

MTH103 (Elementary Mathematics III)

Exam Summary

- Vectors are objects that have both magnitude and direction.
- A Scalar quantity has magnitude but no direction.
- Vector quantity can be represented by lines, drawn such that the length of the line denotes the magnitude of the vector, while the direction of the line represents the direction of the vector.
- Two vectors are said to be equal if and only if they have the same magnitude and direction.
- The types of vectors include position vector, line vector, and free vector.
- The vector sum is given by a single equivalent vector joining the beginning of the first vector to the end of the last.
- If the vector diagram is closed, then the end of the last vector, joins or meets with the beginning of the first, so that the resultant sum is a vector with no magnitude.
- A single vector can be used to replace any number of component vectors so long as they form a chain in the vector. i.e., $AB + BC + CD + DE = AE$
- Vector AB is defined by its magnitude (r) and its angle (θ)
- If r is the Hypotenuse of a right-angled triangle with sides a and b , then $r = ai + bj$, where i and j are unit vectors in the horizontal and vertical directions.
- The symbols i , j and k represent unit vector in the directions OX , OY , OZ respectively. If $OP = ai + bj + ck$, then $|OP| = r = \sqrt{a^2 + b^2 + c^2}$
- Direction cosines are the cosines of the angles between a line and the coordinate axes.
- The direction cosines $[l, m, n]$ are the cosines of the angles between the vector and the axes OX , OY , OZ respectively.
- For the vector $r = ai + bj + ck$, where i , j , k are unit vectors, you have that $l = a/r$, $m = b/r$, $n = c/r$, $l^2 + m^2 + n^2 = 1$ and $r = \sqrt{a^2 + b^2 + c^2}$
- A scalar quantity is a quantity that has magnitude but no direction.
- The scalar product (dot product) $a \cdot b$, is equal to $ab \cos \theta$ and θ is the angle between a and b .
- If $a = a_1i + a_2j + a_3k$ and $b = b_1i + b_2j + b_3k$, then $a \cdot b = a_1b_1 + a_2b_2 + a_3b_3$.
- The dot product of two vectors at a right angle to each other is zero.
- Vector product, also known as the cross product of a and b is represented by $a \times b$. Where $a \times b = |a||b| \sin \theta$.
- i. If $a = a_1i + a_2j + a_3k$ and $b = b_1i + b_2j + b_3k$, then $a \times b = (a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1)j + (a_1b_2 - a_2b_1)k$
- **Gradient** (m) = dy/dx
- **Angle of Slope** $\theta = \tan^{-1} dy/dx$
- **Distance between** (x_1, y_1) and (x_2, y_2) is $(x_2 - x_1)^2 + (y_2 - y_1)^2$
- **Midpoint** of (x_1, y_1) and (x_2, y_2) is $\frac{1}{2}(x_1 + x_2)$, $\frac{1}{2}(y_1 + y_2)$
- Gradient and y-intercept form is $y = mx + c$
- Gradient and one point form is $y - y_1 = m(x - x_1)$
- Two-point form is $(y - y_1)/(y_2 - y_1) = (x - x_1)/(x_2 - x_1)$
- Acute angle β between lines of gradients m_1, m_2 is given by $\tan \beta = (m_2 - m_1)/(1 + m_2m_1)$
- For parallel lines, $m_1 = m_2$. For perpendicular line, $m_1m_2 = -1$
- The equation of a circle with center O , radius r is: $x^2 + y^2 = r^2$

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- Circle, centre (a, b) , radius r has equation : $(x - h)^2 + (y - k)^2 = r^2$
- General equation of circle: $x^2 + y^2 + 2gx + 2fy + c = 0$
- A parabola is defined as the locus of a point which moves so that it is always the same distance from the fixed point (called the focus) and a given line (called the directrix).
- The equation of a parabola with a horizontal axis is $y^2 = 4px$
- The equation of a parabola with a vertical axis is $x^2 = 4py$
- If the axis of a parabola is vertical and the vertex is at (h, k) , the equation of the parabola is: $(x - h)^2 = 4p(y - k)$
- If the axis of a parabola is horizontal and the vertex is at (h, k) , the equation of the parabola is: $(y - k)^2 = 4p(x - h)$
- An Ellipse can be defined as the locus of all points that satisfy the equation: $x^2/a^2 + y^2/b^2 = 1$
- By placing a hyperbola on an x - y graph (centered over the x -axis and y -axis), the equation of the curve is: $x^2/a^2 - y^2/b^2 = 1$