

MYUNGSUNG MEDICAL COLLEGE(MMC)
Math 100: Mathematics for Natural Sciences
Exercises on Logic and Set Theory

1. Which of the following sentences are propositions? What are the truth values of those that are propositions?

- (a) $5 < 9$ and there are infinitely many prime numbers.
- (b) Nairobi is the capital of Kenya.
- (c) Thank you for your consideration.
- (d) Answer this question.
- (e) $x^2 = y^2$ provided that $x = y$.
- (f) How many courses are you taking this semester?
- (g) $x + 5 = 11$.
- (h) The first human kind lived in Ethiopia.

2. What is the negation of each of the following propositions?

- (a) Today is Friday.
- (b) There is no pollution in Addis Ababa.
- (c) $6 + 4 = 10$.
- (d) The summer in Dire Dawa is hot and sunny.

3. Let p and q be the propositions

p : I bought a lottery ticket this week.

q : I won the million birr jackpot on Monday.

Express each of the following propositions as an English sentence.

- a) $\neg p$
- b) $p \vee q$
- c) $p \Rightarrow q$
- d) $p \wedge q$
- e) $p \Leftrightarrow q$
- f) $\neg p \Rightarrow \neg q$
- g) $\neg p \wedge \neg q$
- h) $\neg p \vee (p \wedge q)$

4. Consider the following propositions

r : Rabbits have been seen in the area.

b : Berries are ripe along the path w : Walking on the path is safe.

Write the following propositions using r , b and w and logical connectives.

- a) Berries are ripe along the path, but rabbits have not been seen in the area.
- b) Rabbits have not been seen in the area, and walking on the path is safe, but berries are ripe along the path.
- c) If berries are ripe along the path, then walking is safe if and only if rabbits have not been seen in the area.
- d) It is not safe to walk along the path, but rabbits have not been seen in the area and the berries along the path are ripe.

- e) For walking on the path to be safe, it is necessary but not sufficient that berries not be ripe along the path and for rabbits not to have been seen in the area.
- c) Walking is not safe on the path whenever rabbits have been seen in the area and berries are ripe along the path.

5. Let p and q be the propositions

p : It is below freezing.

q : It is snowing.

Write the following propositions using p and q and logical connectives.

- a) It is below freezing and snowing.
- b) It is below freezing but not snowing.
- c) It is not below freezing and it is not snowing.
- d) It is either snowing or below freezing (or both).
- e) If it is below freezing, it is also snowing.
- f) It is either below freezing or it is snowing, but it is not snowing if it is below freezing.
- g) That it is below freezing is necessary and sufficient for it to be snowing.

6. Given the following propositions

p : You have the flu. r : You pass the course.

q : You miss the final examination.

Express each of the following propositions as an ordinary English sentence.

- a) $p \Rightarrow q$.
- a) $\neg q \Leftrightarrow r$ b) $\neg(p \wedge q)$ c) $(p \Rightarrow \neg r) \vee (q \Rightarrow \neg r)$.
- c) $p \vee q \vee r$.
- e) $(p \wedge q) \vee (\neg q \wedge r)$.

7. Determine the truth value of p if

- a) $(q \Rightarrow \neg p) \vee r$ is False b) $\neg q$ and $p \Rightarrow q$ are True
- c) $(q \Rightarrow p) \Rightarrow (r \Rightarrow q)$ is False.
- d) $(p \vee \neg q) \Rightarrow (\neg r \vee p)$ is False.

8. If $\neg[\neg r \Rightarrow \neg(p \wedge q)]$ is true, then find the truth value of $[(p \Leftrightarrow r) \vee q] \Leftrightarrow (\neg p \wedge r)$.

9. Prove that

- a) $(p \vee q) \wedge (r \vee p) \wedge (\neg q \vee \neg r \vee p) \equiv p$.
- c) $(p \wedge q) \Rightarrow r \equiv p \Rightarrow (q \Rightarrow r)$ b) $(q \vee r) \Rightarrow p \equiv (q \Rightarrow p) \wedge (r \Rightarrow p)$.

10. For the following propositions, indicate whether it is a tautology, a contradiction, or neither. Use a truth table to decide.

- a) $(p \Rightarrow q) \wedge (q \Rightarrow \neg p) \Rightarrow p$.
- b) $[p \wedge (p \Rightarrow q)] \Rightarrow q$ c) $(p \Rightarrow q) \wedge \neg(\neg q \Rightarrow \neg p)$.
- d) $(\neg q \Rightarrow \neg p) \Rightarrow ((\neg q \Rightarrow p) \Rightarrow q)$.

11. Determine whether the following statements are valid or invalid.

- a) If Addis Ababa is a big city, then Addis Ababa has no tall buildings.
Addis Ababa has tall buildings.
Therefore, Addis Ababa is a big city.
- b) If I do not wake up, then I cannot go to work.
If I cannot go to work, then I will not get paid.
Therefore, if I do not wake up, then I will not get paid.
- c) If I study, then I will not fail Math 100.
If I do not play cards too often, then I will study.
I failed Math 100.
Therefore, I played cards too often.
- d) Today is either Thursday or Friday.
On Thursdays I have to go to a lecture.
Today is not Friday.
Therefore, I have to go to a lecture today.

12. Let $P(x) : x$ is an integer greater than 5 $Q(x) : x$ is a natural number.
Determine the truth value of the following propositions.

- a) $P(3) \wedge Q(-2)$ b) $P(2) \Rightarrow Q(2)$ c) $[P(3) \Rightarrow Q(4)] \vee Q(9)$.
- d) $[P(6) \vee Q(7)] \wedge P(0)$.

13. Let $P(x) : x$ is a prime number $Q(x) : x$ is an even number
 $R(x) : x$ is an odd number. $S(x) : x$ is an integer.
Write a sentence, which corresponds to each of the following:

- a) $S(x) \Rightarrow Q(x) \vee R(x)$.
- b) $R(x) \vee Q(x) \Rightarrow P(x)$
- c) $P(x) \Leftrightarrow Q(x) \vee R(x)$.

14. If $U = \mathbb{R}$, then find the truth value of $(\forall x)(\exists y)(\sqrt{x^2} = 16 \Rightarrow y + x = 10)$.

15. Let $P(x) : x$ is a composite number $Q(x) : x$ is a prime number.
Find the truth values of (i) $(\exists x)[P(x) \Leftrightarrow Q(x)]$ (ii) $(\forall x)[P(x) \Leftrightarrow Q(x)]$.

16. Find the truth values of the following where $U = \mathbb{R}$

- a) $(\exists x)(3x^2 - 2x + 1 = 0)$.
- b) $(\forall x)(x + 2x^2 = x^2 + 4x + 4)$
- c) $(\exists x)(\forall y)(x^2 < y^2)$
- d) $(\forall x)(\exists y)(x^2 - y^2 = (x - y)(x + y))$.
- e) $0(\forall x)[x \neq 0 \Rightarrow (\exists y)(xy = 4)]$.
- f) $(\exists x)(\forall y)(x + y = 2x)$.

17. Write the following sets as an interval or union of intervals. In each case $x \in \mathbb{R}$.

- a) $A = \{x | x \neq 5 \wedge x \neq 8\}$.
- b) $B = \{x | x^2(x^3 - 1) \neq 0\}$.
- c) $C = \{x | x^3 - 2x \neq 0\}$.

d) $D = \{x | (x - 2)(x + 3)(x + 4) = 0 \Rightarrow x \geq 2\}.$

18. Simplify each of the following sets and give the final result as an interval or union of intervals

- a) $([-3, 2] \cap (0, 5))^c.$
 b) $((-3, 5.6] \cap (-5, 2.\bar{4}3)) \cup (0.\bar{3}, 11).$
 c) $(2, 4]^c \cap ((-\infty, 5)) \cup [0, \infty)).$
 d) $(-0.\bar{3}, 0.0\bar{3}) \cup ((-\infty, \frac{1}{30}) \cap [5, \infty)).$

19. Write the following sets in complete listing or partial listing method.

- a) $A = \{x | x \text{ is an integer and } 3 < x \leq 10\} \cap \{x | x \text{ is even integer}\}.$
 b) $B = \{x \in \mathbb{N} | (x - 1)(x - 3) \neq 0 \Leftrightarrow x + 1 = x\}.$
 c) $C = \{x \in \mathbb{N} | x < 4 \Rightarrow x \geq 6\}.$
 d) $C = \{x \in \mathbb{N} | x \geq 3 \Leftrightarrow x < 0\}.$
 e) $D = \{x | x \in \mathbb{N} \wedge (x - 1)(x - 3) \neq 0 \Leftrightarrow x + 1 = x\}.$
 f) $F = \{x \in \mathbb{Z} | x \neq 0 \Leftrightarrow x(x + 1)(x + 2) = 0\}.$
 g) $G = \{x \in C | \frac{2}{x} < 3 \Rightarrow x - 1 \leq 2\}, \text{ where } C = \{1, 2, 3, 4, 5\}.$

20. If $n(A \setminus B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

21. For all subsets A, B and C of some Universal set U , prove or disprove the following statements.

- a) $(A \subseteq B \wedge A \subseteq C) \Rightarrow A \subseteq B \cap C$ b) $A \subseteq B \Leftrightarrow A \cap B = A$
 c) $A^c \setminus B^c = B \setminus A$ d) $A \subseteq B \Rightarrow A \cup (B \setminus A) = B.$

22. Let $U = \{x \in \mathbb{Z} | -12 \leq x \leq 6\}.$

$$A = \{x \in \mathbb{Z} | x = 2n \wedge x = 3m, \text{ for } -4 \leq n \leq 8 \text{ and } -2 \leq m \leq 3\}.$$

$$B = \{0, 1, 3\} \quad C = \{x \in \mathbb{Z} | x = \frac{n}{3}, n \in A\}.$$

Then find

- a) $A \setminus [B \cup C]^c$ b) $(A \setminus B) \setminus (A \cup C)^c$ c) $(A \cap B) \setminus C.$

23. (a) For every natural number n , define $A_n = \{x \in \mathbb{Z} | \frac{-24}{n} \leq 2x + 1 \leq \frac{24}{n}\}.$
 Then find

$$\text{i) } \bigcup_{n=1}^6 A_n \quad \text{ii) } [(\bigcup_{n=1}^4 A_n) \setminus (\bigcap_{n=1}^3 A_n)]^c \quad \text{iii) } \bigcap A_n, n \in \mathbb{N}.$$

(b) For every natural number n , define

$$A_n = \{x \in \mathbb{N} | x = n \Rightarrow x < n\}.$$

Then find

$$\text{i) } \bigcup_{n=1}^{10} A_n \quad \text{ii) } \bigcup A_n, n \in \mathbb{N}.$$

24. In a group of 100 students, 35 like tea, 45 like coffee and 15 like both. How many like either of them and how many like neither?

25. There are 30 students in a class. Among them, 8 students are learning both English and French. A total of 18 students are learning English. If every student is learning at least one language, how many students are learning French in total?

26. Among a group of students, 50 played cricket, 50 played hockey and 40 played volley ball. 15 played both cricket and hockey, 20 played both hockey and volley ball, 15 played cricket and volley ball and 10 played all three. If every student played at least one game, find the number of students who played only cricket, only hockey and only volley ball.