MATH GRADE 12 - Questions from Derivatives

1. Which of the following is the derivative of $f(x) = tanx + 3^x$? (UEE 2014/15)
   A. $-tanx + 3^x lnx$
   B. $sec^2x + 3^x$
   C. $sec^2x + 3^x lnx$
   D. $-csc^2x + 3^x lnx$

2. Let f and g be two functions. Which one of the following statements is true about f or g? (UEE 2014/15)
   A. If $f'(a) = 0$, then the tangent line to the graph of f at $(a, f(a))$ is $y = a$
   B. If f and g are differentiable at a, then f/g is differentiable at a.
   C. The line y=0 is the tangent line to the graph of $f(x) = |x|$ at 0.
   D. If f is differentiable at a, then it continuous at a.

3. What is the derivative of the function $f(x) = \frac{xe^x}{cosx}$ $at\ x = 0$? (UEE 2014/15)
   A. -1
   B. 2
   C. 0
   D. 1

4. If the perimeter of a rectangle is 120m, what are the length l and width w of the rectangle that maximizes the area? (UEE 2014/15)
   A. L=35m and w=25m
   B. L=40m and w=20m
   C. L=50m and w=10m
   D. L=30m and w=30m

5. Suppose f is a continuous function on an interval [a, b] and differentiable on (a, b) with the property f(a)=f(b), which of the following must be true? (UEE 2014/15)
   A. The function has maximum value in (a, b)
   B. The graph of f has horizontal tangent line at (c, f(c)) for some $c\epsilon(a, b)$
   C. The function has zero in (a, b)
   D. The function has minimum value in (a, b)

6. Let f be a continuous function on $\mathbb{R}$ and c be a number in the domain of f. which one of the following must be true about the function? (UEE 2014/15)
   A. If f(c) is a local minimum value of f, then $f'(c) = 0$
   B. An absolute maximum value of f which is attained at a critical number is also a local maximum value of f.
   C. If c is a critical number of f, then f(c) is a local extreme value of f.
   D. If $f''(c) = 0$, then (c, f(c)) is an inflection point of f.

7. Which of the following is the same as the derivative of a function f at point P, where P is on the graph of f? (UEE 2013/14)
   A. The slope of a secant line through P
   B. The continuity of the function at P
   C. The graph of f has no jump at P
   D. The slope of the tangent line through P

8. What is the derivative of $f(x) = 3x^2 + 2\sqrt{x} - 4x$ at x=1? (UEE 2013/14)
   A. -3
   B. -1
   C. 1
   D. 3

9. What are the 2nd and the nth derivative of $f(x) = e^{3x}$ respectively? (UEE 2013/14)
   A. 2f(x) and nf(x)
   B. 4f(x) and 2^nf(x)
   C. 9f(x) and n^2f(x)
   D. 9f(x) and 3^nf(x)

10. If $f'(1) = 4$, $f(1) = 3$ and $g'(3) = 5$, then $(gof)'(1)$ is equal to (UEE 2013/14)
    A. 3
    B. 5
    C. 15
    D. 20

11. Which of the following is true about the function defined by $f(x) = x^3 - 3x^2$ on [-1, 3]? (UEE 2013/14)
    A. Its critical numbers are 0, 2, 3 and its maximum value is 0.
    B. Its critical numbers are 0 and 2 and its maximum value is 15
    C. Its critical numbers are 0 and 2 and its maximum value is -4
    D. It has critical numbers at 0 and 2 and its maximum value is 0

12. What are the length L and width W of a rectangle with perimeter 10,000m that maximizes the area? (UEE 2013/14)
    A. L=25,000m and W=2,500m
    B. L=2,500m and W=3,000m
    C. L=3,000m and W=2,500m
    D. L=5,000m and W=5,000m

13. If the radius r of a sphere is increasing at the rate of 2cm/min, then the rate of change of the volume of the sphere when r=1cm is (UEE 2013/14)
    A. $4\pi cm^3/min$
    B. $6\pi cm^3/min$
    C. $8\pi cm^3/min$
    D. $12\pi cm^3/min$

14. A position of a particle is given by the equation $S(t) = 4t^4 - 4t + 1$ where t is measured in seconds and S in meters. Which of the following is NOT true about the motion of the particle? (UEE 2013/14)
    A. The velocity V at any time t is given by $V(t) = 16t^3 - 4$
    B. The velocity at 2 seconds is given by 124m/s
    C. The particle is at rest at t=0.5 seconds
    D. The acceleration of the particle at 1sec is $48m/s^2$

15. The function $f(x) = (x-1)^{2/3}$ is differentiable on (UEE 2012/13)
    A. $(-\infty, 1) \cup (1, \infty)$
    B. $(-\infty, 1) \cup (-1, \infty)$
    C. $(-\infty, 1)$
    D. $(1, \infty)$

16. Which of the following is an interval at which the function $f(x) = \frac{1}{x-1} - \sqrt{4-x^2}$ is differentiable? (UEE 2012/13)
    A. $[-2, 1) \cup (1, 2]$
    B. $(-\infty, -2) \cup (2, \infty)$
    C. $(-2, 2)$
    D. $(-2, 1) \cup (1, 2)$

17. The second derivative of the function $f(x) = xe^x$ is (UEE 2012/13)
    A. $(x+2)^2 e^{-x}$
    B. $(x-2)e^x$
    C. $(2-x)e^{-x}$
    D. $-xe^{-x}$

18. What is the slope of the line tangent to the graph of $f(x) = x^2 + tanx$ at $(\pi, f(\pi))$? (UEE 2012/13)
    A. $2\pi - 1$
    B. $2\pi + 1$
    C. 2
    D. 2π

19. Let $f(x) = \frac{1}{(1-\sqrt{x})^2}$. Then $f'(4)$ is equal to (UEE 2012/13)
    A. $\frac{1}{2}$
    B. $-\frac{1}{2}$
    C. 1
    D. 2

20. The product of two positive real numbers is 100, such that the sum of two times the first number and eight times the second number is minimum. Which of the following pairs of numbers are the first and the second number respectively ? (UEE 2012/13)
    A. 50 and 2
    B. 1 and 99
    C. 20 and 5
    D. 25 and 4

21. If $f(x) = {ln2}/{x^2}$, then $f'(x)$ is equal to (UEE 2012/13)
    A. $\frac{1}{2}x^2$
    B. $-\frac{2ln2}{x^3}$
    C. $-\frac{2}{x^4}$
    D. $\frac{(1+2lnx)}{x^3}$

22. Let $f(x) = x^2 + 2x + 3$ find a number $c\epsilon(1,3)$ such that $f'(c) = \frac{f(3)-f(1)}{2}$? (UEE 2012/13)
    A. 2
    B. -2
    C. 6
    D. -6

23. The volume of a cube is increasing at a rate of $9cm^3/sec$. How fast is the surface area increasing when the length of an edge is 10cm. (UEE 2012/13)
    A. $6cm^2/sec$
    B. $90cm^2/sec$
    C. $36cm^2/sec$
    D. $3.6cm^2/sec$

24. The maximum profit that a company can make if the profit function is given by $P(x) = 36 + 72x - 18x^2$ is (UEE 2012/13)
    A. 124
    B. 31
    C. 2232
    D. 108

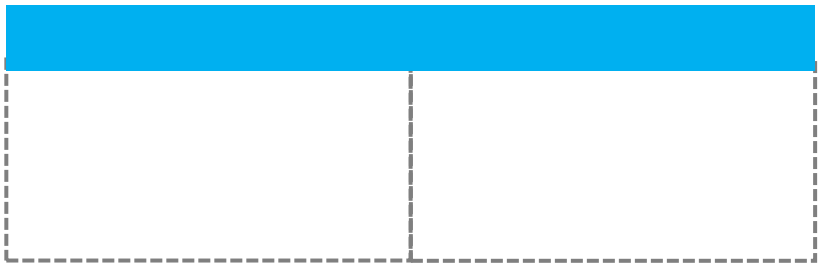
25. If $f(x) = \ln(2^{tanx})$, then what is the value of $f'(0)$? (UEE 2011)
    A. -2ln2
    B. ln2
    C. $\ln\frac{2}{2}$
    D. 1

26. If $h(x) = \sqrt{1+\sqrt{x}}$, then which of the following is equal to $h'(x)$? (UEE 2011)
    A. $\frac{1}{2\sqrt{1+\sqrt{x}}}$
    B. $\frac{1x}{4\sqrt{x+x\sqrt{x}}}$
    C. $\frac{x}{2\sqrt{1+\sqrt{x}}}$
    D. $\frac{1}{4\sqrt{x+x\sqrt{x}}}$

27. At what value of x does the function $f(x) = \frac{4x^3}{3} - x^4$ attains its maximum value? (UEE 2011)
    A. -1
    B. 1
    C. 0
    D. $\frac{4}{3}$

28. A water tank is a rectangular parallelepiped with base length 3m, width 2m and height 2.5m. if water is flowing into the tank at the rate of $0.12m^3/sec$, then how fast does the level of water rise up in the tank? (UEE 2011)
    A. 0.03 m/s
    B. 0.02 m/s
    C. 0.04 m/s
    D. 0.06 m/s

29. A man wants to fence two identical rectangular enclosures in a file alongside a straight river as shown in the following figure.



What is the maximum area of enclosure that he can make with 192 meter fencing material if the side along the river does not need a fence? (UEE 2011)

    A. $1530\ m^2$
    B. $1536\ m^2$
    C. $1664\ m^2$
    D. $1564\ m^2$

30. Let $f(x) = 2e^x - ksinx + 1$. If the equation of the tangent line to the graph of f at $(0, 3)$ is $y = 5x + 3$, then what is the value of k? (UEE 2011)
    A. 3
    B. 2
    C. -5
    D. -3

31. Which of the following is the set of critical numbers of $f(x) = \frac{3}{8}x^{\frac{8}{3}} - 6x^{\frac{2}{3}}$? (UEE 2011)
    A. {-2, 2}
    B. {-2, 0, 2}
    C. {-1, 0, 1}
    D. {-1, 1}

32. What is the slope of the tangent line to the graph of $f(x) = 3e^x + sinx + 2$ at $(0, 5)$? (UEE 2010)
    A. 2
    B. 3
    C. 5
    D. 4

33. If $f(x) = klnx + e^{sinx}$ and $f''(\pi) = -1$, then what is the value of k? (UEE 2010)
    A. $2\pi^2$
    B. π^2
    C. π
    D. 2π

34. Let $f(x) = \ln(x\sqrt{x})$. Then what is $f'(x)$ equal to? (UEE 2010)
    A. $\frac{2x}{3}$
    B. $\frac{\sqrt{x}}{2}$
    C. $\frac{2}{x\sqrt{x}}$
    D. $\frac{3}{2x}$

35. Which of the following is equal to $\frac{d}{dx}\log_2\sqrt{6x}$? (UEE 2010)
    A. $\frac{1}{2xln(2)}$
    B. $\frac{3}{2xln(2)}$
    C. $\frac{3x}{2xln(2)}$
    D. $\frac{1}{6xln(2)}$

36. Which of the following is not true about the function $f(x) = 3x^4 - 4x^3$ (UEE 2010)
    A. $(0, 0)$ is a point of inflection of f. (UEE 2010)
    B. 0 and 1 are critical number of f.
    C. f is concave upward on $(0, {}^2/_3)$ and concave downward on $(-\infty, 0)$ and $({}^2/_3, \infty)$
    D. f is decreasing on $(-\infty, 1)$ and increasing on $(1, \infty)$

37. if $f(x) = \frac{1}{3}x^3 + cx + ax + 5$ has a local minimum value at $x = 1$, then which of the following is true about the possible values of a and c? (UEE 2010)
    A. a=3; c=-2
    B. a=-2c-1, c any real number
    C. a=-2c-1, c<-1
    D. a=-2c-1, c>-1

38. A cylindrical tank whose inner diameter is 2m contains $4\pi m^3$ oil. If the oil is discharged from the tank at the rate of $\frac{2\pi}{3} {}^{m^3}/_{min}$, then how long (in min) does it take for the tank to be empty? (UEE 2010)
    A. ${}^{4}/_{3}$
    B. 4
    C. 12
    D. 6

39. What is the maximum possible area of a rectangle in square units with diagonal of length 16 units? (UEE 2010)
    A. 48
    B. 128
    C. 64
    D. 256

40. If $f(x) = \pi^2 + 1$, then what is the value of $f'(x)$? (UEE 2009)
    A. $2\pi + 1$
    B. 2π
    C. 2
    D. 0

UNIT EXRECISES

41. Compute the average rate of change of
    a. $f(x) = 3x - 2$ over the interval $0 \le x \le 2$
    b. $f(x) = x^3 - 3x$ over the interval $1 \le x \le 6$
    c. $f(x) = 3x - 2$ on the interval $[1, 5]$
    d. $f(x) = x^2 + 3x + 1$ on the interval $[0, k]$

42. In the following table, the average cost in Birr of a gallon of gasoline for the years 2005–2013 is illustrated. Using the data in the table, find the average rate of change of the price of gasoline between 2007 and 2009.

x	2005	2006	2007	2008	2009	2010	2011	2012	2013
$c(x)$	2.31	2.62	2.84	3.30	2.41	2.84	3.58	3.68	4.21

43. A candle has a starting length of 10 cm. Thirty minutes after lighting it, the length became 7 cm. Determine the rate of change in the length of the candle as it burns. Determine how long the candle takes to completely burn out.

44. Evaluate the instantaneous rate of change of $f(x) = 6x^2 - 3$ at $x = 1$.

45. Find the instantaneous rate of change of $f(x) = 2\sqrt{x}$ at $x = x_0 > 0$.

46. The biomass of a bacterial culture in milligrams as a function of time in minutes is given by $f(t)=81t^3+90$. What is the instantaneous rate of growth of the culture when $t=3$ minutes?

47. Find the slope of a line passing through the two given points.

a. $(-3,-5)$ and $(3,11)$ b. $(4,-9)$ and $(5,-7)$

48. For each of the following functions, find the slope of a secant line to the graph of the functions over the given intervals.

a. $f(x)=x^2+1;[-1,3]$ b. $g(x)=x^3-4x+6;\ [1,2]$.

49. Find the gradients of the tangent lines to the graph of the following function at specified points.

a. $f(x)=x^2$ at $x=-1$ and $x=4$. b. $f(x)=3x^2-5x+4$ at $x=2$ and $x=-\sqrt{2}$.

c. $g(x)=\sqrt{x}$ and $x=4$. d. $f(x)=\frac{1}{x}$ at the point $(2,\frac{1}{2})$.

50. If $f(x)=x^3$, find the gradient of the tangent line to the graph of $f(x)$ at the point $P(a,a^3)$.

51. Find the derivative of each of the following at the given value

a. $f(x)=9x-7,\ a=2$ b. $f(x)=x^2-1,\ a=2$

c. $f(x)=x^3+2x,\ a=3$ d. $f(x)=\sqrt{3x-6},\ a=3$

52. Find the derivative of the following functions respect to x.

a. $f(x)=x$ b. $f(x)=x^3$ c. $f(x)=\frac{1}{x}$ d. $f(x)=2\sqrt{x}$

53. Find the derivative of each of the following functions.

a. $f(x)=x^8$ b. $f(x)=x^{27}$

c. $f(x)=2x^6$ d. $f(x)=10x^{15}$

e. $f(x)=4x^{-7}$ f. $f(x)=3x^{-20}$

g. $f(x)=4x^{-\frac{3}{2}}$ h. $f(x)=-10x^{-\frac{2}{5}}$

54. Let $f(x)=x^{-9}$, find

a. $f'(1)$ b. $f'\left(\frac{1}{3}\right)$ c. $f'(k)=k, k\neq 0$.

55. Find the derivative of each of the following functions

a. $f(x) = 5x^4 + \sqrt{x}$ b. $f(x) = x + \frac{1}{x^2}$ c. $f(x) = \frac{1}{6}x^6 - 2x - 4$

d. $f(x) = \sqrt{x} - \frac{1}{\sqrt{x}}$ c. $f(x) = x^{10} + \frac{1}{x} + 6x^{\frac{5}{3}}$ f. $f(x) = 6x^{\frac{4}{3}} - \frac{1}{3x^3} - \frac{1}{2}x^2$

56. Find the derivatives of the following functions, and compute the indicated values.

a. $y = 2\sqrt{x}$, evaluate $\left.\frac{dy}{dx}\right|_{x=1}$ and $\left.\frac{dy}{dx}\right|_{x=4}$.

b. $y = \frac{9}{x^3}$, evaluate $\left.\frac{dy}{dx}\right|_{x=-1}$ and $\left.\frac{dy}{dx}\right|_{x=3}$.

c. $y = x^3 + \frac{1}{2}x^2 - \frac{2}{\sqrt{x}}$, evaluate $\left.\frac{dy}{dx}\right|_{x=1}$ and $\left.\frac{dy}{dx}\right|_{x=4}$.

57. Find $f'(x)$ of the following functions using product rule.

a. $f(x) = (2x+1)(x^2 + x - 5)$ b. $f(x) = (x^2 - 2)(x^3 + 4x)$

c. $f(x) = x(x^2 + \sqrt{x})$ d. $f(x) = (x + 2\sqrt{x})(x^2 - 2)$

58. Find the derivative of each of the following functions.

a. $f(x) = \frac{1}{x^2+1}$ b. $f(x) = \frac{2x^3+3}{x-5}$

c. $f(x) = \frac{x^3+3x}{x^2}$ d. $f(x) = \frac{2x^4}{\sqrt{x}}$

59. Find the derivative of the following functions using the chain rule.

a. $f(x) = (2x+1)^4$ b. $f(x) = (x^3 + 2x)^{25}$

c. $f(x) = \sqrt{x^2 + 3x - 1}$ d. $f(x) = \frac{1}{(3x+2)^2}$

e. $f(x) = \sqrt{x^2 + \frac{1}{x^2}}$ f. $f(x) = \left(\frac{3x-5}{2-7x}\right)^6$

60. Find all critical numbers of the following functions

a. $f(x) = x^3 + 6x^2$ b. $f(x) = \frac{2x^2}{1-x^2}$

c. $f(x) = 2x^3 + 9x^2 - 24x$ d. $f(x) = 8x\sqrt{1-x^2}$

61. Find the interval on which of the following functions are decreasing and increasing

a. $f(x)=x^3+6x^2$ b. $f(x)=2x^2+6x-10$

c. $f(x)=-2x^3+3x^2+12x+6$ d. $f(x)=x-6\sqrt{x-1}$

e. $f(x)=(x+3)^2(x-1)^2$

62. Find the absolute maximum and minimum values of the following functions on a given interval.

a. $f(x)=x^2+2x+3,\ [-2,2]$

b. $f(x)=x^3+3x^2-9x+5,[-2,2]$

c. $f(x)=3x^4-26x^3-60x^2-11$ on $[1,5]$.

d. $f(x)=\dfrac{x^2-2x+4}{x-2},[-3,1]$

63. Find the extreme value of the following functions (if any)

a. $f(x)=2x^3-9x^2-24$ b. $f(x)=\dfrac{1}{4}x^3-3x$ c. $f(x)=\dfrac{x^2+x+1}{x^2-x+1}$

64. If $f(x)=x^3+ax+b$ has a relative minimum value of 4 at $x=2$, find a and b.

65. Find the equation of the line tangent and the normal line to the graph of the following functions at the given point.

a. $f(x)=x^3+x+2;\ (0,2)$ b. $f(x)=(1-x^3)\sqrt{x+2};\ (-1,2)$.

66. Find all points at where the following functions has vertical tangent line.

a. $f(x)=\sqrt{x^2-x}$ b. $f(x)=2\sqrt{x}$

67. If $f(x)=x^2+ax+b$ and $g(x)=x^3-c$ have the same tangent line at $(1,2)$ find the values of the constants a,b and c and the equation of their normal lines.

68. A circular plate of metal is being expanded by heating. The radius of plate increases at the rate of 0.3 cm per second. Find the rate of increase of area when its radius becomes 15 cm.

69. The volume of a sphere is increasing at the rate of 0.2π cm^3/sec. Find the rate of increase of radius when the radius is 10 cm.

70. A square plate is being expanded by heating. If its side is increasing at the rate of 0.2 cm per second, then at what rate the area is increasing when the side is 20 cm?

71. A man 2 m tall walks at a uniform speed of 8 m/minute from a lamp post 6 m high. Find the rate at which the length of his shadow increases.

72. A varying force is applied on a body of unit mass and if the force increases at the rate of 2 N/sec., then find the rate at which the acceleration in the body is changing.

73. Given $C(x)=62x^2+27{,}500$ and $R(x)=x^3-12x^2+40x+10$ where $C(x)$ and $R(x)$ represent, respectively, the total cost and revenue from the production and sale of x items. Find each of the following:
    a. Total profit , $P(x)$.
    b. Total cost, revenue, and profit from the production and sale of 50 units of the product.
    c. The marginal cost, revenue, and profit when 50 units are produced and sold.

74. A rectangular area of 3200 ft^2 is to be fenced off. Two opposite sides will use fencing costing 1 dollar per foot and the remaining sides will use fencing costing 2 dollars per foot. Find the dimensions of the rectangle of the least cost.

75. A manufacturer wants to design an open box that has a square base and a surface area of 48 square unit. What dimensions will produce a box with a maximum volume?

76. A closed cylindrical can having a volume of $128\pi\ cm^3$ is to be formed. Find the dimensions (the radius and height) of the cylinder that will minimize the amount of material to be used?

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