

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Applied Mathematics III (Math 2042)

Exercise 3

1. Find $\lim_{t \rightarrow 0} f(t)$ if it exists.
 - a) $f(t) = \frac{\sin 3t}{2t} \vec{i} + e^{-2t} \vec{j} + \frac{t^2}{e^t} \vec{k}$
 - b) $f(t) = (t^2 - 1) \vec{i} - \frac{1 - \cos t}{3t^2} \vec{j} + \frac{t}{t-1} \vec{k}$
2. Let $\vec{u}(t) = 3t \vec{i} - t \vec{k}$, $\vec{v}(t) = t^2 \vec{i} + 2 \vec{j} - (t+1) \vec{k}$ and $\vec{w}(t) = 3t^2 \vec{i} + t^3 \vec{j} - t \vec{k}$. Find
 - a) $(\vec{u} \cdot \vec{v})'$
 - b) $(\vec{u} \times \vec{v})'$
 - c) $[\vec{v} \times (\vec{u} + \vec{w})]'$
3. Find a parametric representation of
 - a) the curve $(x+2)^2 + y^2 = 9, z = -1$ in counterclockwise direction.
 - b) the ellipse $4x^2 + y^2 = 4$ in the xy -plane in clockwise direction.
4. Find the length of the curve parametrized by
 - a) $\vec{r}(t) = e^t \cos t \vec{i} + e^t \sin t \vec{j} + 5 \vec{k}$ for $0 \leq t \leq 1$.
 - b) $\vec{r}(t) = e^t \vec{i} + e^{-t} \vec{j} + \sqrt{2t} \vec{k}$ for $0 \leq t \leq 1$
5. Find the definite integrals
 - a) $\int_0^\pi \vec{r}(t) dt$ for $\vec{r}(t) = \sin t \vec{i} + \cos t \vec{j} - t \vec{k}$
 - b) $\int_0^1 \vec{r}(t) dt$ for $\vec{r}(t) = t \vec{i} - \sqrt{1+t} \vec{j} + e^{-t} \vec{k}$
6. Find the unit tangent vector $\vec{T}$, the principal normal vector $\vec{N}$, the binormal vector, the curvature and radius of curvature to the curve traced out by
 - a) $\vec{r}(t) = e^t \vec{i} + e^t \sin t \vec{j} + e^t \cos t \vec{k}$ at $t = 0$
 - b) $\vec{r}(t) = t^2 \vec{i} + (\frac{1}{2}t^2 - 1) \vec{j} - t \vec{k}$ at $t = 0$
 - c) $\vec{r}(t) = t \vec{i} + \frac{1}{2}e^{2t} \vec{j}$ at $t = \ln 2$
7. Prove that the following vector functions are solenoidal (or incompressible)
 - a) $f(x, y, z) = yz \vec{i} + xz \vec{j} + xy \vec{k}$
 - b) $g(x, y, z) = 3x^2 \vec{i} - y^2 \vec{j} + (2yz - 6xz) \vec{k}$
8. Find the $\text{div} \vec{F}$, $\text{curl} \vec{F}$ and $\text{curl}(\text{curl} \vec{F})$ of the following vector fields
 - a) $\vec{F}(x, y, z) = xe^y \vec{i} - 2e^{yz} \vec{j} - xe^y \vec{k}$
 - b) $\vec{F}(x, y, z) = xy^2 \vec{i} + (y - 2z) \vec{j} - (3xz) \vec{k}$
 - c) $\vec{F}(x, y, z) = \text{grad}(x^2 + y^2 + z^2 - xyz)$
9. Determine whether or not $\vec{F}$ is a gradient of some function f . If $\vec{F} = \nabla f$, find f .
 - a) $\vec{F}(x, y) = y^3 \vec{i} + 3xy^2 \vec{j}$
 - b) $\vec{F}(x, y, z) = yz \vec{i} + xz \vec{j} + (xy - 1) \vec{k}$
 - c) $\vec{F}(x, y, z) = yze^{xy} \vec{i} + xze^{xy} \vec{j} + (e^{xy} + z^2) \vec{k}$
10. Show that $\text{curl}(f\vec{F}) = f \text{curl} \vec{F} + \nabla f \times \vec{F}$, where all the partial derivatives of the scalar valued function f exist.
11. Show that $\vec{F}(x, y, z) = y \vec{i} + (z \cos yz + x) \vec{j} + (y \cos yz) \vec{k}$ is irrotational and if it exists find the potential function of $\vec{F}$.
12. Show that the curvature of the helix $\vec{r}(t) = a \cos t \vec{i} + a \sin t \vec{j} + bt \vec{k}$ where $a > 0, t \in \mathbb{R}$ is a constant.