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MTH501 Linear Algebra

Mid Term Examination - November 2004

Time Allowed: 90 Minutes

Instructions

Please read the following instructions carefully before attempting any of the questions:

1. Attempt all questions. Marks are written adjacent to each question.
2. Do not ask any questions about the contents of this examination from anyone.

a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.
b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

****WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will get an `F` grade in this course.**

Total Marks: 45 **Total**
Questions: 11

Question No. 1 **Marks : 1**

Consider the system of Linear equations

$$x_1 + 4x_2 - 5x_3 = 0$$

$$2x_1 - x_2 + 8x_3 = 9$$

$$x_1 + x_2 - 8x_3 = 2$$

Then determinant of the Matrix of Coefficients of the above system is (a) 91
(b) 123
(c) 141
(d) **81**

Question No. 2 **Marks : 2**

Consider the matrix $A = \begin{bmatrix} 12 & x+y \\ 23 & 4 \\ 23 & x-y \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & x-5y \\ 2 & 1 & 1 \\ 0 & 2 & 3x \end{bmatrix}$ such that $AB = \begin{bmatrix} 2020 \\ 44 & 5 \\ 2512 \end{bmatrix}$ Then the values of x and y are

(a) x = - 2 and y = 4
(b) x = 2 and y = - 6
(c) x = 0 and y = 0
(d) x = 2 and y = -4

Question No. 3 **Marks : 1**

Which statement about the set S is false where S = {(1, 1, 3), (2, 3, 7),(2, 2, 6)}

(a) The set S contain an element which is solution of the equation 5x – y- z =
(b) The Set S is linearly independent.
(c) The set S contain two elements which are multiple of each
(d) The set S is linearly dependent.

Question No. 4 **Marks : 2**

Consider a linear transformation T such that $T\begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$ Then $T\begin{bmatrix} 5 \\ 25 \\ 15 \end{bmatrix} =$

(a) $\begin{bmatrix} 9 \\ 18 \end{bmatrix}$

(b) $\begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}$

(c) $\begin{bmatrix} 30 \\ 45 \end{bmatrix}$

(d) $\begin{bmatrix} 12 \\ 9 \end{bmatrix}$

Question No. 5 **Marks : 1**

How many subspaces R^2 have?

(a) only two: {0} and R^2
(b) Only four: {0} x- axis and y -axis and R^2
(c) Infinitely many.
(d) None of the above.

Question No. 6 **Marks : 8**

An oil refinery produces low-sulfur and high-sulfur fuel. Each ton of low-sulfur requires 5 minutes in the blending plant and 4 minutes in the refining plant; each ton of high sulfur fuel requires 4 minutes in the blending plant and 2 minutes in the refining plant. If the blending plant is available for 3 hours and the refining plant is available for 2 hours, how many tons of each type of fuel should be manufactured so that the plants are fully utilized?

Solution:

The data given in the question can be formed into system of linear equations as

	Low-sulfur	High-sulfur
Blending plant	5	4
Refining Plant	4	2

As we are given in the question that we have blending plant and refining plant available for 3 and 2 hours respectively. Let x tons of low-sulfur and y tons of high sulfur be the amount should be manufactured so that plants are fully utilized. Thus from the above data we must have the system,

$$5x + 4y = 180$$
$$4x + 2y = 120$$

Augmented matrix for the above system is

$$\begin{bmatrix} 5 & 4 & 180 \\ 4 & 2 & 120 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 60 \\ 4 & 2 & 120 \end{bmatrix} \text{ by } R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 2 & 60 \\ 0 & -6 & -120 \end{bmatrix}$$

So we have $x + 2y = 60$ and $-6y = -120$, thus we have $y = 20$ tons and $x = 20$tons are the required manufactured tons of each low-sulfur and high-sulfur so that we can utilize both plants for the given time.

Question No. 7 **Marks : 5**

Consider the matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 1 & 1 \\ 7 & 0 & 2 \end{bmatrix}$ then find the entry $\mathbf{a_{33}}$ of the matrix $\mathbf{A^2}$ without calculating the matrix $\mathbf{A^2}$.

Solution:

$$\text{Required entry } a_{33} \text{ of } A^2 = \begin{bmatrix} - & - & - \\ - & - & - \\ 7 & 0 & 2 \end{bmatrix} \begin{bmatrix} - & - & 2 \\ - & - & 1 \\ - & - & 2 \end{bmatrix} \text{marks: } 3 = \begin{bmatrix} - \\ - \\ 14+0+4 \end{bmatrix} = \begin{bmatrix} - \\ - \\ 18 \end{bmatrix}$$

Question No. 8 **Marks : 8**

Find LU decomposition for the matrix $A = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 1 & 1 \\ 7 & 0 & 2 \end{bmatrix}$.

Solution:

$$U \qquad\qquad L$$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{2}{3} \\ 4 & 1 & 1 \\ 7 & 0 & 2 \end{bmatrix} \text{multiplier } \frac{1}{3} \begin{bmatrix} 3 & 0 & 0 \\ * & * & 0 \\ * & * & * \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & -\frac{1}{3} & -\frac{5}{3} \\ 0 & -\frac{7}{3} & -\frac{8}{3} \end{bmatrix} R_2 - 4R_1, R_3 - 7R_1 \begin{bmatrix} 3 & 0 & 0 \\ 4 & * & 0 \\ 7 & * & * \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 5 \\ 0 & -\frac{7}{3} & -\frac{8}{3} \end{bmatrix} \text{multiplier } -3 \begin{bmatrix} 3 & 0 & 0 \\ 4 & -\frac{1}{3} & 0 \\ 7 & * & * \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 5 \\ 0 & 0 & 9 \end{bmatrix} R_2 + \frac{7}{3} R_2 \begin{bmatrix} 3 & 0 & 0 \\ 4 & -\frac{1}{3} & 0 \\ 7 & -\frac{7}{3} & * \end{bmatrix}$$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \text{multiplier } -\frac{1}{9} \begin{bmatrix} 3 & 0 & 0 \\ 4 & -\frac{1}{3} & 0 \\ 7 & -\frac{7}{3} & 9 \end{bmatrix} .$$

The required LU factorization is

$$L = \begin{bmatrix} 3 & 0 & 0 \\ 4 & -\frac{1}{3} & 0 \\ 7 & -\frac{7}{3} & 9 \end{bmatrix} \quad U = \begin{bmatrix} 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Question No. 9 **Marks : 8**

Write $v = \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix}$ as linear combination of $u_1 = \begin{bmatrix} 1 \\ 2, \\ -1 \end{bmatrix}$ $u_2 = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$ *and* $u_3 = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$. (That is you have to find out the constants such that $v = c_1u_1 + c_2u_2 + c_3u_3$)

Solution:

We will try to find out the constants such that we can write $v = c_1u_1 + c_2u_2 + c_3u_3$ and we get the augmented matrix of the system of linear equations correspond to the linear combination as

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 2 & 4 & -3 & -9 \\ -1 & 2 & 2 & 2 \end{bmatrix}$$ And echelon form of that matrix is $\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \Rightarrow x_3 = 3, x_2 = -1, x_1 = 2$

Question No. 10 **Marks** **: 5**

Show that the $M: R^3 \to R^3$ defined by $M \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is a linear transformation. Also write the matrix of that linear transformation.

Solution:

We will prove that the mapping satisfies the conditions (i) $M(u + v) = M(u) + M(v)$ (ii) $M(cu)$

(i) $M\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix}\right\}$ Now as we know that matrices are distributed over addition that is if A, B and C are matrices then A (B + C) = AB + BC, so we can write,

$M\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\left\{\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix}\right\} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\begin{bmatrix} d \\ e \\ f \end{bmatrix}$ also by definition we have $M\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ *and* $M\begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\begin{bmatrix} d \\ e \\ f \end{bmatrix}$. Hence we have,

$$M\begin{bmatrix} a+d \\ b+e \\ c+f \end{bmatrix} = Mb\begin{bmatrix} a \\ Me \\ c \end{bmatrix} + \begin{bmatrix} d \\ f \end{bmatrix}.$$

$$M\begin{bmatrix} cd \\ ce \\ cf \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\begin{bmatrix} cd \\ ce \\ cf \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\left\{c\,e\begin{bmatrix} d \\ f \end{bmatrix}\right\} = c\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}\begin{bmatrix} cd \\ ce \\ cf \end{bmatrix}$$

As we have A c(B) = cAB where c is a real number .

(ii) $Mce\begin{bmatrix} cd \\ cf \end{bmatrix} = cMe\begin{bmatrix} d \\ f \end{bmatrix}$

Hence M is a linear transformation and matrix of transformation is $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & \\ & 1 & \end{bmatrix}$

Question No. 11 Marks : 5

Solve the system 3x + y = 12 by using Jacobi's Method starting $\overset{0}{x}$ = (0, 0, 0) and only three iterations.

2x – 4y = -6

Solution:

Iteration 1
(12/3 , 3/2)
iteration 2
(3.5 , 3.5)
iteration 3

(2.833, 3.25)

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Question # 1:-

Let A= $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ Write 5A.Is det5A=5detA?

Let A = $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$5A = 5\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 20 & 10 \end{bmatrix}$

$\det 5A = ((15 \cdot 10) - (20 \cdot 5))$

$\det 5A = 150 - 100 = 50$

$\det 5A = 50$

$\det A = ((3 \cdot 2) - (1 \cdot 4))$

$\det A = 6 - 4 \quad 2$

$5\det A = 5 \cdot 2 \quad 10$

Hence proved det5A not equal to 5detA

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Question #2:-

Let A = $\begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$.Show that these matrices do not commute.

Solution:-

$AB = \begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 14 & 3 \\ -2 & 6 - \end{bmatrix}$

$BA = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 3 & 2 - \end{bmatrix} = \begin{bmatrix} 10 & 2 \\ 29 & 2 - \end{bmatrix}$

$AB \neq BA$

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Question # 3:-

Use Cramer's rule to solve the system $3x_1 - 2x_2 = 6$, $-5x_1 + 4x_2 = 8$

Note : In order to get full marks do all necessary steps

Solution:-

$3x_1 - 2x_2 = 6$

$-5x_1 + 4x_2 = 8$

View the System as Ax =b. Using the notation introduced above

$$A = \begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$$

$$A_1(b) = \begin{bmatrix} 6 & -2 \\ 8 & 4 \end{bmatrix}$$

$$A_2(b) = \begin{bmatrix} 3 & 6 \\ -5 & 8 \end{bmatrix}$$

Since detA = 2, the system has a unique solution. By Cramer's rule,

$$x_1 = \frac{\det A_1(b)}{\det A} = \frac{24+16}{2} = 20$$

$$x_2 = \frac{\det A_2(b)}{\det A} = \frac{24+30}{2} = 27$$

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Question # 4:-

Find the matrix of linear transformation $T: R^3 \to R^4$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 - x_3, x_1)$ with respect to the standard basis of R^3 ?

Note : In order to get full marks do all necessary steps

Solution:-

The standard basis of R^3 is $\{e_1, e_2, e_3\}$

$E_1 = (1, 0, 0)$

$E_2 = (0, 1, 0)$

$E_3 = (0, 0, 1)$

$T(e_1) = T(1, 0, 0) = (1, 0, 1, 1)$

$T(e_2) = T(0, 1, 0) = (1, 1, 0, 0)$

$T(e_3) = T(0, 0, 1) = (0, 1, -1, 0)$

The matrix of linear transformation is

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

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$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & k & 2 \\ 0 & 4 & 2 \end{bmatrix}$$

If such that $|A| = 0$ then the value of k is

- 8
- **4**
- 0
- None of these

$$x + y = 1$$
$$x + y = 0$$

The system of linear equations has

- **No solution**
- Infinitely many solutions
- Infinitely many solutions
- d) None of these

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$$A = \begin{bmatrix} 4 & 3 \\ 7 & 2 \\ 9 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 5 \\ 1 & 6 \end{bmatrix}$$

Let then AB is

$$\begin{bmatrix} 11 & 38 \\ 16 & 47 \\ 18 & 45 \end{bmatrix}$$

- **a)**
- b) A and B are not comfortable for multiplication

$\begin{bmatrix} 11 & 38 \\ 16 & 47 \\ 8 & 45 \end{bmatrix}$

c)

d) None of these

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$\begin{bmatrix} -4 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

Determine whether the set S={ } is a spanning set for R^2 or not.

Note : In order to get full marks do all necessary steps

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$R^2 \rightarrow R^3$

Let T : be a linear transformation defined by T(x)=Ax , where

$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}$ $\begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$

A= ,then find an x in R^2 whose image under T is b.Where

Note : In order to get full marks do all necessary steps

$\begin{bmatrix} 12 \\ 51 \end{bmatrix}$

The inverse of the matrix is

$\begin{bmatrix} 1 & -2 \\ -5 & 1 \end{bmatrix}$

(a)

$\frac{1}{9}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$

b)

$\frac{1}{9}\begin{bmatrix} -1 & 2 \\ 5 & -1 \end{bmatrix}$

(c)

d) None of these

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The set of vectors {(5,0,0), (7,2,-6), (9,4,-8)} is,

a) Linearly

b) Linearly dependent

c) Basis of R^3

Q: let $A=\begin{bmatrix}2 & 3 & 4\\ 1 & 7 & 6\\ 3 & 1 & 2\end{bmatrix}$ find A^t, also show that (A^t)^t =A **2Marks**

Q: If the following equations of linear equation is solved using the Jacob's method then, Construct the matrix M from the coefficient matrix of the following linear system: **2Marks**

$$9x_1 + 2x_2 + 3x_3 = 5$$
$$3x_1 + 7x_2 + 2x_3 = 7$$
$$2x_1 + 3x_2 + 6x_3 = 5$$

Q: let $A=\begin{bmatrix}1 & 0 & 0\\ 3 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ find inverse of A using inversion algorithm. **3Marks**

Q: Construct partitions of the following matrix into three 2×2 blocks **:** **3Marks**

$$\mathbf{B}=\begin{bmatrix}1 & 2 & 3 & 4 & 1 & 3\\ 3 & 4 & 5 & 6 & 3 & 4\end{bmatrix}$$

$$A=\begin{bmatrix}2 & 3\\ 1 & 2\end{bmatrix} \qquad \vec{b}=\begin{bmatrix}4\\ 5\end{bmatrix}$$

Q: Let and **.** Define $T:R^2 \to R^2$ by $T(\vec{x})=A\vec{x}$ **.** Then find an $\vec{x}$ in R^2 whose image under T is $\vec{b}$ **.**Also show that $\vec{x}$ is unique**.** **5 Marks**

Q: Show that the coefficient matrix of the following linear system is strictly diagonally dominant. Also calculate the second iteration using Gauss – Seidel method if the first iteration is $(x_1^{(1)}, x_2^{(1)}, x_3^{(1)})=(1.3, 1.04, 0.936)$. **5Marks**

$$10x_1 + 2x_2 + x_3 = 13$$
$$2x_1 + 10x_2 + x_3 = 13$$
$$2x_1 + x_2 + 10x_3 = 13$$

Mth501 subjective for mid term
Latest spring 2013
By
~"Librainsmine"~

$$E=\begin{bmatrix}1 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Q1. Applying certain elementary row operation to the elementary matrix $I_{3\times 3}$ to produce an identity matrix . [2 marks]

Solution:

Multiply R_2 by 1/5

$$E=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

We get the identity matrix $I_{3\times 3}$

Q2. Why is it NOT possible to solve the following system of linear equations applying the Cramer'rule? [2 marks]

$$3x_1+2x_2=10$$
$$9x_1+6x_2=30$$

Solution:

$let\ \text{A}=\begin{bmatrix}3 & 2\\ 9 & 6\end{bmatrix}, x=\begin{bmatrix}x_1\\ x_2\end{bmatrix}, b=\begin{bmatrix}10\\ 30\end{bmatrix}$

$\det(A)=\begin{bmatrix}3 & 2\\ 9 & 6\end{bmatrix}=18-18=0$

$\det(A)=0$

since it is an singular matric and its determinant is zero.

we need determinant of A f.or applying crammers rule

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 1 & 0 & 0\end{bmatrix}$$

Q3. Determine whether or not the inverse of the matrix exists? Justify your answer with appropriate reason. [3 marks]

Solution:

$$|A|=\begin{vmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 1 & 0 & 0\end{vmatrix}$$

$|A|=1(0-0)+2(0-6)-3(0-4)$

$|A|=1(0)+2(-6)-3(-4)$

$|A|=0+(-12)+12$

$|A|=0$

As the determinant of the given matrix is zero its mean it is a singular matrix the inverse of the singular matrix does not exist.

$$A=\begin{bmatrix}3 & 2 & 1\\ 2 & 1 & 2\\ 0 & 2 & 0\end{bmatrix}$$

Q4. Compute $\det(A)$ by using a cofactor expansion across the third row, where .

[3 marks]

Solution:

Using cofactor expansion along the first column:

$$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 1 & 2 \\ 0 & 2 & 0 \end{vmatrix} = (0)(-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + (2)(-1)^{2+1}\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + (0)(-1)^{3+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$$

now if we compare it with the formula

det A= $(0)C_{31} + (2)C_{32} + (0)C_{33}$

$$= (0)(-1)^{1+1}\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + (2)(-1)^{2+1}\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + (0)(-1)^{3+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix}$$

$$= 0 + (2)(-1)^{3}\begin{vmatrix} 3 & 1 \\ 2 & 2 \end{vmatrix} + 0$$

=0+ {-2(6-2)}+0

=0-2(4)+0

= -8

$$A = \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 3 & 4 & 0 \\ \hline 0 & 0 & 2 \end{array}\right] \qquad B = \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 1 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Q5. Compute AB using block multiplication, where a .

[5 marks]

Solution:

Let

$$A=\begin{bmatrix}1 & 2 & 1\\ 3 & 4 & 0\\ 0 & 0 & 2\end{bmatrix}=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}, B=\begin{bmatrix}1 & 2 & 3 & 1\\ 4 & 5 & 6 & 1\\ 0 & 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}$$

let

$$A_{11}=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, A_{12=}\begin{bmatrix}1\\ 0\end{bmatrix}, A_{21}=\begin{bmatrix}0 & 0\end{bmatrix}, A_{22}=\begin{bmatrix}2\end{bmatrix}$$

$$B_{11}=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}, B_{12}=\begin{bmatrix}1\\ 1\end{bmatrix}, B_{21}=\begin{bmatrix}0 & 0 & 0\end{bmatrix}, B_{22}=\begin{bmatrix}1\end{bmatrix}$$

now

$$AB=\begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}B_{11} & B_{12}\\ B_{21} & B_{22}\end{bmatrix}=\begin{bmatrix}A_{11}B_{11}+A_{12}B_{21} & A_{11}B_{12}+A_{12}B_{22}\\ A_{21}B_{11}+A_{22}B_{21} & A_{21}B_{12}+A_{22}B_{22}\end{bmatrix}$$

the number of colums of A equals numbers of rows of B

so we can performed multiplication operation:

$$A_{11}B_{11}+A_{12}B_{21}=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}\begin{bmatrix}0 & 0 & 0\end{bmatrix}=\begin{bmatrix}9 & 12 & 15\\ 19 & 26 & 33\end{bmatrix}$$

$$A_{11}B_{12}+A_{12}B_{22}=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}+\begin{bmatrix}1\\ 0\end{bmatrix}\begin{bmatrix}1\end{bmatrix}=\begin{bmatrix}4\\ 7\end{bmatrix}$$

$$A_{21}B_{11}+A_{22}B_{21}=\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}+\begin{bmatrix}2\end{bmatrix}\begin{bmatrix}0 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\end{bmatrix}$$

$$A_{21}B_{12}+A_{22}B_{22}=\begin{bmatrix}0 & 0\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}+\begin{bmatrix}2\end{bmatrix}\begin{bmatrix}1\end{bmatrix}=\begin{bmatrix}2\end{bmatrix}$$

$$AB=\begin{bmatrix}A_{11}B_{11}+A_{12}B_{21} & A_{11}B_{12}+A_{12}B_{22}\\ A_{21}B_{11}+A_{22}B_{21} & A_{21}B_{12}+A_{22}B_{22}\end{bmatrix}=\begin{bmatrix}9 & 12 & 15 & 4\\ 19 & 26 & 33 & 7\\ 0 & 0 & 0 & 2\end{bmatrix}$$

Q6. Consider the vector space $V=R^3$ and the set W consists of all vectors in R^3 whose entries are equal : that is ,

$$W = \{(a, b, c) : a = b = c\}$$

Show that W is a subspace of R^3. [5 marks]

Solution:

To check if W is the subspace of R^3,we 1st check that axiom 1 and 6 of a vector space holds.

Let

$u = (a_1, b_1, c_1)$ and v=(a_2, b_2, c_2)be vectors in W then

u+v=$(a_1, b_1, c_1) + (a_2, b_2, c_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$is in W

and also if k ia any scalar and u=(a_1, b_1, c_1)in any vector W,then ku=k(a_1, b_1, c_1)

Hence W is a subspace.

MIDTERM EXAMINATION
Spring 2013
(MTH501- Linear Algebra (Session - 3)

Q1: If A = B, then determine the values of x and y; where

$$A = \begin{bmatrix} 1 & y+2 \\ x+2 & 3 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

Solution:

As we know that matrix A=B

its mean that the every entry in A = to the coresponding entry in B

so its clearly seen that value of

x+2=4 .....x=4/-2=x=-2

y+2=2.....y=-1

Q2: Determine which of the following condition(s) hold(s) for a vector space V over R. Justify your answer with appropriate reason.

$a)\left\{\vec{x}+\vec{y}\mid \vec{x}\in V,\vec{y}\in V\right\}=V$

$b)\left\{\vec{x}+\vec{y}\mid \vec{x}\in V,\vec{y}\in V\right\}=VxV$

$c)\left\{\lambda\vec{x}\mid \vec{x}\in V,\lambda\in R\right\}=RxV$

Solution:

(b) and (c) both are truefor vector space V over a field R is a set V equipped with an operation called (vector) addition, which takes vectors u and v and produces another vector .

There is also an operation called scalar multiplication, which takes an element and a vector and produces a vector .

Q3: Determine whether or not the solution of the following system of linear equations is possible using inversion algorithm? Justify your answer with appropriate reason.

$2x_1+4x_2+3x_3=3$

$4x_1+8x_2+6x_3=4$

$6x_1+12x_2+9x_3=4$

Solution:

let

$$A=\begin{bmatrix}2&4&3\\4&8&6\\6&12&9\end{bmatrix},x=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix},b=\begin{bmatrix}3\\4\\4\end{bmatrix}$$

$\det(A)=2(72-72)-4(36-36)+3(48-48)$

$\det(A)=2(0)-4(0)+3(0)$

$\det(A)=0$

sin ce matrix A is non invertible matrix so we can not apply inversion algorithm here.

Q4: Find a Matrix A such that W=ColA

$$W=\left\{\begin{bmatrix}2b+2c\\a+b-2c\\4a+b\\3a-b-c\end{bmatrix};a,b,c\ are\ real\right\}$$

Solution:

1st we write as a set of linear combinations:

$$\text{w}=\{\text{a}\begin{bmatrix}0\\1\\4\\3\end{bmatrix}+b\begin{bmatrix}2\\1\\1\\-1\end{bmatrix}+c\begin{bmatrix}2\\-2\\0\\-1\end{bmatrix}:a,b,c \text{ in R}\}=\text{span}\left\{\begin{bmatrix}0\\1\\4\\3\end{bmatrix},\begin{bmatrix}2\\1\\1\\-1\end{bmatrix},\begin{bmatrix}2\\-2\\0\\-1\end{bmatrix}\right\}$$

let

$$A=\begin{bmatrix}0&2&2\\1&1&-2\\4&1&0\\3&-1&-1\end{bmatrix}$$

Q5: Find an LU – decomposition of the Matrix

$$A=\begin{bmatrix}2&3\\5&7\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}2&3\\5&7\end{bmatrix} \qquad \begin{bmatrix}*&0*&*\end{bmatrix}$$

$$=\begin{bmatrix}1&\frac{3}{2}\\5&7\end{bmatrix} multipler=\frac{1}{2} \qquad \begin{bmatrix}2&0*&*\end{bmatrix}$$

$$=\begin{bmatrix}1&\frac{3}{2}\\0&-\frac{1}{2}\end{bmatrix} multipler=-5 \qquad \begin{bmatrix}2&0\\5&*\end{bmatrix}$$

$$=\begin{bmatrix}1&\frac{3}{2}\\0&1\end{bmatrix} multipler=-2 \qquad \begin{bmatrix}2&0\\5&1\end{bmatrix}$$

Q6: Consider the vector space V=R^2 and the set W consists of all points in R^2 such that, W={(a,b) : a,b ≥ 0}Show that W is not a subspace of R^2.

Solution:

This is not subspace because it is not enclosed under scalar multiplication. So ,Vector space $V=R^2$ where R^2 not passing through origin is not a subspace of $R^{2.}$

MIDTERM EXAMINATION
Spring 2013
MTH501- Linear Algebra (Session - 2)

Q1. If $A=\left[\begin{array}{cc|cc} 1 & 3 & 2 & 4 \\ \hline 2 & 4 & 1 & 3 \\ 5 & 3 & 2 & 4 \\ \hline 9 & 8 & 6 & 5 \end{array}\right]$ and $B=\begin{bmatrix} 2 & 1 \\ 1 & 3 \\ 3 & 6 \\ 4 & 5 \end{bmatrix}$ then partition B in such a way that the multiplication can be possible?

Solution:

No as matrix A has 6 partitions it will only able to multiply with matrix B if ad only if:
No of columns of A =no of rows of B

Q2. File the determinant and tell that the given matrix is singular or no singular

$$A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 0 & 0 \end{bmatrix}$$

Solution:

Det (A)=1(0-0)-2(0-6)+3(0-4)

Det (A)=1+12-12

Det (A)=1 it is non singular matrix

Q3. Determine whether the inverse in possible or not of the give matrix and justify your answer

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 1 & 0 & 0\end{bmatrix}$$

Solution:

As

Det (A)=1(0-0)-2(0-6)+3(0-4)

Det (A)=1+12-12

Det (A)=1 it is non singular matrix

Determinant of the matrix is non singular so its inverse is possible.

Q4. Apply Cramer's rule and find the inverse of $A=\begin{bmatrix}2 & 3\\ -1 & 5\end{bmatrix}$

Solution:

For the matrrix say

$$A=\begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix} \Rightarrow \det A = 10-(-3)=13$$

$\Rightarrow A^{-1}$ will also be a 2×2 matrix

As

$Aji =$ submatrix of A formed by deleting row ***j*** and column ***i***

So in this case

$A_{11} =$ submatrix of A formed by deleting row ***1*** and column ***1*** $=[5]$

$A_{12} =$ submatrix of A formed by deleting row ***1*** and column ***2*** $=[-1]$

$A_{21} =$ submatrix of A formed by deleting row ***2*** and column ***1*** $=[3]$

$A_{22} =$ submatrix of A formed by deleting row ***2*** and column ***2*** $=[2]$

and

$$\det A_i(e_j) = (-1)^{i+j} \det(A_{ji}) = C_{ji}$$

where e_j is the jth column of identity matrix $I_{n\times n}$

So in this case

$$C_{11} = \det A_1(e_1) = (-1)^{1+1} \det A_{11} = (+1)\det[5] = 5$$

$$C_{12} = \det A_2(e_1) = (-1)^{1+2} \det A_{12} = (-1)\det[-1] = (-1)(-1) = 1$$

$$C_{21} = \det A_1(e_2) = (-1)^{2+1} \det A_{21} = (-1)\det[3] = -3$$

$$C_{22} = \det A_2(e_2) = (-1)^{2+2} \det A_{22} = (+1)\det[2] = 2$$

By Cramer's rule,

$$\left\{(i,j) - entry\ of\ A^{-1}\right\} = x_{ij} = \frac{\det A_i(e_j)}{\det A} = \frac{C_{ji}}{\det A}$$

So for the current matrix;

$$\left\{(1,1) - entry\ of\ A^{-1}\right\} = x_{11} = \frac{\det A_1(e_1)}{\det A} = \frac{C_{11}}{\det A} = \frac{5}{13}$$

$$\left\{(1,2) - entry\ of\ A^{-1}\right\} = x_{12} = \frac{\det A_1(e_2)}{\det A} = \frac{C_{21}}{\det A} = \frac{-3}{13}$$

$$\left\{(2,1) - entry\ of\ A^{-1}\right\} = x_{21} = \frac{\det A_2(e_1)}{\det A} = \frac{C_{12}}{\det A} = \frac{1}{13}$$

$$\left\{(2,2) - entry\ of\ A^{-1}\right\} = x_{22} = \frac{\det A_2(e_2)}{\det A} = \frac{C_{22}}{\det A} = \frac{2}{13}$$

Hence by using equation # 4, we get

$$A^{-1} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} \frac{C_{11}}{\det A} & \frac{C_{21}}{\det A} \\ \frac{C_{12}}{\det A} & \frac{C_{22}}{\det A} \end{bmatrix} = \frac{1}{\det A}\begin{bmatrix} C_{11} & C_{21} \\ C_{12} & C_{22} \end{bmatrix} = \begin{bmatrix} \frac{5}{13} & \frac{-3}{13} \\ \frac{1}{13} & \frac{2}{13} \end{bmatrix}$$

Q5. If $A = \begin{bmatrix} 4 & 5 \\ 6 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -\frac{7}{2} & \frac{5}{2} \\ 3 & -2 \end{bmatrix}$ then show that B is multiplicative of A?

Solution:

$$AB = \begin{bmatrix} 4 & 5 \\ 6 & 6 \end{bmatrix} \begin{bmatrix} -\frac{7}{2} & \frac{5}{2} \\ 3 & -2 \end{bmatrix}$$

$$AB = \begin{bmatrix} \frac{47}{5} & 0 \\ \frac{48}{5} & 3 \end{bmatrix} \neq \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

As AB is not equal to I so ,A is not multiplicative identity of B

MIDTERM EXAMINATION
Spring 2010
(MTH501- Linear Algebra (Session - 3)

Q1. Find vector and parametric equation of the plane that passes through the origin of R^3 and is parallel to the vectors V_1= (1, 2, 5) and V_2 = (5, 0, 4).

Solution:

As vector equation of the plane passing through origin is $x = t_1 v_1 + t_2 v_2$
Let x = (x, y, z) then this equation can be expressed in component form as
$(x, y, z) = t_1 (1, 2, 5) + t_2 (5,0,4)$
This is the **vector equation of the plane**.

Equating corresponding components, we get
$x = t_1 + 5 t_2$, $y = 2 t_1$, $z = 5 t_1 + 4 t_2$
These are the **parametric equations of the plane.**

Q2. Which of the following is true? If V is a vector space over the field F.(justify your answer)

a) $\{\frac{x+y}{x} \varepsilon V, y \varepsilon V\}=V$

b) $\{\frac{x+y}{x} \varepsilon V, y \varepsilon V\}=VxV$

c) $\{\frac{\lambda V}{V} \varepsilon V, \lambda \varepsilon F\}FxV$

Solution:

(b) and (c) both are correct vector space V over a field F is a set V equipped with an operation called (vector) addition, which takes vectors u and v and produces another vector .

There is also an operation called scalar multiplication, which takes an element and a vector and produces a vector .

Q3. *let*

$v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, v_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$ and $y=\begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$

for what value (s) of h is y in the plane is generated by

v_1 and v_2 ?

Solution:

we can write in matrix form as

$$\begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ -2 & 7 & -5 \end{bmatrix}$$

$$2R_1 + R_3 \longrightarrow R_3' \begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 3 & 2h-5 \end{bmatrix}$$

$$2R_2 + R_1 \longrightarrow R_1' \begin{bmatrix} 1 & 0 & h-6 \\ 0 & 1 & -3 \\ 0 & 3 & 2h-5 \end{bmatrix}$$

$$-3R_2 + R_3 \longrightarrow R_3' \begin{bmatrix} 1 & 0 & h-6 \\ 0 & 1 & -3 \\ 0 & 0 & 2h+4 \end{bmatrix}$$

for h=2

y is in the plane generated.

Q8.given A and b ,write the augmented matrix for the linear system that corresponds to the matrix equation Ax=b. then solve the system and write the solution as a vector.

$$A=\begin{bmatrix}1 & 2 & 1\\ -3 & -1 & 2\\ 0 & 5 & 3\end{bmatrix}, b=\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}$$

Solution:

we can write the given ab in the matrix equation form Ax=b

$$Ax=\begin{bmatrix}1 & 2 & 1\\ -3 & -1 & 2\\ 0 & 5 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}=b$$

or

$$\begin{bmatrix}1 & 2 & 1\\ -3 & -1 & 2\\ 0 & 5 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\ 1\\ -1\end{bmatrix}$$

Q9. Q5.Find the AREA of parallelogram of the vertices (1,2,4)(2,4,-7) and (-1,-3,20). PG# 239

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Virtual University	**MIDTERM EXAMINATION(Solution File)** **SEMESTER SPRING 2004** **MTH501- Linear Algebra**	**Total Marks:40** **Duration: 90min**

Instructions

1. **The duration of this examination is 90 minutes.**
2. **This examination is closed book, closed notes, closed neighbors.**
3. **Answer all questions.**
4. **Do not ask any questions about the contents of this examination from anyone.**
5. **All the questions are descriptive in that paper, you can't answer yes or no only and if in certain questions your answer is yes or no then you have to justify your answer.**
6. **There is no mistake in the question paper and if you feel that there is something wrong with the question then made the best assumptions which you think and then give the answer.**

Question No: 1 **Marks: 5**

Determine the value(s) of h such that the matrix $\begin{bmatrix} 1 & 4 & -2 \\ 3 & \boldsymbol{h} & -6 \end{bmatrix}$ is augmented matrix of a consistent system.

Solution:

$$\begin{bmatrix} 1 & 4 & -2 \\ 3 & \boldsymbol{h} & -6 \end{bmatrix} \square \begin{bmatrix} 1 & 4 & -2 \\ 0 & -12+\boldsymbol{h} & 0 \end{bmatrix} \text{ By } \boldsymbol{R}_2 - 3\boldsymbol{R}_1$$

It implies, $x_1 + 4x_2 = -2$

$(-12 + h)\, x_2 = 0$

Therefore, the above system of equations is consistent for all h.

Question No: 2 **Marks: 5**

Let $\boldsymbol{u} = \begin{bmatrix} 8 \\ 2 \\ 3 \end{bmatrix}$ and $\boldsymbol{A} = \begin{bmatrix} 4 & 3 & 5 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$. Is $\boldsymbol{u}$ in the subset of $\boldsymbol{R^3}$ spanned by the columns of $\boldsymbol{A}$? Why or why not?

Solution:

Consider $\boldsymbol{u} = a_1 \boldsymbol{c_1} + a_2 \boldsymbol{c_2} + a_3 \boldsymbol{c_3}$

Where a_1, a_2 and a_3 are any scalars.

It implies that

$$\begin{bmatrix} 8 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4a_1 + 3a_2 + 5a_3 \\ a_2 - a_3 \\ a_1 + 2a_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{aligned} 4a_1 + 3a_2 + 5a_3 &= 8 \\ a_2 - a_3 &= 2 \qquad (1) \\ a_1 + 2a_2 &= 3 \end{aligned}$$

Now $\begin{vmatrix} 4 & 3 & 5 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{vmatrix} = 4(0+2) - 0 + 1(-3-5) = 8 - 8 = 0$

Therefore, the system (1) is inconsistent. Hence $\boldsymbol{u}$ is not in the subset of $\boldsymbol{R^3}$ spanned by the columns of A.

Question No: 3 **Marks: 5**

Assume that T is a linear transformation. Find the standard matrix of T.
T: $\boldsymbol{R^2} \rightarrow \boldsymbol{R^4}$, T($\boldsymbol{e_1}$) = (1, 2, 0, 5) and T($\boldsymbol{e_2}$) = (3, -6, 1, 0), where $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ are the columns of 2 x 2 identity matrix.

Solution:

The standard matrix is:

$$A = \begin{bmatrix} T(\boldsymbol{e_1}) & T(\boldsymbol{e_2}) \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & -6 \\ 0 & 1 \\ 5 & 0 \end{bmatrix}$$

Question No: 4 **Marks: 5**

Find the entries in the second row of $\boldsymbol{AB}$, where

$$\boldsymbol{A} = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ -3 & 0 & 9 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix}$$

Solution:

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$$\rightarrow \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ -3 & 0 & 9 \end{bmatrix} \begin{bmatrix} \downarrow & \downarrow \\ 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} * & * \\ -4+21-12 & 6+3-8 \\ * & * \\ * & * \end{bmatrix} = \begin{bmatrix} * & * \\ 5 & 1 \\ * & * \\ * & * \end{bmatrix}$$

Question No: 5 **Marks: 5**

Solve the equation $\boldsymbol{Ax} = \boldsymbol{b}$ by using the $\boldsymbol{LU}$ factorization given for $\boldsymbol{A}$.

$$\boldsymbol{A} = \begin{bmatrix} 4 & 3 & -5 \\ -4 & -5 & 7 \\ 8 & 6 & -8 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

Solution:

Consider

$$\begin{cases} Ly = b \\ Ux = y \end{cases}$$

$$Ly = b \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix} \Rightarrow y_1 = 2, y_2 = -2, y_3 = 2$$

$$Ux = y \Rightarrow \begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} \Rightarrow x_1 = 1/4, x_2 = 2, x_3 = 1$$

Question No: 6 **Marks: 5**

Determine the matrix $\begin{bmatrix} 9 & -5 & 2 \\ 5 & -8 & -1 \\ -2 & 1 & 4 \end{bmatrix}$ is strictly diagonally dominant.

Solution:

Consider

$$|9| > |-5| + |2|$$
$$|-8| > |5| + |-1|$$
$$|4| > |-2| + |1|$$

Therefore, the given matrix is strictly diagonally dominant.

Question No: 7 **Marks: 5**

Let S be the parallelogram determined by the vectors $\boldsymbol{b}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\boldsymbol{b}_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$, and let $\boldsymbol{A} = \begin{bmatrix} 1 & -.1 \\ 0 & 2 \end{bmatrix}$. Compute the area of the image of S under the mapping $\boldsymbol{x} \rightarrow \boldsymbol{Ax}$.

Solution:

The area of S is $\left| \det \begin{bmatrix} 1 & 5 \\ 3 & 1 \end{bmatrix} \right| = 14$, and det A = 2. Therefore, by definition, the area of image of S under the mapping $\boldsymbol{x} \rightarrow \boldsymbol{Ax}$ is |det A|.{area of S}= 2 . 14 = 28

Question No: 8 **Marks: 5**

Let $\boldsymbol{A} = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Determine if $\boldsymbol{w}$ is in Col $\boldsymbol{A}$?

Solution:

Consider $\boldsymbol{w} = a_1 \boldsymbol{c_1} + a_2 \boldsymbol{c_2} + a_3 \boldsymbol{c_3}$
Where a_1, a_2 and a_3 are any scalars.
The Augmented matrix is:

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$$\begin{bmatrix} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{bmatrix} \square \begin{bmatrix} 1 & 0 & 1 & -1/2 \\ 0 & 1 & 1/2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

It implies, $a_1 + a_3 = -1/2$

$a_2 + (1/2)a_3 = 1$

Hence ***w*** is in Col A.

MTH501-Linear Algebra
Midterm Special 2006
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Question # 1:-

Let A= $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$ Write 5A.Is det5A=5detA?

Let A = $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$

$$5A = 5\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 20 & 10 \end{bmatrix}$$

$$\det 5A = ((15 \times 10) - (20 \times 5))$$

$$\det 5A = 150 - 100 = 50$$

$$\det 5A = 50$$

$$\det A = ((3 \times 2) - (1 \times 4))$$

$$\det A = 6 - 4 = 2$$

$$5 \det A = 5 \times 2 = 10$$

Hence proved det5A not equal to 5detA

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Question #2:-

Let A = $\begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}$ .Show that these matrices do not commute.

Solution:-

$$AB = \begin{bmatrix} 5 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 14 & 3 \\ -2 & -6 \end{bmatrix}$$

$$BA = \begin{bmatrix} 2 & 0 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 10 & 2 \\ 29 & -2 \end{bmatrix}$$

$$AB \neq BA$$

==========

Question # 3:-

Use Cramer's rule to solve the system $3x_1 - 2x_2 = 6 \qquad -5x_1 + 4x_2 = 8$,

Note : In order to get full marks do all necessary steps

Solution:-

$3x_1 - 2x_2 = 6$

$-5x_1 + 4x_2 = 8$

View the System as Ax =b. Using the notation introduced above**.**

$$A\begin{bmatrix} 3 & -2 \\ -5 & 4 \end{bmatrix}$$

$$A_1(b) = \begin{bmatrix} 6 & -2 \\ 8 & 4 \end{bmatrix}$$

$$A_2(b) = \begin{bmatrix} 3 & 6 \\ -5 & 8 \end{bmatrix}$$

Since detA = 2, the system has a unique solution. By Cramer's rule,

$$x_1 = \frac{\det A_1(b)}{\det A} = \frac{24+16}{2} = 20$$

$$x_2 = \frac{\det A_2(b)}{\det A} = \frac{24+30}{2} = 27$$

==========

Question # 4:-

Find the matrix of linear transformation $T : R^3 \to R^4$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 - x_3, x_1)$ with respect to the standard basis of R^3 ?

Note : In order to get full marks do all necessary steps

Solution:-

The standard basis of R^3 is $\{e_1, e_2, e_3\}$
$E_1 = (1, 0, 0)$
$E_2 = (0, 1, 0)$
$E_3 = (0, 0, 1)$

$T(e_1) = T(1, 0, 0) = (1, 0, 1, 1)$
$T(e_2) = T(0, 1, 0) = (1, 1, 0, 0)$
$T(e_3) = T(0, 0, 1) = (0, 1, -1, 0)$
The matrix of linear transformation is

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$$

==========

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 0 & k & 2 \\ 0 & 4 & 2 \end{bmatrix}$$

If such that $|A| = 0$ then the value of k is

- 8
- **4**
- 0
- None of these

$$x + y = 1$$
$$x + y = 0$$

The system of linear equations has

- **No solution**
- Infinitely many solutions
- Infinitely many solutions
- d) None of these

==========

$$A = \begin{bmatrix} 4 & 3 \\ 7 & 2 \\ 9 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 5 \\ 1 & 6 \end{bmatrix}$$

Let then AB is

$$\begin{bmatrix} 11 & 38 \\ 16 & 47 \\ 18 & 45 \end{bmatrix}$$

- **a)**
- b) A and B are not comfortable for multiplication

$$\begin{bmatrix} 11 & 38 \\ 16 & 47 \\ 8 & 45 \end{bmatrix}$$

- c)
- d) None of these

==========

$$\begin{bmatrix} -4 \\ 6 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

Determine whether the set S={ } is a spanning set for R^2 or not.

Note : In order to get full marks do all necessary steps

==========

Let T : $R^2 \to R^3$ be a linear transformation defined by T(x)=Ax , where

$$\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix} \qquad \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$$

A= ,then find an x in R^2 whose image under T is b.Where b=

Note : In order to get full marks do all necessary steps

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$$\begin{bmatrix} 1 & 2 \\ 5 & 1 \end{bmatrix}$$

The inverse of the matrix is

$$\begin{bmatrix} 1 & -2 \\ -5 & 1 \end{bmatrix}$$

- (a) $\frac{1}{9}\begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$
- b) $\frac{1}{9}\begin{bmatrix} -1 & 2 \\ 5 & -1 \end{bmatrix}$
- **(c)**
- **d) None of these**

==========

The set of vectors {(5,0,0), (7,2,-6), (9,4,-8)} is,

- a) Linearly independent
- **b) Linearly dependent**
- c) Basis of R^3