MTH 603

Iqra ch

Numerical Analyssis

Formuals:-

Lagrane's Formuala:-

$$y = f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} y_1 + \frac{(x-x_0)(x-x_1)\cdots}{}$$

Simplification Form:-

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_1)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

Divided Dipperences

Zero order :-

$$x = y_0$$

1st

~~2nd~~ order:-

$$x = \frac{y_1 - y_0}{x_1 - x_0}$$

2nd order :-

$$f(x) = \frac{y_1 - y_0}{x_2 - x_0}$$

3rd order :-

$$f(x) = \frac{y_1 - y_0}{x_3 - x_0}$$

and up to so on.

Newton Divided formula

$$y(x_0) = f(x_0) = y_0 = a$$

$$y(x_1) = f(x_1) = y_1 = a_0 + a_1(x_1 - x_0)$$

$$\star \longrightarrow y(x_2) = f(x_2) = y_2 = a_0 + a_1(x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$$

for first Eq :-

$$a_0 = y(x_0) = y_0 \text{ (zero order).}$$

for Second Eq :-

$$a_1 = y(x_0, x_1) \text{ (First order)}$$

2nd order :-

$$a_2 = y(x_0, x_1, x_2).$$

Error term

$$E(x) = y(x) - P_n(x)$$

$p(x)$ is polynomial of degree n, its $(n+1)$th derivation is zero.

$$k = \frac{y^{(n+1)}(\xi)}{(n+1)!}$$

Two and three point formulas:-

$$= \frac{y(x_1 - h) - 2y(x_i) + y(x_i + h)}{h}$$

Difference using interpolation :-

$$P_n(x) = y(x_0) + (x - x_0)\, y(x_0, x_1) + (x - x_0)(x - x_1)\, y[x_0, x_1, x_2] + \ldots\ldots$$

Error term:-

$$E_n(x) = y(x) - P_n(x) = \frac{\Pi(x)}{(n+1)!}\, y^{n+1}(\xi)$$

Richarson's Extrapolation :-

$$y'(x) = \frac{y(x+h) - y(x-h)}{2h} + E_T.$$

4th order:-

$$y'(x) = F_1\left[\frac{h}{2}\right] + a_1 h^4 + O(h^6).$$

$$\frac{f(h)}{2}$$

6th order:-

$$F_2\left[\frac{h}{4}\right] = \frac{4^2 F_1[h/2^2) - F_1[h/2]}{4^2 - 1}$$

Integration

$$\text{Area} = \int_a^b f(x)\, dx$$

$$\text{Volume} = \int_c^d \int_a^b g(x,y)\,dx\,dy$$

Newton - cotes integration

$x_0 = a$, $x_n = b$

$h = \frac{b-a}{n}$ and $x_k = x_0 + kh$

Trapezoidal Rule:-

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}(y_0 + y_6 + 2(y_1 + y_2 + y_3 + y_4 + y_5))$$

$h_2 \quad \frac{b-a}{n}$

Simpson's 1/3 rule:-

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}\left\{y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right\}$$

Multiply by 4 (odd term) and multiple by 2 (even term).

Simpson's 3/8 rule:-

$$\int_{x_0}^{x_1} f(x)\,dx = \frac{3}{8}h\left\{y_0 + y_3 + 3(y_1 + y_2)\right\}$$

Romberg's integral :-

$$I_{im}\left[\frac{h}{2^m}\right] = \frac{4^m I_{I(m-1)}\left[h/2^m\right] - I_{I(m-1)}\left[h/2^{m-1}\right]}{4^{m-1}}$$

Taylor's Series :-

$$y(t) = y_0 + (t-t_0)y_0' + \frac{(t-t_0)^2}{2}y_0'' + \frac{(t-t_0)^3}{6}y_0''' + \frac{(t-t_0)^4}{24}y_0^{iv} + \frac{(t-t_0)}{120}y_0^{v} + \ldots$$

Euler method :-

$$y_{m+1} = y_m + hf(t_m, y_m).$$

Modified Euler method :-

$$y_{m+1} = y_m + h\left[\frac{f(t_m, y_m) + f(t_{m+1} + y'_{m+1})}{2}\right]$$

RK method :-

2nd order :-

$$y_{n+1} = y_n + \frac{1}{3}(2k_1 + 1k_2)$$

4th order :-

$$y_{n+} = y_n + \frac{1}{6}(k_1 + 2k + 2k_3 + k_4),$$

k_1 in 4th order :-

$$k_1 = hf(t_n + y_n)$$

Adam-Moultan's Predictor:-

→ (differential equation).

$$y_{n+1} = y_n + \frac{h}{24}\left[55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right] + \frac{251}{720}h\nabla^4 y'$$

Milne's Predictor:-

correctness

To find y_4 ——— ?

$$y_4 = y_0 + \frac{4h}{3}(2y'_1 - y'_2 + 2y'_3), \quad y_4 = y_2 + \frac{h}{3}(y'_2 + 4y'_3 + y'_4)$$

RK Method

(4th order).

$k_1 = ?$

$k_1 = hf(x_1, y_1)$

$k_2 = ?$

$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$

$k_3 = ?$

$k_3 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2}\right)$

$k_4 = ?$

$k_4 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_3}{2}\right)$

Adam → (correctness)

$$y_{n+1} = y_n + \frac{h}{24}\left(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right)$$

Main Points :-

* Number of interval must be 6 in 1/3 Simpson's (Even)
* Simpson's rule is vaild for number of interval is 8.
* In 3/8 Simpson's rule interval must be multiple of 3.
* Traperzoidal rule → derived $Ax + b$
* 3/8 Simpson's → derived $Ax^2 + bx + c$
* Moller's method required 3 starting point.

☺

Interpolation two dimens:-

$$P = \frac{y - y_0}{h}$$

$$f_0 + P\Delta f_0 + \frac{P(P-1)}{2!}\Delta^2 f_0$$

Differentiation Using difference operation:-

Δ term:- (Forward)

$$D = \frac{1}{h}\left[\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \frac{\Delta^4}{4} + \frac{\Delta^5}{5} + \cdots\right]$$

1st derivation:-

$$Dy_0 = y_0' = \frac{1}{h}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4} + \cdots\right]$$

2nd derivation:-

$$D^2 y_0 = y_0'' = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots\right]$$

Δ term (backward)

$$D = \frac{1}{h}\left[\nabla - \frac{\nabla^2}{2} + \frac{\nabla^3}{3} + \frac{\nabla^4}{4} + \cdots\right]$$

1st $$y' = \frac{1}{h}\left[\nabla y_0 + \frac{\nabla^2 y_0}{2} + \frac{\nabla^3 y_0}{3} + \frac{\nabla^4 y_0}{4}\right]$$

?

2nd $$y'' = \frac{1}{h^2}\left[\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\nabla^4 y_0\right]$$