

LITTORAL MOCK

General Certificate of Education Examination

**MARCH 2019****ADVANCED LEVEL**

Subject Title	Further Mathematics
Paper number	2
Subject Code	0775

THREE HOURS**INSTRUCTIONS TO CANDIDATES****Answer ALL questions.**

For your guidance the approximate mark allocation for parts of each question is indicated bold.

Mathematical formulae and tables, published by the Board, and noiseless non – programmable electronic calculators are allowed.

In calculations you are advised to show all the steps in your working giving your answer at each stage.



1. Using the definition $\tanh y = \frac{e^y - e^{-y}}{e^y + e^{-y}}$,

a) show that, for $|x| < 1$, $\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$

4 marks

b) Hence, or otherwise, show that $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

3 marks

2. (i) Sketch, using the same axes, the curves $y^2 = 4x$ and $y^2 = x^3$.

Shade the region for which $(y^2 - 4x)(y^2 - x^3) \leq 0$.

4 marks

(ii) Solve the differential equation $(x+1)\frac{dy}{dx} - 3y = (x+1)^5$, given that $y = \frac{3}{2}$ when $x = 0$

4 marks

3.(i) The arc of a curve with equation $y = \cosh x$, between the points where $x = 0$ and $x = \ln \left(\frac{9}{4} \right)$, is rotated through 2π

radians about the x -axis. Show that the surface area generated is $\pi \left[\ln \left(\frac{9}{4} \right) + p \right]$, where p is a number to be determined

5 marks

(ii) Sketch the graph of the function $f(x) = |x^2 - 4| + 3$

Hence, find the solution of the inequality $|x^2 - 4| > 2$

4 marks

4.(i) The Cartesian equation of the curve C is

$$x^2 + y^2 - 2x = (x^2 + y^2)^{\frac{1}{2}}$$

a) Find the polar equation of the curve for $-\pi < \theta \leq \pi$

2 marks

b) Find the equations of the tangents to the curve at the Pole

2 marks

c) Sketch the curve C.

2 marks

(ii) Given that $e^{-x} + \frac{2x+3}{x+2} \leq f(x) \leq \frac{3-7x+2x^2}{x+2}$, find $\lim_{x \rightarrow +\infty} f(x)$

3 marks

5. (i) When the number 900 is divided by another number b , it gives a quotient 14 and a remainder r .

a) Write down the relationship between 900, 14, b and r ,

1 mark

b) What are the possible values of b and r

5 marks

(ii) A linear transformation, T , maps the points $(1, 1)$ onto the point $(2, -3)$ and the point $(0, 1)$ on to $(1, 2)$.

Find $T \begin{pmatrix} a \\ b \end{pmatrix}$ and hence $T \begin{pmatrix} 3 \\ -19 \end{pmatrix}$

4 marks

- 6.(i) Find the equation of the tangent and normal of the parabola $x^2 = 8y$ at the point $(4t, 2t^2)$. The tangent meets the x -axis at A and the y -axis at B.

- a) Find the coordinates of the mid-point M of AB
b) Hence find the locus of M as t varies.

4 marks

3 marks



4 marks

- (ii) If $I_n = \int (\ln x)^n dx$, show that $I_n = x \ln x - nI_{n-1}, n \geq 1$

Hence find I_3

- 7.(i) Prove that $(\mathbb{R}, \circ)$ is a group where $\circ$ is defined by $a \circ b = a + b - 1$

3 marks

- (ii) Define a map from $(\mathbb{R}^*, \times)$ to $(\mathbb{R}, +)$ by $f(x) = \ln x$, where $\mathbb{R}^*$, is the set of positive non-zero real numbers.

- a) Show that f is a homomorphism.

3 marks

- b) Show, also that, f is an isomorphism.

3 marks

- 8.(i) Determine the *kelf* and *Im* f in the linear transformation

$$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$(x, y) \mapsto (2x + y, x - y)$$

5 marks

- (ii) Given that $\frac{z-1}{z-2} = e^{i\theta}$, where θ is real, prove that $z = \frac{1}{2} \left[3 - i \cot \left(\frac{\theta}{2} \right) \right]$

4 marks

9. A function f , is defined by

$$f(x) = \begin{cases} \frac{1}{2} x e^{\frac{1}{x}}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

- a) State the domain of f
b) Show that f is not continuous at the point where $x = 0$
c) Study the differentiability of f and state its sign in the domain
d) Find $\lim_{x \rightarrow +\infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$
e) State the asymptote of f
f) Establish a variation table for f
g) Interpret your results graphically.

1 mark

2 marks

3 marks

3 marks

1 mark

2 marks

2 marks



10. Consider the sequence (u_n) given by
$$\begin{cases} u_0 = -1, & u_1 = \frac{1}{2} \\ u_{n+2} = u_{n+1} - \frac{1}{4}u_n \end{cases}$$

a) By letting (v_n) and (w_n) be two other sequences defined by $v_n = u_{n+1} - \frac{1}{2}u_n$ and $w_n = \frac{u_n}{v_n}$

i. Calculate v_0 and w_0

2 marks

ii. Show that the sequence (v_n) is a geometric progression with common ratio $r = \frac{1}{2}$.

2 marks

iii. Express v_n in terms of n and hence evaluate $\lim_{n \rightarrow \infty} v_n$

2 marks

iv. Show that $w_{n+1} = w_n + 2$ and then characterize the sequence (w_n)

2 marks

v. Express w_n in terms of n .

2 marks

b) Show that $\forall n \in \mathbb{N}$, we have $u_n = \frac{2n-1}{2^n}$

2 marks

c) Let $S_n = \sum_{k=0}^n u_k$. Prove by mathematical induction that $\forall n \in \mathbb{N}$ $S_n = 2 - \frac{2n+2}{2^n}$ by mathematical induction.

3 marks