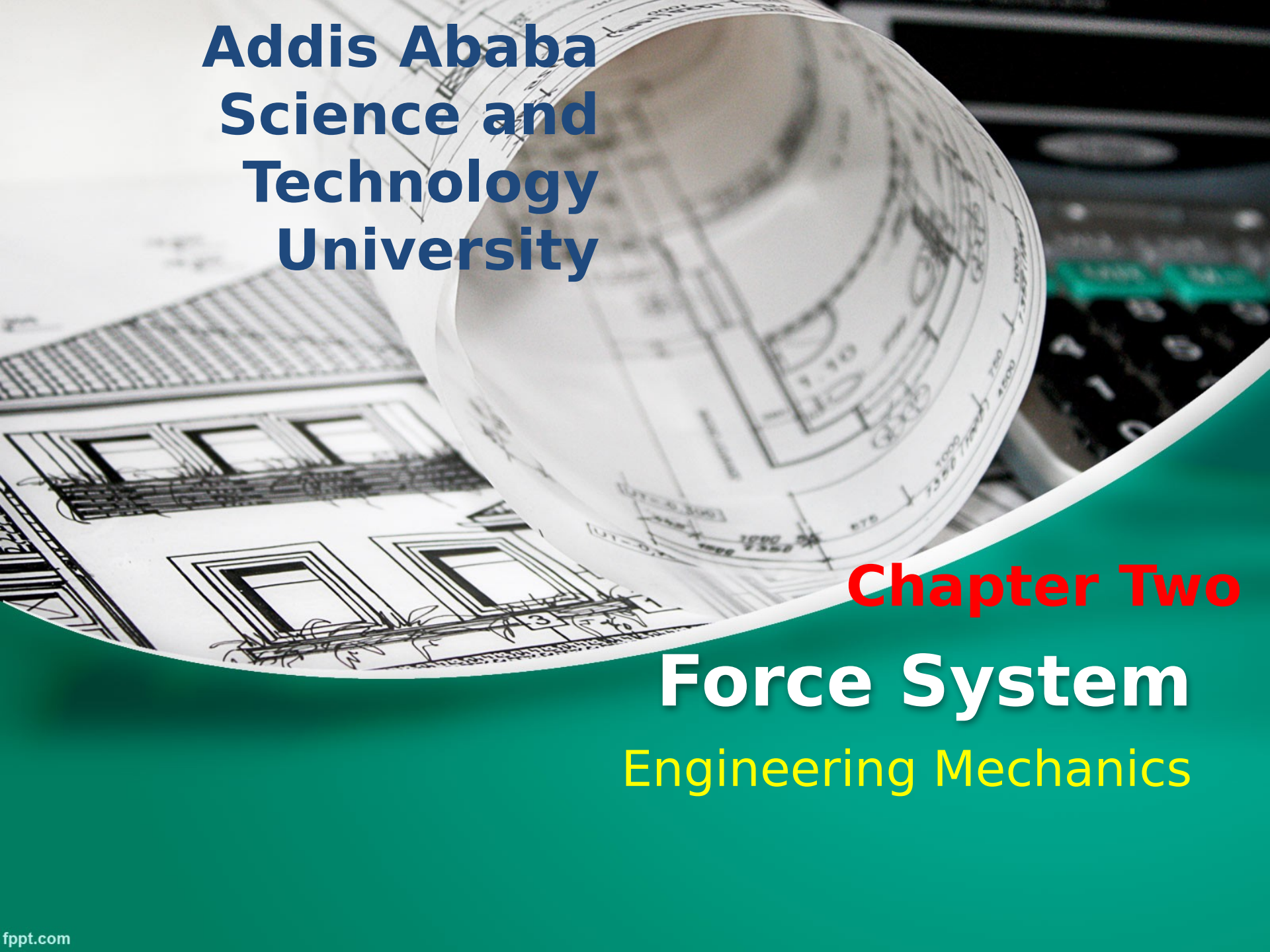




Addis Ababa Science and Technology University

Chapter Two

Force System

Engineering Mechanics



Content

2. FORCE SYSTEMS

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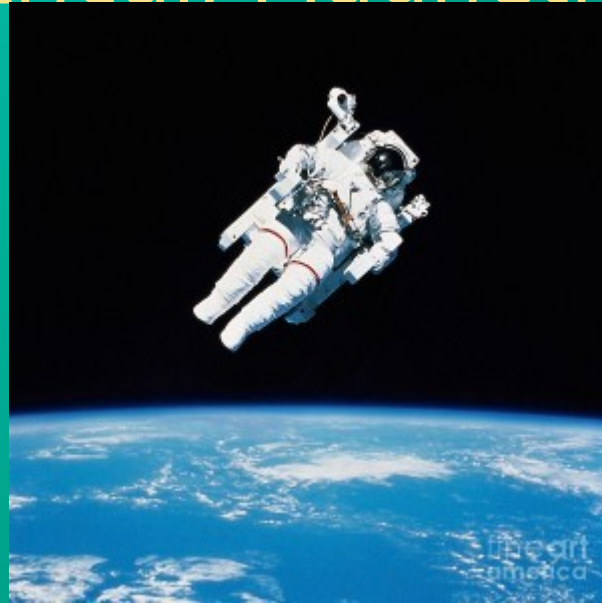
2.1 Introduction

- A force can be defined as the action of one body on another that changes/tends to changes the state of the body acted on.
- **A force can be applied on a body as;**
- **Contact force**:- Applied by direct mechanical contact between two bodies. It can be pushing or pulling.



2.1 Introduction

- **Remote action (Body force)**:- Applied by remote action as in gravitational, electrical, Magnetic, etc. forces.



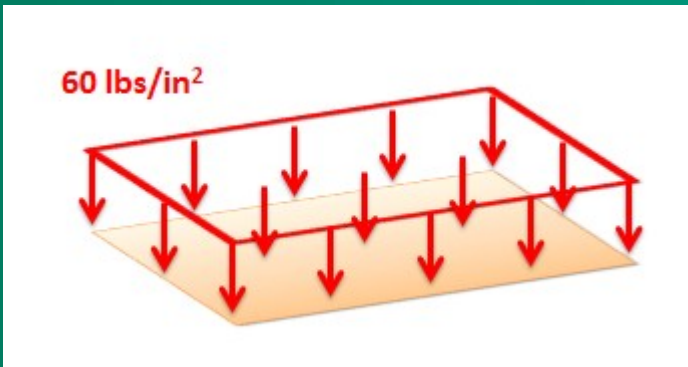
A technical drawing of a mechanical part, possibly a gear or a pulley, is visible on the left side of the slide. It shows various dimensions and a cross-section of a cylindrical component.

2.1 Introduction

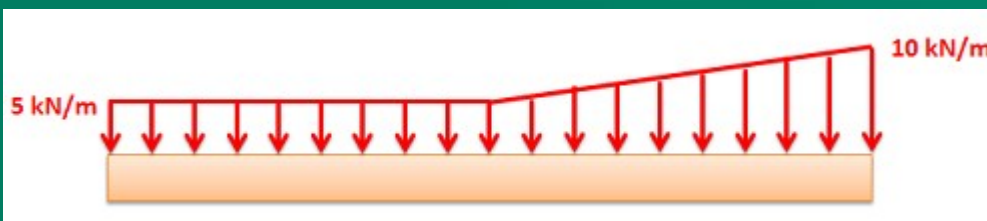
- A force may also be classified as
 - **Concentrated and**
 - **Distributed**
- **Concentrated** when area of application is negligible
- A force can be distributed over
 - **Area**, as in the case of mechanical contact
 - **Volume**, when considering body forces such as weight
 - **A line**, in the case of the weight of a suspended cable.



The gravitational force this plane is distributed over the entire volume of the airplane
 KN/m^3



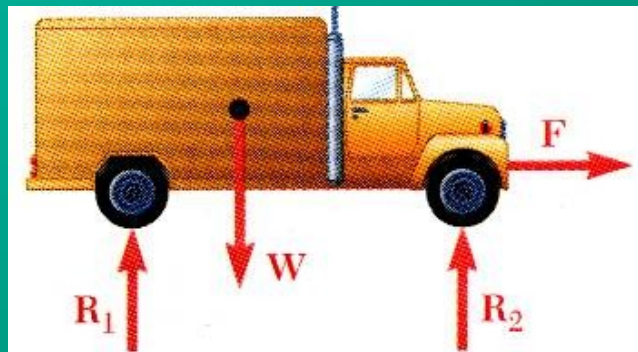
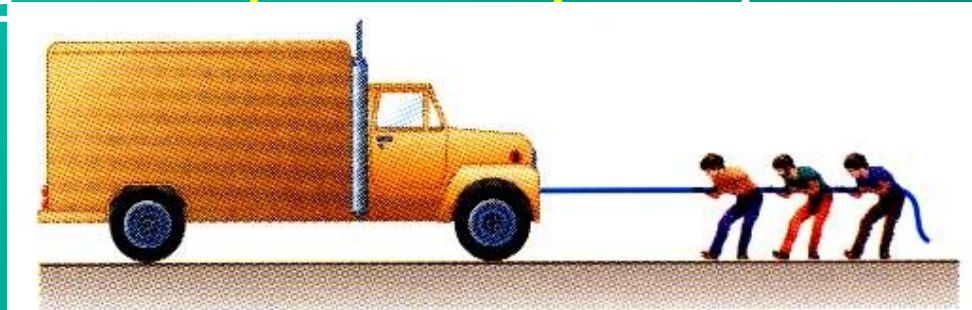
Representation of a force distributed over an area
 KN/m^2



Representation of a force distributed over a line
 KN/m

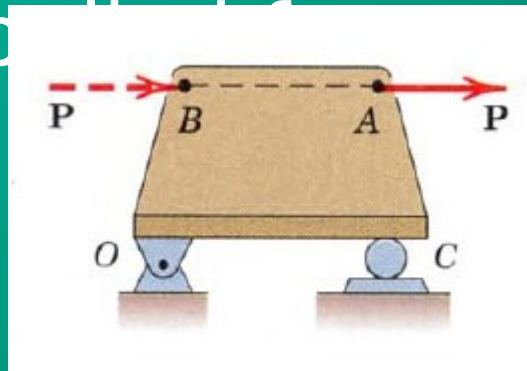
2.1 Introduction

- **External and Internal Effects**
- We can separate the action of a force on a body into two effects: **external** and **internal**.



2.1 Introduction

- **Principle of Transmissibility**
- When dealing with **mechanics of rigid body**, we are only concerned about the **net effects of external**
- In such cases it is not necessary to restrict the action of an applied force to a given point.





2.1 Introduction

- **Principle of transmissibility** states that a force may be **applied at any point on its given line of action** without altering the resultant effect of the force **external** to the rigid body on which it acts.

2.1 Introduction

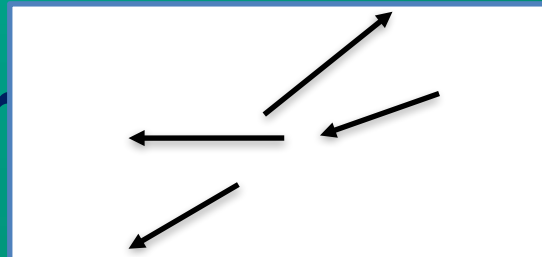
- **Force System**

- A system of forces can be grouped into different categories **depending on their arrangement** in space

- **Collinear forces**:- are coplanar forces acting on the same line

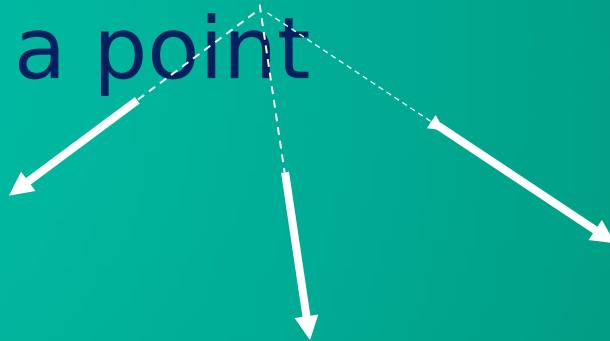


- **Coplanar** forces which act in the same plane.

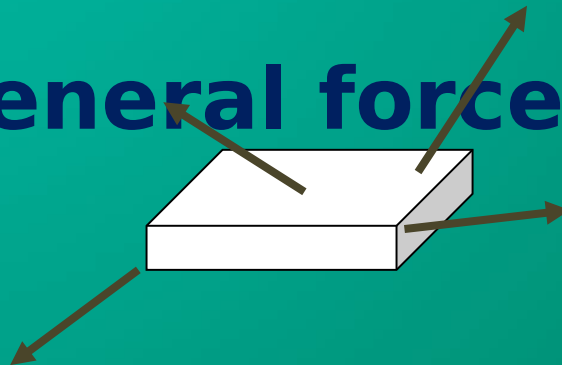


2.1 Introduction

- **Concurrent forces:-** Are forces whose lines of action intersect at a point



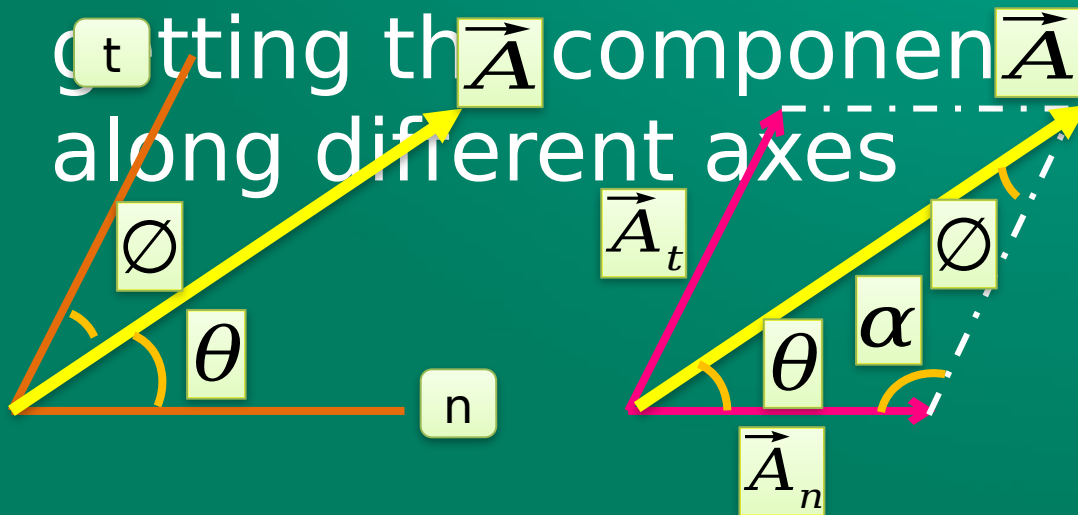
- **General forces:-**



2.2. Resolution of a Force

2.2.1. Coplanar Force Systems (2-D)

Resolution is the reverse of composition. It is the process of getting the components of a force along different axes



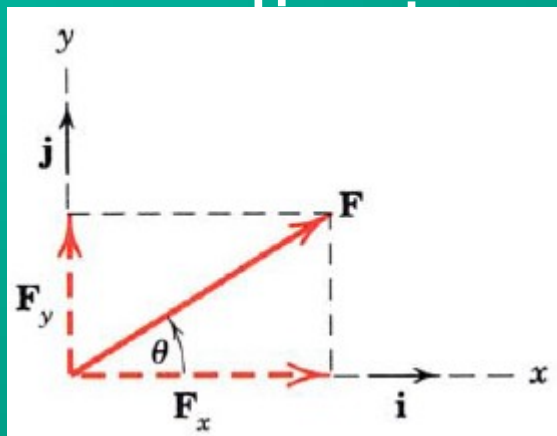
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For rectangular coordinate system

2.2.1. Resolution of a Force

- **Rectangular Component**
- The **most common** two-dimensional resolution
- Rectangular components of a force are the components of the force along the rectangular



$$F_x = F \cos \theta \quad F = \sqrt{F_x^2 + F_y^2}$$

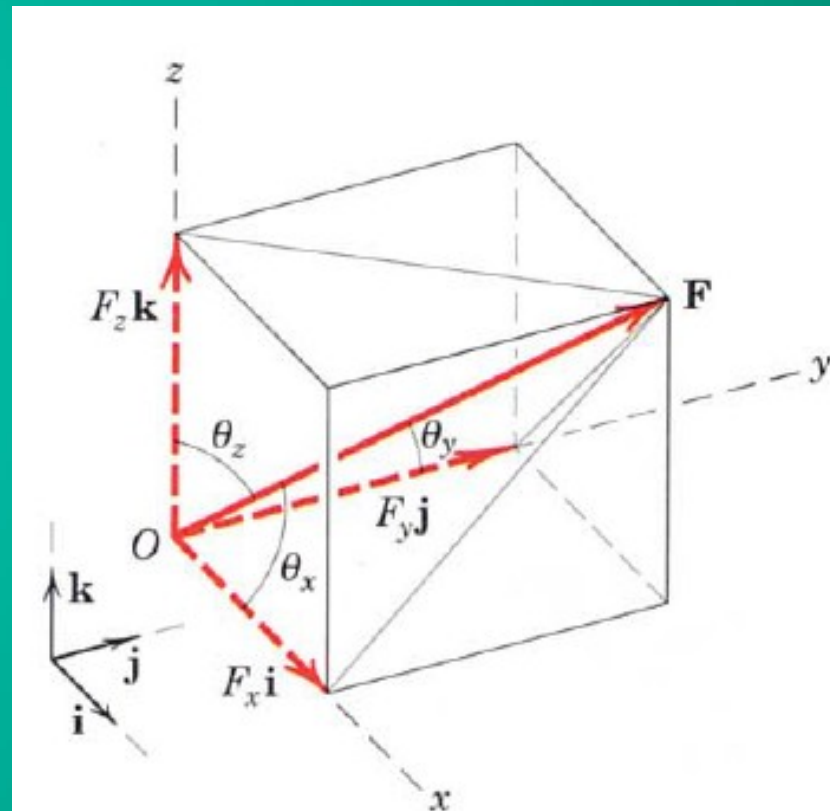
$$F_y = F \sin \theta \quad \theta = \tan^{-1} \frac{F_y}{F_x}$$

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

2.2.2. Non-Coplanar Force Systems (3-D)

- **Rectangular components**
- Many problems in mechanics requires analysis in **3 dimension**
- To solve such problems it is necessary to resolve the force into its three mutually perpendicular components.
- A force F in 3-d system has a rectangular components F_x , F_y , F_z

Rectangular Components



$$F_x = F \cos \theta_x \quad F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F_y = F \cos \theta_y \quad \mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{k}$$

$$F_z = F \cos \theta_z \quad \mathbf{F} = F(\mathbf{i} \cos \theta_x + \mathbf{j} \cos \theta_y + \mathbf{k} \cos \theta_z)$$



Rectangular Components

- In solving three-dimensional problems, one must usually find the x , y and z scalar components of a given or unknown force. In most cases, the direction of a force is described;

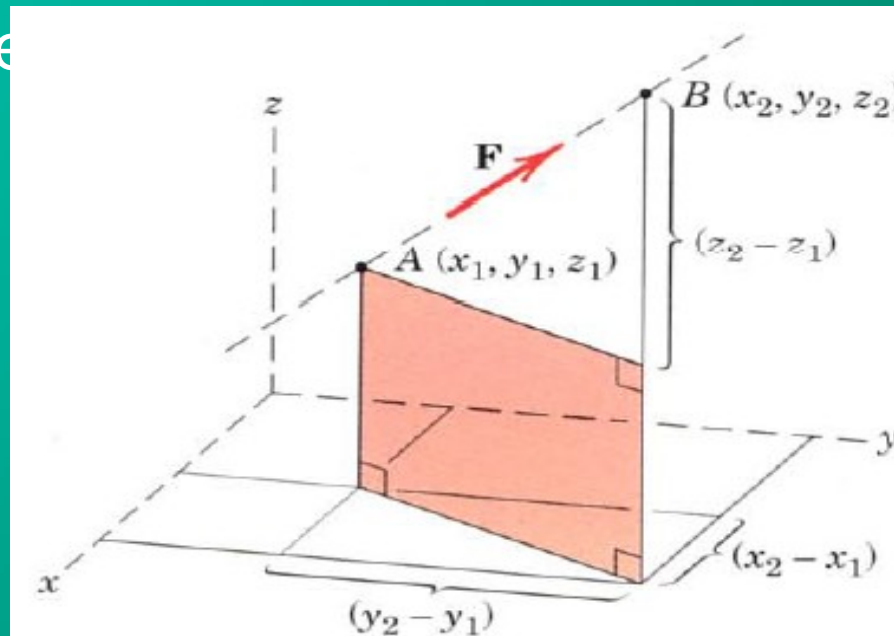
A. By two points on the line of action of the force, or

B. By two angles which orient the line of action.

Rectangular Components

A. By two points on the line of action of the force, or

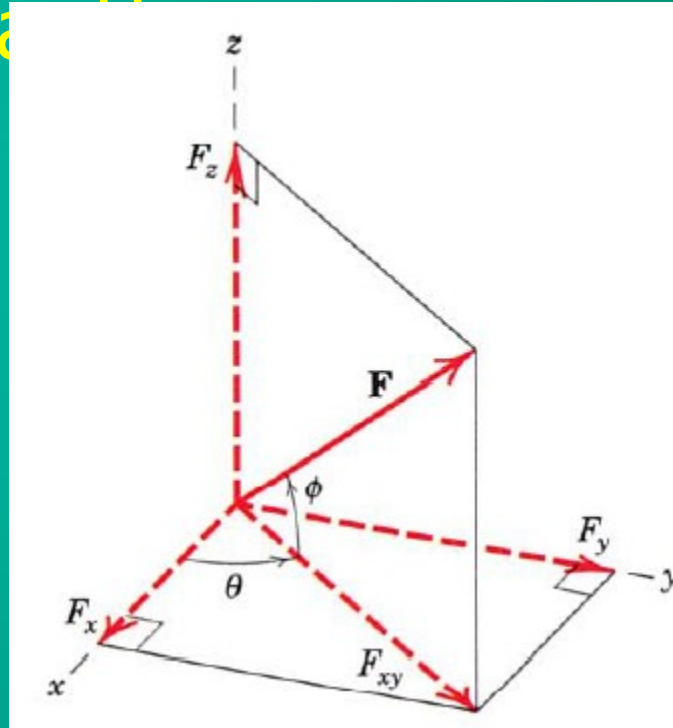
- If the coordinates of two point, say A&B, on the line of action of the force are known and the direction of the force is from A to B, the force



$$\mathbf{F} = F\mathbf{n}_F = F \frac{\overrightarrow{AB}}{AB} = F \frac{(x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

Rectangular Components

- B. By two angles which orient the line of action



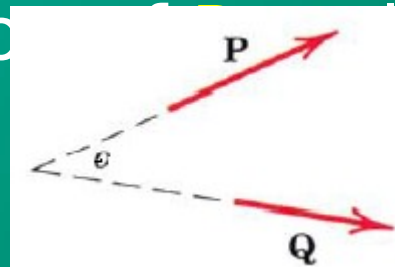
$$\begin{aligned} F_{xy} &= F \cos \phi \\ F_z &= F \sin \phi \end{aligned}$$

$$\begin{aligned} F_x &= F_{xy} \cos \theta = F \cos \phi \cos \theta \\ F_y &= F_{xy} \sin \theta = F \cos \phi \sin \theta \end{aligned}$$

Rectangular Components

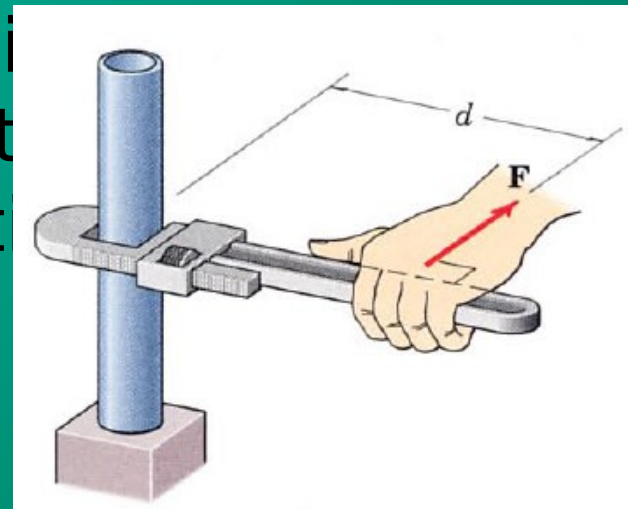
Dot product

- It is possible to express the rectangular components of a force with the aid of the vector operation known as the dot or scalar product.
- This product can be viewed as either
 1. The orthogonal projection of P on the direction of Q multiplied by Q , or
 2. As the orthogonal projection of Q in the direction of P multiplied by P



2.3. Moment, Couple & Force-Couple systems

- **Moment**
- In addition to moving a body in the direction of application, a **force also tends to rotate**
- This **tendency of a force to rotate** a body about a given axis is known as the **moment**
- The axis of rotation is perpendicular to the plane in which neither intersects nor is parallel to the line of action of the force

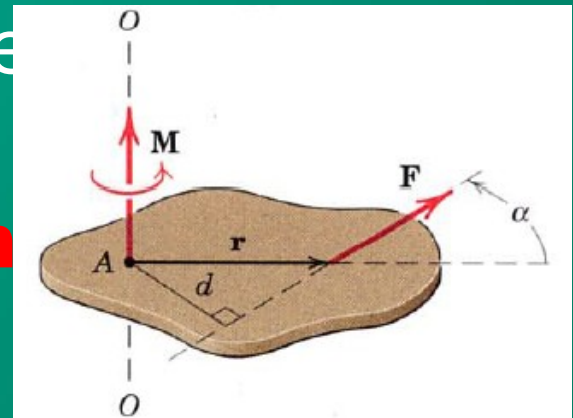


Moment

- The moment or rotational tendency of a force is proportional to magnitude of the force and the moment arm d , which is the perpendicular distance from the axis to the line of action of the force.
- The axis of rotation, axis O-O, of the body is normal to the plane of the body formed by the position vector.

**magnitude of moment
obtained by:**

$$M = F \times d$$



Moment

The cross product

- Useful when dealing with complex 2-dimensional and 3-dimensional problems.
- The moment of a force F about a point maybe represented by the cross-product expression.

$$M = r \times F$$

- Where r is the position vector from point of rotation to any point on the line of application of the force
- The magnitude of this expression is

$$M = rF\sin$$

Moment

Varignon's theorem

- One of the most important principles in mechanics
- states “the moment of a force about any point is equal to the sum of the moments of the components of the force about the same

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

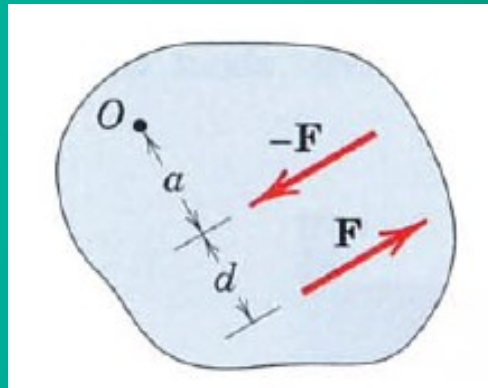
$$\text{Because } \mathbf{R} = \mathbf{P} + \mathbf{Q}$$

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$

Couple

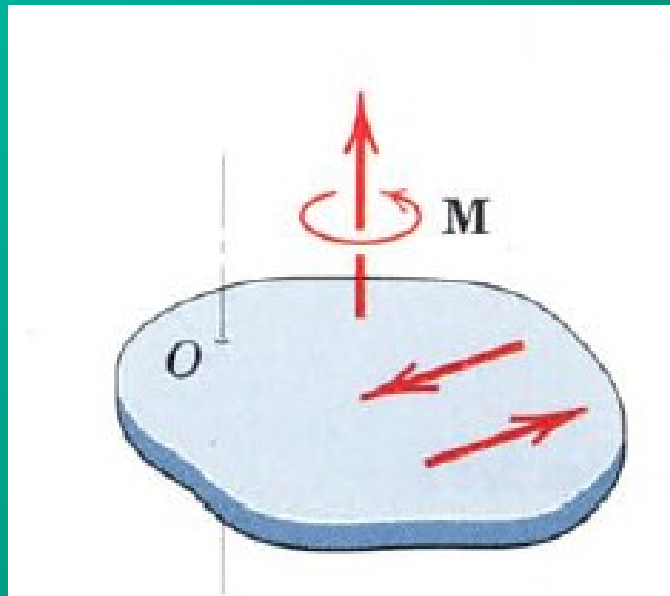
- The moment produced by two equal and opposite and non-collinear forces



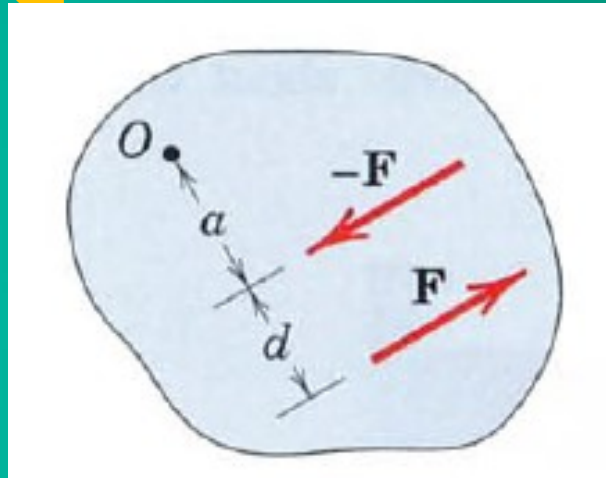
- These two forces can't be combined in to a single force of the same effect on the body, as their sum in every direction is zero
- Their only effect is to create a tendency of rotation

Couple

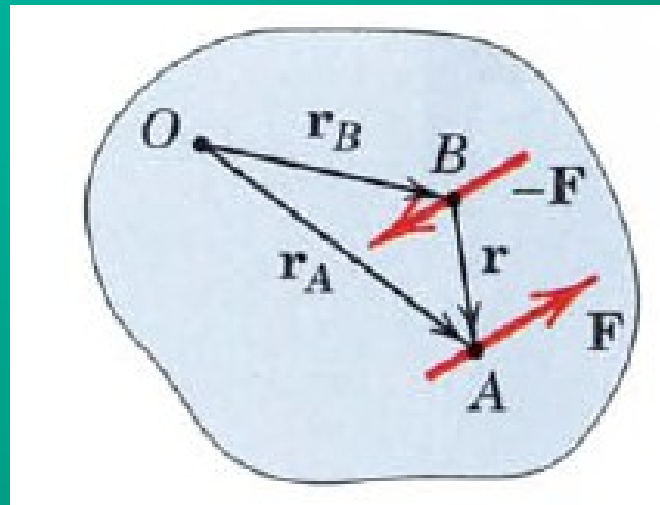
- The combined **moment** of the two forces **about an axis normal** to their plane and passing through any point such as O in their plane is **the couple**



Couple



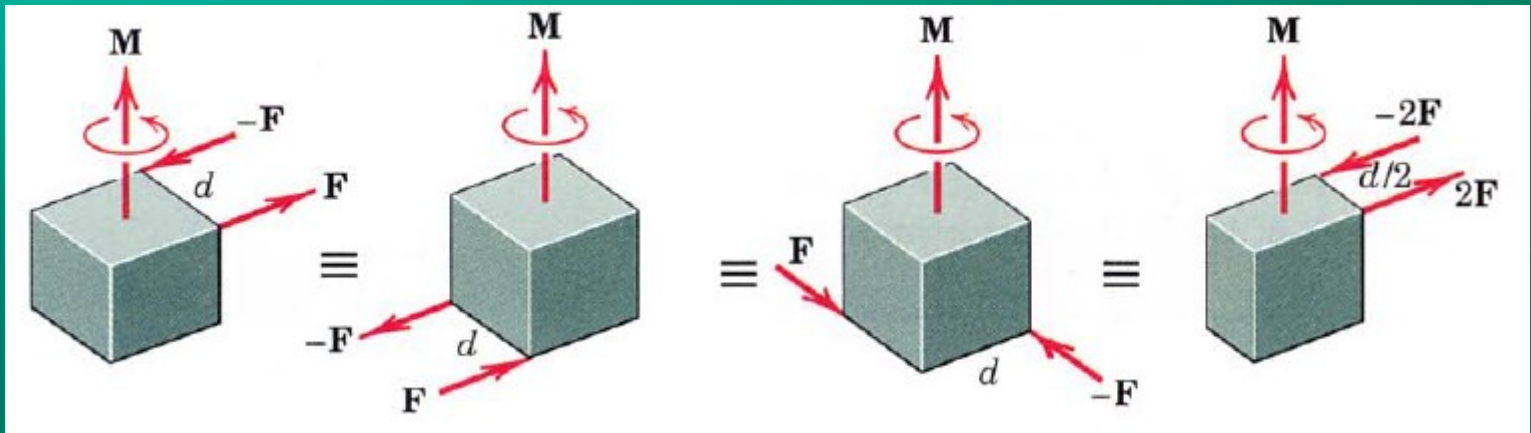
The magnitude of the couple $M = F(a + d) - Fa$
 $= Fd$



$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

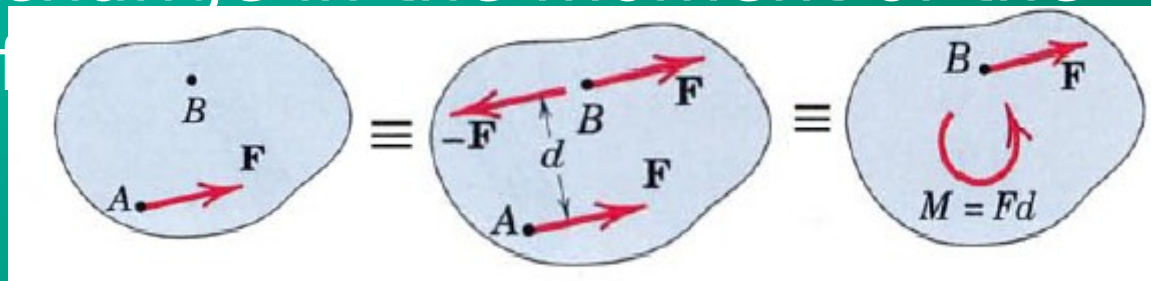
Couple

- **Equivalent Couples**
- Changing the value of F and d does not change a given couple as long as the product Fad remains the same.
- Likewise, a couple is not affected if the forces act in a



Force-Couple System

- The effect of a force on a body is the tendency to push or pull and to rotate the body.
- The representation of this dual effect can be facilitated by replacing the given force by an equal and parallel force and a couple to compensate for the change in the moment of the



2.3.2. Moment and Couple

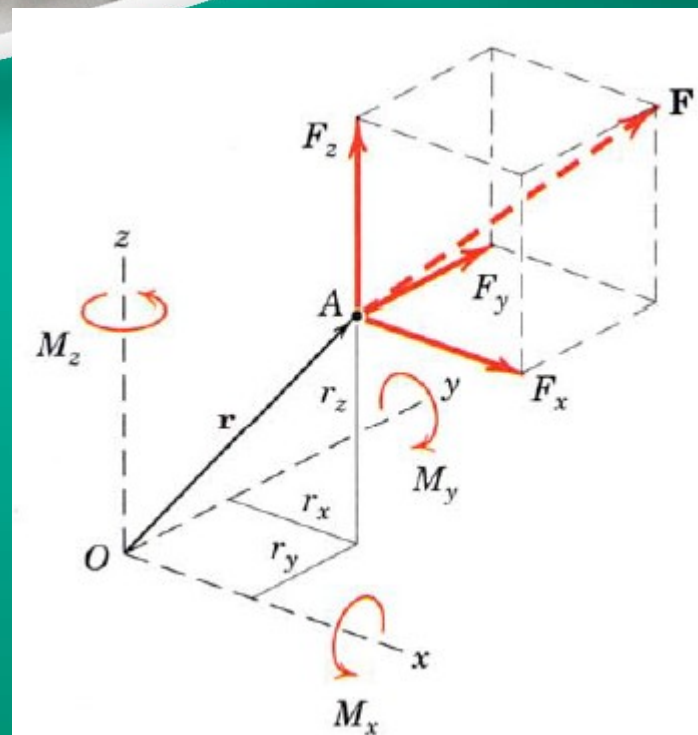
3-D

- Moments in three dimensions
- In two dimensional analysis it is convenient to use the moment arm rule ($M = Fd$)
- However, In three dimensions, the determination of the perpendicular distance between a point or line and the line of action of the force can be a tedious computation.
- In such cases the use of a vector approach with cross-product multiplication becomes

Cross product

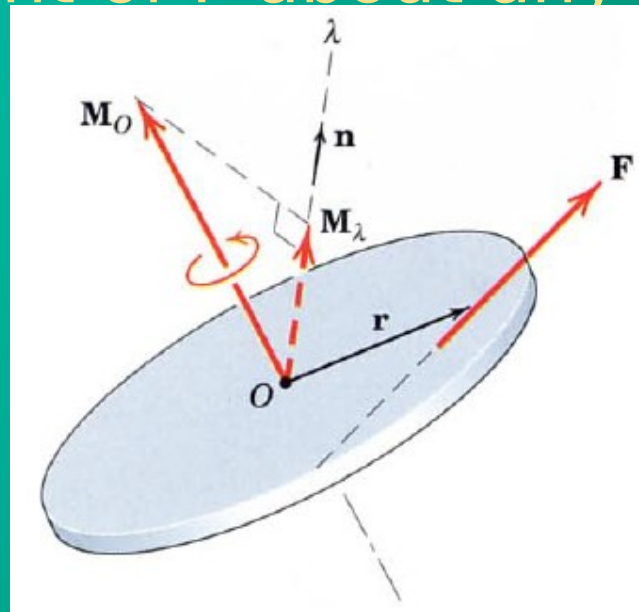
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$



2.3.2. Moment and Couple 3-D

- Moment about an Arbitrary Axis
- It is possible to obtain an expression for moment of F about any axis λ through O

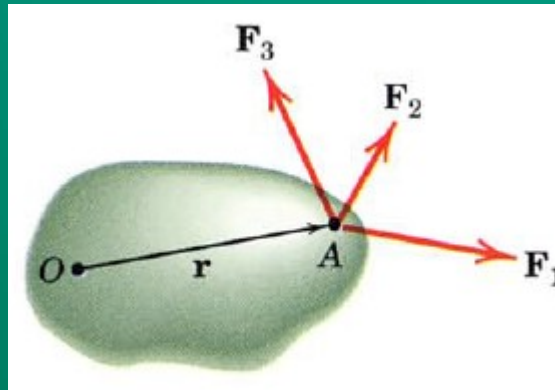


- We can use dot product expression
-
- Vector expression

2.3.2. Moment and Couple 3-D

- **Varignon's theorem**

- Varignon's theorem introduced in two-dimension can be easily extended to three dimensions.



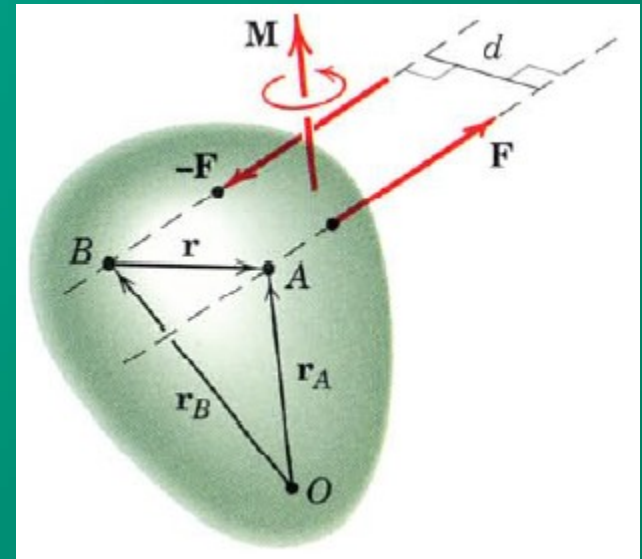
$$\begin{aligned}\mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2 + \mathbf{r} \times \mathbf{F}_3 + \cdots &= \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots) \\ &= \mathbf{r} \times \Sigma \mathbf{F}\end{aligned}$$

2.3.2. Moment and Couple 3-D

- **Couples in Three Dimensions**

- If the vector $\mathbf{r}$ joins any point B on the line of action of $-\mathbf{F}$ to any point A on the line of action of $\mathbf{F}$. The combined moment (couple) of the two forces about O is;

- **Magnitude of M**



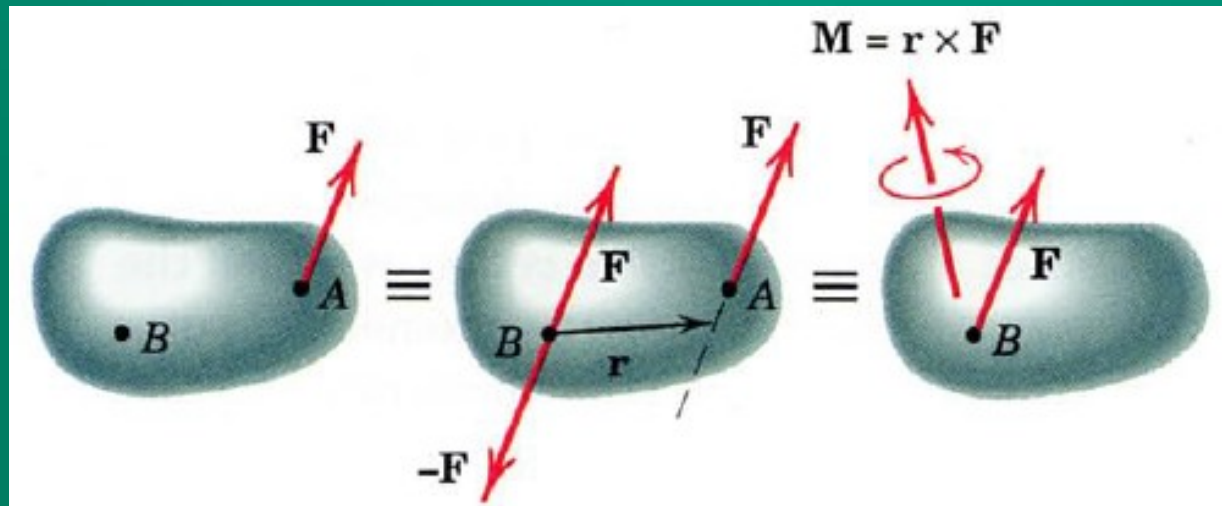
2.3.2. Moment and Couple 3-D

- The moment of the couple, M , is the same about all points. Thus,
 - ✓ The moment of a couple is a free vector,
 - ✓ whereas the moment of a force about a point (which is also the moment about a defined axis through the point) is a sliding vector whose direction is along the axis through the point.
 - ✓ A couple tends to produce a pure rotation of the body about an axis normal to the plane of the forces which constitute the couple.

2.3.2. Moment and Couple 3-D

- Force-Couple System

- It is possible to carry out replacement of a force by its equivalent force-couple system

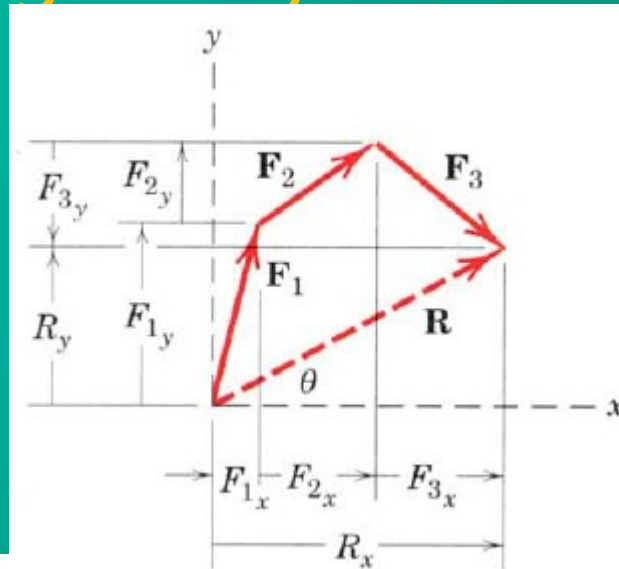


2.4. Resultants

- The **resultant** of a force system is the **simplest force combination** that can replace the original forces **without altering the external effect** of the system on the rigid body to which the forces are applied.
- The **equilibrium of a body** is the condition **where the resultant** of all forces that act on it **is zero**.

2.4. Resultants

- The resultant can be computed by using the **parallelogram rule** or **using analytical methods**.



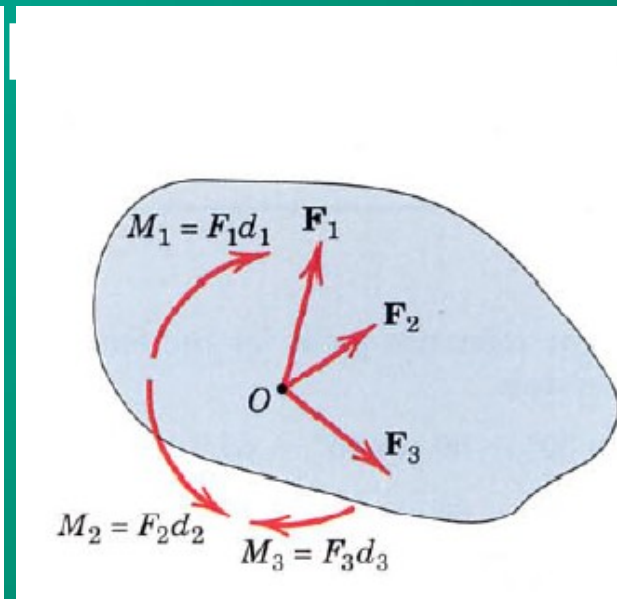
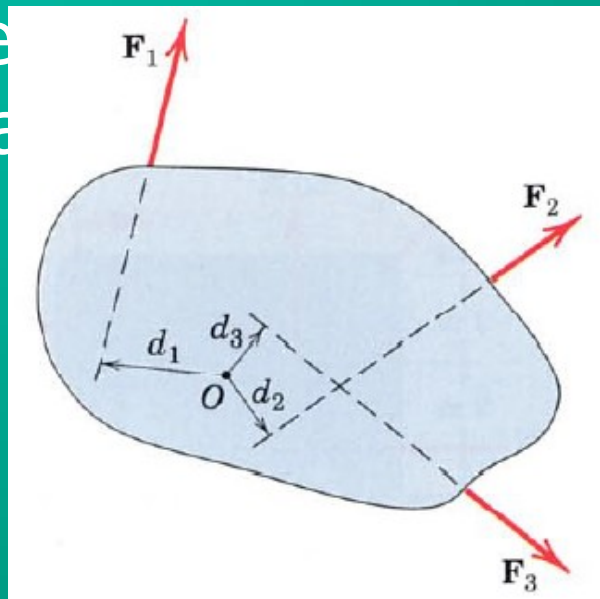
$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma \mathbf{F}$$
$$R_x = \Sigma F_x \quad R_y = \Sigma F_y \quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$\theta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x}$$

2.4. Resultants

- The **algebraic method** can be used to obtain the resultant force and its line of action.

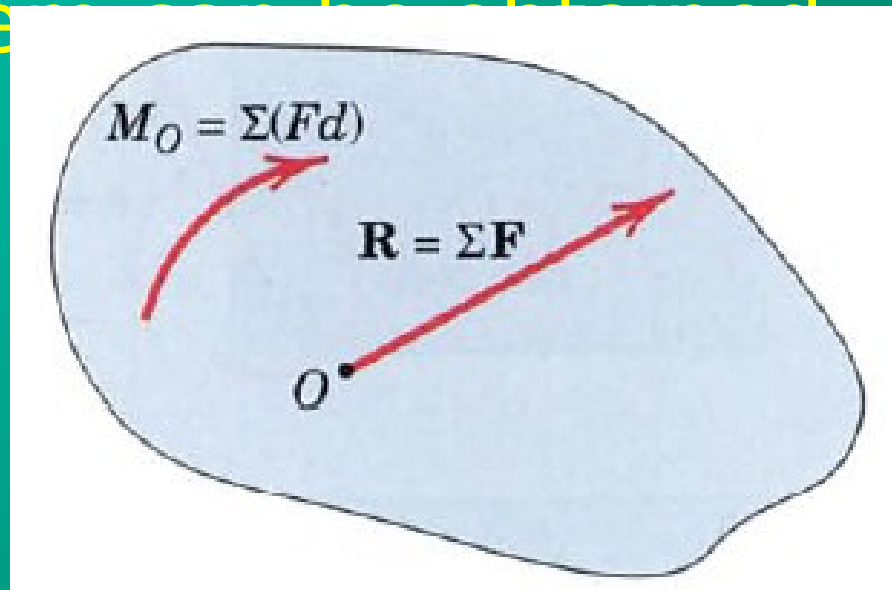
Steps

1. Choose a convenient reference point and move all forces to that point. Also determine the moments of the forces about the reference point.



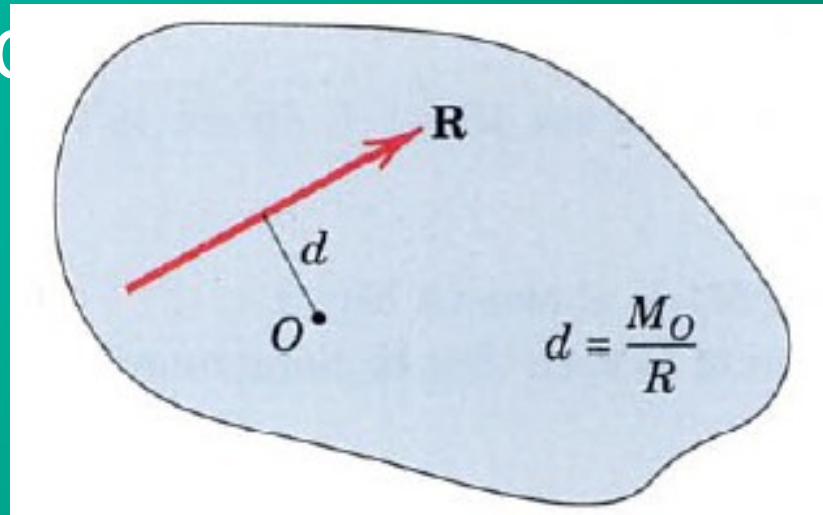
2.4. Resultants (2D)

2. Add all forces at the point chosen to form the resultant force **R** and add all the couples to form the resultant couple **M**. Then the single force-couple system



2.4. Resultants

3. The line of action of the resultant force **R** can be obtained by determining the arm distance **d** required to find the resultant **M**



$$\mathbf{R} = \Sigma \mathbf{F}$$

$$M_O = \Sigma M = \Sigma (Fd)$$

$$Rd = M_O$$

2.4. Resultants

- The **resultant of a force system** is the **simplest force combination**
- Any system of forces may be replaced by its resultant force R and the resultant couple M

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma \mathbf{F}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \cdots = \Sigma(\mathbf{r} \times \mathbf{F})$$

- The magnitude of the resultants and the

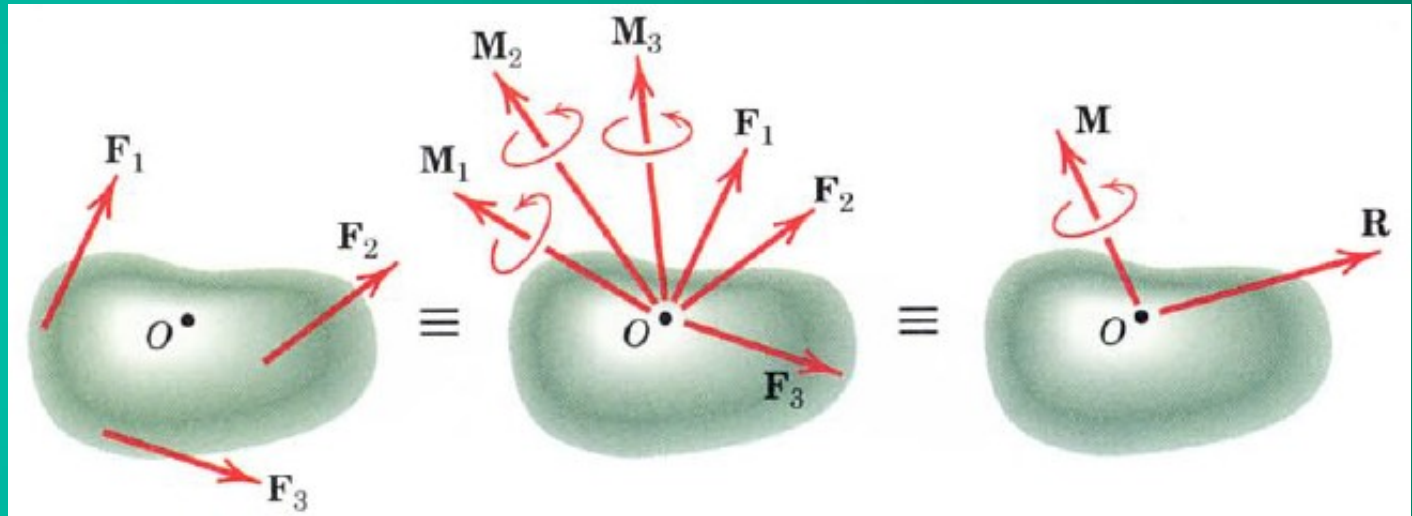
$$R_x = \Sigma F_x \quad R_y = \Sigma F_y \quad R_z = \Sigma F_z$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

$$\mathbf{M}_x = \Sigma(\mathbf{r} \times \mathbf{F})_x \quad \mathbf{M}_y = \Sigma(\mathbf{r} \times \mathbf{F})_y \quad \mathbf{M}_z = \Sigma(\mathbf{r} \times \mathbf{F})_z$$

$$M = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

2.4. Resultants



- The point O selected as the point of concurrency for the force is arbitrary
- The magnitude and direction of M depend on the particular point O selected
- However the magnitude and direction of R are the same no matter which point is selected.

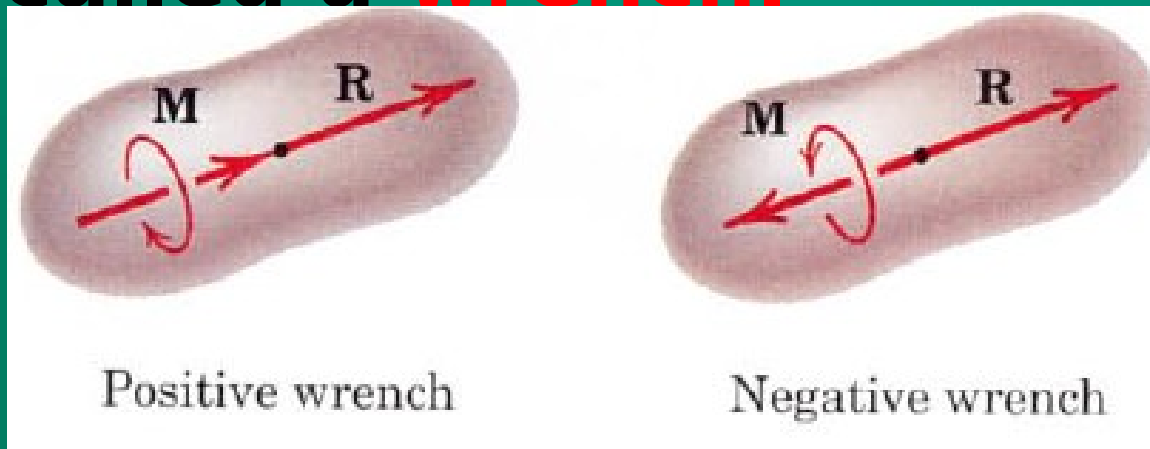
2.4. Resultants

- **Resultant of special force system**
- **Concurrent Forces.** When forces are concurrent at a point only is used because there is no moment about the point of concurrency.
- **Parallel Forces.** For a of parallel forces not all in the same plane, the magnitude of the resultant force R is simply the algebraic sum of the given forces.

The position of its line of action is obtained from the principle of moments by requiring that

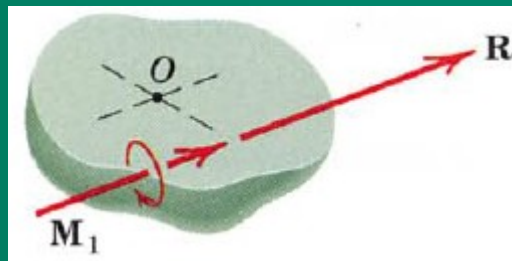
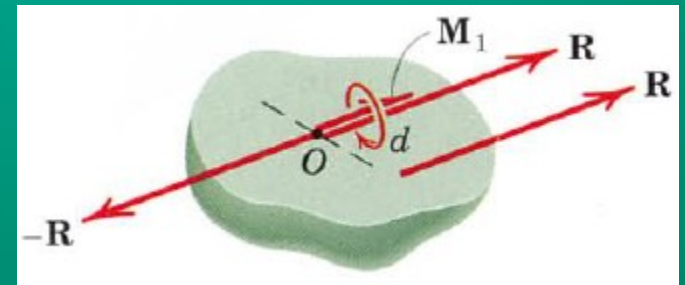
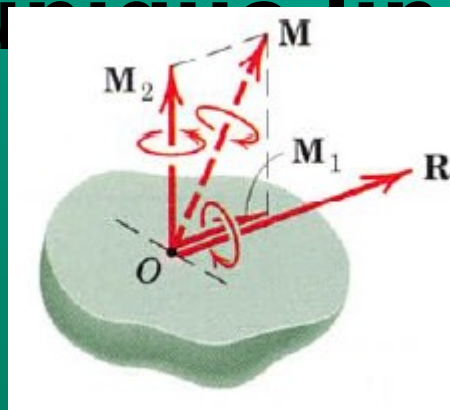
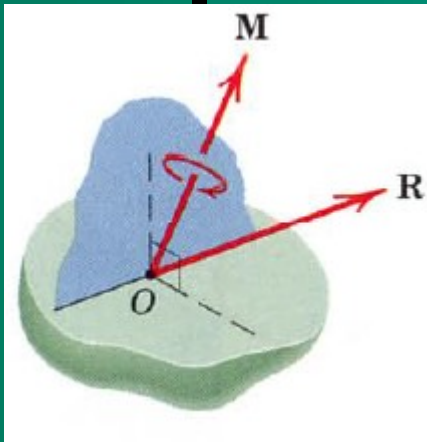
2.4. Resultants

- **Wrench Resultant.** When the resultant couple vector is parallel to the resultant force R the resultant is called a **wrench**.



2.4. Resultants

- Any general force system may be represented by a wrench applied at a unique line of action.





THANK YOU