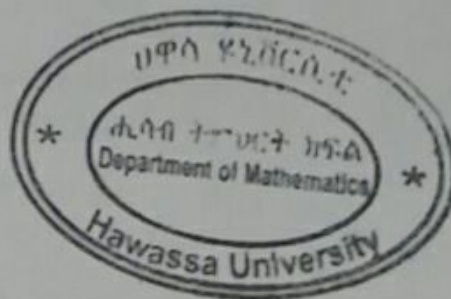
- Which of the following pair of functions are equal?
 A) $h(x) = \frac{1}{x}$ and $k(x) = \frac{x+1}{x^2+x}$ B) $m(x) = \frac{x}{x^2}$ and $n(x) = \frac{1}{x}$
 C) $f(x) = \frac{1}{x}$ and $g(x) = \log_2 x$ D) A and B
- Suppose $f(x) = 2x^3 + 4x^2 + 5x - 9$. A root of f lies between
 A) 1 and 2 B) -2 and -1 C) 0 and 1 D) -1 and 0
- The simplest form of $\log_3 4 \times \log_4 5 \times \log_5 6 \times \log_6 7 \times \log_7 8 \times \log_8 9$ is
 A) 2 B) 3 C) 1 D) $\log_8 9$
- Which of the following is equal to $\sin 280^\circ$?
 A) $\cos -10^\circ$ B) $-\cos 80^\circ$ C) $-\cos 10^\circ$ D) $\sin 80^\circ$
- Which of the following compound statement is contradiction?
 A) $(\neg p \wedge \neg q) \Rightarrow \neg(p \vee q)$ B) $(p \Leftrightarrow q) \Leftrightarrow (\neg p \Leftrightarrow \neg q)$
 C) $(p \Rightarrow q) \Leftrightarrow (p \vee q)$ D) $(p \Rightarrow q) \Leftrightarrow (p \wedge \neg q)$
- The equation of the circle with center at (2, 1) and tangent to the line $l: 3y - 4x = 5$ is
 A) $x^2 + y^2 + 4x + 2y + 4 = 0$ B) $x^2 + y^2 - 4x - 2y + 1 = 0$
 C) $x^2 + y^2 - 4x + 2y - 1 = 0$ D) $x^2 + y^2 + 4x - 2y - 16 = 0$
- Which of the following is a one-to-one correspondence function?
 A) $g: \mathbb{R} \mapsto [0, \infty): g(x) = 2^x + 1$ B) $h: \mathbb{R} \setminus \{0\} \mapsto \mathbb{R}: h(x) = \ln x^2$
 C) $f: (-\infty, 1] \mapsto \mathbb{R}: f(x) = x^2 - 2x + 3$ D) $k: (-\infty, 2] \mapsto [0, \infty): k(x) = \sqrt{2-x}$
- If $\cos \theta = \frac{5}{6}$ and $0 < \theta < \frac{\pi}{2}$, then $\csc \theta$ is A) $\frac{6}{\sqrt{11}}$ B) $\frac{36}{11}$ C) $\frac{\sqrt{11}}{6}$ D) $\frac{11}{36}$
- Which of the following is not a correct identity?
 A) $\frac{\csc^2 x - 1}{\csc^2 x} = \cos^2 x$ B) $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$
 C) $(\cot^2 x + 1)(\sin^2 x - 1) = \cot^2 x$ D) $\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x} = 2\csc^2 x$
- Which of the following is not true about the graph of $f(x) = -\left(\frac{1}{3}\right)^{x-1} - 2$?
 A) Its y -intercept is (0, -5)
 B) The graph of $f(x)$ is increasing in its domain
 C) Its horizontal asymptote is $y = -2$
 D) The range of $f(x)$ is $(-2, \infty)$

Part II. Give the simplified answers to each of the following questions on the space provided in the answer sheet [1.5 points each]

1. If $z = 1 - \sqrt{3}i$, then the principal argument of $z^4 =$ _____
2. Give the equation of the line that passes through the point $(1, 0)$ and perpendicular to the line $-2x + 4y = 2$ _____
3. If $GCF(a, b) = 5$ and $ab = 250$, then $LCM(a, b) =$ _____
4. The inverse of $f(x) = \sqrt{2-x} + 5$ is _____
5. Let $l_1 : 2y + x = 3$ and $l_2 : 2y + x = 1$. The distance between l_1 and l_2 is _____
6. If $x = 1$ and $x + 1$ are a root and a factor of $p(x) = 2ax^5 - x^3 + bx^2 + 1$ respectively, then $a =$ _____ and $b =$ _____
7. Suppose $f(x) = \frac{x}{1-x}$ and $g(x) = \frac{x-1}{x+1}$ then $f \circ g(x) =$ _____
8. The value of $\coth(\ln 5)$ is _____
9. The domain of the function $f(x) = \frac{\sqrt{x-2}}{\sqrt{3-x}}$ is _____
10. The amplitude and period of $y = -2\sin(\frac{3}{2}x)$ respectively are $A =$ _____, $P =$ _____
11. Let $f = \{(2, 4), (3, 6), (4, 2), (6, 0)\}$ and $g = \{(0, 3), (3, 5), (2, -4), (4, 8), (6, 0)\}$.
Then $\frac{f}{g} =$ _____
12. If $\sqrt{3}^{[4 + \log_{\sqrt{5}} k]} = \frac{1}{3}$, then $k =$ _____

Part III. Work on the following problems by showing all the information.

1. If $2 + 3i$ is a root of $p(x) = x^4 - 4x^3 + 12x^2 + 4x - 13$, then find all the other roots.
2. Discuss whether the lines $l_1 : y = -x + 3$, $l_2 : 3y = 5x + 15 = 0$ and $l_3 : y - 2x + 1 = 0$ or one point or no point in common with the circle $x^2 + y^2 + 2x - 4y + 3 = 0$
3. Find equation of the circle passing through the points $(1, 1)$, $(-1, 1)$ and $(-2, -1)$
4. Let $f(x) = \frac{x^2 - x - 6}{x - 2}$. Find
 - a. Its intercepts
 - b. Asymptotes
 - c. Sketch the graph



Part I: - Write true if the statement is correct or false if it is incorrect (1 pt each)

- ___ 1. The real part of any complex number z and its conjugate are the same.
- ___ 2. Every integer power of a complex number i is a member of the set $\{-i, -1, i, 1\}$.
- ___ 3. The subset of a Cartesian product of any two nonempty set is a function.
- ___ 4. The inverse of any one to one correspondence function is also function.
- ___ 5. The Exponential function has horizontal asymptote but not vertical.
- ___ 6. The function $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = 3x$ is onto function.
- ___ 7. $10^{2n-1} + 1$ is divisible by 11 for all $n \in \mathbb{N}$.
- ___ 8. Logarithmic function has y -intercept but not x - intercept.
- ___ 9. Trigonometric functions are periodic functions with period $p = 2\pi$.
- ___ 10. The value of x which satisfies $\log x + \log(x + 3) = 1$ is 2.

Parts II: - Fill the simplified answer in the given space for the following questions (1 pts each)

1. The range of the function $f(x) = \frac{x}{|x|}$ is _____
2. The radian measure of 25° is _____
3. If $f(x) = x^3 - \frac{1}{x^3}$, then $f(x) + f\left(\frac{1}{x}\right)$ is equal to _____
4. Let $f(x) = x + 1$ and $g(x) = \sqrt{x - 1}$, then $(f \circ g)(5)$ is equal to _____
5. The modulus of $z = \frac{1-i}{3+i} + \frac{4i}{5}$ is _____
6. the principal argument of the complex number $z = -2 + 2\sqrt{3}i$ is _____
7. The amplitude of a function $f(x) = \cos\left(\frac{x}{4}\right)$ is _____
8. The polar form of the complex number $z = (2 + 2i)^{1000}$ is _____
9. If $f(x) = x - 3$ and $g(x) = x^2 + 3$, then $(gf)(x) =$ _____
10. If $f(x) = \frac{x^3 - 1}{(x^2 - 1)(x^4 - 16)}$ then the vertical asymptote(s) of f is(are) _____

Parts III:- Write the simplified answer from the given alternative in a given space (1.5 pts each)

1. Which one of the following is differ from the other?

A. $y = 2x$

B. $y = 1$

C. $x = 2$

D. $y = x^2 + 3$
2. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \begin{cases} x+2, & \text{if } x < -1 \\ x^2, & \text{if } -1 \leq x \leq 1 \\ 2-x, & \text{if } x > 1 \end{cases}$, then the value of $f(-1.75) + f(0.5) + f(1.5)$ is

A. 0

B. 1

C. 2

D. 3
3. Which one of the following is a real valued function

A. $f(x) = \frac{2-x}{x-4}$

B. $f(x) = \sin x$

C. $f(x) = x^4 + 3x^3 + 2x^2 + x - 4$

D. all
4. The value of x which satisfies the equation $\log_6(x+2) - \log_6 x = 2$ is

A. $\frac{2}{35}$

B. $\frac{35}{2}$

C. -6

D. $\frac{1}{18}$
5. Suppose $f(x) = \log(x-3)$ and $g(x) = x+4$, then the domain of $(f \circ g)(x)$ is

A. $(-3, \infty)$

B. $(-\infty, -3)$

C. $(-1, \infty)$

D. $[-1, \infty)$
6. All the zeros of the polynomial $p(x) = 2x^3 - 9x^2 - 6x + 40$ are

A. $-2, \frac{5}{2}, 4$

B. $0, 2, \frac{5}{2}$

C. $2, \frac{5}{2}, -4$

D. $2, -9, -6$
7. Which one of the following functions has an inverse function?

A. $f(x) = |x-4|$

B. $g(x) = (x+4)^3$

C. $h(x) = (x+4)^4$

D. $h(x) = \frac{1}{(x-4)(x+4)}$
8. Let $f(x) = \frac{1}{x+3}$, then which one of the following is a formula for $f^{-1}(x)$