MOTION IN STRAIGHT LINE

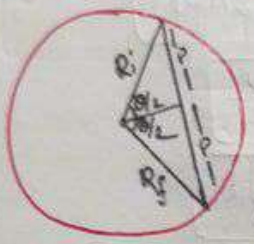
Distance → Actual path travelled
↳ scalar.

Displacement → Shortest distance bet° 2 points.
↳ change in position = $\vec{x}_f - \vec{x}_i$

|Distance| ≥ |displacement|

Displacement	Displacement must be zero	Displacement may or may not zero	If displacement is zero	If displacement is not zero
Distance	If distance is zero	If distance is not zero	distance may or may not be zero	distance must not be zero

Distance & Displacement on circular path



Arc = distance = $R\theta$

Displacement = $2P = 2R\sin(\theta/2)$

$\theta \to$ small $\Rightarrow$ displacement = $R\theta$

Ins velocity → $\frac{d\vec{x}}{dt}$ # Average velocity → $\frac{\text{total displacement}}{\text{total time}}$

Avg speed → $\frac{\text{total distance}}{\text{total time}} = \frac{\int |\text{Ins}|\, dt}{\int dt}$

|Avg velocity| = |Avg speed| → Only when direction not change.

|Ins speed| = |Ins velocity| → Always.

Object moving in straight line cover x_1 distance in t_1 time and x_2 distance in t_2 time

$$\text{Avg speed} = \frac{x_1 + x_2}{t_1 + t_2}$$

v_1 for t_1 & v_2 for t_2

$$\text{Avg speed} = \frac{x_1 + x_2}{t_1 + t_2} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$$

x_1 with v_1 & x_2 with v_2

$$\text{Avg } v = \frac{x_1 + x_2}{t_1 + t_2} = \frac{x_1 + x_2}{\frac{x_1}{v_1} + \frac{x_2}{v_2}}$$

Acceleration → Rate of change of velocity

$$\vec{a}_{ins} = \frac{d\vec{v}}{dt} = \vec{v}\frac{d\vec{v}}{dx} \qquad \vec{a}_{avg} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{\Delta v}{\Delta t} = \frac{\int a_{ins}\, dt}{\int dt}$$

(Area of v-t graph)
Change in pos^n = $x_f - x_i$ = disp = $\int_{t_1}^{t_2} \vec{v}\, dt$

(Area of a-t graph)
$\Delta v = \vec{v}_f - \vec{v}_i = \int_{t_1}^{t_2} a\, dt$ = change in velocity

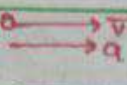
Position ($\vec{x}$) ⇄ Velocity ($\vec{v}$) ⇄ acceleration ($\vec{a}$)

$\vec{v}_{ins} = \frac{d\vec{x}}{dt}$ (slope of x-t graph)

$\vec{a}_{ins} = \frac{d\vec{v}}{dt}$ (slope of v-t graph)

$\vec{a}_{ins} = \vec{v}\frac{d\vec{v}}{dx}$

Acceleration = 0	Acc° = constant	Acc = variable
$\vec{a} = 0 = \frac{d\vec{v}}{dt}$	eq^ns of motion	Then we will use integration and differentiation
Velocity = constant (uniform motion)	$v = u + at$	
$v_{inst} = v_{avg}$	$s = ut + \frac{1}{2}at^2$	
disp = $\int v\,dt$	$v^2 - u^2 = 2as$	
dist = v × time		

		
Angle bet° $\vec{v}$ & $\vec{a}$ 180°	Angle bet° $\vec{v}$ & $\vec{a}$ 0°	Angle bet° $\vec{v}$ and $\vec{a}$ 90°
speed → decrease	speed → increase	at this instant speed → const
$\vec{a}\cdot\vec{v} = av\cos 180°$ = −ve	$\vec{a}\cdot\vec{v} = av$ = +ve	
No change in direction	No change in direction	direction will change

Constant acceleration

acceleration will be constant

if $a \propto t^2$ (or) $v \propto t$ (or) $v \propto \sqrt{a}$

Motion with const acc°

$\vec{v} = \vec{u} + \vec{a}t$ $\quad v^2 - u^2 = 2as$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$S_{nth} = u + \frac{a}{2}(2n-1)$

$v_{avg} = \frac{u+v}{2}$

$\vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t$

Transitional acc° (a_T) = $\frac{d|\vec{v}|}{dt}$ = Rate of change in magnitude of velocity (speed) per time

object start moving from rest with const acc°

$S_{1st\,sec} : S_{2nd\,sec} : S_{3rd\,sec} : \ldots = 1 : 3 : 5 : \ldots$

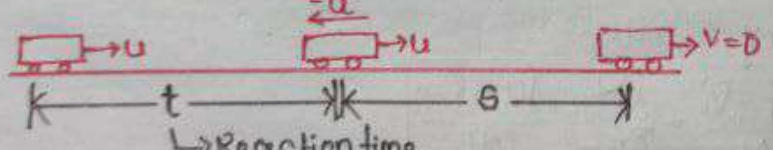
$S_{1sec} : S_{2sec} : S_{3sec} : \ldots = 1 : 4 : 9 : \ldots$

#



velocity of mid point = $\sqrt{\frac{v^2+u^2}{2}}$

Reaction time

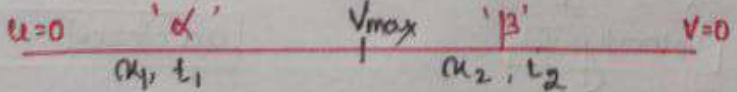


s → stopping distance

$s = \frac{u^2}{2a}$

Rest to Rest Motion

object start from rest and constant acc° 'α' for time 't₁', and then retard with 'β' and comes to rest in time 't₂'.



$v_{max} = \alpha t_1$ $\quad v_{max} = \beta t_2$

$v_{max}^2 = 2\alpha\alpha_1$ $\quad v_{max}^2 = 2\beta\alpha_2$

$\alpha t_1 = \beta t_2$ $\quad \alpha\alpha_1 = \beta\alpha_2$

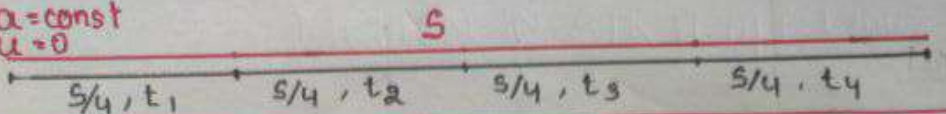
$v_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)T$

T → total time

$s = \alpha_1 + \alpha_2$

$s = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)T^2$

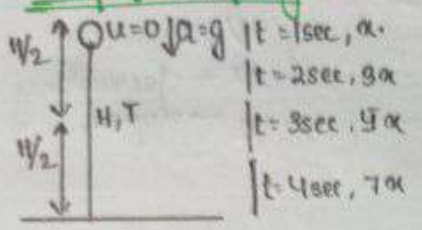
a = const
u = 0

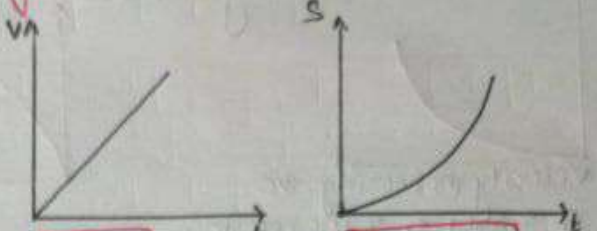


$t_1 : t_2 : t_3 : t_4 : \ldots = 1 : \sqrt{2}-1 : \sqrt{3}-\sqrt{2} : \sqrt{4}-\sqrt{3} : \ldots$

Motion under Gravity

Drop from Height

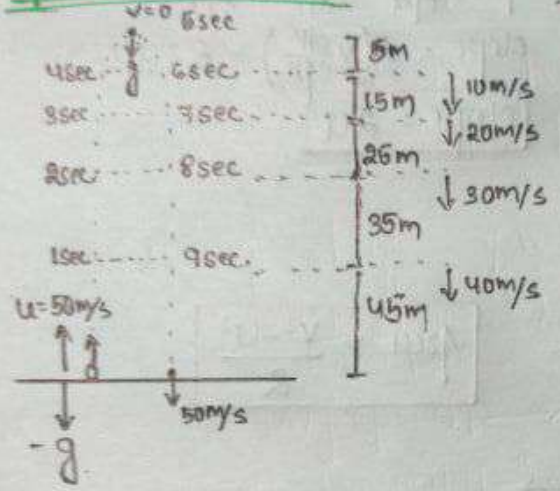




$V = gt$ | $s = \frac{1}{2}gt^2$ | $V = \sqrt{2gs}$ | $a = g$ (const)

$T = \sqrt{\frac{2h}{g}}$

Ground to Ground



$H_{max} = \frac{u^2}{2g}$

Time of flight

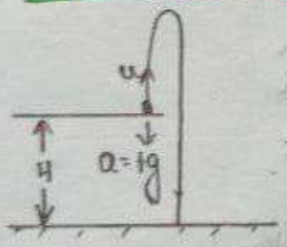
$T_{up} = T_{down} = u/g$

$T_{flight} = 2u/g$

If air resistance ≠ 0

$$\frac{T_{up}}{T_{down}} = \sqrt{\frac{g-a}{g+a}}$$

Motion under Gravity from some height

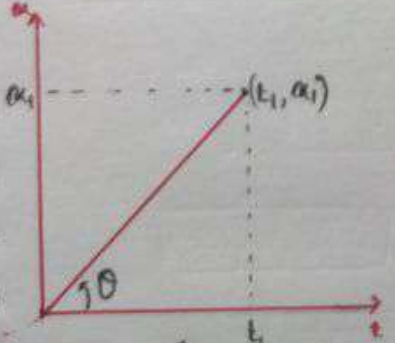


$s = H = -ut + \frac{1}{2}gt^2$

$$T_f = \frac{u}{g} + \sqrt{\frac{u^2}{g^2} + \frac{2H}{g}}$$

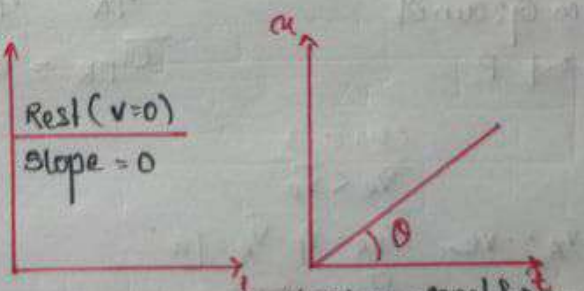
Graphs

Position-time Graph



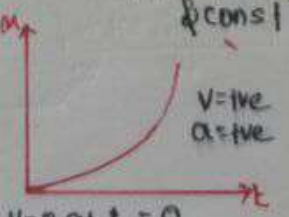
slope = $\frac{dx}{dt}$ = velocity = $\tan\theta$

Velocity (at t=0) ⇒ +ve & const

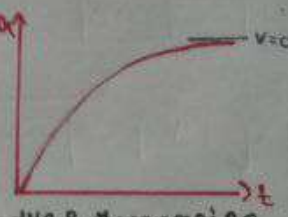


V = slope = const & +ve
Motion = start from rest & const velocity

slope = -ve & const

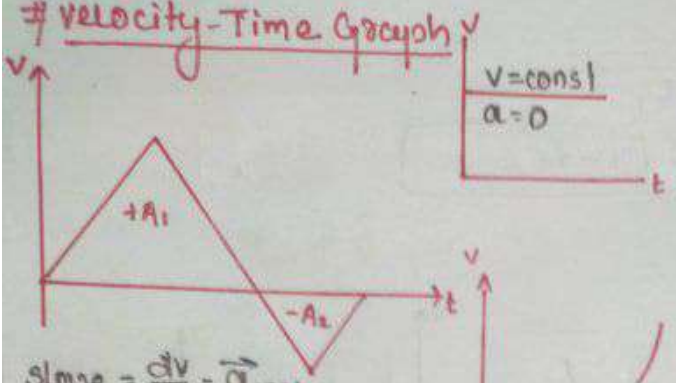


V = 0 at t = 0
& increasing velocity



V = +ve & decreasing
⇒ Retardation
'a' opposite to 'V'

Velocity-Time Graph

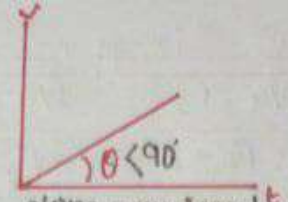


slope = $\frac{dv}{dt} = \vec{a}_{inst}$

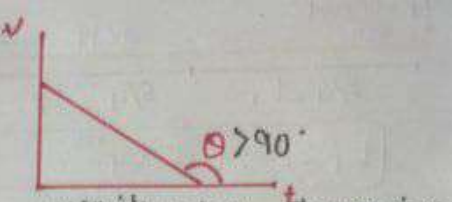
area = $\int v\,dt$ = displacement

disp = $A_1 - A_2$

dist = $A_1 + A_2$

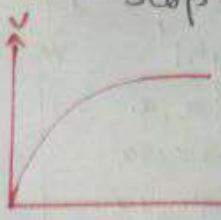


slope = +ve & const
a = +ve & const
velocity increase



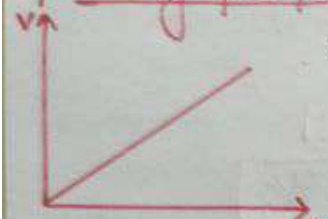
velocity → +ve, decreasing
slope = a = -ve, const

velocity increasing, +ve
a = slope = increasing



V increasing
a → decreasing
↳ slope

Velocity - position Graph



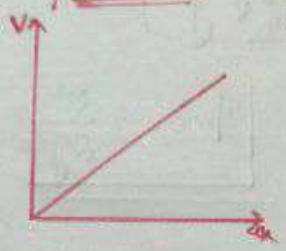
slope = $\frac{dv}{dx}$

$a = v\frac{dv}{dx}$

$a = v \times slope$

$\boxed{slope = \frac{a}{v}}$

$v^2 - x$

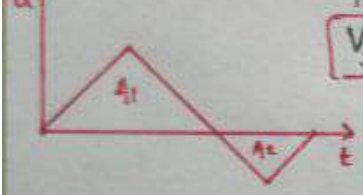


$slope = \frac{dv^2}{dx} = \frac{dv^2}{dv} \times \frac{dv}{dx}$

$slope = 2\left(v\frac{dv}{dx}\right) = 2 \times acceleration$

$\boxed{slope = 2a}$

a - t

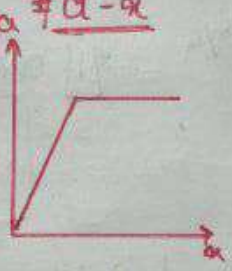


area under (a-t) = $\int a\,dt = \Delta v$

$\boxed{v_f - v_i = \text{Area of graph}}$

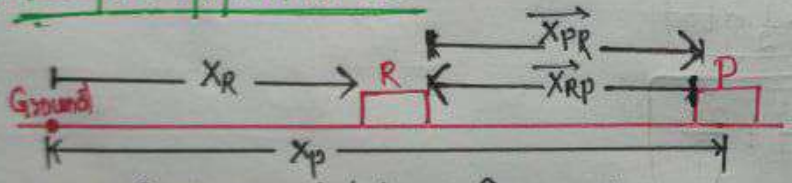
$v_f - v_i = A_1 - A_2$

a - x



$\boxed{\text{Area} = \frac{v^2 - u^2}{2}}$

Relative motion in 1D



- x_R → Position of 'R' from Ground
- x_P → Position of P from Ground
- x_{RP} → Position of R w.r.t P

$\vec{x}_{PR} = \vec{x}_P - \vec{x}_R$

$\vec{v}_{PR} \xrightarrow{\text{diff}} \vec{v}_P - \vec{v}_R$

$\vec{v}_{PR} = \vec{v}_{Pg} + (-\vec{v}_{Rg})$

$\vec{a}_{PR} = \vec{a}_P - \vec{a}_R$

Case-1

$\vec{v}_R = \vec{v}_P$

$\vec{v}_{PR} = \vec{v}_P - \vec{v}_R = 0$

Case-2

$v_R > v_P$

$v_{PR} = v_P - v_R = -ve$

$v_{RP} = +ve$

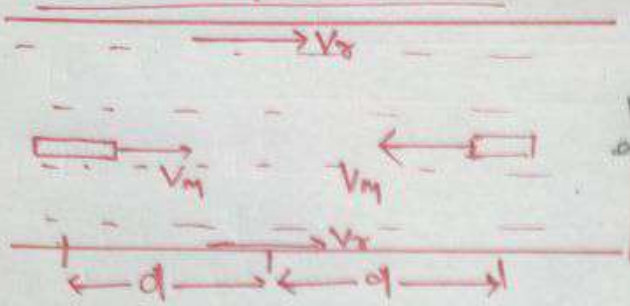
Case-3

$v_R < v_P$

$v_{PR} = v_P - v_R = +ve$

$v_{RP} = -ve$

#Relative motion in River



man swimming downstream

$V_{m\,w.r.t\,Ground}$ (effective) $= V_m + V_r$

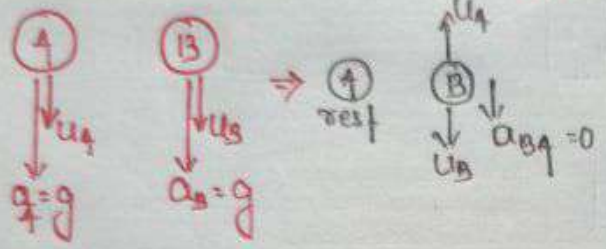
$$t = \frac{d}{V_m + V_r}$$

man swimming upstream

V_{m} (effective) $_{w.r.t\,Ground} = V_m - V_r$

$$t = \frac{d}{V_m - V_r}$$

#Relative motion under Gravity



$$\vec{u}_{BA} = \vec{u}_B - \vec{u}_A$$

$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A = 0$$

Rel. acc. $\rightarrow 0$

Rel. vel $\rightarrow$ const.