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MOTTO: KNOWLEDGE FOR CAREER CONSCIOUSNESS

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MTH102: ELEMENTARY MATHEMATICS II

Many quantities in everyday life depend on one or more_____

Changing variables

_____ is a phenomenon that relates how one variable or quantity depends on the other variable or quantity

A function OR

The relationship between two variables such that, to each value of the dependent there is exactly one corresponding value of the dependent variable

A function

_____of the function is the set of all values of the independent variable for which the function is defined. **The domain the** set of all values taken on by the dependent variable. Is called

The Range

A _____is a correspondence from a first set, called the domain**,**

Function

A Function is a correspondence to a second set, called_____

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The Range

A function is mostly specified by _____

Equations or formulas

_____ can Also be described using tables, graphs and diagram

A function

$$y = 4x^2 + 3$$

The equation above describes Y as a function of _____

Function of X

From the equation above is the independent variable and y is_____

The dependent variable.

Why is it helpful to isolate the dependent variable on the left side?

To decide whether an equation defines a function

FOR EXAMPLE

In the equation. X+Y =1

In the above equation to determine where the equation defines Y and a function of X,

The equation is written as Y= V~X

From this form, you can see that for any value of X , there is exactly one value of Y . So, Y is a function of X .

equations that assign two values (_+) to the dependent variable for a given value of the independent variable do not_____ **define functions of x**

You generally isolate the dependent variable on the left. When using_____

an equation to define a function,

THEREFOR.....

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The equation X+3y=1 is then written as _____

$$y = \frac{1-x}{3}$$

From the equation above Y is _____

The dependent variable

In function notation the above formula is written as_____

$$f(x) = \frac{1-x}{3}$$

from this equation_____is The independent variable (input).

X the name of the function is _____

''f''.

The symbol f(x) is read as _____

''f of x'' or ''f at x''

The symbol f(x) represent _____ **the**

value of the function at the number x.

What is the value of f when x =3?

$$f(3) = \frac{1-(3)}{3} = \frac{-2}{3} = \frac{-2}{3}$$

_____is the value of f (3) in the above value of F when x=3?

A function value

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it allows you to be less wordy is an advantage of _____

Function notation

For instance, instead of asking ''what is the value of y when x=3?'' you can ask ''what is f(3)?

Note that the function f(x) does not mean_____

F times X

_____is the opposite of differentiation

Integration

_____is the graph of its ordered pairs.

The graph of a function

The horizontal axis represents the independent variable and the vertical axis represents_____

The dependent variable

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Example Evaluating a function

Find the value of the function

$f(x) = 2x^2 - 4x + 1$

When x= −1, 0, and 2. Is f one to one?

Solution

When x= −1, the value of is

$f(x) = 2x^2 - 4x + 1$ **This is the given function.**

$f(-1) = 2(-1)^2 - 4(-1) + 1$ **Replace each occurrence of x with −1.**

$= 2(1) - 4(-1) + 1$ **Evaluate the exponential expression.** $(-1)^2 = 1$

$= 2 + 4 + 1$ **Perform the multiplications.**

$= 7 \therefore f(-1) = 7$

When x =0, the value of f is

$f(0) = 2(0)^2 - 4(0) + 1$

$f(0) = 0 - 0 + 1 = 1$

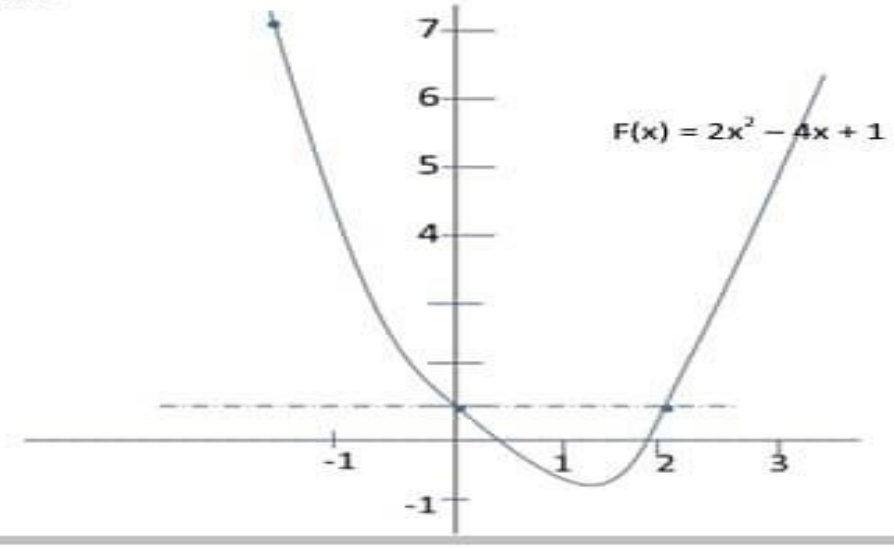
When x=2, the value of f is

$f(2) = 2(2)^2 - 4(2) + 1$

$= 2(4) - 4(2) + 1$

$= 8 - 8 + 1 = 1$

Diagram



Just like the example above, a function is not one to one if_____ **two**

different values of x yield the same value of f(x),

_____is the graph of its ordered pairs.

The graph of a function

_____ states that if every vertical line intersects the graph of an equation at most once, then the equation defines y as a function of x.

vertical line test.

A function defined by two or more equations over a specified domain is called_____

piecewise – defined function

A function is _____ if to each value of the dependent variable in the range there corresponds exactly one value of the independent variable.

one to one

Geometrically, a function is one- to- one if,,,,,,,,,,,,,,,,

to each value of the dependent variable in the range there corresponds exactly one value of the independent variable.

a graph that represents a one- to –one test must satisfy_____and _____

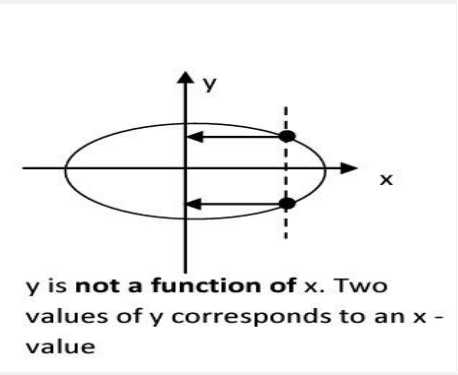
The vertical line test and the horizontal line test.

What happens If any vertical line intersects a graph in more than one point,?

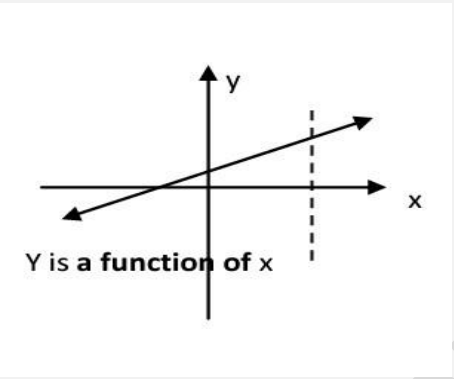
the graph does not define y as a function of x.

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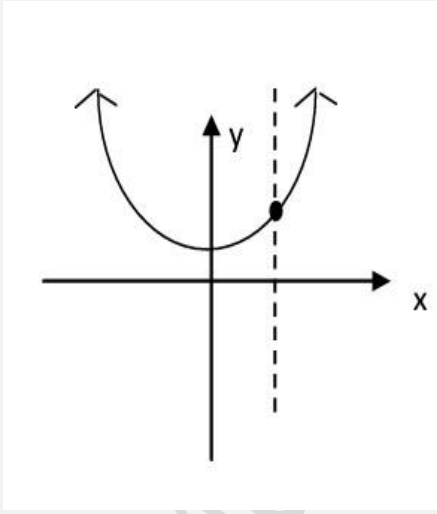
CONSIDER THESE GRAPHS BELOW



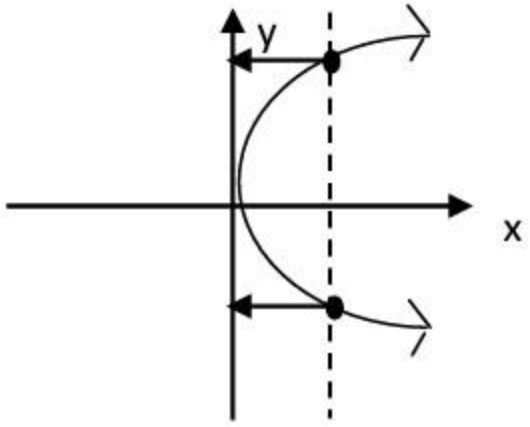
y is **not a function of** x. Two values of y corresponds to an x - value



Y is **a function of** x



Y is a function of X



Y is not a function of x. Two values

Of Y corresponds to an X value

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Example

Find the value of $f(x) = x^2 - 5x + 1$ when x is 0, 1, and 4. Is f one to one?

Solution

When x = 0, the value of f is

$f(0) = 0^2 - 5(0) + 1 = 1$

When x = 1, the value of f is

$f(1) = 1^2 - 5(1) + 1$

$= 1 - 5 + 1 = -5$

When x = 4, the value of f(x) is

$= 4^2 - 5(4) + 1$

$= 16 - 20 + 1 = -3$

The function given by (f.g) (x) = f (g(x)) is _____

The composite of f with g.

_____is the set of all x in the domain of g such that g(x) is the domain of f.

The domain of (f.g)

Let f and g be two functions such that: f (g(x)) = x for each x in the domain of g and g(f(x)) = x for each x in the domain of f Under these conditions, the function g is_____

T**he inverse of the function f.**

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DIFFERENTIATION TECHNIQUES

- Product rule; y = uv ; $\frac{dy}{dx} = U\frac{dv}{dx} + V\frac{du}{dx}$

- Quotient rule; $y = \frac{u}{v}$; $\frac{dy}{dx} = \frac{V\frac{du}{dx} - U\frac{dv}{dx}}{V^2}$

- Function of a function ; $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$: this is also referred to as the chain rule.

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DIFFERENTIATION

Derivative of a function of a real variable measures the sensitivity to change of a quantity(dependent) which is determined by another quantity(independent)

Derivative of power of x^n

If $y = ax^n$, $\frac{dy}{dx} = nax^{n-1}$

If y = k(where k is a constant), $\frac{dy}{dx} = 0$

Example; $y = x^3 + 5x^2 - 4x + 2$

$= 3x^{3-1} + (2)5x^{2-1} - 4x^{1-1} + 0$

$= \frac{dy}{dx} = 3x^2 + 10x - 4$

STANDARD DERIVATIVES

$y = \sin x$, $\frac{dy}{dx} = \cos x$

$y = \cos x$, $\frac{dy}{dx} = -\sin x$

$y = \tan x$, $\frac{dy}{dx} = sec^2 x$

Two functions can be combined in various ways to create new functions.

For instance, if f (x) = 2x – 3 and g(x) = x^2 + 1, you can form the following functions.

f(x) + g(x) = (2x – 3) + (x^2 + 1) = x^2 + 2x – 2 **Sum.**

f(x) – g(x) = (2x - 3) – (x^2 + 1) = - x^2 + 2x – 4 **Difference.**

f(x) g(x) = (2x – 3) (x^2 + 1) = $2x^3 - 3x^2 + 2x - 3$ **Product.**

f(x)/g(x) = $\frac{2x-3}{x^2}$ **Quotient/ division**

Example

Forming composite functions

Let f(x) = 2x – 4 and g(x) =x^2+ 3, and find

a. f (g(x)) b. g(f(x))

Solution

a. The composite of f with g is given by

g (f(x)) = 2(g(x)) – 4 **Evaluate f at g(x).**

= 2(x^2 + 3) – 4 **Substitute x^2+ 3 for g(x).**

= 2x^2 + 6 – 4 **Simplify.**

= 2x^2 – 2

= 2(x^2 - 1)

b. The composite of g with f is given by

g (f(x)) = (f(x)) ^{2} + 1 **Evaluate g at f(x).**

= $(2x - 4)^2$ + 1 **Substitute 2x – 4 for f(x).**

= $4x^2 - 16x + 16 + 1$ **Simplify.**

= $4x^2 - 16x + 17$

Checkpoint

Let $f(x) = 2x + 1$ and $g(x) = x^2 + 2$, and find

a) $f(g(x))$. b) $g(f(x))$.

If f and g are two functions such that: f (g(x)) = x for each x in the domain of g and g(f(x)) = x for each x in the domain of f

The function of g here is called _____

Inverse of the function

The inverse of a function is denoted by_____

$$f^{-1}$$

The inverse of a function F^{-1} is read as_____

"f- inverse"

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Example : Finding the inverse function

Find the inverse function of f(x) = $\sqrt{2x-3}$.

Solution

Begin by substituting f(x) with y. Then interchange x and y and solve for y.

$f(x) = \sqrt{2x-3}$ Write original function.

$y = \sqrt{2x-3}$ Replace f(x) with y.

$x = \sqrt{2y-3}$ Interchange x and y.

$x^2 = 2y-3$ Squaring each side.

$x^2+3 = 2y$ Add 3 to each side.

$\frac{x2+3}{2}$ =y Divide each side by 2.

So, the inverse function has the form

$$f^{-1}(__) = \frac{(__)^2+3}{2}$$

Using x as the independent variable, you can write $f^{-1}(x) = \frac{x^2+3}{2}$ as $x \geq 0$.

In the figure 1.24, note that the domain of f^{-1} coincides with the range of f.

Diagram

Not every function has_____

An inverse function.

For a function to have an inverse function, it must be_____

one- to- one.

FOR EXAMPLE.....

A function that has no inverse function.

Show that the function $f(x) = x^2 - 1$ has no inverse function. (Assume that the domain of f is the set of all real numbers)

Solution

Begin by sketching the graph of f, as shown in figure 1.25 Note that

$f(x) = x^2 - 1$

$f(2) = 2^2 - 1 = 3$

And

$f(-2) = (-2)^2 - 1 = 3$

So, f does not pass the horizontal line test, which implies that if is not one- to- one and therefore has no inverse function. The same conclusion can be obtained by trying to find the inverse of f algebraically.

$f(x) = x^2 - 1$	**Write original function.**
$y = x^2 - 1$	**Replace f(x) with y**
$x = y^2 - 1$	**Interchange x and y**
$x + 1 = y^2$	**Add 1 to each side**
$\pm\sqrt{x+1} = y$	**Take square root of each side**

The last equation does not define y as a function of x, and so f has no inverse function

y

$F(x) = x^2 - 1$	write original function.
$Y = x^2 - 1$	Replace f(x) with y
$X = y^2 - 1$	Interchange x and y
$X + 1 = y^2$	Add1 to each side
$+\sqrt{x+1=y}$	Take square root of each side

The last equation does not define y as a function of x, and so f has no inverse function

_____states that provided an elastic limit of a spring is not exceeded; the extension (e) is directly proportional to the tension or force acting on it.

Hooke's law

THE LIMIT OF A FUNCTION IS DEFINED MATHEMATICALLY BY SAY…….……

If $f(x)$ becomes arbitrarily close to a single number **L** as x approaches **c** from either side, $\lim_{x\to c} f(x) = L$, then $\text{Lim}\, f(x)$ which is read as **"the limit of $f(x)$ as x approaches c is L"**

Limits can be evaluated by_____

Direct substitution

PROPERTIES OF LIMIT

Let b and c be real numbers, and let n be a positive integer

i. $\lim_{x\to c} b = b$

ii. $\lim_{x\to c} x = c$

iii. $\lim_{x\to c} x^n = x^c$

iv. $\lim_{x\to c} x^n = c^n$

In the property IV, if n is even, then c must be_____

positive.

OPERATIONS WITH LIMITS

Let b and c be real numbers let n be a positive integer, and

let f and g be functions with the following limits.

$\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = k$

I.	Scalar multiple:	$\lim_{x\to c}[b\,f(x)] = b\,L$.
II.	Sum or Difference:	$\lim_{x\to c}[f(x) \pm g(x)] = L \pm k$.
III.	Product:	$\lim_{x\to c}[f(x)\,.\,g(x)] = L\,.\,k$.
IV.	Quotient:	$\lim_{x\to c} f(x)/g(x) = L/k$ provided that $k \neq 0$.
V.	Power:	$\lim_{x\to c}[f(x)]^n = L^n$
VI.	Radical:	$\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{L}$

In property VI, if n is even, then L must be_____ **positive**

Example 1: Find the limit:

$\lim_{x\to 1}(x^2+1)$

Solution.

Using direct substitution by substituting 1 for x

$\lim_{x\to 1}(x^2+1) = 1^2+1 = 2$

$\lim_{x\to c} p(x) = p(c)$ if _____

If p is a polynomial function and c is any real number

FOR EXAMPLE

Finding the limit of a polynomial function

Find the limit: $\lim_{x\to 2}(x^2 + 2x - 3)$

Solution:

$\lim_{x\to 2} x^2 + 2x - 3 = \lim_{x\to 2} x^2 + \lim_{x\to 2} 2x - \lim_{x\to 2} 3$ **Applying property II.**

$= 2^2 + 2(2) - 3$ **Use direct substitution.**

$4 + 4 - 3 = 5$ **Simplify.**

$= 5$

The example above show or state that the limit of polynomial can be evaluated by_____ **direct substitution.**

TECHNICS FOR EVALUATING LIMITS

- **The Replacement Theorem/Technique**

Let c be a real number and f(x) = g(x) for all $x \neq c$. if the limit of g(x) exists as $x \to c$, then the limit of f(x) also exists and

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} g(x)$$

Dividing Out Technique.

Example

Find the limit: $\lim_{x\to -3} \frac{x^2+x+6}{x+3}$

Solution.

Using direct substitution will fails because both the numerator and thedenominator are zero when x = -3.

$$\lim_{x\to -3} \frac{x^2+x+6}{x+3}.$$

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Rationalizing The Numerator Technique.

Example 6

Find the limit: $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$.

Solution

Direct substitution fails because both the numerator and the denominator are zero when x = 0.

In this case, you can rewrite the fraction by rationalizing the numerator by taking the conjugate of the numerator and using it to both the numerator and denominator.

Taking the conjugate of the numerator of the numerator $\sqrt{x+1}-1$ will be $\sqrt{x+1}+1$.

Conjugate of $\sqrt{x+1}-1$ is $\sqrt{x+1}+1$.

$$\therefore \frac{\sqrt{x+1}-1}{x}=\left(\frac{\sqrt{x+1}-1}{x}\right)\left(\frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}\right)$$

$$=\frac{x+1+\sqrt{x+1}-\sqrt{x+1}-1}{x(\sqrt{x+1}+1)}$$

$$=\frac{x+1-1}{x(\sqrt{x+1}+1)}=\frac{x}{x(\sqrt{x+1}+1)}=\frac{1}{\sqrt{x+1}+1},\ x\neq 0$$

Now, using the replacement theorem, you can evaluate the limit as follows:

$$\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}=\lim_{x\to 0}\frac{1}{\sqrt{x+1}+1}=\frac{1}{\sqrt{0+1}+1}=\frac{1}{\sqrt{1}+1}=\frac{1}{1+1}=\frac{1}{2}$$

A Limit can fail to exist when_____ **f(x)**

increases or decrease without bound as x approaches c.

In mathematics,_____refers to rigorous formulation of the intuitive concept of function that varies with no abrupt breaks or jumps.

continuity

if the X values are closed together, then the y value of the function will also be close. This expression implied_____ **Continuity**

if its graph on the interval can be traced using a pencil and paper without lifting the pencil from the paper (i.e. the graph of f is unbroken at c, and there are no holes, jumps, or gaps or to say that a function is continuous at x=c when there is no interruption in the graph of f at c. Then we can say_____

That the function is continuous

There are _____types of functions

Two (2)

_____and _____ are the two types of functions **polynomial**

functions and rational functions.

A _____ function is continuous at every real number.

polynomial

A _____function is continuous at every number in its domain.

rational

FOR EXAMPLE ; FIND THE CONTINUITY OF THE FOLLOWING

$f(x) = \frac{1}{x}$ (b) $f(x) = \frac{x^2-1}{x-1}$ (c) $f(x) = \frac{1}{x^2+1}$

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Solution

Each of these functions is a rational function and is therefore continuous at every number in its domain.

(a) The domain of $f(x) = \frac{1}{x}$ consist of all real numbers except x = o. So, this function is continuous on the intervals $(-\infty, 0)$ and $(0,\infty)$.

(b) The domain of $f(x) = (x^2 - 1)/ (x - 1)$ consists of all real numbers except x = 1. So, this function is continuous on the intervals $(-\infty, 1)$ and $(1,\infty)$.

(c) The domain of $f(x) = \frac{1}{x^2+1}$ consists of all real numbers. So, this function is continuous on the entire real line.

Discuss the continuity of each function:

(a) $f(x) = \frac{1}{x-1}$ (**Answer**: Continuous $(-\infty, 1)$ and $(1, \infty)$)

(b) $f(x) = \frac{x^2-4}{x-2}$ (**Answer**: Continuous $(-\infty, 2)$ and $(2, -\infty)$)

(c) $f(x) = \frac{1}{x^2+2}$ (**Answer**: Continuous on the entire real line)

A function is said to be_____ if and only if it is continuous at every point of its domain.

continuous

Consider an open interval **I** that contains a real number **c.** If a function **f** is defined on **I** (except possibly at c), and **f** is not continuous at **c**, then **f** is said to have a _____ at **c.**

discontinuity

There are _____ category of discontinuity

Two (2)

_____and_____are the categories of discontinuity

Removable and Non removable

A discontinuity is called_____ if f can be made continuous by appropriately defining (or redefining) f(c). **removable**

A discontinuity at x = c is _____ if the function cannot be made continuous at x=c by defining or redefining the function at x=c. **non removable**

_____ are fundamental tools of calculus

Derivatives

_____is the slope of the tangent line to the graph of the function at that point.

The derivative of a function of a single variable at a chosen input value

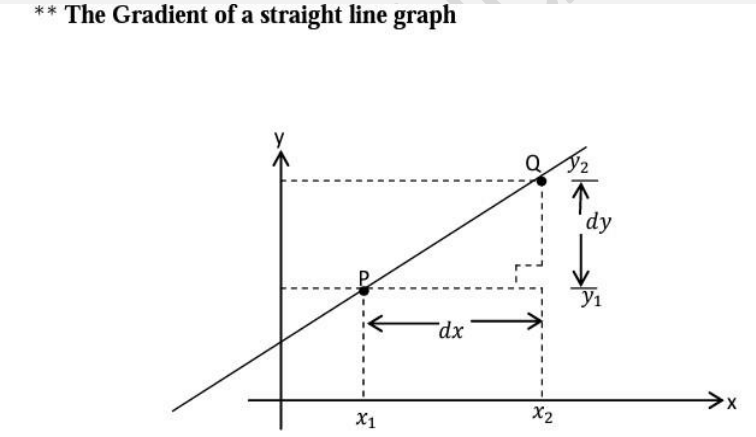
_____ is often described as the "instantaneous rate of change", **Derivative**

_____ is the action of computing a derivative.

Differentiation

The process of finding a derivative is called _____ **differentiation.**

** **The Gradient of a straight line graph**



_____ is defined as the ratio of the vertical distance the line rises or falls between two points P and Q to the horizontal distance between P and Q. **Gradient**

m is the symbol used denoting gradient of a straight line graph

i.e. $m = \frac{\delta y}{\delta x} = \frac{\Delta x}{\Delta y} = \frac{y_2 - y_1}{x_{2-} x_1}$

Example

If $y = x^2 + 3x$, find $\frac{\delta y}{\delta x}$ using first principle

$$y = x^2 + 3x \quad \text{----------} \quad (1)$$

$$y+\delta y = (x + \delta x)^2+3(x + \delta x) \text{----------} \quad (2)$$

Expanding the bracket in equation (2)

$$y+\delta y = x^2 + 2x\,dx + \delta x + |\delta x|^2 + 3x + 3\delta x$$

Subtracting y from both sides

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$y + \delta y - y = x^2 + 2x\,dx + \delta x + |\delta x|^2 + 3x + 3\delta x - y$

$\delta y = x^2 + 2x\,dx + |\delta x|^2 + 3x + 3\delta x - y$

Replacing the value of y in equation (1) in the term above

$\delta y = x^2 + 2x\,dx + |\delta x|^2 + 3x + 3\delta x - (x^2 + 3x)$

$\delta y = x^2 + 2x\delta x + |\delta x|^2 + 3x + 3\delta x - x^2 - 3x$

Colleting like terms

$\delta y = x^2 - x^2 + 2x\delta x + |\delta x|^2 + 3x - 3x + 3\delta x$

$8y = 2x\delta x + |\delta x|^2 + 3\delta x$

Dividing throughout by δx

$\frac{\delta y}{\delta x} = 2\,x\,\frac{\delta x}{\delta x} + \frac{|\delta x|^2}{\delta x} + \frac{3\delta x}{\delta x}$

$\frac{\delta y}{\delta x} = 2\,x + |\delta x| + 3$

Lim $\delta y \to 0$, $\delta x \to 0$

$\frac{\delta y}{\delta x} = 2x + 0 + 3$

$\frac{\delta y}{\delta x} = 2x + 3$

Example

If $y = x^4 + 6x^3 - 4x^2 + 7x - 2$, *find* $\frac{\delta y}{\delta x}$ *and the value of* $\frac{\delta y}{\delta x}$ *at* $x = 2$

Solution

$y = x^4 + 6x^3 - 4x^2 + 7x - 2$

$\frac{\delta y}{\delta x} = 4x^{4-1} + 3.6x^{3-1} - 2.4x^{2-1} + 1.7x^{1-1} - 0$

$\frac{\delta y}{\delta x} = 4x^3 + 18x^2 - 8x^1 + 7$

At $x = 2$, $\frac{\delta y}{\delta x} = 4(2)^3 + 18(2)^2 - 8(2)^1 + 7$

$= 32 + 72 - 16 + 7$

$= 95$

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Hence the other standard derivative is:

Y	$\delta y/\delta x$
x^n	$n .x^{n-1}$
c	0
$a .x^n$	$an. x^{n-1}$
sin x	cos x
cos x	- sin x
tan x	sec^2x
e^x	e^x
a^x	a^x .lna
e^{kx}	ke^{kx}
a^x	a^x . lna
lnx	1/x
Cot x	- $cosec^2x$
Sec x	Sec x tan x
Cosec x	- Cosec x .Cotx
Sinh x	cosh x
Cosh x	sinh x

Differentiation of products of functions include:

- **Product rule**
- **quotient rule**
- **composite function**

The Product rule is also called _____ **Differentiation of products of functions** differentiation of a quotient of two function is called_____ **quotient rule**

A Function of a function is called_____ **composite function**

Differentiation of Products of Functions (Product Rule)

Let y = uv, where u and v are functions of x

If x → $x + \delta x$, u → u+δu, v →v + ∂v and as a result,y → y+ δy.

The above expression shows that an increment ∂x in x will turn produce increments ∂u in u and also producing a change ∂v in v and a change ∂y in y.

Using first principle:

If y = uv. . . . . (1)

Therefore, y+δy= (u+δu)(v+δv). . . . . (2)

Expanding the left hand side in equation (2) gives:

y + δy = uv+uδv+vδu+δu. δv . . . . . (3)

Subtracting y = uv in equation (3) gives

y +δy – y=uv+uδv+vδu+δu.δv-y

δy=uv+uδv +vδu + δu.δv-y

δy=uv+uδv+vδu+δu.δv-uv (where y=uv)

δy=uδv+vδu+δu. δv

Divide throughout by δx

$\frac{\delta y}{\delta x} = \frac{u\delta v}{\delta x} + \frac{v\delta u}{\delta x} + \delta u.\delta v$ . . . . . (4)

Limit if $\delta x \rightarrow 0$, $\frac{\delta y}{\delta x} \rightarrow \frac{\delta y}{\delta x}, \frac{\delta u}{\delta x} \rightarrow \frac{\delta u}{\delta x}, \frac{\delta v}{\delta x} \rightarrow \frac{\delta v}{\delta x}$

(NB tends to turns to)

Therefore, (4) now gives:

$$\frac{\delta y}{\delta x} = \frac{u\delta v}{\delta x} + \frac{v\delta u}{\delta x} + \frac{0.\delta v}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{u\delta v}{\delta x} + \frac{v\delta u}{\delta x}$$

Product Rule:

If y=uv

$$\frac{\delta y}{\delta x} = \frac{u\delta v}{\delta x} + \frac{v\delta u}{\delta x}$$

Example 1

Differentiate with respect to x: $y = x^3 \sin x$

$y = x^3 \sin x$

Let $u = x^3$ and $v = \sin x$

$\frac{\delta y}{\delta x} = \cos x, \frac{\delta u}{\delta x} = 3x^2$

$$\frac{\delta y}{\delta x} = \frac{u\delta v}{\delta x} + \frac{v\delta u}{\delta x} = x^3\cos x + \sin x\, 3x$$

$$= x^3\cos x + 3x\sin x$$

$$= x\,(x^2\cos x + 3\sin x)$$

Differentiation of a quotient of two function (Quotient Rule)

Let $y = \frac{u}{v}$ where u and v are functions of x.

An increment δx *in* x *will turn to produce increment and a change* δu *in* u and a change δv *in* v *and a change* δy *in* y

If $x \rightarrow x + \delta x, u \rightarrow u + \delta u, v \rightarrow v + \delta v$ *and as a result,* $y \rightarrow y + \delta y$

Using the first principle

If $y = \frac{u}{v}$ . . . . . .(1)

Then $y + \delta y = \frac{u+\delta u}{v+\delta v}$ . . . . . .(2)

Subtract y from both side of the term in equation (2) above

$$y + \delta y - y = \frac{u + \delta u}{v + \delta v} - y$$

$$\delta y = \frac{u + \delta u}{v + \delta v}\frac{-u}{v} \quad [where\ y = \frac{u}{v} \quad in\ (i)$$

$$\delta y = \frac{v(u + \delta u) - u(v + \delta v)}{(v + \delta v)v}$$

$\delta y = \frac{uv+v\delta u-uv-u\delta v}{v^2+v\delta v}$ Simplify by collecting like terms.

$$\delta y = \frac{v\delta u - v\delta u}{v^2 + v\delta v}$$

$$\delta y = \frac{v\delta u + u\delta v}{v^2 + v\delta v}$$

dividing both side by δx

$$\frac{\delta y}{\delta x} = \frac{\frac{v\delta u}{\delta x} + \frac{u\delta v}{\delta x}}{v^2 + v\,dv}$$

if $\delta x \rightarrow 0, \delta u \rightarrow 0$ *and* $\delta x \rightarrow 0$, *then equation* (3)

becomes

$$\frac{\delta y}{\delta x} = \frac{\frac{v\delta u}{\delta x} - \frac{u\delta v}{\delta x}}{V^2 + v(0)}$$

$\frac{\delta y}{\delta x} = \frac{\frac{v\delta u}{\delta x} - \frac{u\delta v}{\delta x}}{v^2}$ . . . . . .(4)

Then equation (4) is the quotient rule of differentiation

Example :

If $y=\frac{5e^x}{cos^x}$, dfferentiate with respect to x.

Solution

$$y = \frac{4e^x}{cosx}$$

Let u=$4e^x$ and =cos x

$\frac{\delta u}{\delta x} = 4e^x, \frac{\delta v}{\delta x} = -sinx$

$$\frac{\delta y}{\delta x} = \frac{\frac{v\delta u}{\delta x} - \frac{u\delta v}{\delta x}}{v^2} = \frac{cosx(4e^x) - 4e^x(-sinx)}{cos^2x}$$

$$\frac{cosx(4e^x) + 4e^x sinx}{cos^2x}$$

$$\frac{4e^x(cosx + sinx)}{cos^2x}$$

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FUNCTION OF A FUNCTION (COMPOSITE FUNCTION)

When differentiating a function of a function, we will have to introduce the chain rule

$$let\ say, if\ y = \frac{1}{(5x-6)^2} = (5x-6)^{-2}.$$

This cannot be expressed as a polynomial and it leave us to devise a very important techniques to solve such a function. To solve such a function in $\frac{dy}{dx}$, we have the differentiate $(5x-6)^{-2}$ as a **composite function** or as a **function of a function** which can be differentiated from two main functions i.e

(5x-6) by putting u=(5x-6)

And let y $=u^{-2}\ where\ u = 5x-6$

Hence,if **x**has increase in dx,**u** will have an increase δu and then **y** will have an increase δy i.e.

$x \to x+\delta x, u \to u+\delta u\ and\ y \to y+\delta y.$

At this stage, the increase $\delta x, \delta u\ and \delta y$ are all finite values and therefore we can say that $\frac{\delta y}{\delta x} = \frac{\delta y}{\delta x}.\frac{\delta u}{\delta x}$ Because the $\delta u\ in \frac{\delta y}{\delta u}$ cancels the δu in $\frac{\delta u}{\delta x}$.

Also $\frac{\delta y}{\delta x} \to \frac{\delta u}{\delta x}$ and the previos statement now becomes:

$$\boxed{\frac{\delta y}{\delta x} = \frac{\delta y}{\delta u}.\frac{\delta u}{\delta x}}$$

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Example :

If $y=\frac{5e^x}{cos^x}$, dfferentiate with respect to x.

Solution

$$y = \frac{4e^x}{cosx}$$

Let $u=4e^x$ and $=\cos x$

$\frac{\delta u}{\delta x} = 4e^x, \frac{\delta v}{\delta x} = -sinx$

$$\frac{\delta y}{\delta x} = \frac{\frac{v\delta u}{\delta x} - \frac{u\delta v}{\delta x}}{v^2} = \frac{cosx(4e^x) - 4e^x(-sinx)}{cos^2x}$$

$$\frac{cosx(4e^x) + 4e^x sinx}{cos^2x}$$

$$\frac{4e^x(cosx + sinx)}{cos^2x}$$

IMPLICIT FUNCTIONS

When functions that are differentiated which in the form $y= x^3 + 5x -3$ are called an **explicit function** of x. i.e. y is stated directly in terms of x.

A relationship in which x and y is more involved may not be possible to separate y completely on the left hand side.

For example $xy + \cos y = 5$, in this case, y is called an **implicit function**of x, because a relationship of the form $y = f(x)$ is implied in the given equation.

Steps for Differentiating Implicit function

When differentiating implicit function, it is important to determine the derivate y with respect to x and while doing so, it is the derivative of the function. It is very important to multiply the differential function of y by the derivative of the function. It is very important to also notice that derivative of constant number is zero.

Example 1

If $2x^2 + 3y^2 = 16$, find $\frac{\delta y}{\delta x}$

Solution

$2x^2+3y^2 =16$

Differentiating the above term with respect to x

$4x + 6y\ \delta y/\delta x = 0$

Subtracting L. H. S. from R. H. S. by 4x

$4x + 6y\ \delta y/\delta x - 4x = 0 - 4x$

$6y\ \delta y/\delta x = -4x$

Making $\delta y/\delta x$ subject of the formula by divding both side by 6y

$$\delta y/\delta x = \frac{-4x}{6y}$$

$$\delta y/\delta x = \frac{-4x}{6y}$$

_____ is a way of adding slices to find the whole.

Integration

Integration is like filling a tank from a tap.

_____ can be used to find areas, volumes, central points and many useful things.

Integration

_____can also be used for areas under curve surfaces, Centre of mass – displacement and velocity, fluid flow, modelling the behavior of object under stress

Integration

Integration involves bringing 2 or more separate lives together

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Integration

Integration simply means the inverse operation to differentiation. Let say when $y = x^n$, the derivate i.e. $\frac{\delta y}{\delta x} = n\,x^{n-1}$. If we differentiate $\frac{1}{n+1}x^{n+1} = \frac{x^{n+1}}{n+1}$ with respect to x.

$\therefore \frac{\delta y}{\delta x} = n + 1.\ \frac{1}{n+1}.\ x^{n+1-1}.$

$$\frac{\delta y}{\delta x} = x^n$$

Therefore, when $\frac{\delta y}{\delta x} = x^n$, then $y = \frac{x^{n+1}}{n+1}$, that is to say the integral of x^n with respect to x is $\frac{x^{n+1}}{n+1}$ (where $n \neq 1$).

When integrating, it is very important that you show a constant term called an **arbitrary constant c**. The reason is that when we integrate $3x^2 - 1$, it might be for the following function as their differentiatione.g. $x^3 - x + 5, x^3 - x$in each case. When integrating$3x^2 - 1$, the constant is not always recovered. In order to show there is a constant term in the integral, the arbitrary constant is added.

6.4 Notation for Integration

$\int$ is theSymbolof integration (integral sign) and both $\int$ and δx must be written. **Integrand** is the function to be integrated and it is place in between the $\int$ and δx. δx *is* written to illustrate that the integrand is to be integrated.

If $\frac{\delta y}{\delta x} = f(x)$, $v = \int f(x)\,dx + c$ where c is any constant.

Example1

Integrate the following:

a. $\int (x^3 + 3x^2 + 2x + 4)\, dx$ b. $\int t^3 + 4t^2 - 2\, dt$ c. $\int (s^3 + 4s) ds$

Solution

Using the general formula: $\int x^n\ dx = \frac{x^{n+1}}{n+1} + c$

1. $\int (x^3 + 3x^2 + 2x + 4)\, dx$

$= \frac{x^{3+1}}{3+1} + \frac{3x^{2+1}}{2+1} + \frac{2x^{1+1}}{1+1} + \frac{4x^{0+1}}{0+1} + c$ **In indices law: $a^0 = 1$, i.e. $4x^0 = 1$.**

$= \frac{x^4}{4} + \frac{3x^3}{3} + \frac{2x^2}{2} + \frac{4x}{1} + c$ **Simplifying.**

$= \frac{x^4}{4} + x^3 + x^2 + 4x + c$

2. $\int (s^3 + 4s) ds$

$= \frac{s^{3+1}}{3+1} + \frac{4s^{1+1}}{1+1} + c$

$= \frac{s^4}{4} + \frac{4s^2}{2} + c$

$= \frac{s^4}{4} + 2s^2 + c$

3. $\int (t^3 + 4t^2 - 2) dt$

$= \frac{t^{3+1}}{3+1} + \frac{4t^{2+1}}{2+1} - \frac{2t^{0+1}}{0+1} + c$

$= \frac{t^4}{4} + \frac{4t^3}{3} - 2t + c$

Standard Integral.

I. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$ (provided $n \neq 1$)

II. $\int \frac{1}{x}dx = \ln x + c$

(3) $\int e^x\,dx = e^x + c$

(4) $\int e^{kx}dx = \frac{e^{kx}}{k} + c$

(5) $\int \sin x dx = -\cos x + c$

(6) $\int cosxdx = \sin x + c$

(7) $\int \sec^2 xdx = \tan x + c$

(8) $\int \sinh x dx = \cosh x + c$

IX. $\int \cosh x dx = \sinh x + c$

X. $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1} x + c$

XI. $\int \frac{-1}{\sqrt{1-x^2}}\,dx = \cos^{-1} x + c$

XII. $\int \frac{1}{\sqrt{1+x^2}}\,dx = \tan^{-1} x + c$

XIII. $\int \frac{1}{\sqrt{x^2+1}}\,dx = \sinh^{-1} x + c$

XIV. $\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1} x + c$

XV. $\int \frac{1}{1-x^2}dx = \tanh^{-1} x + c$

XVI. $\int a^x dx = \frac{a^x}{\ln a} + c$

_____is A solid figure generated by revolving a given curve around an axis of revolution **solid of revolution.**

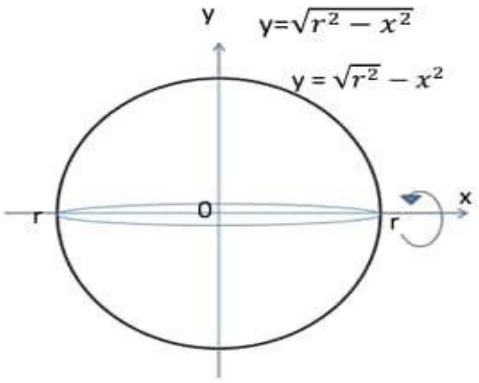
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The Volume of a sphere

Find the volume of a sphere generated by a semicircle $y=\sqrt{r^2-x^2}$ revolving around the x-axis.

Solution.

Since the end points of the diameter lying on the x-axis and –r and r as shown below, the



Since $y=\sqrt{r^2-x^2}$

Squaring both side

$y^2 = (r^2 - x^2)^{½ \times 2}$

$y^2 = r^2 - x^2$

$v_x = \pi \int_a^b y^2\,\delta x = \pi \int_{-r}^{r}(r^2 - x^2)\,\delta x = \pi\left[r^2x - \frac{x^3}{3}\right]_{-r}^{r}$

$= \pi[(r^2.r - r^3/3) - (r^2.-r - (-r)^3/3)]$

$= \pi[(r^3-r^3/3) - (-r^3 + r^3/3)] = \pi\left[\frac{(3r^3 - r^3)}{3} - \frac{(-3r^3 + r^3)}{3}\right]$

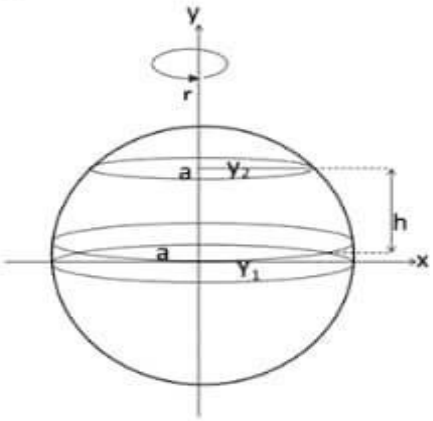
$= \pi\left[\left(\frac{2r^3}{3}\right) - \left(\frac{-2r^3}{3}\right)\right] = \pi\left[\frac{2r^3}{3} + \frac{2r^3}{3}\right] = \pi\left[\frac{2r^3 + 2r^3}{3}\right]$

$= \pi \cdot 4r^3/3 = 4\pi r^3/3$

$V_x = 4\pi r^3/3$

The Volume of a Spherical segment

Example: Find the volume of a spherical segment generated by the portion of the right semi circle between y=a and y=a+h, revolving around the y-axis, is as shown in the below figure:



Solution

Since the right semi circle equation $x = \sqrt{r^2 + y^2}$ then

$V_y = \int_c^d x^2 \delta y = \int_a^{a+h} (r^2 - y^2) \delta y$

$= \pi \left[r^2 y - \frac{y^3}{3} \right]_a^{a+h}$

$V_y = \pi \left[r^2(a+h) - \frac{1}{3}(a+h)^3) - (r^2.a - \frac{1}{3}(a^3)) \right]$

$= \pi[(r^2a + r^2h - \frac{1}{3}(a^3 + 3a^2h + 2ah^2 + h^2a + h^3) - (r^2a - \frac{1}{3}a^3)]$

$= \pi[r^2a + r^2h - \frac{1}{3}a^3 - a^2h - \frac{2}{3}ah^2 - \frac{1}{3}h^2a - \frac{1}{3}h^3 - r^2a + \frac{1}{3}a^3]$

$= \pi[r^2a + r^2h - \frac{1}{3}a^3 - a^2h - \frac{2}{3}ah^2 - \frac{1}{3}h^2a - \frac{1}{3}h^3 - r^2a + \frac{1}{3}a^3]$

Collecting like or common terms

$= \pi\ [r^2a - r^2a + r^2h - \frac{1}{3}a^3 + \frac{1}{3}a^3 - a^2h - \frac{2}{3}ah^2 - \frac{1}{3}ah^2 - \frac{1}{3}h^3]$

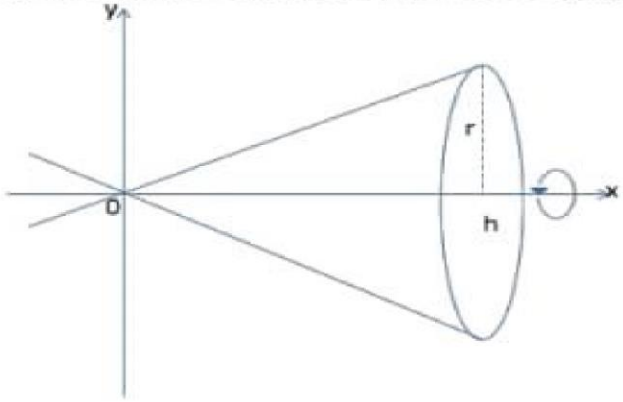
$= \pi[\ r^2h - a^2h - ah^2 - \frac{1}{3}h^3]$

$v_y = \pi h\ \ [r^2 - a^2 - ah - \frac{1}{3}h^2]$

The volume of a cone

Example 1

Find the volume of a right circular cone generated by the line (segment) passing through the origin and the point (h,r), where h denotes the height of the cone and r is the radius of its base, revolving around the x-axis , as shown in the figure below.



Solution

The equation of the generating line

$y = mx$ ..... (1)

$m = \frac{\Delta x}{\Delta y} = \frac{r}{h}$ ..... (2)

Substituting (2) into (1)

(1) becomes: $y = \frac{r}{h}x$ ..... (3)

$V_x = \pi \int_a^b y^2 \delta x$ ..... (4)

sub (3) into (4)

$= \pi \int_o^h (\frac{r}{h}x)^2 \delta x = \pi \int_o^h \frac{r2x2}{h2} dx$

$= \pi \frac{r^2}{h^2} \int_o^h x^2 \delta x = \frac{\pi r^2}{h^2} \left[\frac{x^3}{3}\right]_0^h$

$= \frac{\pi r^2}{h^2} \left[\frac{h^3}{3}\right] = \frac{\pi r^2 h^3}{3h} = \frac{\pi r^2 h}{3}$

$V_x = \frac{\pi r^2 h}{3}$

$V_{cone} = \frac{1}{3}\pi r^2 h$

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