

Addis Ababa University
Department of Mathematics
Math 231- B (Applied Mathematics I)
Mid-Semester Examination

Date: December 26, 2004.

Time allowed 3 hours.

Name: _____

ID.Nº: _____

Instructions:

1. Calculators or any other electronic devices are **NOT** allowed.
2. You are **NOT** allowed to have any paper of your own. You will be provided with papers for rough work.
3. Check that your booklet has **6** pages including the cover page. Check also that there are **10** short answer problems in part - I and there are **5** workout problems in part - II.
4. Write yours answers as clear and neat as possible. Unclear writings may not be graded.
5. Results you might need: $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ and $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$.

DO NOT START WORKING BEFORE YOU ARE TOLD TO DO SO

AND GOOD LUCK.

For instructor's use		
Part I	Part II	Total

Part I. Write *only* your simplified solutions for the following problems (2 pts. each.)

1. For what value of c will the planes

$$\begin{aligned} 2x - 7y + z + 5 &= 0 \\ \text{and} \quad x - y + cz - 4 &= 0 \end{aligned}$$

be perpendicular? _____

2. Let $\mathbf{A}$ be a 2×1 matrix and $\mathbf{B}$ be a 1×2 matrix. Then $\mathbf{AB}$ is not invertible matrix

(True/False)_____

3. Let $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ be non – zero vectors in $\mathbf{R}^3$.

a. $\mathbf{A} \cdot \mathbf{B} = 0 = \mathbf{A} \cdot \mathbf{C} \Rightarrow \mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = 0$. (True/ False) _____.

b. $\mathbf{A} \times \mathbf{B} = \mathbf{A} \times \mathbf{C} \Rightarrow \mathbf{B} \cdot \mathbf{C} = 0$ (True/ False) _____.

4. If $\mathbf{A} = \begin{pmatrix} -1 & 0 & 3 \\ 2 & 4 & 1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 5 & 2 & -1 \\ -3 & 6 & 4 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 0 & -1 & 3 \\ -1 & 0 & 2 \end{pmatrix}$, then

$$(\mathbf{A} + \mathbf{B}) \cdot {}^t \mathbf{C} = \underline{\hspace{2cm}}.$$

5. Find the point located $\frac{2}{3}$ of the way from the point $P(1,0,7)$ to the point $Q(-5,3,4)$

6. Find the **parametric** equation of the line passing through the point $(3,-5, 2)$ and perpendicular to the plane with equation $7x - 11y = 2$.

_____.

7. Find the distance of the point $P = (-2,1,0)$ from the line with symmetric equation

$$-x = \frac{y-3}{2} = \frac{z+2}{-2}$$

_____.

8. If the projection of vector $\mathbf{A}$ along vector $\mathbf{B}$ is $(1, \frac{3}{2}, \frac{-5}{2})$ and if $\|\mathbf{A}\| = 2$, $\|\mathbf{B}\| = \sqrt{2}$,

then the norm of the projection of $\mathbf{B}$ along $\mathbf{A}$ i.e. $\|\text{Proj}_{\mathbf{A}}^{\mathbf{B}}\| = \underline{\hspace{2cm}}$

9. Suppose the angle between two vectors $\mathbf{A}$ and $\mathbf{B}$ in $\mathbf{R}^3$ be $\frac{\pi}{3}$ and $\|\mathbf{A}\| = 2$, $\|\mathbf{B}\| = 1$.

Let $\mathbf{C} = \mathbf{A} \times \mathbf{B}$, then $\|\mathbf{A} + \mathbf{B} - \mathbf{C}\| = \underline{\hspace{2cm}}$

10. Find the rank of the matrix

$$\begin{pmatrix} -2 & 0 & 1 & 1 \\ 1 & -2 & 0 & 1 \\ 4 & 4 & -3 & -5 \\ -3 & -2 & 2 & 3 \end{pmatrix}$$

Part - II Workout problems. Show all the necessary steps.

1. Solve the system of linear equations using Gauss – Jordan (Gaussian) elimination.

(4 points)

$$\begin{aligned} x - y + w &= 4 \\ 2x + y - z + 2w &= 1 \\ x + y + z - 2w &= 2 \\ 2y + z + 3w &= 4 \end{aligned}$$

2. Given the matrix $\mathbf{A} = \begin{pmatrix} -3 & 6 & 5 \\ 2 & 1 & -1 \\ -2 & 0 & 4 \end{pmatrix}$, (4 points)
- Find $\mathbf{A}^{-1}$

3. Show that $W = \{(x, y, z) \mid y + 3x = 0\}$ is a subspace of the vector space $\mathbf{R}^3$ with the usual addition and scalar multiplication of vectors. (3 points)

4. Let $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 5 & 3 & 1 \\ 0 & 2 & 4 \end{pmatrix}$.

Determine all the eigenvalues and corresponding eigenvectors for $\mathbf{A}$. (4 points)

5. Show that $B = \{ (1,0,-1), (1,2,1), (0,-3,2) \}$ is a basis for the vector space $\mathbf{R}^3$. (5 pts.)