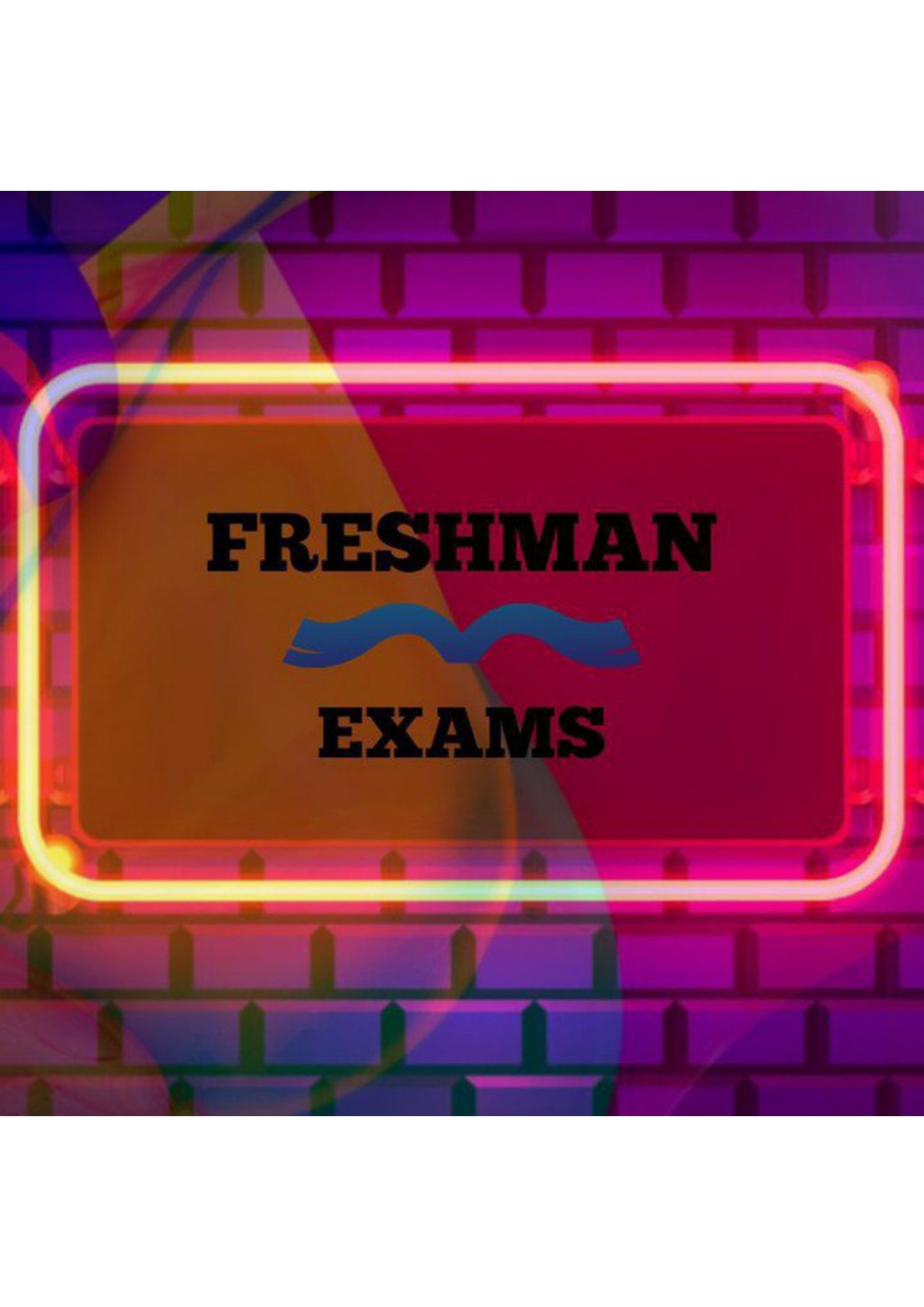




FRESHMAN

EXAMS

Wollo University, KICET
Department Of Pre-Engineering
Mathematics for Natural Science Mid-Examinations.

Wolde 10%
Time Allowed: 1:00

Instruction:

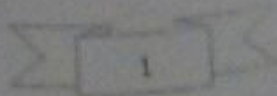
- ❖ Read all the instructions carefully.
- ❖ Don't detach any paper from the exam paper.
- ❖ Any attempt of cheating will make your result disqualify.
- ❖ Switch off your cell phone

PART I: Choose the best answer from the given alternatives and put your choice in the provided space. (5%)

- 1) Which one of the following statements is **not** true?
 - A) $\emptyset \in P(\emptyset)$
 - B) For any set A , $\emptyset \subseteq P(A)$
 - C) For any set A , $A \in P(A)$
 - D) For any set A , $P(A) \subset A$
- 2) One of the following is an open proposition.
 - A) $x + 9 = 9$
 - B) There is no real number x such that $x^2 = 4$.
 - C) For all $x \in \mathbb{R}$, $x^2 + 6x - 12 = 0$.
 - D) For every real number x , $x^2 + 1 \geq 0$.
- 3) The negation of a proposition $(P \wedge \neg Q) \Rightarrow Q$ is
 - A) $\neg P \vee Q$
 - B) $\neg P \wedge Q$
 - C) $P \wedge \neg Q$
 - D) $P \vee \neg Q$
- 4) If $B \subseteq A$, $A \cap B = \{1, 4, 5\}$ and $A \cup B = \{1, 2, 3, 4, 5, 6\}$, then $B =$ _____.
 - A) $\{1, 4, 5\}$
 - B) $\{2, 3, 6\}$
 - C) $\{1, 2, 3, 4, 5, 6\}$
 - D) $\{1, 2, 3, 4, 5\}$
- 5) Which one of the following statement is correct?
 - A) The sum of any two rational numbers is rational.
 - B) The sum of any two irrational numbers is irrational.
 - C) The product of any two irrational numbers is rational.
 - D) The product of any two irrational numbers is irrational.

PART II: Write "True" if the statement is correct and "False" if it is incorrect. (5%)

- 1) The truth value of a proposition "2 is an odd and even number if Addis Ababa is the Capital city of Ethiopia." is false.
- 2) The set of natural numbers is closed under addition and subtraction.
- 3) Two sets A and B are said to be disjoint if $A \cap B = \{\emptyset\}$.
- 4) For some real numbers x , such that $|x| = -1$ is an example of proposition.
- 5) The argument of product of two complex numbers is the product of their arguments.



9) Express each of the following in polar form:

a) $2 + 2\sqrt{3}i$ b) $-5 + 5i$ c) $-\sqrt{6} - \sqrt{2}i$ d) $1 - i$ e) $-3i$ f) -10

10) Write each of the following in rectangular form (i.e. $x+yi$).

a) $(\sqrt{3} + i)^7$ b) $\frac{3i^{20} - i^{19}}{2i - 1}$ c) $(-\sqrt{3} - i)^5$ d) $(\sqrt{3} - i)^6$

12) Find the modulus and principal argument of $(1 + i)^{10}$

13) Find all values of z such that: a) $z^5 = -32$ b) $z^2 = 1 + i$ c) $z^2 + (2i - 3)z + 5 - i = 0$

d) $(z + 1)^4 = 1 - i$ e) $e^z = 1 + \sqrt{3}i$ f) $z^2(2 - z^2) = 2$

14) Find the values of $(\sqrt{3} + i)^{1/2}$ and express each in rectangular form.

11) If A, B , and C are any three sets, then show that,

(a) $(B - A) \cup (C - A) = (B \cup C) - A$,

(b) $(A - B) - (B - C) = A - B$

(c) $P(A) \subseteq P(B) \Leftrightarrow A \subseteq B$

(d) $A \Delta (B \Delta C) = (A \Delta B) \Delta C$

11. Determine whether each of the following arguments is valid or not:

(a) If the moon is made of bananas, then there are monkeys on the moon. The moon is not made of bananas. Therefore, there are no monkeys on the moon.

(b) A sufficient condition for this year to be 2020 is that last year was 2019. This year is 2020. Therefore last year was 2019.

Section 16,4

STUDENT COPY

Jimma University
College of Natural Sciences
Department of Mathematics

Math1011 (Mathematics for natural sciences) Worksheet I
Chapter I: Propositional Logic and Set Theory

- 1) Let p : 387 is a composite number.
a. q : 123 is a prime number.

Write each of the following in words, and determine the truth value of each.

- a. $q \vee \neg p$ b. $\neg q \wedge p$ c. $\neg p \Rightarrow \neg q$ d. $\neg q \Leftrightarrow \neg p$

- 2) Suppose p, q , and r are propositions. Use truth table to show that each of the following pairs of propositions is logically equivalent.

- a. a. $(p \vee q) \Rightarrow r$ and $(p \Rightarrow r) \wedge (q \Rightarrow r)$
b. $p \Rightarrow (q \vee r)$ and $(\neg r) \Rightarrow (p \Rightarrow q)$
c. $p \Rightarrow (q \vee r)$ and $((\neg r) \wedge p) \Rightarrow q$

- 3) Let $P(x)$: x is an integer, $Q(x)$: x is a rational number,

$R(x)$: x is an irrational number and $S(x)$: x is a real number.

Express each of the following in symbolic form.

- (a) x is an irrational number if x is a real number
(b) x is not an integer or x is not a rational number.
(c) x is a real sufficient condition for x is an irrational number.
(d) x is an irrational number only if x is a real number.
4) Let U = the set of real numbers. Determine the truth value of the following.
(a) $(\forall x)(x^2 + x + 1 > 0)$
(b) $(\exists x)(x^3 + x = 0)$
(c) $(\forall x)(\forall y)[x^2 - y^2 = (x + y)(x - y)]$

MAWASSA UNIVERSITY
DEPARTMENT OF MATHEMATICS

December 25, 2019

Name: _____ ID.No. _____ Sec. _____

Instructor's name: _____

TIME allowed 2 : 30 hrs.

Part I. Choose the best answer to each of the following questions and write your answer on the space provided in the answer sheet [1.5 points each]

- Let U = the set of natural numbers.
 $P(x) = x$ is prime. $q(x) = x$ is even. $r(x) = x$ is odd.
Then which of the following statement is false?
A) $(\exists x)[p(x) \vee r(x)]$ B) $(\exists x)[p(x) \wedge q(x)]$
C) $(\exists x)[p(x) \Rightarrow q(x)]$ D) $(\forall x)[(p(x) \wedge r(x)) \Rightarrow q(x)]$
- Which of the following is not necessarily true for any sets A and B of the universal set U ?
A) If $A \subseteq A'$, then $A = \emptyset$ B) $A \cap B = A \setminus (A \setminus B)$
C) $B \setminus A = B' \cap A$ D) If $A \setminus B = B \setminus A$ then $A = B$
- Which of the following pair of statements are not equivalent to each other?
A) $p \Rightarrow (q \Rightarrow r), (p \wedge q) \Rightarrow r$ B) $p \Rightarrow (q \vee r), (r \wedge p) \Rightarrow q$
C) $p \Rightarrow q, q \vee \neg p$ D) $\neg(\neg p \wedge q), p \vee \neg q$
- The polar form of $z = \frac{1 + i\sqrt{3}}{1 + i}$ is
A) $\sqrt{2}(\cos(\frac{\pi}{3}) + i \sin(\frac{\pi}{3}))$ B) $\sqrt{2}(\cos(\frac{\pi}{12}) + i \sin(\frac{\pi}{12}))$
C) $\frac{\sqrt{2}}{2}(\cos(105^\circ) + i \sin(105^\circ))$ D) $2(\cos(\frac{\pi}{6}) + i \sin(\frac{\pi}{6}))$
- If $2 \in A$ but $-2 \notin A$ and $-2 \in B$ but $2 \notin B$, then which of the following is necessarily true?
A) $\{-2, 2\} \subseteq A' \cup B'$ B) $A \setminus B = \{2\}$ C) $A \subseteq B$ D) $A \cap B = \emptyset$
- Which of the following is not true on the set $M = \{1, \{a\}, \emptyset, \{\emptyset\}\}$
A) $\{\emptyset\} \subseteq M$ B) $\{\emptyset\} \in M$ C) $\{\emptyset, \{\emptyset\}\} \in M$ D) $\emptyset \subseteq M$

Part II. Give the most simplified short answers to each of the following questions on the space provided in the answer sheet [1.5 points each]

1. When $\{x \in \mathbb{R} : x(x^2 - 2) \neq 0\}$ expressed as a union of intervals is _____
2. Let $U = \{n \in \mathbb{N} : n \leq 10\}$,
 $B = \{2, 3, 4, 6, 8\}$, $A = \{1, 2, 3, 4, 9\}$,
 $C = \{2, 4, 6, 7, 8, 9\}$
 Then a) $(A - B) \cup C' =$ _____, b) $(A - C)' \cap B' =$ _____
3. Let $w = 2 - i$ and $z = -1 + i$, then $z^2 w \bar{z}^2$ is equal to _____
4. If $(5 - 2i)(a + bi) = 11i + 16$, Then $a =$ _____ and $b =$ _____
5. Let $p(x) : "x \text{ is negative}"$, $q(x) : "x < 5"$. Then give the negation of $(\forall x)[q(x) \Rightarrow p(x)]$

6. The fraction form of $2.2\overline{34}$ is _____
7. If $A = \left\{ (-1)^n \left(\frac{n-1}{n} \right) \right\}_{n=1}^{\infty}$, then
 a) $\text{lub of } A =$ _____, b) $\text{glb of } A =$ _____
8. If $\neg r \wedge (p \Leftrightarrow q)$ has a truth value of True, then the truth value of $p \Leftrightarrow (q \Rightarrow r)$ is _____
9. If $z = (\sqrt{3} + i)^6$, then $\text{Re}(z) =$ _____
10. Determine the truth value of the following statements if $x, y \in \mathbb{R}$
 a) $(\forall y)(\exists x)(x^2 + y^2 = 1)$ _____ b) $(\exists x)(\forall y)(x < y^2 - y)$ _____

Part III. Work on the following problems by showing all the pertinent information.

1. For the statements p, q and r , use the truth table to show that the statements $p \Rightarrow (q \vee r)$ and $\neg r \Rightarrow (p \Rightarrow q)$ are logically equivalent. [4 points]
2. Using principle of Mathematical induction Prove that
 $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$ for $n \in \mathbb{N}$ and $n \geq 1$ [4 points]
3. Suppose p : The rain comes. q : Crops are ruined. r : The people will starve.
 Give the symbolic argument form and determine the validity of
 "If the rain does not come, the crops are ruined and the people will starve. The crops are not ruined or the people will not starve. Therefore, the rain comes"
 using formal proof. [4 points]
4. Find the 4th roots of z if $z = -1 - \sqrt{3}i$ [4 points]

33. Part II: (2 point each) (Choose the best answer for the following questions:

1. Let A be the set of even natural numbers less than or equal to 15. Which of the following is true.

A. $\{2, 6\} \in A$

C. $0 \in A$

B. $\emptyset \in A$

D. $\{2, 4\} \subset A$

2. Let $P(x, y) : xy = 0, x, y \in \mathbb{Z}$. Which of the following is false.

A. $(\forall x)(\forall y)P(x, y)$

C. $(\forall x)(\exists y)P(x, y)$

B. $(\exists x)(\exists y)P(x, y)$

D. $(\exists x)(\forall y)P(x, y)$

3. Which of the following sentence is not a proposition?

A. The sum of two prime numbers is even.

C. The sun is shining.

B. Come to class!

D. $3 + 4 = 7$.

4. Which of the following statement is true?

A. The sum of any two irrational numbers is irrational.

B. The product of any two rational numbers is irrational.

C. The product of any two rational numbers is rational.

D. The sum of any two irrational numbers is rational.

5. The GCF and LCM of 80 and 120 is ___ and ___ respectively.

A. 40, 240

B. 120, 80

C. 240, 40

D. 80, 120

Part IV: Workout: (Show all necessary steps neatly and clearly.)

1. (3 point) Prove that $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \dots + \frac{1}{n(n+1)} = \frac{n}{n+1}, n \geq 1$ using principle of mathematical induction.

2. (4 point) Consider the argument

"If the professor offers chocolate for an answer, then you answer the professor's question. The professor offers chocolate for answer. Therefore, you answer the professor's question."

Identify all premises, conclusion and verify its the validity using truth table.

3. (3 point) Find all cubic roots of $z = -i$.

PART II. Choose the best answer and write the letter of your choice on the space provided.
(1.5pts each)

- Which one of the following is always true?
A) The sum of any two irrational numbers is irrational.
B) The sum of any two rational numbers is rational.
C) The product of any two irrational numbers is irrational.
D) The product of a rational and an irrational numbers is irrational.
- For natural numbers x and y , which statement is false?
A. $(\forall x) 2x$ is even. C. $(\forall x)(\exists y) y < x$.
B. $(\exists x)(\forall y) x \leq y$. D. $(\forall x)(\forall y) (x + y)^2$ is composite.
- For any set A , if $A * A' = A$, then the set operation $*$ is:
A. $\cup$ B. $\cap$ C. Δ D. $\setminus$ E. None of the above
- Let $A = \{ \{ \{ \phi \} \}, \{ \phi \}, \phi \}$. Then which one is not true?
A. $\{ \{ \phi \} \} \subseteq A$ B. $\{ \{ \phi \} \} \in A$ C. $\{ \phi, \{ \phi \} \} \subset A$ D. $\{ \{ \{ \phi \} \} \} \in P(A)$ E. None of the above
- Which one of the following is false about any two sets A and B ?
A. $A \subset B \Rightarrow B' \subset A'$ C. $A' - B' = B - A$
B. $A = B \Rightarrow A' = B'$ D. $(A \cap B)' \cap B = B$

PART III. Write the most simplified form of your answer on the space provided. (1.5 pts each)

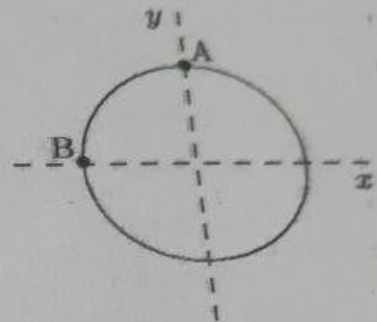
- In polar form, $(1 - i)^{12} = 64(\cos 3\pi - i \sin 3\pi)$
- The complex number(s) z for which $\operatorname{Re}(z^2) = 0$ and $\operatorname{Im}(z^2) = 2$ is (are) $\pm 2i$
- Write one of the fifth roots of $-32i$. $2(\cos \frac{2\pi}{5} + i \sin \frac{2\pi}{5})$
- The identity element in the operation union of sets is empty set
- Let $z = \sqrt{6} - 6\sqrt{3}i$, then the polar form of z is $2\sqrt{3}(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6})$

Two trains are moving at 30 m/s in opposite directions on the same track. The engineers see simultaneously that they are on a collision course and apply the brakes when they are 1000 m apart. Assuming both trains have the same acceleration, what must this acceleration be if the trains are to stop just before colliding?

- A. -1.0 m/s^2 B. -0.9 m/s^2 C. -30.0 m/s^2 D. -0.03 m/s^2

7. A particle is moving around a circular path with a constant speed as shown below. The coordinates of the particle when it is at point B are $x = 5\text{m}$, $y = 0\text{m}$ and its velocity is $(10 \text{ m/s})\hat{j}$. What will be the particle's velocity and acceleration when it is at point A respectively?

- A. $(10 \text{ m/s})\hat{i}$ and $(-20 \text{ m/s}^2)\hat{j}$
 B. $(6 \text{ m/s})\hat{i}$ and $(-12 \text{ m/s}^2)\hat{j}$
 C. $(6 \text{ m/s})\hat{j}$ and $(12 \text{ m/s}^2)\hat{i}$
 D. $(10 \text{ m/s})\hat{j}$ and $(20 \text{ m/s}^2)\hat{i}$



DIRECTION II. Give short answers that best fill the blank spaces following each question and give your answers on the answer sheet.

1. A unit vector in the direction of the momentum vector $P = (3\hat{i} - 5\hat{j} + 4\hat{k}) \text{ kgm/s}$ is _____ (2 pts.)

2. A missile is fired towards an enemy target with a velocity of 120m/s at an angle of 53° from the horizontal. Find the values for the following questions based on this information.

a) The velocity of the missile at 4s: $V(t = 4\text{s}) = ______ \hat{i} + ______ \hat{j}$ (2 pts.)

b) If the enemy target is at horizontal distance of 500m, will the missile hit the target? _____ (2 pts.)

3. A boy is about to cross over a rail way but unfortunately a 300m long train arrived and passes by him. How long does he need to wait to cross? _____ (2 pts.)

Part II. Give the most simplified short answers to each of the following questions on the space provided in the answer sheet [1.5 points each]

1. When $\{x \in \mathbb{R} : x(x^2 - 2) \neq 0\}$ expressed as a union of intervals is _____
2. Let $U = \{n \in \mathbb{N} : n \leq 10\}$,
 $B = \{2, 3, 4, 6, 8\}$, $A = \{1, 2, 3, 4, 9\}$,
 $C = \{2, 4, 6, 7, 8, 9\}$
 Then a) $(A - B) \cup C' =$ _____, b) $(A - C)' \cap B' =$ _____
3. Let $w = 2 - i$ and $z = -1 + i$, then $z^2 w \bar{z}^2$ is equal to _____
4. If $(5 - 2i)(a + bi) = 11i + 16$, Then $a =$ _____ and $b =$ _____
5. Let $p(x) : "x \text{ is negative}"$, $q(x) : "x < 5"$. Then give the negation of $(\forall x)[q(x) \Rightarrow p(x)]$

6. The fraction form of $2.2\overline{34}$ is _____
7. If $A = \left\{ (-1)^n \left(\frac{n-1}{n} \right) \right\}_{n=1}^{\infty}$, then
 a) $\text{lub of } A =$ _____, b) $\text{glb of } A =$ _____
8. If $\neg r \wedge (p \Leftrightarrow q)$ has a truth value of True, then the truth value of $p \Leftrightarrow (q \Rightarrow r)$ is _____
9. If $z = (\sqrt{3} + i)^6$, then $\text{Re}(z) =$ _____
10. Determine the truth value of the following statements if $x, y \in \mathbb{R}$
 a) $(\forall y)(\exists x)(x^2 + y^2 = 1)$ _____ b) $(\exists x)(\forall y)(x < y^2 - y)$ _____

Part III. Work on the following problems by showing all the pertinent information.

1. For the statements p, q and r , use the truth table to show that the statements $p \Rightarrow (q \vee r)$ and $\neg r \Rightarrow (p \Rightarrow q)$ are logically equivalent. [4 points]
2. Using principle of Mathematical induction Prove that
 $1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$ for $n \in \mathbb{N}$ and $n \geq 1$ [4 points]
3. Suppose p : The rain comes. q : Crops are ruined. r : The people will starve.
 Give the symbolic argument form and determine the validity of
 "If the rain does not come, the crops are ruined and the people will starve. The crops are not ruined or the people will not starve. Therefore, the rain comes"
 using formal proof. [4 points]
4. Find the 4th roots of z if $z = -1 - \sqrt{3}i$ [4 points]

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
Mathematics for Natural Sciences (Math 1011)
worksheet I - November 2019

Part I: Propositional Logic.

(1) Which of the following sentences are propositions? What are the truth values of those that are propositions?

- (a) $5 < 9$ and there are infinitely many prime numbers.
- (b) $x^2 = y^2$ provided that $x = y$.
- (c) How many courses are you taking this semester?
- (d) The first human kind lived in Ethiopia.

(2) Consider the following propositions

r : Rabbits have been seen in the area.

b : Berries are ripe along the path w : Walking on the path is safe.

Write the following propositions using r , b and w and logical connectives.

- a) It is not safe to walk along the path, but rabbits have not been seen in the area and the berries along the path are ripe.
- b) For walking on the path to be safe, it is necessary but not sufficient that berries not be ripe along the path and for rabbits not to have been seen in the area.
- c) Walking is not safe on the path whenever rabbits have been seen in the area and berries are ripe along the path.

(3) Given the following propositions

p : You have the flu.

r : You pass the course.

q : You miss the final examination.

Express each of the following propositions as an ordinary English sentence.

a) $\neg q \Leftrightarrow r$

b) $\neg(p \wedge q)$

c) $(p \Rightarrow \neg r) \vee (q \Rightarrow \neg r)$.

(4) Determine the truth value of p if

a) $(q \Rightarrow \neg p) \vee r$ is False

b) $\neg q$ and $p \Rightarrow q$ are True

c) $(p \vee \neg q) \Rightarrow (\neg r \vee p)$ is False.

(5) If $\neg[\neg r \Rightarrow \neg(p \wedge q)]$ is true, then find the truth value of $[(p \Leftrightarrow r) \vee q] \Leftrightarrow (\neg p \wedge r)$.

(6) Prove that

a) $(p \wedge q) \Rightarrow r \equiv p \Rightarrow (q \Rightarrow r)$

b) $(q \vee r) \Rightarrow p \equiv (q \Rightarrow p) \wedge (r \Rightarrow p)$.

(7) For the following propositions, indicate whether it is a tautology, a contradiction, or neither. Use a truth table to decide.

a) $[p \wedge (p \Rightarrow q)] \Rightarrow q$

b) $(p \Rightarrow q) \wedge \neg(\neg q \Rightarrow \neg p)$.

c) $(\neg q \Rightarrow \neg p) \Rightarrow ((\neg q \Rightarrow p) \Rightarrow q)$.

(8) Determine whether the following statements are valid or invalid.

a) If I do not wake up, then I cannot go to work.

If I cannot go to work, then I will not get paid.

Therefore, if I do not wake up, then I will not get paid.

b) If I study, then I will not fail Math 1011.

If I do not play cards too often, then I will study.

I failed Math 1011.

Therefore, I played cards too often.

Part II: The Complex number system.

- (1) Change the following complex numbers to polar form
(a) $-2 - 4i$. (b) $1 - \sqrt{2}i$.
(c) -27 . (d) $5i$.
- (2) Find the modulus and argument of
(a) $(1 - \sqrt{2}i)^2$. (b) $(1 + i)^{11}$. c) $(\frac{1}{2} - \frac{\sqrt{3}i}{2})^{15}$.
- (3) Find $z \in \mathbb{C}$ such that
(a) $\operatorname{Re}(z(1 + i)) + z\bar{z} = 0$; (b) $\operatorname{Re}(z^2) + i\operatorname{Im}(\bar{z}(1 + 2i)) = -3$;
(c) $\operatorname{Im}(2 - i)z = 1$.
- (4) Express the following complex numbers in the form $a + bi$.
(a) $(1 + i)^3$. (c) $i^5 + i^{16}$.
(b) $\frac{2+3i}{3-4i}$. (d) $\frac{1}{2}(1 + i)(1 + i^{-8})$.
- (5) Compute the absolute value and the conjugate of
(a) $z = (1 + i)^6$. (b) $w = i^{17}$.
- (6) In each case, determine all real x and y which satisfy the given relation.
(a) $x + iy = |x - iy|$. (b) $x + iy = (x - iy)^2$.
(c) $\sum_{k=0}^{100} i^k = x + iy$.
- (7) Solve the following equations.
(a) $z^2 = 1 + i$. (b) $z^2 - 4z + 6 = 0$.
(c) $z^4 + 4z^2 + 16 = 0$. (d) $(\bar{z} - i)^3 = 1$. (e) $z^5 = 32$.
- (8) Compute
(a) the square roots of $z = -1 - i$.
(b) the three cube roots of $z = -8$.
(c) the four fourth roots of $z = -1 + \sqrt{3}i$.
- (9) Write $\sin 4\theta$ and $\cos 4\theta$ in terms of $\sin \theta$ and $\cos \theta$. (Hint: use $(\cos \theta + i \sin \theta)^4$)
- (10) Find the locus of points $z = x + iy$ that satisfy
(a) $|z + 1| = |z - i|$. (b) $3|z| = |z + 3i|$.
(c) $|z - 1| = 2$. (d) $z^2 \bar{z} = z$.
- (11) If $\alpha = \cos \theta + i \sin \theta$ for $0 < \theta < 2\pi$, show that $\frac{1+\alpha}{1-\alpha} = i \cot \frac{\theta}{2}$.
- (12) If $z \neq -1$ and $\operatorname{Im}(z) \neq 0$, show that $\frac{z}{(1+z)^2}$ is real iff $z \cdot \bar{z} = 1$.
- (13) If for $z = x + iy$, $f(z) = e^x(\cos y + i \sin y)$, then show that
(a) $f(z_1 + z_2) = f(z_1)f(z_2)$. (b) $\overline{f(z)} = f(\bar{z})$.
- (14) Find $z \in \mathbb{C}$ such that $z^2 \in \mathbb{R}$.

(9) Let $P(x) : x$ is an integer greater than 5

$Q(x) : x$ is a natural number

Determine the truth value of the following propositions.

- a) $P(3) \wedge Q(-2)$ b) $P(2) \Rightarrow Q(2)$ c) $[P(3) \Rightarrow Q(4)] \vee Q(9)$.

(10) Let $P(x) : x$ is a prime number

$Q(x) : x$ is an even number

$R(x) : x$ is an odd number.

$S(x) : x$ is an integer.

Write a sentence, which corresponds to each of the following:

a) $R(x) \vee Q(x) \Rightarrow P(x)$

b) $P(x) \Leftrightarrow Q(x) \vee R(x)$.

(11) If $U = \mathbb{R}$, then find the truth value of $(\forall x)(\exists y)(\sqrt{x^2} = 16 \Rightarrow y + x = 10)$.

(12) Let $P(x) : x$ is a composite number

$Q(x) : x$ is a prime number.

Find the truth values of (i) $(\exists x)[P(x) \Leftrightarrow Q(x)]$ (ii) $(\forall x)[P(x) \Leftrightarrow Q(x)]$.

(13) Find the truth values of the following where $U = \mathbb{R}$

a) $(\exists x)(\forall y)(x^2 < y^2)$

b) $(\forall x)[x \neq 0 \Rightarrow (\exists y)(xy = 4)]$.

Part II: Set Theory.

(1) Write the following sets in complete listing or partial listing method.

a) $A = \{x | x \text{ is an integer and } 3 < x \leq 10\} \cap \{x | x \text{ is even integer}\}$.

b) $B = \{x \in \mathbb{N} | (x-1)(x-3) \neq 0 \Leftrightarrow x+1 = x\}$.

c) $C = \{x \in \mathbb{N} | x \geq 3 \Leftrightarrow x < 0\}$.

d) $D = \{x | x \in \mathbb{N} \wedge (x-1)(x-3) \neq 0 \Leftrightarrow x+1 = x\}$.

(2) If $n(A \setminus B) = 18$, $n(A \cup B) = 70$ and $n(A \cap B) = 25$, then find $n(B)$.

(3) For all subsets A, B and C of some Universal set U , prove or disprove the following statements.

a) $(A \subseteq B \wedge A \subseteq C) \Rightarrow A \subseteq B \cap C$

b) $A \subseteq B \Leftrightarrow A \cap B = A$

c) $A^c \setminus B^c = B \setminus A$

d) $A \subseteq B \Rightarrow A \cup (B \setminus A) = B$.

(4) Let $U = \{x \in \mathbb{Z} | -12 \leq x \leq 6\}$.

$A = \{x \in \mathbb{Z} | x = 2n \wedge x = 3m, \text{ for } -4 \leq n \leq 8 \text{ and } -2 \leq m \leq 3\}$.

$B = \{0, 1, 3\}$

$C = \{x \in \mathbb{Z} | x = \frac{n}{2}, n \in A\}$.

Then find

a) $A \setminus (B \cup C)^c$

b) $(A \setminus B) \setminus (A \cup C)^c$

c) $(A \cap B) \setminus C$.

(5) (a) For every natural number n , define $A_n = \{x \in \mathbb{Z} | -\frac{n}{2} \leq 2x+1 \leq \frac{n}{2}\}$.

Then find

i) $\bigcup_{n=1}^{\infty} A_n$

ii) $[(\bigcup_{n=1}^{\infty} A_n) \setminus (\bigcap_{n=1}^{\infty} A_n)]^c$

iii) $\bigcap_{n \in \mathbb{N}} A_n$

(b) For every natural number n , define

$A_n = \{x \in \mathbb{N} | x = n \Rightarrow x < n\}$.

Then find

i) $\bigcup_{n=1}^{\infty} A_n$

ii) $\bigcup_{n \in \mathbb{N}} A_n$

iii) $\bigcap_{n \in \mathbb{N}} A_n$

as given in the question
the set is empty
the set is empty
the set is empty

$$\begin{aligned} & 0 \leq 2x+1 \leq 0 \\ & -1 \leq 2x \leq -1 \\ & -\frac{1}{2} \leq x \leq -\frac{1}{2} \\ & -0.5 \leq x \leq -0.5 \end{aligned}$$

$$\begin{aligned} & -2n \leq 2x+1 \leq 2n \\ & -2n-1 \leq 2x \leq 2n-1 \\ & -\frac{2n+1}{2} \leq x \leq \frac{2n-1}{2} \\ & -n-0.5 \leq x \leq n-0.5 \end{aligned}$$

ADDIS ABABA UNIVERSITY
DEPARTMENT OF MATHEMATICS
 Mathematics for Natural Sciences (Math 1011)
worksheet II - November 2019

Part I: The real number system.

- (1) Two incandescent lamps are turned on at the same time. One blinks every 4 seconds and the other blinks every 6 seconds. In 60 seconds, how many times will they blink at the same time?
- (2) Write the following sets as an interval or union of intervals. In each case $x \in \mathbb{R}$.
 - (a) $A = \{x | x \neq 5 \wedge x \neq 8\}$.
 - (b) $B = \{x | x^2(x^3 - 1) \neq 0\}$.
 - (c) $C = \{x | x^3 - 2x \neq 0\}$.
 - (d) $D = \{x | (x - 2)(x + 3)(x + 4) = 0 \Rightarrow x \geq 2\}$.
- (3) Simplify each of the following sets and give the final result as an interval or union of intervals
 - (a) $([-3, 2] \cap (0, 5))^c$.
 - (b) $((-3, 5.6] \cap (-5, 2.4\bar{3})) \cup (0.\bar{3}, 11)$.
 - (c) $(2, 4]^c \cap ((-\infty, 5)) \cup [0, \infty))$.
 - (d) $(-0.\bar{3}, 0.0\bar{3}) \cup ((-\infty, \frac{1}{30}) \cap [5, \infty))$.
- (4) Find the supremum and infimum of each of the following sets of real numbers:
 - (a) All numbers of the form $2^{-p} + 3^{-q} + 5^{-r}$, where p, q , and r take on all positive integer values.
 - (b) $S = \{3x^2 - 10x + 3 < 0\}$.
- (5) Find the smallest and the largest elements of the given set. Also the lub and glb whenever they exist.
 - (a) $A = \{x | x = 5 - \frac{1}{n}; n \in \mathbb{N}\}$.
 - (b) $B = \{x | x \in (-1, 1) \cup \{\frac{1}{n} | n \in \mathbb{N}\}\}$.
 - (c) $C = \{x | x = \frac{n}{n+1}; n \in \mathbb{N}\}$.
 - (d) $D = \{x | x = \frac{n+1}{n}; n \in \mathbb{N}\}$.
 - (e) $E = \{x | x = \frac{n^3}{2n^2+1}; n \in \mathbb{N}\}$.
- (6) Using the Principle of Mathematical Induction, prove that
 - (a) $1 + 4 + 7 + \dots + (3n - 2) = \frac{n(3n-1)}{2}$ for $n \geq 1$.
 - (b) $1 \times 2 + 2 \times 3 + 3 \times 4 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$ for $n \geq 1$.
 - (c) $2^{2n+1} + 7$ is a multiple of 3 for $n \geq 1$.
 - (d) $n^2 - n$ is even for $n \geq 1$.
 - (e) $6^n - 1$ is divisible by 5 for $n \geq 1$.
 - (f) $n! > 3^n$ for $n \geq 7$.
- (7) Consider the sequence of real numbers defined by the relations

$$x_1 = 1 \text{ and } x_{n+1} = \sqrt{1 + 2x_n} \text{ for } n \geq 1.$$

Use the Principle of Mathematical Induction to show that $x_n < 4$ for all $n \geq 1$.

- (8) Prove that $\sqrt{2} + \sqrt{3}$ is irrational number.
- (9) If $\frac{a}{b} < \frac{c}{d}$ with $b > 0, d > 0$, prove that $\frac{a+c}{b+d}$ lies between the two fractions $\frac{a}{b}$ and $\frac{c}{d}$.

ADDIS ABABA UNIVERSITY
COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCES
DEPARTMENT OF MATHEMATICS
Mathematics for Natural Sciences (Math 1011)
Mid-Term Examination - December 19, 2019

Name _____ ID. No. _____

Select Campus (AAiT, CHS, CNCS, EiABC) Section _____ Time allowed: $2\frac{1}{2}$ Hours

Instructions: This examination has 10 short answer questions and 5 workout problems. Attempt each question on the space provided for each question. Using Calculators and Mobiles are strictly forbidden.

For Instructor's use only:

PART I	PART II	TOTAL

Part I: (Short Answer) Give the most simplified answers on the space provided.

- (1) Let p and q be propositions. If $p \Rightarrow q$ is False, then the truth value of $((\neg p) \wedge q) \Leftrightarrow (p \vee q)$ is _____.
- (2) If $\frac{x+yi}{i} = 7+9i$, where x and y are real numbers, then the value of $(x+yi)(x-yi)$ is equal to _____.
- (3) Let the set $S = \{x \in \mathbb{R} : |x-2| < 3\}$. Then the $\text{lub}(S) =$ _____ and $\text{glb}(S) =$ _____.
- (4) The simplified form of the set $(2, 4]^c \cap ((-\infty, 5) \cap [0, \infty))$ as an interval or union of intervals is equal to _____.
- (5) Let the set $B = \{y : y \in \mathbb{N} \text{ and } 1 \leq y \leq \sqrt{5}\}$. Then the power set of B , $P(B)$ is equal to _____.
- (6) The smallest positive integer m for which $(\frac{1+i}{1-i})^m = 1$ is equal to _____.
- (7) Let $GCF(x, 400) = 20$ and $LCM(x, 400) = 1200$. Then the value of x is equal to _____.
- (8) Determine the truth value of the following propositions
(a) $(\exists x \in \mathbb{R})(\exists y \in \mathbb{R})[|x| + y = 0]$ is _____.

- (b) $(\exists x \in \mathbb{N})(\forall y \in \mathbb{R})[x \leq y^2]$ is _____.
- (9) In a class of 40 students, 12 registered for both Mathematics and Physics, 22 registered for Physics. If the students of the class registered for at least one of the two subjects, then how many students registered only for Mathematics? _____.
- (10) The values of real numbers x and y that satisfy the equation $x - yi = |x + yi|$ respectively are equal to _____ and _____.

Part II: (Workout Problems) Attempt each of the following questions by showing all the necessary steps.

- (1) Using the principle of mathematical induction, prove that
 $2^1 + 2^2 + 2^3 + \dots + 2^n = 2^{n+1} - 2$, for any $n \in \mathbb{N}$. [4 Pts.]

- (2) Let U be a universal set and A and B be any two subsets of U . Prove formally that
 $A - (A - B) = A \cap B$. [4 Pts.]

Addis Ababa University
Department of Mathematics
Mathematics for Natural Sciences (Math 1011): Test 1

Name: _____

Allocated mark: 10%

Allocated time: 50min

Id. No: _____

Section: _____

Target group: Section 5 & 7

Part 1: Multiple choice problems. (0.5pt. each)

- Let $\mathcal{P}(A)$ denote the power set of A . If $\mathcal{P}(A) \subseteq B$ then
A. $2^{|A|} \leq |B|$ B. $2^{|A|} \geq |B|$ C. $2^{|A|} < |B|$ D. $|A| + 2 \leq |B|$ E. $2^{|A|} \geq 2^{|B|}$
 - Which of the following statements is FALSE?
A. $\{2,3,4\} \subseteq A$ implies that $2 \in A$ and $\{3,4\} \subseteq A$
B. $\{2,3,4\} \in A$ and $\{2,3\} \in B$ implies that $\{4\} \subseteq A - B$
C. $A \cap B \supseteq \{2,3,4\}$ implies that $\{2,3,4\} \subseteq A$ and $\{2,3,4\} \subseteq B$
D. $A - B \supseteq \{3,4\}$ and $\{1,2\} \subseteq B$ implies that $\{1,2,3,4\} \subseteq A \cup B$
E. $\{2,3\} \subseteq A \cup B$ implies that if $\{2,3\} \cap A = \emptyset$ then $\{2,3\} \subseteq B$
 - Consider the following assertion: "The two statements (1) $\exists x \in D, (P(x) \wedge Q(x))$ and (2) $(\exists x \in D, P(x)) \wedge (\exists x \in D, Q(x))$ have the same truth value". Which of the following is correct? The assertion is:
A. False. A counter example is $D = \mathbb{N}$, $P(x) = "x \text{ is divisible by } 6"$, $Q(x) = "x \text{ is divisible by } 3"$.
B. True. The proof follows from the distributive law of $\wedge$.
C. False. A counter example is $D = \mathbb{Z}$, $P(x) = "x < 0"$, $Q(x) = "x \geq 0"$.
D. True. To see why, let $D = \mathbb{N}$, $P(x) = "x \text{ is divisible by } 6"$, $Q(x) = "x \text{ is divisible by } 3"$. If $x = 6$, then x is divisible by both 3 and 6 so both statements in the assertion have the same truth value for this x .
E. False. A counter example is $D = \mathbb{N}$, $P(x) = "x \text{ is a square}"$, $Q(x) = "x \text{ is odd}"$.
 - The statement form $(p \Leftrightarrow r) \Rightarrow (q \Leftrightarrow r)$ is equivalent to
A. $[(\neg p \vee r) \wedge (p \vee \neg r)] \vee \neg [(\neg q \vee r) \wedge (q \vee \neg r)]$ D. $[(\neg p \vee r) \wedge (p \vee \neg r)] \vee [(\neg q \vee r) \wedge (q \vee \neg r)]$
B. $\neg [(\neg p \vee r) \wedge (p \vee \neg r)] \wedge [(\neg q \vee r) \wedge (q \vee \neg r)]$ E. $\neg [(\neg p \vee r) \wedge (p \vee \neg r)] \vee [(\neg q \vee r) \wedge (q \vee \neg r)]$
C. $[(\neg p \vee r) \wedge (p \vee \neg r)] \wedge [(\neg q \vee r) \wedge (q \vee \neg r)]$
- Which of the following statement is the contrapositive of the statement, "You win the game if you know the rules but are not overconfident"?
- If you lose the game then you don't know the rules or you are overconfident.
 - If you don't know the rules or are overconfident you lose the game.
 - If you know the rules and are overconfident you win the game.
 - A sufficient condition that you win the game is that you know the rules or you are not overconfident.
 - A necessary condition that you know the rules or you are not overconfident is that you win the game.
- Which of the following is irrational?
- $\sqrt{196}$
 - $0.14\overline{16}$
 - 0.1416
 - $0.1428514285 \dots$
 - 0.40140014000

(3) Find the three cube roots of the complex number $z = -8i$.

[4 Pts.]

(4) Without using truth table, show that $(q \wedge (\neg q \vee p)) \vee (p \wedge \neg(q \wedge p)) \equiv p$ for any propositions p and q .

[4 Pts.]

(5) Determine the validity of the argument:

[4 Pts.]

If there was a great run, then traveling was difficult.

If they arrived on time, then traveling was not difficult.

They arrived on time.

Therefore, there was no great run.

Addis Ababa University

Department of Mathematics

Basic Math for Natural Sciences (Math 101N) Test (10%)

Time allowed: 1hr

Name _____ I.D.No. _____ Section _____

Part 1: True/False (1 point each)

- _____ 1. If $(\exists x)P(x)$ is true then $(\forall x)P(x)$ is true.
- _____ 2. Let A be a set and a be an object. Then " $a \in A$ " is a proposition.
- _____ 3. Every composite natural number can be expressed as a product of its prime factors.

Part 2: Short answer (2 points)

1. Let $A = \{1, 2, \{3\}\}$. Then the power set of A

$P(A) =$ _____

Part III: Work out: Show all the necessary steps.

1. Use prime factorization method to find GCF and LCM of 450 and 98. (2 points)
2. Use mathematical induction to show that

$$1^3 + 2^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4} \text{ for all } n \geq 1. \quad (3 \text{ points})$$