Secondary

MATHEMATICS

Form Three Teachers' Guide

Third Edition

Consider Mr Mbuyu who secured a loan of Ksh 3 780 000 from Maendeleo Building Society for the purchase of a residential house. The amount was repayable in fourteen years at a compound interest rate of 14% per annum.

This means he had to pay a monthly instalment of



KENYA LITERATURE BUREAU

Approved by the Ministry of Education

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Chapter One

QUADRATIC EXPRESSIONS AND EQUATIONS (II)

Quadratic expressions and equations are not new to the learner, having been covered in book two. In this topic, the learner will be introduced to other methods of solving quadratic equations.

Objectives

By the end of the topic, the learner should be able to:

(i) factorise quadratic expressions.
(ii) identify perfect squares.
(iii) complete the square.
(iv) solve quadratic equations by completing the square.
(v) derive the quadratic formula.
(vi) solve quadratic equations using the formula.
(vii) form and solve quadratic equations from roots and given situations.
(viii) make tables of values from quadratic functions.
(ix) draw graphs of quadratic functions.
(x) solve quadratic equations using graphs.
(xi) solve simultaneous equations (one linear and one quadratic) analytically and graphically.
(xii) apply the knowledge of quadratic equations to real life situations.

Time: Twenty two lessons.

Teaching/Learning Activities

Factorisation of Quadratic Expressions

- The teacher should take the learner through a review of factorisation of quadratic expressions, as in the students' book.
- The teacher should lead the learner to identify perfect squares.
- The learner should do exercise 1.1.

Completing the Square

- The learner should be involved in obtaining missing terms to make a given expression a perfect square as, in examples 1, 2, 3 and 4 in the students' book.
- The learner should be given exercise 1.2.

Solution of Quadratic Equations by Completing the Square

- The learner should be guided in using the method of completing the square to solve quadratic equations, as in examples 5 and 6 in the students' book.
- The learner to do exercise 1.3.

The Quadratic Formula

- The teacher should guide the learner in deriving the quadratic formula as developed in the students' book.
- The learner should be led through example 7.
- The learner should do exercise 1.4.
- The learner should be led to determine the nature of the roots of a quadratic equation using the discriminant.

Evaluation

- The teacher should give a written test covering all aspects of the topic.

Further Practice

- The teacher should give an assignment on solving quadratic equations using the method of completing the square, the formula method and the graphical method. The results should then be compared.

Answers

Exercise 1.1

1. (a) $(x-2)(x-4)$ (b) $(x+6)^2$
2. (a) $(2x-3)(x+2)$ (b) $(2x+3)^2$
3. (a) $(x+1)(x-9)$ (b) $(x-7)^2$
4. (a) $(2x+1)(x+1)$ (b) $(4x-1)^2$
5. (a) $(3x+2)^2$ (b) $(5x-1)(x+3)$

Exercise 1.2

1. (a) 16 (b) 49 (c) 121

2. (a) x^2 (b) x^2 (c) $48x$

3. (a) $28x$ (b) x (c) 1

4. (a) $\frac{1}{8}x$ (b) 9 (c) $\pm 4xy$

5. (a) $9b^2$ (b) $2(x-1)(y-1)$ (c) $25a^2$

6. (a) $36x^2$ (b) $4x^2$ (c) $\frac{1}{4}K^2$

7. (a) $25x^2$ (b) $49x^2$

Exercise 1.3

1. (a) – 2.414 or 0.414 (b) 0.4385 or 4.5615
   (c) –5.450 or –0.550
2. (a) 2.268 or 5.732 (b) –1.193 or 4.193
   (c) 0.2762 or 21.72
3. (a) –4.303 or –0.697 (b) –2.783 or 10.78
   (c) –6.542 or –0.4585
4. 0.3792 or 0.8792
5. (a) –2.766 or 1.266 (b) –2.808 or 0.4748
   (c) –3.621 or 0.6213
6. (a) –1.580 or –0.3798 (b) –1.554 or 0.8043
   (c) 0.3930 or –3.393
7. (a) –0.5 or 1.333 (b) –3 or –1.5
   (c) –2 or 1
8. –5.742 or 1.742

Exercise 1.4

1. (a) –6.542 or –0.4585 (b) 1 or 3
   (c) –4.766 or –0.7345
2. (a) –0.8673 or 1.153 (b) –0.4574 or 1.458
   (c) 0.2764 or 0.7236

3. (a) -1 or $-\frac{3}{4}$ (b) $\frac{4}{3}$ (c). -1.379 or -0.1209

4. -1.264 or 0.2638

Exercise 1.5

1. (a) -2 or 3 (b) $-\frac{3}{2}$ or $-\frac{1}{3}$ (c) $\frac{-3}{2}$ or $\frac{-2}{3}$

   (d) 0 or $\frac{-1}{2}$ (e) -0.1761 or 5.676 (f) $\frac{-2}{5}$ or $\frac{2}{5}$

   (g) 0 or $\frac{-1}{8}$ (h) 0.634 or 2.366 (i) -1 or $\frac{5}{3}$

   (j) 0.5538 or 14.45 (k) $\frac{-2}{5}$ or $\frac{1}{2}$ (l) -1 or $\frac{5}{3}$

   (m) -0.1629 or 0.8771 (n) -1 or 6

   (p) -0.449 or 4.449 (q) -0.3891 or 2.056

   (r) -1.652 or 0.652

2. $50\ cm^2$ 3. 12

4. (a) 20 cm or 10 cm (b) 15 cm or 30 cm

5. $2x^2 - 9x - 5 = 0$, 6. 6 km/h, 3 h

   Area = $84\ cm^2$

7. 4 m, $48\ m^2$ 8. 34

Exercise 1.6

1. Table 1.2 (a) $y = x^2 - 2x - 8$

x	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7
x^2	25	16	9	4	1	0	1	4	9	16	25	36	49
−2x	10	8	6	4	2	0	−2	−4	−6	−8	−10	−12	−14
−8	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8	−8
$y = x^2 - 2x - 8$	27	16	7	0	−5	−8	−9	−8	−5	0	7	16	27
x	−5	−4	−3	−2	−1	0	1	2	3	4	5	6	7
8	8	8	8	8	8	8	8	8	8	8	8	8	8
2x	−10	−8	−6	−4	−2	0	2	4	6	8	10	12	14

$-x^2$	–25	–16	–9	–4	–1	0	–1	–4	–9	–16	–25	–36	–49
$y = 8 + 2x - x^2$	–27	–16	–7	0	5	8	9	8	5	0	–7	–16	–27

Equations of lines of symmetry; $x = 1$ and $y = 0$

2. *Line of symmetry* *Maximum/minimum points*

(a) $x = -1$ minimum value –4

(b) $x = -3$ minimum value 0

(c) $x = 2\frac{1}{2}$ minimum value $\frac{3}{4}$

(d) $x = -2$ maximum value 16

(e) $x = 1$ maximum value –4

3. (a) Minimum points; (i) 0 (ii) 10 (iii) –10

(b) $y = x^2$ onto $y = x^2 + 10$; a translation of vector $\begin{pmatrix} 0 \\ 10 \end{pmatrix}$

$y = x^2$ onto $y = x^2 - 10$; a translation of vector $\begin{pmatrix} 0 \\ -10 \end{pmatrix}$

(c) a translation of vector $\begin{pmatrix} 0 \\ -20 \end{pmatrix}$

4.

x	–3	–2	–1	0	1	2	3	4	5	6	7
y	– 41	–29	–19	–11	–5	–1	1	1	–1	–5	–11

Lines of symmetry; $x = 3\frac{1}{2}$ or $y = 0$

5. Table 1.3

y	–4	–3	–2	–1	0	1	2	3	4
x	4	$\frac{9}{4}$	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1	$\frac{9}{4}$	4

Exercise 1.7

1. For all values of x from the learner's graph, allow a deviation of ± 0.1 from the given values.

(a) 0.3 or 1.7 (b) No real roots

(c) No real roots (d) –2

(e) –4.2 or 1.2 (f) No real roots

Exercise 1.8

For all values of x from the learner's graph, allow a deviation of ± 0.1 from the stated values.

1. (a) -3.8 or 1.8 for x
-2.8 or 2.8 for y
(b) -2.0 or 0 for x
-3.0 or 1 for y
(c) -3.1 or 1.1 for x
5.1 or 0.9 for y
(d) -1.4 or 1.7 for x
6.1 or -3.1 for y
(e) 5.4 or 0.6 for x
8.2 or 1.8 for y

2. (a) $y = x^2$ and $y = 3x - 2$; $x = 1.0$ or 2.0
(b) $y = 3x^2$ and $y = 7x - 4$; $x = 1.0$ or 1.3
(c) $y = x^2$ and $y = 6x + 8$; $x = -1.1$ or 7.1
(d) $y = 2x^2$ and $y = 7x + 8$; $x = -0.9$ or 4.4
(e) $y = x^2$ and $y = 22x + 6$; $x = 0.3$ or 21.7
(f) $y = x^2$ and $y = -7x - 3$; $x = -6.5$ or -0.5

Exercise 1.9

1. *Equation of line* — *Solution*
(a) $y = 0$ — $x = -5.4$ or 0.4
(b) $y = 2$ — $x = -4.5$ or -0.5
(c) $y = x + 1$ — $x = -3$ or -1

2. *Equation of line* — *Solution*
(a) $y = 0$ — $x = -1$ or 0.5
(b) $y = -1$ — $x = -0.5$ or 0
(c) $y = x + 4$ — $x = 1.6$ or -1.6

3. *Equation of line* — *Solution*
(a) $y = 0$ — $x = -0.6$ or 6.8
(b) $y = 2x + 4$ — $x = 0$ or 4
(c) $y = \frac{4}{3}x + 5$ — $x = 0.2$ or 4.4

4. *Equation of line* — *Solution*
(a) $y = 0$ — $x = -0.9$ or 2.2
(b) $y = 2$ — $x = -1.9$ or 3.2
(c) $y = \frac{-x}{3} + 1$ — $x = -1.7$ or 2.4

5. *Equation of line* — *Solution*
(a) $y = 0$ — $x = -4$ or 1.5
(b) $y = 3x$ — $x = -3$ or 2
(c) $y = \frac{x}{3} - 10$ — $x = -2.7$ or 0.4

Chapter Two

APPROXIMATION AND ERRORS

This is not an entirely new topic since the learner has met rounding off of numbers in book one. The learner will be introduced to the use of a calculator in computation.

Objectives

By the end of the topic, the learner should be able to:

(i) use calculators in various computations.
(ii) make reasonable approximations and estimations of quantity computations and measurements.
(iii) express values to a given number of significant figures.
(iv) define absolute, relative, percentage, round off and truncation errors.
(v) determine errors made from computation.
(vi) find maximum and minimum errors from operations.

Time: Sixteen lessons.

Teaching/Learning Activities

Computing using Calculator

- The teacher should introduce a calculator, as in the students' book.
- The teacher should guide the learner on how to use the calculator in computations, as in examples 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10 of the students' book.
- The learner should do exercise 2.1.

Approximations

- The learner should be guided on how to round off, truncate and estimate numbers, as in examples 10, 11, 12 and 13 in the students' book.
- The learner to do exercise 2.2.

Accuracy and Errors

- The teacher should discuss accuracy in measurements, as in the students' book.

- The teacher should guide the learner on how to find absolute errors as illustrated in the students' book.
- The teacher should discuss relative and percentage errors, as illustrated in the students' book.
- The learner should be guided through examples 14 and 15.
- The teacher should discuss round off and truncation errors, as in the students' book.

Propagation of errors

- The teacher should discuss propagation of errors, as in examples 16, 17, 18 and 19.
- The learner should do exercise 2.4.

Evaluation

- A speed test should be administered.

Answers

Exercise 2.1

1. (a) (i) 79 (ii) 997.07 (iii) –238
   (iv) 79 631 (v) –102 643 (vi) 8.373592
   (b) (i) –54 758 (ii) –0.0099 (iii) –2.69793
   (iv) 75 (v) –4 205 (vi) –643
   (c) (i) 110.5 (ii) –1 950 (iii) 1.2015
   (iv) 612 (v) 0.000066768 (vi) 472 926
   (d) (i) –0.6793 (4 s.f.) (ii) 0.2635 (4 s.f.)
   (iii) 4.824 (4 s.f.) (iv) 3.25 (v) 6.731 (4 s.f.)
   (vi) 92 051
   (e) (i) 7 921 (ii) 36.036009 (iii) 50.2681
   (iv) 121 801 (v) 2 116 (vi) 184 041
   (f) (i) 29.47880595 (ii) 298.9331029
   (iii) 26.94438717 (iv) 0.09591663 (v) 0.056302753
   (vi) 9.055385138
   (g) (i) 314 432 (ii) –39651821
   (iii) 0.000323818 (iv) 4 665 834 711
   (v) $6.1489559 \times 10^{-11}$ (vi) 2 197
   (h) (i) 3.732511157 (ii) 8.845085422
   (iii) –8.763380887 (iv) 0.201652967
   (v) 0.193987741 (vi) –0.147514601

(i) (i) 0.693465572 (ii) 0.125534728
(iii) 378.8501611 (iv) 0.569078797

Exercise 2.2

1. (a) (i) 80 000 (ii) 65 000 (iii) 67 000
(iv) 4 000 (v) 380 000 (vi) 13 000
(b) (i) 79 500 (ii) 65 300 (iii) 67 200
(iv) 4 000 (v) 379 900 (vi) 13 200
(c) (i) 79 550 (ii) 65 320 (iii) 67 240
(iv) 4 030 (v) 379 880 (vi) 13 190
2. (a) (i) 7.6 (ii) 8.0 (iii) 17.4
(iv) 0.1 (v) 4.2 (vi) 0
(b) (i) 7.6 (ii) 8.0 (iii) 17
(iv) 0
3. (a) (i) 67.2 (ii) 1.3 (iii) 0
(iv) 0 (v) 32.9 (vi) 9.5
(b) (i) 67.25 (ii) 1.26 (iii) 0.04
(iv) 0.04 (v) 32.93 (vi) 9.54
(c) (i) 67.46 (ii) 1.258 (iii) 0.0379
(iv) 0.037 (v) 32.926 (vi) 9.540
4. (a) 709.3 (b) 3.3 (c) 16 980
(d) 56 (c) 0.062 (f) 56.60
5. (a) (i) 420 (ii) 10 190 (iii) 11
(b) (i) 1 (ii) 320 (iii) 4 500
(c) (i) 3 (ii) 12 000 (iii) 48 000 000
(d) (i) 12.5 (ii) 2 (iii) $8\frac{1}{3}$
(e) (i) 143 (ii) 700 (iii) 0.9286
6. 190
7. 40 h

Exercise 2.3

1. (a) 25.5 – 26.5 cm (b) 25.95 – 26.05 cm
(c) 26.065 – 26.075 cm
2. (a) Lower; 1.0245 kg (b) L; 2.495 m
Upper; 1.0255 kg U; 2.505 m
Absolute error = 0.005 Absolute error = 0.005

(c) L; 157.5 km
U; 158.5 km
Absolute error = 0.05

(d) L ; 12.095 s
U; 12.105s
Absolute error = 0.05

(e) L; 12.05 s
U; 12.15 s
Absolute error = 0.5

(f) L; 11.5 s
U; 12.5 s
Absolute error = 0.5

3. (a) ± 0.5 (b) ± 0.05 (c) ± 0.005
(d) ± 0.05 (e) ± 0.5

4. (a) 0.05 (b) $\frac{0.05}{29.5} = \frac{1}{590}$ (or 0.0017)

(c) $\frac{10}{59}$ = % or 0.1695%

5. (a) 40.5° (b) 39.5° (c) $\frac{0.5}{40} \times 100 = 1.25\%$

6. 10.65 – 10.75 s

7. (a) Absolute error = 0.005; Absolute error $= \frac{0.005}{45.38} = 0.0001$

Percentage error $= \frac{1}{9076} \times 100$

$= 0.011\%$

(b) Absolute error = 0.005; Relative error $= \frac{0.005}{0.45} = 0.0111$

Percentage error $= \frac{10}{9}\%$

$= 1.11\%$

(c) Absolute error = 0.005 ; Relative error $= \frac{0.005}{7.62}$

$= 0.0007$

Percentage error $= \frac{25}{381}\%$

$= 0.066$

(d) Absolute error = 0.0005 ; Relative error $= \frac{0.0005}{12.004} = 0.00004$

Percentage error $= \frac{25}{6002}\%$

$= 0.0042$

(e) Absolute error = 0.5 ; Relative error $= \frac{0.5}{2505} = 0.0002$

Percentage error = $\frac{10}{501}\%$

$= 0.02\%$

8. (a) $2.45 - 2.55$

(b) $\frac{0.05}{2.5} = 0.02$

9. Percentage error = $\frac{0.01}{8.0} \times 100\%$

$= 0.125\%$

10. (a) $\frac{67}{100} - \frac{2}{3} \times 100\% = 0.5\%$

(b) $\left(\frac{2}{3} - \frac{66}{100}\right) \times 100\% = 1\%$

11. (a) (i) 0.33 ; 0.9% (ii) 4.1 ; 1.23%
(iii) 2.7 ; 0.66% (iv) 3.1 ; 1.27%
(v) 10 ; 3.47% (vi) 400 ; 1.27%

(b) (i) 0.32 ; 2.1% (ii) 4.0 ; 1.23%
(iii) 2.7 ; 0.66% (iv) 3.1 ; 1.27%
(v) 10 ; 3.47% (iv) 390 ; 1.27%

12. (a) L; –186.005 (b) L; 7 950
U; –185.995 U; 8 050

(c) L; –43 500 (d) L; 693.95
U; –42 500 U; 694.05

Exercise 2.4

1. (a) Min = 11.170 cm (b) Min =12.140 *l*
Max = 11.190 cm Max = 12.160 *l*

(c) Min = 482.435 m
Max = 483.545 m

2. (a) Min. per = 7 (b) Min per = 143.90 m
Max per = 71.80 cm Max per = 146.10 m

3. Min = 0.10 s
Max = 0.30 s

4. 0.655 – 0.865 m

5. 74.75 cm

6. (a) Min = 11.7 cm (b) Min = 0.0075 km
Max = 11.9 cm Max = 0.0185 km

(c) Min = 2.18 g
Max = 2.20 g

(d) Min = 2.19 l
Max = 2.21 l

7. 0.5

8. (a) Min = 28.00 cm^3
Max = 31.33 cm^3

(b) Min = 64.435 cm^2
Max = 68.635 cm^2

9. (a) $\frac{10}{61}$

(b) $\frac{1}{68}$

(c) $\frac{3}{182}$

(d) $\frac{125}{5292} \simeq 0.02$

(e) $\frac{862}{22491} \simeq 0.038$ (2 s.f.)

(f) 0.023 (2 s.f.)

(g) 0.015 (2 s.f.)

(h) 0.024 (2 s.f.)

(i) 0.023 (2 s.f.)

(j) 0.040 (2 s.f.)

(k) 0.024 (2 s.f.)

(l) –0.068

10. $161.4\pi\ cm^3 - 170.8\pi\ cm^3$

11. $33.75\ cm^2 - 46.75\ cm^2$

12. Min = 330 kg
Max = 390 kg

13. Least = 71.25 cm^2
Max = 236.25 cm^2

14. 4.66 h

15. 3.06 %

16. (a) 0.14 (2 s.f.) (b) 0.092 (2 s.f.)
(c) 0.69 (2 s.f.)

Chapter Three

TRIGONOMETRY (II)

The concept of trigonometry is not new to the learner. In Trigonometry (I), the learner dealt with trigonometric ratios of acute angles only, and this knowledge will be required in this topic. A brief revision of Trigonometry (I) is therefore necessary.

Objectives

By the end of the topic, the learner should be able to:

(i) define and draw the unit circle.
(ii) use the unit circle to find trigonometric ratios in terms of co-ordinates of points for $0° \leq \theta \leq 360°$.
(iii) find trigonometric ratios of negative angles.
(iv) find trigonometric ratios of angles greater than 360° using the unit circle.
(v) use the mathematical tables and calculators to find trigonometric ratios of angles in the range $0° \leq \theta \leq 360°$.
(vi) define radian measure.
(vii) draw graphs of trigonometric functions; $y = \sin x$, $y = \cos x$ and $y = \tan x$, using degrees and radians.
(viii) derive the sine rule.
(ix) derive the cosine rule.
(x) apply the sine and cosine rule to solve triangles (sides and angles).
(xi) apply the knowledge of sine and cosine rules in real life situations.

Time: Twenty two lessons.

Teaching/Learning Activities

The Unit Circle

- The teacher should define the unit circle, as in the students' book, with emphasis on the quadrants and the direction of the measurement of the angles.
- The teacher should then guide the learner to identify the quadrants associated with given angles.

Use of the Unit Circle in Finding the Trigonometric Ratios

- The learner should be led into finding the trigonometric ratios using the unit circle, preferably quadrant to quadrant for angles $\theta° \leq \theta° \leq 360°$, as in the students' book.
- The learner should be given exercise 3.1.
- The teacher should involve the learner in using the unit circle to find the trigonometric ratios of negative angles and angles greater than 360°, as in the students' book.
- The learner should to exercise 3.2.

Note:

The teacher should emphasise the need for the learner to express trigonometric ratios of angles greater than 90° and negative angles as trigonometric ratios of acute angles.

Mathematical Tables and Calculator

- The teacher should guide the learner on how to use the mathematical tables in finding the trigonometric ratios of various angles, as in examples 1, 2, 3 and 4 in the students' book.
- The learner to do exercise 3.3.
- The teacher should guide the learner on how to use the calculator in finding trigonometric ratios of various angles, as in example 5 and 6 in the students' book.
- The learner to do exercise 3.4.

Radian Measure

- The teacher should introduce the radian measure to the learner, as in the students' book.
- The learner should be guided on how to convert radians to degrees and vice versa, as in examples 7, 8 and 9 in the students' book.
- The teacher should discuss the application of the radian measure, as in example 10 in the students' book.
- The learner should do exercise 3.5.

Simple Trigonometric Graphs

- The teacher should involve the learner in the preparation of tables of trigonometric functions and drawing of the simple trigonometric graphs, as in examples 11, 12 and 13 in the students' book.
- The learner to do exercise 3.6.

Solution of Triangles

- The teacher should lead the learner to derive the sine rule, as in the students' book.
- The teacher should discuss the application of the sine rule in solution of triangles, as in examples 14 and 15 in the students' book.
- The learner to do exercise 3.7.
- The learner should be led into deriving the cosine rule, as in the students' book.
- Using examples 16, 17, 18 and 19, the teacher should guide the learner through the application of cosine rule in solving triangles.
- The learner to do exercise 3.8.

Evaluation

- Give a written test covering Approximation and Errors, and Trigonometry II.

Answers

Exercise 3.1

1. (a) 0.3, 0.3, –0.3, –0.3 (b) –0.7, 0.7, 0.7, –0.7
2. (a) 0.9, –0.9, –0.9, 0.9 (b) –0.3, 0.3, –0.3, 0.3
3. (a) 1, –1, 1, –1 (b) –0.7, 0.7, 0.7, –0.7
   (c) –1.6, –1.6, 1.6, 1.6
4.

θ	0°	90°	180°	270°	360°
cos	1	0	–1	0	1
sin	0	1	0	–1	0

5. 0, 1, –1, 0, 1, –1, 0 undefined , undefined
6. (a) 0.8 (b) 1.33
7. θ = 53° , 127°
8. (a) θ is in the 3rd quadrant. (b) θ is in the 2nd quadrant
   (c) θ is in the 1st quadrant.

Exercise 3.2

1. (a) (i) In the 4th quadrant (ii) In the 3rd quadrant
   (iii) In the 2nd quadrant (iv) In the 1st quadrant
   (b) (i) In the 1st quadrant (ii) In the 3rd quadrant
   (iii) In the 3rd quadrant (iv) In the 1st quadrant

2. (a) –sin 10° (b) sin 50°
   (c) sin 88° (d) sin 30°
3. (a) cos 75° (b) cos 20°
   (c) –cos 20° (d) –cos 10
4. –0.6, –0.6, 0.6, 0.6
5. 0.5, –0.5, –0.5, 0.5
6. –0.9, 0.9, –0.9, 0.9
7. 0.98, 0.17, 5.67
8. 0.98, 0.17, 5.67

Exercise 3.3

1. (a) sin 40° (b) –sin 40° (c) –sin 40°
   (d) –sin 40° (e) –sin 40°
2. (a) –cos 50° (b) –cos 50° (c) cos 50°
   (d) cos 50° (e) –cos 50°
3. (a) –tan 48° (b) tan 57° (c) tan 73°
   (d) –tan 40° (e) –tan 30°
4. (a) sin 27.3° = 0.4586 (to 4 d.p.)
   cos 27.3° = 0.8886 (to 4 d.p.)
   tan 27.3° = 0.5161 (to 4 d.p.)
   (b) sin 57°19' = 0.84174' (to 4d.p.)
   cos 57°19' =0.5400' (to 4 d.p.)
   tan 57° 19' = 1.558' (to 4 d.p.)
   (c) sin 97°27' = 0.9915 (to 4 d.p.)
   cos 97°27' = –0.1296 (to 4 d.p.)
   tan 97°27' = –7.6473 (to 4 d.p.)
   (d)sin 115.7° = 0.9011(to 4 d.p.)
   tan115.7°= –0.4331(to 4 d.p.)
   tan115.7°= –2.0778 (to 4 d.p.)
   (e) sin 131°36' = 0.7478 (to 4 d.p.)
   cos 131°36' = –0.6639 (to 4 d.p.)
   tan 131°36' = –1.1263 (to 4 d.p.)
   (f) sin164.7°= 0.2639 (to 4 d.p.)
   cos164.7°= – 0.9646 (to 4 d.p.)
   tan164.7°= – 0.2736 (to 4 d.p.)
   (g) sin 177.2°= 0.0488 (to 4 d.p.)
   cos 177.2° = –0.9988 (to 4 d.p.)
   tan 177.2° = –0.0489 (to 4 d.p)
   (h) sin193°46' = – 0.2380 (to 4 d.p.)
   cos193°46' = – 0.9712 (to 4 d.p)
   tan193°46° = 0.2451(to 4 d.p.)
   (i) sin 207.3° = –0.4586 (to 4 d.p)
   cos 207.3° = –0.8886 (to 4 d.p.)
   tan 207.3°= 0.5161 (to 4 d.p.)
   (j)sin 230° = –0.7660 (to 4 d.p)
   cos 230° = –0.6428 (to 4 d.p.)
   tan 230° = 1.1918 (to 4 d.p.)
   (k) sin 267.6° = –0.9991 (to 4 d.p.)
   cos 267.6° = –0.0419 (to 4 d.p.)
   (l) sin 280° 52' = –0.9821(to 4 d.p.)
   cos 280° 52° = 0.1885' (to 4 d.p.)

tan 267.6° = 23.8592 (to 4 d.p.)

(m) sin 300° = –0.8660 (to 4 d.p.)
cos 300° = 0.5000 (to 4 d.p.)
tan 300° = –1.7321 (to 4 d.p.)

(p) sin 347.23° = –0.2210 (to 4 d.p.)
cos 347.23° = 0.9752 (to 4 d.p.)
tan 347.23° = –0.2266 (to 4 d.p.)

tan 280°52' = –5.2076 (to 4 d.p.)

(n) sin329°30' = – 0.5075 (to 4 d.p.)
cos 329°30' = 0.8616 (to 4 d.p.)
tan 329°30° = –0.5890 (to 4 d.p.)

5. (a) (i) 6.45°, 173.55° (ii) 5.57°, 174.43°
(iii) 338.11°, 201.89° (iv) 23.78°, 156.22°
(v) 330.5°, 209.5° (vi) 34.35°, 145.65°
(vii) 319.62°, 220.38° (viii) 45.43°, 134.57°
(ix) 48.38°, 131.62° (x) 391.5°, 248.5°
(xi) 62.72°, 117.28° (xii) 295.1°, 244.9°
(xiii) 71.75°, 108.25° (xiv) 285.63°, 254.37°
(xv) 270°

(b) (i) 83.55°, 76.45° (ii) 84.43°, 275.57°
(iii) 111.89°, 248.11° (iv) 66.22°, 293.78°
(v) 119.50°, 240.5° (vi) 55.65°, 304.35°
(vii) 130.38°, 229.62° (viii) 44.57°, 315.43°
(ix) 41.62°, 318.38° (x) 158.50°, 201.50°
(xi) 27.28°, 332.72° (xii) 154.90°, 205.10°
(xiii) 18.25°, 341.75° (xiv) 164.37°, 195.63°
(xv) 180°, 180°

6. (a) 0.54°, –179.46° (f) 156.04°, –23.96°
(b) 11.24°, –168.76° (g) –30.05°, 149.95°
(c) –12.36°, 167.64° (h) 35.65°, –144.35°
(d) 14.26°, –165.74° (i) –43.82°, 136.18°
(e) 20.41°, –159.59° (j) –47.6°, 132.4°

7. 306.87°, 233.13°

8. (a) sin 20° (b) –sin 60° (c) sin 30°
(d) –sin 20° (e) –sin 85°

9. 19.75 cm^2 (to 4 s.f.)

10. 55.64 cm^2 (to 4 s.f.)

Exercise 3.4

1. (a) sin 32.5° = 0.5373 (b) sin 64.33° = 0.9013

cos 32.5° = 0.8434
tan 32.5° = 0.6370

(c) sine 100.42° = 0.9835
cos 100.42° = –0.1808
tan 100.42° = –5.4379

(e) sine 177.2° = 0.0488
cos 177.2° = –0.9988
tan 177.2° = –0.0489

(g) sin 313.6° = –0.7242
cos 313.6° = 0.6896
tan 313.6° = –1.0501

(i) sin 550° = –0.1736
cos 550° = –0.9848
tan 550° = 0.1763

(k) sin –175° = –0.0871
cos –175° = –0.9962
tan –175° = 0.0875

(m) sin –220° = 0.6428
cos –220° = –0.7660
tan –220° = –0.8391

(p) sin –80° = –0.9848
cos –80° = 0.1736
tan –80° = –5.6713

cos 64.33° = 0.4332
tan 64.33° = 2.0806

(d) sin 120.6° = 0.8607
cos 120.6° = –0.9001
tan 120.6° = –1.6909

(f) sin 205.83° = –0.4357
cos 205.83° = –0.9001
tan 205.83° = 0.4841

(h) sin 326.42° = –0.5531
cos 326.42° = 0.8331
tan 326.42° = –0.6639

(j) sin 625.67° = –0.9971
cos 625.67° = –0.0755
tan 625.67° = 13.2071

(l) sin –345.5° = 0.2504
cos –345.5° = 0.9681
tan –345.5° = 0.2586

(n) sin –10°15' = –0.1779
cos –10°15' = 0.9840
tan –10°15' = –0.1808

2. (a) (i) 55.996°, 124.004°
(iii) 350.003° , 189.997
(b) (i) 55.996° , 124.004°
(iii) –9.997, –170,003
(c) (i) 34.004° , 325.996°
(iii) 99.997° , 260.003

(ii) 79.997 , 100.003

(ii) 79.997° , 100.003°

(ii) 10.003° , 349.997°

3. (a) 39.912°, 219.912°
(c) 294.748°, 114.748°

(b) 86.530°, 266.530°
(d) 275.17° , 95.159°

Exercise 3.5

1. (a) $\frac{\pi}{6}$ (b) $\frac{3}{4}\pi$ (c) $\frac{5}{12}\pi$

(d) $\frac{5}{4}\pi$ (e) $\frac{7}{12}\pi$ (f) 4π

(g) 5π (h) $\frac{7}{4}\pi$ (i) $\frac{1}{8}\pi$

2. (a) 36° (b) 40° (c) 67.5° (d) 810°
   (e) 660° (f) 108° (g) 315° (h) 292.5°
   (i) 540°
3. (a) 171.9° (171°54') (b) 143.2° (143°12')
   (c) 97.39° (97°24') (d) 143.2° (143°12')
   (e) 14.32° (14°18') (f) 85.94° (85°56')
4. (a) 0.8660 (b) 0.2588 (c) 0.9397
   (d) 1 (e) 2.5721 (f) 0.9635
   (g) 0.9131 (h) –0.2910
5. 11.7 cm
6. (a) 1.25^c (b) 71.61°

Exercise 3.6

1. (a) 45.2°, 134.8° (b) 18°, 162°
   (c) 36.9°, 143.1° (d) 305.9°, 234.1°
2. (a) 1.265^c , 5.018^c (b) 0.141^c, 6.142^c
   (c) 0.4276^c, 5.856^c (d) 2.095^c, 4.189^c
3. (a) 0.61^c, 3.75^c (b) 2.94^c, 6.09^c
   (c) 2.27^c, 5.41^c
4. (a) 0° (b) 66.4°, –66.4° (c) 45.6° , –45.6°
   (d) 78.5° , –78.5° (e) 53.1° , –53.1°
5. (a) 0.7 (b) 0.6 (c) –0.3
   (d) –0.9 (e) –0.8
6. (a) (45° , 0.7) , (225° , –0.7)
   (b) Translation , translation vector $\begin{pmatrix} 45 \\ 0 \end{pmatrix}$
7. Check for accuracy of graph.
8. Check for accuracy of graph.

Exercise 3.7

1. (a) q = 10.08 cm, r = 12.29 cm , P = 64°
   (b) q = 140.5 cm, r = 156.8 cm , P = 38°
   (c) R = 77.5° Q = 78.52°
   (d) R = 31.4°, p = 10.3 cm, P = 85.4°
   (e) Q = 34.8°, r = 4.2 cm, R = 25.2°

2. $A = 16°$, $b = 10.8$ cm, $a = 6.1$ cm
3. $P = 116.7°$, $q = 4.18$ cm, $r = 3.42$ cm
4. $Q = 19.4°$, $q = 38.80$ cm, $r = 43.95$ cm
5. $p = 215.1$ cm, $q = 144.6$ cm, $R = 37.3°$
6. $q = 6.54$ cm, $p = 7.26$ cm, $P = 76°$
7. $X = 70°$, $Y = 70°$ and $y = 11.11$ cm
8. $Q = 62°$, $v = 4.94$ cm, $q = 4.56$ cm
9. $n = 6.27$ cm
10. P is 18.5 km from the ship.
    Q is 24.9 km from the ship.
11. 2.9 cm
12. 475 km
13. 5.4 cm
14. 90 km, 82 km
15. 4.9 cm

Exercise 3.8

1. (a) 104.5° (b) 104.6° (c) 132.8°
   (d) 98.2° (e) 116.8° (f) 80.5°
2. (a) 36.9° (b) 32.2° (c) 41.5° (d) 20°
3. 8.4 cm, 64.7 cm^2
4. 5.1 cm, 42.7°, 56.4°
5. 149°
6. $r = 31.6$ cm, $Q = 46.6°$, $P = 17.4°$
7. (a) 16.3 cm (b) 4.9 cm (c) 12.3 cm (d) 4.2 cm
8. $l = 10.3$ cm, $N = 27.6$, $M = 12.4°$
9. $\angle SPR = 75.8°$ $\angle RPQ = 36.9°$ $\angle SRP = 36.8°$
   $\angle PRQ = 16.2°$ $\angle PSR = 67.4°$ $\angle PQR = 126.9°$
10. 121 km
11. xy = 10.4 km, HZ = 18 km, ZX = 33.1 km
12. 92.3 m, 46.2 m

Chapter Four

SURDS

The learner has met integers in primary school and also in book one. Expressions written in root form are also familiar to the learners, as discussed under square roots and cube roots of numbers. A revision on square roots and cube roots of numbers is therefore necessary.

In this topic, the learner is introduced to rational and irrational numbers. 'Surd' is also a new term.

Objectives

By the end of the topic, the learner should be able to:

(i) define rational and irrational numbers.
(ii) simplify expressions with surds,
(iii) rationalise the denominator.

Time: Nine lessons.

Teaching/Learning Activities

Rational and Irrational Numbers

- The teacher should define rational and irrational numbers, giving examples in each case, as in the students' book.
- The teacher should then probe the learner's understanding of rational and irrational numbers using the provoking questions given in the students' book.
- The teacher should emphasise that even though surds are irrational numbers, the number under the root sign is rational.

Simplification of Surds

- The teacher should discuss various orders of surds.
- The teacher should involve the learner in simplifying surds to their lowest terms, as in example 1.
- The teachers should take the learner through examples 2, 3 and 4 on operation of surds.
- The learner should then be assisted to make the following generalisations, as in the students' book:

(i) $a\sqrt{n} \pm b\sqrt{n} = (a \pm b)\sqrt{n}$

(ii) $\sqrt{a} \times \sqrt{a} = \sqrt{axa}$

$= \sqrt{a^2}$

$= a$

(iii) $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$

(iv) $\sqrt{a} \div \sqrt{b} = \sqrt{\left(\frac{a}{b}\right)}$

- For a start, it is advisable for the teacher to use algebraic symbols to represent surds.
- The learner to do exercise 4.1.

Rationalising the Denominator

- The teacher should define rationalisation.
- The learner should be guided to rationalise the denominators, as illustrated in the students' book.
- The teacher should introduce conjugate surd as discussed in the students' book.
- The learner should be guided through example 5.

Evaluation

- The teacher is advised to give a short test covering all the concepts discussed in the topic.

Further Questions

1. Given that $\sqrt{5} = 2.240$, find $\sqrt{5^2} + 2\sqrt{5}$
2. Simplify $x^2 - x\sqrt{3}$ if $x = 2 - \sqrt{3}$.
3. Simplify $(\sqrt{14} - 2\sqrt{3})(\sqrt{14} + 2\sqrt{3})$. Hence determine the value of;

$$\frac{1}{\sqrt{14} - 2\sqrt{3}} + \frac{1}{\sqrt{14} + 2\sqrt{3}}$$ given that $\sqrt{14} = 3.742$.

Answers

Exercise 4.1

1. (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $4\sqrt{2}$ (d) $2\sqrt{13}$
   (e) $10\sqrt{10}$ (f) $5\sqrt{10}$ (g) $8\sqrt{2}$ (h) $3\sqrt{6}$

2.

$4\sqrt{3}$	$\sqrt{4^2\left(\sqrt{3}\right)^2}$	$\sqrt{(16 \times 3)}$	$\sqrt{48}$
$2\sqrt{5}$	$\sqrt{2^2\left(\sqrt{5}\right)^2}$	$\sqrt{4 \times 5}$	$\sqrt{20}$
$6\sqrt{3}$	$\sqrt{6^2\left(\sqrt{3}\right)^2}$	$\sqrt{(36 \times 3)}$	$\sqrt{108}$
$7\sqrt{2}$	$\sqrt{7^2\left(\sqrt{2}\right)^2}$	$\sqrt{(49 \times 2)}$	$\sqrt{98}$
$13\sqrt{10}$	$\sqrt{13^2\left(\sqrt{10}\right)^2}$	$\sqrt{169 \times 10}$	$\sqrt{1\ 690}$
$20\sqrt{10}$	$\sqrt{20^2\left(\sqrt{10}\right)^2}$	$\sqrt{400 \times 10}$	$\sqrt{4\ 000}$

3. (a) $7\sqrt{3}$ (b) $13\sqrt{7}$ (c) $\frac{5}{2}\sqrt{7}$ (d) $6\sqrt{11}$
   (e) $-6\sqrt{23}$ (f) $5\sqrt{x}$ (g) $6\sqrt{x} + 5\sqrt{3}$
   (h) $42\sqrt{2} - 6\sqrt{3}$ (i) $-67x$ (j) $\sqrt{19}$
   (k) $12\sqrt{3} - 3\sqrt{10}$ (l) $-4\sqrt{6}$ (m) $32\sqrt{10}$ (n) $24\sqrt{2}$
   (p) $\frac{\sqrt{6}}{6}$ (q) $\frac{11}{3}\sqrt{5}$ (r) $\frac{237}{4}\sqrt{6}$ (s) $7\frac{\sqrt{3}}{2}$
   (t) $4\sqrt{7}$

4. (a) $\sqrt{10}$ (b) $10\sqrt{21}$ (c) $3\sqrt{11}$ (d) $15\sqrt{26}$

(e) $6\sqrt{3}$ (f) $16\sqrt{xy}$ (g) $16\frac{\sqrt{345}}{5}$ (h) 5

(i) 12 (j) $20\sqrt{6}$ (k) 12 (m) $9\sqrt{7}$

(n) $15\sqrt{15}$ (p) $13\sqrt{14}$ (q) $14\sqrt{5}$ (r) $25\sqrt{7}$

(s) $84\sqrt{6}$

5. (a) $4\sqrt{14}$ (b) $3\sqrt{2}$ (c) $25\sqrt{30}$ (d) 20

(e) $2\sqrt{31}$ (f) $4\sqrt{6}$ (g) $8\sqrt{23}$ (h) $6\sqrt{6}$

(i) $8\sqrt{13}$ (j) $48x^3$ (k) 6 (l) $\sqrt{53}$

(m) 75 (n) $39\sqrt{2}$ (p) $10\sqrt{39}$ (q) $84\sqrt{2}$

(r) $18\sqrt{3}$ (s) $60\sqrt{15}$ (t) $5\sqrt{10}$ (u) $\sqrt{65}$

6. (a) $1\frac{1}{2}$ (b) $2\frac{1}{2}$ (c) $1\frac{5}{9}$ (d) $1\frac{3}{4}$

(e) $\frac{9\sqrt{3}}{\sqrt{26}}$ (f) 4 (g) $\frac{4\sqrt{2}}{7}$ (h) $1\frac{1}{2}$

(i) $2\frac{\sqrt{3}}{3}$ (j) $\frac{3\sqrt{6}}{10}$ (k) $1\frac{1}{2}$ (l) $\frac{-20\sqrt{2}}{9\sqrt{3}}$

(m) $\frac{17\sqrt{2}}{12}$ (n) 3 (p) $\frac{6x^3y^6}{\sqrt{3xy}}$

(q) $\frac{9}{\sqrt{41}}$ (r) $\frac{7}{12\sqrt{2}}$ (s) $\frac{16}{6}\sqrt{\frac{2}{61}}$

7. (a) $7+4\sqrt{3}$ (b) $5+2\sqrt{6}$ (c) $\sqrt{15}+\sqrt{10}-\sqrt{6}-3$

(d) $7-2\sqrt{10}$ (e) $4\sqrt{3}+\sqrt{42}-2\sqrt{14}-6$

(f) $3-3\sqrt{2}+\sqrt{15}-\sqrt{30}$ (g) $\sqrt{30}+\sqrt{6}-\sqrt{10}-5\sqrt{2}$

(h) $10+2\sqrt{10}-\sqrt{30}-5\sqrt{3}$ (i) $6\sqrt{5}-5\sqrt{2}-\sqrt{10}+6$

(j) $9+4\sqrt{5}$ (k) -9 (l) 60

(m) $6\sqrt{15}-6\sqrt{35}$ (n) $\sqrt{447}-\sqrt{894}+4\sqrt{30}-8\sqrt{15}$

Exercise 4.2

1. (a) $\frac{\sqrt{2}}{2}$ (b) $\sqrt{3}$ (c) $5\frac{\sqrt{11}}{11}$

   (d) $\frac{\sqrt{10}}{20}$ (e) $\frac{\sqrt{2}}{3}$ (f) $\frac{\sqrt{3}}{5}$

2. (a) $3+\sqrt{3}$ (b) $8+2\sqrt{3}$

   (c) $\frac{5(3+\sqrt{2})}{7}$ (d) $\frac{7\sqrt{3}+\sqrt{15}}{22}$

3. (a) $\frac{5\sqrt{35}+5\sqrt{14}}{3}$ (b) $\frac{11+\sqrt{55}}{3}$

   (c) $\frac{\sqrt{14}+\sqrt{10}+\sqrt{21}+\sqrt{15}}{2}$ (d) $6-\sqrt{35}$ (e) $3\sqrt{5}-6$

4. (a) $\frac{3\sqrt{14}+2\sqrt{3}+6\sqrt{6}-\sqrt{7}}{17}$

   (b) $\frac{6\sqrt{30}+3\sqrt{14}-10\sqrt{165}-5\sqrt{77}}{106}$ (c) $8+4\sqrt{5}-2\sqrt{3}-\sqrt{15}$

   (d) $\frac{3\sqrt{2}-2\sqrt{3}}{6}$ (e) $\frac{3\sqrt{3}+2\sqrt{2}}{19}$

5. (a) 0.3536 (b) 0.1836 (c) 1.2623 (d) 0.7394
   (e) 11.41

6. (a) $2a(1-3b)$ (b) $a(3+2b)$ (c) 2 (d) $8a-b$

7. (a) $\frac{-4\sqrt{5}}{3}$ (b) $\frac{4\sqrt{6}-4\sqrt{3}-15\sqrt{7}+15\sqrt{5}}{6}$

8. (a) $4+2\sqrt{2}$ (b) $\frac{-\sqrt{10}-\sqrt{30}}{2}$ (c) $7+4\sqrt{3}$

9. $\frac{25\sqrt{3}}{4}$

10. $\sqrt{6}$

11. (a) $x = 4$ (b) $x = 9$

Further Questions

1. 9.480
2. $10-6\sqrt{3}$
3. 3.742

Chapter Five

FURTHER LOGARITHMS

The learner has met logarithms in book two. A review of common logarithms and use of mathematical tables is therefore necessary. In this topic, the learner will be introduced to laws of logarithms and their applications.

Objectives

By the end of the topic the learner should be able to:

(i) derive logarithmic relations from index form and vice versa.
(ii) state the laws of logarithms.
(iii) use logarithm laws to simplify logarithmic equations and expressions.
(iv) apply laws of logarithms in further computation.

Time: Eleven lessons.

Teaching/Learning Activities

Logarithmic Notation

- The teacher should involve the learner in expressing numbers in index form and relate index form to logarithmic notation, as illustrated in the students' book.

Laws of Logarithms

- The teacher should discuss the laws of indices.
- The learner should be led to establish the laws of logarithms, as illustrated in the students' book.
- The teacher should guide the learners through examples 1, 2 and 3.
- The learner to do exercise 5.1.

Logarithmic Equations and Expressions

- The teacher should discuss the application of index notation in solving logarithmic equations, as in the students' book.
- The teacher should lead the learner through examples 4, 5 and 6.

Application of Laws of logarithms for Further Computation

- The teacher should guide the learner in finding logarithms of numbers to bases other than 10, as illustrated in the students' book.
- The learner should be led through examples 7 and 8.
- The learner to do exercise 5.2.

Evaluation

The teacher should reinforce the learner's understanding by giving a written test covering all the areas discussed in the topic.

Further Questions

1. Show that $\log a + \log ax + \log ax^2 = 3 (\log a + \log x)$
2. If $\log_a 5^2 + \log_5 a = 3$, find the value of a.
3. Solve the equation $\log_3 x - 4 \log_x 3 + 3 = 0$
4. Solve for x and y in the simultaneous equations;
   $xy = 80$
   $\log x - 2\log y = 1$

Answers

Exercise 5.1

1. (a) $\text{Log}_7 49 = 2$ (b) $\text{Log}_4 32 = \frac{5}{2}$ (c) $\text{Log}_{121} 1331 = \frac{3}{2}$

   (d) $\text{Log}_{10} 0.01 = -2$ (e) $\text{Log}_5 1 = 0$ (f) $\text{Log}_8 \frac{1}{2} = \frac{-1}{3}$

   (g) $\text{Log}_{\frac{1}{5}} 125 = -3$ (h) $\text{Log}_9 \frac{1}{27} = \frac{-3}{2}$ (i) $\text{Log}_a 1 = 0$

2. (a) $5^4 = 625$ (b) $10^3 = 1\,000$ (c) $3^3 = 27$

   (d) $10^0 = 1$ (e) $\left(\frac{1}{2}\right)^{-2} = 4$ (f) $25^2 = 5$

   (g) $x^2 = y$

3. (a) $-a$ (b) $2a + b$ (c) $a - b$
   (d) $2a - b$ (e) $3c - 1$ (f) $-a - 1$

   (g) $2 - \frac{1}{2}c$

4. (a) Log 12 (b) Log 3 (c) 1 (d) Log 3
   (e) –Log 3 (f) Log 4 (g) 1 (h) 3
5. (a) –Log (x – 1) (b) 0 (c) 5 Log x (d) 2
   (e) 1 (f) 2
6. (a) 1.0791 (b) 0.6990 (c) 2.0332

Exercise 5.2

1. (a) 3 (b) 4 (c) Log ab
2. (a) 1.2619 (b) 1.4560 (c) 1.6397 (d) 1.8597
   (e) 1.4945 (f) 3.3219 (g) 0.8271
3. (a) 0.7925 (b) 1.2323 (c) 0.8451 (d) 1.5114
   (e) 1.1610 (f) 2.7712 (g) 1.2153 (h) 1.7094
4. $y = 3\sqrt{\left(\frac{6}{x}\right)}$
5. $x = 2$
6. (a) $x = 2, y = 4$ (b) $x = \frac{1}{2}, y = 1$ (c) $x = 1, y = 0$
7. (a) $x = 16$ (d) $y = 405^2$ (g) $y = \frac{1}{21}$ (b) $x = 2$
   (e) $p = \pm 5$ (c) $x = 2.4$ (f) $x = \frac{5}{54}$ (h) $x = 1$

Further Questions

1. 3 log (ax)

   But log(ax) = log a + log x

   = 3(log a + log x)
2. $a = 5$
3. $x = 3$ or $\frac{1}{81}$
4. $x = \frac{1}{2}$, $y = 1$

Chapter Six

COMMERCIAL ARITHMETIC (II)

The learner has been introduced to commercial arithmetic in book one. In this topic, the learner will be exposed to higher concepts involving the same.

Objectives

By the end of the topic, the learner should be able to:

(i) define principal, rate and time in relation to interest.
(ii) calculate simple interest using the simple interest formula.
(iii) calculate compound interest using step by step method.
(iv) derive the compound interest formula.
(v) apply the compound interest formula in calculating interest.
(vi) define appreciation and depreciation
(vii) use compound interest formula to calculate appreciation and depreciation.
(viii) calculate hire purchase.
(ix) calculate income tax, given the income tax bands.

Time: Twenty two lessons.

Teaching/Learning Activities

Simple Interest

- The teacher should guide the learner in defining principal, rate and time, as in the students' book.
- The teacher should lead the learner through examples 1, 2, 3 and 4
- The learner to do exercise 6.1.

Compound Interest

- The teacher should discuss compound interest, as in the students' book.
- The learner should be guided on how to calculate compound interest using the step by step method illustrated in examples 5 and 6.

- The teacher should lead the learner in deriving the compound interest formula.
- The learner should be involved in applying the compound interest formula in solving problems, as in examples 7, 8, 9, 10 and 11.
- The learner to do exercise 6.2.

Appreciation and Depreciation

- The teacher should define appreciation and depreciation, as in the students' book.
- The learner should be led through examples 12, 13 and 14.
- The learner to do exercise 6.3.

Hire Purchase

- The teacher to guide the learner through the meaning of hire purchase, as in the students' book.
- The learner should be involved in calculating hire purchase problems, as in examples 15 and 16.
- The learner to do exercise 6.4.

Income Tax

- The teacher should discuss income tax as in the students' book.
- The teacher to involve the learner in solving problems on income tax, as in examples 17 and 18.
- The teacher should discuss PAYE, as in the students' book.
- The teacher should lead the learner through examples 19 and 20.
- The teacher to do exercise 6.5.

Additional Hints

- The teacher is advised to refer to the latest Kenya Revenue Authority employers' guide to PAYE.
- The teacher is encouraged to organise an educational trip to the nearest hire purchase dealer, or invite a salesman to give the class more insight on the topic.

Evaluation

- The teacher to prepare and give a comprehensive written test on Surds, Further Logarithms and Commercial Arithmetic.

Further questions

1. Mrs. Murungi deposits sh. 50 000 in a fixed deposit account at the beginning of every year for 8 years at 10% p.a. compound interest. Find the amount to her credit at the beginning of the eighth year.
2. Mr. Aden owes a financial institution sh. 300 000 towards which he pays sh. 80 000 every year. If the interest charged is at 12% on the outstanding balance, find:
   (a) the time it will take him to clear the loan.
   (b) the total interest paid.
3. A house is valued at sh. 2 500 000. Mr. Tarayia paid 10% of the value. The balance was financed by a financial institution, which charged interest at 12% p.a. for the first 6 years and 14% p.a. for the remaining 6 years. Find:
   (a) the amount of monthly instalments for the first six years.
   (b) the amount of monthly instalments for the other six years.
   (c) the total amount he paid for the house.

Answers

Exercise 6.1

1.	(a) sh. 5278.50	(b) sh. 8 112	(c) sh. 14516.25
	(d) sh. 2 430	(e) sh. 1 125	(f) sh. 1491.75
2.	(a) 6.186%	(b) 16.67%	(c) 10%
	(d) 6.25%	(e) 26.19%	(f) 15%
3.	(a) 2 years	(b) 3 years	(c) 1 year 7 months
	(d) 1 year 7 months	(e) 2 years 6 months	(f) 11years 3 months
4.	(a) sh. 30 000	(b) sh. 14 000	(c) sh. 48 000
	(d) sh. 40 000	(e) sh. 53 330	(f) sh. 2 268
5.	(a) sh. 16 320	(b) sh. 59 375	(c) sh. 255 000
	(d) sh. 618 750	(e) sh. 68 400	(f) sh. 8 959
6.	sh. 1 800	7. sh. 36 920	8. sh. 129 600
9.	sh. 45 390	10. sh. 4778.75	11. sh. 20 500
12.	sh. 3 040		

Exercise 6.2

1.	(a) sh. 2215.20	(b) sh. 8727.80	(c) sh. 2684.80
	(d) sh. 101 800	(e) sh. 7 634	(f) sh. 749 000

2. (a) sh. 31278.50 (b) sh. 9 382 (b) sh. 22 582
(d) sh. 19 000 (e) sh. 13473.50 (f) sh. 976 425

3. (a) 2 years 5 months (b) 2 years
(c) 3 years 3 months (d) 2 years 2 months
(e) 5 years

4. 10% 5. sh. 7 978
6. sh. 248 370 7. sh. 42 342
8. sh. 262.80 9. K£ 917.40
10. (a) (i) 12.68% (ii) 12.55% (iii) 12.36%
(b) 10.125%
(c) 12.304%

Exercise 6.3

1. K£ 60 750 , K£ 54 675
2. sh. 236 028
3. sh. 64 590
4. 608 326 people
5. sh. 132 684
6. 10%
7. 5 weeks
8. 26 655 people
9. 658 162 people
10. 42 448 km^2
11. 253 900 people
12. sh. 121 440
13. 5%
14. sh. 760437.50

Exercise 6.4

1. sh. 1 680 2. sh. 68
3. sh. 5 250 , $37\frac{1}{3}$% p.a. 4. sh. 1 400 , 41.1% p.a.
5. (a) sh. 35 250 (b) sh. 33 750
Plan (b) is cheaper by sh. 1 500
6. (a) First method sh. 20 750, second method ksh. 20 000
Second plan is cheaper by sh. 750
(b) First method 15.45%, second method 21.11%
Second method is the dearer.

Exercise 6.5

1. (a) sh. 5 520 (b) sh. 18 240
(c) sh. 45 788 (d) sh. 171 264
(e) sh. 65 064
2. (a) sh. 2 080 (b) sh. 2 468
(c) sh. 3 244 (d) No PAYE

3. sh. 4 800 4. No PAYE
5. K£ 6560 p.a. 6. sh. 10 267
7. (a) sh. 1 910 (b) sh. 1 775
8. Mr. Kisaka sh. 26 904, Mrs Kisaka sh. 30 684
   ∴ both sh. 57 588
9. sh. 2 689 10. sh. 55 488
11. (a) sh. 8 484 (b) sh. 21 015

Further Questions

1. sh. 531 831.40
2. (a) 6 years (b) sh. 122 930
3. (a) sh. 30 841 (b) sh. 34 296 (c) sh. 4 939 894

Mixed Exercise 1

1. (a) $3x^2 - 14x - 5 = 0$
   (b) $x^2 + 5x + 6 = 0$
   (c) $x^2 + 4x - 77 = 0$
2. (a) sh. 150 (b) sh. 140 (c) sh. 8 800 (d) sh. 9375
3. 75.9
4. (a) 21.74 (b) 0.007092 (c) 430 900
   (d) 1.205 (e) 0.4729 (f) 0.7143
5. (a) sh. 46 500 (b) sh. 50551.7
6. $\angle B = \angle C = 33.7°$, $\angle A = 112.6°$
8. (a) 0.666 (b) 2.147 (c) 523.974
   (d) 17.348 (e) 0
9. $p = 4, q = 5$
10. sh. 300
11. 30° and 150°
12. (a) 30 cm^2 (b) 29.98^2 (c) 30 cm^2
13. 2 and –3 or 3 and –2
14. (a) x = 3.05 cm (b) 8.75 cm
15. $\frac{3\sqrt{6}}{4}$
16. y = 368
17. (a) $(2x + 3)(4x - 3)$ (b) $\frac{1}{8}(2x - 1)(12x - 1)$

(c) $\frac{1}{6}(x-3)(x+2)$ (d) $4(2+x)(2-x)$

18. (a) $q = 22.0°$ or $158.0°$
(b) $q = 41.4°$ or $318.6°$
(c) $q = 146.3°$ or $326.4°$
(d) $q = 38.7$ or $141.3°$

19. (a) $x^2 \pm 4x + 4$ (b) $x^2 - 12x + 36$

(c) $9x^2 \pm 3x + \frac{1}{4}$ (d) $x^2 - \frac{2}{3}x + \frac{1}{9}$

20. (a) sh. 1 890 (b) sh. 1032.50

21. (a) $\cos x = \frac{12}{13}$ (b) $\sin x = \frac{5}{13}$

(c) $\cos(90 - x) = \frac{5}{13}$ (d) $\frac{5}{13}$

22. Lower limit sh. 53 500
Upper limit sh. 54 500

23. (a) $x = 10.78$ or $x = -2.78$
(b) $x = -1.618$ or $x = 0.618$
(c) $x = -1.309$ or $x = -0.191$

(d) $x = 0$ or $x = -\frac{1}{4}$ (double root)

(e) $x = 2$ or $x = 3$

(f) $x = 5$ or $-\frac{1}{2}$

(g) $x = 2$ or $\frac{1}{2}$

24. (a) $41.3\ cm^3 \leq V \leq 45.1\ cm^3$
(b) $74.40\ cm^2 \leq S.A. \leq 78.86\ cm^2$

25. (a) $x = 1.269$ or $x = 17.73$
    (b) $x = 0.5619$ or -5.562
26. (a) 0.0238 (b) 0.0238 (c) 0.0268 (d) 0.0332
    (e) 0.0540 (f) 0.0201 (h) 0.0703
27. $20\sqrt{2}$
28. (a) $\frac{1}{3}\log a + 6.286$ (b) 7.327
29. sh. 1 692
30. 0.00015
31. $\sin\theta = -\frac{8}{17}$, $\tan\theta = -\frac{8}{15}$
32. 20 cm
33. $x = 9.333$ cm $\angle Y = 66.98°$ $\angle Z = 41.02°$
34. (a) Smallest 11.5 cm, greatest 11.7 cm.
    (b) Smallest 0.0265 km, greatest 0.0375
    (c) Smallest 1.980 kg, greatest 2.000 kg
35. 1.629
36. 5%
37. Case 1: $\angle Q = 32°$, $\angle P = 82.7°$, $\angle R = 65.3°$
    $p = 13.1$ cm, $q = 7$ cm, $r = 12$ cm
    Case 2: $\angle Q = 32°$, $\angle P = 33.3°$, $\angle R = 114.7°$
    $p = 7.3$ cm, $q = 7$ cm, $r = 12$ cm
38. 7.7 cm
39. (a) 1 (b) $1\frac{1}{2}$
40. Check for accuracy of graph.
    (a) 48.6°, 131.4° (b) 19.5°, 160.5°
    (c) 194.5°, 345.5°
41. 1 year
42. (a) $33\sqrt{15}$ (b) $\frac{6-4\sqrt{2}}{7}$ (c) $\frac{11-6\sqrt{2}}{7}$

(d) $\frac{8+2\sqrt{2}}{11}$ (e) $\frac{10+\sqrt{2}}{14}$

43. (a) $x = \frac{2}{3}$ (b) 2.583

44. 119.2 cm $\leq$ P $\leq$ 119.6 cm

45. sh. 125 483 (to the nearest shilling)

46. (a) sh. 34 726 655 (ii) sh. 144 694

47. (a) Check for correct graph.

$x = -1$ or $-\frac{1}{2}$

(b) Check for correct graph.
$x = -2$ or -1

(c) Check for correct graph.
$x = -1$ or 2.33

48. (a) Check for correct graph.
$x = 0, y = 1$

(b) Check for correct graph.
$x = 1.48, y = 3.48$ or $x = 1.81, y = 0.19$

(c) Check for correct graph.
$x = -2.7, y = 0$
$x = 0.7, y = 0$

(d) Check for correct graph.
$x = 0, y = 0$ or $x = -2.73, y = -8.19$
$x = 0.73, y = 2.19$

49. (a) 3.146 (b) 0.3535

50. (a) Minimum 14.48, maximum 14.50
(b) Minimum 650.30, maximum 651.50
(c) Minimum 81.00, maximum 81.20

51. 41.14 km N 2.2°W

52. (a) 86.25 cm^2 $\leq$ A $\leq$ 106.25 cm^2 (b) 0.104

53. (a) $\frac{4-\sqrt{2}}{7}$ (b) 6

54. (a) $b = 10.1$ cm, $\angle C = 70.2°$, $\angle A = 79.8°$
(b) $\angle C = 29°$, $\angle A = 104.4°$, $\angle B = 46.6°$
(c) $\angle R = 6°$, $q = 41.82$ cm, $r = 4.65$ cm

Chapter Seven

CIRCLES: CHORDS AND TANGENTS

The learner has dealt with circles before. In this topic, chords and tangents of circles are looked into in detail.

Objectives

By the end of the topic the learner should be able to:

(i) calculate length of an arc and a chord.
(ii) calculate lengths of tangents and intersecting chords.
(iii) state and use properties of chords.
(iv) construct tangents to a circle.
(v) construct direct and transverse common tangents to two circles.
(vi) relate angles in alternate segment.
(vii) construct circumscribed, inscribed and escribed circles.
(viii) locate centroid and orthocentre of a triangle.
(ix) apply knowledge of circles, tangents and chords to real life situations.

Time: Twenty one lessons.

Teaching/Learning Activities

Length of an Arc

- The teacher should state the formula for finding the length of an arc.
- The teacher should involve the learner in calculations involving arc length, as in examples 1, 2 and 3.
- The learner should do exercise 7.1.

Chords

- The teacher should introduce chords as in the students' book.
- The teacher should discuss perpendicular bisector of a chord, as in the students' book.
- The learner should be led through examples 4 and 5.

- The teacher should discuss parallel chords, as illustrated in the students' book.
- The teacher should guide the learner through examples 6 and 7.
- The teacher should discuss equal chords using figures 7.11 and 7.12 in the students' book.
- The teacher should lead the learner through the relationship between chords which intersect internally, as in the students' book.
- The learner should be led through example 8.
- The teacher should guide the learner through the relationship between chords which intersect externally, as in the students' book.
- The learner should be taken through example 9.
- The learner should do exercise 7.2.

Tangent to a Circle

- The teacher should discuss tangents as in the students' book.
- The learner should be led through construction of a tangent to a circle, as in figure 7.29.
- The learner should be guided through calculations involving tangents, as in examples 10 and 11.
- The teacher should guide the learner to construct tangent to a circle from an external point, as in figure 7.31.
- The learner should be guided through problems involving tangent from an external point, as in examples 12, 13 and 14.
- The learner to do exercise 7.3

Tangents Two Circles

- The learner should be guided on how to construct direct and transverse common tangents, as in the students' book.
- The learner to do exercise 7.4.

Contact of Circles

- The teacher should discuss contact of circles as in the students' book.
- The learner should be led through examples 15, 16 and 17.
- The learner should do exercise 7.5.

Angles in Alternate Segment

- The teacher should discuss the relationship between angles in alternate segments, as in the students' book.

- The learner should be led through examples 18 and 19.
- The learner to do exercise 7.6.

Circles and Triangles

- The teacher should guide the learner to construct inscribed, circumscribed and escribed circles, as in the students' book.

Centroid and Orthocentre

- The learner should be led to locate orthocentre and centroid, as illustrated in figures 7.74 (a) and (b).
- The learner should do exercise 7.7.

Evaluation

- The teacher should give a short test on chords and tangents.

Further Questions

PQ and RS are parallel chords of a circle. If M and N are points of intersection of the perpendicular bisector and chords PQ and RS respectively, show that $PQ^2 - RS^2 = 4\ MN(ON - OM)$.

Answers

Exercise 7.1

1.

Angle subtended by arc at centre, θ	*Circumference in cm*	*Arc length in cm* $\frac{\theta}{360} \times C$
30	48	**4**
250	25	17.36
45	**56**	7
90	**40**	10
120	60	20
180	30	15

2. (a) 43.42 (b) 72
   (c) 35.2 (d) 44.57
   (e) 62.56
3. 72
4. 31.5

5. 41.89
6. 1833.3
7. 10.5
8. 1.723
9. 4.775
10. 83.76
11. 0.7

Exercise 7.2

1. 8
2. 30
3. 2x
4. 13.31
5. 17
6. 491.8
8. 5.966
10. XL = 3.5
    XM = 5.83
11. (a) (i) 52 (ii) 128° (iii) 106°s
    (b) 3 cm
12. 6 or 5
13. 12.5
14. (a) 2.25 (b) 3.125
15. 20 cm

Exercise 7.3

1. (a) 70, 40 (b) 62, 28
   (c) 5 (d) 12
2. 8
3. 138
4. ∠NKL = 91°
   ∠KLM = 74.5°
   ∠LMN = 89°
   ∠MNK = 105.5°
5. 146.5°
6. 10 cm
7. 100

8. 21
10. (a) 60 (b) 58 (c) 62
11. (a) $x = 60°$, $y = 30°$ (b) 2

Exercise 7.5

1. 200
2. 9 cm
3. 8
4. 50
5. (a) 7.86 cm (b) 19.86
6. 4 cm
7. (a) PT = 8 cm (b) PS = 8 cm (c) $O_1O_2 = 21$ cm
8. 24
9. 9.7 cm, 5 cm, 3 cm

Exercise 7.6

1. (a) $a = 30°$, $b = 60°$ (b) $a = 60°$, $b = 15°$
   (c) $a = 43°$, $b = 47°$ (d) $a = 56°$, $b = 34°$
   (e) $a = 84$, $b = 48$ (f) $a = 50°$, $b = 20°$
2. $x = 25°$, $y = 115°$
3. 4.933
4. $x = 52°$, $y = 40°$
5. $\angle ABD = 62°$, $\angle ADB = 30°$, $\angle BAD = 88°$
   $\angle ACD = 62°$, $\angle ADC = 52°$, $\angle CAD = 66°$
   $\angle BCD = 92°$, $\angle BDC = 22°$, $\angle CBD = 66°$
6. $\angle CBX = 53°$ $\angle DBC = 91°$
7. $\angle TXZ = 25°$

Exercise 7.7

1–10: Check for accuracy of constructions

Chapter Eight

MATRICES

This is a new topic to the learner. However, the learner has encountered situations where information is represented in tabular form.

Objectives

By the end of the topic, the learner should be able to:

(i) define a matrix.
(ii) state the order of a matrix.
(iii) define a square matrix.
(iv) determine compatibility in addition and multiplication of matrices.
(v) add matrices.
(vi) multiply matrices.
(vii) identify matrix.
(viii) find determinant of a 2 x 2 matrix.
(ix) find the inverse of a 2 x 2 matrix.
(x) use matrices to solve simultaneous equations.

Time: Twenty one lessons.

Teaching/Learning Activities

Definition and Order of a Matrix

- The teacher should define a matrix using the illustrations in the students' book.
- The learner should be led to state a matrix order, as discussed in the students' book.
- The teacher should guide the learner to define a square matrix, as in the students' book.

Matrix Addition and Subtraction

- The learner should be taken through examples 1 and 2.
- The teacher should give further examples to emphasise compatibility of matrices in addition and subtraction.
- The learner to do exercise 8.1.

Multiplication of Matrices

- The teacher should discuss multiplication of a matrix by a scalar, as discussed in the students' book.
- The learner should be guided through example 3.
- The teacher should discuss multiplication of a matrix by another matrix, as illustrated in the students' book.
- The learner should be guided through examples 4, 5 and 6.
- The teacher should give further examples to show compatibility in multiplication of a matrix by a matrix.
- The learner to do exercise 8.2.

Identity Matrix

- The teacher should discuss the identity matrix, as illustrated in the students' book.
- The teacher should guide the learner to write down identity matrix of order 4 x 4 and 5 x 5.

Determinant and Inverse of a 2 x 2 Matrix

- The teacher should guide the learner on how to find the determinant of a 2 x 2 matrix, as discussed in the students' book.
- The learner should be guided on how to find the inverse of a 2 x 2 matrix whose determinant is 1, as in the students' book.
- The learner should be taken through example 7.
- The teacher should guide the learner to find the determinant of a 2 x 2 matrix where determinant is not equal to 1, as illustrated in the students' book.
- The learner should be taken through example 8.
- The teacher should define a singular matrix, as in the students' book.
- The learner to do exercise 8.3.

Use of Matrix Method to solve Simultaneous Equations

- The teacher should discuss how to solve simultaneous equations using matrix method, as discussed in the students' book.
- The learner should be taken through example 9.
- The learner to do exercise 8.4.

Evaluation

The teacher should probe the learner's understanding by giving a test covering all concepts taught under matrices.

Further Question

1. A train travelling between Nairobi and Kisumu had 3 first class coaches, 4 second class coaches and 10 third class coaches. The table below shows the maximum number of people each coach could carry and the fare for adults and children.

Coach	*Maximum number of people*	*Fare Ksh*	
		Adults	*Children*
1st Class	15	1 200	900
2nd Class	40	750	400
3rd Class	60	300	150

The ratio of the number of children to adults for this particular journey was 1 : 4, 1 : 3 and 1 : 2 for first, second and third class coaches respectively. Calculate the amount collected for the journey if the train had its maximum capacity in the coaches.

Answers

Exercise 8.1

1. (a) 2 x 2 (b) 3 x 1 (c) 3 x 2 (d) 1 x 1 (e) 3 x 3
   (f) 2 x 3 (g) 3 x 4 (h) 4 x 3 (i) 1 x 4 (j) 2 x 3

3. (a) $\begin{pmatrix} 0 & 2 \\ 15 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 20 & 11 \\ -9 & 8 \end{pmatrix}$ (c) $\begin{pmatrix} 11 & 15 \\ 8 & 6 \\ 12 & 18 \end{pmatrix}$ (d) Not possible

   (e) $\begin{pmatrix} 10.6 & 8.5 & 18.1 \\ 4.7 & 14.6 & 7.4 \\ 18.2 & 12.2 & 17.5 \end{pmatrix}$ (f) Not possible (g) Not possible

   (h) Not possible (i) $\begin{pmatrix} 5 & 16 \\ 4 & 14 \\ 3 & 0 \end{pmatrix}$ (j) Not possible

4. (a) $\begin{pmatrix} 9 & -13 \\ -2 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} -9 & 13 \\ 2 & -4 \end{pmatrix}$ (c) $\begin{pmatrix} -9 & 0 & 1 \\ -3 & -15 & 1 \\ -2 & 5 & 8 \end{pmatrix}$

(d) $\begin{pmatrix} 9 & 0 & -1 \\ 3 & 15 & -1 \\ 2 & -5 & -8 \end{pmatrix}$ (e) Not possible

(f) Not possible (g) Not possible

5. (a) $\begin{pmatrix} p-e & -q+h & r+k \\ s+f & t+i & -u-l \\ r+g & -w+j & x+m \end{pmatrix}$ (b) $\begin{pmatrix} -e-p & h+q & k-r \\ f-s & i-t & -l+u \\ g-v & j+w & m-x \end{pmatrix}$

6. (a) $x = 7, y = 2, z = 4$ (b) $a = -5, b = -3, c = -3$
(c) $p = 1, q = 0, r = 3, s = 0$ (d) $x = -3, y = -2$
(e) $a = -12, b = 9, c = 10, d = 1, e = 40, f = -11$
(f) $p = -4, q = -4, r = 5, s = 4, t = 3, u = 2$
(g) $x = 1, y = 2$
(h) $x = 1, y = 2, z = 3$
(i) $x = -2$
(j) $x = 1$ or 5. when $x = 1$, $y = 4$; when $x = 5$, $y = 20$

7. $x = \begin{pmatrix} 0 & 3 \\ -1 & -2 \end{pmatrix}$

8. (a) $P^T = \begin{pmatrix} 4 & -1 \\ 3 & 0 \end{pmatrix}$ (b) $Q^T = \begin{pmatrix} 1 & 4 & 3 \\ -2 & 0 & 1 \end{pmatrix}$ (c) $R^T = \begin{pmatrix} 1 & -4 \\ -3 & 9 \\ 7 & 2 \end{pmatrix}$

(d) $S^T = \begin{pmatrix} 4 & 13 & -1 \\ 7 & 15 & 17 \\ 9 & -2 & 3 \end{pmatrix}$ (e) $T^T = \begin{pmatrix} 3 & -1 & 6 \\ -4 & 7 & 2 \\ 0 & 3 & 5 \\ 8 & -6 & -1 \end{pmatrix}$

(f) $U^T = \begin{pmatrix} 3 & 9 & 16 & 0 \\ 2 & 3 & 5 & 3 \\ -5 & 4 & 1 & -8 \\ 1 & 0 & -15 & 12 \\ 8 & 2 & 7 & -1 \end{pmatrix}$

9. $\begin{matrix} & \textit{Buses} & \textit{Lorries} & \textit{Cars} \\ \text{Day } 1 & 10 & 6 & 15 \\ \text{" } 2 & 7 & 4 & 20 \\ \text{" } 3 & 3 & 1 & 9 \end{matrix}$, 20 buses, 11 lorries and 44 cars

10. First leg

	Won	*Drawn*	*Lost*	*Points*
Tusker	8	4	6	28
Mathare	5	7	6	22
Ulinzi	6	2	8	20
Gor Mahia	9	6	3	33

Second leg

	Won	*Drawn*	*Lost*	*Points*
Tusker	11	5	2	38
Mathare	7	2	9	23
Ulinzi	8	6	2	30
Gor Mahia	10	5	3	35

Final results

$$\begin{matrix} \\ \text{Tusker} \\ \text{Mathare} \\ \text{Ulinzi} \\ \text{Gor Mahia} \end{matrix} \begin{matrix} \textit{1st leg} \\ \begin{pmatrix} 8 & 4 & 6 & 28 \\ 5 & 7 & 6 & 22 \\ 6 & 2 & 8 & 20 \\ 9 & 6 & 3 & 33 \end{pmatrix} \end{matrix} + \begin{matrix} \textit{2nd leg} \\ \begin{pmatrix} 11 & 5 & 2 & 38 \\ 7 & 2 & 9 & 23 \\ 8 & 6 & 2 & 30 \\ 10 & 5 & 3 & 35 \end{pmatrix} \end{matrix}$$

$$\begin{matrix} \\ \text{Tusker} \\ \text{Mathare} \\ \text{Ulinzi} \\ \text{Gor Mahia} \end{matrix} \begin{matrix} \textit{Won} & \textit{Drawn} & \textit{Lost} & \textit{Points} \\ \begin{pmatrix} 19 & 9 & 8 & 46 \\ 12 & 9 & 15 & 45 \\ 14 & 8 & 10 & 50 \\ 19 & 11 & 6 & 68 \end{pmatrix} \end{matrix}$$

11. $a_{12} = 6$, $a_{21} = 3$, $a_{43} = 2$, $a_{42} = 5$, $a_{32} = 4$

12. $$B = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 2 & 3 \\ -2 & -1 & 0 & 1 & 2 \\ -3 & -2 & -1 & 0 & 1 \\ -4 & -3 & -2 & -1 & 0 \end{pmatrix}$$

Exercise 8.2

1. (a) $\begin{pmatrix} 2 & 4 \\ 0 & 8 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 2 \\ \frac{1}{2} & 5\frac{1}{2} \end{pmatrix}$ (c) $\begin{pmatrix} 6 & 2 \\ 3 & 9 \end{pmatrix}$

(d) $\begin{pmatrix} -7 & 6 \\ -6 & 2 \end{pmatrix}$ (e) $\begin{pmatrix} 4 & -6 \\ 4 & -4 \end{pmatrix}$ (f) $\begin{pmatrix} 9 & 2 \\ 6 & 6 \end{pmatrix}$

2. (a) (13) (b) $\begin{pmatrix} 6 & 2 \\ 12 & 4 \end{pmatrix}$ (c) (14) (d) (11)
(e) (47) (f) Non conformable (g) Non conformable
(h) Non conformable (i) (14 6 -19)
(j) Non conformable

3. $\begin{pmatrix} 4 & \frac{1}{2} & 0 \\ -1 & \frac{11}{6} & -\frac{7}{6} \\ -1 & 0 & -\frac{7}{6} \end{pmatrix}$

4. (a) [5] (b) $\begin{pmatrix} 26 \\ -36 \end{pmatrix}$ (c) $\begin{pmatrix} -10 & 91 \\ 13 & 45 \end{pmatrix}$
(d) $\begin{pmatrix} 19 & 17 \\ -39 & -19 \end{pmatrix}$ (e) (12 35 18)
(f) $\begin{pmatrix} 36 & 13 & 14 \\ 24 & 9 & 7 \\ 25 & 9 & 9 \end{pmatrix}$ (g) $\begin{pmatrix} 4 & 21 & 24 \\ 14 & 41 & 49 \end{pmatrix}$
(h) $\begin{pmatrix} 20 \\ 18 \\ 23 \end{pmatrix}$ (i) $\begin{pmatrix} 5 & 4 & -16 \\ 8 & 2 & -8 \\ 5 & -1 & 4 \end{pmatrix}$

5.

Order of A	*Order B*	*Order of AB*
2 x 2	2 x 1	2 x 1
2x 3	3 x 1	2 x 1
2 x 3	3 x 2	2 x 2
2 x 4	4 x 1	2 x 1
3 x 2	2 x 4	3 x 4
3 x 3	3 x 1	3 x 1
3 x 4	4 x 2	3 x 2
m x n	n x p	m x p

6. (a) $\begin{pmatrix} 5 & 4 \\ 13 & 8 \end{pmatrix}$ (b) $\begin{pmatrix} 22 \\ -1 \end{pmatrix}$ (c) (32 50 6) (d) $\begin{pmatrix} -7 \\ 34 \end{pmatrix}$
(e) (-4 19) (f) $\begin{pmatrix} 13 \\ 7 \\ 4 \end{pmatrix}$ (g) $\begin{pmatrix} 2 & 23 \\ -12 & 17 \\ 2 & 12 \end{pmatrix}$

(h) $\begin{pmatrix} 18 & 2 \\ 32 & 3 \\ 54 & 5 \end{pmatrix}$ (i) $(6 \quad 14 \quad 20)$ (j) $\begin{pmatrix} 4 & 5 & -4 \\ 13 & 11 & -4 \\ -3 & 12 & -18 \end{pmatrix}$

7. (a) $x = 2, y = 0$ (b) $x = 0, y = 1$ (c) $a = -13$
   (d) $x = 1, y = 1$ (e) $x = -3$ or 1 (f) $x = 2, y = 3$
   (g) $x = 3, y = 4, z = 2$

8. (a) $\begin{pmatrix} 78 & 28 \\ 98 & 50 \end{pmatrix}$ (b) $\begin{pmatrix} 77 & 28 \\ 99 & 51 \end{pmatrix}$

   $(A+B)^2 \neq A^2 + 2AB + B^2$

9. (a) (i) $\begin{pmatrix} 29 & 25 \\ 38 & 43 \end{pmatrix}$ (ii) $\begin{pmatrix} 29 & 25 \\ 38 & 43 \end{pmatrix}$
   (b) (i) $\begin{pmatrix} 27 & 27 \\ 36 & 36 \end{pmatrix}$ (ii) $\begin{pmatrix} 27 & 27 \\ 36 & 36 \end{pmatrix}$
   (c) (i) $\begin{pmatrix} 54 & 54 \\ 81 & 81 \end{pmatrix}$ (ii) $\begin{pmatrix} 54 & 54 \\ 81 & 81 \end{pmatrix}$

   In each case, (i) = (ii)

10. Theatre A: sh. 85 000
    Theatre B: sh. 78 000

11. Mrs Kamau spent sh. 625
    Mrs Mutua spents sh. 670

12. Nairobi to station K: sh. 51 100
    Nairobi to station M: sh. 61 600
    Total collection sh. 112 700

13. $\begin{pmatrix} 69 & 58 & 43 \\ 80 & 36 & 62 \\ 72 & 54 & 48 \\ 65 & 46 & 73 \\ 47 & 67 & 70 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} 525 \\ 508 \\ 528 \\ 525 \\ 549 \end{pmatrix}$

    E, with 549 marks

14. sh. 104 000

15. Joyce's expenditure: sh. 135
    Margaret's expenditure: sh. 115

16. Manchester, with 46 points.

17. $\mathbf{P} = (5 \quad 20 \quad 10)$

$\mathbf{Q} = \begin{pmatrix} 5\,000 & 900 \\ 2\,000 & 400 \\ 2\,800 & 360 \end{pmatrix}$

$\mathbf{PQ} = (93\,000 \quad 16\,100)$ Represents total units of materials and time needed for the site

$\mathbf{P} = \begin{pmatrix} 100 \\ 50 \end{pmatrix}$

$\mathbf{PQR} = (10 \quad 105\,000)$ This matrix represents the total cost of the completed site.

Exercise 8.3

1. (a) 29 (b) –49 (c) 16 (d) 0 (e) 5
   (f) 2 (g) 14 (h) 1 (i) 10 (j) 2
   (k) $\begin{pmatrix} -19 \\ 96 \end{pmatrix}$ (l) $3x + 2y$ (m) –8 (n) 22 (p) 2

2. (a) $\begin{pmatrix} 4 & -9 \\ -3 & 7 \end{pmatrix}$ (b) $\begin{pmatrix} -2 & \frac{-13}{2} \\ 1 & 3 \end{pmatrix}$ (c) $\begin{pmatrix} \frac{-3}{4} & \frac{7}{4} \\ -1 & 2 \end{pmatrix}$

   (d) $\begin{pmatrix} \frac{1}{7} & \frac{1}{28} \\ \frac{1}{14} & \frac{-3}{28} \end{pmatrix}$ (e) No inverse (f) $\begin{pmatrix} 8 & -13 \\ -3 & 5 \end{pmatrix}$

   (g) $\begin{pmatrix} \frac{-9}{5} & 2 \\ 1 & -1 \end{pmatrix}$ (h) No inverse (i) $\begin{pmatrix} \frac{1}{2} & 1 \\ \frac{3}{4} & 2 \end{pmatrix}$

(j) $\begin{pmatrix} \frac{8}{3} & \frac{-4}{3} \\ \frac{-8}{3} & \frac{16}{3} \end{pmatrix}$ (k) $\begin{pmatrix} \frac{1}{13} & 0 \\ \frac{7}{13} & -1 \end{pmatrix}$ (l) $\begin{pmatrix} \frac{1}{8} & \frac{-1}{24} \\ \frac{1}{4} & \frac{-1}{12} \end{pmatrix}$

(m) $\begin{pmatrix} \frac{1}{8} & \frac{1}{4} \\ \frac{3}{8} & \frac{1}{4} \end{pmatrix}$ (n) $\begin{pmatrix} -3 & 1 \\ 2 & \frac{-1}{2} \end{pmatrix}$ (p) $\begin{pmatrix} \frac{-1}{8} & \frac{1}{2} \\ 1 & -2 \end{pmatrix}$

3. (a) $w = 2, y = -5, \quad x = -3, \quad z = 8$

(b) $a = \frac{-1}{8}, b = \frac{3}{8}, \quad c = \frac{1}{4}, d = \frac{1}{4}$

4. (a) $c = 3.75$ (b) $y = 0$ or $y = 1$

(c) $x = 1$ (d) $x = -3$ or -4

5. (a) $\begin{pmatrix} \frac{27}{13} & \frac{4}{13} \\ 1 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} \frac{3}{19} & \frac{1}{19} \\ \frac{-7}{76} & \frac{1}{19} \end{pmatrix}$

Exercise 8.4

1. (a) $x = 2, y = 3$ (b) $p = \frac{1}{2}, q = 4$ (c) $m = \frac{9}{7}, n = \frac{29}{7}$

(d) $t = 1, r = -2$ (e) $x = \frac{1}{2}, y = \frac{1}{3}$

2. (a) $x = 2, y = 1$ (b) $x = 2, y = 2$ (c) $a = -4, \quad b = -3$

(d) $c = -2, d = -1$ (e) $v = 4,\ u = 2$

3. (a) $x = \frac{8}{11}$, $y = \frac{29}{11}$ (b) $x = 2, y = 2$ (c) $w = 1,\ x = 4$

(d) $p = -4, q = 1$ (e) $x = 18.56, y = 28$

4. (a) $a = 3,\ b = 4$ (b) $m = \frac{1}{2},\ n = \frac{1}{5}$

(c) $g = \frac{-13}{14},\ f = \frac{31}{14}$ (d) $u = \frac{1}{5},\ v = \frac{3}{10}$

(e) $u = 4,\ v = 6$

5. (a) $x = 8, y = -11$ (b) $p = 23, q = -30$

(c) $x = 0,\ y = 2$ (d) $c = -192,\ d = 360$

(e) $t = \frac{1}{7},\ s = \frac{26}{21}$

Further Question

1. Total collection is sh. 307 300.

Chapter Nine

FORMULAE AND VARIATIONS

This is a new topic. However, the learner has encountered situations that require the use of formulae to solve problems.

Objectives

By the end of the topic, the learner should be able to:

(i) rewrite a given formula by changing the subject.
(ii) define direct, inverse, partial and joint variations.
(iii) identify constants of proportionality .
(iv) form and solve equations involving variations.
(v) draw graphs to illustrate direct and inverse proportions.
(vi) use variations to solve everyday life problems.

Time: Twenty one lessons.

Teaching/Learning Activities

- The learner should be guided on how to change the subject of a formula, as illustrated in the students' book.
- The teacher should lead the learner through examples 1, 2 and 3.
- The learner to do exercise 9.1.
- The teacher should lead the learner through the definition of variation, as in the students' book.
- The teacher should discuss direct variation as illustrated in the students' book.
- The learner should be guided through examples 4 and 5.
- The learner to do exercise 9.2.
- The teacher should discuss inverse variation, as in the students' book.
- The teacher should guide the learner through example 6.
- The learner should do exercise 9.3.
- The teacher should guide the learner in discussing partial variation using examples 7, 8 and 9.
- The learner to do exercise 9.4.

- The teacher should discuss joint variation as illustrated in the students' book.
- The teacher should guide the learner through examples 10 and 11.
- The learner to do exercise 9.5.

Evaluation

A written test should be given to probe the learner's understanding in all areas covered on in the topic.

Answers

Exercise 9.1

1. (a) $P = \frac{100I}{RT}$ (b) $a = \frac{2(s - ut)}{t^2}$

(c) $g = \frac{4\pi^2 l}{T^2}$ (d) $S = \frac{V^2 - u^2}{2a}$

(e) $r = \sqrt{\frac{M_1 M_2}{F}}$ (f) $r = 3\sqrt{\left(\frac{3V}{4\pi}\right)}$

(g) $d_1 = \left(\frac{r^2}{r_1}\right)^2 d_2$ (h) $d = \frac{2(s - an)}{n(n-1)}$

(i) $h = \frac{s - 2\pi r^2}{2\pi r}$ (j) $h = \sqrt{\frac{\left(A - \pi r^2\right)^2 - \pi^2 r^4}{\pi^2 r^2}}$

(k) $x = -b \pm \sqrt{\frac{b^2 - 4ac}{2a}}$ (l) $t = \frac{-u \pm \sqrt{(2as + u^2)}}{a}$

2. (a) $l = \frac{T^2 g}{4\pi^2}$ (b) 101.3

3. (a) $g = \frac{2E - mv^2}{2mh}$ (b) 10

4. (a) $P = \frac{RQ}{Q-R}$ (b) 4

5. (a) $n = \frac{\log\left(\frac{a-s+sr}{a}\right)}{\log r}$ (b) 5

6. (a) $n = \frac{\log\left(\frac{c}{p}\right)}{\log v}$ (b) 0.1322

7. $n = \frac{\log\left(\frac{a}{100}\right)}{\log b}$

8. $Z = \sqrt{\left(\frac{d+gx}{d+fx}\right)}$

9. $N = \sqrt{\left(\frac{B^2P}{E^2-B^2}\right)}$

10. $a = \frac{bc}{b+c}$

11. $R = \sqrt{\left(\frac{A+\pi r^2}{\pi}\right)}$

Exercise 9.2

1.

m	2	**3**	5
E	1.2	1.8	**3.0**

2. (a) $P = \frac{1}{6}Q$ (b) $8\frac{1}{3}$ (c) 72

3. 88
4. 108
5. 4
6. M is decreased by 75%
7. 15, 0.67
8. 17.01

9. 16, 4
10. (a) 1 (b) 3.5
11. 0.004, 442.4

Exercise 9.3

1. 180

2. (a) $y = \frac{264}{x}$ (b) 132

3.

Q	15	30	50	75
P	**10**	5	3	**2**

(a) $P = \frac{150}{Q}$

(b) Check for accuracy of graph.

4. $\frac{1}{18}$

5. $P = \frac{240}{A}$

6. (a) $k = 1$

(b) Check for accuracy of graph.

7. (a) 480 (b) 2 (c) 6

8.

P	1	3	**6**	9	12	**15**	18
V	3	**1**	0.5	**0.33**	0.25	0.2	**0.17**

$P = \frac{3}{V}$

9. (a) 18.82 (b) 476.7
10. 160
11. 19.73

Exercise 9.4

1. $y = 5x - 20$
2. 5
3. (a) $v = 12 + 6t$ (b) 48
4. 39 500, $c = 2\,000 + 50s$
5. 430
6. $y = s + \frac{200}{x^2}$; 5.617
7. $y = \frac{3\,000}{x^3} + 12$, $23\frac{1}{9}$
8. $y = 3x + 2x^2$, 2
9. $y = \frac{4}{x} - 3x$, -4

Exercise 9.5

1. (a) $A = kbh$ (b) $A = khr^3$ (c) $y = \frac{kx}{z}$

   (d) $W = \frac{kbd^2}{l}$ (e) $H = k\frac{tE^2}{R}$ (f) $P = k\frac{\sqrt{t}}{\sqrt{v}}$

   k is a constant.

2. $y = \frac{x}{36z^3}$
3. 291.6
4. P = 112.5, R = 9.913, P is multiplied by 3.
5. M decreased by 19.08%
6. (a) 15 (b) 24 (c) P decreases by 1.44%
7. $y \propto t^2$

8. $T \alpha \sqrt{x}$

9. $y = \frac{50x}{z^2}$

10. $6.95\ cm^3$

Chapter Ten

SEQUENCES AND SERIES

This topic is not entirely new to the learner as he/she has dealt with simple sequences in primary school.

Objectives

By the end of the topic, the learner should be able to:

(i) identify simple number patterns.
(ii) define sequences.
(iii) identify the pattern for a given set of numbers and deduce the general rule.
(iv) determine a term in a sequence.
(v) recognise arithmetic and geometric sequences.
(vi) define a series.
(vii) recognise arithmetic and geometric series (progression).
(viii) derive the formula for partial sum of an arithmetic and geometric series.
(ix) apply A.P. and G.P. formula to solve problems in real life situations.

Time: Twenty one lessons.

Teaching/Learning Activities

Simple Sequences

- The teacher should guide the learner to define a sequence, as in the students' book.
- The learner should be guided to identify a pattern in a set of numbers and deduce the general rule, as in examples 1, 2 and 3 in the students' book.
- The learner to do exercise 10.1.

Arithmetic and Geometric Sequences

- The teacher should guide the learner to define an arithmetic sequence, as in the students' book.
- The learner should be guided through examples 4, 5 and 6.

- The learner should do exercise 10.2.
- The teacher should lead the learner to define a geometric sequence.
- The teacher should guide the learner through examples 7, 8 and 9.
- The learner to do exercise 10.3.

Arithmetic and Geometric Series

- The teacher should lead the learner through the definition of series.
- The learner should be led to recognise an arithmetic series.
- The teacher should lead the learner to derive the formula for partial sum of an arithmetic series, as in the students' book.
- The learner should be taken through examples 10 and 11.
- The learner should be given exercise 10.4.
- The learner should be guided to derive the formula for partial sum of a geometric series, as in the students' book.
- The teacher should guide the learner through examples 12, 13 and 14.
- The learner should be given exercise 10.5.

Evaluation

- The learner should be given a comprehensive test on formula and variation, and series and sequence.

Answers

Exercise 10.1

1. (a) 6, 20 (b) 37 (c) 27, 39
   (d) 24, 67 (e) –3, –27 (f) 3, 15
   (g) 4, 8 (h) 42, 132 (i) $\frac{1}{36}$, $\frac{1}{100}$
   (j) $\frac{15}{2}$, $\frac{19}{2}$ (k) $\frac{25}{3}$, 11 (l) $\frac{17}{5}$, $\frac{29}{5}$
2. (a) 2, 4, 6 (b) 3, 5, 7 (c) 1, 4, 9
   (d) 1, $\frac{5}{2}$, 5 (e) 0, 6, 16 (f) 3, 5, 9
3. (a) $3n^2$ (b) $2n - 1$
   (c) $5n + 1$ (d) $n^2 - 1$

Exercise 10.2

1. 21
2. 8, 20
3. (a) -4 (b) -13
4. (a) $a = 10$ (b) 67
5. (a) $a = 4$ (b) $d = 3$
6. 0
7. 21
8. -20 or -10
9. £3 060
10. -80 or 30
11. 12.5
12. 13
13. 18
14. 3 years and 2 months

Exercise 10.3

1. (a) 5 (b) -2 (c) $\frac{1}{8}$ (d) $\frac{-1}{4}$

   (e) a (f) -1 (g) $-n$

2. (a) 64, 1 024

   (b) $\frac{-1}{4}, 32\left(\frac{-1}{2}\right)^{n-1}$

   (c) $\frac{1}{64}, \frac{1}{2^{n-1}}$

   (d) $-1, 1$

   (e) $\frac{1}{8}, 3^{4-n}$

   (f) $\frac{b}{28}, \frac{b}{256}$

3. ± 3

4. $9, 3, 1, \frac{1}{3}$

5. $10, \frac{5}{64}$

7. 2, 512
8. $(a + 1)\, x^{2n-2}$ $(a + 1)\, x^{16}$
9. $y = 3$ or -3
10. sh. 312 500

Exercise 10.4

1. (a) 72 (b) 75 (c) 25 (d) 300
   (e) –364 (f) 0 (g) 770 (9) 525
   (i) 12 459 (j) 38.5
2. (a) 462 (b) 135 (c) 129
3. 975
4. 2.869
5. 396
6. 5
7. 25
8. 2
9. $a = 16,\ d = -3$
10. 16.28

Exercise 10.5

1. (a) 510 (b) 26 240 (c) –6 560
   (d) 40.48 (e) 0.5 (f) $6\frac{1}{9}$
   (g) –7.969 (h) 327 670 (i) 3
   (j) 13.5
2. 31.25
3. 1 456
4. 122
5. 5
6. (a) $r = 4$ (b) $n = 4$
7. $r = 3$ or -4
8. $a = 5$
9. $r = 2$ or -3
10. $a = -16,\ r = \frac{-1}{2}$
11. 3
12. 20
13. 18.6
14. 92587.3
15. $r = 1\frac{1}{2},\ S_n = 394.06$

Chapter Eleven

VECTORS (II)

The learner met vectors in two dimensions in book two. This topic will mainly deal with vectors in three dimensions.

Objectives

By the end of the topic, the learner should be able to:

(i) locate a point in two and three dimension co-ordinate systems.
(ii) represent vectors as column and position vectors in three dimensions.
(iii) distinguish between column and position vectors.
(iv) represent vectors in terms of **i**, **j** and **k**.
(v) calculate the magnitude of a vector in three dimensions.
(vi) use the vector method in dividing a line proportionately.
(vii) use vector method to show parallelism.
(viii) use vector method to show collinearity.
(ix) state and use the ratio theorem.
(x) apply vector methods in geometry.

Time: Twenty four lessons.

Teaching/Learning Activities

Co-ordinates in Two and Three Dimensions

- The teacher should introduce co-ordinates in three dimensions, as illustrated in the students' book.
- The learner should do exercise 11.1.

Column and Position Vectors in Three Dimensions

- The learner should be guided on how to represent column and position vectors in three dimensions, as illustrated in the students' book.
- The learner to do exercise 11.2.

*Column Vectors in terms of Unit Vectors **i**, **j** and **k***

- The teacher should introduce the unit vectors **i**, **j** and **k** using figure 11.4 and 11.5 in the students' book.
- The learner should be involved in representing vectors in terms of **i**, **j** and **k**, as in examples 1 and 2.
- The learner to do exercise 11.3.

Magnitude of a Vector in Three Dimensions

- The teacher should guide the learner to calculate the magnitude of a vector in three dimensions, as shown in the students' book.
- The learner to do exercise 11.4.

Parallel Vectors and Collinearity

- The teacher should discuss parallel vectors, as in the students' book.
- The teacher should guide the learner through example 3.
- The learner should be led to identify collinear points using vector method, as in the students' book.
- The teacher should take the learner through examples 4, 5 and 6.
- The learner should attempt exercise 11.5.

Proportional Division of a Line

- The teacher should discuss proportional division of a line internally and externally, as in the students' book.
- The learner to do exercise 11.6.

The Ratio Theorem

- The teacher should discuss the ratio theorem as in the students' book.
- The learner should be guided through examples 7, 8 and 9.
- The learner to do exercise 11.7.

Applications of Vector Methods in Geometry

- The teacher should discuss applications of vector methods in geometry, as in examples 10 and 11.
- The learner to do exercise 11.8.

Evaluation

- The teacher should probe the learner's understanding by giving a written test.

Answers

Exercise 11.1

1. (a) T (7, 4, 0)
   X (3, 9, 7)
   Z (0, 2, 7)

   (b) (i) O (0, 0, 0)
   A (7, 0, 0)
   B (7, 9, 0)
   C (0, 9, 0)

   (ii) D (7, 0, 14)
   G (0, 0, 14)
   F (7, 9, 14)
   E (7, 9, 14)

   (iii) K (7, 0, 7)
   L (7, 9, 7)
   M (0, 9, 7)
   N (0, 0, 7)

   (c) (3.5, 4.5, 14)

   (d) (3.5, 9, 7)

2. (a) L (b) M (c) E
   (d) T (e) D (f) N
3. (a) x (b) y (c) z
4. (a) (1, 2, 4)
   (b) (4, 6, 6)
   (c) (8, 12, 6)
5. (5.66, 5.66, 1.75)
6. V' (2, 3, –7)
   X' (3, 9, –7)
   M' (0, 9, –7)
   R' (0, 3, 0)
   E' (7, 9, –14)
   G' (0, 0, –14)
7. V' (–2, 3, 7)
   X' (–3, 9, 7)
   M' (0, 9, 7)
   R' (0, 3, 0)
   E' (–7, 9, 14)
   G' (0, 0, 14)
8. W' (5, –7, 7)
   E' (7, –9, 14)
   Q' (4, –3, 0)
   Y' (2, 0, 7)
   M' (0, –9, 7)
9. (i) Q' (–a, b, c)
   (ii) Q' (a, –b, c)

Exercise 11.2

1. (a) $\begin{pmatrix} 3 \\ -2 \\ 18 \end{pmatrix}$ (b) $\begin{pmatrix} 7\frac{1}{3} \\ 2\frac{1}{2} \\ 5\frac{1}{24} \end{pmatrix}$ (c) $\begin{pmatrix} -4 \\ 14 \\ -6 \end{pmatrix}$ (d) $\begin{pmatrix} 2.5 \\ 12.8 \\ 10 \end{pmatrix}$

2. (a) $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 2 \\ -4 \\ 5 \end{pmatrix}$ (c) $\begin{pmatrix} 6 \\ 2 \\ 4 \end{pmatrix}$ (d) $\begin{pmatrix} -4 \\ -1 \\ -5 \end{pmatrix}$ (e) $\begin{pmatrix} -2 \\ -2 \\ -7 \end{pmatrix}$

3. (24 14 26)

4. $\mathbf{OW} = \begin{pmatrix} 5 \\ 7 \\ 7 \end{pmatrix}$ $\mathbf{OQ} = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}$ $\mathbf{OP} = \begin{pmatrix} 2 \\ 8 \\ 0 \end{pmatrix}$ $\mathbf{OX} = \begin{pmatrix} 3 \\ 9 \\ 7 \end{pmatrix}$

   $\mathbf{OG} = \begin{pmatrix} 0 \\ 0 \\ 14 \end{pmatrix}$ $\mathbf{OD} = \begin{pmatrix} 7 \\ 0 \\ 14 \end{pmatrix}$

5. (a) $\begin{pmatrix} 2 \\ 8 \\ 7 \end{pmatrix}$ (b) $\begin{pmatrix} 4 \\ 6 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} 4 \\ 3 \\ 7 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 8 \\ 14 \end{pmatrix}$ (e) $\begin{pmatrix} 0 \\ 9 \\ 14 \end{pmatrix}$

6. (a) $\mathbf{OP} = \begin{pmatrix} 25 \\ 0 \\ 0 \end{pmatrix}$ $\mathbf{PQ} = \begin{pmatrix} 0 \\ 30 \\ 24 \end{pmatrix}$ $\mathbf{QR} = \begin{pmatrix} 0 \\ 0 \\ 38 \end{pmatrix}$ $\mathbf{RS} = \begin{pmatrix} 0 \\ -10 \\ 10 \end{pmatrix}$ $\mathbf{ST} = \begin{pmatrix} -40 \\ 0 \\ 0 \end{pmatrix}$

   (b) $\mathbf{OT} = \begin{pmatrix} -15 \\ 20 \\ 72 \end{pmatrix}$ $\mathbf{OM} = \begin{pmatrix} -7.5 \\ 10 \\ 36 \end{pmatrix}$

7. (a) $\begin{pmatrix} 1 \\ 2 \\ 0.5 \end{pmatrix}$ $\begin{pmatrix} -2 & -2 \\ 3 & -1 \\ -0.25 & 0.4 \end{pmatrix}$ $\begin{pmatrix} 0.5 \\ 2 \\ 0.3 \end{pmatrix}$

   (b) $\begin{pmatrix} -2.5 \\ 6 \\ 0.95 \end{pmatrix}$

(c) Top of hill is 950 m above base camp, 2.5 km west and 6 km north.

(d) $\begin{pmatrix} 2.5 \\ -6 \\ -0.95 \end{pmatrix}$

Exercise 11.3

1. (a) (i) $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (ii) $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ (iii) $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$

   (b) (i) $\begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ (ii) $\begin{pmatrix} 5 \\ -2 \\ 6 \end{pmatrix}$ (iii) $\begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix}$

2. (a) 7, 1, 4) (b) (–1, 3, 0) (c) (–9, 5, –6)
   (d) (–4.5, 0, 0)

3. **LM** = $\begin{pmatrix} (-3.5 \\ -4.5 \\ 0 \end{pmatrix}$ **DF** = $\begin{pmatrix} -7 \\ -9 \\ 0 \end{pmatrix}$

4. $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

Exercise 11.4

1. (a) 8.3 units (b) 5.1 units (c) 10 units (d) 13 units

2. 7.8 units, 3.7 units

3. (a) 7.9 units (b) 10.5 units (c) 5.2 units (d) 5 units
   (e) 6.25 units (f) 11.4 units (g) 14 units (h) 17.6 units

4. (a) 3 units (b) 2.45 units (c) 3 units (d) 4.6 units
   (e) 4.6 units (f) 6 units (g) 9.2 units (h) 5.7 units

5. (a) $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 2 \\ -2.5 \end{pmatrix}$ $\begin{pmatrix} -2 \\ 0 \\ 3 \end{pmatrix}$ (b) 10.4 m (c) 5.3 m

6. (a) 2.3 km (b) 3.6 km (c) 1.9 km (d) 6.6 km

Exercise 11.5

1. (a) Parallel (b) Parallel (c) Not parallel (d) Parallel

(e) Parallel (f) Parallel (g) Not parallel (h) Parallel
(i) Parallel (j) Parallel (k) Not parallel (l) Parallel

2. (a) $y = 3x - 2$, collinear (b) Not collinear
(c) Not collinear (d) Collinear; $y = x + 1$
(e) Collinear; $y = 2x - 1$

3. (a) Not collinear (b) Not collinear (c) Collinear
(d) Not collinear (e) Collinear (f) Collinear

4. (a) 7 (b) 2 (c) –20 (d) 4 (e) 0.8

5. (a) (i) C (1, 4), D (5, 1) (b) $\mathbf{AB} = 2\,\mathbf{CD} \Rightarrow \mathbf{AB} \parallel \mathbf{CD}$
(ii)5 units, 10 units $\mathbf{OA} = 2\mathbf{DE} \Rightarrow \mathbf{OA} \parallel \mathbf{DE}$

6. (1 , 5)

10. (a) (i) $\frac{2}{3}\mathbf{b}$ (ii) $\frac{1}{3}(2\mathbf{a}+\mathbf{b})$ (iii) $\frac{4}{3}\mathbf{a}$

(iv) $\frac{1}{3}(2\mathbf{a}-\mathbf{b})$ (v) $\frac{1}{3}(2\mathbf{a}-\mathbf{b})$

11. $\frac{1}{2}(\mathbf{a}+\mathbf{b})\frac{3}{8}(\mathbf{a}+\mathbf{b})$, $\frac{3}{4}(5\mathbf{b}-3\mathbf{a}),\frac{1}{5}(5\mathbf{b}-3\mathbf{a})$

3 : 5, 3 : 8, 8 : 5.

12. (a) $\mathbf{p}+\frac{2}{3}\mathbf{q}$ (b) $\mathbf{r}+\frac{1}{2}\mathbf{p}$ or $\frac{3}{2}\mathbf{p}+\mathbf{q}$

13. $\frac{2}{3}\mathbf{b}$, $\frac{1}{3}(2\mathbf{a}+\mathbf{b})$, $\frac{4}{3}\mathbf{a}$, $\frac{1}{3}(2\mathbf{a}-\mathbf{b})$

Exercise 11.6

1. 7 : 5, 5 : 1, 1 : 3 and 4 : –1

2. (a) 2:5 (b) 3 : –1 (c) 8 : –11

3. (a) — (e): Check for accuracy of drawing.

4. (a) $\frac{1}{6}\mathbf{c}$ (b) $\frac{1}{2}\mathbf{c}$ (c) $\frac{3}{2}\mathbf{c}$ (d) $\frac{1}{3}\mathbf{c}$

5. Check for accuracy of drawing; (2, –1)

6. (a) 1 : 2 (b) 3 : –2 (c) –3 : 5 (d) 3 : 2
(e) 3 : 5 (f) –2 : 5 (g) 9 : –2

7. 9 : –1

Exercise 11.7

1. (a) $\frac{2}{5}\mathbf{s}+\frac{3}{5}\mathbf{u}$ (b) $\frac{7}{4}\mathbf{s}-\frac{3}{4}\mathbf{q}$

(c) $\frac{11}{14}\mathbf{u}-\frac{3}{11}\mathbf{p}$ (d) $\frac{3}{7}\mathbf{q}+\frac{4}{7}\mathbf{p}$

2. (a) (5, 9)
(b) (1, 3, 2)

3. (a) S (2, 3.8), T (4.5, –0.2)
(b) S (0, 3, 3), T (–5, 8, 3)

4. $\frac{1}{18}(5\mathbf{a}+5\mathbf{b}+8\mathbf{c})$

5. (a) (i) $\frac{2}{3}\mathbf{b}+\frac{1}{3}\mathbf{a}$ (ii) $\frac{1}{2}\mathbf{b}+\frac{1}{2}\mathbf{c}$ (iii) $\mathbf{a}+\mathbf{c}$

(iv) $\mathbf{a}-\mathbf{b}+\mathbf{c}$ (v) $\frac{1}{3}(3\mathbf{a}-2\mathbf{b}+2\mathbf{c})$ (vi) $\frac{1}{18}(16\mathbf{a}-7\mathbf{b}+9\mathbf{a})$

(b) $\frac{1}{2}(\mathbf{a}+\mathbf{c})$

6. m = 4, n = 18
7. h = –1, m = –20
8. (a) $\mathbf{d}-\mathbf{a}$
(b) $\mathbf{OE} = \mathbf{a} + k(\mathbf{d}-\mathbf{a})$
$\mathbf{OB} = 6\mathbf{d} + 4\mathbf{a}$

(c) $h = \frac{1}{10}, k = \frac{3}{5}$

$\mathbf{DE : EA = 2 : 3}$

9. $\mathbf{OS} = \frac{1}{11}(2\mathbf{a} + 3\mathbf{c})$

11. $\mathbf{AM} = \frac{1}{4}\mathbf{c} + \frac{3}{4}\mathbf{b}$

$\mathbf{BN} = -\mathbf{b} + \frac{3}{5}\mathbf{c}$

$\mathbf{AX : XN} = 6 : 1$

12. $\mathbf{QM} = \frac{5}{12}(2\mathbf{q} + 5\mathbf{r})$

$\mathbf{PM} = \frac{1}{28}(38\mathbf{q} + 25\mathbf{r})$

$\mathbf{PL : LM} = 4 : 5$
$\mathbf{QL : LR} = 25 : 38$

13. $\mathbf{CX : XD} = 9 : 4$, $\mathbf{BX : XE} = 10 : 3$
14. $\mathbf{AX : XM} = 1 : 1$
$\mathbf{CX : XN} = 3 : 1$

15. $\mathbf{AM} = \frac{1}{5}(2\mathbf{a} + 3\mathbf{c})$

$\mathbf{AN} = 3\mathbf{a}$

$\mathbf{MN} = \frac{1}{5}(13\mathbf{a} - 3\mathbf{c})$

$\mathbf{AL : LC} = 9 : 4$
$\mathbf{NM : ML} = 13 : 2$

16. $\mathbf{QX} = \frac{2}{9}(\mathbf{a} - 2\mathbf{b})$; $\mathbf{QR} = \frac{1}{6}(3\mathbf{a} - \mathbf{b})$
17. 3 : –2 (externally in the ratio 3 : 2)
18. $\mathbf{AL : LC} = 1 : 3$
19. (a) (i) $\begin{pmatrix}10\\0\end{pmatrix}$, $\begin{pmatrix}10\\10\end{pmatrix}$ (ii) (14, 14)

(c) 1 : 4

Exercise 11.8

7. $\mathbf{O_2G_1} = \frac{1}{3}(\mathbf{a}+\mathbf{b}+\mathbf{c})$, $\mathbf{O_2G_2} = \frac{1}{3}(\mathbf{a}+\mathbf{b}+\mathbf{c})$

They bisect each other in the ratio 2:1

Mixed Exercise 2

1. $\begin{pmatrix} 2 & 9 \\ 2 & 6 \end{pmatrix}$
2. (a) $\begin{pmatrix} 3 & -3 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 9 & 6 \\ 3 & -6 \end{pmatrix}$ (c) $\begin{pmatrix} 9 & 6 \\ 3 & -6 \end{pmatrix}$ (d) $\begin{pmatrix} 7 & 20 \\ 9 & -10 \end{pmatrix}$
   (e) $\begin{pmatrix} -1 & -11 \\ -6 & 3 \end{pmatrix}$
4. 24
5. x = 2, y = –1
6. 2.646
7. (a) 54, 132 (b) –32, –2 048
   (c) 7 x 5, 13 x 11 (d) $\frac{x+5}{11}$, $\frac{x+11}{23}$
8. 9
9. (a) 6 (b) –1 (c) 0 (d) $\frac{-1}{16}$
10. 2.828
11. $\dfrac{0.5\pi^2+(1-E)^2}{\pi^2+(1-E)^2}$
12. (a) $\begin{pmatrix} \frac{1}{7} & \frac{-3}{7} \\ \frac{2}{7} & \frac{1}{7} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{-1}{5} & \frac{3}{5} \\ \frac{1}{5} & \frac{2}{5} \end{pmatrix}$

(c) Singular matrix

(d) $\begin{pmatrix} \frac{4}{3x} & \frac{-2}{3y} \\ \frac{-2}{3x} & \frac{4}{3y} \end{pmatrix}$

13. 1.455

14. $X = \frac{96\sqrt{Y}}{\sqrt[3]{Z}}$

15. 16.44

16. $\angle CBA = 38°$, $\angle CBY = 33°$, $\angle BYC = 114°$

17. (a) $3\mathbf{i}$ (b) $3\mathbf{i} + 3\mathbf{j}$ (c) $\mathbf{i} + \mathbf{j}$

18. $P = 9Q - 26$, 28

19. ar^2, ar^4, 3.984

20. 566.4

21. 4.203

22. 1.055

23. (a) $x \simeq 0.8$ and $x \simeq 1.6$
(b) $x \simeq -2.2$ and $x \simeq 2.9$
(c) $x \simeq 1.9$ and $x \simeq -1.4$
It has no solutions

24. $\mathbf{RQ} = \mathbf{q} - \mathbf{r}$ $\mathbf{OS} = \frac{2}{3}\mathbf{q} + \frac{1}{3}\mathbf{r}$ $\mathbf{RT} = \frac{2}{5}\mathbf{q} - \frac{4}{5}\mathbf{r}$

25. (a) 9b, 11b, 13b (b) 39b (c) 400b

26. (a) $r = 2k$ (b) 1, 2k, $4k^2$ (c) $\frac{1024k^{10} - 1}{2k - 1}$

27. (a) Arithmetic, 210

(b) Geometric, $\frac{b\left(b^{20} - 1\right)}{b - 1}$

(c) Arithmetic $210 + 190\,k$

(d) Neither
(e) Geometric 699 050

28. $\angle RPQ = 32°$
$\angle STD = 148°$

29. $\mathbf{OT} = \frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$ $\mathbf{OP} = \frac{2}{5}\mathbf{b}$

$\mathbf{AP} = \frac{2}{5}\mathbf{b} - \mathbf{a}$ $\mathbf{OQ} = \frac{6}{11}\mathbf{a} + \frac{2}{11}\mathbf{b}$

30. $\mathbf{a} = -3$ or $1\frac{1}{2}$

31. 35

32. $0, \frac{1}{2}, 1$,....

$\frac{-1}{3}, 0, \frac{1}{3}$,....

33. $\mathbf{OT} = \frac{1}{2}\mathbf{p} + \frac{1}{3}\mathbf{q}$, 1 : 1

34. X (5, 0)

35. 60°

36. $2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} = \frac{17}{28}(4\mathbf{i} - \mathbf{j} + 5\mathbf{k}) + \frac{3}{7}(-\mathbf{i} + 9\mathbf{j} + 5\mathbf{k})$

37. 86.49

39. (a) $\begin{pmatrix} 26 & 9 \\ 22 & 23 \end{pmatrix}$ (b) $\frac{1}{40}\begin{pmatrix} 7 & -3 \\ -2 & 6 \end{pmatrix}$ (c) $\frac{1}{10}\begin{pmatrix} 3 & -1 \\ -2 & 4 \end{pmatrix}$

(d) $\frac{1}{400}\begin{pmatrix} 23 & -9 \\ -22 & 26 \end{pmatrix}$ (e) $\frac{1}{400}\begin{pmatrix} 23 & -9 \\ -22 & 26 \end{pmatrix}$

40. $v = 2 \qquad \lambda = 5$

43. 83.63

44. $\angle BOC = 58°$
$\angle AED = 26°$

Chapter Twelve

BINOMIAL EXPANSION

The learner has already met factorisation of algebraic and quadratic expressions. In this topic, the learner will be introduced to higher orders of binomial expressions.

Objectives

By the end of the topic, the learner should be able to:

(i) expand binomial expressions up to the power of four by multiplication.
(ii) build up Pascal's triangle up to the eleventh row (power of 10).
(iii) use Pascal's triangle to determine the coefficient up to the power of 10.
(iv) apply binomial expansion in numerical cases.

Time: Eleven lessons.

Teaching/Learning Activities

Binomial Expansion up to Power Four

The teacher should guide the learner to expand $(a + b)^n$ up to power four through multiplication, as illustrated in the students' book.

Pascal's Triangle

- The learner should be led to build Pascal's triangle (binomial coefficient) as illustrated in the students' book.
- The teacher should involve the learner in using Pascal's triangle to obtain coefficients of expansions of the form $(a + b)^n$, as in examples 1, 2, 3 and 4.
- The learner to do exercise 12.1.

Application to Numerical Cases

- The teacher should guide the learner on application of binomial expansion to numerical cases, as in examples 5, 6 and 7.
- The learner to do exercise 12.2.

Evaluation

- The teacher is advised to give a detailed test covering all aspects of this topic.

Answers

Exercise 12.1

1. (a) $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
   (b) $a^8 + 8a^7b + 28a^6b^2 + 56a^5b^3 + 70a^4b^4 + 56a^3b^5 + 28a^2b^6 + 8ab^7 + b^8$
   (c) $p^7 + 7p^6q + 21p^5q^2 + 35p^4q^3 + 35p^3q^4 + 21p^2q^5 + 7pq^6 + q^7$
   (d) $y^{10} + 10y^9z + 45y^8z^2 + 120y^7z^3 + 210y^6z^4 + 252y^5z^5 + 210y^4z^6 + 120y^3z^7 + 45y^2z^8 + 10yz^9 + z^{10}$
2. (a) $x^3 + 12x^2y + 48xy^2 + 64y^3$
   (b) $1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$
   (c) $16x^4 + 32x^3 + 24x^2 + 8x + 1$
   (d) $32z^5 + 240z^4y + 720z^3y^2 + 1\,080z^2y^3 + 810zy^4 + 243y^5$
3. (a) $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$
   (b) $b^6 - 6b^5c + 15b^4c^2 - 20b^3c^3 + 15b^2c^4 - 6bc^5 + c^6$
   (c) $m^9 - 9m^8n + 36m^7n^2 - 84m^6n^3 + 126m^5n^4 - 126m^4n^5 + 84m^3n^6 - 36m^2n^7 + 9mn^8 - n^9$
   (d) $t^{10} - 10t^9k + 45t^8k^2 - 120t^7k^3 + 210t^6k^4 - 252t^5k^5 + 210t^4k^6 - 120t^3k^7 + 45t^2k^8 - 10tk^9 + k^{10}$
4. (a) $x^5 - 15x^4y + 90x^3y^2 - 270x^2y^3 + 405xy^4 - 243y^5$
   (b) $16x^4 - 96x^3 + 216x^2 - 216x + 81$
   (c) $729 + 1\,458x + 1\,215x^2 + 540x^3 + 135x^4 + 18x^5 + x^6$
   (d) $b^5 - 15b^4c + 90b^3c^2 - 270b^2c^3 + 405bc^4 - 243c^6$
   (e) $1 - 4p + 6p^2 - 4p^3 + p^4$
5. (a) $x^3 + \frac{3x^2}{2} + \frac{3x}{4} + \frac{1}{8}$
   (b) $p^4 - q^3p + \frac{3}{8}p^2q^2 - \frac{1}{16}pq^3 + \frac{1}{256}q^4$.
   (c) $q^3 - q^2p + \frac{1}{3}qp^2 - \frac{1}{27}p^3$
   (d) $y^5 + 5y^3 + 10y + \frac{10}{y} + \frac{5}{y^3} + \frac{1}{y^5}$

(e) $81x^4 + 36x^3 + 6x^2 + \frac{4}{9}x + \frac{1}{81}$.

(f) $x^8 - 8x^6 + 28x^4 - 56x^2 + 70 - \frac{56}{x^2} + \frac{28}{x^4} - \frac{8}{x^6} + \frac{1}{x^8}$

(g) $\frac{r^6}{729} + \frac{2}{27}r^4 + \frac{5r^2}{3} + 20 + \frac{135}{r^2} + \frac{486}{r^4} + \frac{729}{r^6}$

6. (a) $16 + 8\sqrt{5}$

(b) $526 + 256\sqrt{5}$

(c) $217 + 104\sqrt{3}$

(d) $44\ 762 - 19\ 707\sqrt{7}$

(e) $108\frac{1}{4} - 57\sqrt{2}$

(f) $104\ 760 + 42\ 768\sqrt{6}$

7. (a) $1 + 2x + 3x^2 + 2x^3 + x^4$
(b) $1 - 3x - 6x^2 - 7x^3 - 6x^4 - 3x^5 - x^6$

8. $32 + 48x + 2x^2$

9. 24

Exercise 12.2

1. (a) 1.1041 (b) 1.0090 (c) 0.8853
(d) 0.9412 (e) 31.2080 (f) 0.8493

2. (a) 1.0511 (b) 1.0937 (c) 0.9227
(d) 0.9940

3. (a) 650.4 (b) 77.81 (c) 60.25

4. $1 + x + \frac{3}{8}x^2 + \frac{1}{16}x^3 + \frac{1}{256}x^4$; 1.104

5. $1 + \frac{8}{x} + \frac{24}{x^2} + \frac{32}{x^3} + \frac{16}{x^4}$; 1.082

6. $a^5 - 5a^4b + 10a^3b^2 - 10a^2b^3 + 5ab^4 - b^5$; 0.815

7. $32 - 16x + \frac{16}{5}x^2 - \frac{8}{25}x^3 + \frac{2}{125}x^4 - \frac{x^5}{3125}$; 28.926

8. $64 + 576x + 2\,160x^2 + 4\,320x^3 + 4\,860x^4$; 48.5474

Chapter Thirteen

PROBABILITY

The concept of probability is new to the learner. The learner's knowledge in numbers, ratios and fractions will however be useful in this topic.

Objectives

By the end of the topic, the learner should be able to:

(i) define probability.
(ii) determine probability from experiments and real life situations.
(iii) construct a probability space.
(iv) determine theoretical probability.
(v) differentiate between discrete and continuous probability
(vi) differentiate mutually exclusive and independent events.
(vii) state and apply laws of probability.
(viii) use a tree diagram to determine probabilities.

Time: Twenty two lessons.

Teaching/Learning Activities

Probability

- The teacher should lead the learner through the definition of the term probability as in the students' book.

Experimental Probability

- The teacher should discuss experimental probability as highlighted in the students' book.
- The learner should be led through examples 1 and 2.
- The learner to do exercise 13.1.

Range of Probability Measure

- The learner should be led through the range of probability measure, as in the students' book.

Probability space and Theoretical Probability

- The teacher should discuss probability space, as in the students' book.

- The learner should be guided in the construction of the probability space and calculation of probability, as in the students' book.
- The teacher should lead the learner through examples 3, 4, 5 and 6.
- The learner to do exercise 13.2.

Combined Events

- The teacher should discuss mutually exclusive events, as in the students' book.
- The teacher should lead the learner through example 7.
- The teacher should discuss the independent events and guide the learner through examples 8, 9 and 10.
- The learner should be led through examples 11, 12 and 13.
- The learner to do exercise 13.3.

Tree Diagrams

- The teacher should guide the learner in presenting the probability space using a tree diagram, as in the students' book.
- The learner should be guided through examples 14 and 15.
- The learner to do exercise 13.4

Additional Hints

- The teacher is advised to give a practical approach to probability by, for example, availing packs of playing cards, dice, coins, marbles and beads.

Evaluation

- The teacher is advised to give a detailed written test covering all the aspects of the topic.

Answers

Exercise 13.1

1. Check the students' results.
2. Check the students' results.
3. $\frac{481}{500}$
4. Check the students' results.
5. 0.6
6. (a) $\frac{3}{20}$ (b) $\frac{13}{20}$

7. Check the learner's results.

8. (a) $\frac{1}{5}$ (b) 10 (c) $\frac{11}{46}$ (d) $\frac{88}{365}$ (for non leap year)

$\frac{88}{366}$ (for leap year)

9. (a) $\frac{9}{17}$ (b) $\frac{8}{17}$

10. (a) 58 (b) 18 (c) $\frac{9}{29}$

Exercise 13.2

1. $\frac{1}{7}$ 2. $\frac{5}{26}$ 3. $\frac{1}{4}$ 4. $\frac{1}{2}$

5. (a) $\frac{1}{18}$ (b) $\frac{1}{9}$ (c) $\frac{5}{35}$

6. (a) TTT, THT, THH, HHH (b) (i) $\frac{1}{8}$ (ii) $\frac{3}{8}$

HTH, HTT, TTH, HHT

7. $\frac{3}{10}$ 8. (a) $\frac{4}{9}$ (b) $\frac{5}{9}$ 9. $\frac{1}{4}$

10. (a) $\frac{1}{12}$ (b) $\frac{1}{3}$

11. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{13}$ (d) $\frac{1}{52}$ 12. (a) $\frac{5}{7}$ (b) 49

13. $\frac{1}{8}$ 14. (a) $\frac{1}{5}$ (b) $\frac{1}{2}$

15. (a) $\frac{2\pi - 3\sqrt{3}}{12\pi}$ (b) $\frac{10\pi + 3\sqrt{3}}{12\pi}$

16. $\frac{8\sqrt{3} - \pi}{8\sqrt{3}}$ 17. (a) $\frac{1}{4}$ (b) $\frac{3}{4}$

Exercise 13.3

1. $\frac{3}{8}$ 2. $\frac{1}{81}$ 3. (a) $\frac{14}{15}$ (b) $\frac{1}{10}$ (c) $\frac{3}{5}$

4. (a) $\frac{3}{10}$ (b) $\frac{1}{10}$ (c) $\frac{2}{5}$ (d) $\frac{3}{5}$ (e) $\frac{9}{10}$

5. (a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{3}{20}$

6. (a) $\frac{3}{4}$ (b) $\frac{7}{16}$ 7. (a) $\frac{1}{15}$ (b) $\frac{4}{15}$

8. (a) $\frac{1}{10}$ (b) $\frac{3}{5}$ 9. $\frac{1}{35}$ 10. (a) $\frac{4}{35}$ (b) $\frac{4}{35}$

11. $\frac{253}{580}$ 12. (a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{1}{2}$

13. (a) $\frac{9}{12}$ (b) $\frac{7}{10}$ 14. 0.0864

Exercise 13.4

1. (a) $\frac{1}{6}$ (b) $\frac{1}{4}$ (c) $\frac{5}{8}$

2. (a) $\frac{27}{125}$ (b) $\frac{54}{125}$ (c) $\frac{98}{125}$ (d) $\frac{81}{125}$

3. $\frac{7}{16}$ 4. (a) 14 (b) $\frac{1}{24}$ 5. $\frac{11}{36}$

6. (a) $\frac{2}{9}$ (b) $\frac{7}{27}$ (c) $\frac{7}{27}$

7. (a) 0.328 (b) $\frac{729}{10\,000}$ (c) $\frac{107}{12\,500}$

8. (a) $\frac{15}{16}$ (b) $\frac{1}{32}$ (c) $\frac{5}{16}$

9. (a) $\frac{3}{14}$ (b) $\frac{1}{4}$

10. (a) 0.098 (b) 0.988
(c) 0.368 (d) 0.110

11. Check for accuracy of diagram.

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$

12. (a) (i) $\frac{6}{13}$ (ii) $\frac{7}{13}$ (b) $\frac{16}{169}$ (c) $\frac{60}{169}$

13. (a) $\frac{47}{135}$ (b) $\frac{43}{135}$ (c) $\frac{1}{3}$

14. (a) 0.3 (b) 0.31

Chapter Fourteen

COMPOUND PROPORTIONS AND RATES OF WORK

The learner has dealt with rates, ratio and proportion in book one. In this topic, the learner will be introduced to compound proportion and rates of work.

Objectives

By the end of the topic, the learner should be able to:

(i) solve problems involving compound proportions using unitary and ratio methods.
(ii) apply ratios and proportions to real life situations.
(iii) solve problems involving rates of work.

Time: Nine lessons.

Teaching/Learning Activities

Compound Proportion

- The teacher should introduce proportion to the learner, as covered in the students' book.
- The learner should be led through example 1.
- Using examples such as in the students' book, the teacher should discuss continued proportions.
- The teacher should guide the learner through examples 2, 3 and 4.
- The learner to do exercise 14.1.

Proportion Parts

- The teacher should introduce proportional parts as discussed in the students' book.
- The teacher should guide the learner through example 5.
- The learner to do exercise 14.2.

Rates of Work and Mixtures

- The teacher should guide the learner through examples 6, 7, 8 and 9.
- The learner to exercise 14.3.

Additional Hint

The concept of ratio may be used to solve questions on rates of work consider example 6;

$$\text{Time taken} = 8 \times \frac{20}{25} \times \frac{54}{36} \text{ h};$$

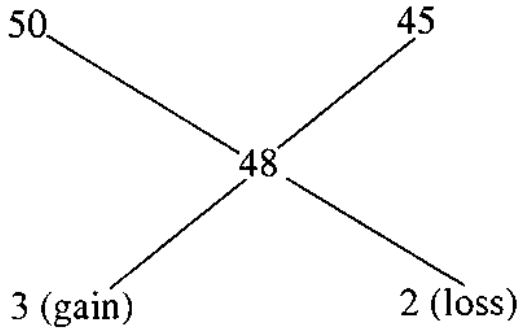
since the time taken is directly proportional to the length of pipe and also inversely proportional to the number of men. Thus, time decreases in the ratio 20 : 25 due to increase in number of men. Also, time increases in the ratio 54 : 36 since length of pipe increases.

Consider also example 9;

This can also be solved by balancing the gain and loss. Thus, the grade costing sh. 45 gains sh. 3 and the grade costing sh. 50 losses sh. 2. To balance the two, we get L.C.M. of 2 and 3, i.e., 6.

$3 \times \boxed{2} = 2 \times \boxed{3}$

So, the two are mixed in the ratio of terms inside the box, i.e., 2 : 3.



Ratio of mixture is 2 : 3

Further Question

1. It takes m tractors n days to plough h hectares. How many more tractors will be required to plough a field which is 20% larger and take 10% less days?

Answers

Exercise 14.1

1. (a) 3, 5, 6, **10** (b) 6, 11, 14, $\frac{\mathbf{77}}{\mathbf{3}}$

   (c) 1.5, 2, 2.5, $\frac{\mathbf{10}}{\mathbf{3}}$ (d) 25, 64, 81, **207.4**

(e) p^2, pq, pr, **qr**

2. (a) $2\sqrt{5}$ (b) $\frac{\sqrt{10}}{2}$ (c) 64 (d) $2\frac{1}{4}$

   (e) $\frac{y^2}{x}$

3. (a) ± 12 (b) ± 3 (c) $\pm \sqrt{10}$
   (d) $\pm ab^2$ (e) ± 1
4. 8 : 3
5. Check for correct verification.
6. 8 : 1
7. $\frac{3}{4}$
8. 4 : 1; $\frac{8}{3}$

Exercise 14.2

1. (a) sh. 375, sh. 675
   (b) 34 kg, 102 kg, 170 kg
   (c) 160 ha, 200 ha, 240 ha, 280 ha
   (d) 1 761 *l*, 661 *l*, 2 971 *l*
2. 72°, 120°, 168°
3. 30 km, 52.5 km, 75 km
4. sh. 75 600, sh. 120 960, sh. 171 360, sh. 226 800
5. 46.25 kg
6. Limo pays sh. 2 160, Koech sh. 8 640
7. 5 months.
8. 1 litre.
9. sh. 501702.10, sh. 418085.10, sh. 390212.80
   Otieno Ouma Makotsi

Exercise 14.3

1. sh. 30
2. $2\frac{6}{11}$

3. sh. 551.25
4. 17 doz costing sh. 184 per doz, 10 doz costing sh. 173.20
5. $10\frac{2}{3}$ days
6. 8.4 days
7. $4\frac{1}{5}$ days
8. 3 : 2
9. C
10. sh. 10 611
11. $3\frac{3}{7}$ min
12. 2 : 3
13. $7\frac{1}{2}$ min
14. 10 more men
15. $7\frac{1}{2}$ min
16. 1: 0.0181
17. Loss 5.41%
18. Akeyo 120 min
    Cheruto 90 min
    Mueni 72 min
19. sh. 400
20. 20 cms^{-1}
21. 6 days
22. 1 : 3

Further Question

1. $\frac{1}{3}$ m more tractors.

Chapter Fifteen

GRAPHICAL METHODS

The learner has met graphs in books one, two and book three chapters 1 and 3. In this topic, the drawing of graphs shall be extended to other functions.

Objectives

By the end of the topic, the learner should be able to:

(i) make a table of values from given relations.
(ii) use tables of values to draw graphs of relations.
(iii) determine and interpret instantaneous rates of change from a graph.
(iv) interpret information from graphs.
(v) draw and interpret graphs from empirical data.
(vi) solve cubic equations graphically.
(vii) draw the line of best fit.
(viii) identify the equation of a circle.
(ix) find the equation of a circle, given the centre and the radius.
(x) determine the centre and radius of a circle and draw the circle on a cartesian plane.

Time: Twenty one lessons.

Teaching/Learning Activities

Tables and Graphs of Given Relations

- The teacher should lead the learner in revision of graphs of linear equations.
- The learner should be guided through examples 1 and 2.
- The learner to do exercise 15.1.

Graphical Solution of Cubic Equations

- The teacher should discuss cubic functions/equations, as in the students' book.
- The teacher to guide the learner through example 3.
- The learner should do exercise 15.2.

Average Rate of Change

- The teacher should discuss average rate of change, as in the students' book.
- The learner to do exercise 15.3.

Rate of Change at an Instant

- The teacher to introduce rate of change at an instant, as in the students' book.
- The learner should be guided through example 4.
- The learner to do exercise 15.4.

Empirical Graphs

- The teacher should introduce empirical graphs, as illustrated in the students' book.
- The learner to do exercise 15.5.

Reduction of Non-Linear Laws to Linear Form

- The teacher should discuss reduction of non-linear laws to linear form, as in examples 5 and 6.
- The learner to do exercise 15.6.

Equation of a Circle

- The teacher should discuss equation of a circle using illustration in the students' book.
- The learner should be guided through examples 7, 8, 9 and 10.
- The learner to do exercise 15.7.

Evaluation

- The learner should be given a test on compound proportion, rates of work and graphical methods.

Further Question

Two variables t and r are believed to obey a law of the form;

$$t = \frac{a}{\sqrt{r}} + b\sqrt{r}$$

The table below shows some values of r against corresponding values of t.

r	1	3	7	9	11
t	15.6	15.5	18.6	20.1	21.6

By plotting a suitable straight line graph on a square grid, determine the values of a and b.

Answers

Exercise 15.1

1.

x 0	1	2	3	4	5
2^x 1	**2**	**4**	8	**16**	32
y 2	**3**	**5**	**9**	**17**	33

2. Check for correct graph.
   x = 0.5, y = –0.29
   x = 2.5, y = 1.83
3. Check for correct graph.
   When (a) x = 1.5, y = 6.
   (b) y = 5.4 x = 2.1
4. Check for correct graph.
5. Check for correct graph.
6. Check for correct graph.

Exercise 15.2

1. $y = 5x^3 + 2x^2 - 5x - 2$

x	–2	–1	0	1	2
–5x	10	5	0	–5	–10
$2x^2$	8	**2**	0	**2**	**8**
$5x^3$	**– 40**	**–5**	0	**5**	40
y	–24	**0**	**–2**	**0**	**36**

   Check for correct graph.
   x = –1, –0.4, 1
2. Check for correct graph.
   x = –2.6, 0.8, 3.8
3. Check for correct graph.
   x = –0.6, 0, 0.6
4. Check for correct graph.
   x = 0.4

5.

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5
$2x^3$	–16	– 6.75	–2	–0.25	0	0.25	2	6.75	16	31.25
$-5x^2$	–20	–11.25	–5	–1.25	0	–1.25	–5	–11.25	–20	–31.25
+5	+5	+5	5	5	5	5	5	5	5	5
$y = 2x^3 - 5x^2 + 5$	–31	–13	–2	3.5	5	4	2	0.5	1	5

x = –0.5, x = 1.3, x = 1.6

Exercise 15.3

1. (a) 4 m/s^2 (b) 8 m/s^2 (c) – 4 m/s^2
2. Check for correct graph (a) 6 °C/s (b) 8 °C/s
3. (a) Check for correct graph.
   (b) (i) 78 500 (ii) 875 persons per year, 2 250 persons per year.
4. (a) 0.15 m/day (b) 0.09 m/day (c) 0.52 m/day
   (d) 0.075 m/day
5. (a) 875 (b) 875 (c) 13th and 15th day.
   (d)

	Average rate of decay
Day 7 to day 9	650
Day 8 to day 10	500
Day 9 to day 11	400
Day 10 to day 12	350
Day 11 to day 13	300
Day 12 to day 14	200
Day 13 to day 15	0

Exercise 15.4

1. (a) –2 (b) 0 (c) –2
2. (a) –0.35 (b) –1.03
3. (a) 38 cm/s (b) 50 cm/s
4. (a) 13.2 cm^3/s (b) 30.5 cm^3/s (c) 18.6 cm^3/s
5. (a) 14.5 (b) 27.6
6. (a) 1.8 m/s^2 (b) –1.5 m/s^2 (c) 0 m/s^2
7. (a) 20 (b) 8.8
8. (a) 0.13 m/s (b) t = 15 object has reached maximum height and therefore it is momentarily at rest.

9. (a) $\frac{-4}{3}$ (b) $y = -\frac{4}{3}x + 8$

Exercise 15.5

1. Density $= 0.91$ g/cm^3, $m = 0.91V$
2. $y = 2x + 5$
3. $y = 0.015x - 0.36$
4. $y = 14.2x + 50$
   (a) Volume of each marble.
   (b) Initial reading of the amount of water in the cylinder.
5. Check for a correctly drawn line of best fit; $\pi = 3.1$
6. Check for correctly drawn line of the best fit.
   (a) Charge per call is sh. 2.00.
   (b) Standing charge is sh. 70.00.
   $S = 2n + 70$

Exercise 15.6

1. (a) V against r^3 or log V against log r.

   (b) P against $\frac{1}{V}$ or log P against + log V.

   (c) S against $\frac{1}{t^2}$ or log S against + log t.

   (d) r against t^3

   (e) A against t^2

2. (a) $y = \frac{k}{x} + r$ y against $\frac{1}{x}$

   (b) $V = c + \frac{d}{u}$ v against $\frac{1}{y}$

   (c) $\frac{1}{s} = \frac{1}{t} + k$ $\frac{1}{s}$ against $\frac{1}{t}$

   (d) $p^2 = \frac{c}{q} + m$, p^2 against $\frac{1}{q}$

(c) $y = mx^2 + n$, y against x^2

3. R against $\frac{1}{v}$

(a) $R = \frac{a}{v} + b$

(b) Check for the correct straight line graph.

(c) $a = 4$, $b = 1$

4 (a) Check for the correct graph.

(b) $P = -0.62$, $B = 2.93$

5. (i) $\log P = \log A + n \log I$

Check for correct graph

$A = 3$, $n = 2$

6. $\log y = \log a + n \log x$

Check for correct graph.

$a = 3.1$, $n = 4$

7. F against $\frac{1}{E}$

Check for correct graph.

$a = 4.9$, $b = 2$.

8. $\log I = \log a + n \log V$

(a) Check for correct graph.

$a = 0.01$, $b = 3.3$

(b) (i) 4 000 lux (ii) 45 volts

9. $\log V = \log A + t \log K$.

Check for correct graph

(a) $t = 0.8$, $V = 65.0$; correct value is 56.2.

(b) $A = 35.5$

(c) $k = 1.68$

10. $\log y = (b + x) \log A = b \log A + x \log A$

(a) Check for correct graph,

$A = 3$, $b = 0.03$

(b) (i) $x = 4.1$ when $y = 24$ (ii) 67

11. $y = \frac{y}{p + x} \Rightarrow \frac{1}{y} = \frac{p + x}{q} = \frac{p}{q} + \frac{x}{q}$

(a) Check for correct graph.

(b) $P = q = 8$

(c) (i) $y = 0.36$ (ii) $x = 9$

12. $\log I = \log k + n\log V$

Check for correct graph.

$k = 1.67, n = 3.59$

13. Check for correct graph.

$A = 0.08, B = 3$

Exercise 15.7

1. (a) $x^2 + y^2 - 4 = 0$

(b) $x^2 + y^2 - 5 = 0$

(c) $x^2 - 8x + y^2 + 2y + 8 = 0$

(d) $x^2 + 2x + y^2 + 6y - 6 = 0$

(e) $x^2 + 2ax + y^2 - 2y + a^2 + 1 - c^2 = 0$

2. (a) $x^2 - 4x + y^2 - 5 = 0$

(b) $x^2 - 4x + y^2 - 10y + 4 = 0$

(c) $x^2 + 4x + y^2 - 2y - 8 = 0$

(d) $x^2 + 6x + y^2 + 4y - 21 = 0$

3. (a) $x^2 - 8x + y^2 - 6y - 15 = 0$

(b) $x^2 - 4x + y^2 - 11y + 28 = 0$

(c) $x^2 - 12x + y^2 + 14y + 60 = 0$

4. Centre $(-4, 1)$, radius $3\sqrt{2}$ units.

5. $(x - a)^2 + (y - b)^2 = r^2$

Here, $a = -3, b = 3, r = 5$.

$(x + 3)^2 + (y - 3)^2 = 5^2$

$x^2 + 6x + 9 + y^2 - 6y + 9 = 25$

$x^2 + 6x + y^2 - 6y + 18 = 25$

$\therefore\ x^2 + y^2 + 6x - 6y = 7$

6. Centre $(-2, 0)$, radius 3 units

7. (a) $x^2 - 6x + 9 = (x - 3)^2$

(b) $y^2 + 14y + 49 = (x + 7)^2$

Centre $(3, -7)$, radius 3 units.

8. (a) Centre $(-5, -9)$, radius 12 units

(b) Centre $(8, -12)$, radius 9 units

(c) Centre $(7.5, 3.5)$, radius 7 units

(d) Centre $(-7, 13)$, radius 16 units

Mixed Exercise 3

1. (a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $\frac{5}{18}$

   (d) $\frac{1}{13}$ (e) $\frac{11}{16}$ (f) $\frac{23}{42}$

2. (a) $\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{6}$

   (d) $\frac{5}{12}$ (e) $\frac{5}{12}$

3. $a = 1.4, b = 2.8$

4. (a) Metal A 36.4 kg
   Metal B 36.4 kg
   Metal C 27.2 kg

   (b) Metal A $\frac{4}{11}(x^2 + 5x - 3)$

   Metal B $\frac{4}{11}(x^2 + 5x - 3)$

   Metal C $\frac{3}{11}(x^2 + 5x - 3)$

   (c) Metal A $\frac{4}{11}(x^3 + y^3)$

   Metal B $\frac{4}{11}(x^3 + y^3)$

Metal C $\frac{3}{11}(x^3 + y^3)$

(d) $\frac{96}{121}\left(x^3+y^3\right)^2 - \frac{9}{11}\left(x^3+y^3\right)$

5. $\frac{2}{3}$

6. (a) 0.04 (b) 0.32
7. sh. 800
8. 1294 . 274304

9. (a) $\frac{7}{44}$ (b) $\frac{1}{22}$

10. 1 : 3
11. 1 : 2
12. a = 2, n = 1.96

13. $\frac{1}{84}$

14. $a = \frac{1}{3}$, c = 1.3; $x \simeq 24$
15. –108 864

16. $1 + \frac{1}{2}x + \frac{5}{48}x^2 + \frac{5}{432}x^3 + \ldots$; 3.75
17. – 4 320
18. (a) $1\,024x^5 - 1\,280\,x^4 y + 640\,x^3\ y^2 - 160x^2\ y^3 + 20\,xy^4 - y^5$

(b) $1+\frac{3}{2}x+\frac{15}{16}x^2+\frac{5}{16}x^3+\frac{15}{256}x^4+\frac{3}{512}x^5+\frac{1}{4\,096}x^6$

19. 3 : 2
20. (i) 33.44 h
(ii) 50.66 h
(a) 7 095 778 (b) 4 688 134 (c) 343 5579

21. (a) $\frac{1}{13}$ (b) $\frac{1}{4}$ (c) $\frac{1}{52}$ (d) $\frac{4}{13}$

22. k = 1.2, n = 0.9
Check for accuracy of graph.

Revision Exercise 1

1. A'(2, 2) B' (–1, 2) C' (–1, 0)
2. (4, 4) and (4, –8)

3 Check for accuracy of graph.

4. $b = \frac{-y}{x}$ 5. 60°

6. (b) 12 7. x = 3, y = $\frac{1}{3}$

8. (a) –3 or $\frac{1}{2}$ (b) $\frac{-1}{3}$ or 1 (c) $\frac{-2}{3}$ or $\frac{-1}{2}$

9. (a) $y = \frac{5}{4}x$ (b) $q = \frac{11}{16}x$

10. 54.91 cm 11. 9 : -1

12. 2

13. (a) $\frac{56}{65}$ (b) $\frac{-33}{65}$ (c) $\frac{16}{5}$ (d) 1

14. (a) (–5, 6) (b) (3, –5)
(c) (–7, –3) (d) (–9, 5)

Revision Exercise 2

1. 5.7 cm
2. (a) x = –1 or 0 or 3

(b) $x = \frac{-1}{2}$ or $\frac{2}{3}$

3. 2 days
4. $x = 1, y = -2$
5. 13 units
6. (a) P'(–3, –3), Q' (–6, –3), R' (–4.5, –6)
   (b) P' (–3, –3), Q' (–6, –3), R' (–4.5, –6)
   (c) Area A = 40.5 sq. units.
7. Check for correct representation.
8. $x = \frac{-(3y + 13)}{15y}$
9. 3 cm
10. (a) $90.25 \leq A \leq 110.25$ $38 \leq p \leq 42$
    (b) $39.08 \text{ cm}^2 \leq A \leq 47.74 \text{ cm}^2$ $28.5 \leq p \leq 31.5$
11. (7, 4) , (7, 3)
    (a) (10, 2) (b) (13, 1) (c) (16, –1)
12. (a) 11
    (b) (i) 10 axes of order 2 (ii) 1 axis of order 10.
    (c) 10
13. 1 : 3
14. (a) Check for a correctly drawn graph.
    $x = 0$ or –2.2 or 3.2
    (i) The graph has a minimum value at (–2, –3)
    (ii) $y = -3$, $y = -1$ and $y = 3$
    (b) (i) (1.5, 10), (2.6, 10), (–5.8, 10)
    (ii) (0.3, 0.4), (– 0.5, –1) , (1.9, 2), (–2.5, –2.1) (–2.5, 3)
    (c) $x = -2$
15. (a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$

Revision Exercise 3

1. (a) $C = \left(\frac{1}{9}(9V^2r^2 - r^3)^2 - 1\right)$

(b) $r = 4kn^2x^2$

2. Minimum 9.9 mm, Maximum 10.9mm.
3. 1.032 km
4. 211 weeks
5. P (3.1, 1.3)
6. 15 cm^2
7. $x = 2$ or $-\frac{1}{2}$
8. $x = 2, y = \pm 1$ For $x = -3$, y has no real values since $y^2 = -4$
9. $x = 9$
10. (a) (i) $\frac{1}{2}$ (ii) $\frac{4}{9}$

    (b) (i) $\frac{5}{12}$ (ii) $\frac{4}{9}$
11. 2 days
12. $\frac{1}{12}, \frac{1}{12}$
13. (a) $l = \sqrt{c^2 - (x - y)^2}$ (b) $l = \sqrt{c^2 - (x - y)^2}$
14. (a) $\sqrt{2} : 1$ (b) 138.9%
15. Check for correct graph.
    (a) 105°, 165°, 285°, 345°
    (b) 45°, 225°
    (c) 13.5°, 103.5°, 193.5°, 283.5°
16. (a) $\log A = \log P + n \log (1 + r)$
    (b) (i) sh. 40364.50
        (ii) sh. 65993.30
        (iii) sh. 75945.15
        (iv) sh. 100577.30
        (v) sh. 26485.00

Revision Exercise 4

1. (a) $P'(-3, -1)$ $Q'(8, -1)$ $S'(-5, -3)$
2. (a) $P'(8, 0)$, $Q'(3, -3)$, $R'(1, 0)$ $S'(3, 3)$
   (b) $p'(0, -8)$ $Q''(-3, -3)$ $R''(0, -1)$ $S''(3, -3)$
3. (a) $P''(-3, 8)$ (b) $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$
4. (a) sh. 393.50 (b) sh. 3 624
   (c) sh. 3 040 (d) sh. 1 520
5. (a) 80 cm (b) 300 cm^3 6. 53.1° or 306.9°
7. $y = -2x^2 - x - 2$
8. (a) 23.6° or 156.4°
   (b) 194.5 or 345.5°; all $\pm 0.1°$
9. 18 cm 10. P(1, 2) 11. 12.95%, 45 h 30 min
12. 174.8 cm 13. (a) (i) $\frac{3}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{1}{4}$

    (b) (i) $\frac{3}{32}$ (ii) $\frac{9}{256}$
14. (a) 2
    (b) CD under MT : $2y = x - 7$ ED under MT : $y = 2x - 8$
    under QMT: $y = -2x + 7$ under QMT : $2y = -x + 8$
    under TMQ : $y = -2x - 13$ under TMQ : $2y = -x - 14$
    $x \leq -1, y \geq -3, 2y \leq x - 3$
    (d) Reflection in the line $y + x = 0$
15. 144°
16. $P'(3.2, 1.6)$ $Q'(6, 2)$, $R'(6, 2)$
    $P''(0.2, 3.6)$ $Q''(3, 4)$ $R''(4, 7)$

Revision Exercise 5

1. $u = \frac{2xw}{V - x}$

2. $P = \frac{100I}{RT}$

3. AC = 5.34 cm
$\angle BAC = 104.9°$ or $75.9°$
$\angle ACB = 40.1°$ or $69.1°$
Area of triangle = 15.5 cm^2

4. $\frac{1}{36}$

5. $-12\frac{1}{2}$

6. (a) $\begin{pmatrix} -9 & 11 & -1 \\ -6 & 4 & 6 \end{pmatrix}$

(b) Not possible

(c) $\begin{pmatrix} 10 & -12 \\ -10 & 14 \end{pmatrix}$

(d) $\begin{pmatrix} -6 & 10 & -6 \\ -3 & 11 & -15 \\ -9 & 1 & 19 \end{pmatrix}$

7. $\frac{1}{12}$

8. (a) $\begin{pmatrix} \frac{5}{6} & \frac{-7}{6} \\ \frac{-1}{3} & \frac{2}{3} \end{pmatrix}$ (b) $\begin{pmatrix} \frac{3}{2q-3p} & \frac{q}{2q-3p} \\ \frac{-2}{2q-3p} & \frac{-p}{2q-3p} \end{pmatrix}$

9. (a) $x = 8$ or $x = -4.8$
(b) $x = 0.3$ or $x = -4.3$

10. (a) $35x^3y^4$ (b) $-48\,384x^5y^3$ (c) $-\frac{448}{9}x^2y^4$
(d) $512x^5$ (e) $\frac{3}{5}x^2$

11. Least breadth 2.3 cm, greatest value 2.9 cm
12. (a) 70 Ohms (b) $T = 33\frac{1}{3}$ °C
13. 275.4 cm
14. (a) A" (0, 5), B" (2, 3) C" (0, 8)
    (b) A" (6, 3) , B" (4, 5) ,C" (6, 0)
15. $x \geq 0$, $y > 2$, $y \leq 8 - 2x$
16. (a) 56 (b) 2.00 a.m. (c) 105 h

Revision Exercise 6

1. 15 ± 0.5 m
2. (a) $a^8 + 8a^7b + 28a^6b^2 _ 56a^5b^3 + \ldots.$
    (b) $a^7 - 7a^6b + 21a^5b^2 - 35a^4b^3 + \ldots$
    (c) $a^6 + 12a^5b + 60a^4b^2 + 160\, a^3b^3 + \ldots..$
3. 193.2 m
4. $\frac{135}{1024}$
5. (a) Enlargement centre (0, 0), s.f. –2

    (b) Enlargement centre (0, 0), s.f. $\frac{-1}{2}$
6. Check for accuracy of graphs
    Exact answer

    (a) x= –0.33 or –1 (b) x = 0.79 or –2.12 (c) x =1.29 or –1.29
    (d) x = 0 or –2
7. 17.26 units
8. 180
9. Least 548.6 m
    Greatest 585.7 m
10. (a) p = –3 q = 2
    (b) p = 1, q = –2, r = 1
11. E = 0.8 Joules
12. a = 1.875, b = 3.75
13. (a) 2 units (b) 4 units (c) 2.828 units (d) 1.414 units
14. (a) $P = t + \frac{1}{16}t^2$
    (b) P = 96; t = 27.55, – 43.55
15. 5

Revision Exercise 7

1. 17.32 cm 2. $\begin{pmatrix} 1 & \frac{-1}{2} \\ \frac{-5}{4} & \frac{3}{4} \end{pmatrix}$

3. (a) 16, $(x + 4)^2$ (b) $\frac{121}{4}$, $(C - \frac{11}{2})^2$ (c) $40xy, (4x+5y)^2$

   (d) $8p, (p + 4)^2$ (e) $64\ x^4, (8x^2 - 1)^2$

4. ksh. 1 540 5. (a) $x = 2, y = -1$, (b) $p = 10, q = 2$

6. 1 250g

8. (a) $\frac{8}{16575}$ (b) $\frac{48}{16575}$

9. Lower limits (a) 5.5 cm (b) 0.7815 km (c) 1.68 g (d) 7.5 m
   Upper limits (a) 5.7 cm (b) 0.7925 km (c) 1.70 g (d) 7.7 m

10. $\frac{1}{204}$ 11. 75%, $p = \frac{4r}{\sqrt{q}}$

12. 7.348 m 13. 87.5% decrease

14. (a) $n = \frac{8}{25}$, $k = 446.7$

    (b) 839.1 Nm

    (c) 11.2°

Revision Exercise 8

1. $p = -26 + 9q$; $p = 28$
2. $5724.5 \leq \text{vol} \leq 5814.6$

3. (a) $\frac{1}{221}$ (b) $\frac{1}{17}$ (c) $\frac{4}{17}$

4. sh. 391 488

5. (a) (4 2) (b) (14 7)

6. (a) $3\frac{3}{4}$ hrs (b) $4\frac{2}{3}$ hrs

7. a = 5.62, n = 1.5
8. 7.211 cm
9. (a) y = 3x – 16
   (b) 3y = 15 – x
   (c) 3y = 15 – x
10. 14 : 5
11. 10^7 or 10^8
13. (a) Coincidental (b) Parallel

14. root $= -1\frac{1}{4}$ B = $-\frac{27}{2}$

15. (a) $\frac{1}{72}$ (b) $\frac{5}{108}$

Revision Exercise 9

1. $2\sqrt{2} = 2.828$ units
2. Check for correct verification.
3. 93 kg to 95 kg
4. 1.6 h
5. 8.66 cm
6. sh. 297, 539.60 to the nearest cents.
7. x : y = 3: 1
8. t = 15
9. (a) 1.0615 (b) 1.0202 (c) 236.24

10. k = $\frac{34}{7}$, a = $\frac{18}{7}$

11. b = $\sqrt{\frac{16c}{3}}$

12. Negative quarter turn about N (–4, –3)
(–5, –10)
13. a = 3, b = 7.5 ± 0.1
P $\simeq$ 180 V = –9.6 or 6.9
14. (a) Neither. (b) Geometric sequence.
(c) Arithmetic sequence (d) Arithmetic sequence.
(e) Neither.

Revision Exercise 10

1. $P^2 - (r-2)^2$
2. (a) $\sqrt{6}$ (b) $\sqrt{35}$ (c) $\sqrt{41}$
(d) $\sqrt{194}$ (e) $3\sqrt{6} + 2\sqrt{35}$
3. The difference lies between 0 and 2 kg
4. (a) (12, 10) (b) (3, 3) (c) (6, 11)
(d) (10, 12)
5. 4.6 ± 0.1 cm
6. sh. 720
7. Minimum = 15.0675 m^2
Maximum = 15.9375 m^2
8. 2.47 cm or 2.5 cm
9. 2 : 1
10. (a) $-2+\sqrt{3}$ (b) $-4-2\sqrt{2}$ (c) $6-3\sqrt{6}$ (d) $1-\sqrt{2}-\sqrt{3}+\sqrt{6}$
11. sh. 2 296.3
12. $\begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$
13. (a) (1, 3) (b) P"(1, 6) Q"(3, 6) R"(3, 4) S"(1, 4)
14. (a) 8 or –8 (b) 380 or –340
15. 31.0464 litres
16. n = – 0.96, a = 501.2

Revision Exercise 11

1. a = 5, b = 5
2. 437.5 rev
3. $11 \leq \angle \leq 7.6$

4. (a) sh. 22 500
   (b) sh. 25 500
5. sh. 9672.30 (to nearest 10 cents)
6. (–3, –8), (8, 3)

7. (a) $s = \frac{888x}{w^2} + \frac{3}{5}w$

   (b) $s = 25\sqrt[3]{\frac{2220}{61}x}$

8. (a) $\begin{pmatrix} 77 & -64 \\ 16 & 45 \end{pmatrix}$ (b) $\begin{pmatrix} 24 & -2 & -3 \\ 25 & 37 & 22 \end{pmatrix}$

   (c) $\begin{pmatrix} 11 & 23 & 5 \\ 0 & 41 & 7 \end{pmatrix}$ (d) $\begin{pmatrix} 22 & 2 & 12 \\ -44 & 23 & -10 \end{pmatrix}$
9. 1 035 N

10. $\frac{5}{12}$

11. 4
12. (a) 226.1 cm^2
    (b) 82.1 cm^2
13. sh. 211 000
14. 23 yrs 2 months
15. 48.57 km/h
16. Check for correctly drawn graph.
    Log P = log c – m log V
    m = –1.4 and c = 141.3
    Actual law; $PV^{-1.4} = 141.3$ or $P = cV^{1.4}$

Revision Exercise 12

1. $\begin{pmatrix} 2 \\ -3 \\ -4 \end{pmatrix}$, 5.385 units
2. 2.38%
3. (–4, 7), (–4, 8), (–10, 13)

4. 64
5. 12 cm and 9 cm, 54 cm^2
6. $\frac{2}{3}, \frac{-3}{\sqrt{13}}$
7. 9.292 km, 8.748 km, 8.615 km
8. (a) P'(5, 2) Q'(5, 0) R'(7, 2) S'(7, 0)
   (b) P'(2, –1), Q'(2, –3), R'(4, –1), S'(4, –3)
9. 24 h
10. sh. 85
11. 2 seconds and 4 seconds
12. $42\frac{17}{24}$
13. $a^9 - 9a^8x + 36a^7x^2 - 84a^6x^3 + \ldots$; 19 100. 322
14. 1 : 4
15. Between 1 hour 32 minutes and 1 hour 49 minutes.
16. (a) 70 (b) 0.9412