ADDIS ABABA CITY GOVERNMENT OF EDUCATION BUREAU YEKA SUB– CITY EDUCATION OFFICE PREPARATORY SCHOOLS MATHEMATICS MODEL EXAMINATION FOR GRADE 12TH TO NATURAL SCIENCE STREAM, GINBOT, 2012 E.C / MAY 2020 G.C

TIME ALLOWED: 3 HOURS

GENERAL INSTRUCTIONS:

THIS BOOKLET CONTAINS **MATHEMATICS** EXAMINATION. THE EXAMINATION CONTAINS **65** QUESTIONS. ATTEMPT ALL THE QUESTIONS. FOLLOW THE INSTRUCTIONS ON THE ANSWER SHEET AND THE EXAMINATION PAPER EXACTLY. USE ONLY **PENCIL** TO MARK YOUR ANSWERS.

THERE IS ONLY **ONE BEST** ANSWER FOR EACH QUESTION. CHOOSE THE BEST ANSWER FROM THE SUGGESTED OPTIONS AND BLACKEN THE LETTER OF YOUR CHOICE ON THE ANSWER SHEET. YOUR ANSWER MARK SHOULD BE HEAVY AND DARK, COVERING THE ANSWER SPACE COMPLETELY. PLEASE ERASE ALL UNNECESSARY PENCIL/PEN MARKS COMPLETELY FROM YOUR ANSWER SHEET.

YOU WILL BE LOWED TO WORK FOR **3:00** (OR THREE HOURS). IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY GO BACK AND REVISED. WHEN TIME IS CALLED, YOU MUST IMMEDIATELY STOP WORKING, LAY YOUR PENCIL DOWN, AND WAIT FOR FURTHER INSTRUCTIONS.

ANY FORM OF CHEATING OR AN ATTEMPT TO CHEAT IN THE EXAMINATION HALL WILL RESULT IN AN AUTOMATIC DISMISSAL FROM THE EXAMINATION HALL AND CANCELLATION OF YOUR SCORE(S).

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO

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Time Allowed: 3 hours

DIRECTIONS: For each of the following questions, choose the best answers from the given alternatives and carefully **blacken** the letter of your best choice on the separate answer sheet provided.

1. Let $f(x) = \ln\frac{x}{x+1}$. Then what is the inverse of f(x)?
   A/ $f^{-1}(x) = \frac{x}{x-1}$ C/ $f^{-1}(x) = \frac{-e^x}{e^x-1}$
   B/ $f^{-1}(x) = \frac{e^x}{1-e^x}$ D/ $f^{-1}(x) = \frac{-x}{1-x}$
2. Let $f(x) = \sqrt{16-x^2}$ and $h(x) = \sqrt{x^2-9}$. Then what is the domain of $(foh)(x)$?
   A/ $[-3,-5] \cup [3,5]$ C/ $[-\infty,-3] \cup [3,\infty]$
   B/ $[-3,-4] \cup [3,4]$ D/ $[-\infty,-4] \cup [4,\infty]$
3. Which one of the following is **not true** about the rational function $f(x) = \frac{x^3+27}{x^2-9}$?
   A/ $y = x$ is an oblique asymptote C/ The point $(0,-3)$ crosses graph of f
   B/ $x = -3$ is the x intercept D/ $x = -3\ and\ 3$ are vertical asymptote
4. Let $x^2 + y^2 - 8x + 6y + C = 0$. If the radius of the circle is 6 c.m. Then what is the value of C?
   A/ -11 B/ 6 C/ 11 D/ -6
5. What are the intersections points of a circle with equation $(x+1)^2 + (y-2)^2 = 32$ and a line with equation $x - y + 3 = 0$?
   A/ $T(5,\ 2) and\ S(-3,-6)$ C/ $T(8,\ 5) and\ S(0,-3)$
   B/ $T(-5,-2) and\ S(3,\ 6)$ D/ $T(-8,-5) and\ S(0,-3)$
6. Let $16x^2 - 9y^2 + 96x - 108y - 324 = 0$ is the general equation of the hyperbola. Then which one of the following are the center and the foci of the hyperbola respectively?
   A/ $C(3,6), F(2,1) and\ F'(2,\ 11)$ C/ $C(-3,-6), F(2,-6) and\ F'(-8,-6)$
   B/ $C(-3,-6), F(-3,-1) and\ F'(-3,-11)$ D/ $C(3,6), F(2,6) and\ F'(-8,6)$
7. Let $r, s\ and\ t$ be compound proposition with the truth value of $r \Rightarrow (\neg s \vee t)$ is **False.** Then which one of the following compound propositions has a truth value True?
   A/ $r \Rightarrow t$ B/ $\neg r \Leftrightarrow t$ C/ $s \Rightarrow t$ D/ $t \Leftrightarrow s$
8. The mark that students scored in an examination is grouped shown in the following table.

Mark	31-40	41-50	51-60	61-70	71-80	81-90	91-100
No of students	6	10	13	6	1	2	2

   Then what is the mode of the mark?
   A/ 54.5 B/ 53.5 C/ 64.5 D/ 63.5

9. Let the variance of $x_1 + x_2 + x_3 + \cdots + x_n$ is 9. Then what is the variance of $2x_1 + 4 + 2x_2 + 4 + 2x_3 + 4 + \cdots + 2x_n + 4$?

A/ 27 B/ 18 C/ 22 D/ 36

10. Which one of the following combined quantifiers is **not true** in the set of real number?

A/ $\forall x \exists y(x^2 + y \geq 0)$ C/ $\forall x \exists y(x + y = x)$

B/ $\exists x \forall y(x + y = y)$ D/ $\forall x \forall y(x^2 + y^2 > 0)$

11. Suppose there are 6 seats arranged in a row. In how many different ways can 6 people be seated on the seats?

A/ 120 B/ 720 C/ 540 D/ 700

12. A committee with consists of 6 students is to be selected from 10 candidates among which are 4 males. What is the probability that at least three boys are selected?

A/ $\frac{5}{28}$ B/ $\frac{11}{42}$ C/ $\frac{19}{42}$ D/ $\frac{25}{28}$

13. If $C(n, 5) = C(n, 3)$. Then what is the value of n?

A/ $\{10, -1\}$ B/ $\{8\}$ C/ $\{8, -1\}$ D/ $\{10\}$

14. What is the constant term in the expansion of $\left(x - \frac{3}{x^5}\right)^6$?

A/ -18 B/ 18 C/ 24 D/ -24

15. A box contains 4 red and 3 yellow balls. If one ball is drawn at random, is not replaced, and a second ball is drawn. What is the probability that the first ball is red and the second is yellow?

A/ $\frac{4}{11}$ B/ $\frac{8}{49}$ C/ $\frac{7}{12}$ D/ $\frac{2}{7}$

16. Let A= $\begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$. What is the inverse of A?

A/ $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ C/ $\begin{bmatrix} -1 & 2 & 2 \\ -2 & 0 & 1 \\ 4 & -5 & -1 \end{bmatrix}$

B/ $\begin{bmatrix} 1 & 2 & 4 \\ -2 & 0 & 1 \\ 4 & -1 & -1 \end{bmatrix}$ D/ $\begin{bmatrix} -1 & 2 & 2 \\ 2 & 0 & -6 \\ 4 & -1 & -1 \end{bmatrix}$

17. Let the system be given $\begin{cases} x + 2y - 3z = 5 \\ 2x - y - z = 8 \\ 3x + ky + 2z = 14 \end{cases}$. Which one of the following is not true about the system?

A/ If the value of $k = -5$, the system has no solution

B/ If the value of $k \neq -5$, the system has unique solution

C/ If the value of $k = -5$ & $k = 31$, the system has infinitely many solution

D/ If the value of $k = 31$, the system has unique solution

18. Let the system be given $\begin{cases} 2x - 4y + 2z = 14 \\ 6x + 2y - 2z = 4 \\ 4x + 6y + 4z = 14 \end{cases}$. What is the solution set of the system?

A/ $\{-2, -1, -3\}$ B/ $\{-2, -1, 3\}$ C/ $\{-2, 1 - 3\}$ D/ $\{2, -1, 3\}$

19. Which one of the following is the simplified form of $\frac{50i}{4+3i} - |8+6i|$?

A/ $-4-8i$ B/ $4+8i$ C/ $4-8i\}$ D/ $\{-4+8i\}$

20. Which one of the following is the solution set of $z^3+27=0$ in the set of complex number?

A/ $\{\pm 3i, \frac{3+3\sqrt{3}i}{2}\}$ C/ $\{-3, \frac{3+3\sqrt{3}i}{2}, \frac{3-3\sqrt{3}i}{2}\}$

B/ $\{3, \frac{3+3\sqrt{3}i}{2}, \frac{3-3\sqrt{3}i}{2}\}$ D/ $\{\pm 3i, \frac{3-3\sqrt{3}i}{2}\}$

21. Which one of the following is not true about a matrix $C=(c_{ij})_{m\times n}$?

A/ $(C^t)^{-1} = (A^{-1})^t$ if A is inverse of C C/ $\det C \neq \det C^t$

B/ CA is defined if $A=(a_{ij})_{n\times m}$ D/ If $\det C^t \neq 0$, then C is non-singular matrix

22. Let Z = 1+i, then what is the polar representation of z^6?

A/ $8(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$ C/ $\sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$

B/ $-8(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2})$ D/ $-\sqrt{2}(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4})$

23. Let $\vec{A}=2i+6j+3k$ and $\vec{B}=-i+2j+k$, i,j,k are the standard unit vectors in the direction of positive x, y, z axis respectively. Then which one of the following is the unit vector in the direction of $\vec{A}+\vec{B}$?

A/ $\frac{i}{3}+\frac{8j}{3}+\frac{4k}{3}$ B/ $\frac{-i}{5}+\frac{-8j}{5}+\frac{4k}{5}$ C/ $\frac{4i}{3}-\frac{8j}{3}+\frac{4k}{3}$ D/ $\frac{6i}{3}+\frac{8j}{3}-\frac{4k}{3}$

24. Let the vectors $\vec{A}=12j+9k$ and $\vec{B}=6j+8k$. If θ is an angle between A and B , Then what is $\cos\theta$?

A/ $\frac{25}{24}$ B/ $\frac{24}{25}$ C/ $-\frac{24}{25}$ D/ $-\frac{25}{24}$

25. Let $|\vec{a}|=2$ and $|\overrightarrow{3a}+\vec{b}| = |\vec{a}-\vec{b}|$ for vectors $\vec{a}$ & $\vec{b}$> Then what is the value of $\vec{a}.\vec{b}$?

A/ -6 B/ -5 C/ -4 D/ -3

26. Which one of the following is the image of $(4,1)$ after rotated by $90°$ about the point $(3,-2)$?

A/ $(3,1)$ B/ $(-1,0)$ C/ $(1,3)$ D/ $(0,-1)$

27. A committee of 5 students has to be formed from 6 boys and 4 girls. In how many ways can this be done when a committee contains at most one girl?

A/ 66 B/ 60 C/ 54 D/ 56

28. What is the quadratic function $y=ax^2+bx+c$ that contains the points $(1,12,), (2,6), (3,2)$?

A/ $x^2+9x+20$ C/ $x^2-9x+20$

B/ $-x^2+9x+20$ D/ $x^2-9x-20$

29. What is the value of $\lim_{x\to 1}\frac{\sqrt{1+\sqrt{x+3}}-\sqrt{3}}{x-1}$?

A/ $\frac{\sqrt{3}}{8}$ B/ $\frac{\sqrt{3}}{24}$ C/ $\frac{\sqrt{3}}{12}$ D/ $\frac{\sqrt{3}}{16}$

30. What is the largest interval for which f is continuous if $f(x) = \sqrt{\frac{x^2-25}{36-x^2}}$?

A/ $[-6, -5] \cup [5, 6]$ C / $(-\infty, -5) \cup (5, \infty)$

B/ / $(-\infty, -6) \cup (6, \infty)$ D/ $(-6, -5] \cup ([5, 6)$

31. Which one of the following functions is not one to one correspondence?

A/ $f: \mathbb{R}^+ \to \mathbb{R}^+, f(x) = x - 8$ C/ $f: \mathbb{R}^+ \to \mathbb{R}^+, f(x) = 5^x$

B/ $f: \mathbb{R} \to \mathbb{R}, f(x) = \ln x$ D/ $f: \mathbb{R} \to \mathbb{R}, f(x) = x^3 + x^2 + x + 1$

32. Let $f(X) = 3 + \frac{5}{2}\cos(\frac{\pi}{4}x)$ then which one of the following is the amplitude and period of f respectively?

A/ $\frac{5}{2}$ & 8 B/ 8 & $\frac{5}{2}$ C/ 3&$\frac{5}{2}$ D. $\frac{5}{2}$ &3

33. what is the simplest form of $\csc(\frac{\pi}{2} - x)\cos^3 x - \cos 2x$

A/ $\sin x$ B/ $\cos^2 x$ C/ $\sin^2 x$ D. $\cos x$

34. what is the solution set of $2\cos^2 x - \sin x - 1 = 0$ on $[0, \pi]$

A/ $\{\pi, 0\}$ B. $\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}$ C. $\left\{\frac{\pi}{6}, \frac{\pi}{3}\right\}$ D. $\left\{\frac{\pi}{3}, \frac{5\pi}{6}\right\}$

35. A flag pole casts shadow of 20m. if the angle of inclination from the top of the shadow of the top of the pole is 60° then what is the height of flag pole?

A/ 20m B. 40 C. $10\sqrt{2}$ D. $20\sqrt{3}$

36. Let $0 < x < l$, then what is $\tan(\cos^{-1} x)$?

A/ $\frac{X}{\sqrt{1-X^2}}$ B. $\frac{\sqrt{1-X^2}}{X}$ C / $\frac{X}{\sqrt{X^2-1}}$ D / $\frac{\sqrt{X^2-1}}{X}$

37. What is the value of $\lim_{n\to\infty}\left(\frac{2+4n+6n^2+5n^4}{5+3n^2-4n^3+24}\right)$?

A/ 2.5 B/ -1.5 C/ -2.5 D. 0

38. If $\{a_n\}$ be a sequence, so that $a_1 = 3$ and $a_{n+1} = a_n + 3$, then what is the value of $\sum_{n=1}^{20} a_n$?

A/ 680 B. 650 C. 630 D. 600

39. If we insert 4 arithmetic means between 2 and 62, then what are the inserted numbers?

A/ $\{2, 14, 26, 38\}$ C/$\{10,22,34,46\}$

B/ $\{14, 26, 38, 50\}$ D/$\{26, 38, 50, 62\}$

40. What is the same of the series $9 - \frac{10}{7} + \frac{20}{49} - \frac{40}{343} + \cdots$

A/ $\frac{71}{9}$ B/ $-\frac{71}{9}$ C/ $\frac{9}{7}$ D/ $-\frac{9}{7}$

41. What is the value of $\lim_{x\to 1}\frac{x^2-1}{\sqrt{x}-1}$

A/ does not exist B/ 4 C/ 2 D./ ∞

42. Let $\lim_{n\to 2} g(x) = 10, \lim_{x\to 2} f(x) = 4$, then what is the value of $\lim_{n\to 2} \frac{[f(x)-4g(x))(g(x)-4f(x))]}{(g(x)-f(x))^2}$

A/ -36 B/ -6 C/ 6 D/ 36

43. Let $f(x) = \begin{cases} x^2, if\ x \neq 5 \\ x + a\ is\ x = 5 \end{cases}$, then what is the value of a if f is continuous at $x = 5$

A/ 30 B/ 25 C/ 15 D/ 20

44. Which one of the following is true about function f?

A/ If f is differentiable, then it is continuous

B/ If f is continuous, then it is differentiable

C/ If the limit of f exists, then it is continuous

D/ If f is continuous at a point. Then it does throughout the domain

45. Let $f(x) = 2x^2 - 3x + 4$, then what is the equation of the line tangent to the graph of f at $x = 2$?

A/ $y = 6x - 4$ B/ $Y = -5x + 4$ C/ $Y = 5x - 4$ D/ $Y = -6x + 4$

46. If $h(x) = xg(x) - \sqrt{g(x)}\ and\ g(3) = 4$ & $g'(3) = 12$, then which one of the following is the value of $h'(3)$?

A/ 36 B/ 34 C/ 37 D/ -35

47. Which one of the following is not true about the derivative of $f(x) = x^2|x|$?

A/ $f'(0)$ exist C/ $f'(x) = -3x^2, on\ (\infty, 0]$

B/ $f'^{(x)} = 3x^2, on\ [0, \infty)$ D./ $f'(x)$ exist on $(-\infty, \infty)$

48. Let $f(x)$=ln $x + 2e^{2x} + x - 80$. *Then what is the value of* $f''(\frac{\pi}{2})$

A/ $8e^{\pi} + \frac{4}{\pi^2}$ B/ $8e^{\pi} - \frac{4}{\pi^2}$ C/ $8e^{\frac{\pi}{2}} + \frac{4}{\pi^2}$ D/ $8e^{\frac{\pi}{2}} - \frac{4}{\pi^2}$

49. Let $f(x) = x^3 - 6x^2 + 4$, Then which one of the following is not true about the function f?

A/ f is increasing on $(-\infty, 0] \cup [4, \infty)$

B/ f is decreasing on $[0, 4]$

C/ / $(2, -13)$ is the inflection point of f

D/ f has a maximum value -28 & a minimum $value$ 4

50. A box is to have a square base an open top and volume of 108 cubic units. If x is the length of each side of its base and y is its height, what is the dimension $(x\ \&\ y)$ in order to make the box with the smallest amount that made?

A/ 6 & 3 B/ 4 & 2 C/ 10&5 D/ 8&4

51. Let the volume of V of melting ice cube after t seconds is $V = 400 - 50t + 0.3t^2$ $(in\ m^3)$. How fast is the volume changed when $t = 50$ second?

A/ 20 B/ 30 C/ -30 D/ -20

52. what is the anti- derivative of $f(x) = xe^{x^2} + \ln x$?

A/ $\frac{e^{x^2}}{2} - \ln x + x + c$ C/ $\frac{e^{x^2}}{2} - \ln x - x + c$

B/ $\frac{e^{x^2}}{2} + x\ln x - x + c$ D/ $-\frac{e^{x^2}}{2} - \ln x - x + c$

53. What is the integral of $f(x) = \frac{x^2}{x^2-9}$

A/ $x - \frac{3}{2}\ln\left|\frac{x+3}{x-3}\right|$ C/ $x + \frac{3}{2}\ln\left|\frac{x-3}{x+3}\right|$

B/ $x + \frac{3}{2}\ln\left|\frac{x+3}{x-3}\right|$ D/ $-x - \frac{3}{2}\ln\left|\frac{x-3}{x+3}\right|$

54. What is the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2x\sin x dx$?

A/ 2 B/ -2 C/ -4 D/ 4

55. What is the area of the region between the graph of $Y = -X^2$ and $Y = -X - 6$

A/ $\frac{125}{6}$ B/ $\frac{123}{6}$ C/ $\frac{119}{6}$ D/ $\frac{155}{6}$

56. What is the volume of the region bounded by the graph of Y= x^2 & Y =X between?

A/ $-\frac{2\pi}{15}$ B/ $\frac{2\pi}{15}$ C/ $\frac{15\pi}{2}$ D/ $-\frac{15\pi}{2}$

57. Let $f''(x) = xe^x + 6$, with $f'(0) = 3\ and\ f(0) = 4$, then what is $f(x)$?

A/ $xe^x - 2e^x + 3x^2 - 4x + 16$ C/ $xe^x - 2e^x + 4x + 5$

B/ $xe^x - 2e^x + 3x^2 + 4x + 6$ D/ $xe^x - 2e^x - 4x - 5$

58. What is the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x e^{sinx}$?

A/ $\frac{e^2+1}{e}$ B/ $\frac{e^2+e}{e}$ C/$\frac{e^2-e}{e}$ D/ $\frac{e^2-1}{e}$

59. Which one of the following triple points are vertices of equilateral triangle?

A/ $\{(0,4,6),(-2,8,2),(4,6,0)\}$ C/ $\{(0,4,8),(-2,10,5),(2,3,0)\}$

B/$\{(0,4,6),(-3,8,2),(2,3,0)\}$ D/ $\{(-3,8,2),(2,3,0),(0,2,3)\}$

60. If the center and radius of a sphere are $C(2,-2,3)$ & 5 respectively. Then which one of the following is the equation of the sphere?

A/ $x^2+y^2+z^2-4x+4Y-6z-3=0$

B/ $x^2+y^2+z^2-4x+4Y-6z-8=0$

C/ $x^2+y^2+z^2-4x+4Y-6z+3=0$

D/ $x^2+y^2+z^2-4x+4Y-6z+8=0$

61. What are the set of all critical numbers for the function $f(x)=\frac{x^2+2}{x-1}$?

A/ $\{1\pm\sqrt{3}\}$ C/ $\{1,1\pm\sqrt{3}\}$

B/ $\{-1\pm\sqrt{3}\}$ D/ $\{-1,1\pm\sqrt{3}\}$

62. A farmer estimates that if he plants 30 apple trees, the average yield will be 100 apples per tree. But because of the size of the farm, for each removed tree, the yield will be increased by 5 apples per tree. How many apple trees remained and what is the maximum yield respectively?

A/ 25 and 3125 C/ 24 and 2800

B/ 35 and 2625 D/ 30 and 3000

63. What is the value of $\lim_{x\to 0}\frac{\tan 2x\ \sin 2x\cos 2x}{x^3+10x^2}$?

A/ 0 B/ ∞ C/ does not exist D/ $\frac{2}{5}$

64. What are the points on the graph of $y=x^2-1$that closet to $(0\,,4)$?

A/ $\left(\frac{3}{\sqrt{2}},\frac{7}{2}\right)$and $\left(\frac{3}{\sqrt{2}},\frac{7}{2}\right)$ C/ $\left(-\frac{3}{\sqrt{2}},-\frac{7}{2}\right)$ and $\left(\frac{3}{\sqrt{2}},\frac{7}{2}\right)$

B/ $\left(\frac{3}{\sqrt{2}},\frac{7}{2}\right)$and $\left(-\frac{3}{\sqrt{2}},\frac{7}{2}\right)$ D/ $\left(\frac{3}{\sqrt{2}},-\frac{7}{2}\right)$and $\left(\frac{3}{\sqrt{2}},-\frac{7}{2}\right)$

65. If the 5^{th} and 11^{th} terms of a sequence are 20 and 80 respectively. Then what is the sum of the 1^{st} 80 terms of a sequence?

A/ 35,000 B/ 45,000 C/ 30,000 D/ 40,000

END