

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Math1011: Mathematics for Natural Sciences
Worksheet I

Part I: Propositional Logic.

- (1) Which of the following sentences are propositions? What are the truth values of those that are propositions?
- (a) $5 < 9$ and there are infinitely many prime numbers.
 - (b) Nairobi is the capital city of Kenya.
 - (c) Thank you for your consideration.
 - (d) Answer this question.
 - (e) $x^2 = y^2$ provided that $x = y$.
 - (f) How many courses are you taking this semester?
 - (g) $x + 5 = 11$.
 - (h) The first human kind lived in Ethiopia.
- (2) Consider the following sentences
- r : Rabbits have been seen in the area.
 b : Berries are ripe along the path w : Walking on the path is safe.
- Write the following sentences using r , b and w and logical connectives.
- (a) Berries are ripe along the path, but rabbits have not been seen in the area.
 - (b) Rabbits have not been seen in the area, and walking on the path is safe, but berries are ripe along the path.
 - (c) If berries are ripe along the path, then walking is safe if and only if rabbits have not been seen in the area.
 - (d) It is not safe to walk along the path, but rabbits have not been seen in the area and the berries along the path are ripe.
 - (e) For walking on the path to be safe, it is necessary but not sufficient that berries not be ripe along the path and for rabbits not to have been seen in the area.
 - (f) Walking is not safe on the path whenever rabbits have been seen in the area and berries are ripe along the path.
- (3) Given the following propositions
- p : You have the flu. r : You pass the course.
 q : You miss the final examination.
- Express each of the following propositions as an ordinary English sentence.
- a) $\neg q \Leftrightarrow r$ b) $\neg(p \wedge q)$ c) $(p \Rightarrow \neg r) \vee (q \Rightarrow \neg r)$.
- (4) Determine the truth value of p if
- a) $(q \Rightarrow \neg p) \vee r$ is False b) $\neg q$ and $p \Rightarrow q$ are True
 - c) $(p \vee \neg q) \Rightarrow (\neg r \vee p)$ is False.
- (5) If $\neg[\neg r \Rightarrow \neg(p \wedge q)]$ is true, then find the truth value of $[(p \Leftrightarrow r) \vee q] \Leftrightarrow (\neg p \wedge r)$.
- (6) Prove that
- a) $(p \wedge q) \Rightarrow r \equiv p \Rightarrow (q \Rightarrow r)$ b) $(q \vee r) \Rightarrow p \equiv (q \Rightarrow p) \wedge (r \Rightarrow p)$.
- (7) For the following propositions, indicate whether it is a tautology, a contradiction, or neither. Use a truth table to decide.
- a) $[p \wedge (p \Rightarrow q)] \Rightarrow q$ b) $(p \Rightarrow q) \wedge \neg(\neg q \Rightarrow \neg p)$.
 - c) $(\neg q \Rightarrow \neg p) \Rightarrow ((\neg q \Rightarrow p) \Rightarrow q)$.
- (8) Determine whether the following statements are valid or invalid.
- a) If I do not wake up, then I cannot go to work.
If I cannot go to work, then I will not get paid.
Therefore, if I do not wake up, then I will not get paid.

- b) If I study, then I will not fail Math 1011.
 If I do not play cards too often, then I will study.
 I failed Math 1011.
 Therefore, I played cards too often.
- (9) Let $P(x) : x$ is an integer greater than 5 $Q(x) : x$ is a natural number.
 Determine the truth value of the following propositions.
 a) $P(3) \wedge Q(-2)$ b) $P(2) \Rightarrow Q(2)$ c) $[P(3) \Rightarrow Q(4)] \vee Q(9)$.
- (10) Let $P(x) : x$ is a prime number $Q(x) : x$ is an even number
 $R(x) : x$ is an odd number. $S(x) : x$ is an integer.
 Write a sentence, which corresponds to each of the following:
 a) $R(x) \vee Q(x) \Rightarrow P(x)$ b) $P(x) \Leftrightarrow Q(x) \vee R(x)$.
- (11) If $U = \mathbb{R}$, then find the truth value of $(\forall x)(\exists y)(\sqrt{x^2} = 16 \Rightarrow y + x = 10)$.
- (12) Let $P(x) : x$ is a composite number $Q(x) : x$ is a prime number.
 Find the truth values of (i) $(\exists x)[P(x) \Leftrightarrow Q(x)]$ (ii) $(\forall x)[P(x) \Leftrightarrow Q(x)]$.
- (13) Find the truth values of the following where $U = \mathbb{R}$
 a) $(\exists x)(\forall y)(x^2 < y^2)$ b) $(\forall x)[x \neq 0 \Rightarrow (\exists y)(xy = 4)]$.

Part II: Set Theory.

- (1) Write the following sets as an interval or union of intervals. In each case $x \in \mathbb{R}$.
 a) $A = \{x | x \neq 5 \wedge x \neq 8\}$.
 b) $B = \{x | x^2(x^3 - 1) \neq 0\}$.
 c) $C = \{x | x^3 - 2x \neq 0\}$.
 d) $D = \{x | (x - 2)(x + 3)(x + 4) = 0 \Rightarrow x \geq 2\}$.
- (2) Simplify each of the following sets and give the final result as an interval or union of intervals
 a) $([-3, 2] \cap (0, 5))^c$.
 b) $((-3, 5.6] \cap (-5, 2.4\bar{3})) \cup (0.\bar{3}, 11)$.
 c) $(2, 4]^c \cap ((-\infty, 5)) \cup [0, \infty))$.
 d) $(-0.\bar{3}, 0.0\bar{3}) \cup ((-\infty, \frac{1}{30}) \cap [5, \infty))$.
- (3) Write the following sets in complete listing or partial listing method.
 a) $A = \{x | x \text{ is an integer and } 3 < x \leq 10\} \cap \{x | x \text{ is even integer}\}$.
 b) $B = \{x \in \mathbb{N} | (x - 1)(x - 3) \neq 0 \Leftrightarrow x + 1 = x\}$.
 c) $C = \{x \in \mathbb{N} | x \geq 3 \Leftrightarrow x < 0\}$.
 d) $D = \{x | x \in \mathbb{N} \wedge (x - 1)(x - 3) \neq 0 \Leftrightarrow x + 1 = x\}$.
- (4) If $n(A \setminus B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.
- (5) For all subsets A, B and C of some Universal set U , prove or disprove the following statements.
 a) $(A \subseteq B \wedge A \subseteq C) \Rightarrow A \subseteq B \cap C$ b) $A \subseteq B \Leftrightarrow A \cap B = A$
 c) $A^c \setminus B^c = B \setminus A$ d) $A \subseteq B \Rightarrow A \cup (B \setminus A) = B$.
- (6) Let $U = \{x \in \mathbb{Z} | -12 \leq x \leq 6\}$.
 $A = \{x \in \mathbb{Z} | x = 2n \wedge x = 3m, \text{ for } -4 \leq n \leq 8 \text{ and } -2 \leq m \leq 3\}$.
 $B = \{0, 1, 3\}$ $C = \{x \in \mathbb{Z} | x = \frac{n}{3}, n \in A\}$.
 Then find
 a) $A \setminus [B \cup C]^c$ b) $(A \setminus B) \setminus (A \cup C)^c$ c) $(A \cap B) \setminus C$.
- (7) (a) For every natural number n , define $A_n = \{x \in \mathbb{Z} | \frac{-24}{n} \leq 2x + 1 \leq \frac{24}{n}\}$.
 Then find
 i) $\cup_{n=1}^6 A_n$ ii) $[(\cup_{n=1}^4 A_n) \setminus (\cap_{n=1}^3 A_n)]^c$ iii) $\cap A_n, n \in \mathbb{N}$.
 (b) For every natural number n , define
 $A_n = \{x \in \mathbb{N} | x = n \Rightarrow x < n\}$.
 Then find
 i) $\cup_{n=1}^{10} A_n$ ii) $\cup A_n, n \in \mathbb{N}$.