



Addis Ababa Science and Technology University
Department of Mathematics
Mathematics for Natural Science
Worksheet-III

Chapter-3: Functions

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1. The Cartesian product $A \times A$ has 9 elements among which $(-1, 0)$ and $(0, 1)$ are found. Find the set A and the remaining elements of $A \times A$.
2. If the ordered pairs $(x^2 - 3x, y^2 + 4y)$ and $(-2, 5)$ are equal, then find x and y .
3. Let R be a relation on the set $A = \{1, 2, 3, 4, 5, 6\}$ defined by $R = \{(a, b) : a + b \leq 11\}$. Then
 - (a) list the elements of R
 - (b) find R^{-1}
 - (c) find $\text{Dom}(R)$ and $\text{Range}(R)$
 - (d) find $\text{Dom}(R^{-1})$ and $\text{Range}(R^{-1})$
4. Write the domain and range of the following relations. Determine which relations are functions.
 - (a) $R_1 = \{(4, 3), (6, 8), (4, 8), (0, 9), (0, 10)\}$
 - (b) $R_2 = \{(\frac{1}{2}, \frac{1}{6}), (-1, 1), (\frac{1}{3}, \frac{1}{8})\}$
 - (c) $R_3 = \{(3, 7), (3, 11), (4, 9), (5, 11)\}$
 - (d) $R_4 = \{(a, 2), (b, 3), (c, 2), (a, 3), (d, 4), (b, 4)\}$
 - (e) $R_5 = \{(-\sqrt{3}, 3), (-1, 1), (0, 0), (1, 1), (\sqrt{3}, 3)\}$
 - (f) $R_6 = \{(5, 0), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5)\}$
5. Find the domain and range of the following relations
 - (a) $(x, y) : x^2 + y^2 \leq 4$
 - (b) $(x, y) : x + y < 1$
 - (c) $\begin{cases} 2x + y = 1 \\ x + 2y = 1 \end{cases}$
 - (d) $(x, y) : |x| + y \geq 2$
6. Given that
$$f(x) = \begin{cases} \sqrt{x-1}, & x \geq 1 \\ 4, & x < 1. \end{cases} \quad \text{Find}$$
 - (a) $f(0)$
 - (b) $f(3)$
 - (c) $f(a+1)$ in terms of a . (Given that $a \geq 0$)

7. Let $A = \{9, 10, 11, 12, 13, 14, 15, 16, 17\}$, and $f : A \rightarrow \mathbb{N}$ be defined by $f(n) =$ the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f .

8. Find the domain and range of the following functions.

(a) $f(x) = \sqrt{x^2 - 6x + 8}$

(c) $f(x) = \begin{cases} 3x + 4, & -1 \leq x < 2 \\ 1 + x, & 2 \leq x < 5 \end{cases}$

(b) $f(x) = x^2 + 1, x \in \mathbb{W}$

(d) $f(x) = 2 - x, x \in \mathbb{N}$

9. for $y = (x - 1)(x - 2)$, state the range if

(a) the domain is considered to be $\{x | 1 \leq x \leq 2\}$

(a) the domain is considered to be $\{x | (-\infty, 1] \cup [2, \infty)\}$

10. For the following functions form $f \circ g$, $g \circ f$, and state the domain.

(a) $f(x) = 2 - \frac{1}{x} + |x|, g(x) = x^2 - 4$ (b) $f(x) = \sqrt{x}, g(x) = |x|$

11. Let $f(x) = 5x - 3$. Find $g(x)$ so that $(f \circ g)(x) = 2x + 7$.

12. If f is a real function defined by $f(x) = \frac{x-1}{x+1}$. Show that $f(2x) = \frac{3f(x)+1}{f(x)+3}$.

13. Which of the following functions are one-to-one? on to? and one to one correspondence? Explain.

(a) $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 - 2x^2 + 1$

(d) $f : \mathbb{Z} \rightarrow \mathbb{Z}, f(n) = n^3 + 1$

(b) $p : \mathbb{R} \rightarrow \mathbb{R}, p(x) = |1 - 3x|$

(e) $g : \mathbb{R} \rightarrow \mathbb{R}, g(x) = 5^x$

(c) $g : [2, \infty) \rightarrow \mathbb{R}, g(x) = x^3 - 2x^2 + 1$

(f) $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = \sqrt{x^2}$

14. Find the inverse function of $f(x)$. State the domain of the inverse function $f^{-1}(x)$

(a) $f(x) = -10 + 8x$

(d) $f(x) = \frac{4 - \sqrt[3]{4x}}{2}$

(g) $f(x) = \frac{5x+3}{1-2x}$

(b) $f(x) = (x - 2)^5 + 3$

(e) $f(x) = \sqrt[3]{x+1} + 2$

(h) $f(x) = -1 + x^3$

(c) $f(x) = \frac{-2x-2}{x+2}$

(f) $f(x) = \frac{\sqrt[5]{2-x}}{2}$

15. State which of the following define polynomial functions:

(a) $y = x^2 + 2^x$.

(d) $y = \frac{x+i}{2-i}$.

(g) $y = (x + 2)^{100}$.

(b) $y = \frac{4}{x}$.

(e) $f(x) = \sqrt[3]{x+1} + 2$

(h) $y = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$.

(c) $y = \sqrt{x}$.

(f) $y = x - \sqrt{6x^2} + \pi x^0$.

16. State which of the following define rational functions:

- (a) $y = \frac{1}{x}$. (d) $y = 1$. (g) $y = x + x^2$.
 (b) $y = |x|$. (e) $y = \frac{1-3x^2+x}{x^4-2x^5+1}$. (h) $y = \frac{x-1}{\pi+i}$.
 (c) $y = \sqrt{x}$. (f) $y = (-1)^x$. (i) $y = \frac{\sqrt{2}x+1}{x-1}$.

17. Find a polynomial equation of lowest degree which has the given roots:

- (a) 1, 1, 1. (b) 1, 2, 1 + i, 1 - i. (c) $1, -\frac{1}{2} - \frac{i\sqrt{3}}{2}, -\frac{1}{2} + \frac{i\sqrt{3}}{2}$.

18. Use the Remainder Theorem to find:

- (a) $P(1)$ when $P(x) = x^2 - x + 3$. (c) $P(0)$ when $P(x) = 3x^3 + x^2 - 5$.
 (b) $P(i)$ when $P(x) = x^2 + 1$.

19. Given that 3 is a double zero of $P(x) = x^4 - 3x^3 - 19x^2 + 87x = 90$, find all the zeros of $P(x)$.

20. Write the general polynomial $P(x)$ whose only zeros are 1, 2 and 3, with multiplicity 3, 2 and 1 respectively. What is its degree?

21. In the following equations find the rational roots, and, where possible, solve completely.

- (a) $P(x) = x^3 - 2x^2 - 2x + 3 = 0$. (c) $P(x) = 3x^4 + 2x^3 + 2x^2 - x = 0$.
 (b) $P(x) = 2x^3 - 3x^2 - 3x + 2 = 0$.

22. How many zeros does the polynomial $x(x - r_1)(x - r_2)(x - r_3) \cdots (x - r_k)$ have?

23. Find the domain and the real zeros of the given functions and sketch their graphs.

- (a) $f(x) = \frac{3x}{x^2+2x-3}$. (c) $f(x) = \frac{(x+2)(x-3)}{(x+1)^2(x-2)}$.
 (b) $f(x) = \frac{x^3+2x^2-x-2}{x^2+6x+8}$. (d) $f(x) = \frac{(x+2)^2(x-2)}{2(x-1)^2(x-3)}$.

24. For what values of k is $x^2 + 2x + k$ exactly divisible by $x - k$?

25. If the polynomial $P(x) = x^5 + Ax^3 + Ax + 4$ is such that $P(2) = 6$, find $P(-2)$.

26. Sketch the graph of the given function. Identify the domain, range, intercepts, and asymptotes.

- (a) $f(x) = \log_2(x - 3)$. (d) $f(x) = 3\log_2(x + 5)$. (g) $f(x) = 16 - 4^x$.
 (b) $f(x) = -3 + \log_2 x$. (e) $\log_{\frac{1}{3}} x$. (h) $f(x) = 1 - e^x$.
 (c) $f(x) = -\log_x(-3)$. (f) $f(x) = 2^{-x}$. (i) $f(x) = e^{x-3}$.

27. Solve the following equations.

- (a) $16^{3a-2} = \frac{1}{4}$. (d) $\log_2(\log_3(x)) = 1$. (g) $9^{-3x+2} \cdot 3^{-x} = 27^{-2x-1}$.
(b) $\log_2(1 + \sqrt{x}) = 6$. (e) $\frac{1}{2}(10^{x-1})^x + 3 = 53$.
(c) $2^{x^2} = 8$. (f) $e^{2x} - 7e^x + 10 = 0$.
28. Let $f(x) = 3^x$. Show that $\frac{f(x+2)-f(2)}{2} = 4(3^x)$.
29. Find the inverse of (a) $f(x) = e^{3x-1}$ (b) $g(x) = \ln(\frac{x}{x-1} + 2)$, for $x > 1$
30. Sketch the graph of the following functions
(a) $f(\theta) = 2 + \cos \theta$ (b) $f(x) = \sin(x + \frac{\pi}{3})$ (c) $f(x) = \tanh(x)$
31. Show that $\cos(\frac{3}{12}\pi) = -\frac{1}{4}(\sqrt{2} + \sqrt{6})$
32. Given $\sin \alpha = -\frac{4}{5}$ (α in fourth quadrant) and $\tan \beta = -\frac{5}{12}$ (β in second quadrant), find $\sin(\alpha - \beta)$ and $\cos(\alpha - \beta)$.
33. Let $\sin 4\theta = \frac{1}{2}$ and $\tan \theta > 0$. Then find (a) $\sin 2\theta$ (b) $\cos 8\theta$
34. Prove that (a) $\sin(\frac{\pi}{4} - x) = \frac{\cos x - \sin x}{\sqrt{x}}$ (b) $\sin(\frac{\pi}{4} - x) = \frac{\cos x + \sin x}{\sqrt{x}}$
35. Prove the following identities
(a) $\sin x(\cot x + \csc x) = \cos x + 1$ (c) $\sinh(x - y) = \sinh(x) \cosh(y) - \cosh(x) \sinh(y)$
(b) $\sec 2x = \frac{\csc^2 x}{\csc^2 x - 2}$ (d) $\sec^2 x - \csc^2 x = \tan^2 x - \cot^2 x$
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