ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics I (Math 1041)
Exercise 2 on matrices and Determinants AY2021/2022

1. Let $A=\begin{pmatrix}-1 & 2\\ 3 & 0\\ 1 & 5\end{pmatrix}$ and $B=\begin{pmatrix}3 & -5 & 7\\ 1 & 1 & -3\end{pmatrix}$. Find

   (a) $2A-B^t$, A^t+3B (b) AB, $(AB)^t$, A^tB^t, B^tA^t

2. (a) Find x, y, z and w if $3\begin{pmatrix}x & y\\ z & -w\end{pmatrix}=\begin{pmatrix}x & 6\\ -1 & 2w\end{pmatrix}+\begin{pmatrix}4 & x+y\\ z+w & 3\end{pmatrix}$.

   (b) Let $A=\begin{pmatrix}2 & -1 & 3\\ 1 & 5 & 0\\ 0 & 2 & 4\end{pmatrix}$. If $(2A-3B)^t=6I_3$, then find matrix B.

3. Find a matrix D such that A = DB where $A=\begin{pmatrix}1 & -1\\ -2 & 3\\ 1 & 0\end{pmatrix}$ and $B=\begin{pmatrix}3 & 1\\ -4 & 2\end{pmatrix}$.

4. Let $A=\begin{pmatrix}4 & 5 & 3 & 7\\ -1 & 3 & 2 & -1\\ 2 & 3 & 2 & 1\end{pmatrix}$ Find the reduced row echelon form of A

5. Let $A=\begin{pmatrix}\frac{1}{2} & \frac{-1}{2}\\ \frac{-1}{2} & \frac{1}{2}\end{pmatrix}$. Then find A^n for any positive integer n.

6. Let $A=\begin{pmatrix}1 & 2 & 4\\ -1 & 3 & 0\\ -2 & -4 & -2\end{pmatrix}$. Write A as a sum of symmetric and skew-symmetric matrices.

7.

8. Let A and B be square matrices of order n. Express the following in terms of $\det A$ and $\det B$

   a) $\det(4A^tB^5)$ b) $\det(adjA)$

9. Given that $\begin{pmatrix}1 & -2 & -3\\ 2 & 1 & -2\\ 0 & 2 & 0\end{pmatrix}$ A $\begin{pmatrix}1 & 0 & 3\\ -2 & 1 & -8\\ 3 & 0 & 7\end{pmatrix}=2I_3$, find det(A).

10. Find the determinant of the following matrices

a) $\begin{pmatrix}3 & 1\\ 2 & 5\end{pmatrix}\begin{pmatrix}-1 & 2\\ 0 & 5\end{pmatrix}\begin{pmatrix}5 & -2\\ 4 & -1\end{pmatrix}$ b) $\begin{pmatrix}abc & a^2 & ac\\ bc & ac & c^2\\ b^2 & ab & bc\end{pmatrix}$ c) $\begin{pmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{pmatrix}$ d) $\begin{pmatrix}a & b & c\\ c & a & b\\ b & c & a\end{pmatrix}$

11. Find the rank of the following matrices

(a) $\begin{pmatrix}-1 & 0\\ 2 & 5\end{pmatrix}$ (b) $\begin{pmatrix}2 & -3 & 1\\ -4 & 6 & -2\end{pmatrix}$ (c) $\begin{pmatrix}-1 & 0 & -1\\ 2 & 1 & 2\\ 1 & 2 & 1\end{pmatrix}$ (d) $\begin{pmatrix}0 & 1\\ -1 & 0\\ 2 & 3\\ 2 & 3\end{pmatrix}$

12. If it exists, find the inverse of the following matrices.

(a) $\begin{pmatrix}2 & 4\\ 6 & -1\end{pmatrix}$ (b) $\begin{pmatrix}2 & 1 & -1\\ 0 & 2 & 1\\ 5 & 2 & 3\end{pmatrix}$ (c) $\begin{pmatrix}4 & -2 & 5\\ 1 & 1 & -2\\ 6 & 1 & -1\end{pmatrix}$

13. Let $A=\begin{pmatrix}2 & 2 & t\\ 0 & -t & -3\\ t & 0 & -1\end{pmatrix}$. Find the set of values of t for which the homogeneous system of linear equations $AX=0$ has non trivial solution.

14. Let $M=\begin{pmatrix}a & 0 & b & 2\\ a & a & 4 & 4\\ 0 & a & 2 & b\end{pmatrix}$ be the augmented matrix for a linear system. For what values of a and b the system will have: a) no solution? b) unique solution? c) infinitely many solutions?

15. Solve the following system of equations using either Gauss's Method or Cramer's Rule, whenever possible.

(a) $\begin{cases}3x-7y=2\\ 2x-5y=-1\end{cases}$ (b) $\begin{cases}4x+6y+z=2\\ 2x+y-4z=3\\ 3x-2y+5z=8\end{cases}$ (c) $\begin{cases}3x+3y+4z=3\\ 4x+6z=2\\ 8x+z=2\end{cases}$

(d) $\begin{cases}3x-9y=1\\ -2x+6y=-3\end{cases}$ (e) $\begin{cases}2x+3y+z=1\\ x+4y-z=0\end{cases}$ (f) $\begin{cases}x+y+2z-w=-2\\ 2x+y-2z-2w=-2\\ 3x-3w=-3\end{cases}$

16. Find the characteristic polynomial, eigenvalues and eigenvectors of

a) $A=\begin{pmatrix}-2 & 0\\ 1 & -2\end{pmatrix}$ b) $A=\begin{pmatrix}3 & 4\\ 1 & 3\end{pmatrix}$ c) $B=\begin{pmatrix}0 & -2\\ 1 & 3\end{pmatrix}$ d) $B=\begin{pmatrix}2 & 0 & 1\\ 0 & 1 & 0\\ -2 & 0 & -1\end{pmatrix}$

17. Find the determinant, adjoint and inverse (if exists) of each of the following matrices

(a) $\begin{bmatrix}-1 & 3\\ 8 & 5\end{bmatrix}$ b) $\begin{bmatrix}2 & 5\\ 1 & 3\end{bmatrix}$ c) $\begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}$ D) $\begin{bmatrix}1 & 0 & 1\\ 2 & 1 & 2\\ 1 & 2 & 1\end{bmatrix}$