

Worksheet on Integration

1. Suppose $f''(x) = x - \frac{1}{x^2}$ such that $f(-1) = 4$ and $f'(2) = 3$, find an expression for $f(x)$.

2. Evaluate the following indefinite integrals

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| a) $\int (x^3 + 2x^2 + \frac{x}{2} + 4)dx$ | b) $\int (xe^{(x^2+1)} + x)dx$ | c) $\int x^2 \cos(x^3)dx$ |
| d) $\int \frac{x}{\sqrt{x^2+1}}dx$ | e) $\int \sin^3 x \cos x dx$ | f) $\int x^2 e^x dx$ |
| g) $\int \frac{1}{x^2 + 3x + 2}dx$ | h) $\int \frac{\sin(\ln x)}{x}dx$ | i) $\int \frac{1}{16-9x^2}dx$ |
| j) $\int \frac{x^3}{\sqrt{4-x^2}}dx$ | k) $\int \frac{x^3}{x^2-2x-3}dx$ | l) $\int \sin 2x \cos 3x dx$ |
| m) $\int \frac{1}{x(1-\ln x)}dx$ | n) $\int \frac{\sin 3x}{\cos x}dx$ | o) $\int \frac{1}{4x^2-4x+5}dx$ |
| p) $\int \frac{3x+2}{(1+x)^2(x-2)}dx$ | q) $\int \frac{x^3+2x^2}{(x-1)(x^2+2x+2)}dx$ | r) $\int \frac{2x-1}{x^4+x^2+1}dx$ |
| s) $\int \frac{\cosh 2x}{1+\sinh 2x}dx$ | t) $\int e^x \cosh x dx$ | u) $\int \frac{\sinh(\frac{1}{x})}{x^2}dx$ |

3. Evaluate the following definite integrals

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| a) $\int_0^1 \frac{x}{\sqrt{1+x^4}}dx$ | b) $\int_0^2 (x^2 + 1)dx$ | c) $\int_{-\pi/2}^{\pi/2} x^2 \cos x dx$ |
| d) $\int_2^5 x - 3 dx$ | e) $\int_{-1}^1 x(1 - x^2)^5 dx$ | f) $\int_1^2 (\sqrt{x} + \frac{1}{\sqrt{x}})dx$ |
| g) $\int_0^\pi \sin^2 x \cos^3 x dx$ | h) $\int_0^1 \sin^{-1} x dx$ | i) $\int_0^4 \sqrt{16 - x^2}dx$ |
| j) $\int_{-1}^2 \frac{x}{x^2+2x+10}dx$ | k) $\int_a^x e^t dt$ | |

4. Show that $\int_0^\pi \frac{x}{1+\sin x}dx = \pi$

5. Evaluate the following improper integrals whenever converges.

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|--------------------------------------|--|--|
| a) $\int_3^4 \frac{1}{(x-3)^2}dx$ | b) $\int_0^2 \frac{x}{\sqrt{4-x^2}}dx$ | c) $\int_0^1 \frac{1}{x^3+x}dx$ |
| d) $\int_0^\infty \frac{1}{e^x+1}dx$ | e) $\int_0^1 \frac{1}{e^x-e^{-x}}dx$ | f) $\int_2^\infty \frac{1}{x(\ln x)^3}dx$ |
| g) $\int_{\pi/4}^{\pi/2} \sec x dx$ | h) $\int_{-\infty}^\infty xe^{-x^2}dx$ | i) $\int_{-\infty}^\infty \frac{1}{x^2-6x+10}dx$ |

6. Find the area of the region bounded by each of the given equations

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| a) $y = x, x = 2$ and $y = 0$ | b) $y = x^2, x = -2, x = 2$ and $y = 0$ |
| c) $y = \sin x, x = -\pi, x = \pi$ and $y = 0$ | d) $y = x^2, y = x$ |
| e) $y = x^3, y = x$ | f) $y = x^2 - 4, y = 4 - x^2$ |
| g) $y = x(x - 2)^2, y = 0$ | h) $y^2 = 6x, x^2 = 6y$ |

7. Find the area of the region between the graphs of

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| a) $f(x) = x^2 + x$ and $g(x) = 3x^2 - x$ | b) $y^2 = x$ and $y = x - 2$ |
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8. Determine the area of the region enclosed by the ellipse $b^2x^2 + a^2y^2 = (ab)^2$.

Worksheet on Functions of Several Variables

- Find the domain, range and sketch the domain of
 - $f(x, y) = \frac{1}{x^2 + y^2}$
 - $f(x, y) = \frac{2}{\sqrt{9 - x^2 - y^2}}$
- Identify the level curves and sketch some of them.
 - $f(x, y) = \frac{1}{x - y^2}$
 - $f(x, y) = \frac{1}{x^2 + y^2}$
 - $f(x, y) = xy$
 - $f(x, y) = |y|$
- Sketch the graph of f
 - $f(x, y) = x + 2y$
 - $f(x, y) = \sqrt{4 - x^2 - y^2}$
 - $f(x, y) = e^{-x}$
- Sketch the level surface $f(x, y, z) = c$.
 - $f(x, y, z) = 2x - 4y + z; c = 1$
 - $f(x, y, z) = x^2 + y^2 + z^2; c = 2$
 - $f(x, y, z) = 4x^2 + 4y^2 + z^2; c = 1$
- The given surfaces intersect in a space curve C. Determine the the projection of C onto the xy -plane.
 - The sphere $x^2 + y^2 + (z - 1)^2 = \frac{7}{4}$ and the paraboloid $x^2 + y^2 - z = 1$.
 - The cone $x^2 + y^2 = z^2$ and the plane $y + z = 2$.
 - The paraboloid $x^2 + y^2 + z = -2$ and the surface $x^2 - 3y^2 = z$.
- Check whether or not the limit exists. If it exists, evaluate it.
 - $\lim_{(x,y) \rightarrow (-2,1)} \frac{x^3 + 2x^2y - xy - 2y^2}{x + 2y}$
 - $\lim_{(x,y) \rightarrow (0,2)} \frac{(y - 2) \cos x - y + 2}{x^2y^2 - 4x^2}$
 - $\lim_{(x,y) \rightarrow (3,1)} \frac{x^2 - xy - 6y^2}{\sqrt{x - y} - \sqrt{2y}}$
 - $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}$
- Using the definition show that
 - $\lim_{(x,y) \rightarrow (0,0)} \frac{x(x - 2y)}{\sqrt{x^2 + y^2}} = 0$
 - $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{y \sin(x^2 + y^2 + z^2)}{x^2 + y^2 + z^2} = 0$
- Prove that f is not continuous at $(0, 0)$
 - $f(x, y) = \begin{cases} \frac{\sin xy}{x^2 + y^2} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$
 - $f(x, y) = \begin{cases} \frac{x^2y^3}{x^{12} + y^4} & \text{for } (x, y) \neq (0, 0) \\ 0 & \text{for } (x, y) = (0, 0) \end{cases}$
- Find the first and second partial derivatives of f .
 - $f(x, y) = \ln\left(\frac{x}{x - y}\right)$
 - $f(x, y) = \int_y^x e^{-t^2} dt$
- A function is said to be harmonic if it satisfies Laplace's equation: $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$. Show that the following functions are harmonic:

a) $f(x, y) = x^3 - 3xy^2$

b) $f(x, y) = e^x \sin y$

c) $f(x, y) = \ln \sqrt{x^2 + y^2}$

11. Verify $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$ given that

a) $f(x, y) = g(x) + h(y)$, where g and h are differentiable functions.

b) $f(x, y) = g(x)h(y)$, where g and h are differentiable functions.

12. Let g be a twice differentiable function of one variable. If $h(x, y) = g(x + y) - g(x - y)$, then prove that $\frac{\partial^2 h}{\partial x^2} = \frac{\partial^2 h}{\partial y^2}$.

13. a) Find $\frac{dz}{dt}$ where $z = xy^2 + x^2y$, $x = ct^2$ and $y = 2ct$, where c is a constant.

b) If $z = f(x, y)$ with $x = e^u + e^{-v}$; $y = e^{-u} + e^v$. Then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$.

14. Find the equation of the tangent plane to each of the following at the indicated point.

a) $f(x, y) = 3x^2 + 4y^2$; at $(-2, 1, 16)$

b) $e^{xy} \cos(zy) = 1$; at $(1, 0, 1)$.

15. Find the directional derivative of $f(x, y, z) = x^3 - \tan^{-1}(y + 2z)$ at $(1, 3, -1)$ in the direction of $v = 2i + j - k$.

16. Find the directions in which f increases and decreases most rapidly at the given point. Find the rate of changes in these directions.

a) $f(x, y) = \arctan(x - y)$; $P(2, -2)$.

b) $f(x, y) = 2x^2 + 3xy + 4y^2$; $P(1, 2)$.

17. The temperature at each point of a metal plate is given by the function $T(x, y) = e^x \cos y + e^y \cos x$. In what direction does the temperature increase most rapidly at point $(0, 0)$? What is this maximum rate?

18. Find the local extreme points and values of

a) $z = x^2 + xy + y^2 - 2x - y$

b) $f(x, y) = (y - 2) \ln(xy)$

19. Find extreme values in the given set D .

a) $f(x, y) = 2x^2 + y^2 + 2x$; $D = \{(x, y) | x^2 + y^2 \leq 1\}$.

b) $f(x, y) = e^{x^2 - y^2}$; D is the region bounded by $x^2 + y^2 = 1$ and $x^2 + y^2 = 2$.

c) $f(x, y) = x^2 + y^2 - x - y$; D is the triangular region with vertices at $(0, 0)$, $(2, 0)$ and $(0, 2)$.

20. Find the maximum and minimum values of f subject to the given constraint

a) $f(x, y) = 4x^2 + 9y^2$, subject to $x^2 + y^2 = 1$.

b) $f(x, y) = xy$, subject to $4x^2 + 9y^2 = 36$.

c) $f(x, y, z) = x^2 + y^2 + z^2$; subject to $3x + 2y + z = 6$.

d) $f(x, y, z) = xyz$ subject to $x^2 + y^2 + z^2 = 1$.

21. Use Lagrange Multipliers to

a) find the points on the circle $x^2 + y^2 = 45$ that are closest to and furthest from $(1, 2)$.

b) find the points on the sphere $x^2 + y^2 + z^2 = 1$ that are closest to or farthest from the $(4, 2, 1)$.

c) find the points on the sphere $xy - z^2 = 1$ that are closest to the origin.

22. The sum of three positive numbers is 120. What is the maximum possible value of their product?

Worksheet on Multiple Integrals

1. Evaluate the following integrals

a) $\int_0^1 \int_0^x \cos(x^2) dy dx$

b) $\int_0^1 \int_0^y x \sqrt{x^2 + y^2} dx dy$

c) $\int_0^{\ln 3} \int_0^1 \int_0^y (z^3 - y) e^{y^2} dx dz dy$

d) $\iint_R (x^2 + 3y^2) dA$ where $R = \{(x, y) | x^2 + y^2 \leq 1\}$.

e) $\iiint_D y e^{-xy} dV$, where D is the cube bounded by the planes
 $x = 1, x = 3, y = 0, y = 2, z = -2$ and $z = 0$.

f) $\iiint_D x(3 - y) dV$, where D is the solid in the first octant bounded by the sphere
 $x^2 + y^2 + z^2 = 4$ and the coordinate planes.

2. By changing the order of integration, when necessary, evaluate

a) $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \sin(5x^2) dx dy$

b) $\int_0^1 \int_x^1 x^2 e^{y^4} dy dx$

c) $\int_1^4 \int_{\sqrt{x}}^2 \cos(y^3 - 3y) dy dx$

d) $\int_1^3 \int_0^{\ln y} e^{x+y} dx dy$

3. Using polar or cylindrical coordinates, evaluate

a) $\int_{-2}^2 \int_2^{\sqrt{8-x^2}} \frac{1}{\sqrt{x^2 + y^2}} dy dx$

b) $\int_0^1 \int_0^{\sqrt{1-y^2}} \frac{1}{x^2 + y^2 + 1} dx dy$

c) $\int_0^2 \int_0^{\sqrt{2y-y^2}} \int_0^1 z \sqrt{x^2 + y^2} dz dx dy$

4. Using double integrals, find the area of the region R bounded by

a) the curves $y = x^2$ and $y = \frac{2}{1+x^2}$

b) the cardioid $r = a(1 + \sin \theta), a > 0$

5. Find the surface area of

a) the paraboloid $z = 4 - x^2 - y^2$ above the plane $z = 3$.

b) the portion of the sphere $x^2 + y^2 + z^2 = 5$ that is inside the paraboloid $x^2 + y^2 = 4z$.

6. Find the volume of the solid D that is

a) in the first octant bounded by the paraboloid $z = x^2 + y^2$, the plane $x + y = 1$ and the coordinate planes.

b) inside the cylinder $x^2 + y^2 = 4x$ bounded above by the upper nappe of the cone $z^2 = x^2 + y^2$ and below by the xy-plane.

c) inside the sphere $x^2 + y^2 + z^2 = 4$ outside the cylinder $x^2 + y^2 = 1$.

d) bounded above by $x^2 + y^2 + z^2 = 25$ and below by $z = \sqrt{x^2 + y^2}$.

7. a) Find the volume of the solid inside the cylinder $x^2 + z^2 = 1$ bounded on both sides by the sphere $x^2 + y^2 + z^2 = 4$.

b) Write only the iterated integrals to find the volume of the solid bounded by the paraboloid $z = x^2 + y^2$ and the plane $2x + y + z = 5$.

8. Show that

a) $\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3} dy dx = \frac{1}{2}$

b) $\int_{-\infty}^{\infty} \int_0^{\infty} e^{-x^2-y^2} dx dy = \frac{\pi}{2}$

9. Evaluate $\iiint_D x e^{x^2+y^2+z^2} dV$ where $D = \{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}$.
10. Find the centroid of the solid region of constant mass density bounded below by the paraboloid $z = 2x^2 + 2y^2$ and above by the plane $z = 8$.
11. Evaluate using a suitable change of variables
- a) $\iint_R \left(\frac{x-y}{x+y} \right)^4 dA$ where R is the region bounded by the graphs of $x+y=1$, $y=0$ and $x=0$.
- b) $\iint_R e^{\left(\frac{2x-y}{x+y} \right)} dA$ where R is the region bounded by the graphs of $x=2y$, $y=2x$, $x+y=1$ and $x+y=2$.
- c) $\iint_R x^3 y dA$ where R is bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ for $x, y \geq 0$.
(Hint: use $x = au \cos v$, $y = bu \sin v$)
- d) $\int_0^2 \int_0^{\sqrt{2x-x^2}} \int_0^a z \sqrt{x^2+y^2} dz dy dx$ (use cylindrical coordinates).
- e) $\iiint_D y dV$, where D is the ellipsoid $\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1$ (Hint: Let $x = 3u$, $y = 2v$, $z = w$).
12. Find the area of the region bounded by the curves $xy = 1$, $xy = 2$, $y = x$ and $y = 4x$ in the first quadrant. (Hint: Let $x = \frac{u}{v}$, $y = v$)

Worksheet on Sequences

1. Using the definition verify

$$\text{a) } \lim_{n \rightarrow \infty} \frac{2n-1}{n+1} = 2 \qquad \text{b) } \lim_{n \rightarrow \infty} \frac{3n^2}{5n^2+1} = \frac{3}{5} \qquad \text{c) } \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) = 0$$

2. Suppose $\{a_n\}$ and $\{b_n\}$ are sequences such that $|a_n| \leq |b_n| \forall n \geq 1$. Show that if $\{b_n\}$ converges to 0, then $\{a_n\}$ converges to 0.

3. Let $\{a_n\}$ and $\{b_n\}$ be sequences. If $\{a_n\}$ converges to 0 and $\{b_n\}$ is bounded, show that $\{a_n b_n\}$ converges to 0.

4. Find $\lim_{n \rightarrow \infty} a_n$ if it exists

$$\begin{array}{lll} \text{a) } a_n = \left(1 + \frac{3}{n}\right)^n & \text{b) } a_n = \sqrt[3]{n+2} - \sqrt[3]{n+1} & \text{c) } a_n = \left(\frac{n^2-1}{n}\right)^n \\ \text{d) } a_n = \frac{n}{n+1} \sin \frac{n\pi}{2} & \text{e) } a_n = \tanh n & \text{f) } a_n = \frac{n^2(n!)}{(n+2)!} \end{array}$$

5. Determine whether the given sequence is monotonic or not, and convergent or not

$$\begin{array}{lll} \text{a) } \left\{ \frac{3^n}{2+3^n} \right\} & \text{b) } \left\{ \frac{n!}{n^n} \right\} & \text{c) } \{\cos n\pi\} \\ \text{d) } \left\{ \ln \left(\frac{3n}{n+1} \right) \right\} & \text{e) } \left\{ \sqrt[n]{3^n + 4^n} \right\} & \end{array}$$

6. Determine the convergence or divergence of the sequence $\{a_n\}$ if

$$\begin{array}{ll} \text{a) } a_n = \frac{\ln n}{n+1} & \text{b) } a_n = \left(1 + \frac{1}{n^2}\right)^{3n} \\ \text{c) } a_n = \frac{1+3+5+\dots+(2n-1)}{n^2} & \text{d) } a_n = \sum_{k=0}^n 2^{-k} \end{array}$$

7. a) Prove that $\lim_{n \rightarrow \infty} r^n = \begin{cases} 0 & \text{if } |r| < 1 \\ \infty & \text{if } |r| > 1 \end{cases}$

$$\text{b) Using (a) find} \quad \text{i) } \lim_{n \rightarrow \infty} \frac{1+3^n}{2^n} \quad \text{ii) } \lim_{n \rightarrow \infty} \frac{1+(-1)^n}{e^{2n}}$$

$$\text{c) Find } \lim_{n \rightarrow \infty} \sqrt[n]{4^n + 5^n}$$

8. Let $\{a_n\}$ be sequence with $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2+a_n}, n \geq 1$.

$$\text{a) Show that } \{a_n\} \text{ is convergent.} \qquad \text{b) Find } \lim_{n \rightarrow \infty} a_n$$

9. Let

$$\text{a) } a_n = \frac{1}{1.2} + \frac{1}{2.3} + \dots + \frac{1}{n(n+1)}. \text{ Show that } \lim_{n \rightarrow \infty} a_n = 1.$$

b) $b_n = \frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \dots + \frac{1}{\sqrt{n^2+n}}$. Show that $\lim_{n \rightarrow \infty} a_n = 1$.

c) $c_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{(-1)^{n+1}}{n}$. Show that $\lim_{n \rightarrow \infty} c_n = \ln 2$.
(Hint: Use Maclaurin's Formula on $f(x) = \ln(x+1)$)

10. Find two limit points and subsequences which converge to each of these points

a) $\left\{ (-1)^n \left(2 - \frac{1}{n} \right) \right\}$ b) $\left\{ \sin \frac{n\pi}{4} \right\}$

Worksheet on Series

1. Find the sum of the following series

a) $\sum_{n=0}^{\infty} 3^{-2n}$

b) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{2^{(n-1)}}}$

d) $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

2. A ball is dropped from a height of 3 meters above a smooth surface, it bounces $\frac{2}{3}$ of its previous height. Find the total length the ball moves until it gets to rest.

3. Check the convergence or divergence of the following series using

i. Integral test

a) $\sum_{n=2}^{\infty} \frac{n}{n^2 - 1}$

b) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^3}$

c) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2 + 1}$

d) $\sum_{n=1}^{\infty} \frac{1}{n \ln(n+1)}$

ii. Comparison test

a) $\sum_{n=1}^{\infty} \frac{\sqrt{n} - 1}{10 + n}$

b) $\sum_{n=1}^{\infty} \frac{3n^2 + 2}{4n^2 + 5n - 1}$

c) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{\sqrt{n^2 + 3}}$

iii. D'Alembert ratio test or Cauchy's root test

a) $\sum_{n=1}^{\infty} \frac{n^4}{3^n}$

b) $\sum_{n=1}^{\infty} \frac{n^n}{2^{nn!}}$

c) $\sum_{n=1}^{\infty} \left(\frac{n-1}{4n+1} \right)^n$

d) $\sum_{n=1}^{\infty} \left(\frac{2n+5}{3n-1} \right)^{2n-1}$

e) $\sum_{n=1}^{\infty} \frac{n!}{1.3.5.....(2n+1)}$

f) $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}$

iv. Alternating series test

a) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{5n-2}$

b) $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{\ln n}$

c) $\sum_{n=1}^{\infty} (-1)^n \left(\frac{n}{2n+1} \right)^n$

4. Test the series for absolute or conditional convergence, or divergence

a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n(n+1)}}$

b) $\sum_{n=1}^{\infty} \frac{\cos \pi n}{\sqrt{n}}$

c) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$

d) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{5^n}$

e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n \ln n}$

f) $\sum_{n=1}^{\infty} (-1)^n \ln \left(\frac{n}{n+1} \right)$

5. For what values of p does the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^p}$ converge absolutely and conditionally?

6. Find the value(s) of x each of the following series converge absolutely, conditionally, or diverge.

a) $\sum_{n=1}^{\infty} \frac{x^n}{n+1}$

b) $\sum_{n=1}^{\infty} (nx^n)^2$

7. For each of the following find the interval and radius of convergence

a) $\sum_{n=0}^{\infty} \frac{4^n}{3^{2n}} x^n$

b) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2} x^n$

c) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2 + 1} x^n$

d) $\sum_{n=1}^{\infty} (-1)^n n e^{-\frac{n}{2}} x^{2n}$

e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$

f) $\sum_{n=1}^{\infty} \frac{\ln n}{2^n} (x + 3) x^n$

8. Find the radius of convergence of

a) $\sum_{n=1}^{\infty} \frac{n!}{n^n} x^n$

b) $\sum_{n=1}^{\infty} \frac{n}{n^2 + 2} (x - 1)^{n+1}$

c) $\sum_{n=1}^{\infty} (-1)^n \frac{(\ln n)^2}{n^2} x^n$

d) $\sum_{n=1}^{\infty} \frac{1.3.5.....(2n-1)}{2.4.6.....(2n)} x^n$

9. Find the power series representation of $\ln(1 + x^2)$ in powers of x .

10. Use power series expansion to show that

a) $\frac{x}{(1-x)^2} = \sum_{n=1}^{\infty} n x^n, |x| < 1$

b) $\frac{t^2}{1+t^4} = \sum_{n=0}^{\infty} (-1)^n t^{4n+2}, |t| < 1$

11. Find the Taylor series expansion of

a) $f(x) = \ln(1 - 2x)$, at $x_0 = 0$ (Maclaurin's series)

b) $\frac{x-1}{x+1}$, at $x_0 = 0$

c) $f(x) = \frac{-3x+2}{2x^2-3x+1}$, at $x_0 = 0$

d) $f(x) = \sin(2x)$, at $x_0 = \pi$

e) $f(x) = \cosh x$, at $x_0 = 0$

f) $f(x) = \sqrt{x}$, at $x_0 = 1$

12. Show that

a) $\ln 2 = \sum_{n=1}^{\infty} \frac{1}{n^2}$

b) $\sum_{n=0}^{\infty} \frac{1}{n!(n+2)} = 1$

13. Using Taylor series expansion show that $\sqrt{e} = \sum_{n=0}^{\infty} \frac{1}{2^n n!}$.

14. Find the sum of the following series

a) $x + \frac{x^5}{5} + \dots + \frac{x^{4n-3}}{4n-3} + \dots$ [Hint: Differentiate]

b) $e^{-x} + 2e^{-2x} + \dots + ne^{-nx} + \dots$ [Hint: Integrate]