

MATHEMATICS

REVISION EXAMINATION QUESTIONS
WITH ANSWERS



10-12

SUCCESS IS IN YOUR HANDS, PLAY A
PART IN SHAPING YOUR FUTURE
TODAY AND GOD WILL DO THE REST
TOMORROW



$$A = \pi r^2$$

$$S_{\infty} = \frac{a}{1-r}$$

$$a^2 + b^2 = c^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = \frac{1}{2}(a + b)h$$

$$\frac{dy}{dx}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \sin \theta$$

$$V = \frac{1}{3}\pi(R^2H - r^2h)$$

TRAINED TO SAVE, SAVE TO REMEMBER AND REMEMBER TO TESTIFY

PRICE: K100.00

COMPILED AND SOLVED BY KUNDA ALLAN CEPHAS

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@2019

Gratitudes goes to ALMIGHTY GOD for conceding me a good breathy and a repetitious heart to have the work accomplished.

*Special thanks goes to my sister **CHALWE MELBA** for the support, both financial and effusive she rendered during the authorial period.*

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Contributors and editors have made a sincere effort to produce an accurate and useful resource. Should you have suggestions, find mistakes or be prepared to donate material for inclusion, please don't hesitate to contact the author. I intend to work with all who are willing to help make this a continuously evolving resource!

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@06/2020

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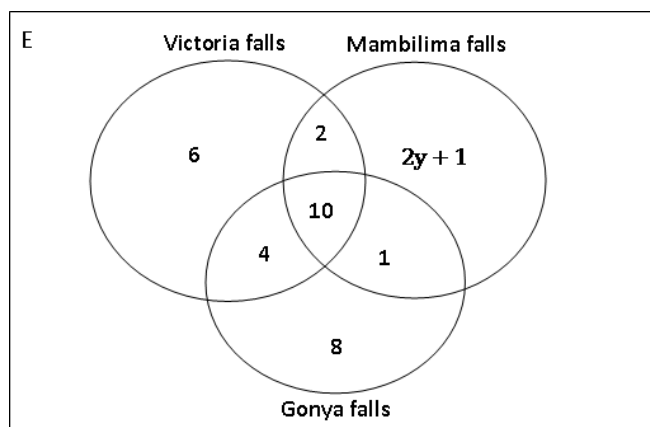
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SETS

1. The Venn diagram below shows tourist attractions visited by students in a certain week.

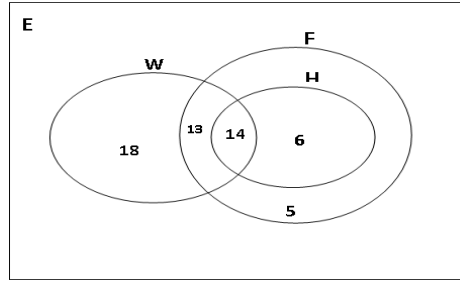


- a) Find the value of y , if 7 students visited Mambilima falls only [2]
- b) How many students visited
 - Victoria Falls but not Gonya Falls, [1]
 - Two tourist attractions only, [1]
 - One tourist attraction only? [1]

2. A survey carried out at Kamulima Farming Block showed that 44 Farmers planted maize, 32 planted sweet potatoes, 37 planted Cassava, 14 planted both Maize and sweet potatoes, 24 planted both sweet potatoes and cassava, 20 planted both maize and cassava, 9 planted all the three crops and 6 did not plant any of the three Crops.

- Illustrate this information in a Venn diagram [2]
- How many farmers
 - Where at this farming Block, [1]
 - Planted maize only, [1]
 - Planted two different crops only? [1]

3. The Venn diagram below shows the results of a survey conducted at Mayuka clinic on patients who were attended to on a particular day. Set W represents those who complained of body weakness, set H those who complained of headache and set F those who complained of fever.



- I. Use the information in the Venn diagram to find the number of people who complained of
 - a) Body weakness only, [1]
 - b) Body weakness and fever only, [1]
 - c) Headache. [1]
- II. Calculate the percentage of those who complained of all the three ailments. [2]

4. At Senkani School, there are 60 learners in grade 12. A survey was carried out to determine how many of these learners liked Netball, Football or Volleyball and the following were the findings:-

5 liked all the three

8 liked football only 4 liked volleyball only

3 liked netball only

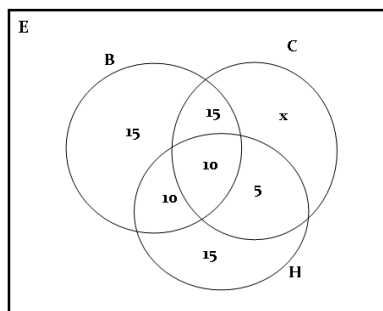
12 liked football and netball

21 liked football and volleyball

18 liked netball and volleyball

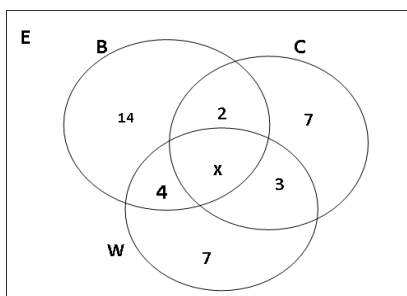
- a) Illustrate this information in a Venn diagram [2]
- b) How many learners did not like any of the three games? [1]
- c) How many learners did not like volleyball? [1]
- d) How many learners liked two games only? [1]

5. The Venn diagram below shows the number of students who took Business studies (B), Human resource (H) and Community resource (C) at Mafundisho College. 100 students took these courses.



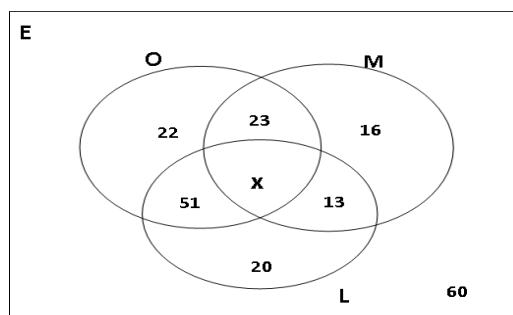
- a) Find the value of x [2]
- b) The number of students who took Human resource [1]
- c) $n(B \cap C) \cap H'$ [1]
- d) $n(B \cup C) \cap H'$ [1]

6. The diagram below shows how learners in a grade 12 class at Twaenda School travel to school. The learners use either Buses (B), cars (C) or Walk (W) to school.



- I. If 22 learners walk to school, find the value of x . [2]
- II. How many learners use
 - a) Only one mode of transport, [1]
 - b) Two different modes of transport, [1]
 - c) How many learners are there in the class? [1]

7. In a certain month, a survey was conducted on 250 high school pupils to find out the number of pupils that bought Oranges (O), Mangoes (M) and Lemons (L). Their responses were as shown in the Venn diagram below.



- I. Find the value of x. [2]
- II. How many pupils bought Mangoes and Lemons but not Oranges? [2]
- III. How many pupils bought one type of fruit only? [1]

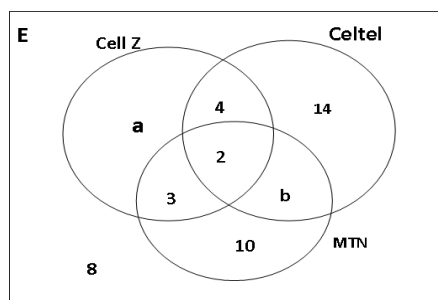
8. Of the 50 villagers who can tune in to Kambani radio station, 29 listen to news, 25 listen to sports, 22 listen to music, 11 listen to both news and sports, 9 listen to both music and sports, 12 listen to both news and music, 4 listen to all the three programs and 2 do not listen to any program.

- I. Draw a Venn diagram to illustrate this information. [2]
- II. How many villagers
 - a) Listen to music only, [1]
 - b) Listen to one type of program only, [1]
 - c) Listen to two types of programs only? [1]

9. At Sambilileni College, 20 students study at least one of the three subjects; Mathematics (M), Chemistry (C) and Physics (P). All those who study chemistry also study Mathematics. 3 students study all the three subjects. 4 students study Mathematics only, 8 students study Chemistry and 14 students study Mathematics.

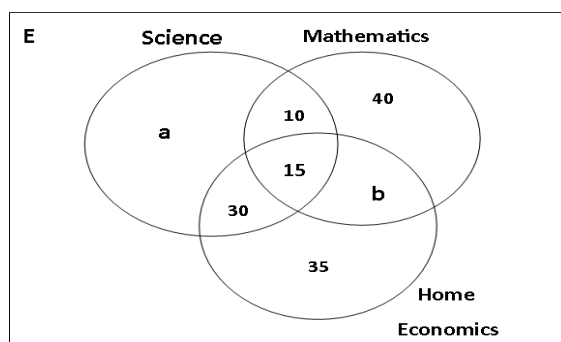
- I. Draw a Venn diagram to illustrate the information above. [2]
- II. How many students study
 - a) Physics only, [1]
 - b) two types of subjects only, [1]
 - c) Mathematics and physics but not chemistry? [1]

10. A survey was conducted on 60 women concerning the type of sim cards used in their cell phones for the past two years. Their responses are given in the Venn diagram below.



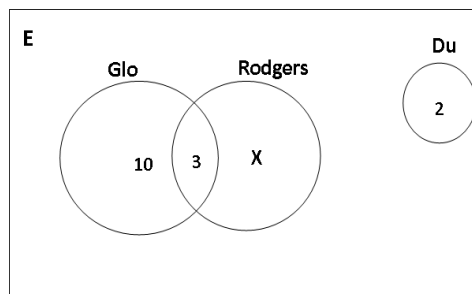
- I. Given that 23 women have used cell Z sim cards, find the value of a & b [3]
- II. How many women have used only two different sim cards? [1]
- III. How many women did not use MTN and cell Z sim cards? [2]
- IV. How many women used either Celtel or MTN sim cards but not cell Z? [2]

11. In a particular year, 150 candidates applied to be trained as teachers of Mathematics, Science and Home economics at COSETCO. The Venn diagram below illustrates the number of candidates for each subject.



- I. Given that 70 candidates applied for science, find the value of a and b [3]
- II. How many candidates applied for two different subjects only? [1]
- III. How many candidates did not apply for Science or Mathematics? [1]
- IV. How many candidates applied either for Science or Home economics but not Mathematics? [1]

12. At one college, a group of 25 students were asked which cell phone service providers they subscribed to. The results are shown in the Venn diagram below.



- I. Calculate the value of x [1]
- II. Given that $G = (Glo)$, $R = (Rodgers)$ and $D = (Du)$. Find
- a) $n(G \cap R)$. [1]
- b) $n(D \cup G')$. [2]

ALGEBRA

1. Simplify $\frac{12dn^3}{15cd^3} \div \frac{9c^3n}{10c^2d^2}$ [2]
2. Simplify $\frac{14x^3}{9y^2} \div \frac{7x^4}{18y^3}$ [2]
3. Express $\frac{3}{5x-2} - \frac{2}{x+2}$ as a single fraction in its lowest terms. [3]
4. Simplify $\frac{17k^2}{20a^2} \div \frac{51k^2}{5a}$ [2]
5. Express $\frac{3}{x+1} - \frac{4}{x-1}$ as a single fraction in its lowest terms [3]
6. Simplify $\frac{2x^2-8}{x+2}$ [2]
7. Simplify $\frac{7st^3}{15u^3v^2} \times \frac{5u^3}{28s^3t^2}$ [3]
8. Simplify $\frac{b-a}{a^2-b^2}$ [2]
9. Express $\frac{3}{2x-5} - \frac{4}{x-3}$ as a single fraction in its lowest terms [3]
10. Simplify $\frac{p^2q^2}{4} \times \frac{8}{pq} \div 2p^2q$ [3]
11. Simplify $\frac{2y^2-3y-5}{y^2-1}$ [3]

12. Express $\frac{4}{3r-2} - \frac{3}{r+5}$ as a single fraction in its lowest terms. [3]
13. Simplify $\frac{x-1}{x^2-1}$ [2]
14. Simplify $\frac{3a}{5c^2} \times \frac{10c^3}{a^2}$ [2]
15. Express $\frac{2}{2x-1} - \frac{1}{3x+1}$ as a single fraction in its lowest term [3]
16. Factorize completely $4x^2 - 16y^2$ [2]
17. Evaluate $2\frac{1}{2} + \frac{3}{4} \div \frac{1}{18}$ [2]
18. Express $\frac{1}{x-4} - \frac{2}{5x-1}$ as a single fraction in its lowest terms. [3]
19. Simplify $\frac{m^2-1}{m^2-m}$ [2]
20. Evaluate $1.3^2 + 1.3 \times 0.3$ [2]
21. Simplify $\frac{a-2}{a^2-4}$ [2]
22. Factorize completely $5px - 5py + 3qx - 3qy$ [2]
23. Given that $x = 8$ and $y = -2$, find $x - y^2$ [2]
24. Factorize completely $3x - 12x^3$ [2]
25. Express as a single fraction $\frac{x+5}{3} - \frac{x+2}{4}$ [3]
26. Factorize completely $\frac{4u^2}{a^2} - \frac{1}{9}$ [3]
27. If $\omega = \frac{u+v}{uv}$,
 - a. Find the value of ω when $u = 3$ and $v = -4$ [2]
 - b. Express u in terms of v and ω . [2]

PROBABILITY

1. Twelve identical balls are labeled 1 to 12. These are placed in a bag. One ball is selected at random and then replaced. Find the probability that the ball selected is labeled;

- a) Even number, [1]
- b) Less than 4, [1]
- c) Prime number. [1]

2. A box contains Blue, Black, Green and Red Marbles. Given that $P(\text{Green}) = 0.4$, $P(\text{Red}) = 0.25$ and $P(\text{Black}) = 0.2$.

- a) Find $P(\text{Blue})$. [1]
- b) Find the least number of Marbles that must be in the box to suit these Probabilities. [2]

3. A survey was carried out at certain hospital indicated that the probability that patient tested positive for malaria is 0.6. What is the probability that two patients selected at random

- a) One tested negative while the other positive, [1]
- b) Both patients tested negative. [2]

4. A box contains identical buttons of different colors. There are 20 black, 12 red and 4 white buttons in the box. Two buttons are picked at random one after the other and not replaced in the box.

- a) Draw a tree diagram to show all the possible outcomes. [1]
- b) What is the probability that both buttons are white? [2]

5. A grade one pupil has a certain number of Fanta, Coca-Cola and Sprite bottle tops in her bag. She takes one bottle top at random from the bag and the probability that it is the Fanta bottle top is 0.25 and the probability that it is the Sprite bottle top is 0.4.

- I. Find the probability that it is
 - a) a Coca-Cola bottle top [1]
 - b) Not a Sprite bottle top [1]
- II. Originally they were 16 Sprite bottle tops in her bag. Find the total number of bottle tops that she had. [2]

6. A box of chalk contains 5 white, 4 blue and 3 yellow pieces of chalk. A piece of chalk is selected at random from the box and not replaced. A second piece of chalk is then selected.

- a) Draw a tree diagram to show all the possible outcomes. [1]
- b) Find the probability of selecting pieces of chalk of the same color. [2]

7. In a box of 10 bulbs, 3 are faulty. If two bulbs are drawn at random one after the other, find the probability that

- a) Both are good. [1]
- b) One is faulty and the other one is good. [2]

8. A box contains 3 green apples and 5 red apples. An apple is picked from the box and not replaced then a second apple is picked. Expressing the answer as a fraction in its simplest form, calculate

- a) The probability that both apples picked are green, [2]
- b) The probability that the two apples picked are of different colors. [3]

9. A small bag contains 6 black and 9 red pens of the same type. Two pens are taken at random one after the other without replacement. Calculate the probability that both pens

- a) are black, [2]
- b) are of different color. [3]

10. Lillian has eight blouses: two green, two blue one red and three white. She has five skirts two blue, two green and one grey. One dark morning she selects a blouse and a skirt at random. Expressing each answer as a fraction in its lowest terms, find the probability that Lillian selects

- a) a blouse which is not blue, [1]
- b) a white blouse and a blue skirt, [2]
- c) A blouse and skirt of the same color. [2]

MATRICES

1. Given that $M = \begin{bmatrix} 3 & -2 \\ 5 & x \end{bmatrix}$,

- a. Find the value of x for which the determinant of **M** is 22, [2]
- b. Hence find the inverse of **M**. [3]

2. Given that $A = \begin{bmatrix} 5 & -2 \\ 3 & x \end{bmatrix}$ and $B = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

- I. Given that the determinant of A is 21 find x. [2]
Hence find;
- II. A^{-1} [2]

III. AB [2]

3. Given that $Q = \begin{pmatrix} 3 & -2 \\ x & 4 \end{pmatrix}$, Find

a) The value of x, given that the determinant of Q is 2, [2]

b) The inverse of Q. [2]

4. Given that $A = \begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix}$,

a) Find the positive value of x if the determinant of A is 12, [3]

a) Hence or otherwise find A^{-1} . [2]

5. Given that $A = \begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & y \\ 3 & 5 \end{pmatrix}$,

a) Find the value of y, for which the determinant of A and B are equal, [3]

b) Hence find the inverse of B. [2]

6. Given that $K = \begin{pmatrix} 10 & -2 \\ 11 & -2 \end{pmatrix}$, Find

a) The determinant of K, [1]

b) The inverse of K. [2]

c) Express as a single matrix $\begin{pmatrix} -3 \\ 4 \end{pmatrix} \begin{pmatrix} -1 & 0 \end{pmatrix}$ [2]

d) The determinant of the matrix $\begin{pmatrix} 6 & -a \\ 3 & -2 \end{pmatrix}$ is -9, calculate the value of a. [2]

7. Given that $\begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & n \end{bmatrix} \begin{bmatrix} m \\ -4 \\ 1 \end{bmatrix} = \begin{pmatrix} 15 \\ 2 \end{pmatrix}$, find the value of **m** and **n**. [5]

8. Given that $A = \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$, find the

a) Determinant of A, [1]

b) Inverse of A, [2]

c) Value of $A^{-1} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$. [2]

9. Given that $A = \begin{pmatrix} 1 & x \\ -1 & 2 \end{pmatrix}$,

a) Write an expression in terms of x, the determinant of A, [2]

b) Find the value of x, given that the determinant of A is 5, [1]

c) Write A^{-1} . [2]

10. Given that $\begin{pmatrix} 2 & x \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ -8 \end{pmatrix}$, find the value of x. [3]

11. If $P = \begin{pmatrix} 2 & 0 \\ 6 & 1 \end{pmatrix}$ and $Q = \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$, find

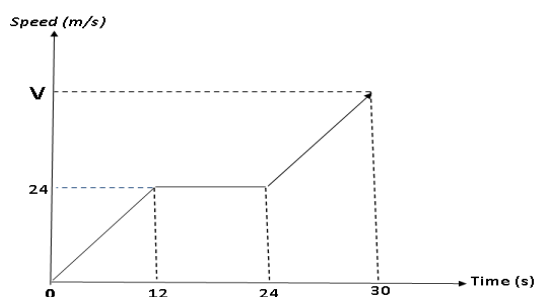
a) PQ, [2]

b) The values of a and b, given that $PQ = P - Q$

[3]

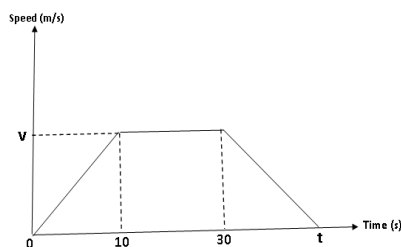
KINEMATICS

1. The diagram below is a speed-time graph of a particle which accelerates uniformly from rest for 12 seconds until it reaches a speed of 24 ms^{-1} . It moves for a further 12 seconds before it accelerates uniformly before it accelerates for 6 seconds to a speed of $V \text{ m/s}$.



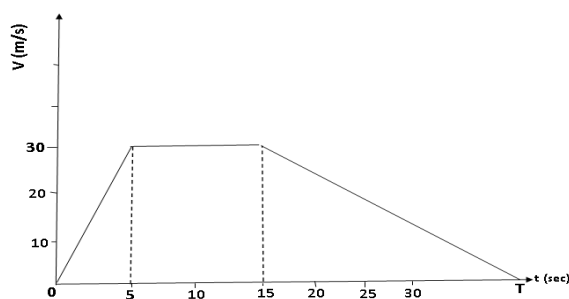
- a) Calculate its acceleration for the first 12 seconds. [1]
- b) Find the distance which the particle covered in the first 24 seconds. [2]
- c) Given that the total distance covered was 600m, calculate the value of V [3]

2. The diagram below shows the speed-time graph of a particle. The particle started off from rest and accelerated uniformly for 10 seconds. It then travelled at constant speed for 20 seconds and then decelerated to rest.



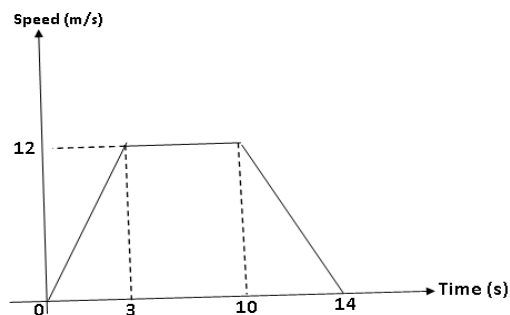
- a) Find the speed V the particle reached if its acceleration was 2 ms^{-2} in the first 10 seconds. [2]
- b) Given that the total distance was 750m, find the value of t . [2]
- c) What is the speed at 40 seconds? [2]

3. The diagram below shows the speed -time graph of a car journey.



- Find the acceleration during the first 5 seconds. [1]
- If the total distance travelled was 825m, find the value of T. [3]
- Find the average speed for the whole journey. [2]

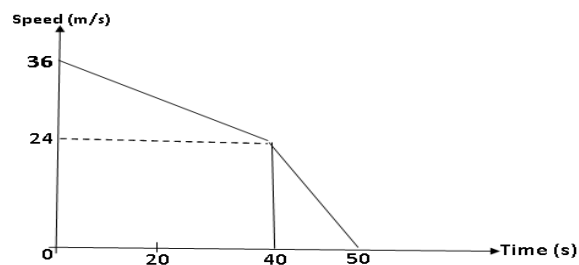
4. The diagram below shows the speed -time graph of a 100m sprinter who accelerates uniformly for 3 seconds until he reaches a speed of 12m/s. He maintains the speed for 7 seconds and then uniformly retards for a further 4 seconds and comes to a stop.



Calculate the

- Acceleration during the first 3 seconds. [2]
- Retardation at the end of the race. [2]
- Distance covered in the first 10 seconds. [2]

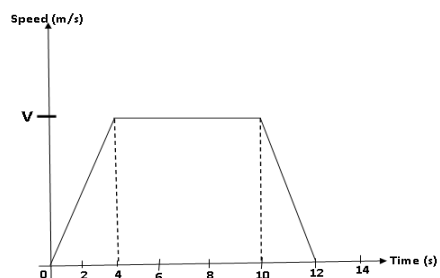
5. The figure below shows how a car is retarded uniformly from a speed of 30m/s to a speed of 24m/s in a time of 40 seconds and then uniformly brought to rest after a further 10 seconds.



Calculate

- a) The retardation of the car during the first 40 seconds. [2]
- b) The speed of the car after 47 seconds. [2]
- c) The distance travelled in the first 20 seconds. [2]

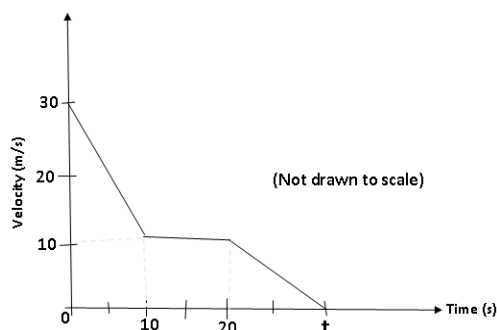
6. The diagram below shows the speed-time graph of a car which starts from rest and increases its speed at a constant rate for 4 seconds and then travels at a constant speed of V m/s for a further 6 seconds before it comes to a halt in 2 seconds.



Given that the car traveled 80m in the first 4 seconds, calculate the

- a) Maximum speed V , the car reached, [2]
- b) acceleration of the car in the first 4 seconds, [1]
- c) retardation in the last 2 seconds, [1]
- d) Total distance travelled in the period of 12 seconds. [2]

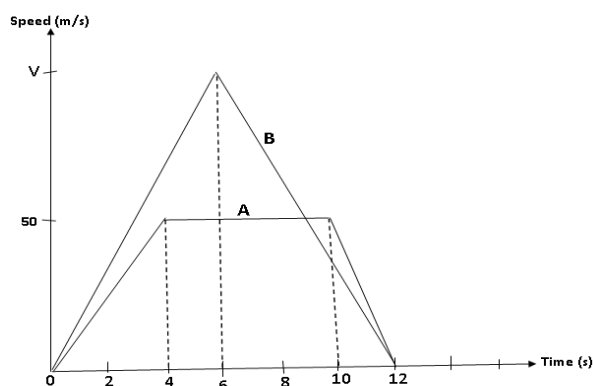
7. The diagram below shows the velocity-time graph of a particle during a period of t seconds.



Calculate

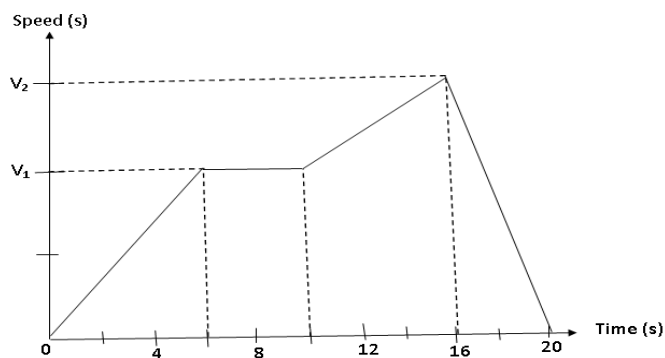
- the acceleration of the particle in the first 10 seconds, [1]
- the value of t , if it traveled 50m from the 20th second, [2]
- The average speed of the particle for the whole journey. [2]

8. The diagram below shows the speed-time graphs of two trains leaving a station at the same time. Train A accelerates uniformly from rest to a speed of 50m/s in 4 seconds. It moves at this speed for 6 seconds before coming to a rest in a further 2 seconds. Train B accelerates uniformly from rest to a speed of V m/s in 6 seconds then comes to rest in further 6 seconds.



- Given that the trains moved the same distance, calculate the maximum speed V , of train B. [4]
- Find the distance between the trains during the first 6 seconds. [3]

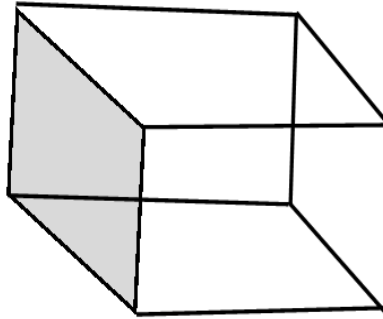
9. The diagram below shows the speed-time graph of a lorry which starts off from rest and increases its speed at a constant rate for 6 seconds. Then it travels at a constant speed of V_1 m/s for 4 seconds. It again increases its speed constantly for 6 seconds reaching a maximum speed of V_2 m/s before it eventually comes to rest a further 4 seconds.



- a) If the Lorry traveled 120m in the first 6 seconds, calculate the value of V_1 . [2]
- b) Given that the acceleration of the Lorry between 10 and 16 seconds is 2m/s^2 , find the value of V_2 . [2]
- c) Calculate the total distance covered by the Lorry. [2]

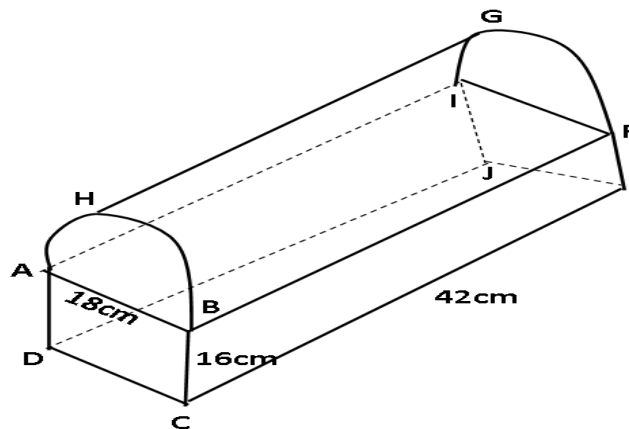
MENSURATION

1. The cross section of a rectangular tank measures 1.2m by 0.9m. If it contains fuel to a depth of 10m, find the number of litres of fuel in the tank. ($1\text{m}^3 = 1000\text{litres}$). [4]
2. A cone has a perpendicular height of 12 cm and slant height of 13 cm; calculate its total surface area. (Take $\pi = 3.142$). [3]
3. The diagram below shows a solid metal cube of volume 125cm^3 . Given that 1cm^3 of the metal has the mass of 9 grams.



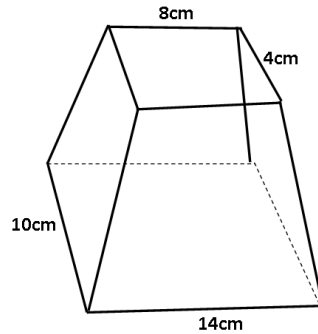
Calculate

- a) the length of the edge of the cube, [1]
 - b) the mass of the cube giving your answer in kilograms, [2]
 - c) The total surface area of the cube. [2]
4. The cover of the sewing machine is made up of half a cylinder (curved part) and a box (cuboid) as shown in the diagram below.



Given that **AHB** is a semicircle **AB** = 18cm, **BC** = 16cm and **CE** = 42cm. Take $\pi = 3.142$, calculate

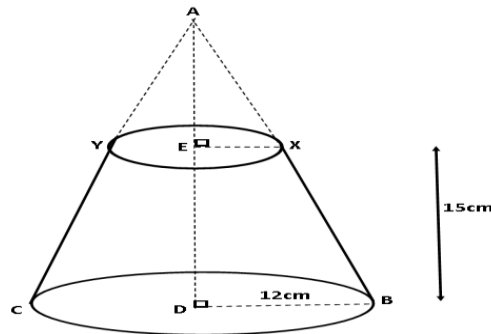
- a) the curved surface area of the cover, [2]
 - b) the surface area of the face **ADCBH**, [3]
 - c) The volume of the cover of the sewing machine. [3]
5. The diagram below is a frustum of a rectangular pyramid with a base 14cm long and 10cm wide. The top of a frustum is 8cm long and 4cm wide.



Given that the height of the frustum is 11.4cm, calculate its volume.

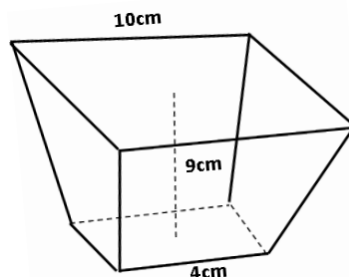
[3]

6. The figure below is a cone **ABC** from which **BCXY** remained after the small cone **AXY** was cut of [Take $\pi = 3.142$]. Given that **EX** = 4cm, **DB** = 12cm and **DE** = 15cm,



Calculate;

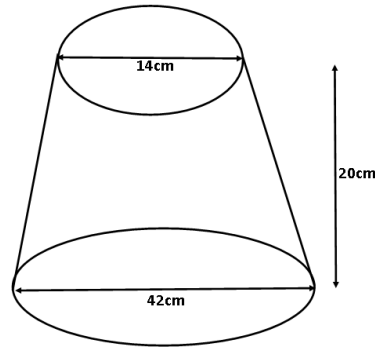
- the height AE, of the smaller cone AXY, [3]
 - The volume of XBCY, the shape that remained. [4]
7. The diagram below shows a bin in the form of a frustum with square base ends of t sides 4cm and 10cm respectively. The height of the bin is 9cm.



Take $\pi = 3.142$. Find the volume of the bin.

[5]

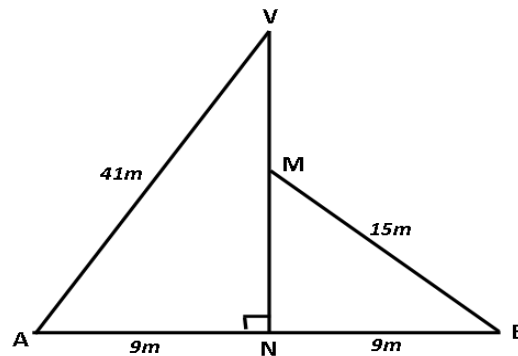
8. The figure below is a frustum of a cone. The base diameter and top diameter are 42cm and 14 cm respectively, while the height is 20cm. (Take $\pi = 3.142$)



Calculate its volume.

[3]

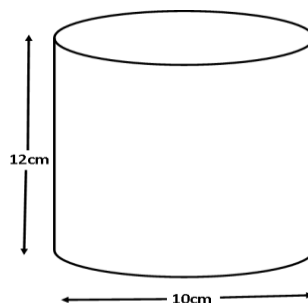
9. In the diagram below, **ANB** is a horizontal line and **VN** is a vertical mast supported by two stays **AV** and **BM**.



Given that **AV**=42m, **BM**=15m and **AN**=**NB**=9m. Calculate the length of **MV**.

[3]

10. The diagram below represents a glass of water of uniform cross section and thickness.



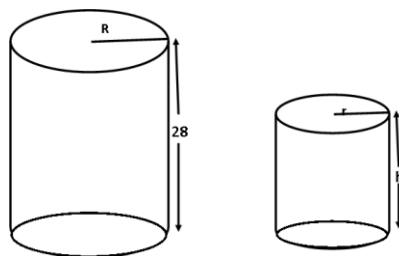
Given that its height is 12cm, diameter is 10cm and taking $\pi = 3.142$,

- a) Calculate the outer surface area of the glass correct to two decimal places,

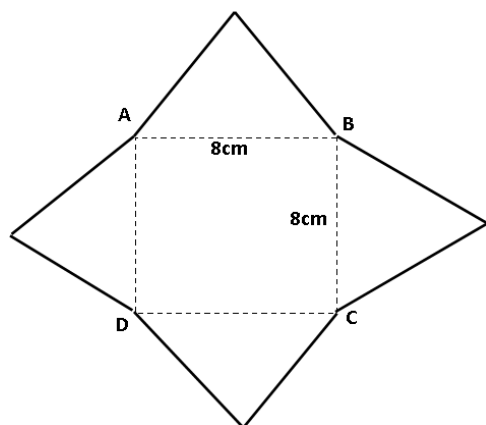
[3]

- b) Given that the thickness of the glass of water is 2mm. Calculate the volume of the water that the glass can hold when completely full, giving your answer correct to two decimal places. [4]

11. The two tins below are geometrically similar and the ratio of their volumes is 64:27

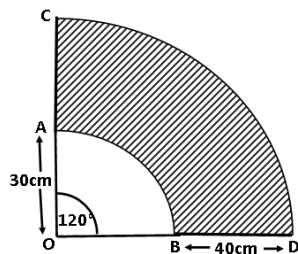


- a) Calculate the ratio of their curved surface areas. [3]
 b) Given that the height of the larger tin is 28cm, calculate the height of the smaller tin. [3]
12. The figure below shows the net of the pyramid with a square base **ABCD** of size 8cm.



Given that each of the triangles is an equilateral triangle. Calculate

- a) the perimeter of the net, [1]
 b) the area of the figure in centimeters correct to 3 significant figures, [3]
 c) The volume of the pyramid when the shape is folded along the shaded lines. [3]
 (Volume of pyramid = $\frac{1}{3}$ base area $\times$ perpendicular height)
13. The shaded part of the diagram is that of a path of the car windscreen wiper. The wiper rotates through 120° about O. (Take $\pi = \frac{22}{7}$).



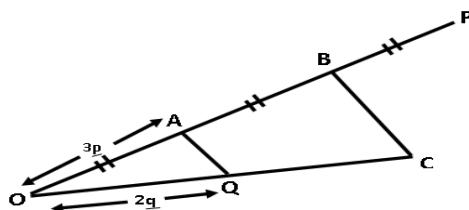
Given that $OA = OB = 30\text{cm}$, $AC = BD = 40\text{cm}$. Calculate

- The perimeter of the sector OCD correct to 1 decimal place, [2]
- The area of the shaded region correct to 1 decimal place. [3]

VECTORS

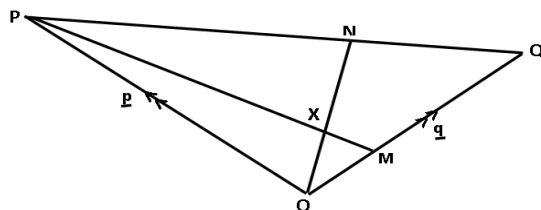
- In the diagram below Q is the midpoint of OC and OABP is a straight line $OA = AB = BP$.

$$\vec{OA} = 3\vec{p} \text{ and } \vec{OQ} = 2\vec{q}.$$



Find in terms of $\vec{p}$ and or $\vec{q}$

- $\vec{OB}$ [1]
 - $\vec{BC}$ [1]
 - $\vec{AQ}$ [1]
 - Hence state the relationship between $\vec{BC}$ and $\vec{AQ}$. [2]
- In the diagram below $\vec{OP} = \vec{p}$, $\vec{OQ} = \vec{q}$, $\vec{OM} = \frac{1}{3}\vec{OQ}$ and PN: NQ = 3:1.



Express in terms of $\vec{p}$ and or $\vec{q}$

a) $\overrightarrow{PQ}$ [1]

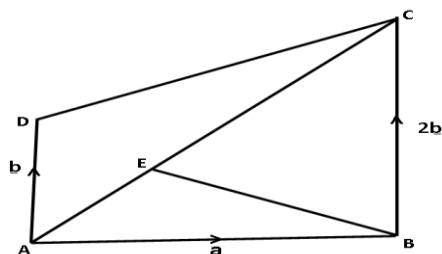
b) $\overrightarrow{ON}$ [1]

c) $\overrightarrow{PM}$ [1]

d) Given that $\overrightarrow{OX} = h\overrightarrow{ON}$, express $\overrightarrow{OX}$ in terms of $\vec{p}$, $\vec{q}$ and h . [2]

e) Given also that $\overrightarrow{PX} = k\overrightarrow{PM}$, show that $\overrightarrow{OX} = (1 - k)\vec{p} + \frac{1}{3}k\vec{q}$. [2]

3. In the quadrilateral **ABCD**, $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{AD} = \vec{b}$, $\overrightarrow{BC} = 2\vec{b}$ and $AE:AC = 1:3$.



Find in terms of $\vec{a}$ and or $\vec{b}$

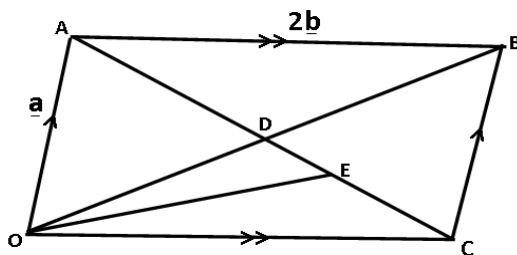
a) $\overrightarrow{AE}$ [1]

b) $\overrightarrow{BE}$ [1]

c) $\overrightarrow{BD}$ [1]

d) Hence or otherwise show that the points B, D and E are collinear. [2]

4. In the diagram below, OABC is a parallelogram in which $\overrightarrow{OA} = \vec{a}$ and $\overrightarrow{AB} = \vec{b}$. OB and AC intersect at D. E is the midpoint of CD.



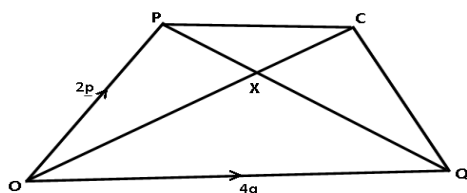
Express in terms of $\vec{a}$ and $\vec{b}$

a) $\overrightarrow{OB}$ [1]

b) $\overrightarrow{OE}$ [2]

c) $\overrightarrow{CD}$ [2]

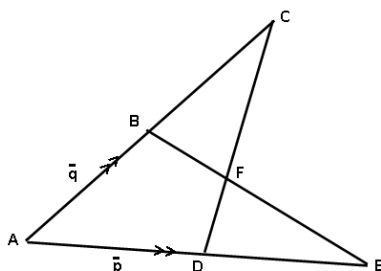
5. In the diagram below, $\overrightarrow{OP} = 2\vec{p}$, $\overrightarrow{OQ} = 4\vec{q}$ and $PX:XQ = 1:2$.



Express in terms of $\vec{p}$ and or $\vec{q}$,

- $\overrightarrow{PQ}$ [1]
- $\overrightarrow{PX}$ [1]
- $\overrightarrow{OX}$ [1]
- Given that $\overrightarrow{OC} = h\overrightarrow{OX}$, show that $\overrightarrow{CQ} = 4(1 - \frac{h}{3})\vec{q} - \frac{4h}{3}\vec{p}$. [2]

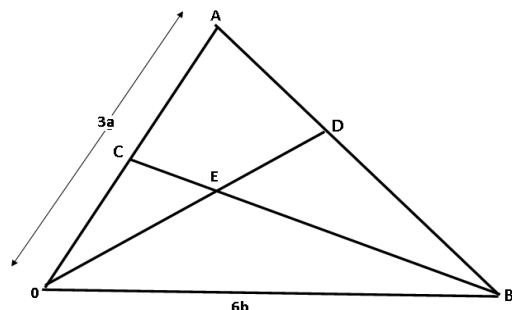
6. In the diagram below, ABC and ADE are straight lines. CD and BE intersect at F so that $BF:FE = 1:2$. B is the midpoint of AC and D is the midpoint of AE.



Given that $\overrightarrow{AB} = \vec{q}$ and $\overrightarrow{AD} = \vec{p}$. Express as simply as possible in terms of $\vec{p}$ and or $\vec{q}$,

- $\overrightarrow{BE}$ [1]
- $\overrightarrow{FE}$ [1]
- $\overrightarrow{DF}$ [1]
- Given that $\overrightarrow{DC} = h\overrightarrow{DF}$, write down an expression for $\overrightarrow{DC}$, and hence show that $\overrightarrow{AC} = (1 - \frac{h}{3})\vec{p} + \frac{2h}{3}\vec{q}$. [2]

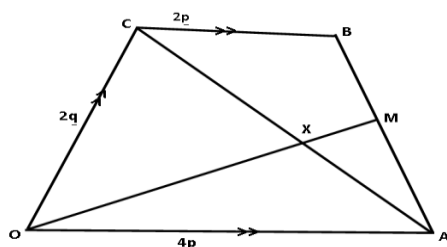
7. In the diagram below, OAB is a triangle in which $\overrightarrow{OA} = 3\vec{a}$ and $\overrightarrow{OB} = 6\vec{b}$. OC: CA = 2: 3 and AD: DB = 1: 2. OD meets CB at E.



Express each of the following in terms of $\vec{a}$ and or $\vec{b}$

- a) $\vec{AB}$ [1]
- b) $\vec{OD}$ [1]
- c) $\vec{BC}$ [1]
- d) Given that $\vec{BE} = h\vec{BC}$, express $\vec{BE}$ in terms of $\vec{a}$ and or $\vec{b}$. [2]

8. The diagram below is a trapezium OABC. M is the midpoint of AB, OM and CA meet at X. $\vec{OA} = 4\vec{p}$, $\vec{OC} = 2\vec{q}$ and $\vec{CB} = 2\vec{p}$



Express as simply as possible, in terms of $\vec{p}$ and or $\vec{q}$

- a) $\vec{CA}$ [1]
- b) $\vec{BA}$ [1]
- c) $\vec{OM}$ [1]
- d) Given that $CX = hCA$, express $\vec{CX}$ in terms of $\vec{p}$, $\vec{q}$ and h. [1]
- e) Hence show that $\vec{OX} = 4h\vec{p} + 2(1-h)\vec{q}$. [2]

ARITHMETIC AND GEOMETRIC PROGRESSION

1. For the sequence -10, -7, -4, -1, ... find the

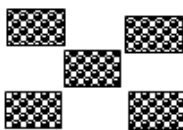
- a. 17th term, [1]
- b. nth term, [2]
- c. Sum of the first 20 terms. [2]
- 2. For the sequence 25, 22, 19, 16, ..., Find the
 - a. formula for the nth term, [2]
 - b. 10th term, [1]
 - c. Sum of the first 20 terms. [2]
- 3. The first three terms in an arithmetic progression are 5, 7 and 9. Find the
 - a) common difference, [1]
 - b) nth term, [2]
 - c) Sum of the first 12 terms. [2]
- 4. The first and second terms of an arithmetic progression are 100 and 95, respectively. Find the
 - a. tenth term, [1]
 - b. nth term, [2]
 - c. Sum of the first ten terms. [2]
- 5. For the sequence 11, 13, 15, 17, ..., find the
 - a) 13th term, [1]
 - b) nth term, [2]
 - c) Sum of the first 20 terms. [2]
- 6. If the arithmetic mean of 5 and c is 11, what is the value of c? [2]
- 7. The first five terms of the sequence are 9, 16, 25, 36, 49, ... Find the
 - a) 10th term, [1]
 - b) Formula for the nth term. [3]
- 8. For the sequence 4, 7, 10, 13, ... Write down the
 - a. 30th term, [1]
 - b. nth term, [2]
 - c. Sum of the first 24 terms. [2]
- 9. For the sequence 7, 9, 11, 13 ... Find the
 - a. Eleventh term, [1]
 - b. Expression for the nth term. [2]
 - c. sum of the 20 terms [2]
- 10. Consider the sequence -1, 4, 21, 56, ... Find the
 - a) 6th term, [2]
 - b) Expression for the nth term. [4]
- 11. The first three terms of a Geometric progression are $6 + n$, $10 + n$ and $15 + n$. Find
 - a. The value of n, [2]
 - b. The common ratio, [2]
 - c. The sum of the first 6 terms of this sequence. [3]

12. For the Geometric progression 20, 5, 1.25, ... Find the
- Common ratio, [2]
 - n^{th} term, [2]
 - Sum of the first 8 terms. [3]
13. In a Geometric progression, the third term is $\frac{2}{9}$ and the fourth term is $\frac{2}{27}$. Find
- the common ratio and the first term, [3]
 - the sum of the first 5 terms of the geometric progression, [2]
 - The sum to infinity. [2]
14. The first 3 terms of a geometric progression are $x + 1$, $x - 3$ and $x - 1$. Find the
- value of x , [3]
 - the first term, [1]
 - Sum to infinity. [3]
15. The first three terms of a geometric progression are $k + 4$, k and $2k - 15$ where k is a positive integer.
- Find the value of k , [3]
 - list the first three terms of the geometric progression, [2]
 - Find the sum to infinity. [2]

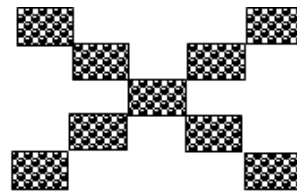
16. The patterns below were made using floor tiles.



Pattern No. 1



Pattern No. 2

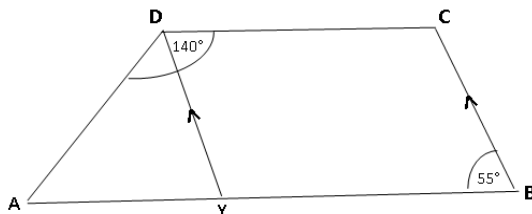


Pattern No. 3

- Find the number of floor tiles in pattern number 10, [1]
 - Given that m is the number of tiles in pattern number n , find the formula for m in terms of n . [2]
17. Find the geometric mean of 2, 4 and 8. [2]
18. The geometric mean of 12 and x is 6. Find the value of x . [2]

CIRCLE PROPERTY

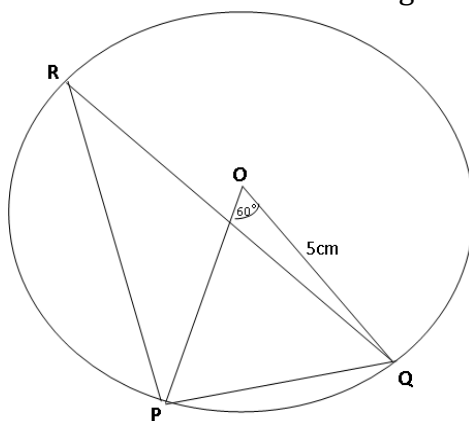
1. In the diagram, $ABCD$ is a quadrilateral in which $\widehat{ADC} = 140^\circ$, $\widehat{ABC} = 55^\circ$ and the point Y on AB is such that YD is parallel to BC .



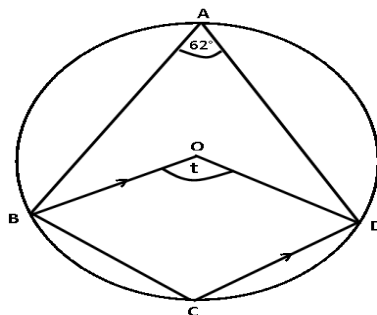
Calculate

- | | |
|--------------------|-----|
| a) $\widehat{BYD}$ | [1] |
| b) $\widehat{BAD}$ | [2] |

2. P , Q and R are points on a circle Centre O . Angle $POQ = 60^\circ$ and $OQ = 5\text{cm}$.



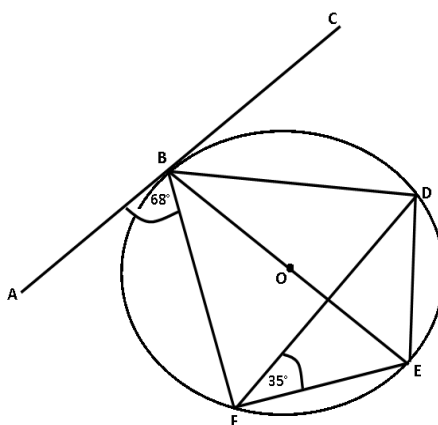
- | | |
|--|-----|
| a) Write down the size of angle $\widehat{PRQ}$ | [1] |
| b) Find the size of angle $\widehat{OPQ}$ | [2] |
| c) Given that the radius of the circle is 5cm and $\pi = 3.14$. Calculate | |
| i. The length of the major arc $\widehat{PRQ}$ | [2] |
| ii. The area of the minor sector OPQ | [2] |
3. In the diagram below, A , B , C and D lie on the circumference of the circle, center O . BO is parallel to CD and $\widehat{BAD} = 62^\circ$.



Calculate

- | | |
|--------------------|-----|
| a) Angle t | [1] |
| b) $\widehat{BCD}$ | [2] |
| c) $\widehat{OBC}$ | [2] |

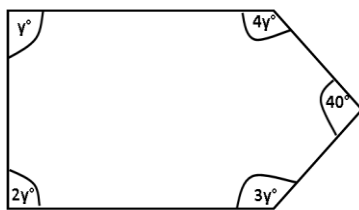
4. In the diagram below, ABC is the tangent to the circle Centre O and BE is the diameter. $\widehat{ABF} = 68^\circ$ and $\widehat{DFE} = 35^\circ$.



Calculate

- | | |
|--------------------|-----|
| a) $\widehat{BFD}$ | [1] |
| b) $\widehat{DBC}$ | [1] |
| c) $\widehat{FED}$ | [2] |
| d) $\widehat{FDB}$ | [2] |

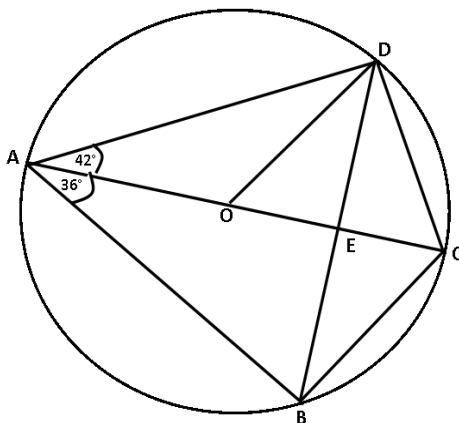
5. The five angles of a pentagon below are y° , 40° , $2y^\circ$, $3y^\circ$ and $4y^\circ$.



Calculate the value of y .

[2]

6. In the diagram below, $ABCD$ is a circle with Centre O . Given that $\widehat{OAD} = 42^\circ$ and $\widehat{OAB} = 36^\circ$.



Calculate

a) $\widehat{AOD}$

[1]

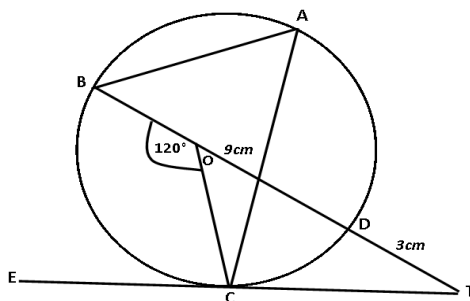
b) $\widehat{ABC}$

[1]

c) $\widehat{ODB}$

[2]

7. In the diagram below, O is the Centre of the circle $ABCD$. $BD = 9\text{cm}$, $DT = 3\text{cm}$, $\widehat{BOC} = 120^\circ$ and ECT is a tangent at C .



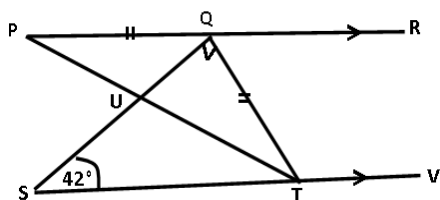
a) Explain why $\widehat{OCT} = 90^\circ$.

[1]

b) Calculate the length of CT .

[2]

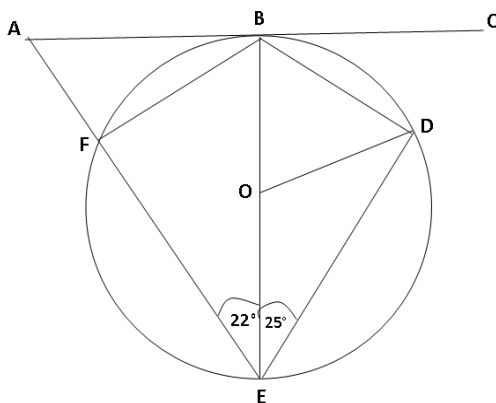
8. In the diagram below, PQR is parallel to STV , $\widehat{SQT} = 90^\circ$, $\widehat{QST} = 42^\circ$ and $PQ = QT$.



Calculate

- | | |
|--------------------|-----|
| a) $\widehat{TQR}$ | [1] |
| b) $\widehat{PTS}$ | [1] |
| c) $\widehat{PUS}$ | [1] |

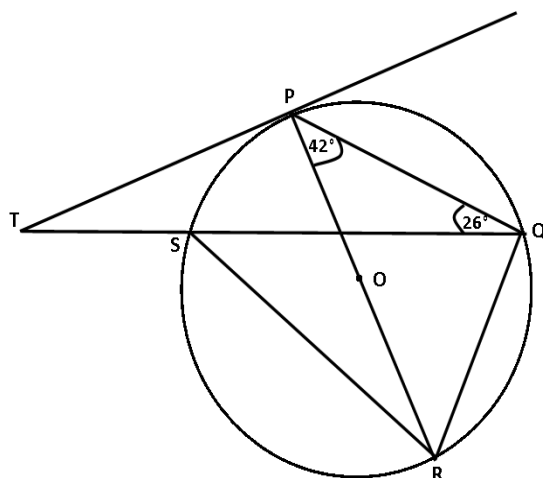
9. In the diagram below, ABC is the tangent and O is the Centre of the circle. BE is the diameter $\widehat{DEO} = 25^\circ$ and $\widehat{FEO} = 22^\circ$.



Calculate

- | | |
|--------------------|-----|
| a) $\widehat{BOD}$ | [1] |
| b) $\widehat{CBD}$ | [1] |
| c) $\widehat{DBF}$ | [1] |

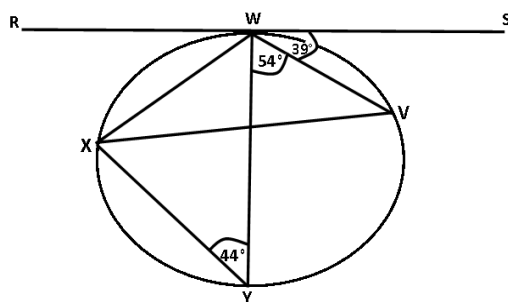
10. In the diagram below, PR is a diameter of a circle with Centre O . Q and S are points on the circumference. The tangent to the circle at the point P meets QS produced at T , $\widehat{PQS} = 26^\circ$ and $\widehat{QPR} = 42^\circ$



Calculate

- | | |
|--------------------|-----|
| a) $\widehat{PRS}$ | [1] |
| b) $\widehat{PRQ}$ | [1] |
| c) $\widehat{PTQ}$ | [2] |

11. The diagram below shows a circle with a tangent **RWS**. **V, W, X** and **Y** are on the circle such that $\widehat{XYW} = 44$, $\widehat{VWY} = 54$ and $\widehat{SWV} = 39$

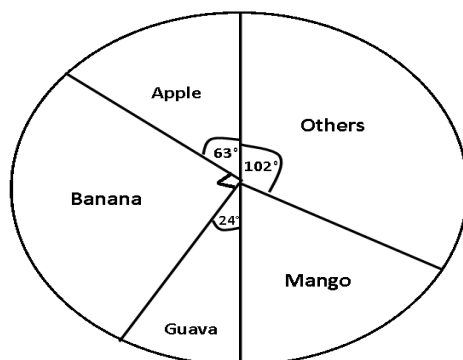


Calculate

- | | |
|--------------------|-----|
| a) $\widehat{RWX}$ | [1] |
| b) $\widehat{XVW}$ | [1] |
| c) $\widehat{YXW}$ | [2] |

PROPORTION AND VARIATIONS

1. The ratio of the heights of the two containers that are geometrically similar is 2:3. If the surface area of the smaller container is 80cm^2 . Find the surface area of the bigger container. [2]
2. y varies directly as z and inversely as w^2 when $y = 6$, $z = 2$ and $w = 3$, find the values of
 - a. k , [1]
 - b. y when $z = 6$ and $w = 9$, [1]
 - c. W when $y = 3$ and $z = 4$. [2]
3. The pie chart below shows the results of a survey that was conducted on 600 AIDS patients in one province on the fruit they preferred to eat.



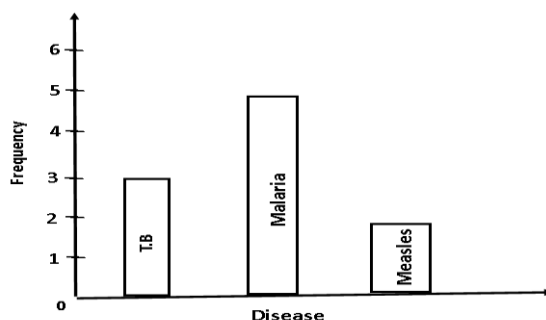
- a) What fraction of parents chose Banana? [1]
 - b) How many more chose Banana than Mango? [2]
4. Bupe is a street vendor who sells handbags. One day he sold the bag at K42 000 and made a loss of 30%. Calculate the coast price of the bag. [2]
5. During the 2017 FAZ elections, presidential candidates received the following votes:

Name of candidate	Number of votes received
Kwasha	120
Naluba	45
Nkoshi	X

- a) Express as a ratio in its simplest form Naluba's votes to Kwasha's. [1]
 - b) Given that Naluba got 25% of the total votes cast, calculate the total number of delegates who voted. [2]

c) Find the value of x . [2]

6. The bar chart below shows the number of people who were treated for Malaria, Measles and Tuberculosis (TB) at an urban Clinic in one week.



- a) If 120 people were treated for Malaria, how many were treated for Measles? [2]
 b) Calculate the total number of people treated for the three diseases at the clinic. [2]

7. It is given that x varies inversely as the square of y and directly as z , and $x = 2$ when $y = 3$ and $z = 4$.

- a. Find k (the constant of variation). [1]
 b. Find y when $x = 3$ and $z = 24$. [2]

8. The ratio of the surface areas of two similar bottles is 4:9. What is the ratio of their volumes? [2]

9. Given that x varies directly proportional to y and inversely proportional to z^2 and that

$x = 12$ when $y = 3$ and $z = 2$. Find

- a. The equation connecting x , y and z , [2]
 b. The value of x when $y = 3$ and $z = 4$, [1]
 c. The value of z when $x = 4$ and $y = 25$. [2]

10. The diagram below shows a simple electricity bill for Bana Kantwa.

Name: Mrs. Kantwa B.		Billing date: 10/06/2015	
Previous reading	Present reading	Rate/Unit	Charge
71879	74279		K184 800.00

Find the cost per unit of electricity. [2]

11. Given that 4% of the bricks get damaged when they reach the building site. How many bricks should a builder order if 4 800 are needed to finish the job? [2]
12. Tickets for a variety show held at Movi secondary school were sold at K50.00 each. Half of the money collected at the show was donated to Charity. If Charity received K2 500.00, how many tickets were sold? [3]
13. Ladies were invited to a kitchen party. The ratio of those who took Fanta to those who did not was 13:5 respectively. Given that 18 others came in an uninvited and 8 of those took Fanta.
Find the new ratio of those who took Fanta to those who did not. [3]
14. Given that y varies directly as x and inversely as the square of z and $y = 10$ when $x = 32$ and $z = 4$. Find
- The value of k , the constant of variation, [1]
 - y when $x = 20$ and $z = 5$, [1]
 - z when $x = 9$ and $y = 5$. [2]
15. The ratio of the volumes of two similar solids is 64:27 the surface area of the smaller solid is 180cm^2 . What is the surface area of the bigger solid? [2]
16. Two variables x and y have corresponding values as shown in the table below.

X	2	3	a
Y	20	40	104

Given that y varies directly as $(x^2 + 1)$, find the

- Constant of variation, k [1]
 - Equation connecting y and x , [1]
 - Values of a . [2]
17. It is given that $t = kv^2$ where K is the constant of variation.

v	1	b	5
t	4	36	a

Use the information in the table to find the value(s) of

- k , [1]
- b , [2]

- c. a. [2]
18. A furniture shop conducted a sale. In this sale, prices were reduced by 15%. Find the price of a table before reduction that was reduced by K120. 00 [2]
19. Given that y is proportional to x^3 and $y = 250$ when $x = 10$. Find
- a) The value of the constant K , [1]
- b) y when $x = 4$, [1]
- c) X when $y = 54$. [2]
20. The table below shows the relationship between two variables x and y . It is given that y varies inversely as the square root of x , where x is positive.

Y	2	8	$\frac{8}{9}$
X	16	1	a

- a. Write an expression for y in terms of x and the constant k , [1]
- b. Find the value of
- i. k [1]
- ii. a [2]
21. Y varies directly as the square of x and $y = 96$ when $x = 4$. Find the
- a) value of the constant k , [1]
- b) value of y when $x = 5$, [1]
- c) Values of x when $y = 24$. [2]
22. It is given that w varies directly as the square of x and inversely as y
- a. Write an expression for w , in terms of x , y and a constant k , [1]
- b. If $y = -6$, $x = 12$ and $w = 15$, find k . [1]
- c. Find the value of y when $x = 8$ and $w = 20$. [2]

FUNCTIONS AND SYSTEM OF EQUATIONS

1. The functions g and f are defined as $f(x) = \frac{5x+4}{5}$ and $g(x) = x - 1$. Find
- a) $f^1(x)$, [1]
- b) $f^1(-2)$, [1]
- c) $f[g(x)]$ in its simplest form. [2]

2. Solve the equations

- a) $x^2 = 3x$, [2]
 b) $25^x = 5$. [2]
3. The functions g and f are defined as $g: x \rightarrow \frac{x-1}{2}$ and $f(x) = 3x - 5$. Find
 a) $g^{-1}(x)$, [1]
 b) x if $f(x) = g(x)$, [2]
 c) $g^{-1}[f(x)]$. [2]
4. The functions f and g are defined by $f(x) = 2x + 1$ and $g(x) = 5x - 1$. Find
 a) $g^{-1}(x)$, [2]
 b) $f[g(x)]$, [2]
 c) $f[g(-3)]$. [1]
5. Find the value(s) of x in each of the following:
 a) $y = (1 - 2x)(1 - x) - 2$, when $y = -2$. [2]
 b) $(2x - 1)^2 = 25$ [2]
 c) $2x^3 - 60 = -6$. [2]
6. Given that $f(x) = 2x - 5$ and $g(x) = x + 3$. Find
 a) $f^1(x)$, [1]
 b) $f^1(-10)$, [1]
 c) $f[g(x)]$. [2]
7. Solve the equation $x^2 - 4x - 2 = 0$, giving your answers correct to two decimal places. [5]
8. Solve the equation
 a) $2^{2x-1} = 16^{-2x}$. [2]
 b) $x^3 = 4$, [2]
 c) $x^3 - 6x^2 = 16x$. [4]
9. Given that $f(x) = 8x$ and $g(x) = \frac{3x-2}{4}$, find
 a) $f^1(x)$, [1]
 b) an expression for $f[g(x)]$, [2]
 c) the value of x for which $f[g(x)] = 20$. [1]
10. Solve the equation $3x^2 + 5x + 2 = 0$. [3]
11. Solve the simultaneous equation

$$\begin{aligned} 2y &= 3x - 12 \\ 5x - 6y &= 16. \end{aligned} \quad [3]$$

12. Solve the equation $(2x - 1)(3x - 2) = 3$, giving your answers correct to two decimal places. [6]

13. A function is defined as $f(x) = \frac{3}{5x+1}$, $x \neq -\frac{1}{5}$. Find

a) $f(1)$, [1]

b) x , if $f(x) = 6$. [2]

14. Solve the simultaneous equations

$$\begin{aligned} 2x + y &= 1 \\ 3x - 2y &= -9. \end{aligned} \quad [3]$$

15. Solve the equation $2y^2 + 6y - 1 = 0$ giving your answers correct to two decimal places. [5]

16. For the function $g(x) = \frac{4x-3}{2x-5}$. Find

a) $g(-1)$, [1]

b) $g^{-1}(x)$. [2]

17. Solve the simultaneous equations

$$\begin{aligned} 2x + y &= 4 \\ x - 2y &= 2 \end{aligned} \quad [3]$$

18. Find

a) The equation of line l passing through a point $(0, 5)$ whose gradient is 3. [2]

b) If B is a point $(6, -3)$ and C is $(-2, 1)$, find the

I. gradient of line BC , [2]

II. Equation of line BC . [2]

19. Given that $f(x) = 12 - 3x$, find

a) $f(-3)$, [1]

b) $f^{-1}(-3)$. [2]

20. Solve the simultaneous equations

$$\begin{aligned} 2x &= 5y \\ x - 2y &= 3. \end{aligned} \quad [3]$$

21. Given that $f(x) = x^3 - x$, find the largest integer which is a common factor of $f(4)$ and $f(5)$. [3]

22. Solve the simultaneous equations

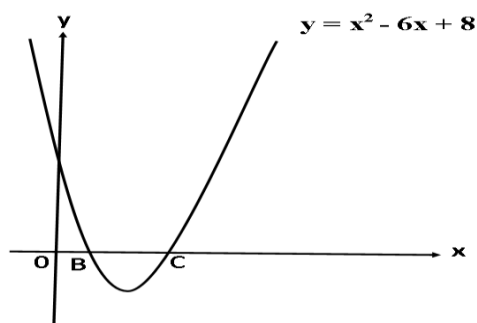
$$\begin{aligned} 3x - 3y &= 6 \\ x - 4y &= 8. \end{aligned} \quad [3]$$

23. For the function $f(x) = \frac{2x-3}{x}$, $x \neq 0$. find

- a) $f(-3)$, [1]
- b) $f^{-1}(4)$. [2]

24. Show that the points $L(-2, -10)$, $M(2, 2)$ and $N(5, 11)$ are Collinear. [5]

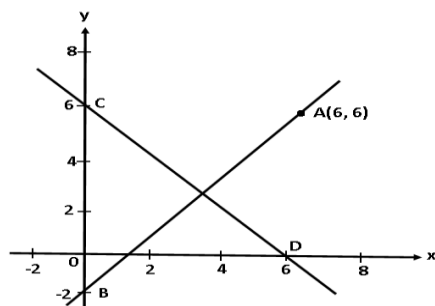
25. The diagram below shows a sketch of the graph of $y = x^2 - 6x + 8$ cutting the X-axis at B and C.



Find the coordinates of

- a) B and C, [4]
- b) the turning point of the graph. [2]

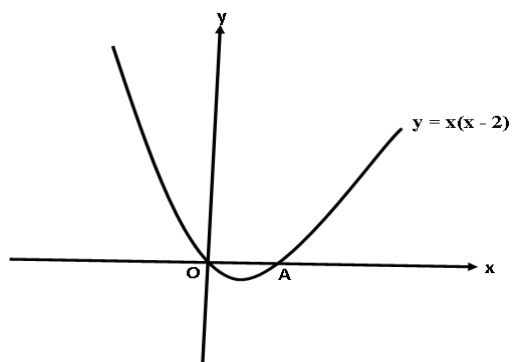
26. The diagram below shows a Cartesian plane with points $A(6, 6)$, $B(0, -2)$, $C(0, 6)$ and $D(6, 0)$.



Find the

- a) equation of the line CD, [2]
- b) distance AB. [2]

27. The sketch shown below represents a section of the curve $y = x(x - 2)$.

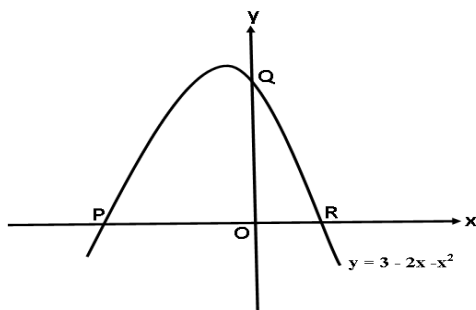


- a) Find the coordinates of the points where the curve cuts the X-axis, [2]
- b) What is the minimum value of the function? [2]
- c) The gradient of the line joining the points $(-2, K)$ and $(K, -14)$ is 2. Calculate the value of k. [2]

28. Given that $f(x) = \frac{2x+7}{8}$ and $g(x) = \frac{3x-6}{6}$, Find

- a) $g(6)$ [1]
- b) $f^{-1}(x)$, [1]
- c) The value of x if $f(x) = g(x)$ [2]

29. The diagram below shows a sketch of the graph of $y = 3 - 2x - x^2$ passing through P, Q and R.



Find the

- a) equation of the axis of symmetry of the graph, [2]
- b) Coordinates of the turning point of the graph. [2]

30. Find the equation of the straight line passing through $(-4, 4)$ and is perpendicular to the straight line whose equation is $y + \frac{x}{7} = 1$. [3]

CALCULUS

1. The equation of the curve is $y = x^3 - \frac{3}{2}x^2$. Find
 - a) equation of the normal where $x = 2$, [2]
 - b) The coordinates of the stationary points. [2]
2. Evaluate $\int_{-1}^3 (3x^2 - 2x)dx$ [3]
3. Find the coordinates of the points on a curve $y = 2x^3 - 3x^2 - 36x - 3$ where the gradient is zero. [3]
4. Find the equation of the normal to the curve $y = x + \frac{4}{x}$ and the point where $x = 4$. [3]
5. Evaluate $\int_2^5 (3x^2 + 2)dx$ [3]
6. Determine the equation of the normal to the curve $y = 2x^2 - 3x - 2$ that passes through $(3, 7)$. [4]
7. Evaluate $\int_0^1 (x^2 - 2x + 3)dx$ [3]

CONSTRUCTION

1. Answer the whole of this question on a sheet of graph paper.
 - a) Construct a quadrilateral ABCD in which $AB = 10\text{cm}$, and $\widehat{ABC} = 120^\circ$, $\widehat{BAD} = 60^\circ$, $BC = 7\text{cm}$ and $AD = 11\text{cm}$. [1]
 - b) Measure and write the length of CD. [1]
 - c) Within the quadrilateral ABCD, draw the locus of points which are
 - I. 8cm from A, [1]
 - II. Equidistant from BC and CD. [2]
 - d) A point P, within the quadrilateral ABCD, is such that it 8cm from A and equidistant from BC and CD. Label point P. [1]

- e) Another point Q, within the quadrilateral ABCD, is such that, it is nearer to CD than BC and greater than or equal to 8cm from A. indicate, by shading, the region in which Q must lie. [2]

2. Answer the whole of this question on a sheet of graph paper.

- a) Construct triangle XYZ in which XY = 9cm, YZ = 7cm and angle XYZ = 38°. [1]
b) Measure and write the length of XZ. [1]
c) On your diagram within triangle XYZ, construct the locus of points which are
I. 6cm from Y, [1]
II. Equidistant from XY and XZ. [1]
d) Mark clearly with letter P, within triangle XYZ, a point which is 6cm from Y and equidistance from XY and XZ. [2]
e) A point T within triangle XYZ is such that its distance from Y is less than or equal to 6cm and it is nearer to XY than XZ. Indicate clearly by shading the region in which T must lie. [2]

3. Answer the whole of this question on a sheet of graph paper.

- a) Construct triangle PQR in which PQ = 9cm, angle PQR = 60° and QR = 10cm. [1]
b) Measure and write the length of PR. [1]
c) On your diagram, draw the locus of points with triangle PQR which are
I. 3cm from PQ, [1]
II. 7cm from R, [1]
III. Equidistant from P and R. [2]
d) A point M, within triangle PQR, is such that it is nearer to R than P, less than or equal to 7cm from R and less than or equal to 3cm from PQ. Shade the region in which M must lie. [2]

4. Answer the whole of this question on a sheet of graph paper.

- a) Construct triangle ABC where AB = BC = CA = 7cm. [1]
b) Measure and write the size of $\angle CAB$. [1]
c) Within triangle ABC construct the locus of points which are
I. Equidistant from AB and BC, [1]
II. 4cm from B, [1]
III. 3cm from AB. [2]
d) A point R, within triangle ABC, is such that it is nearer to BC than AB, less than 3cm from AB and less than 4cm from B. Shade the region in which R must lie. [2]

5. Answer the whole of this question on a sheet of graph paper.

- a) Construct triangle PQR in which $PQ = 10\text{cm}$, $QR = 8\text{cm}$ and angle $PQR = 50^\circ$. [1]
- b) Measure and write the length of PR . [1]
- c) On your diagram, within triangle PQR , construct the locus of points which are
 - I. Equidistant from P and Q . [1]
 - II. Equidistant from PR and PQ . [1]
 - III. 5cm from R . [1]
- d) A point T within triangle PQR is such that it is 5cm from R and equidistant from P and Q . Label point T . [1]
- e) Another point X is such that it is less than or equal to 5cm from R , nearer to Q than P and nearer to PQ than PR . Indicate by shading, the region in which X must lie. [2]

LINEAR PROGRAMMING

1. Answer the whole of this question on a sheet of graph paper.

Himakwebo orders maize and groundnuts for sale. The order price for a bag of maize is K75.00 and that of a bag of groundnuts is K150.00. He is ready to spend up to K7 500.00 altogether. He intends to order at least 5 bags of maize and at least 10 bags of groundnuts. He does not want to order more than 70 bags altogether.

- a) If x and y are the number of bags of maize and groundnuts respectively, write four inequalities which represent these conditions. [4]
- b) Using a scale of 2cm to represent 10 bags on each axis, draw the x and y axes for $0 \leq x \leq 70$ and $0 \leq y \leq 70$ respectively and shade the unwanted region to show clearly the region where the solution of the inequalities lie. [4]
- c) Given that a profit on the bag of maize is K25.00 and on the bag of groundnuts is K50.00, how many bags of each type should he order to have the maximum profit? [2]
- d) What is this estimate of the maximum profit? [2]

2. Answer the whole of this question on a sheet of graph paper.

A tailor at a certain market intends to make dresses and suits for sale. Let x represent the number of dresses and y the number of suits.

- a) Write the inequalities which represent each of the conditions below.
 - I. The number of dresses should not exceed 50, [1]
 - II. The number of dresses should not be more than the number of suits. [1]
 - III. The cost of making a dress is K140.00 and that of a suit is K210.00. The total should be at least K10 500.00. [2]

- b) Using a scale of 2cm to represent 10 units on both axes, draw x and y axes for $0 \leq x \leq 60$ and $0 \leq y \leq 80$. Shade the unwanted region to indicate clearly the region where (x, y) must lie. [4]
- c) The profit on a dress is K160.00 and on a suit is K270.00. Find the number of dresses and suits the tailor must make for maximum profit. [2]
- d) Calculate this maximum profit. [2]

3. Answer the whole of this question on a sheet of graph paper.

A hired bus is used to take learners and teachers on a trip. The number of learners and teachers must be more than 60. There must be at least 35 people on the trip. There must be at least 6 teachers on the trip. The number of teachers on the trip should not be more than 14. Let x be the number of learners and y the number of teachers.

- a. Write four inequalities which present the information above. [2]
- b. Using a scale of 2cm to represent 10 units on both axes, draw the x and y axes for $0 \leq x \leq 70$ and $0 \leq y \leq 70$ respectively. Shade the unwanted region to indicate clearly where the solution of the inequalities lies. [4]
- c. If the group has 25 learners, what is the minimum number of teachers that must accompany them? [2]
- d. If 8 teachers go on the trip, what is the maximum number of learners that can be accommodated on the bus? [2]
- e. If T is the amount in Kwacha paid by the whole group, what is the cost per learner if $T = 30x + 50y$. [2]

4. Answer the whole of this question on a sheet of graph paper.

Bwalya prepares two types of sausages, Hungarian and beef, daily for sale. She prepares at least 40 Hungarian and at least 10 beef sausages. She prepares not more than 160 sausages altogether. The number of beef sausages prepared is not more than the number of Hungarian sausages.

- a) Given that x represents the number of Hungarian sausages and y the number of beef sausages, write four inequalities which represent these conditions. [4]
- b) Using a scale of 2cm to present 20cm sausages on both axes, draw the x and y axes for $0 \leq x \leq 160$ and $0 \leq y \leq 160$ respectively and shade the unwanted region to show clearly the region where the solution of the inequalities lie. [4]
- c) The profit on the sale of each Hungarian sausage is K3.00 and on each beef sausage is K2.00.
How many of each type of sausages are required to make maximum profit? [2]

d) Calculate this maximum profit.

[2]

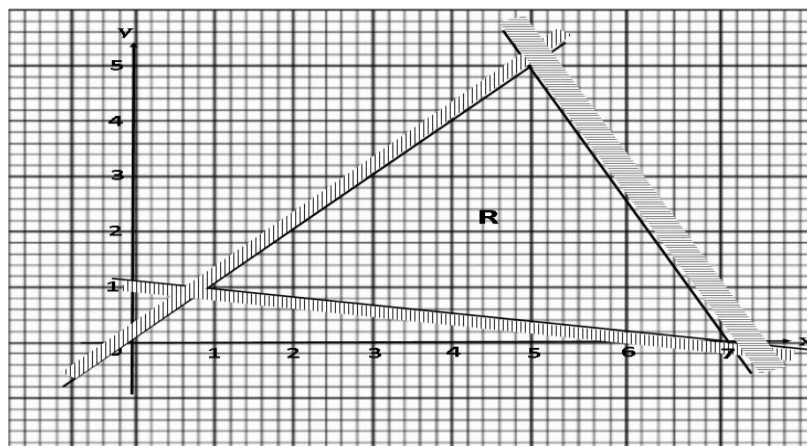
5. Answer the whole of this question on a sheet of graph paper.

A Health Lobby group produced a guide to encourage healthy living among local community. The group produced the guide in two formats: a short video and a printed book. The group needs to decide the number of each format to produce for sale to maximize profit. Let x represent the number of videos produced and y the number of printed books produced.

- a) Write the inequalities which represent each of the following conditions
 - I. the total number of copies produced should not be more than 800, [2]
 - II. the number of video copies to be at least 100, [1]
 - III. the number of printed books to be at least 100. [1]
- b) Using a scale of 2cm to represent 100 copies on both axes, draw the x and y axes for $0 \leq x \leq 800$ and $0 \leq y \leq 800$ respectively and shade the unwanted region to indicate clearly the region where the solution of the inequalities lie. [4]
- c) The profit on the sale of each video copy is K15.00 while the profit on each printed book is K8.00. How many of each type were produced to make maximum profit. [4]

6. Answer the whole of this question on a sheet of graph paper.

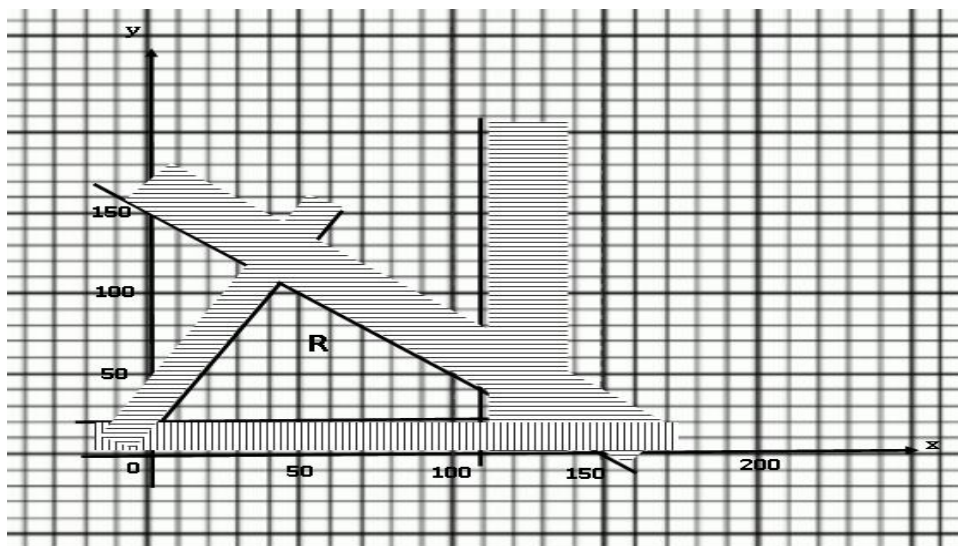
The graph below shows three inequalities that satisfies region R



- a) Write the three inequalities that defines the un-shaded region R. [6]
- b) Find the largest value of $4x - 5y$ within the region. [1]

7. Answer the whole of this question on a sheet of graph paper.

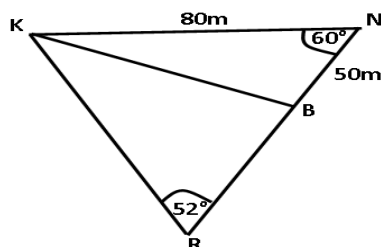
The region R in the diagram below shows the net of points (x, y) satisfying four inequalities representing the number of chairs (x) and tables (y) a carpenter made.



- a) If the two of the inequalities are $y \geq 20$ and $y \leq 2x + 20$, write the other two inequalities. [3]
- b) The carpenter makes a profit of K100.00 on a chair and K60.00 on a table sold. Given that the carpenter sold all the chairs and tables he made, find the
 - I. Value of x and y which would give him maximum profit. [2]
 - II. Maximum profit. [2]

TRIGONOMETRY

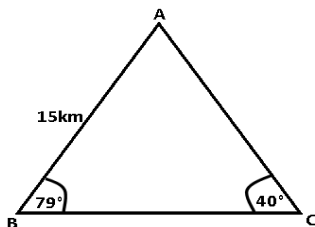
1. In the diagram below, K, N, B and R are places on horizontal surface. $KN = 80\text{m}$, $NB = 50\text{m}$ and $\widehat{KRN} = 52^\circ$



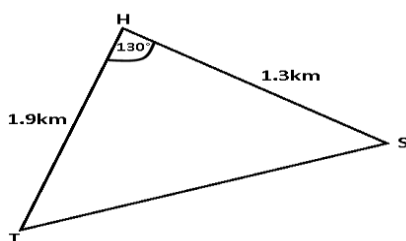
- a) Calculate
 - I. KR , [5]
 - II. the area of triangle KNB . [2]

- b) Given that the area of triangle KNR is equal to $3\,260\text{m}^2$, calculate the shortest distance from **R** to **KN**. [3]
- c) Sketch the graph of $y = \cos \theta$ for $0^\circ \leq \theta \leq 360^\circ$. [2]

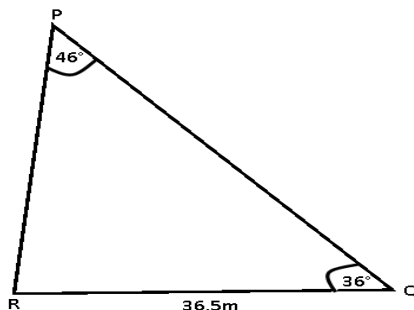
2. Three villages **A**, **B** and **C** are connected by straight paths as shown in the diagram below.



- a) Given that **AB** = 15km, angle **ABC** = 79° and angle **ACB** = 40° , calculate the
- Distance **AC**, [4]
 - area of triangle **ABC**, [2]
 - Shortest distance from **B** to **AC**. [2]
- b) Solve the equation $\cos \theta = 0.937$ for $0^\circ \leq \theta \leq 360^\circ$. [2]
- c) Sketch the graph of $y = \sin \theta$ for $0^\circ \leq \theta \leq 360^\circ$. [2]
3. The diagram below shows the Location of houses for a village Headman (H), his Secretary (S) and a Trustee (T). **H** is 1.3 km from **S**, **T** is 1.9 km from **H** and **THS** = 130°

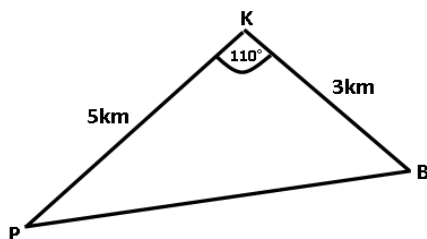


- a) Calculate
- the area of triangle **THS**. [3]
 - the distance **TS**. [5]
 - the shortest distance from **H** to **TS**. [2]
- b) Find the angle between 0° and 90° which satisfies the equation $\cos \theta = \frac{2}{3}$ [2]
4. In Triangle **PQR** below, **QR** = 36.5m, angle **PQR** = 36° and angle **QPR** = 46° .

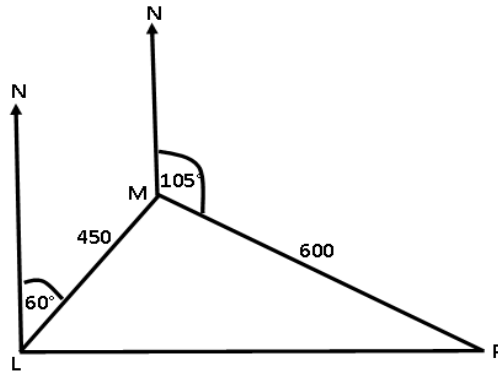


- a) Calculate
- I. **PQ**, [4]
 - II. the area of triangle **PQR**, [2]
 - III. the shortest distance from **R** to **PQ**. [2]
- b) Solve the equation $\sin \theta = 0.6792$ for $0^\circ \leq \theta \leq 360^\circ$. [2]

5. The diagram below shows the location of three secondary schools, namely Kamubala (K), Belengani (B) and Pendeni (P) in a district. **P** is 5km from **K**, **B** is 3km from **K** and angle **PKB** is 110°



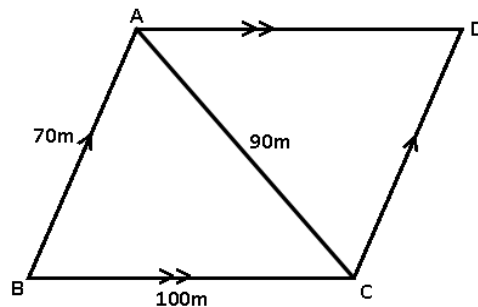
- a) Calculate
- I. **BP**, [5]
 - II. the area of triangle **BKP**, [2]
 - III. the shortest distance from **K** to **BP**. [2]
- b) Solve the equation $\tan \theta = 0.7$ for $0^\circ \leq \theta \leq 180^\circ$. [1]
6. Three towns Luzi (L), Muzi (M) and Puzi (P) are spaced as shown in the diagram. **M** is on a bearing of 060° from **L** and **P** is on a bearing of 105° from **M**.



Given that **M** is 450km from **L** and 600km from **P**, calculate

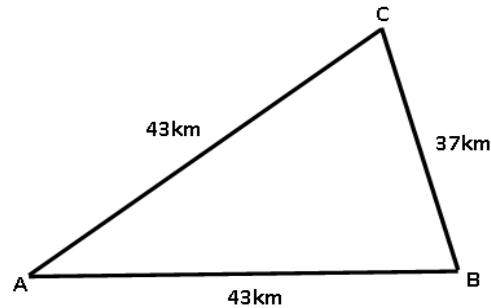
- The distance between **L** and **P**, [5]
- The area of triangle **LMP**, [3]
- Angle **MPL**. [2]

7. A farm is in a shape of a parallelogram **ABCD**. To accommodate two different crops, the farm is divided into two parts as shown in the diagram below.



- Given that **AB** = 70m, **AC** = 90m and **BC** = 100m, calculate
 - Angle BAC, [5]
 - The area of triangle **ABC**, [3]
 - The shortest distance from **D** to **AC**. [2]
- Solve the equation $\sin \theta = 0.766$ for $0^\circ \leq \theta \leq 180^\circ$. [2]

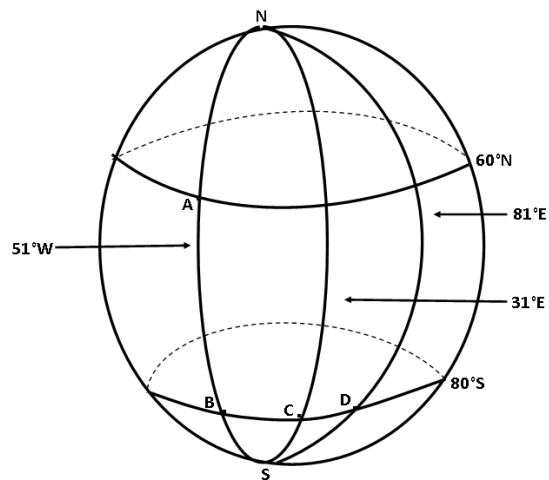
8. In Votani constituency, **A**, **B** and **C** are polling stations as shown in the diagram below.



- a) Calculate
- I. Angle **BAC**, [5]
 - II. Angle **ACB**. [1]
- b) The area of triangle **ABC** correct to 1 decimal place. [2]
- c) The shortest distance from **C** to **AB**. [2]

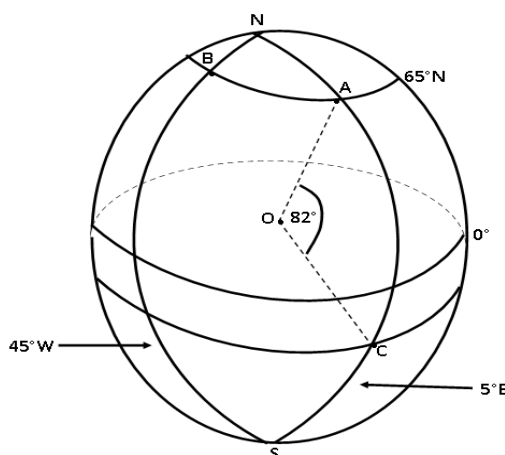
EARTH GEOMETRY

1. A, B, C and D are points on the surface of the earth as shown in the diagram below.

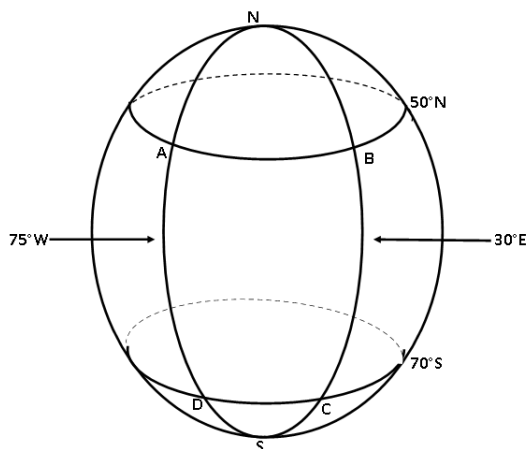


- a) Using latitudes and longitudes, write the positions of the points A and B. [2]
- b) Find the difference in longitude between points C and D. [1]
- c) Calculate the distance CD in nautical miles [$\pi = 3.143$ and $R = 3437\text{nm}$]. [2]
- d) Given that the local time at D is 13: 05hours find the time at C. [1]

2. In the diagram below, A(65°N , 5°E), B(65°SN , 45°W) and C are three points on the surface of the model of the earth and O is the Centre of the model. The point C, due south of A, is such that $\angle AOC = 82^{\circ}$. [$\pi = 3.142$, $R = 3437 \text{ nm}$]



- State the longitude of A. [1]
 - Calculate the latitude of C. [1]
 - Calculate in nautical miles the shortest distance
 - Between A and C measured along the common longitude, [2]
 - Between A and B measured along the circle of latitude. [2]
3. The points A, B, C and D are on the surface of the earth. (Take $\pi = 3.142$ and $R = 3437 \text{ nm}$).



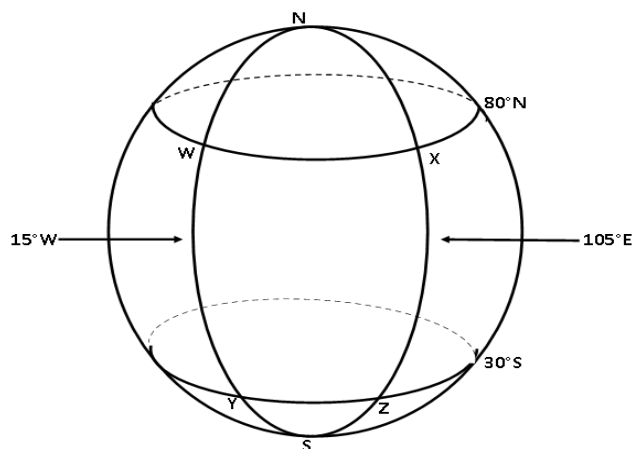
- Find the difference in latitude between points C and B. [1]
 - Calculate the length of the circle of latitude 50°N in nautical miles. [2]
 - Find the distance AD in nautical miles. [3]
4. Q(80°N , 10°E), P(80°N , 70°E), R(85°S , 10°E) and S(85°S , 70°E) are four points on the surface of the earth.
- Show the points on a clearly labeled sketch of the surface of the earth. [2]

b) Find in nautical miles

- I. The distance QR along the longitude, [2]
- II. The circumference of latitude 85°S . [2]

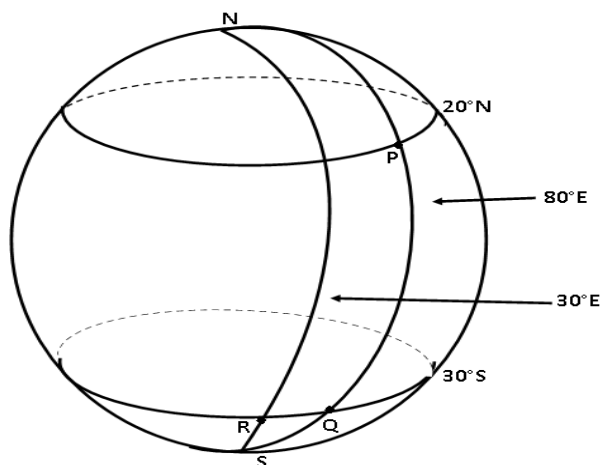
[Take $\pi = 3.142$ and $R = 3437\text{nm}$]

5. W, X, Y and Z are four points on the surface of the earth as shown in the diagram below. (Take $\pi = 3.142$ and $R = 3437\text{nm}$)

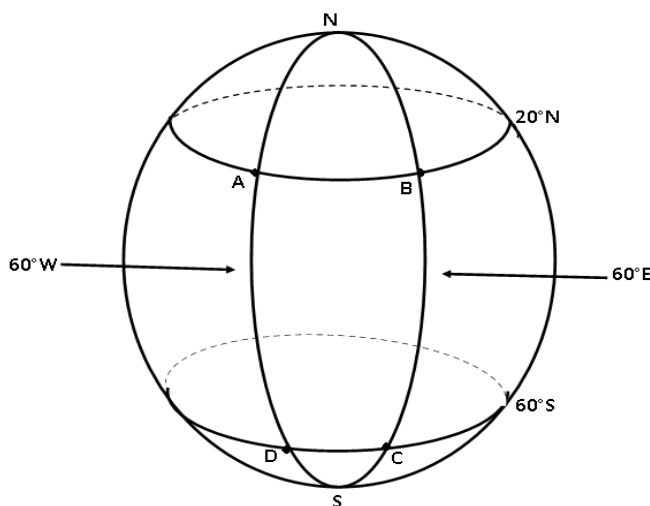


- a) Calculate the difference in latitudes between W and Y. [2]
- b) Calculate the distance in nautical miles between
 - I. X and Z along the longitudes 105°E , [2]
 - II. Y and Z along the circle of latitude 30°S . [2]

6. The diagram below is the sketch of the earth and on it are the points P(20°N , 80°E), Q(40°S , 80°E) and R(40°S , 30°E). Use [$\pi = 3.142$ and $R = 6370\text{km}$]



- a) Calculate the distance QR in kilometers. [2]
- b) An Aero plane starts from P and flies due west on the same latitude covering a distance of 1 232km to point T.
- I. Calculate the difference in angles between P and T, [2]
- II. Find the position of T. [2]
7. The points A (15°N , 40°E), B(35°N , 70°E)and C (35°S , 40°E) are on the surface of the Earth. [Use $\pi = 3.142$ and $R = 6370\text{km}$]
- a) Calculate the distance AC in kilometers. [2]
- b) An Aero plane takes off from point B and flies due west on the same latitude covering a distance of 900km to point Q.
- I. Calculate the difference in longitudes between B and Q. [2]
- II. Find the position of Q. [2]
8. In the diagram below, A and B are points on latitude 60°N while C is a point on latitude 60°S . [$\pi = 3.142$ And $R = 3437\text{nm}$].



- a) Calculate the distance BC along the latitude 60° in nautical miles. [2]
- b) A ship sails from C to D in 12 hours. Find its speed in knots. [4]

QUADRATIC FUNCTIONS

1. Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = x^2 - 2x + 1$. Some of the corresponding values of x and y correct to 1 decimal place are given in the table below.

X	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
Y	p	4	2.3	1	0.3	0	0.3	1	2.3	4

- a) Calculate the value of p , [1]
- b) Using a scale of 2cm to represent 1 unit on both axes, draw the graph of $y = x^2 - 2x + 1$ for $-2 \leq x \leq 3$ and $0 \leq y \leq 10$. [3]
- c) Calculate an estimate of the gradient of the curve at the point $(0, 1)$. [2]
- d) Showing your method clearly, use your graph to solve the equation $x^2 - 2x + 1 = 1.5$. [3]

2. Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = \frac{x^2}{5} + \frac{2}{x}$.

Some corresponding values of x and y correct to 1 decimal place, where necessary, are given in the table below.

X	0.3	0.6	1	1.5	2	3	4	5	6
Y	6.7	3.4	2.2	1.8	1.8	2.5	3.7	5.4	Q

- a) Calculate the value of q , giving your answer to 1 decimal place. [1]
- b) Taking 2cm to represent 1 unit on each axes, draw the x and y axes for $0 \leq x \leq 6$ and $0 \leq y \leq 8$. Draw the graph of $y = \frac{x^2}{5} + \frac{2}{x}$. [3]
- c) Use your graph to solve the equation $\frac{x^2}{5} + \frac{2}{x} = 3$. [2]
- d) On the same axes, draw the graph of $y = \frac{x+7}{2}$. [1]
- e) Hence or otherwise, solve the equation $\frac{x^2}{5} + \frac{2}{x} = \frac{x+7}{2}$. [2]

3. Answer the whole of this question on a sheet of graph paper.

The values of x and y are connected by the equation $y = x^3 - 6x + 2$ as shown in the table below.

X	-3	-2	-1	0	1	2	3
Y	-7	6	7	2	-3	-2	K

- a) Calculate the value of k, [1]
- b) Using 2cm to represent 1 unit on the x axis and 1cm to represent 2 units on the y axes for $-3 \leq x \leq 3$ and $-8 \leq y \leq 12$, draw the graph of $y = x^3 - 6x + 2$. [3]
- c) Use your graph to solve the equation $x^3 - 6x + 2 = 0$. [2]
- d) On the same axes, draw the graph of $y = 2x - 2$. [1]
- e) Hence or otherwise solve the equation $x^3 - 6x + 2 = 2x - 2$. [2]

4. Answer the whole of this question on a sheet of graph paper.

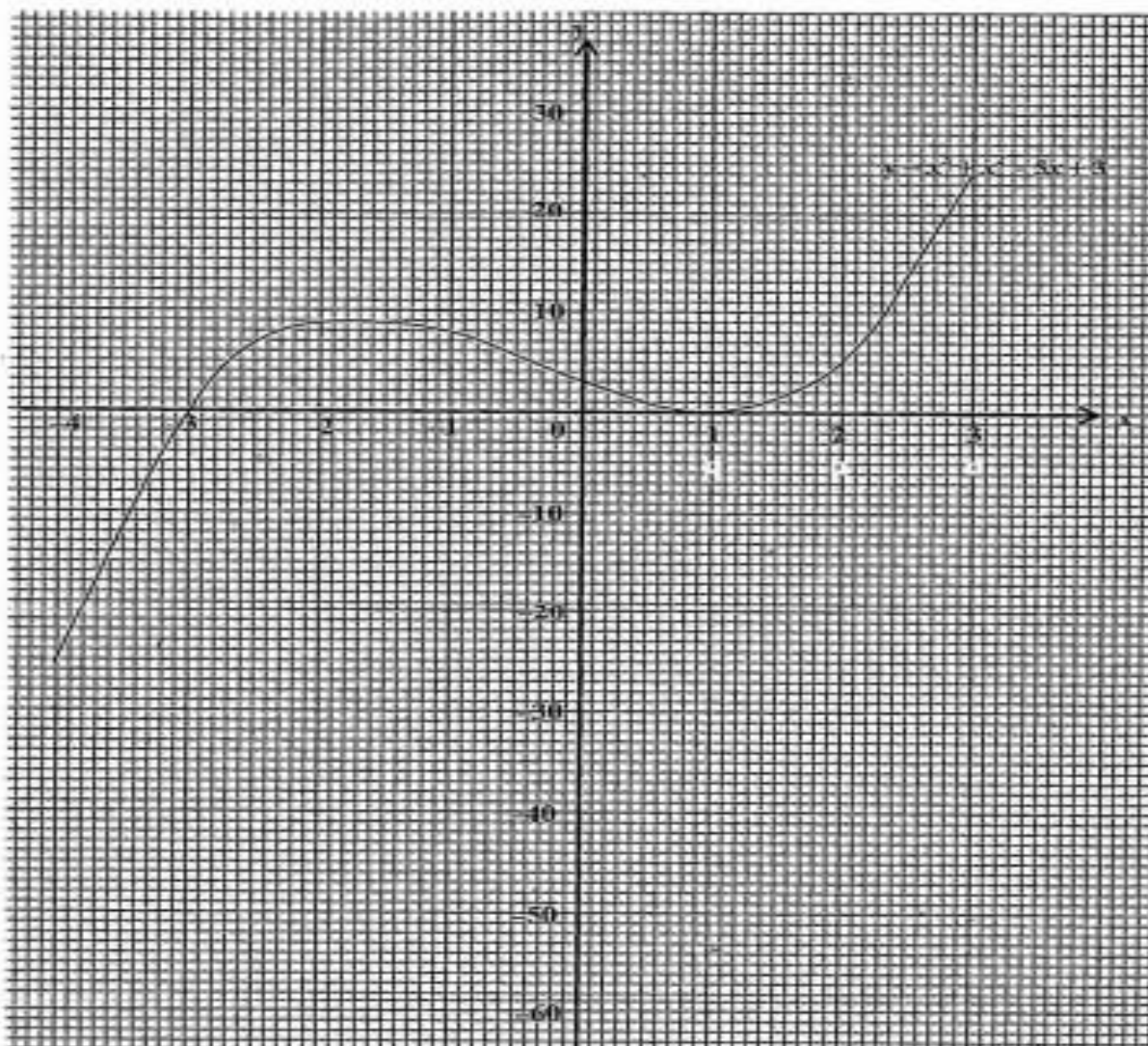
The values of x and y are connected by the equation $y = x(x - 2)(x + 2)$.
Some corresponding values of x and y are given in the table below.

X	-3	-2	-1	0	1	2	3
Y	-15	0	3	0	-3	0	K

- a) Calculate the value of k, [1]
- b) Using a scale of 2cm to 1 unit on the x axes for $-3 \leq x \leq 3$ and 2cm to represent 5 units on the y axes for $-16 \leq y \leq 16$. Draw the graph of $y = x(x - 2)(x + 2)$. [3]
- c) Use your graph to solve the equations
 - I. $x(x - 2)(x + 2) = 0$, [2]
 - II. $x(x - 2)(x + 2) = x + 2$. [3]

5. Answer the whole of this question on a sheet of graph paper.

The diagram below shows the graph of $y = x^3 + x^2 - 5x + 3$.

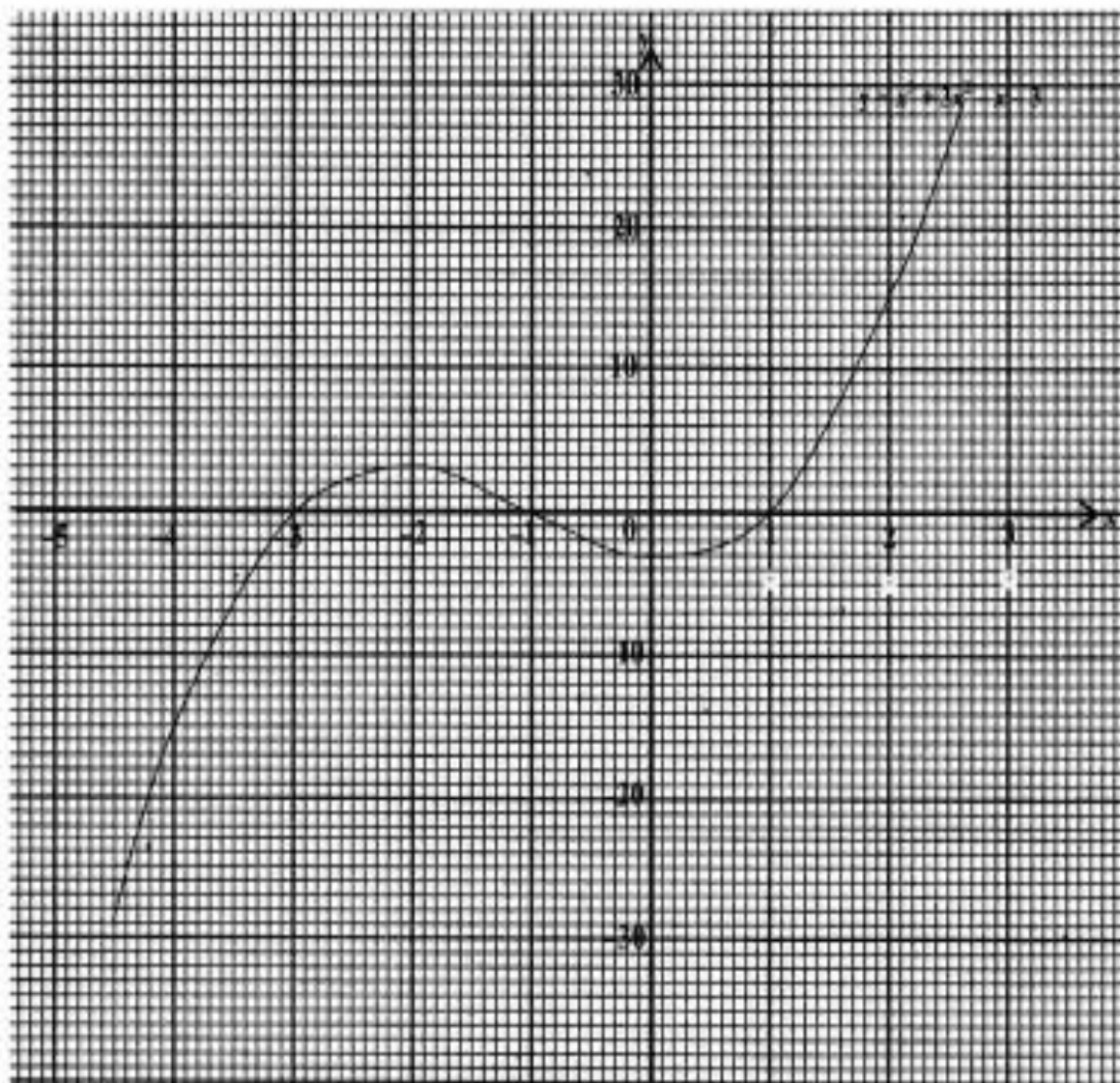


Use the graph

- a) To calculate an estimate of the gradient of the curve at the point (2, 5). [2]
- b) To solve the equations
 - I. $x^3 + x^2 - 5x + 3 = 0$. [2]
 - II. $x^3 + x^2 - 5x + 3 = 5x$. [3]
- c) To calculate an estimate of the area bounded by the curve $x = 0$, $y = 0$ and $x = -2$ [2]

6. Answer the whole of this question on a sheet of graph paper.

The diagram below shows the graph of $y = x^3 + 3x^2 - x - 3$.



Use the graph to find the solutions to the equations

I. $x^3 + 3x^2 - x - 3 = 0$, [2]

II. $x^3 + 3x^2 - x = 5$. [2]

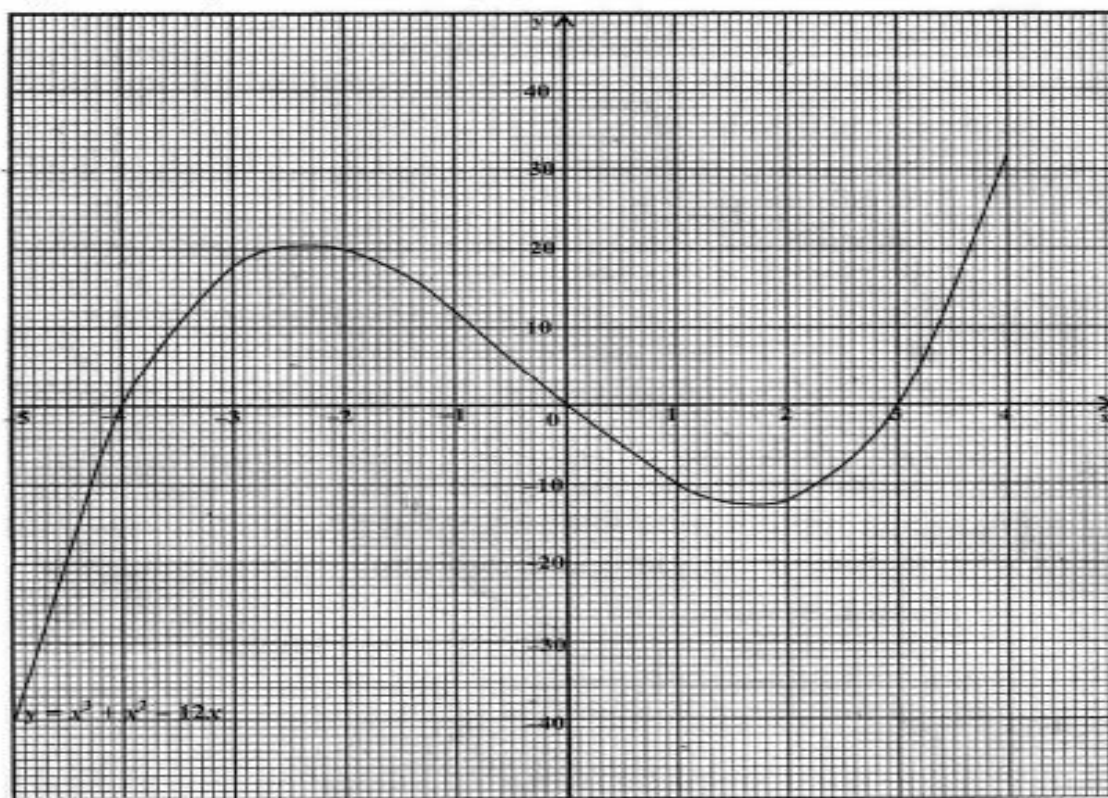
a) Calculate an estimate of

I. The gradient of the curve at the point $(-3, 0)$, [2]

II. The area bounded by the curve $x = 0$, $y = 0$ and $y = 20$. [3]

7. Answer the whole of this question on a sheet of graph paper.

The diagram below shows the graph of $y = x^3 + x^2 - 12x$.



a) Use the graph to solve the equation

I. $x^3 + x^2 - 12x = 0$, [2]

II. $x^3 + x^2 - 12x = x + 10$. [3]

b) Calculate an estimate of the

I. Gradient of the curve at the point where $x = -3$, [2]

II. Area bounded by the curve $x = -3$, $x = -1$ and $y = -10$. [2]

8. Answer the whole of this question on a sheet of graph paper.

The values of x and y are connected by the equation $y = 2x^3 - 3x^2 + 5$.

Some corresponding values of x and y are given in the table below.

X	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
Y	P	-8.5	0	4	5	4.5	4	5	9

a) Calculate the value of p , [1]

- b) Using a scale of 4cm to represent 1 unit on the x axes for $-2 \leq x \leq 2$ and a scale of 2cm to represent 5 units on the y axes for $-25 \leq y \leq 10$, draw the graph of $y = 2x^3 - 3x^2 + 5$. [3]
- c) Use your graph to solve the equation $2x^3 - 3x^2 + 5 = x$. [3]
- d) Calculate an estimate of the gradient of the curve at the point where $x = 1.5$. [2]

STATISTICS

1. Answer the whole of this question on a sheet of graph paper.

On a particular day, a tuck shop owner recorded the expenditure of 350 boys and the results were as shown in the table below.

Amount (K)	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$	$60 < x \leq 70$	$70 < x \leq 80$	$80 < x \leq 90$	$90 < x \leq 100$
No. of boys	20	50	55	70	60	45	35	10	5

- a) Calculate the amount of the money spent. [3]
- b) Copy and complete the cumulative frequency distribution. [1]

Amount (K)	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70	≤ 80	≤ 90	≤ 100
No. of boys	0	20	70	125	195	255	300			350

- c) Using a horizontal scale of 2cm to represent K10.00 and a vertical scale of 2cm to represent 50 boys, draw a smooth cumulative frequency curve. [3]
- d) Showing your method clearly, use your graph to estimate
- The median, [1]
 - The semi-interquartile range. [2]
- e) Given that those who spent K75.00 or more qualified for a draw in a competition to win a prize, find the number of boys who qualified for a draw. [2]

2. Answer the whole of this question on a sheet of graph paper.

A farmer had 260 potatoes in his sack. The masses, in grams, of the potatoes are given in the table below.

Mass (x grams)	$100 < x \leq 150$	$150 < x \leq 200$	$200 < x \leq 250$	$250 < x \leq 300$	$300 < x \leq 350$	$350 < x \leq 400$
No. of potatoes	15	45	75	80	40	5

- a) Calculate an estimate of the mean mass of the potatoes. [3]
 b) Copy and complete the cumulative frequency table below. [1]

Mass (x grams)	≤ 100	≤ 150	≤ 200	≤ 250	≤ 300	≤ 350	≤ 400
Cumulative frequency	0	15	60				260

- c) Using a horizontal scale of 2cm to represent 50 grams for masses between 0 and 400 grams and a vertical scale of 2cm to represent 25 potatoes, draw a smooth cumulative frequency curve. [3]
 d) Use your graph to find
 I. The median, [1]
 II. The interquartile range. [2]

3. Answer the whole of this question on a sheet of graph paper.

The ages of people living at Pamodzi Village are recorded in the frequency table below.

Ages	$0 < x \leq 10$	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$
Number of People	7	22	28	23	15	5

- a) Calculate the standard deviation. [6]

- b) Answer part of this question on a sheet of a graph paper

Age	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60
Number of people	7	29				100

- I. Using the table above, copy and complete the cumulative frequency table below. [1]
 II. Using the scale of 2cm to represent 10 units on both axes, draw a smooth cumulative frequency curve where $0 \leq x \leq 60$ and $0 \leq y \leq 100$. [3]
 III. Showing your method clearly, use your graph to estimate the Semi-interquartile range. [2]

4. Answer the whole of this question on a sheet of graph paper.

The frequency table below shows the number of copies of newspapers allocated to 48 newspaper vendors.

Copies of Newspaper	$25 < x \leq 30$	$30 < x \leq 35$	$35 < x \leq 40$	$40 < x \leq 45$	$45 < x \leq 50$	$50 < x \leq 55$	$55 < x \leq 60$
Number of vendors	6	4	7	11	12	8	1

(a) Calculate the standard deviation. [6]

(b) Answer this part of the question on a sheet of graph paper

I. Using the table above, copy and complete the cumulative frequency table below. [1]

Copies of newspaper	≤ 25	≤ 30	≤ 35	≤ 40	≤ 45	≤ 50	≤ 55	≤ 60
Number of Vendors	0	5	9	16	27			

II. Using a horizontal scale of 2cm to represent 10 newspapers on the x -axis for $0 \leq x \leq 60$ and a vertical scale of 4cm to represent 10 vendors on the y - axis for $0 \leq y \leq 50$, draw a smooth cumulative frequency curve. [3]

III. Showing your method clearly, use your graph to estimate the 50th Percentile. [2]

5. Answer the whole of this question on a sheet of graph paper.

The table below shows the amount of money spent by 100 learners at school on a particular day.

Amount in Kwacha	$0 < x \leq 5$	$5 < x \leq 10$	$10 < x \leq 15$	$15 < x \leq 20$	$20 < x \leq 25$	$25 \leq x \leq 30$
Frequency	13	27	35	16	7	2

a) Calculate the standard deviation. [6]

b) Answer this part of the question on a sheet of graph paper.

I. Using the table above, copy and complete the cumulative frequency table below. [1]

Amount in Kwacha	≤ 0	≤ 5	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30
Cumulative frequency	0	13	40				100

- II. Using a scale of 2cm to represent 5 units on the horizontal axis and 2cm to represent 10 units on the vertical axis, draw a smooth cumulative frequency curve. [3]
- III. Showing your method clearly, use your graph to estimate the Semi-interquartile range. [2]

6. Answer the whole of this question on a sheet of graph paper.

The frequency table below shows the distribution of marks obtained by 90 learners on a test.

Marks(x)	$10 < x \leq 20$	$20 < x \leq 30$	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$	$60 < x \leq 70$
frequency	2	10	15	23	30	10

- a) Calculate the standard deviation. [6]

b) Answer this part of this question on the sheet of graph paper

- I. Copy and complete the cumulative frequency table. [1]

Marks(x)	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70
Cumulative frequency	0	2	12	27	50	80	90
Relative Cumulative frequency	0	0.02	0.13	0.3			

- II. Using a scale of 2cm to represent 10 units on the x- axis for $0 \leq x \leq 70$ and a scale of 2cm to represent 0.1 units on the y – axis for $0 \leq y \leq 1$, draw a smooth relative cumulative frequency curve. [3]
- III. Showing your method clearly, use your graph to estimate the 65th Percentile. [2]

7. Answer the whole of this question on a sheet of graph paper.

A farmer planted 60 fruit trees. In a certain month, the number of fruits per tree was recorded and the results were as shown in the table below.

Fruits per tree	2	3	4	5	6	7	8
frequency	1	5	4	6	10	16	18

- a. Calculate the standard deviation. [6]

b. Answer this part of the question on the sheet of graph paper.

- i. Using the table above, copy and complete the relative cumulative frequency table below. [1]

Fruits per tree	2	3	4	5	6	7	8
Cumulative frequency	1	6	10	16	26	42	60
Relative cumulative frequency	0.02	0.1	0.17	0.27			

- ii. Using a scale of 1cm to represent 1 unit on the x-axis for $0 \leq x \leq 8$ and a scale of 2cm to represent 0.1 unit on the y – axis for $0 \leq y \leq 1$, draw a smooth relative frequency curve. [3]
- iii. Showing your method clearly, use your graph to estimate the 70th Percentile. [2]

PSEUDO CODES AND FLOW CHARTS

I. The program below is given in the form of a pseudo code.

Start

Enter radius

If radius < 0

Then display “error message” and re-enter positive radius

Else enter height

If height < 0

Then display “error message” and re-enter positive height

Else Volume = $\frac{1}{3} * \pi * \text{square radius} * \text{height}$

End if

Display volume

Stop

Draw the corresponding flowchart for the information given above. [5]

2. Study the pseudo code below.

Start

Enter a, r, n

$R = 1 - r$

If $R = 0$ THEN

Print “the value of r is not valid”

Else $S_n = \frac{a(1-r^n)}{R}$

End if

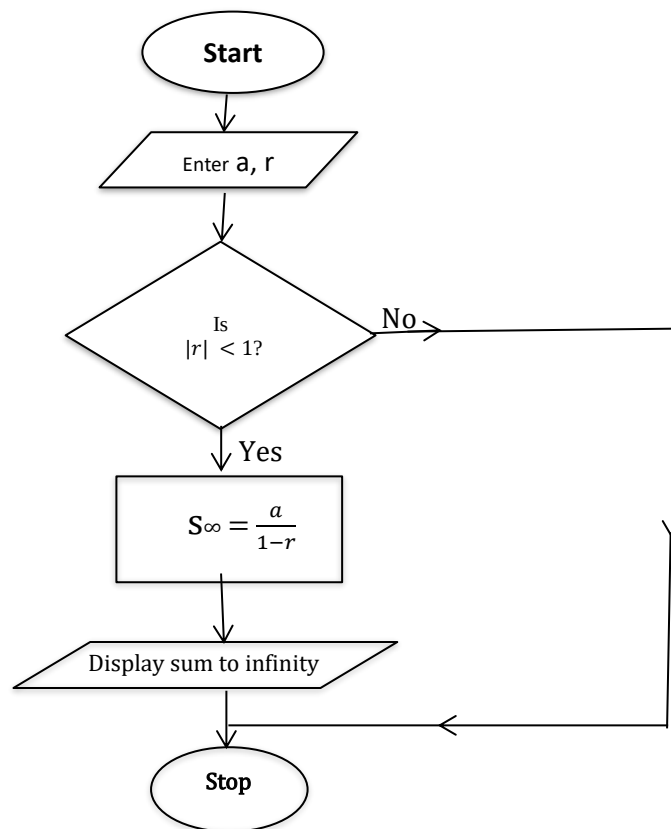
Print S_n

Stop

Construct a flow chart corresponding to the Pseudo code above.

[5]

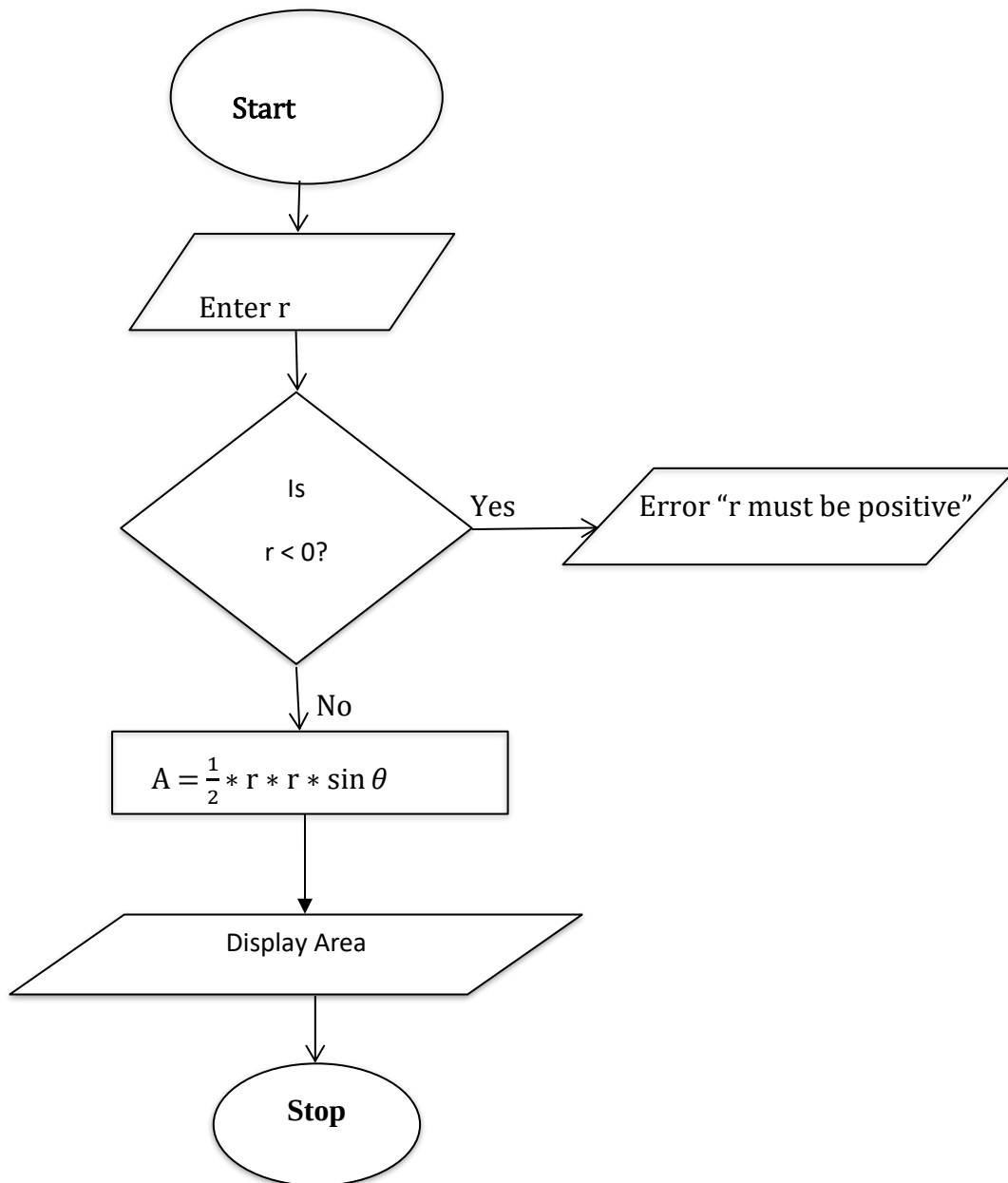
3. The diagram below is given in the form of a flow chart.



Write the pseudo code corresponding to the flow chart program above.

[5]

4. Study the flow chart below.



Write a pseudo code corresponding to the flow chart above.

[5]

5. The program below is given in a form of a pseudo code

Start

Enter x, y

Let $M = \text{Square root}(x^2 + y^2)$

If $M < 0$

Then display error message "M must be positive"

ELSE

END IF

Display M

Stop

Draw a corresponding flow chart for the information given above.

[5]

TRANSFORMATION

1. Answer this part of the question on the sheet of graph paper.

Using a scale of 1cm to represent 1 unit, on both axes, draw x and y axes for $-8 \leq x \leq 12$ and $-6 \leq y \leq 14$.

- a) Draw and label triangle X with vertices (2, 4), (4, 4) and (4, 1). [1]
- b) Triangle X is mapped onto triangle U with vertices (6, 12), (12, 12) and (12, 3) by a single transformation.
 - I. Draw and label triangle U. [1]
 - II. Describe fully this transformation. [3]
- c) A 90° clockwise rotation about the origin maps triangle X onto triangle W. Draw and label triangle W. [2]
- d) A shear with X-axis as the invariant line and shear factor -2 maps triangle X onto triangle S. Draw and label triangle S. [2]
- e) Triangle X is mapped onto triangle M with vertices (4, 4), (8, 4) and (8, 1).
 - I. Draw and label triangle M. [1]
 - II. Find the matrix which represents this transformation. [2]

2. Answer this part of the question on the sheet of graph paper.

Using a scale of 1cm to represent 1 unit on each axis, draw x and y axes for $-6 \leq x \leq 10$ and $-10 \leq y \leq 8$.

- a) A quadrilateral ABCD has vertices A(-5, 7), B(-4, 8), C(-3, 7) and D(-4, 4) while its image has vertices A₁(-5, -3), B₁(-6, -2), C₁(-5, -1) and D₁(-2, -2).

I. Draw and label the quadrilateral ABCD and its image A₁B₁C₁D₁. [2]

II. Describe fully a single transformation which maps the quadrilateral ABCD onto quadrilateral A₁B₁C₁D₁. [3]

- b) The matrix $\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$, maps quadrilateral ABCD onto quadrilateral A₂B₂C₂D₂

I. Find the coordinates of quadrilateral A₂B₂C₂D₂. [3]

II. Draw and label quadrilateral A₂B₂C₂D₂. [1]

- c) The quadrilateral ABCD is mapped onto quadrilateral A₃B₃C₃D₃ where A₃ is (4, -8), B₃ is (2, -10), C₃ is (0, -8) and D₃ is (2, -2).

Describe fully this transformation. [3]

3. Answer this part of the question on the sheet of graph paper.

Using a scale of 1cm to represent 1 unit on each axis, draw x and y axes for $-6 \leq x \leq 10$ and $-6 \leq y \leq 12$.

- a) A quadrilateral ABCD has vertices A(1, 2), B(2, 1), C(3, 2) and D(2, 3). Quadrilateral A₁B₁C₁D₁ has vertices A₁(3, 2), B₁(6, 1), C₁(9, 2) and D₁(6, 3).

i. Draw and label the quadrilateral ABCD and its image A₁B₁C₁D₁. [2]

ii. Describe fully a single transformation which maps the quadrilateral ABCD onto quadrilateral A₁B₁C₁D₁. [3]

- b) The matrix $\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$, maps quadrilateral ABCD onto quadrilateral A₂B₂C₂D₂

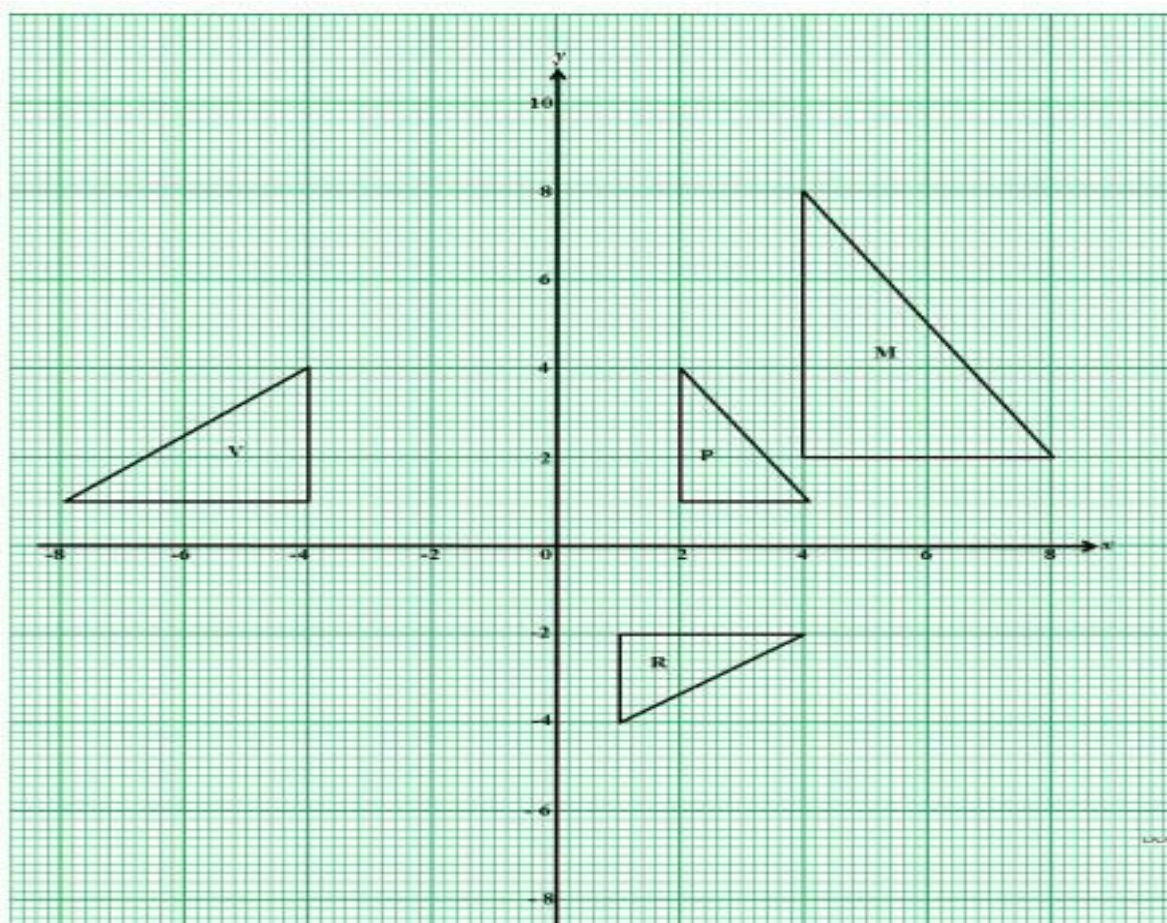
i. Find the coordinates of quadrilateral A₂B₂C₂D₂. [3]

ii. Draw and label quadrilateral A₂B₂C₂D₂. [1]

- c) The quadrilateral ABCD is mapped onto quadrilateral A₃B₃C₃D₃ where A₃ is (-2, -4), B₃ is (-4, -2), C₃ is (-6, -4) and D₃ is (-4, -6). Describe fully the transformation which maps quadrilateral ABCD onto quadrilateral A₃B₃C₃D₃. [3]

4. Answer this part of the question on the sheet of graph paper.

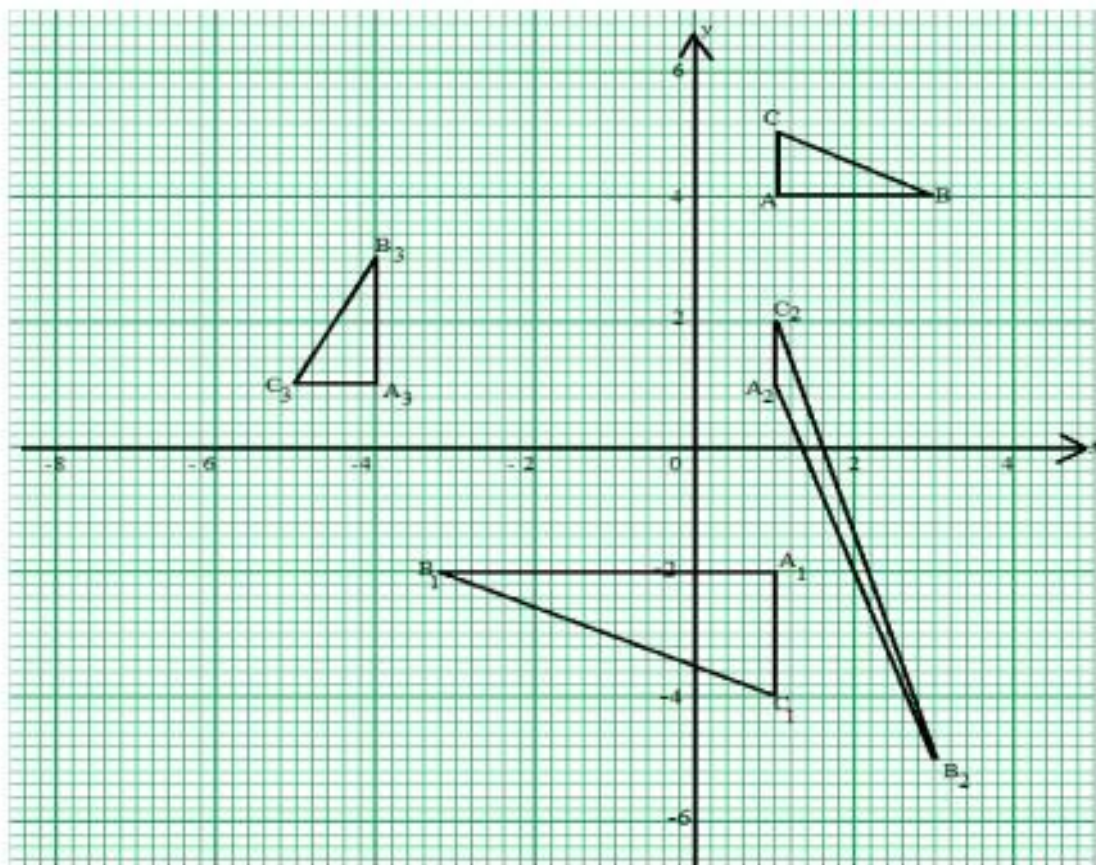
Study the diagram below and answer the questions that follow.



- Triangle R is the image of triangle P under a rotation. Find the coordinates of the Centre, angle and the direction of the rotation. [3]
- A single transformation maps triangle P onto triangle M. describe fully this transformation. [3]
- Triangle P maps onto triangle V by a stretch.
Find the matrix of this transformation. [3]
- If triangle P is mapped onto triangle S by a shear represented by the matrix $\begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$, find the coordinates of S. [3]

5. Answer this part of the question on the sheet of graph paper.

Study the diagram below and answer questions that follow.



- a) An enlargement maps triangle ABC onto triangle $A_1B_1C_1$. Find
 - I. the Centre of enlargement, [1]
 - II. the scale factor. [2]
- b) Triangle ABC is mapped onto triangle $A_2B_2C_2$ by a shear. Find the matrix which presents this transformation. [2]
- c) Triangle ABC is mapped onto triangle $A_3B_3C_3$ by a single transformation. Describe this transformation fully. [3]
- d) A transformation with matrix $\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}$ maps triangle ABC onto triangle $A_4B_4C_4$ not drawn on the diagram. Find
 - I. The scale factor of this transformation, [1]
 - II. The coordinates of A_4 , B_4 and C_4 . [3]

SOLUTIONS

SETS

Question one

a) $2y + 1 = 7$

$$2y = 7 - 1$$

$$2y = 6$$

$$\frac{2y}{2} = \frac{6}{2}$$

$$\therefore y = 3$$

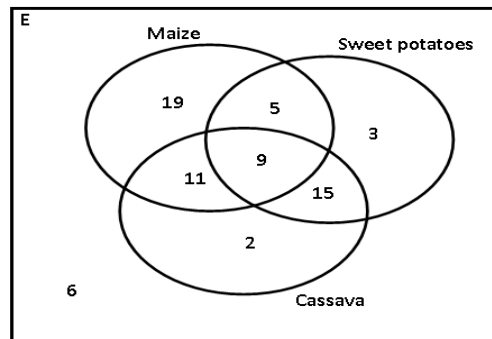
b) I) Victoria falls but not Gonya falls = $6 + 2 = 8$

II) Two tourist attraction only = $4 + 1 + 2 = 7$

III) One tourist attraction only = $8 + 6 + 7 = 21$

Question two

I)



II) a). Total farmers = $6 + 19 + 5 + 11 + 9 + 2 + 15 + 3 = 70$

b). Farmers that planted maize only = **19**

c). Farmers that planted two different crops = $5 + 11 + 15 = 31$

Question three

I).

a) Body weakness only = **18**

b). Body weakness and fever only = **13**

c). Headache = $14 + 6 = \mathbf{20}$

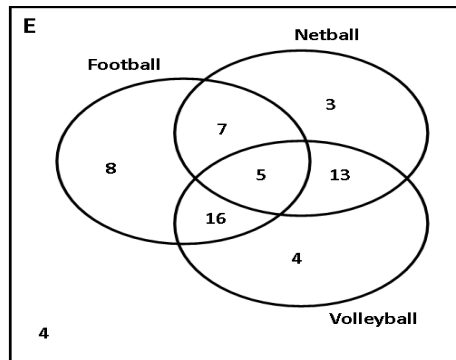
II). Total patients attended to = $18 + 14 + 13 + 6 + 5 = \mathbf{56}$

Complained of all the three ailments = **14**

$$\text{Percentage} = \frac{14}{56} \times 100\% = 0.25 \times 100\% = \mathbf{25\%}$$

Question four

a).



b) Learners that that did not like any of the three games = **4**

c) Learners that did not like volleyball = learners in grade twelve class – learners that liked volleyball = $60 - 38 = \mathbf{22}$

d) Learners that liked two games only = $16 + 13 + 7 = \mathbf{36}$

Question five

a) $15 + 15 + 15 + 10 + 10 + 5 + x = 100$

$$70 + x = 100$$

$$x = 100 - 70$$

$$\therefore x = \mathbf{30}$$

b) Number of students who took human resource = $15 + 10 + 10 + 5 = 40$

c) $n(B \cap C) \cap H' = 15$

d) $n(B \cup C) \cap H' = 15 + 15 + 30 = 60$

Question six

I) $4 + 7 + x + 3 = 22$

$$14 + x = 22$$

$$x = 22 - 14$$

$$\therefore x = 8$$

II) a). Learners used only one mode of transport = $14 + 7 + 7 = 28$

b). Learners that used two different modes of transport = $4 + 3 + 2 + 8 = 17$

c). Learners all together = $14 + 2 + 7 + 8 + 7 + 4 + 3 = 45$

Question seven

I) $60 + 22 + 23 + 16 + 51 + x + 13 + 20 = 250$

$$205 + x = 250$$

$$x = 250 - 205$$

$$\therefore x = 45$$

II) Pupils that bought mangoes and lemons but not oranges = 13

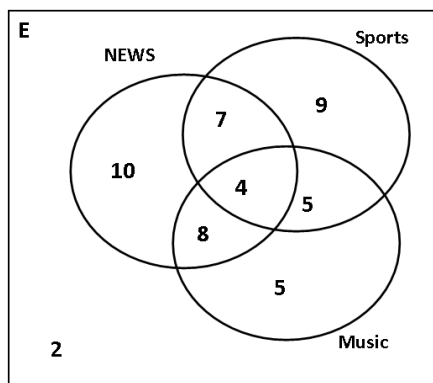


HINT: **And** is used to mean the intersection, **OR** is used to mean the union and **not** is used to mean complement. The question is simply to find the intersection of the pupils that bought Mangos and Lemons but not oranges. Which is no other than 13.

III) Pupils that bought one fruit only = $22 + 20 + 16 = 58$

Question eight

I)



II) a). Villagers who listen to music only = 5

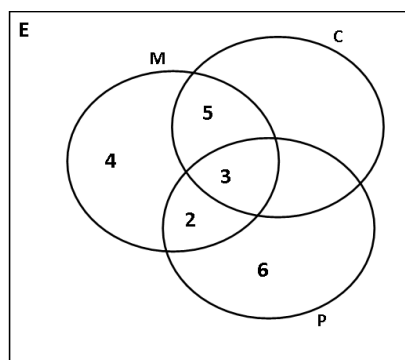
b). Villagers who listen to one program only = $5 + 9 + 10 = 24$

c). Villagers who listen to two programs only = $5 + 7 + 8 = 20$

Question nine

I)

Mathematics (M), Chemistry (C) and Physics (P).



II) a). Physics only = 6

b). Two types of subjects = $2 + 5 = 7$

c). Mathematics and Physics but not Chemistry = 2

Question ten

I) Total women that used Cell Z sim card = $a + 4 + 3 + 2 = 23$

$$a + 9 = 23$$

$$a = 23 - 9$$

$$\therefore a = 14$$

And total women = $4 + 2 + 3 + 14 + 14 + 10 + b + 8 = 60$

$$55 + b = 60$$

$$b = 60 - 55$$

$$\therefore b = 5$$

II) Women that have used only two different sim cards = $3 + 4 + 5 = 12$

III) Women that did not use MTN and Celtel sim cards = $8 + 14 = 22$

IV) Women that used either Celtel or MTN sim card = $14 + 10 + 5 = 29$

Question eleven

I) $a + 10 = 15 + 30 = 70$

$$a + 55 = 70$$

$$a = 70 - 55$$

$$\therefore a = 15$$

And $15 + 10 + 40 + 15 + 30 + 35 + b = 150$

$$b + 145 = 150$$

$$b = 150 - 145$$

$$\therefore b = 5$$

II) Candidates applied for two different subjects only = $30 + 10 + 5 = 45$

III) Candidates that did not apply for Science or Mathematics = **35**

- IV) Candidates applied either for Science or Home Economics but not Mathematics
 $= 15 + 30 + 35 = 80$

Question twelve

- I) Total number of students $= 10 + 3 + 2 + x = 25$

$$x + 15 = 25$$

$$x = 25 - 15$$

$$\therefore x = 10$$

- II). a). $n(G \cap R) = 3$

$$b). n(D \cup G) = 2 + 10 = 12$$

ALGEBRA

$$\begin{aligned} 1. \quad \frac{12dn^3}{15cd^3} \div \frac{9c^3n}{10c^2d^2} \\ &= \frac{12dn^3}{15cd^3} \times \frac{10c^2d^2}{9c^3n} \\ &= \frac{8d^3c^2n^3}{9d^3c^4n} \\ &= \frac{8n^2}{9c^2} \end{aligned}$$

$$\therefore \frac{12dn^3}{15cd^3} \div \frac{9c^3n}{10c^2d^2} = \frac{8n^2}{9c^2}$$

$$\begin{aligned} 2. \quad \frac{14x^3}{9y^2} \div \frac{7x^4}{18y^3} \\ &= \frac{14x^3}{9y^2} \times \frac{18y^3}{7x^4} \\ &= \frac{4y}{x} \end{aligned}$$

$$\therefore \frac{14x^3}{9y^2} \div \frac{7x^4}{18y^3} = \frac{4y}{x}$$

$$\begin{aligned} 3. \quad \frac{3}{5x-2} - \frac{2}{x+2} \\ &= \frac{3(x+2) - 2(5x-2)}{(5x-2)(x+2)} \\ &= \frac{3x+6-10x+4}{(5x-2)(x+2)} \\ &= \frac{-7x+10}{(5x-2)(x+2)} \end{aligned}$$

$$\therefore \frac{3}{5x-2} - \frac{2}{x+2} = \frac{10-7x}{(5x-2)(x+2)}$$

$$\begin{aligned} 4. \quad \frac{17k^2}{20a^2} \div \frac{51k^2}{5a} \\ &= \frac{17k^2}{20a^2} \times \frac{5a}{51k^2} \\ &= \frac{1}{12a} \end{aligned}$$

$$\therefore \frac{17k^2}{20a^2} \div \frac{51k^2}{5a} = \frac{1}{12a}$$

$$\begin{aligned}
 5. \quad & \frac{3}{x+1} - \frac{4}{x-1} \\
 &= \frac{3(x-1) - 4(x+1)}{(x+1)(x-1)} \\
 &= \frac{3x - 3 - 4x - 4}{(x+1)(x-1)} \\
 &= \frac{-x - 7}{(x+1)(x-1)}
 \end{aligned}$$

$$\therefore \frac{3}{x+1} - \frac{4}{x-1} = \frac{-1(x+7)}{(x+1)(x-1)}$$

$$\begin{aligned}
 7. \quad & \frac{7st^3}{15u^3v^2} \times \frac{5u^3}{28s^3t^2} \\
 &= \frac{st^3u^3}{12u^3v^2s^3t^2} \\
 &= \frac{t}{12v^2s^2} \\
 \therefore \frac{7st^3}{15u^3v^2} \times \frac{5u^3}{28s^3t^2} &= \frac{t}{12v^2s^2}
 \end{aligned}$$

$$\begin{aligned}
 9. \quad & \frac{3}{2x-5} - \frac{4}{x-3} \\
 &= \frac{3(x-3) - 4(2x-5)}{(2x-5)(x-3)} \\
 &= \frac{3x - 9 - 8x + 20}{(2x-5)(x-3)} \\
 &= \frac{-5x + 11}{(2x-5)(x-3)} \\
 \therefore \frac{3}{2x-5} - \frac{4}{x-3} &= \frac{11-5x}{(2x-5)(x-3)}
 \end{aligned}$$

$$\begin{aligned}
 6. \quad & \frac{2x^2-8}{x+2} \\
 &= \frac{2(x^2-4)}{x+2} \\
 &= \frac{2[(x+2)(x-2)]}{x+2}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \frac{2x^2-8}{x+2} &= 2(x-2) \\
 &= 2x - 4
 \end{aligned}$$

$$\begin{aligned}
 8. \quad & \frac{b-a}{a^2-b^2} \\
 &= \frac{b-a}{(a+b)(a-b)} \\
 &= \frac{-1(a-b)}{(a+b)(a-b)} \\
 &= \frac{-1}{a+b} \\
 \therefore \frac{b-a}{a^2-b^2} &= \frac{-1}{a+b}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad & \frac{p^2q^2}{4} \times \frac{8}{pq} \div 2p^2q \\
 &= \frac{p^2q^2}{4} \times \frac{8}{pq} \times \frac{1}{2p^2q} \\
 &= \frac{8p^2q^2}{8p^3q^2} \\
 &= \frac{1}{p}
 \end{aligned}$$

$$\therefore \frac{p^2q^2}{4} \times \frac{8}{pq} \div 2p^2q = \frac{1}{p}$$

HINT: BODMAS

$$\begin{aligned}
 11. \quad \frac{2y^2-3y-5}{y^2-1} &= \frac{2y^2+2y-5y-5}{y^2-1^2} \\
 &= \frac{2y(y+1)-5(y+1)}{(y+1)(y-1)} \\
 &= \frac{(2y-5)(y+1)}{(y-1)(y+1)} \\
 &= \frac{2y-5}{y-1} \\
 \therefore \frac{2y^2-3y-5}{y^2-1} &= \frac{2y-5}{y-1}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \frac{x-1}{x^2-1} &= \frac{x-1}{x^2-1^2} \\
 &= \frac{x-1}{(x-1)(x+1)} \\
 &= \frac{1}{x+1} \\
 \therefore \frac{x-1}{x^2-1} &= \frac{1}{x+1}
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \frac{2}{2x-1} - \frac{1}{3x+1} &= \frac{2(3x+1)-1(2x-1)}{(2x-1)(3x+1)} \\
 &= \frac{6x+2-2x+1}{(2x-1)(3x+1)} \\
 &= \frac{4x+3}{(2x-1)(3x+1)} \\
 \therefore \frac{2}{2x-1} - \frac{1}{3x+1} &= \frac{4x+3}{(2x-1)(3x+1)}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \frac{4}{3r-2} - \frac{3}{r+5} &= \frac{4(r+5)-3(3r-2)}{(3r-2)(r+5)} \\
 &= \frac{4r+20-9r+6}{(3r-2)(r+5)} \\
 &= \frac{-5r+26}{(3r-2)(r+5)} \\
 &= \frac{26-5r}{(3r-2)(r+5)} \\
 \therefore \frac{4}{3r-2} - \frac{3}{r+5} &= \frac{26-5r}{(3r-2)(r+5)}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \frac{3a}{5c^2} \times \frac{10c^3}{a^2} &= \frac{30ac^3}{5c^2a^2} \\
 &= \frac{6c}{a} \\
 \therefore \frac{3a}{5c^2} \times \frac{10c^3}{a^2} &= \frac{6c}{a}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad 4x^2 - 16y^2 &= 4(x^2 - 4y^2) \\
 &= 4(x^2 - 2^2y^2) \\
 &= 4[(x+2y)(x-2y)] \\
 \therefore 4x^2 - 16y^2 &= 4[(x+2y)(x-2y)]
 \end{aligned}$$

$$17. 2\frac{1}{2} + \frac{3}{4} \div \frac{1}{18}$$

$$= \frac{5}{2} + \left[\frac{3}{4} \times \frac{18}{1} \right]$$

$$= \frac{5}{2} + \frac{27}{2}$$

$$= \frac{5 + 27}{2}$$

$$= \frac{32}{2}$$

$$= 16$$

$$\therefore 2\frac{1}{2} + \frac{3}{4} \div \frac{1}{18} = \mathbf{16}$$

$$19. \frac{m^2 - 1}{m^2 - m}$$

$$= \frac{m^2 - 1^2}{m(m - 1)}$$

$$= \frac{(m - 1)(m + 1)}{m(m - 1)}$$

$$= \frac{m + 1}{m}$$

$$\therefore \frac{m^2 - 1}{m^2 - m} = \frac{\mathbf{m + 1}}{\mathbf{m}}$$

$$18. \frac{1}{x-4} - \frac{2}{5x-1}$$

$$= \frac{1(5x - 1) - 2(x - 4)}{(x - 4)(5x - 1)}$$

$$= \frac{5x - 1 - 2x + 8}{(x - 4)(5x - 1)}$$

$$= \frac{3x + 7}{(x - 4)(5x - 1)}$$

$$\therefore \frac{1}{x-4} - \frac{2}{5x-1} = \frac{\mathbf{3x + 7}}{(x-4)(5x-1)}$$

$$20. 1.3^2 + 1.3 \times 0.3$$

$$= 1.69 + (1.3 \times 0.3)$$

$$= 1.69 + 0.39$$

$$= 2.08$$

$$\therefore 1.3^2 + 1.3 \times 0.3 = \mathbf{2.08}$$



HINT: When multiplying decimal numbers, you first ignore the decimal points, then multiply. After multiplication, you count the number of decimals in each number and sum them up. Lastly, write your product to the sum decimals. For example $1.3 \times 1.4 \rightarrow [13 \times 14] = 182$

now 1.3 and 1.4 are correct to one decimal point $1 + 1 = 2$

finally, write 182 correct to 2 decimal points

$$\therefore 1.3 \times 1.4 = \mathbf{1.82}$$

$$21. \frac{a-2}{a^2-4}$$

$$= \frac{a-2}{a^2-2^2}$$

$$= \frac{a-2}{(a-2)(a+2)}$$

$$= \frac{1}{a+2}$$

$$\therefore \frac{a-2}{a^2-4} = \frac{1}{a+2}$$

$$23. x = 8 \text{ and } y = -2$$

$$x - y^2$$

$$= 8 - (-2)^2$$

$$= 8 - 4$$

$$= 4$$

$$25. \frac{x+5}{3} - \frac{x+2}{4}$$

$$= \frac{4(x+5) - 3(x+2)}{12}$$

$$= \frac{4x + 20 - 3x - 6}{12}$$

$$= \frac{x + 14}{12}$$

$$\therefore \frac{x+5}{3} - \frac{x+2}{4} = \frac{x+14}{12}$$

$$22. 5px - 5py + 3qx - 3qy$$

$$= 5p(x - y) + 3q(x - y)$$

$$= (5p + 3q)(x - y)$$

$$\therefore 5px - 5py + 3qx - 3qy = (5p + 3q)(x - y)$$

$$24. 3x - 12x^3$$

$$= 3x(1 - 4x^2)$$

$$= 3x[1^2 - (2x)^2]$$

$$= 3x(1 - 2x)(1 - 2x)$$

$$\therefore 3x - 12x^3 = 3x(1 - 2x)(1 - 2x)$$

$$26. \frac{4u^2}{a^2} - \frac{1}{9}$$

$$= \frac{9 \times 4u^2 - 1 \times a^2}{9a^2}$$

$$= \frac{36u^2 - a^2}{9a^2}$$

$$= \frac{(6u)^2 - a^2}{(3a)^2}$$

$$= \frac{(6u - a)(6u + a)}{(3a)^2}$$

$$= \left(\frac{6u - a}{3a}\right) \left(\frac{6u + a}{3a}\right)$$

$$\therefore \frac{4u^2}{a^2} - \frac{1}{9} = \left(\frac{6u - a}{3a}\right) \left(\frac{6u + a}{3a}\right)$$

27. $\omega = \frac{u+v}{uv}, u = 3 \text{ and } v = -4$

$$\begin{aligned} \text{a). } w &= \frac{3+(-4)}{3(-4)} \\ &= \frac{3-4}{-12} \\ &= \frac{-1}{-12} \\ &= \frac{1}{12} \\ \therefore w &= \frac{1}{12} \end{aligned}$$

b). $\omega = \frac{u+v}{uv}$

$$\frac{w}{1} = \frac{u+v}{uv}$$

$$wuv = u + v$$

$$wuv - u = v$$

$$u(wv - 1) = v$$

$$\frac{u(wv - 1)}{wv - 1} = \frac{v}{wv - 1}$$

$$= \frac{v}{wv - 1}$$

$$\therefore u = \frac{v}{wv - 1}$$

PROBABILITY

1. Total number of balls = 12

Total even numbers = 6 (2, 4, 6, 8, 10, 12)

Total numbers less than 4 = 3 (1, 2, 3)

Total prime numbers = 4 (3, 5, 7, 11)

a) $P(\text{even number}) = \frac{6}{12} = \mathbf{0.5}$

b) $P(\text{less than 4}) = \frac{3}{12} = \mathbf{0.25}$

c) $P(\text{prime number}) = \frac{4}{12} = \mathbf{0.33}$

2. a). $P(\text{black}) + P(\text{blue}) + P(\text{green}) + P(\text{red}) = 1$

$$0.2 + 0.4 + 0.25 + P(\text{blue}) = 1$$

$$P(\text{blue}) + 0.85 = 1$$

$$\therefore P(\text{blue}) = 1 - 0.85 = \mathbf{0.15}$$

b). $P(\text{black}) = 0.2 = \frac{2}{10} = \frac{1}{5}$, $P(\text{blue}) = 0.15 = \frac{15}{100} = \frac{3}{20}$, $P(\text{red}) = 0.25 = \frac{25}{100} = \frac{1}{4}$ and
 $P(\text{green}) = 0.4 = \frac{4}{10} = \frac{2}{5}$.

Hint: we need to find the smallest possible value which when multiplied by each of the probabilities, the answer must be a whole number. That can be done by re-writing the probabilities in fraction in their simplest form... The **LOWEST COMMON MULTIPLE** of the denominators is the smallest possible number to suit the probabilities.

From the above fractions we can see clearly that 20 is the lowest common multiple of the denominators, thus, the least possible number of marbles in the box is **20**.

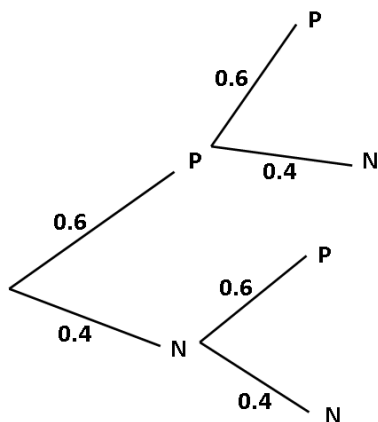
3. We can easily find the probabilities using a tree diagram.

$$P(\text{positive malaria}) = 0.6$$

$$P(\text{negative malaria}) = 1 - P(\text{positive malaria}) = 1 - 0.6 = 0.4$$

Tree diagram

Let P = P(positive malaria) and N = P(negative malaria)



$$P(PP) = 0.6 \times 0.6 = \mathbf{0.36}$$

$$P(PN) = 0.6 \times 0.4 = \mathbf{0.24}$$

$$P(NP) = 0.4 \times 0.6 = \mathbf{0.24}$$

$$P(NN) = 0.4 \times 0.4 = \mathbf{0.16}$$

a) $P(PN) + P(NP) = 0.24 + 0.24 = 0.48$

$\therefore$ Probability of one patient tested positive while the other negative is **0.48**

b) $P(NN) = 0.16$

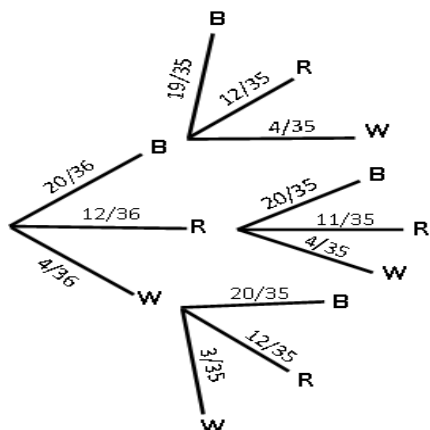
$\therefore$ Probability of both patients tested negative is **0.16**

4. Total number of buttons = $20 + 12 + 4 = 36$

$$P(\text{black}) = \frac{20}{36} = \frac{5}{9}, \quad P(\text{red}) = \frac{12}{36} = \frac{1}{3} \quad \text{and} \quad P(\text{white}) = \frac{4}{36} = \frac{1}{9}$$

Let B = Black, R = Red and W = White

a) TREE DIAGRAM



$$\begin{aligned} \text{b), } P(WW) &= \frac{4}{36} \times \frac{3}{35} \\ &= \frac{12}{1260} \\ &= \frac{1}{105} \end{aligned}$$

$\therefore$ Probability that both buttons are white is $\frac{1}{105}$

5. I) a). $P(\text{Fanta}) + P(\text{Coca-Cola}) + P(\text{Sprite}) = 1$

$$0.25 + 0.4 + P(\text{Coca-Cola}) = 1$$

$$0.65 + P(\text{Coca-Cola}) = 1$$

$$P(\text{Coca-Cola}) = 1 - 0.65 = 0.35$$

$\therefore$ Probability of Coca-Cola is **0.35**

b). $P(\text{not Sprite}) = 1 - P(\text{Sprite})$

$$= 1 - 0.4 = 0.6 \quad \text{Or}$$

$$P(\text{Not Sprite}) = P(\text{Fanta}) + P(\text{Coca-Cola}) = 0.25 + 0.35 = 0.60$$

$\therefore$ Probability that it is not a sprite bottle top is **0.6**

II) $P(\text{Sprite}) = 0.4 = \frac{4}{10} = \frac{2}{5}$



Let the total number of bottle tops be x , then

$$\frac{2}{5}x = 16$$

$$2x = 80$$

$$x = 40$$

$\therefore$ Total number of bottle tops = **40**

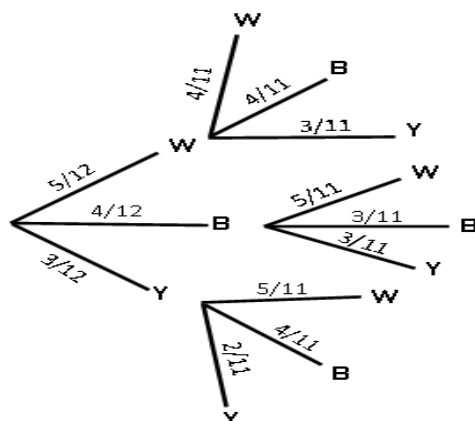
6. Total number of pieces of chalk = $5 + 4 + 3 = 12$

$$P(\text{White}) = \frac{5}{12}, P(\text{Blue}) = \frac{4}{12} \text{ and } P(\text{Yellow}) = \frac{3}{12}$$

Let W = White, B = P(Blue) and Y = P(Yellow)

HINT: use ratios to find the total number of bottle tops

a) **Tree diagram**



$$\begin{aligned} \text{b) } P(WW) + P(BB) + P(Y Y) &= \frac{5}{12} \times \frac{4}{11} + \frac{4}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{2}{11} = \frac{20}{132} + \frac{12}{132} + \frac{6}{132} = \\ &= \frac{20+12+6}{132} = \frac{38}{132} = \frac{19}{66} \end{aligned}$$

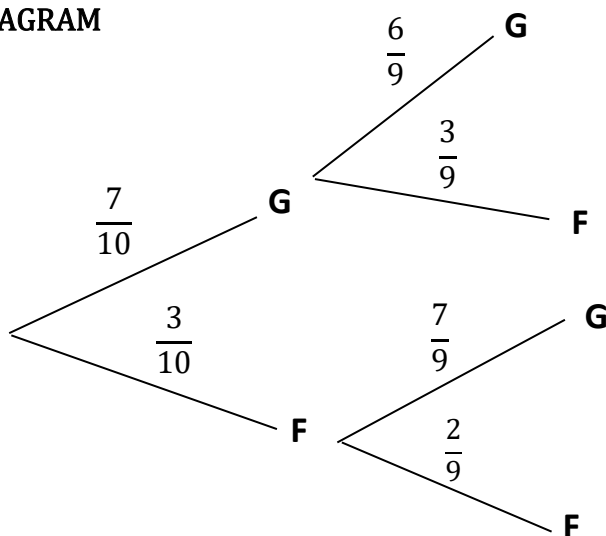
$\therefore$ Probability of selecting pieces of chalk of the same color is $\frac{19}{66}$.

7. Total number of bulbs = 10, fault bulbs = 3 and Good bulbs = 10 - 3 = 7

$$P(\text{Fault bulb}) = \frac{3}{10} \text{ and } P(\text{Good bulb}) = \frac{7}{10}$$

Let G = good bulb and F = fault bulb

TREE DIAGRAM



$$\text{a) } P(\text{both being good}) = P(G) \times P(G) = \frac{7}{10} \times \frac{6}{9} = \frac{42}{90} = \frac{7}{15}$$

∴ Probability of both being good is $\frac{7}{15}$

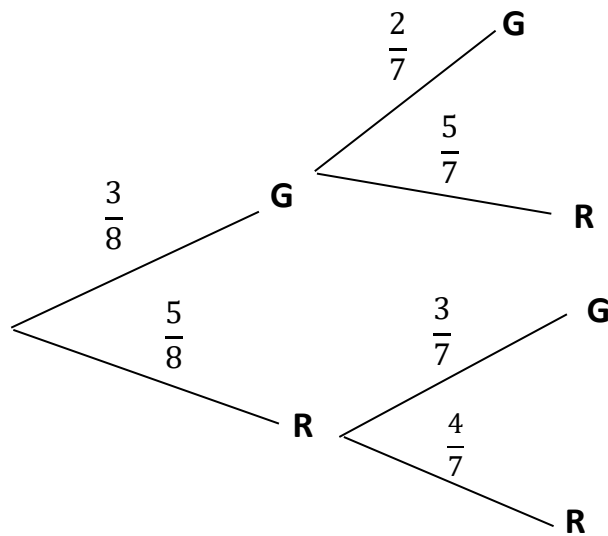
$$b) P(\text{one fault and one good}) = P(GF) + P(FG) = \frac{7}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{7}{9} = \frac{21}{90} + \frac{21}{90} = \frac{42}{90} = \frac{7}{15}$$

∴ Probability of one good and one fault is $\frac{7}{15}$

8. Total apples = 3 + 5 = 8

$P(G) = \frac{3}{8}$ and $P(R) = \frac{5}{8}$ where G = green apples and R = red apples

TREE DIAGRAM



$$a) P(\text{Both apples picked are green}) = P(G) \times P(G) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$$

∴ Probability that both apples picked are green is $\frac{3}{28}$

$$c) P(\text{Apples picked are of different colors}) = P(GR) + P(RG) \\ = \frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7} = \frac{15}{56} + \frac{15}{56} = \frac{15+15}{56} = \frac{30}{56} = \frac{15}{28}$$

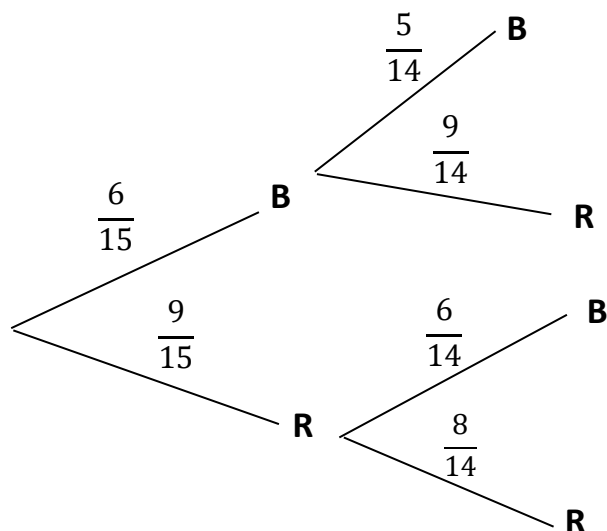
∴ Probability that the two apples picked are of different color is $\frac{15}{28}$

9. Total number of Pens = 6 + 9 = 15

Let B = Black pen and R = Red pen, then

$$P(B) = \frac{6}{15} \text{ and } P(R) = \frac{9}{15}$$

TREE DIAGRAM



$$\text{a) } P(\text{Both pens black}) = P(B) \times P(B) = \frac{6}{15} \times \frac{5}{14} = \frac{30}{210} = \frac{1}{7}$$

$\therefore$ Probability of both pens being black is $\frac{1}{7}$

$$\begin{aligned} \text{b) } P(\text{Pens of different colors}) &= P(BR) + P(RB) = \frac{6}{15} \times \frac{9}{14} + \frac{9}{15} \times \frac{6}{14} = \frac{54}{210} + \frac{54}{210} \\ &= \frac{108}{210} = \frac{18}{35} \end{aligned}$$

$\therefore$ Probability of different color is $\frac{18}{35}$

$$10. \text{ Total blouses} = 2 + 2 + 1 + 3 = 8$$

Let Gb = green blouse, Bb = blue blouse, Rb = red blouse and Wb = white blouse, Then

$$P(\text{Gb}) = \frac{2}{8}, P(\text{Bb}) = \frac{2}{8}, P(\text{Rb}) = \frac{1}{8} \text{ and } P(\text{Wb}) = \frac{3}{8}.$$

$$\text{Total skirts} = 2 + 2 + 1 = 5$$

Let Gs = green skirt, Bs = blue skirt Gr = grey skirt

$$P(\text{Gs}) = \frac{2}{5}, P(\text{Bs}) = \frac{2}{5} \text{ and } P(\text{Gr}) = \frac{1}{5}.$$

a) $P(\text{Blouse not blue}) = 1 - P(\text{Blue blouse}) = 1 - \frac{2}{8} = \frac{3}{4}$ or 0.75

∴ Probability of a blouse which is not blue is **0.75**

b) $P(\text{White blouse and Blue skirt}) = P(\text{Wb}) \times P(\text{Bs}) = \frac{3}{8} \times \frac{2}{8} = \frac{6}{40} = \frac{3}{20}$

∴ Probability of a white blouse and a blue skirt is $\frac{3}{20}$.

c) $P(\text{Blouse and skirt of the same color})$.

Hint: Look at the colors involved. The only colors that are found on both the blouses and the skirts are green and blue and at the same time notice that, their probabilities are the same.

$$P(\text{Blouse and skirt of the same color}) = P(\text{Green blouse \& Green skirt}) + P(\text{Blue blouse \& Blue skirt}) = P(\text{Gb}) \times P(\text{Gs}) + P(\text{Bb}) \times P(\text{Bs}) = \left(\frac{2}{8} \times \frac{2}{5}\right) + \left(\frac{2}{8} \times \frac{2}{5}\right) = \frac{4}{40} + \frac{4}{40} = \frac{1}{5}$$

∴ Probability of picking a blouse and skirt of the same color is $\frac{1}{5}$

MATRICES

1. $M = \begin{bmatrix} 3 & -2 \\ 5 & x \end{bmatrix}$

a) $\text{Det } M = 3(3 \times x) - (5 \times -2)$
 $= 3x - (-10)$
 $= 3x + 10$

$$22 = 3x + 10$$

$$3x = 22 - 10$$

$$\frac{3x}{3} = \frac{12}{3} = 4$$

$$\therefore x = 4$$

b) $M = \begin{bmatrix} 3 & -2 \\ 5 & x \end{bmatrix}$

$$M^{-1} = \frac{1}{\det M} \begin{bmatrix} x & 2 \\ -5 & 3 \end{bmatrix}$$

$$= \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix}$$

$$\therefore M^{-1} = \frac{1}{22} \begin{bmatrix} 4 & 2 \\ -5 & 3 \end{bmatrix}$$

2. $A = \begin{bmatrix} 5 & -2 \\ 3 & x \end{bmatrix}$ and $B = \begin{pmatrix} 6 \\ 4 \end{pmatrix}$

I) $\text{Det } A = 5 \times x - (3 \times -2)$
 $= 5x - (-6) = 5x + 6$
 $21 = 5x + 6$
 $5x = 21 - 6 = 15$
 $\frac{5x}{5} = \frac{15}{5} = 3$

$\therefore x = 3$

II) $A^{-1} = \frac{1}{\det A} \begin{bmatrix} x & 2 \\ -3 & 5 \end{bmatrix}$
 $= \frac{1}{21} \begin{bmatrix} 3 & 2 \\ -3 & 5 \end{bmatrix}$
 $\therefore A^{-1} = \frac{1}{21} \begin{bmatrix} 3 & 2 \\ -3 & 5 \end{bmatrix}$

III). $AB = \begin{bmatrix} 5 & -2 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 4 \end{bmatrix}$
 $= \begin{bmatrix} 5 \times 6 + (-2) \times 4 \\ 3 \times 6 + 3 \times 4 \end{bmatrix}$
 $= \begin{bmatrix} 30 - 8 \\ 18 + 12 \end{bmatrix} = \begin{bmatrix} 22 \\ 30 \end{bmatrix}$

$\therefore AB = \begin{bmatrix} 22 \\ 30 \end{bmatrix}$

HINT: Before you start calculating the product of two matrices, always do a checkup of their order.

3. $Q = \begin{pmatrix} 3 & -2 \\ x & 4 \end{pmatrix}$

a) $\text{Det } Q = 3 \times 4 - x \times (-2)$
 $= 12 + 2x$
 $2 = 2x + 12$
 $2x = 2 - 12 = -10$
 $\frac{2x}{2} = \frac{-10}{2} = -5$

$\therefore x = -5$

b). $Q = \begin{pmatrix} 3 & -2 \\ x & 4 \end{pmatrix}$
 $Q^{-1} = \frac{1}{\det Q} \begin{bmatrix} 4 & 2 \\ -(-5) & 3 \end{bmatrix}$
 $= \frac{1}{2} \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$
 $\therefore Q^{-1} = \frac{1}{2} \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$

4. $A = \begin{pmatrix} 2x & 2 \\ 3 & x \end{pmatrix}$

a) $\text{Det } A = (2x \times x) - (3 \times 2)$
 $= 2x^2 - 6 = 12$
 $2x^2 = 12 + 6 = 18$
 $\frac{2x^2}{2} = \frac{18}{2}$
 $x^2 = 9$

$\sqrt[2]{x^2} = \sqrt[2]{9}$
 $= x = \pm 3$
 $\therefore x = 3$ (positive integer)

$$\begin{aligned} \text{b) } A^{-1} &= \frac{1}{\det A} \begin{bmatrix} x & -2 \\ -3 & 2x \end{bmatrix} \\ &= \frac{1}{12} \begin{bmatrix} 3 & -2 \\ -3 & 6 \end{bmatrix} \\ \therefore A^{-1} &= \frac{1}{12} \begin{bmatrix} 3 & -2 \\ -3 & 6 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} 5. \quad A &= \begin{pmatrix} 4 & -5 \\ 1 & 2 \end{pmatrix} \text{ and } B = \begin{pmatrix} 8 & y \\ 3 & 5 \end{pmatrix} \\ \text{a). } \det A &= \det B \\ 4 \times 2 - 1 \times (-5) &= 8 \times 5 - 3 \times y \\ 8 + 5 &= 40 - 3y \\ 13 &= 40 - 3y \\ 3y &= 40 - 13 = 27 \\ \frac{3y}{3} &= \frac{27}{3} = 9 \\ \therefore y &= 9 \end{aligned}$$

$$\begin{aligned} 6. \quad K &= \begin{pmatrix} 10 & -2 \\ 11 & -2 \end{pmatrix} \\ \text{a) } \det K &= 10 \times (-2) - 11 \times (-2) \\ &= -20 - (-22) \\ &= -20 + 22 = 2 \\ \therefore \det K &= 2 \end{aligned}$$

$$\begin{aligned} \text{c). } \begin{pmatrix} -3 \\ 4 \end{pmatrix} \begin{pmatrix} -1 & 0 \end{pmatrix} & \begin{matrix} (2 \times 1) & (1 \times 2) \end{matrix} \\ &= \begin{bmatrix} -3 \times -1 & -3 \times 0 \\ 4 \times -1 & 4 \times 0 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 0 \\ -4 & 0 \end{bmatrix} \\ \therefore \begin{pmatrix} -3 \\ 4 \end{pmatrix} \begin{pmatrix} -1 & 0 \end{pmatrix} &= \begin{bmatrix} 3 & 0 \\ -4 & 0 \end{bmatrix} \\ & \begin{matrix} (2 \times 2) \end{matrix} \end{aligned}$$

$$\begin{aligned} \text{b). } B^{-1} &= \frac{1}{\det B} \begin{bmatrix} 5 & -y \\ -3 & 8 \end{bmatrix} \\ &= \frac{1}{13} \begin{bmatrix} 5 & -9 \\ -3 & 8 \end{bmatrix} \\ \therefore B^{-1} &= \frac{1}{13} \begin{bmatrix} 5 & -9 \\ -3 & 8 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{b). } K^{-1} &= \frac{1}{\det K} \begin{bmatrix} -2 & 2 \\ -11 & 10 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} -2 & 2 \\ -11 & 10 \end{bmatrix} \\ \therefore K^{-1} &= \begin{bmatrix} -2 & 2 \\ -11 & 10 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{d). } \text{Let } A &= \begin{pmatrix} 6 & -a \\ 3 & -2 \end{pmatrix} \text{ and } \det A = -9 \\ \det A &= 6 \times (-2) - 3 \times (-a) \\ &= -12 - (-3a) = -12 + 3a \\ -9 &= -12 + 3a \\ 3a &= -9 + 12 = 3 \\ \frac{3a}{3} &= \frac{3}{3} = 1 \end{aligned}$$

$$\therefore a = 1$$

$$7. \begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & n \end{bmatrix} \begin{bmatrix} m \\ -4 \\ 1 \end{bmatrix} = \begin{pmatrix} 15 \\ 2 \end{pmatrix}$$

$$\begin{bmatrix} 2 \times m + 0 \times (-4) + 3 \times 1 \\ 0 \times m + 3 \times (-4) + n \times 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 2 \end{bmatrix}$$

$$2m + 0 + 3 = 15$$

$$\text{and } 0 + (-12) + n = 2$$

$$2m = 15 - 3 = 12$$

$$-12 + n = 2$$

$$\frac{2m}{2} = \frac{12}{2} = 6$$

$$n = 2 + 12 = 14$$

$$\therefore m = 6$$

$$\therefore n = 14$$

$$8. A = \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$$

$$\begin{aligned} \text{a) } \det A &= 3 \times 5 - (2 \times 7) \\ &= 15 - 14 = 1 \end{aligned}$$

$$\therefore \det A = 1$$

$$\begin{aligned} \text{b) } A^{-1} &= \frac{1}{\det A} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \\ &= \frac{1}{1} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \end{aligned}$$

$$\therefore A^{-1} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\text{c). } A^{-1} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \times -2 + (-7 \times 1) \\ (-2 \times -2) + 3 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} -10 - 7 \\ 4 + 3 \end{bmatrix}$$

$$= \begin{bmatrix} -17 \\ 7 \end{bmatrix}$$

$$\therefore A^{-1} \begin{pmatrix} -2 \\ 1 \end{pmatrix} = \begin{bmatrix} -17 \\ 7 \end{bmatrix}$$

$$9. A = \begin{pmatrix} 1 & x \\ -1 & 2 \end{pmatrix}$$

$$\begin{aligned} \text{a) } \det A &= 1 \times 2 - (-1 \times x) \\ &= 2 - (-x) \\ &= 2 + x \end{aligned}$$

$$\therefore \det A = x + 2$$

$$\text{b) } \det A = 5$$

$$x + 2 = 5$$

$$x = 5 - 2 = 3$$

$$\therefore x = 3$$

$$\text{c). } A^{-1} = \frac{1}{\det A} \begin{bmatrix} 2 & -x \\ 1 & 1 \end{bmatrix}$$

$$= \frac{1}{5} \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{5} \begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}$$

$$10. \begin{pmatrix} 2 & x \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 10 \\ -8 \end{pmatrix}$$

$$\begin{aligned} \begin{pmatrix} 2 & x \\ -4 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} &= \\ \begin{bmatrix} 2 \times 3 + x \times 4 \\ -4 \times 3 + 1 \times 4 \end{bmatrix} &= \\ \begin{bmatrix} 6 + 4x \\ -12 + 4 \end{bmatrix} &= \\ \begin{bmatrix} 6 + 4x \\ -8 \end{bmatrix} &= \begin{bmatrix} 10 \\ -8 \end{bmatrix} \end{aligned}$$

we compare the corresponding entries

$$\text{thus } 6 + 4x = 10$$

$$4x = 10 - 6 = 4$$

$$\frac{4x}{4} = \frac{4}{4} = 1$$

$$\therefore x = 1$$

$$11. P = \begin{pmatrix} 2 & 0 \\ 6 & 1 \end{pmatrix} \text{ and } Q = \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$$

$$\begin{aligned} \text{a) } PQ &= \begin{pmatrix} 2 & 0 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix} \\ &= \begin{bmatrix} 2 \times a + 0 \times 1 & 2 \times 0 + 0 \times b \\ 6 \times a + 1 \times 1 & 6 \times 0 + 1 \times b \end{bmatrix} \\ &= \begin{bmatrix} 2a + 0 & 0 + 0 \\ 6a + 1 & 0 + b \end{bmatrix} \\ &= \begin{bmatrix} 2a & 0 \\ 6a + 1 & b \end{bmatrix} \end{aligned}$$

$$\therefore PQ = \begin{bmatrix} 2a & 0 \\ 6a + 1 & b \end{bmatrix}$$

$$\text{b). } P - Q = \begin{pmatrix} 2 & 0 \\ 6 & 1 \end{pmatrix} - \begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}$$

$$= \begin{bmatrix} 2 - a & 0 - 0 \\ 6 - 1 & 1 - b \end{bmatrix}$$

$$= \begin{bmatrix} 2 - a & 0 \\ 5 & 1 - b \end{bmatrix}$$

$$\therefore P - Q = \begin{bmatrix} 2 - a & 0 \\ 5 & 1 - b \end{bmatrix}$$

$$\text{Now } PQ = P - Q$$

$$\begin{bmatrix} 2a & 0 \\ 6a + 1 & b \end{bmatrix} = \begin{bmatrix} 2 - a & 0 \\ 5 & 1 - b \end{bmatrix}$$

Equating the corresponding

entries we have, $2a = 2 - a$

$$2a + a = 2$$

$$3a = 2$$

$$\frac{3a}{3} = \frac{2}{3}$$

$$a = \frac{2}{3}$$

$$\text{Or } 6a + 1 = 5$$

$$6a = 5 - 1 = 4$$

$$\frac{6a}{6} = \frac{4}{6} = \frac{2}{3}$$

$$a = \frac{2}{3}$$

$$\text{And } b = 1 - b$$

$$b + b = 1$$

$$2b = 1$$

$$\frac{2b}{2} = \frac{1}{2}$$

$$b = \frac{1}{2}$$

$$\therefore a = \frac{2}{3} \text{ and } b = \frac{1}{2}$$

KINEMATICS

1. DATA

- ✚ $U = 0\text{m/s}$
- ✚ $V_1 = 24\text{m/s}$
- ✚ $t = 12\text{s}$
- ✚ $a = ?$
- ✚ $V = ?$
- ✚ $d = ?$

a). From $v = u + at$,

$$a = \frac{v - u}{t} = \frac{24}{12} = 2$$

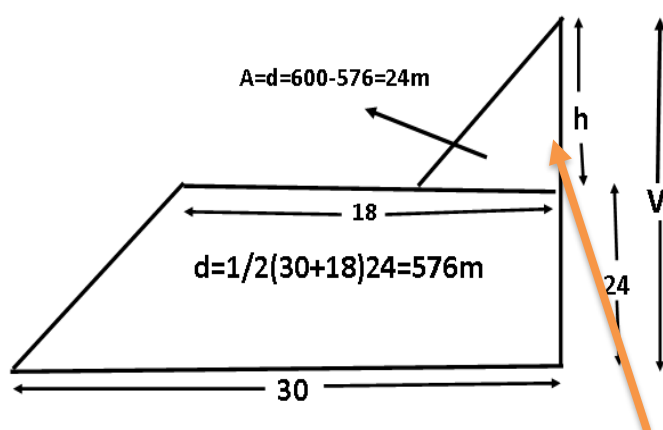
$$\therefore a = 2\text{m/s}^2$$

b). Distance covered is equal to the area under the graph

$$d = \frac{1}{2}(12 + 24)24 = 36 \times 12$$

$$\therefore d = 432\text{m}$$

c).



HINT: To find the value of V, we need to cut the graph in two shapes as shown in the diagram to the left. From the two figures, the area of the trapezium is 576. And area of triangle is $600 - 576 = 24$. Find the height of the triangle and add h to 24 to get V.

$$V = h + 24 = 8 + 24 = 32$$

$$\therefore V = 32\text{m/s}$$

$$24 = \frac{1}{2}(6)h,$$

$$h = \frac{24}{3} = 8$$

2.

DATA

- ✚ $U = 0\text{m/s}$
- ✚ $a = 2\text{m/s}^2$
- ✚ $t = 10\text{s}$
- ✚ $v = ?$
- ✚ $d = ?$

a). $v = u + at$

$$= 0 + 2(10)$$

$$= 20\text{ m/s}$$

$$\therefore v = 20\text{ m/s}$$

b). Total distance is equal to the area under the graph.

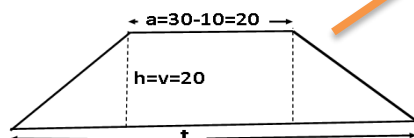
$$750 = \frac{1}{2}(20 + t)20$$

$$= 200 + 10t$$

$$10t = 750 - 200 = 550$$

$$\frac{10t}{10} = \frac{550}{10} = 55$$

$$\therefore t = 55\text{s}$$



$$c). a = \frac{v-u}{t} = \frac{0-20}{25} = -0.8\text{m/s}^2$$

$$v = u + at$$

$$= 20 + (-0.8)10 = 20 - 8 = 12$$

$$\therefore v = 12 \text{ m/s}$$

HINT: To find the speed of the car at 40s we find its velocity. That can be made easier by first find the deceleration of the car using $t=55-30=25\text{s}$. Note that the t value that will be considered at 40s will be $40-30=10$.

$$3. a) a = \frac{v-u}{t} = \frac{30-0}{5} = 6$$

$$\therefore a = 6\text{m/s}^2$$

$$b) d = \frac{1}{2}(a + b)h$$

$$825 = \frac{1}{2}(10 + t)30$$

$$825 = 150 + 15t$$

$$15t = 825 - 150 = 675$$

$$\frac{15t}{15} = \frac{675}{15} = 45$$

$$\therefore t = 45\text{s}$$

DATA

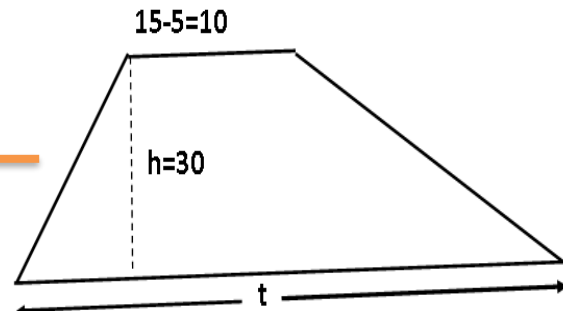
$$a=?$$

$$V=20\text{m/s}$$

$$U=0\text{m/s}$$

$$Tt=?$$

$$Td=825\text{m}$$



$$c) \text{ Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{825}{45} = 18.33$$

$$\therefore \text{Average Speed} = 18.33 \text{ m/s}$$

4.

DATA

- ✚ a=?
- ✚ V=12m/s
- ✚ U=0m/s
- ✚ d=?
- ✚ t=3s

$$\begin{aligned} \text{a) } a &= \frac{v-u}{t} \\ &= \frac{12-0}{3} = 4 \\ \therefore a &= 4 \text{ m/s}^2 \end{aligned}$$

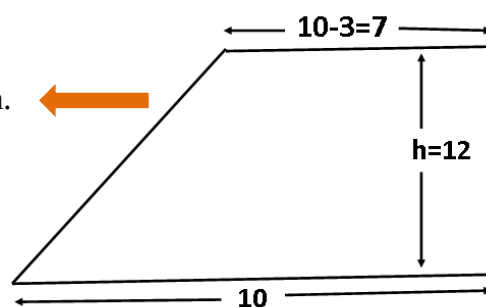
$$\begin{aligned} \text{b) } a &= \frac{v-u}{t} \\ &= \frac{0-12}{4} = -3 \\ \therefore a &= -3 \text{ m/s}^2 \end{aligned}$$

DATA

- ✚ v=0m/s
- ✚ t=4s
- ✚ u=12m/s

c). Distance covered is the area under the graph. 

$$\begin{aligned} d &= \frac{1}{2}(a+b)h \\ &= \frac{1}{2}(7+10)12 \\ &= 17 \times 6 = 102 \\ \therefore d &= 102\text{m} \end{aligned}$$



5.

a).

$$a = \frac{v-u}{t}$$

$$= \frac{24-36}{40}$$

$$= \frac{-12}{40} = -0.3$$

$$\therefore a = -0.3 \text{ m/s}^2$$

DATA

- ✚ u=36m/s
- ✚ v=24m/s
- ✚ t=40s
- ✚ a=?

$$\text{b). } a = \frac{v-u}{t}$$

$$= \frac{0-24}{10} = -2.4 \text{ m/s}$$

$$v = u + at$$

$$= 24 + (-2.4)7$$

$$= 24 - 16.8 = 7.2$$

$$\therefore v = 7.2 \text{ m/s}$$

DATA

- ✚ u=24m/s
- ✚ v=0m/s
- ✚ a=?
- ✚ t=7s

$$\text{c). } d = ut + \frac{1}{2}at^2$$

$$= 36 \times 20 + \frac{1}{2}(-0.3)20^2$$

$$= 720 - 60 = 660$$

$$\therefore d = 660\text{m}$$

DATA

- ✚ u=36m/s
- ✚ a = -0.3m/s²
- ✚ t=20s

$$a = \frac{24-36}{40} = \frac{-12}{40}$$

$$= -0.3 \text{ m/s}^2$$

6. a) Distance is equal to the area under the graph

$$d = \frac{1}{2}(4)v$$

$$80 = 2v$$

$$\frac{2v}{2} = \frac{80}{2} = 40$$

$$\therefore v = 40\text{m/s}$$

b). $a = \frac{v-u}{t}$

$$= \frac{40-0}{4} = \frac{40}{4} = 10$$

$$\therefore a = 10\text{m/s}^2$$

c). $a = \frac{v-u}{t}$

$$= \frac{0-40}{2}$$

$$= \frac{-40}{2} = -20$$

$$\therefore a = -20\text{m/s}^2$$

d). $d = \frac{1}{2}(a+b)h$

$$= \frac{1}{2}(4+12)40$$

$$= 18 \times 20 = 360$$

$$\therefore d = 360\text{m}$$

7. a). $a = \frac{v-u}{t}$

$$= \frac{10-30}{10} = \frac{-20}{10} = -2$$

$$\therefore a = -2\text{m/s}^2$$

b). Distance = Area under the graph

$$d = \frac{1}{2}(b)h$$

$$= \frac{1}{2}(t-20)10 = 5t - 100$$

$$50 = 5t - 100$$

$$5t = 50 + 100 = 150$$

$$\frac{5t}{5} = \frac{150}{5} = 30$$

$$\therefore t = 30\text{s}$$

c). Average Speed = $\frac{\text{Total distance travelled}}{\text{Total time time}}$

Total distance = $A_1 + A_2$

$$= \frac{1}{2}(20+30)10 + \frac{1}{2}(10)20$$

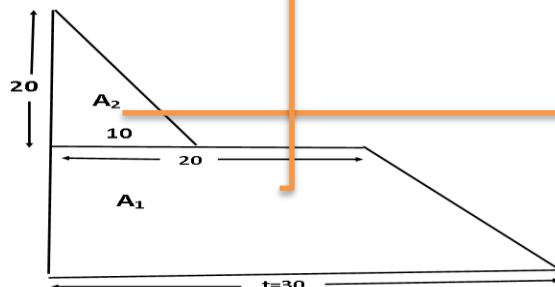
$$= 250 + 100 = 350$$

$$\therefore d = 350\text{m}$$

Total time = 30s

Average speed = $\frac{350\text{m}}{30\text{s}} = 11.67\text{m/s}$

$$\therefore \text{Average speed} = 11.67\text{m/s}$$



8. a) Distance for Train A = $\frac{1}{2}(a + b)h$
 $= \frac{1}{2}(6 + 12)50$
 $= 18 \times 25 = \mathbf{450m}$

Now for train B, since train A and train B traveled the same distance,

$$450 = \frac{1}{2}bh$$

$$= \frac{1}{2}(12)V = 6V$$

$$\frac{6V}{6} = \frac{450}{6} = 75$$

$$\therefore V = \mathbf{75m/s}$$

b) We need to find the distance traveled by each train in the first 6s and find their difference.

Train A: $d = \frac{1}{2}(2 + 6)50 = 8 \times 25 = 200m$

Train B: $d = \frac{1}{2}(6)75 = 3 \times 75 = 225m$

Thus the distance between the two Trains A and B is $225m - 200m = \mathbf{25m}$

9. a) Distance covered is the area under the graph

$$d = \frac{1}{2}bh$$

$$120 = \frac{1}{2}(6)V_1$$

$$3V_1 = 120$$

$$\frac{3V_1}{3} = \frac{120}{3} = 40$$

$$\therefore V_1 = \mathbf{40m/s}$$

b) $V_2 = V_1 + at$

$$= 40 + 2(6)^2$$

$$= 40 + 72 = 112$$

$$\therefore V_2 = \mathbf{112m/s}$$

DATA

$$u = v_1 = 40m/s$$

$$a = 2m/s^2$$

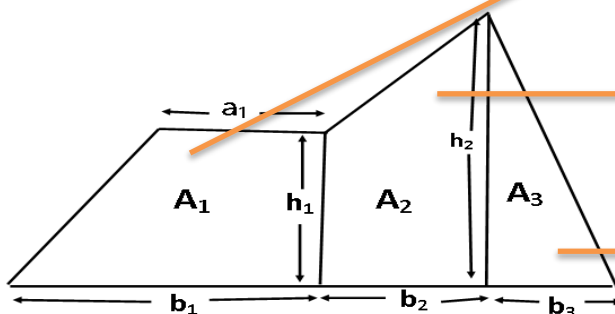
$$t = 6s$$

c) Total distance = $A_1 + A_2 + A_3$

$$= \frac{1}{2}(a_1 + b_1)h_1 + \frac{1}{2}(h_1 + h_2)b_2 + \frac{1}{2}b_3h_2$$

$$= \frac{1}{2}(4 + 10)40 + \frac{1}{2}(40 + 112)6 + \frac{1}{2}(4)112$$

$$= 280m + 456m + 224m = \mathbf{960m}$$



MENSURATION

1. $V = lbh$

$$= 1.2m \times 0.9m \times 10m = 10.8m^3$$

Now $1m^3 = 1000l$

$$10.8m^3 = x$$

$$xm^3 = 10800m^3l$$

$$\frac{xm^3}{m^3} = \frac{10800m^3l}{m^3} = 10800l$$

$\therefore$ The number of litres of fuel in the tank is **10800l**

2. Area of cone $A = \pi r(l + r)$. We need to find the value of r first

$$13^2 - 12^2 = r^2$$

$$r^2 = 169 - 144 = 25$$

$$\sqrt{r^2} = \sqrt{25} = \pm 5$$

$$\therefore r = 5cm$$

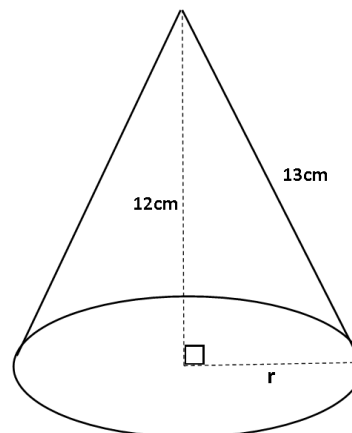
$$A = \pi r(l + r)$$

$$= \pi \times 5cm(13cm + 5cm)$$

$$= 15.71cm \times 18cm$$

$$= 282.78cm^2$$

$$\therefore A = 282.78cm^2$$



3. a) $V = l^3$

$$125 = l^3$$

$$\sqrt[3]{125} = \sqrt[3]{l^3}$$

$$l = 5$$

$$\therefore l = 5cm$$

b) $1cm^3 = 9grams$

$$125cm^3 = x$$

$$xcm^3 = 1125grams\ cm^3$$

$$\frac{xcm^3}{cm^3} = \frac{1125grams\ cm^3}{cm^3}$$

$$= 1125grams$$

$$\therefore \text{Mass of the cube} = 1.125kg$$

c) $A = 6 \times l^3$

$$= 6 \times (5cm)^3$$

$$= 6 \times 125cm^3$$

$$= 750cm^3$$

$$\therefore A = 750cm^3$$

4. a) Surface area of a cylinder $A = 2\pi r(h + r)$

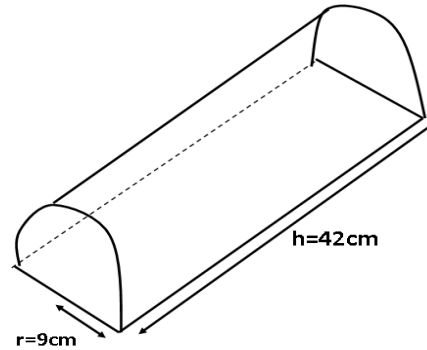
Since the curved part(cover) is $\frac{1}{2}$ a cylinder, its area $A = \pi r(h + r)$

$$A = \pi \times 9\text{cm}(9\text{cm} + 42\text{cm})$$

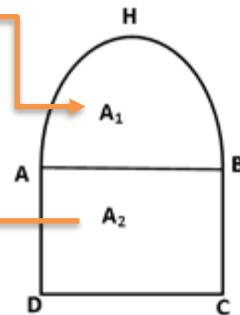
$$= 28.278\text{cm} \times 51\text{cm}$$

$$= 1442.178\text{cm}^2$$

$$\therefore A = 1442.178\text{cm}^2$$



b) $A_T = A_1 + A_2$
 $= \frac{1}{2}\pi r^2 + lb$
 $= \frac{1}{2} \times \pi \times (9\text{cm})^2 + 16\text{cm} \times 18\text{cm}$
 $= 127.251\text{cm}^2 + 288\text{cm}^2$
 $= 415.251\text{cm}^2$
 $\therefore A_T = 415.251\text{cm}^2$



- c) Volume of a cylinder $V = \pi r^2 h$

since the cover is half a cylinder, its volume is also half of that of a cylinder

$$V = \frac{1}{2}\pi r^2 h$$

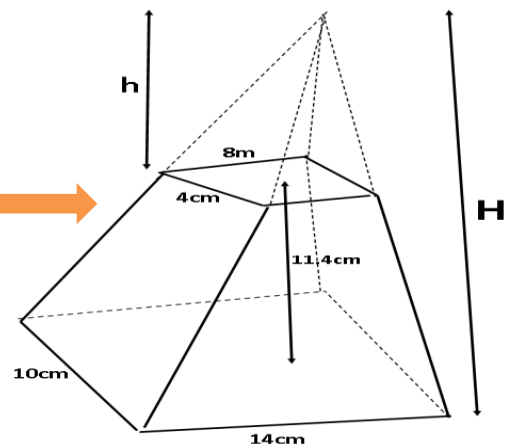
$$= \frac{1}{2} \times \pi \times (9\text{cm})^2 \times 42\text{cm}$$

$$= 5344.542\text{cm}^3$$

$$\therefore V = 5344.5\text{cm}^3$$

5. Volume of a frustum $V = \frac{1}{3}(LBH - lbh)$

HINT: Before you can proceed you need to discover that actually the figure here has frustum and a cone. You need to find the value of H and h. you can do this by using the fact the two frustums are similar.



$$\frac{14}{8} = \frac{H}{H-11.4}$$

$$8H = 14(H - 11.4) = 14H - 159.6$$

$$14H - 8H = 159.6$$

$$\frac{6H}{6} = \frac{159.6}{6} = 26.6\text{cm}$$

$$h = H - 11.4 = 26.6 - 11.4 = 15.2\text{cm}$$

$$\therefore H = \mathbf{26.6\text{cm}} \text{ and } h = \mathbf{15.2\text{cm}}$$

$$V = \frac{1}{3}(LBH - lbh) =$$

$$\frac{1}{3}(14 \times 10 \times 26.6 -$$

$$8 \times 4 \times 15.2)$$

$$= \frac{1}{3}(3724 - 486.4)$$

$$= \frac{1}{3}(3237.6)$$

$$= 1079.2$$

$$\therefore V = \mathbf{1079.2\text{cm}^3}$$

6. $\frac{12}{4} = \frac{15+h}{h}$

$$12h = 4(15 + h)$$

$$12h = 60 + 4h$$

$$12h - 4h = 60$$

$$\frac{8h}{8} = \frac{60}{8} = 7.5\text{cm}$$

$$H = 15 + h = 15 + 7.5 = 22.5\text{cm}$$

$$\therefore H = \mathbf{22.5\text{cm}} \text{ and } h = \mathbf{7.5\text{cm}}$$

The volume that remained is the volume of the frustum,

$$V = \frac{1}{3}\pi(R^2H - r^2h)$$

$$= \frac{1}{3}\pi(12^2 \times 22.5 - 4^2 \times 7.5)$$

$$= \frac{1}{3}\pi(3240 - 120)$$

$$= \frac{1}{3}\pi(3120)$$

$$\therefore V = \mathbf{3267.68\text{cm}^3}$$

7.

$$\frac{10}{4} = \frac{H}{H-9}$$

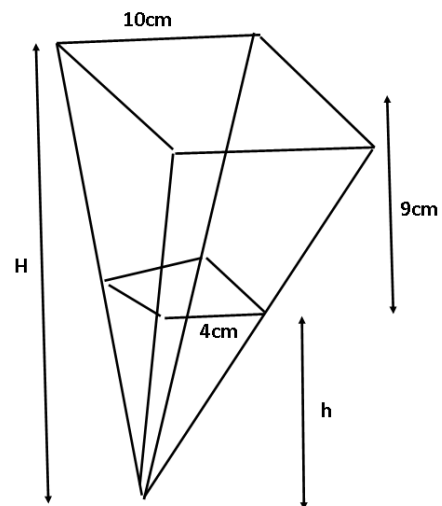
$$10H - 90 = 4H$$

$$6H = 90$$

$$\frac{6H}{6} = \frac{90}{6} = 15 \text{ and}$$

$$h = H - 9 = 15 - 9 = 6$$

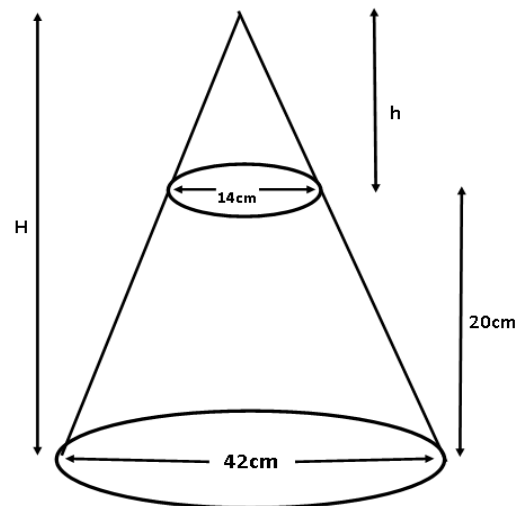
$$\therefore H = \mathbf{15\text{cm}} \text{ and } h = \mathbf{6\text{cm}}$$



$$\begin{aligned}
 V &= \frac{1}{3}(L^2H - l^2h) \\
 &= \frac{1}{3}(10^2 \times 15 - 4^2 \times 6) \\
 &= \frac{1}{3}(1500 - 96) \\
 &= \frac{1}{3}(1404) = 468 \\
 \therefore V &= \mathbf{468cm^3}
 \end{aligned}$$

HINT: Take note that the formula for the volumes has L^2 and l^2 . This is because the figures under discussion are squares.

$$\begin{aligned}
 8. \quad \frac{H}{H-20} &= \frac{42}{14} \\
 14H &= 42H - 840 \\
 28H &= 840 \\
 \frac{28H}{28} &= \frac{840}{28} = 30 \\
 H = 30 \text{ and } h = H - 20 &= 30 - 20 = 10 \\
 \therefore H &= \mathbf{30cm} \text{ and } h = \mathbf{10cm}
 \end{aligned}$$



$$\begin{aligned}
 V &= \frac{1}{3}\pi(R^2H - r^2h) \\
 &= \frac{1}{3}\pi(21^2 \times 30 - 7^2 \times 10) \\
 &= \frac{1}{3}\pi(13230 - 490) \\
 &= \frac{1}{3}\pi(12740) = 13343.02667 \\
 \therefore V &= \mathbf{13300cm^3}
 \end{aligned}$$

9. For us to calculate MV, we need to find NV and MN first and then subtract MN from NV to get MV.

$$\begin{aligned} NV^2 &= AV^2 - AN^2 \\ &= NV^2 = 41^2 - 9^2 = 1681 - 81 = 1600 \end{aligned}$$

$$\sqrt{NV^2} = \sqrt{1600}$$

$$NV = \mp 40$$

$$\therefore NV = \mathbf{40m}$$
 (Positive value since its distance)

$$MN^2 = BM^2 - BN^2$$

$$= 15^2 - 9^2 = 225 - 81 = 144$$

$$\sqrt{MN^2} = \sqrt{144} = 12$$

$$\therefore MN = \mathbf{12m}$$

$$MV = NV - MN = 40 - 12 = 28 \therefore MV = \mathbf{28m}$$

$$\begin{aligned} 10. a) A &= 2\pi r(h + r) \\ &= 2 \times \pi \times 5(5 + 12) \\ &= 10\pi(17) = 31.42 \times 17 = 534.14 \end{aligned}$$

$$\therefore A = \mathbf{534.14cm^2}$$

HINT: To solve this question we need to find the volume of the cylinder and the volume of the thickness of the cylinder then lastly find their difference

b). *Volume of the cylinder*

$$\begin{aligned} V &= \pi r^2 h \\ &= 3.142 \times 5^2 \times 12 = 942.6 \end{aligned}$$

$$\therefore V = 942.6cm^3$$

Volume of the thickness

$$\begin{aligned} V &= \pi r^2 h \\ &= 3.142 \times 0.2^2 \times 12 \\ &= 1.50816 \end{aligned}$$

$$\therefore V = 1.50816cm^3$$

Volume of water = Volume of cylinder –
Volume of thickness

$$= 942.6 - 1.50816 = 941.09184$$

$$\therefore V = \mathbf{941.09cm^3}$$

$$\begin{aligned} 11. a) \quad 64:27 &= 4^3:3^3 \\ a &= 4 \text{ and } b = 3 \\ \text{thus ratio of their surface areas is} \\ 4^2:3^2 &= 16:9 \\ \therefore \text{The areas of the two figures} \\ &\text{are in the ratio } \mathbf{16:9} \end{aligned}$$

HINT: If the ratio of the two solids is $a:b$, the ratio of their surface areas is $a^2:b^2$

and the ratio of their volumes is
 $a^3:b^3$

b).

$$\begin{array}{ccc} 3 \rightarrow x & & 3 \times 28 = 4 \times x \\ \times & & \frac{4x}{4} = \frac{84}{4} = 21 \\ 4 \rightarrow 28 & & \therefore x = \mathbf{21cm} \end{array}$$

12. a) To find the perimeter count the number of sides surrounding the figure. Since it is a square base, then all sides are equal to 8cm.

$$P = 8 \times 8cm = 64cm$$

$$\therefore p = \mathbf{64cm}$$

b). Note that there are four equilateral triangles $A = 4 \times \text{area of one equilateral} + \text{area of square}$.

$$A = 4 \times \frac{\sqrt{3}}{4} l^2 + l^2 = l^2(\sqrt{3} + 1) = 8^2 \times 2.732050808 = 174.8512517$$

$$\therefore A = \mathbf{17500cm} \text{ (correct to 3 s. f).}$$

c).

$$h^2 = 8^2 - 4^2 = 64 - 16 = 48$$

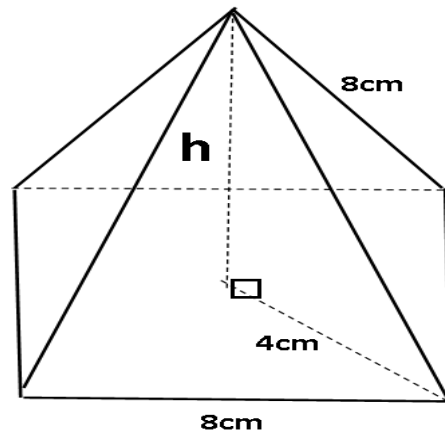
$$h = \sqrt{h^2} = \sqrt{48} = \mathbf{6.928cm}$$

$$\text{Base area} = 8 \times 8 = \mathbf{64cm^2}$$

$$V = \frac{1}{3} \times \text{Base area} \times \text{height}$$

$$= \frac{1}{3} \times 64 \times 6.928 = 443.405$$

$$\therefore V = \mathbf{443.41cm^3}$$



13. a). Perimeter(P) of sector OCD = OC + CD + DO

$$= 70cm + \frac{\mu \times r \times \theta}{180} + 70cm$$

$$= 140cm + \frac{22}{7} \times \frac{70 \times 120}{180} cm$$

$$= 140cm + 146.6666667cm = 286.6666667cm$$

$$\therefore P = \mathbf{286.7cm} \text{ (1 d. p)}$$

b). Area of the shaded region = Area of a bigger sector – Area of the smaller sector

$$= \frac{\pi \times \theta \times R^2}{180} - \frac{\pi \times \theta \times r^2}{180} = \frac{\pi \times \theta}{180} (R^2 - r^2)$$

$$= \frac{22}{7} \times \frac{120}{180} (70^2 - 30^2) = 2.09524(4900 - 900)$$

$$= 2.09524(4000) = 8\,380.95$$

$\therefore$ Area of shaded region = **8 381.0cm²** (correct to 1d.p)

VECTORS

QUESTION ONE

a). $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$$= 3\vec{p} + 3\vec{p}$$

$$= 6\vec{p}$$

$$\therefore \overrightarrow{OB} = 6\vec{p}$$

b). $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$

$$= -\overrightarrow{OB} + \overrightarrow{OQ} + \overrightarrow{QC}$$

$$= -6\vec{p} + 2\vec{q} + 2\vec{q}$$

$$= 4\vec{q} - 6\vec{p}$$

$$\therefore \overrightarrow{BC} = 2(2\vec{q} - 3\vec{p})$$

c). $\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ}$

$$= -\overrightarrow{OA} + \overrightarrow{OQ}$$

$$= -3\vec{p} + 2\vec{q} = 2\vec{q} - 3\vec{p}$$

$$\therefore \overrightarrow{AQ} = 2\vec{q} - 3\vec{p}$$

d). $\overrightarrow{AQ} : \overrightarrow{BC} = 1 : 2$

$\therefore \overrightarrow{BC}$ is twice $\overrightarrow{AQ}$ in magnitude.

QUESTION TWO

a). $\overrightarrow{QP} = \overrightarrow{PO} + \overrightarrow{OQ}$

$$= -\overrightarrow{OP} + \overrightarrow{OQ}$$

$$= -\vec{p} + \vec{q} = \vec{q} - \vec{p}$$

$$\therefore \overrightarrow{QP} = \vec{q} - \vec{p}$$

b). $\overrightarrow{ON} = \overrightarrow{OP} + \overrightarrow{PN}$

$$= \overrightarrow{OP} + \frac{3}{4}\overrightarrow{PQ}$$

$$= \vec{p} + \frac{3}{4}(\vec{q} - \vec{p}) = \frac{3}{4}\vec{q} + \frac{1}{4}\vec{p}$$

$$\therefore \overrightarrow{ON} = \frac{1}{4}(3\vec{q} + \vec{p})$$

$$c). \overrightarrow{PM} = \overrightarrow{PO} + \overrightarrow{OM}$$

$$= -\overrightarrow{OP} + \frac{1}{3}\overrightarrow{OM}$$

$$= -\bar{p} + \frac{1}{3}\bar{q}$$

$$\therefore \overrightarrow{PM} = \frac{1}{3}\bar{q} - \bar{p}$$

$$e). \overrightarrow{PX} = k\overrightarrow{PM}$$

$$\overrightarrow{OX} = \overrightarrow{OP} + \overrightarrow{PX}$$

$$= \overrightarrow{OP} + k\overrightarrow{PM}$$

$$= \bar{p} + k \left[\frac{1}{3}\bar{q} - \bar{p} \right]$$

$$= \bar{p} + \frac{k}{3}\bar{q} - k\bar{p}$$

$$= \bar{p} - k\bar{p} + \frac{1}{3}k\bar{q}$$

$$= (1 - k)\bar{p} + \frac{1}{3}k\bar{q}$$

$$\therefore \overrightarrow{OX} = (1 - k)\bar{p} + \frac{1}{3}k\bar{q} \quad (\text{As Required})$$

$$d). \overrightarrow{OX} = h\overrightarrow{ON}$$

$$= h \left[\frac{1}{4}(3\bar{q} + \bar{p}) \right]$$

$$= \frac{1}{4}(3\bar{q} + \bar{p})h$$

$$\therefore \overrightarrow{OX} = \frac{1}{4}(3\bar{q} + \bar{p})h$$

QUESTION THREE

$$a). \overrightarrow{AE} = \frac{1}{3}\overrightarrow{AC}$$

$$= \frac{1}{3}(\overrightarrow{AB} + \overrightarrow{BC})$$

$$= \frac{1}{3}(\bar{a} + 2\bar{b})$$

$$\therefore \overrightarrow{AE} = \frac{1}{3}(\bar{a} + 2\bar{b})$$

$$b). \overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AC}$$

$$= -\overrightarrow{AB} + \overrightarrow{AE}$$

$$= -\bar{a} + \frac{1}{3}(\bar{a} + 2\bar{b})$$

$$= \frac{2}{3}(\bar{b} - \bar{a})$$

$$\therefore \overrightarrow{BE} = \frac{2}{3}(\bar{b} - \bar{a})$$

$$c). \overrightarrow{BD} = \overrightarrow{BA} + \overrightarrow{AD}$$

$$= -\overrightarrow{AB} + \overrightarrow{AD}$$

$$= -\bar{a} + \bar{b} = \bar{b} - \bar{a}$$

$$\therefore \overrightarrow{BD} = \bar{b} - \bar{a}$$

$$d) \text{ We can see that } \overrightarrow{BE} = \frac{2}{3}\overrightarrow{BD}.$$

This means that $\overrightarrow{BE}$ and $\overrightarrow{BD}$ are parallel and the points B, D and E lie on the same plane.

$\therefore$ The points **B**, **D** and **E** are collinear.

QUESTION FOUR

$$\text{a) } \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$$

$$= \bar{a} + 2\bar{b}$$

$$\therefore \overrightarrow{OB} = \bar{a} + 2\bar{b}$$

$$\text{c) } \overrightarrow{CD} = \overrightarrow{CO} + \overrightarrow{OD}$$

$$= -\overrightarrow{OC} + \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$$

$$= -2\bar{b} + \bar{a} + \frac{1}{2}(-\bar{a} + 2\bar{b})$$

$$= -2\bar{b} + \bar{a} - \frac{1}{2}\bar{a} + \bar{b}$$

$$= -\bar{b} + \frac{1}{2}\bar{a} = \frac{1}{2}\bar{a} - \bar{b}$$

$$\therefore \overrightarrow{CD} = \frac{1}{2}\bar{a} - \bar{b}$$

$$\text{b) } \overrightarrow{OE} = \overrightarrow{OC} + \overrightarrow{CE}$$

$$= \overrightarrow{OC} - \frac{1}{4}\overrightarrow{AC}$$

$$= \overrightarrow{OC} - \frac{1}{4}(-\overrightarrow{OA} + \overrightarrow{OC})$$

$$= 2\bar{b} - \frac{1}{4}(-\bar{a} + 2\bar{b})$$

$$= 2\bar{b} + \frac{1}{4}\bar{a} - \frac{1}{2}\bar{b}$$

$$= \frac{1}{4}\bar{a} + \frac{3}{2}\bar{b}$$

$$= \frac{1}{4}(\bar{a} + 6\bar{b})$$

$$\therefore \overrightarrow{OE} = \frac{1}{4}(\bar{a} + 6\bar{b})$$

QUESTION FIVE

$$\text{a) } \overrightarrow{PQ} = \overrightarrow{PO} + \overrightarrow{OP}$$

$$= -\overrightarrow{OP} + \overrightarrow{OQ}$$

$$= -2\bar{p} + 4\bar{q} = 2(2\bar{q} - \bar{p})$$

$$\therefore \overrightarrow{PQ} = 2(2\bar{q} - \bar{p})$$

$$\text{b) } \overrightarrow{PX} = \frac{1}{3}\overrightarrow{PQ}$$

$$= \frac{1}{3}[2(2\bar{q} - \bar{p})]$$

$$= \frac{2}{3}(2\bar{q} - \bar{p})$$

$$\therefore \overrightarrow{PX} = \frac{2}{3}(2\bar{q} - \bar{p})$$

$$c) \quad \overrightarrow{OX} = \overrightarrow{OP} + \overrightarrow{PX}$$

$$= 2\bar{p} + \frac{2}{3}(2\bar{q} - \bar{p})$$

$$= 2\bar{p} - \frac{2}{3}\bar{p} + \frac{4}{3}\bar{q}$$

$$= \frac{4}{3}\bar{p} + \frac{4}{3}\bar{q}$$

$$\therefore \overrightarrow{OX} = \frac{4}{3}(\bar{p} + \bar{q})$$

$$d) \quad \overrightarrow{OC} = h\overrightarrow{OX}$$

$$\overrightarrow{CQ} = \overrightarrow{CO} + \overrightarrow{OQ} = -\overrightarrow{OC} + \overrightarrow{OQ}$$

$$= -h\overrightarrow{OX} + \overrightarrow{OQ} = -h\left[\frac{4}{3}(\bar{p} + \bar{q})\right] + 4\bar{q}$$

$$= -\frac{4}{3}h\bar{p} - \frac{4}{3}h\bar{q} + 4\bar{q}$$

$$= 4\bar{q} - \frac{4}{3}h\bar{q} - \frac{4}{3}h\bar{p}$$

$$= 4\left(1 - \frac{h}{3}\right)\bar{q} - \frac{4}{3}h\bar{p}$$

$$\therefore \overrightarrow{CQ} = 4\left(1 - \frac{h}{3}\right)\bar{q} - \frac{4}{3}h\bar{p} \text{ (As Required)}$$

QUESTION SIX

$$a). \quad \overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE}$$

$$= -\overrightarrow{AB} + 2\overrightarrow{AD}$$

$$= -\bar{q} + 2\bar{p}$$

$$= 2\bar{p} - \bar{q}$$

$$\therefore \overrightarrow{BE} = 2\bar{p} - \bar{q}$$

$$b) \quad \overrightarrow{FE} = \frac{2}{3}\overrightarrow{BE}$$

$$= \frac{2}{3}(2\bar{p} - \bar{q})$$

$$\therefore \overrightarrow{FE} = \frac{2}{3}(2\bar{p} - \bar{q})$$

$$c) \quad \overrightarrow{DF} = \overrightarrow{DE} + \overrightarrow{EF}$$

$$= \overrightarrow{DE} - \overrightarrow{FE}$$

$$= \bar{p} - \left[\frac{2}{3}(2\bar{p} - \bar{q})\right]$$

$$= \bar{p} - \frac{4}{3}\bar{p} + \frac{2}{3}\bar{q}$$

$$= \frac{2}{3}\bar{q} - \frac{1}{3}\bar{p}$$

$$\therefore \overrightarrow{DF} = \frac{1}{3}(2\bar{q} - \bar{p})$$

$$d) \quad \overrightarrow{DC} = h\overrightarrow{DF} = h\left[\frac{1}{3}(2\bar{q} - \bar{p})\right]$$

$$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC} = \bar{p} + h\left[\frac{1}{3}(2\bar{q} - \bar{p})\right]$$

$$= \bar{p} - \frac{h}{3}\bar{p} + \frac{2}{3}h\bar{q} = \left(1 - \frac{h}{3}\right)\bar{p} + \frac{2h}{3}\bar{q}$$

$$\therefore \overrightarrow{AC} = \left(1 - \frac{h}{3}\right)\bar{p} + \frac{2h}{3}\bar{q} \text{ (As required)}$$

QUESTION SEVEN

$$a) \quad \overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$$

$$= -\overrightarrow{OA} + \overrightarrow{OB}$$

$$= -3\bar{a} + 6\bar{b} = 3(2\bar{b} - \bar{a})$$

$$\therefore \overrightarrow{AB} = 3(2\bar{b} - \bar{a})$$

$$b) \quad \overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \frac{1}{3}\overrightarrow{AB}$$

$$= 3\bar{a} + \frac{1}{3}[3(2\bar{b} - \bar{a})]$$

$$= 3\bar{a} + 2\bar{b} - \bar{a} = 2\bar{a} + 2\bar{b}$$

$$\therefore \overrightarrow{OD} = 2(\bar{a} + \bar{b})$$

$$c) \quad \overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$$

$$= -\overrightarrow{OB} + \frac{2}{5}\overrightarrow{OA}$$

$$= -6\bar{b} + \frac{2}{5}(3\bar{a})$$

$$= \frac{6}{5}\bar{a} - 6\bar{b}$$

$$\therefore \overrightarrow{BC} = 6\left(\frac{1}{5}\bar{a} - \bar{b}\right)$$

$$d) \quad \overrightarrow{BE} = h\overrightarrow{BC}$$

$$= h\left[6\left(\frac{1}{5}\bar{a} - \bar{b}\right)\right]$$

$$= 6\left(\frac{1}{5}\bar{a} - \bar{b}\right)h$$

$$\therefore \overrightarrow{BE} = 6\left(\frac{1}{5}\bar{a} - \bar{b}\right)h$$

QUESTION EIGHT

$$\text{a). } \overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA}$$

$$= -\overrightarrow{OC} + \overrightarrow{OA}$$

$$= -2\bar{q} + 4\bar{p} = 4\bar{p} - 2\bar{q}$$

$$\therefore \overrightarrow{CA} = 2(2\bar{p} - \bar{q})$$

$$\text{b). } \overrightarrow{BA} = \overrightarrow{BC} + \overrightarrow{CA}$$

$$= -\overrightarrow{CB} + \overrightarrow{CA}$$

$$= -2\bar{p} + 4\bar{p} - 2\bar{q} = 2\bar{p} - 2\bar{q}$$

$$\therefore \overrightarrow{BA} = 2(\bar{p} - \bar{q})$$

$$\text{c). } \overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$$

$$= \overrightarrow{OA} - \frac{1}{2}\overrightarrow{BA}$$

$$= 4\bar{p} - \frac{1}{2}[2(\bar{p} - \bar{q})]$$

$$= 4\bar{p} - \bar{p} + \bar{q} = 3\bar{p} + \bar{q}$$

$$\therefore \overrightarrow{OM} = 3\bar{p} + \bar{q}$$

$$\text{d). } \overrightarrow{CX} = h\overrightarrow{CA}$$

$$= h[2(2\bar{p} - \bar{q})]$$

$$\therefore \overrightarrow{CX} = 2(2\bar{p} - \bar{q})h$$

$$\text{e). } \overrightarrow{OX} = \overrightarrow{OC} + \overrightarrow{CX}$$

$$= 2\bar{q} + 2(2\bar{p} - \bar{q})h$$

$$= 2\bar{q} + 4\bar{p}h - 2h\bar{q}$$

$$= 4h\bar{p} + 2\bar{q} - 2h\bar{q}$$

$$= 4h\bar{p} + 2(1 - h)\bar{q}$$

$$\therefore \overrightarrow{OX} = 4h\bar{p} + 2(1 - h)\bar{q} \text{ (As required)}$$

ARITHMETIC AND GEOMETRIC PROGRESION

QUESTION ONE

a). $a_n = a + (n - 1)d$

$$d = -7 - (-10) = 3, a = -10 \text{ and } n = 17$$

$$a_{17} = -10 + (17 - 1)3$$

$$= -10 + (16)3$$

$$= -10 + 48 = 38$$

$$\therefore a_{17} = \mathbf{38}$$

b). $a_n = a + (n - 1)d$

$$= -10 + (n - 1)3$$

$$= -10 + 3n - 3 = -13 + 3n$$

$$\therefore a_n = \mathbf{3n - 13}$$

c). $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{20} = \frac{20}{2}[2(-10) + (20 - 1)3]$$

$$= 10[-20 + (19)3]$$

$$= 10(-20 + 57)$$

$$= 10(37) = 370$$

$$\therefore S_{20} = \mathbf{370}$$

QUESTION TWO

a). $a = 25 \text{ and } d = 22 - 25 = -3$

$$a_n = a + (n - 1)d$$

$$= 25 + (n - 1)(-3)$$

$$= 25 - 3n + 3 = 28 - 3n$$

$$\therefore a_n = \mathbf{28 - 3n}$$

c). $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{20} = \frac{20}{2}[2(25) + (20 - 1)(-3)]$$

$$= 10[50 + 19(-3)]$$

$$= 10(50 - 57)$$

$$= 10(-7) = -70$$

$$\therefore S_{20} = \mathbf{-70}$$

b). $a_n = 28 - 3n$

$$a_{10} = 28 - 3(10)$$

$$= 28 - 30 = -2$$

$$\therefore a_{10} = \mathbf{-2}$$

QUESTION THREE

$$\text{a). } d = a_2 - a_1$$

$$= 7 - 5 = 2$$

$$\therefore d = 2$$

$$\text{b). } a_n = a + (n - 1)d$$

$$= 5 + (n - 1)2$$

$$= 5 + 2n - 2 = 3 + 2n$$

$$\therefore a_n = 2n + 3$$

$$\text{c). } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{12} = \frac{12}{2}[2(5) + (12 - 1)2]$$

$$= 6[10 + 11(2)]$$

$$= 6[10 + 22]$$

$$= 6(32) = 192$$

$$\therefore S_{12} = \mathbf{192}$$

QUESTION FOUR

$$\text{a). } a_n = a + (n - 1)d$$

$$a = 100 \text{ and } d = 95 - 100 = -5$$

$$a_{10} = 100 + (10 - 1)(-5)$$

$$= 100 + 9(-5) = 100 - 45 = 55$$

$$\therefore a_{10} = \mathbf{55}$$

$$\text{c). } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{10} = \frac{10}{2}[2(100) + (10 - 1)(-5)]$$

$$= 5[200 + 9(-5)]$$

$$= 5(200 - 45)$$

$$= 5(155) = 775$$

$$\therefore S_{10} = \mathbf{775}$$

$$\text{b). } a_n = a + (n - 1)d$$

$$= 100 + (n - 1)(-5)$$

$$= 100 - 5n + 5$$

$$= 105 - 5n$$

$$\therefore a_n = \mathbf{105 - 5n}$$

QUESTION FIVE

a). $a_n = a + (n - 1)d$

$$d = 13 - 11 = 2 \text{ and } a = 11$$

$$\begin{aligned} a_{13} &= 11 + (13 - 1)2 \\ &= 11 + 24 = 35 \end{aligned}$$

$$\therefore a_{13} = \mathbf{35}$$

c). $S_n = \frac{n}{2}[2a + (n - 1)d]$

$$S_{20} = \frac{20}{2}[2(11) + (20 - 1)2]$$

$$= 10[22 + (19)2]$$

$$= 10(22 + 38)$$

$$= 10(60) = 600$$

$$\therefore S_{20} = \mathbf{600}$$

b). $a_n = a + (n - 1)d$

$$\begin{aligned} &= 11 + (n - 1)2 \\ &= 11 + 2n - 2 = 9 + 2n \end{aligned}$$

$$\therefore a_n = \mathbf{2n + 9}$$

QUESTION SIX

$$\frac{5+c}{2} = 11$$

$$\frac{5+c}{2} = \frac{11}{1}$$

$$5 + c = 22$$

$$c = 22 - 5 = 17$$

$$\therefore c = \mathbf{17}$$

HINT: The arithmetic mean is computed by dividing the sum of the values or scores by their number (how many they are).

Cross multiply

QUESTION SEVEN

This sequence do not have a common difference. for such sequences, employ other means for example try and error method.

$$\begin{aligned}\text{Note that } a_1 &= 9 = 3^2 = (2 + 1)^2, \\ a_2 &= 16 = 4^2 = (2 + 2)^2, \\ a_3 &= 25 = 5^2 = (2 + 3)^2 \dots\end{aligned}$$

$$\text{a). } a_{10} = (2 + 10)^2 = 12^2 = 144$$

$$\therefore a_{10} = \mathbf{144}$$

$$\text{b). } a_n = (2 + n)^2$$

$$\therefore a_n = \mathbf{(2 + n)^2}$$

QUESTION EIGHT

$$\text{a). } a_n = a + (n - 1)d$$

$$a = 4 \text{ and } d = 7 - 4 = 3$$

$$\begin{aligned}a_{30} &= 4 + (30 - 1)3 \\ &= 4 + 29(3) = 4 + 87 = 91\end{aligned}$$

$$\therefore a_{30} = \mathbf{91}$$

$$\text{c). } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\begin{aligned}S_{24} &= \frac{24}{2}[2(4) + (24 - 1)3] \\ &= 12[8 + 23(3)] \\ &= 12(8 + 69) \\ &= 12(77) = 924\end{aligned}$$

$$\text{b). } a_n = a + (n - 1)d$$

$$\begin{aligned}&= 4 + (n - 1)3 \\ &= 4 + 3n - 3 = 1 + 3n\end{aligned}$$

$$\therefore a_n = \mathbf{3n + 1}$$

$$\therefore S_{24} = \mathbf{924}$$

QUESTION NINE

$$\begin{aligned} \text{a). } a_n &= a + (n - 1)d \\ d &= 9 - 7 = 2 \text{ and } a = 7 \\ a_{11} &= 7 + (11 - 1)2 \\ &= 7 + 20 = 27 \end{aligned}$$

$$\begin{aligned} \text{b). } a_n &= a + (n - 1)d \\ &= 7 + (n - 1)2 \\ &= 7 + 2n - 2 = 5 + 2n \\ \therefore a_n &= \mathbf{2n + 5} \end{aligned}$$

$$\text{c). } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\begin{aligned} S_{20} &= \frac{20}{2}[2(7) + (20 - 1)2] \\ &= 10[14 + 19(2)] \\ &= 10(14 + 38) = 10(52) = 520 \end{aligned}$$

$$\therefore S_n = \mathbf{520}$$

QUESTION TEN

This sequence do not have a common difference. for such sequences, employ other means for example try and error method.

$$\begin{aligned} a_1 &= 1^3 - 2(1) = -1, \\ a_2 &= 2^3 - 2(2) = 4, \\ a_3 &= 3^3 - 2(3) = 21 \dots \\ a_6 &= 6^3 - 2(6) = 216 - 12 = 204 \\ \therefore a_6 &= \mathbf{204} \text{ and } a_n = \mathbf{n^3 - 2n} \end{aligned}$$

QUESTION ELEVEN

HINT: since we know that the ratio of the terms in a GP is equal for every proceeding numbers that can be picked, we build our argument there and relate the two ratios to find the missing variable.

Where a is the first term,
 $T_1, T_2, T_3, \dots$ are $1^{\text{st}}, 2^{\text{nd}}, 3^{\text{rd}}, \dots$
 Terms respectively. And r is
 the common ratio.

$$a). \quad r = \frac{T_2}{T_1} = \frac{T_3}{T_2}, \quad \frac{10+n}{6+n} \neq \frac{15+n}{10+n}$$

$$(10+n)(10+n) = (6+n)(15+n)$$

$$100 + 20n + n^2 = 90 + 21n + n^2$$

$$100 + 20n + n^2 - 90 - 21n - n^2 = 0$$

$$10 - n = 0, \quad n = 10$$

$$\therefore n = 10$$

$$b). \quad G.P = 16, 20, 25, \dots$$

$$r = \frac{T_2}{T_1} = \frac{20}{16} = \frac{5}{4} = 1.25$$

$$\therefore r = 1.25$$

$$c). \quad S_n = \frac{a(r^n-1)}{r-1} \text{ for } r > 1$$

$$S_6 = \frac{16((1.25)^6-1)}{1.25-1}$$

$$= \frac{16(3.814697266-1)}{0.25}$$

$$= \frac{45.03515625}{0.25}$$

$$= 180.140625$$

$$\therefore S_6 = 180$$

QUESTION TWELVE

$$a). \quad r = \frac{T_2}{T_1} = \frac{5}{20} = \frac{1}{4}$$

$$\therefore r = \frac{1}{4} \text{ or } 0.25$$

$$b). \quad T_n = ar^{n-1} = 20 \left(\frac{1}{4}\right)^{n-1}$$

$$\text{where } a = 20 \text{ and } r = 0.25$$

$$= 20 \times \frac{1^{n-1}}{4^{n-1}} = \frac{20 \times 1}{4^{n-1}}$$

$$= \frac{20 \times 4}{4^n} = \frac{80}{4^n}$$

$$\therefore T_n = \frac{80}{4^n}$$

$$c). \quad S_n = \frac{a(1-r^n)}{1-r} \text{ for } r < 1$$

$$S_8 = \frac{20(1 - (0.25)^8)}{1 - 0.25}$$

$$= \frac{20(1 - 0.000015258)}{0.75}$$

$$= \frac{20(0.999984741)}{0.75}$$

$$= \frac{19.99969482}{0.75}$$

$$= 26.66625977$$

$$\therefore S_8 = 26.7$$

Rounding off must be at the end of calculations.

QUESTION THIRTEEN

$$\text{a). } r = \frac{\frac{2}{27}}{\frac{2}{9}} = \frac{2}{27} \times \frac{9}{2} = \frac{1}{3} = 0.3\bar{3}$$

$$\text{b). } S_n = \frac{a(1-r^n)}{1-r} \text{ for } r < 1$$

$$T_3 = ar^{3-1} = ar^2, \frac{2}{9} = a \times \left[\frac{1}{3}\right]^2$$

$$\frac{2}{9} = \frac{a1^2}{3^2} = \frac{a}{9}, \frac{2}{9} \neq \frac{a}{9}$$

$$\frac{9a}{9} = \frac{18}{9} = 2$$

$$\therefore a = 2 \text{ and } r = \frac{1}{3}$$

$$\text{c). } S_\infty = \frac{a}{1-r}$$

$$= \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}}$$

$$= 2 \times \frac{3}{2} = 3$$

$$\therefore S_\infty = 3$$

$$S_5 = \frac{2(1 - (0.3333)^5)}{1 - 0.3333}$$

$$= \frac{2(1 - 0.004115226)}{0.6666667}$$

$$= \frac{2(0.995884773)}{0.6666667}$$

$$= \frac{1.961769547}{0.6666667} = 2.987652827$$

$$\therefore S_5 = 3$$

QUESTION FOURTEEN

$$\text{a). } \frac{T_2}{T_1} = \frac{T_3}{T_2}, \frac{x-3}{x+1} = \frac{x-1}{x-3}$$

$$\text{b). } T_1 = x + 1 = \frac{5}{3} + 1$$

$$(x-3)(x-3) = (x+1)(x-1),$$

$$= \frac{5+3}{3}$$

$$x^2 - 6x + 9 = x^2 - 1 \quad x^2 - 6x + 9 - x^2 + 1 = 0$$

$$= \frac{8}{3} \text{ or } 2.6666667$$

$$-6x = -10, \frac{-6x}{-6} = \frac{-10}{-6}$$

$$\therefore T_1 = 2.67$$

$$x = \frac{5}{3} = 1.6666667$$

c). $S_{\infty} = \frac{a}{1-r}, a = \frac{8}{3} \text{ and}$

$$r = \frac{x-3}{x+1} = \frac{\frac{5}{3}-3}{\frac{5}{3}+1} = \frac{-\frac{4}{3}}{\frac{8}{3}}$$

$$= \frac{-4}{3} \times \frac{3}{8} = \frac{-1}{2}$$

$$S_{\infty} = \frac{\frac{8}{3}}{1 - \left(-\frac{1}{2}\right)} = \frac{\frac{8}{3}}{\frac{3}{2}}$$

$$= \frac{8}{3} \times \frac{2}{3} = \frac{16}{9}$$

$$\therefore S_{\infty} = 1\frac{7}{9}$$

QUESTION FIFTEEN

a). $\frac{T_2}{T_1} = \frac{T_3}{T_2}, \frac{k}{k+4} = \frac{2k-15}{k},$

$$k(k) = (k+4)(2k-15),$$

$$k^2 = 2k^2 - 7k - 60,$$

$$k^2 - 7k - 60 = 0$$

$$k^2 + 5k - 12k - 60 = 0$$

$$k(k+5) - 12(k+5) = 0$$

$$k+5 = 0 \text{ or } k-12 = 0,$$

$$k = -5 \text{ or } 12$$

$$\therefore k = 12 \text{ (since } k \text{ is positive)}$$

b). $T_1 = k + 4 = 12 + 4 = 16$

$$T_2 = k = 12$$

$$T_3 = 2k - 15 = 2(12) - 15 = 24 - 15 = 9$$

$\therefore$ The three terms are **16, 12 and 9**.

c). $S_{\infty} = \frac{a}{1-r}, a = 16 \text{ and}$

$$r = \frac{T_2}{T_1} = \frac{12}{16} = \frac{3}{4}$$

$$S_{\infty} = \frac{16}{1 - \frac{3}{4}} = \frac{16}{\frac{1}{4}}$$

$$= 16 \times 4 = 64$$

$$\therefore S_{\infty} = \mathbf{64}$$

QUESTION SIXTEEN

a). $a = 1, d = 5 - 1 = 4$ and $n = 10$

$$a_n = a + (n - 1)d$$

$$a_{10} = 1 + (10 - 1)4$$

$$= 1 + 9(4) = 1 + 36 = 37$$

$\therefore$ Pattern number 10 will have **37** floor tiles.

b). $m = a_n = a + (n - 1)d$

$$= 1 + (n - 1)4$$

$$= 1 + 4n - 4 = 4n - 3$$

$$\therefore m = 4n - 3$$

QUESTION SEVENTEEN

The numbers are 2, 4 and 8

$$\text{Geometric mean} = (2 \times 4 \times 8)^{\frac{1}{3}} = (64)^{\frac{1}{3}} = \sqrt[3]{64} = 4$$

$\therefore$ Geometric mean of 2, 4 and 8 is **4**.

QUESTION EIGHTEEN

$$\text{Geometric mean} = (12 \times x)^{\frac{1}{2}}$$

$$6 = (12x)^{\frac{1}{2}}$$

$$6^{1 \times 2} = (12x)^{\frac{1}{2} \times 2}$$

$$12x = 6^2 = 36$$

$$\frac{12x}{12} = \frac{36}{12} = 3$$

$$\therefore x = 3.$$

CIRCLE PROPERTY

$$\begin{aligned} 1. \text{ a). } \widehat{BYD} &= 180^\circ - \widehat{BY} \\ &= 180^\circ - 55^\circ = 125^\circ \\ \therefore \widehat{BYD} &= \mathbf{125^\circ} \end{aligned}$$

$$\begin{aligned} \text{b). } \widehat{CBY} + \widehat{BAD} + \widehat{ADC} + \widehat{DCB} &= 360^\circ \\ \widehat{BAD} &= 360^\circ - (\widehat{CBY} + \widehat{ADC} + \widehat{DCB}) \\ &= 360^\circ - [55^\circ + 140^\circ + (180^\circ - \widehat{BYD})] \\ &= 360^\circ - [195 + (180^\circ - 125^\circ)] \\ &= 360^\circ - 250^\circ = 110^\circ \end{aligned}$$

$$\therefore \widehat{BAD} = \mathbf{110^\circ}$$

$$\begin{aligned} 2. \text{ a). } 2\widehat{PRQ} &= \widehat{POQ} \\ \widehat{PRQ} &= \frac{60^\circ}{2} = 30^\circ \\ \therefore \widehat{PRQ} &= \mathbf{30^\circ} \end{aligned}$$

$$\begin{aligned} \text{b) } \widehat{OPQ} &= \widehat{PQO} \text{ and} \\ \widehat{POQ} + \widehat{OPQ} + \widehat{PQO} &= 180^\circ \\ 2\widehat{OPQ} + 60^\circ &= 180^\circ \\ 2\widehat{OPQ} &= 180^\circ - 60^\circ = 120^\circ \\ \widehat{OPQ} &= \frac{120^\circ}{2} = 60^\circ \\ \therefore \widehat{OPQ} &= \mathbf{60^\circ} \end{aligned}$$

$$\text{c) i). } \theta = 360^\circ - 60^\circ = 300^\circ, r = 5\text{cm}$$

$$\begin{aligned} \text{Lenth}(L) &= \frac{\pi \times r \times \theta}{180^\circ} \\ &= \frac{3.14 \times 5\text{cm} \times 300^\circ}{180^\circ} \\ &= \frac{4710^\circ\text{cm}}{180^\circ} = 26.166667\text{cm} \end{aligned}$$

$$\therefore \text{Lenth of arc } PRQ = \mathbf{26\text{cm}}$$

$$\text{ii). } A = \frac{\pi \times r^2 \times \theta}{360^\circ}, \text{ where } r = 5\text{cm}, \theta = 60^\circ$$

$$A = \frac{3.14 \times (5\text{cm})^2 \times 60^\circ}{360^\circ}$$

$$= \frac{78.50\text{cm}^2 \times 60^\circ}{360^\circ}$$

$$= \frac{4710^\circ\text{cm}^2}{360^\circ} = 13.083333333$$

$$\therefore \text{Area of sector } OPQ = \mathbf{13.08\text{cm}^2}$$

$$\begin{array}{lll}
 3 \quad \text{a). } \angle t = 2\hat{B}\hat{A}\hat{D} & \text{b). } \hat{B}\hat{C}\hat{D} + \hat{B}\hat{A}\hat{D} = 180^\circ & \text{c). } \hat{O}\hat{B}\hat{C} + \hat{B}\hat{C}\hat{D} = 180^\circ \\
 = 2 \times 62^\circ = 124^\circ & & \\
 \therefore \text{Angle } t = \mathbf{124^\circ} & \hat{B}\hat{C}\hat{D} = 180^\circ - \hat{B}\hat{A}\hat{D} & \hat{O}\hat{B}\hat{C} = 180^\circ - \hat{B}\hat{C}\hat{D} \\
 & = 180^\circ - 62^\circ = 118^\circ & = 180^\circ - 118^\circ = 62^\circ \\
 & \therefore \hat{B}\hat{C}\hat{D} = \mathbf{118^\circ} & \therefore \hat{B}\hat{C}\hat{D} = \mathbf{62^\circ}
 \end{array}$$

$$\begin{array}{ll}
 4 \quad \text{a). } \hat{B}\hat{F}\hat{D} + \hat{D}\hat{F}\hat{E} = 90^\circ & \text{b). } \hat{D}\hat{B}\hat{C} = 90^\circ - \hat{D}\hat{B}\hat{E} \\
 \hat{B}\hat{F}\hat{D} = 90^\circ - \hat{D}\hat{F}\hat{E} & = 90^\circ - \hat{D}\hat{F}\hat{E} \\
 = 90^\circ - 35^\circ = 55^\circ & = 90^\circ - 35^\circ = 55^\circ \\
 \therefore \hat{B}\hat{F}\hat{D} = \mathbf{55^\circ} & \\
 & \therefore \hat{D}\hat{B}\hat{C} = \mathbf{55^\circ}
 \end{array}$$

$$\begin{array}{ll}
 \text{c). } \hat{F}\hat{E}\hat{D} = \hat{F}\hat{E}\hat{B} + \hat{B}\hat{E}\hat{D} & \text{d). } \hat{F}\hat{D}\hat{B} = 90^\circ - \hat{E}\hat{D}\hat{F} \\
 = (90^\circ - \hat{F}\hat{B}\hat{E}) + (90^\circ - \hat{D}\hat{B}\hat{E}) & = 90^\circ - (180^\circ - (123^\circ + 35^\circ)) \\
 = (90^\circ - 22^\circ) + (90^\circ - 35^\circ) & = 90^\circ - (180^\circ - 158^\circ) \\
 = 68^\circ + 55^\circ = 123^\circ & = 90^\circ - 22^\circ = 68^\circ \\
 \therefore \hat{F}\hat{E}\hat{D} = \mathbf{123^\circ} & \therefore \hat{F}\hat{D}\hat{B} = \mathbf{68^\circ}
 \end{array}$$

$$\begin{array}{l}
 5. \text{ A pentagon has } 540^\circ \text{ thus} \\
 y^\circ + 2y^\circ + 3y^\circ + 4y^\circ + 40^\circ = 540^\circ \\
 10y^\circ + 40^\circ = 540^\circ \\
 10y^\circ = 540^\circ - 40^\circ = 500^\circ \\
 \frac{10y^\circ}{10^\circ} = \frac{500^\circ}{10^\circ} = 50 \\
 \therefore y = \mathbf{50}(\text{unit less})
 \end{array}$$

$$\begin{array}{ll}
 6. \quad \text{a). } \hat{A}\hat{O}\hat{D} + 2\hat{O}\hat{A}\hat{D} = 180^\circ & \text{c). } \hat{O}\hat{D}\hat{B} = \hat{A}\hat{D}\hat{O} + \hat{O}\hat{D}\hat{B} + \hat{B}\hat{D}\hat{C} = 90^\circ \\
 \hat{A}\hat{O}\hat{D} = 180^\circ - 2 \times 42^\circ & \\
 = 180^\circ - 84^\circ = 96^\circ & \hat{O}\hat{D}\hat{B} = 90^\circ - (\hat{A}\hat{D}\hat{O} + \hat{B}\hat{D}\hat{C}) \\
 \therefore \hat{A}\hat{O}\hat{D} = \mathbf{96^\circ} & = 90^\circ - (42^\circ + 36^\circ) \\
 & = 90^\circ - 78^\circ = 12^\circ \\
 \text{b). } \hat{A}\hat{B}\hat{C} \text{ is an angle in a semi-circle} & \\
 \therefore \hat{A}\hat{B}\hat{C} = \mathbf{90^\circ} & \therefore \hat{O}\hat{D}\hat{B} = \mathbf{12^\circ}
 \end{array}$$

7. a). A tangent to a circle is perpendicular to the radius at the point of contact

b). $Diameter = 9cm, r = 4.5cm$ and $OT = 4.5cm + 3cm = 7.5cm$

$$OT^2 = OC^2 + CT^2$$

$$CT^2 = OT^2 - OC^2$$

$$= (7.5)^2 - (4.5)^2$$

$$= 56.25 - 20.25 = 36$$

$$\sqrt{(CT)^2} = \sqrt{36} = \pm 6$$

$\therefore CT = 6cm$ (distance is always Positive).

8. a) $P\hat{Q}S + S\hat{Q}T + T\hat{Q}R = 180^\circ$

$$T\hat{Q}R = 180^\circ - (P\hat{Q}S + S\hat{Q}T)$$

$$= 180^\circ - (90^\circ + 42^\circ)$$

$$= 180^\circ - 132^\circ = 48^\circ$$

$$\therefore T\hat{Q}R = 48^\circ$$

b). $P\hat{T}S = Q\hat{P}T$

$$= \frac{1}{2}[180^\circ - (90^\circ + 42^\circ)]$$

$$= \frac{48^\circ}{2} = 24^\circ$$

$$\therefore P\hat{T}S = 24^\circ$$

c). $P\hat{U}S = 180^\circ - S\hat{U}T$

$$= 180^\circ - [180^\circ - (Q\hat{S}T + S\hat{T}P)]$$

$$= 180^\circ - [180^\circ - (42^\circ + 24^\circ)]$$

$$= 180^\circ - 114^\circ = 66^\circ$$

$$\therefore P\hat{U}S = 66^\circ$$

9. a). $B\hat{O}D = 180^\circ - D\hat{O}E$

$$= 180^\circ - (180^\circ - 50^\circ) = 50^\circ$$

$$\therefore B\hat{O}D = 50^\circ$$

b). $C\hat{B}D = 180^\circ - D\hat{B}E$

$$= 90^\circ - 65^\circ = 25^\circ$$

$$\therefore C\hat{B}D = 25^\circ$$

10. a). $P\hat{R}S = P\hat{Q}S = 26^\circ$

$$\therefore P\hat{R}S = 26^\circ$$

b). $P\hat{R}Q + Q\hat{P}R = 90^\circ$

$$P\hat{R}Q = 90^\circ - Q\hat{P}R$$

$$= 90^\circ - 42^\circ = 48^\circ$$

$$\therefore P\hat{R}Q = 48^\circ$$

c). $D\hat{B}F = D\hat{B}E + E\hat{B}F$

$$= 65^\circ + 68^\circ = 133^\circ$$

$$\therefore D\hat{B}F = 133^\circ$$

c). $P\hat{T}Q + P\hat{Q}T + Q\hat{P}T = 180^\circ$

$$P\hat{T}Q = 180^\circ - (P\hat{Q}T + Q\hat{P}T)$$

$$= 180^\circ - (26^\circ + 132^\circ)$$

$$= 180^\circ - 158^\circ = 22^\circ$$

$$\therefore P\hat{T}Q = 22^\circ$$

11. a). the angle which the tangent makes with the cord is equal to any angle in the alternate segment thus

$$R\hat{W}X = W\hat{V}X = X\hat{Y}W = 44^\circ$$

$$\therefore R\hat{W}X = 44^\circ$$

b). $X\hat{V}W = X\hat{Y}W = 44^\circ$

$$\therefore X\hat{V}W = 44^\circ$$

c). $Y\hat{X}W = Y\hat{X}V + W\hat{X}V$

$$= Y\hat{W}V + S\hat{W}V$$

$$= 54^\circ + 39^\circ = 93^\circ$$

$$\therefore Y\hat{X}W = 93^\circ$$

PROPORTIONS AND VARIATIONS

1. The areas are in the ratio $2^2:3^2 \rightarrow 4:9$.

$$4 \rightarrow 9,$$



$$80 \rightarrow x$$

$$4x = 9 \times 80$$

$$x = \frac{9 \times 80}{4} = 9 \times 20 = \mathbf{180}.$$

$$\therefore \text{area of the bigger container is } \mathbf{180cm^2}$$

2. a). $y \propto \frac{z}{w^2}, y = k \frac{z}{w^2},$

$$w^2 y = kz, \frac{kz}{z} = \frac{w^2 y}{z},$$

$$k = \frac{w^2 y}{z} = \frac{3^2 \times 6}{2} = \frac{54}{2} = 27$$

$$\therefore k = \mathbf{27}$$

b). $y = \frac{kz}{w^2} = \frac{27 \times 6}{9 \times 9}$

$$= \frac{162}{81} = 2$$

$$\therefore y = \mathbf{2}$$

c). $y = \frac{kz}{w^2}, yw^2 = kz$

$$\frac{yw^2}{y} = \frac{kz}{y}, w^2 = \frac{kz}{y}$$

$$w = \sqrt{\frac{kz}{y}} = \sqrt{\frac{27 \times 4}{3}}$$

$$= \sqrt{36} = \pm 6$$

$$\therefore w = \mathbf{6 \text{ or } -6}$$

3. a). Parents that chose banana 90° , fraction of parents who chose

$$\text{banana} = \frac{\text{parents chose banana}}{\text{total number of parents}} = \frac{90}{360} = \frac{1}{4}$$

$$\therefore \text{Fraction of parents who chose banana} = \frac{1}{4}$$

b). Parents that chose mango $= \frac{81}{360} \times 600 = \mathbf{135}$

$$\text{parents that chose banana} = \frac{90}{360} \times 600 = \mathbf{150}$$

$$\therefore \text{parents that chose Banana than Mango} = 150 - 135 = \mathbf{15}$$

4. Cost price(CP) = $\left[\frac{100}{100 - \text{loss}\%} \right] \times \text{Selling price(SP)}$

$$CP = \left[\frac{100}{100 - 30} \right] \times 42000 = 1.428571429 \times 42\,000 = 60\,000$$

$\therefore$ Cost price of the bag is **K60 000**.

5. a). Naluba : Kwesha

45: 120

3: 8

b). for Naluba = 25%,

$$\frac{25}{100} \times x = 45$$

$$25x = 4500$$

$$\frac{25x}{25} = \frac{4500}{25} = 180$$

$\therefore$ Deligates voted = **180**

c). Total = 120 + 45 + x = 180

$$x = 180 - 165 = 15$$

$$\therefore x = \mathbf{15}$$

6. a). 120 $\begin{array}{c} \xrightarrow{\quad} 5 \\ \xrightarrow{\quad} 2 \end{array}$

x $\begin{array}{c} \xrightarrow{\quad} 5 \\ \xrightarrow{\quad} 2 \end{array}$

$$240 = 5x \cdot \frac{5x}{5} = \frac{240}{5} = 48$$

$\therefore$ **48** people were treated for measles.

b). Treated for TB,

$$360 = 5x \cdot \frac{5x}{5} = \frac{360}{5} = 72$$

$\therefore$ Treated for TB = **72**

120 $\begin{array}{c} \xrightarrow{\quad} 5 \\ \xrightarrow{\quad} 3 \end{array}$

x $\begin{array}{c} \xrightarrow{\quad} 5 \\ \xrightarrow{\quad} 3 \end{array}$

Total treated = Treated for Malaria +
treated for measles + treated for TB

$$= 120 + 48 + 72 = 240$$

$\therefore$ Total treated = **240**

7. a). $x \propto \frac{z}{y^2}, x = k \frac{z}{y^2},$

$$xy^2 = kz, \frac{kz}{z} = \frac{xy^2}{z},$$

$$k = \frac{xy^2}{z} = \frac{2 \times 3^2}{4} = \frac{9}{2}$$

$\therefore k = \frac{9}{2}$ is the constant
of variation

b). $x = k \frac{z}{y^2}, xy^2 = kz$

$$y^2 = \frac{kz}{x} \text{ and}$$

$$y = \sqrt{\frac{kz}{x}}$$

$$= \sqrt{\frac{9}{2} \times \frac{24}{3}} = \sqrt{36} = \pm 6$$

$\therefore y = \mathbf{6}$ or $y = \mathbf{-6}$

8. $4:9 \longrightarrow 2^2:3^2$ the bottles are in the ratio $2:3$ and the ratio of their volumes is $2^3:3^3 \longrightarrow 8:27$
 $\therefore$ the ratio of their volumes is $8:27$

HINT: If the ratio of the two solids is $a:b$, the ratio of their surface areas is $a^2:b^2$
 and the ratio of their volumes is $a^3:b^3$

9. a). $x \propto \frac{y}{z^2}, x = k \frac{y}{z^2},$
 $xz^2 = ky, \frac{ky}{y} = \frac{xz^2}{y},$
 $k = \frac{xz^2}{y} = \frac{12 \times 2^2}{3}$
 $= 4 \times 4 = 16$
 $\therefore x = 16 \frac{y}{z^2}$ is the equation connecting x, y and z .

b). $x = 16 \frac{y}{z^2} = \frac{16 \times 3}{2^2}$
 $= \frac{48}{4} = 12$
 $\therefore x = 12$

c). $x = 16 \frac{y}{z^2}, xz^2 = ky$
 $\frac{xz^2}{x} = \frac{ky}{x} \quad z^2 = \frac{ky}{x}$
 $z = \sqrt{\frac{ky}{x}} = \sqrt{\frac{16 \times 25}{4}}$
 $= \sqrt{100} = \pm 10$
 $\therefore z = 10 \text{ or } z = -10$

10. Find the difference in the readings and divide the charge by the difference in the reading to get the rate/unit.

$$\text{Present reading} - \text{previous reading} = 74279 - 71879 = 2400$$

$$\text{Rate/unit} = \frac{184\,800.00}{2\,400} = 77$$

$$\therefore \text{Cost per unit} = \mathbf{K77.00}$$

11. $\text{Bricks to order} = \left[\frac{100}{100 - \text{Loss}\%} \right] \times \text{Bricks needed}$
 $= \left[\frac{100}{100 - 4} \right] \times 48000 = 1.041666667 \times 48000 = 50000$
 $\therefore$ bricks to order is **50 000**.

12. Total money collected = $2 \times$ money that Charity collected
 $= 2 \times \text{K}2\,500.00 = \text{K}5\,000.00$

1 Ticket sold $\longrightarrow$ K50.00

$x \longrightarrow$ K5 000.00

$$1 \text{ Ticket sold} \times \text{K}5\,000.00 = x \times \text{K}50.00$$

$$x = \frac{x \times \text{K}50.00}{\text{K}50.00} = \frac{1 \text{ Ticket sold} \times \text{K}5\,000.00}{\text{K}50.00} = 100 \text{ Tickets sold.}$$

$$\therefore \text{Tickets that were sold is } \mathbf{100}.$$

13. Find the ratios separately, sum them up to get the new ratio.

$$13:5 \text{ and } 8:10 \quad (13 + 8): (5 + 10)$$

$$21: 15$$

$$7: 5$$

$\therefore 7:5$ is the new ratio of those who took Fanta to those who did not.

14. a). $y \propto \frac{x}{z^2}, y = k \frac{x}{z^2}$

$$yz^2 = kx$$

$$\frac{kx}{x} = \frac{yz^2}{x}$$

$$k = \frac{yz^2}{x} = \frac{10 \times 4^2}{32}$$

$$= \frac{160}{32} = 5$$

$$\therefore k = 5$$

b). $y = k \frac{x}{z^2} = 5 \times \frac{20}{5^2}$

$$= \frac{20}{5} = 4$$

$$\therefore y = 4$$

c). $y = k \frac{x}{z^2}, yz^2 = kx$

$$\frac{yz^2}{y} = \frac{kx}{y}, z^2 = \frac{kx}{y} \text{ or}$$

$$z = \sqrt{\frac{kx}{y}} = \sqrt{\frac{5 \times 9}{5}}$$

$$= \sqrt{9} = \pm 3$$

15. Find their ratio of their areas first and relate them to find the area of the bigger solid.

$$64:27 \rightarrow 4^3:3^3 \text{ the two solids are in the ratio } 4:3.$$

$$\text{And their areas is in the ratio } 4^2:3^2 \rightarrow 16:9$$

$$\begin{array}{ccc} 16 & \longrightarrow & x \\ & \searrow & \nearrow \\ 9 & & 180 \end{array} \quad 16 \times 180 = 9 \times x$$

$$\frac{9 \times x}{9} = \frac{16 \times 180}{9} = 16 \times 20 = 320$$

$\therefore$ Surface area of the bigger solid = **320cm²**.

16. a). $y \propto x^2 + 1, y = k(x^2 + 1)$

$$\frac{k(x^2+1)}{(x^2+1)} = \frac{y}{(x^2+1)}$$

$$k = \frac{y}{(x^2+1)} = \frac{20}{2^2+1}$$

$$= \frac{20}{5} = 4$$

$$\therefore k = 4$$

b). $y = k(x^2 + 1), y = 4(x^2 + 1)$

$\therefore y = 4(x^2 + 1)$ is the equation connecting x and y .

c). $y = k(x^2 + 1), \frac{k(x^2+1)}{k} = \frac{y}{k}$

$$x^2 + 1 = \frac{y}{k}, \quad x^2 = \frac{y}{k} - 1$$

$$\sqrt{x^2} = \sqrt{\frac{y}{k} - 1}$$

$$x = \sqrt{\frac{y}{k} - 1} = \sqrt{\frac{104}{4} - 1}$$

$$= \sqrt{26 - 1} = \sqrt{25} = \pm 5$$

$\therefore$ values of a are 5 and -5

HINT: If the ratio of the two solids is $a:b$, the ratio of their surface areas is $a^2:b^2$

and the ratio of their volumes is

$$a^3:b^3$$

$$17. a). t = kv^2, \quad b). t = kv^2, \quad \frac{kv^2}{k} = \frac{t}{k}$$

$$k = \frac{t}{v^2}$$

$$= \frac{4}{1^2} = 4$$

$$\therefore k = 4$$

$$v^2 = \frac{t}{k} \quad \sqrt{v^2} = \sqrt{\frac{t}{k}}$$

$$v = \sqrt{\frac{t}{k}} = \sqrt{\frac{36}{4}}$$

$$= \sqrt{9} = \pm 3$$

$\therefore$ values of b are **3 and -3**

$$18. \quad \frac{15}{100} \times x = 120,$$

$$0.15x = 120$$

$$\frac{0.15x}{0.15} = \frac{120}{0.15} = 800$$

$\therefore$ the price of the table was **K800.00** before reduction

$$19. a). y \propto x^3, y = kx^3$$

$$\frac{kx^3}{x^3} = \frac{y}{x^3}$$

$$k = \frac{y}{x^3} = \frac{250}{10^3}$$

$$= \frac{250}{1000} = 0.25$$

$$\therefore k = 0.25$$

$$b). y = kx^3 = 0.25 \times 4^3$$

$$= 0.25 \times 64 = 16$$

$$\therefore y = 16$$

$$c). y = kx^3, \frac{kx^3}{k} = \frac{y}{k}$$

$$x^3 = \frac{y}{k} \quad \sqrt[3]{x^3} = \sqrt[3]{\frac{y}{k}}$$

$$x = \sqrt[3]{\frac{y}{k}} = \sqrt[3]{\frac{54}{0.25}}$$

$$= \sqrt[3]{216} = 6$$

$$\therefore x = 6$$

$$20. a). y \propto \frac{1}{\sqrt{x}},$$

$$\therefore y = \frac{k}{\sqrt{x}}$$

$$b). i) y = \frac{k}{\sqrt{x}}$$

$$k = y \times \sqrt{x}$$

$$= 2 \times \sqrt{16}$$

$$= 2 \times 4 = 8$$

$$\therefore k = 8$$

$$ii). y = \frac{k}{\sqrt{x}} \quad y\sqrt{x} = k$$

$$\frac{y\sqrt{x}}{y} = \frac{k}{y} \quad (\sqrt{x})^2 = \left(\frac{k}{y}\right)^2$$

$$x = \left(\frac{k}{y}\right)^2 = \left(\frac{8}{\frac{8}{9}}\right)^2$$

$$= \left(8 \times \frac{9}{8}\right)^2 = 9^2 = 81$$

$$\therefore a = 81$$

HINT: find a value say x , whose 15% is 120. That value will be the price of the good under discussion before the reduction.

CHECK: note that the corresponding values of x and y are giving the same solution. Chose corresponding values of your choice.

21. a). $y \propto x^2, y = kx^2$

$$\frac{k \times x^2}{x^2} = \frac{y}{x^2}$$

$$k = \frac{y}{x^2} = \frac{96}{4^2}$$

$$= \frac{96}{16} = 6$$

$$\therefore k = 6$$

b). $y = kx^2 = 6 \times 5^2$

$$= 6 \times 25 = 150$$

$$\therefore y = 150$$

c). $y = kx^2, \frac{k \times x^2}{k} = \frac{y}{k}$

$$x^2 = \frac{y}{k}, \quad \sqrt[2]{x^2} = \sqrt[2]{\frac{y}{k}}$$

$$x = \sqrt[2]{\frac{y}{k}} = \sqrt[2]{\frac{24}{6}}$$

$$= \sqrt[2]{4} = \pm 2$$

$\therefore$ values of x are **2** and **-2**

22.a). $w \propto \frac{x^2}{y}$

$$\therefore w = k \frac{x^2}{y}$$

b). $w = k \frac{x^2}{y}, wy = kx^2$

$$\frac{kx^2}{x^2} = \frac{wy}{x^2}$$

$$k = \frac{wy}{x^2} = \frac{(-6) \times 15}{12^2}$$

$$= \frac{-90}{144} = -\frac{5}{8}$$

$$\therefore k = -\frac{5}{8}$$

c). $w = k \frac{x^2}{y}, wy = kx^2$

$$\frac{wy}{w} = \frac{kx^2}{w}$$

$$y = \frac{kx^2}{w} = \frac{-5}{8} \times \frac{8^2}{20}$$

$$= \frac{-5 \times 8}{20} = \frac{-8}{4} = -2$$

$$\therefore y = -2$$

FUNCTIONS AND SYSTEMS OF EQUATION

1. $f(x) = \frac{5x+4}{5}$ and $g(x) = x - 1$

a). let $y = f(x)$,

then $y = \frac{5x+4}{5}$,

$$x = \frac{5y+4}{5}, 5x = 5y + 4$$

$$5y = 5x - 4$$

$$\frac{5y}{5} = \frac{5x-4}{5}$$

$$y = \frac{5x-4}{5}$$

$$\therefore f^{-1}(x) = \frac{5x-4}{5}$$

b). $f^{-1}(2) = \frac{5(-2)-4}{5}$

$$= \frac{-10-4}{5} = \frac{-14}{5}$$

$$\therefore f^{-1}(2) = -2\frac{4}{5}$$

c). $f[g(x)] = \frac{5(x-1)+4}{5}$

$$= \frac{5x-5+4}{5}$$

$$= \frac{5x-1}{5}$$

$$\therefore f[g(x)] = \frac{5x-1}{5}$$

2. a). $x^2 = 3x$
 $x^2 - 3x = 0$
 $x(x - 3) = 0$
 $x = 0$ or $x - 3 = 0, x = 3$
 $\therefore x = 3$ or $x = 0$

b). $25^x = 5$
 $(5^2)^x = 5^1$
 $5^{2x} = 5^1$
 $2x = 1$
 $\frac{2x}{2} = \frac{1}{2}, x = \frac{1}{2}$
 $\therefore x = \frac{1}{2}$ or **0.5**

HINT: resolve it to a common base and equate the powers to get the value of x

3. $g: x \rightarrow \frac{x-1}{2}$ and $f(x) = 3x - 5$

a). let $y = \frac{x-1}{2}$
 $x = \frac{y-1}{2}$
 $2x = y - 1$
 $y = 2x + 1$

$\therefore g^{-1}(x) = 2x + 1$

HINT: equate the two functions, cross multiply and find the value of x

b). $g(x) = f(x)$
 $\frac{x-1}{2} = 3x - 5$
 $x - 1 = 6x - 10$
 $x - 6x = -10 + 1$
 $-5x = -9$
 $\frac{-5x}{-5} = \frac{-9}{-5} = 1\frac{4}{5}$
 $\therefore x = 1\frac{4}{5}$

b). $g[f(x)] = \frac{3x-5-1}{2} = \frac{3x-6}{2}$

let $y = \frac{3x-6}{2}, x = \frac{3y-6}{2}$

$2x = 3y - 6$

$3y = 2x + 6$

$\frac{3y}{3} = \frac{2x+6}{3}$
 $y = \frac{2x+6}{3}$

$\therefore g^{-1}[f(x)] = \frac{2x+6}{3}$

4. $f(x) = 2x + 1$ and $g(x) = 5x - 1$

a). let $y = 5x - 1$
 $x = \frac{y+1}{5}$
 $5y = x + 1$
 $\frac{5y}{5} = \frac{x+1}{5}$
 $y = \frac{x+1}{5}$

$\therefore g^{-1}(x) = \frac{x+1}{5}$

b). $f[g(x)] = 2(5x - 1) + 1$
 $= 10x - 2 + 1$
 $= 10x - 1$
 $\therefore f[g(x)] = 10x - 1$

HINT: replace $g(x)$ for x in $f(x)$ and simplify.

c). $f[g(-3)] = 10(-3) - 1$
 $= -30 - 1$
 $= -31$

$\therefore f[g(-3)] = -31$

5. a). $y = (1 - 2x)(1 - x) - 2$

when $y = -2$

$$y = 1 - x - 2x + 2x^2 - 2$$

$$2x^2 - 3x - 1 = -2$$

$$2x^2 - 3x = -1$$

$$x^2 - \frac{3}{2}x = -\frac{1}{2}$$

$$\left(x - \frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^2 = -\frac{1}{2}$$

$$\left(x - \frac{3}{4}\right)^2 = -\frac{1}{2} + \frac{9}{16} = \frac{1}{16}$$

$$x = \pm \frac{1}{4} + \frac{3}{4} = 1 \text{ or } \frac{1}{2}$$

$$\therefore x = 1 \text{ or } x = \frac{1}{2}$$

b). $(2x - 1)^2 = 25$

$$\sqrt{(2x - 1)^2} = \sqrt{25}$$

$$2x - 1 = \pm 5$$

$$2x - 1 = 5 \text{ or } 2x - 1 = -5$$

$$2x = 6 \text{ or } 2x = -4$$

$$\frac{2x}{2} = \frac{6}{2} = 3 \text{ or } \frac{2x}{2} = \frac{-4}{2} = -2$$

$$\therefore x = 3 \text{ or } x = -2$$

c). $2x^3 - 60 = -6$

$$2x^3 = -6 + 60 = 54$$

$$\frac{2x^3}{2} = \frac{54}{2}$$

$$x^3 = 27$$

$$\sqrt[3]{x^3} = \sqrt[3]{27} = 3$$

$$\therefore x = 3$$

6. $f(x) = 2x - 5$ and $g(x) = x + 3$

a). let $y = 2x - 5$

$$x = 2y - 5$$

$$2y = x + 5$$

$$\frac{2y}{2} = \frac{x+5}{2}$$

$$y = \frac{x+5}{2}$$

$$\therefore f^{-1}(x) = \frac{x+5}{2}$$

b). $f^{-1}(x) = \frac{x+5}{2}$

$$f^{-1}(-10) = \frac{-10+5}{2}$$

$$= \frac{-5}{2} = -2\frac{1}{2}$$

$$\therefore f^{-1}(-10) = -2\frac{1}{2}$$

c). $f[g(x)] = 2(x + 3) - 5$

$$= 2x + 6 - 5$$

$$= 2x + 1$$

$$\therefore f[g(x)] = 2x + 1$$

7. $x^2 - 4x - 2 = 0$ $a = 1, b = -4$ and $c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2(1)} = \frac{4 \pm \sqrt{16+8}}{2}$$

$$= \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 4.898979486}{2}$$

$$= \frac{4+4.898979486}{2} \text{ or } \frac{4-4.898979486}{2}$$

$$= \frac{8.898979486}{2} \text{ or } \frac{0.898979486}{2}$$

$$= 4.449489743 \text{ or } 0.449489742$$

$$\therefore x = 4.45 \text{ or } 0.45 \text{ (Rounded off to 2d.p)}$$

NOTE: rounding off must be at the end of the calculation.

8. a). $2^{2x-1} = 16^{-2x}$

$$2^{2x-1} = (2^4)^{-2x}$$

$$2^{2x-1} = 2^{-8x}$$

$$2x - 1 = -8x$$

$$2x + 8x = 1$$

$$10x = 1$$

$$\frac{10x}{10} = \frac{1}{10}$$

$$\therefore x = \frac{1}{10}$$

b). $x^{-3} = 4$

$$x^{-3 \times (-\frac{1}{3})} = 4^{1 \times (-\frac{1}{3})}$$

$$x = 4^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{4}}$$

$$\therefore x = \frac{1}{\sqrt[3]{4}}$$

c). $x^3 - 6x^2 = 16x$

$$x^3 - 6x^2 - 16x = 0$$

$$x(x^2 - 6x - 16) = 0$$

$$x = 0 \text{ or } x^2 - 6x - 16 = 0$$

$$x^2 + 2x - 8x - 16 = 0$$

$$x(x + 2) - 8(x + 2) = 0$$

$$x + 2 = 0 \text{ or } x - 8 = 0$$

$$x = -2 \text{ or } x = 8$$

$$\therefore x = 0 \text{ or } x = -2 \text{ or } x = 8$$

9. $f(x) = 8x$ and $g(x) = \frac{3x-2}{4}$

a). let $y = 8x, x = 8y$

$$\frac{8y}{8} = \frac{x}{8}$$

$$y = \frac{x}{8}$$

$$\therefore f^{-1}(x) = \frac{x}{8}$$

b). $f[g(x)] = 8 \left[\frac{3x-2}{4} \right]$

$$= \frac{8}{4} (3x - 2)$$

$$= 2(3x - 2)$$

$$= 6x - 4$$

$$\therefore f[g(x)] = 6x - 2$$

c). $f[g(x)] = 20$

$$6x - 4 = 20$$

$$6x = 20 + 4 = 24$$

$$\frac{6x}{6} = \frac{24}{6}$$

$$x = 4$$

$$\therefore x = 4$$

10. $3x^2 + 5x + 2 = 0$

$$3x^2 + 3x + 2x + 2 = 0$$

$$3x(x + 1) + 2(x + 1) = 0$$

$$(3x + 2)(x + 1) = 0$$

$$3x + 2 = 0 \text{ or } x + 1 = 0$$

$$\frac{3x}{3} = \frac{-2}{3}$$

$$x = -\frac{2}{3} \text{ or } x = -1$$

$$\therefore x = -\frac{2}{3} \text{ or } x = -1$$

11. $2y = 3x - 12 \quad \left| \begin{array}{l} \times 3 \\ + \end{array} \right| \quad \begin{array}{l} 6y - 9x = -36 \\ 5x - 6y = 16 \end{array}$

$$[6y + (-6y)] - 9x + 5x = -20$$

$$-4x = -20, \frac{-4x}{-4} = \frac{-20}{-4} = 5,$$

$$x = 5$$

replacing for x in one equation, we have

$$2y - 3(5) = -12$$

$$2y = -12 + 15 = 3$$

$$\frac{2y}{2} = \frac{3}{2}, y = 1\frac{1}{2}$$

$$\therefore x = 5 \text{ and } y = 1\frac{1}{2}$$

12. $(2x - 1)(3x - 2) = 3$

$$6x^2 - 7x + 2 = 3$$

$$6x^2 - 7x - 1 = 0$$

$a = 6, b = -7 \text{ and } c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-1)}}{2(6)}$$

$$= \frac{7 \pm \sqrt{49 + 24}}{12} = \frac{7 \pm \sqrt{73}}{12}$$

$$= \frac{7 \pm 8.544003745}{12}$$

$$= \frac{15.544003745}{12} \text{ or } \frac{-1.544003745}{12}$$

$$= 1.295333645 \text{ or } -0.128666978$$

$\therefore x = 1.30 \text{ or } x = -0.13$

14. $2x + y = 1 \quad \times 2 \quad 4x + 3x + 2y - 2y = 2 + 9$

$$3x - 2y = 9$$

$$\left(\begin{array}{l} 4x + 2y = 2 \\ 3x - 2y = 9 \end{array} \right) +$$

$$7x = 11$$

$$\frac{7x}{7} = \frac{11}{7}$$

$$x = 1\frac{4}{7}$$

15. $2y^2 + 6y - 1 = 0$

$a = 2, b = 6 \text{ and } c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(6) \pm \sqrt{6^2 - 4(2)(-1)}}{2(2)}$$

$$= \frac{-6 \pm \sqrt{36 + 8}}{4} = \frac{-6 \pm \sqrt{44}}{4}$$

$$= \frac{-6 \pm 6.633249581}{4}$$

$$= \frac{0.63324958}{4} \text{ or } \frac{-12.63324958}{4}$$

$$= 0.158312395 \text{ or } -3.15831239$$

$\therefore x = 0.16 \text{ or } -3.16$

13. a). $f(x) = \frac{3}{5x+1}, x \neq -\frac{1}{5}$ b). $x, \text{ if } f(x) = 6$

$$f(1) = \frac{3}{5(1)+1}$$

$$= \frac{3}{5+1}$$

$$= \frac{3}{6} = \frac{1}{2}$$

$\therefore f(1) = \frac{1}{2} \text{ or } 0.5$

$$\frac{3}{5x+1} = \frac{6}{1}$$

$$6(5x+1) = 3(1)$$

$$30x + 6 = 3$$

$$30x = 3 - 6$$

$$= -3$$

$$\frac{30x}{30} = \frac{-3}{30}$$

$$x = \frac{-1}{10}$$

$\therefore x = \frac{-1}{10}$

$$2x + y = 1$$

$$y = 1 - 2x = 1 - 2 \times \frac{11}{7}$$

$$= 1 - \frac{22}{7} = \frac{7-22}{7}$$

$$= \frac{-15}{7} = -2\frac{1}{7}$$

$\therefore y = -2\frac{1}{7} \text{ and } x = 1\frac{4}{7}$

b). $g(x) = \frac{4x-3}{2x-5}, \text{ let } y = g(x)$

$$y = \frac{4x-3}{2x-5}$$

$$x = \frac{4y-3}{2y-5}, \quad \frac{x}{1} = \frac{4y-3}{2y-5}$$

$$x(2y-5) = 1(4y-3)$$

$$2xy - 5x = 4y - 3$$

$$2xy - 4y = 5x - 3$$

$$y(2x-4) = 5x-3$$

$$\frac{y(2x-4)}{2x-4} = \frac{5x-3}{2x-4}, y = \frac{5x-3}{2x-4}$$

$\therefore g^{-1}(x) = \frac{5x-3}{2x-4}$

19. $2x + y = 4$
 $x - 2y = 2$ from the first equation
 $2x + y = 4, y = 4 - 2x$. Replace for y in the second equation to get
 $x - 2(4 - 2x) = 2,$
 $x - 8 + 4x = 2$
 $5x = 8 + 2 = 10$
 $\frac{5x}{5} = \frac{10}{5} = 2, x = 2.$
 replcae for x in the first equation to get $2(2) + y = 4,$
 $y = 4 - 4 = 0, y = 0$
 $\therefore x = 2$ and $y = 0$

20. a). $y - y_0 = m(x - x_0)$
 where $m = 3, x_0 = 0$ and $y_0 = 5$
 $y - 5 = 3(x - 0) = 3x$
 $y = 3x + 5$
 $\therefore$ the equation of the line is $y - 3x = 5$ or $y = 3x + 5$

b) I). $\text{Gradient}(m) = \frac{y_2 - y_1}{x_2 - x_1}$ II). $y = mx + c, 1 = -\frac{1}{2}(-2) + c$
 $= \frac{1 - (-3)}{-2 - 6}$
 $= \frac{4}{-8}$
 $= -\frac{1}{2}$
 $\therefore m = -\frac{1}{2}$
 $1 = 1 + c, \quad c = 1 - 1 = 0$
 $\therefore c = 0$
 And the equation of line BC is $y = mx$

21. $f(x) = 12 - 3x$ let $y = f(x) = 12 - 3x$ b). $f^{-1}(x) = \frac{12-x}{3}$
 a). $f(-3) = 12 - 3(-3)$ $y = 12 - 3x$
 $= 12 - (-9)$
 $= 12 + 9 = 21$ $x = 12 - 3y$
 $\therefore f(-3) = 21$ $3y = 12 - x$
 $\frac{3y}{3} = \frac{12-x}{3}$
 $y = \frac{12-x}{3}$
 $f^{-1}(-3) = \frac{12 - (-3)}{3}$
 $= \frac{15}{3} = 5$
 $\therefore f^{-1}(-3) = 5$

22. $2x = 5y$
 $x - 2y = 3$ making x the subject in $x - 2y = 3$ and substituting in

$2x = 5y$, we have $x = 3 + 2y$ and

$2(3 + 2y) = 5y$

$6 + 4y = 5y$

$5y - 4y = 6$ $y = 6$ and substituting for y in $2x = 5y$ we have

$2x = 5(6)$ $2x = 30$

$\frac{2x}{2} = \frac{30}{2} = 15$

$\therefore x = 15$ and $y = 6$

23. $f(x) = x^3 - x$,

$f(4) = 4^3 - 4 = 64 - 4 = 60$

$f(5) = 5^3 - 5 = 125 - 5 = 120$

$\therefore$ largest common factor of 60 and 120 is **60**

24. $3x - 3y = 6$
 $x - 4y = 8$ making x the subject in $x - 4y = 8$, we have $x = 8 + 4y$.

substituting for x in $3x - 3y = 6$ we have

$3(8 + 4y) - 3y = 6$

$24 + 12y - 3y = 6$

$9y = 6 - 24 = -18$

$\frac{9y}{9} = \frac{-18}{9} = -2$

$y = -2$

replacing for y in $x - 4y = 8$ we have

$x - 4(-2) = 8$

$x + 8 = 8$

$x = 8 - 8 = 0$

$\therefore x = 0$ and $y = -2$

25. $f(x) = \frac{2x-3}{x}$, $x \neq 0$

a). $f(-3) = \frac{2(-3)-3}{-3}$
 $= \frac{-6-3}{-3}$
 $= \frac{-9}{-3} = 3$

$\therefore f(-3) = 3$

let $y = f(x) = \frac{2x-3}{x}$, then

$x = \frac{2y-3}{y}$

$xy = 2y - 3$

$xy - 2y = -3$

$y(x - 2) = -3$

$\frac{y(x-2)}{x-2} = \frac{-3}{x-2}$

$y = \frac{-3}{x-2}$

$\therefore f^{-1}(x) = \frac{-3}{x-2}$

b). $f^{-1}(4) = \frac{-3}{4-2}$

$= \frac{-3}{2} = -1\frac{1}{2}$

$\therefore f^{-1}(4) = -1\frac{1}{2}$

26. The points L, M and N will be collinear (lying on the same plane) if

gradient LM = gradient MN = gradient LN.

Gradient (m)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient LM} = \frac{2 - (-10)}{2 - (-2)} = \frac{12}{4} = 3$$

$$\text{Gradient MN} = \frac{11 - 2}{5 - 2} = \frac{9}{3} = 3$$

$$\text{Gradient LN} = \frac{11 - (-10)}{5 - (-2)} = \frac{21}{7} = 3$$

NOTE: You can also show that the three points are collinear by showing that length of LM + length of MN = length of LN. $\text{Length} = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

it can be seen clearly that gradient LM = gradient MN = gradient LN = 3.

Thus the three points L, M and N lie on the same plane.

$\therefore$ L, M and N are collinear.

27. a). The equation $y = x^2 - 6x + 8$ cuts the x – axis at points B and C.

Thus the values of the y component is zero on both points. To get the two values of x , we equate y to zero and find the values of x .

$$x^2 - 6x + 8 = 0$$

$$x^2 - 2x - 4x + 8 = 0$$

$$x(x - 2) - 4(x - 2) = 0$$

$$(x - 4)(x - 2) = 0$$

$$x - 4 = 0 \text{ or } x - 2 = 0$$

$$x = 4 \text{ or } x = 2$$

$\therefore$ The coordinates are **B(2, 0)** and **C(4, 0)**.

$$\text{b). } f'(x) = \frac{d}{dx}(x^2 - 6x + 8) = 2x - 6 = 0$$

$$2x = 6, \quad \frac{2x}{2} = \frac{6}{2} = 3, \quad x = 3$$

$$y = x^2 - 6x + 8$$

$$= 3^2 - (6)3 + 8 = 9 - 18 + 8 = -1$$

$$y = -1$$

$\therefore$ Turning point is **(3, -1)**

$$28. \text{ a). } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{6 - 0} = \frac{-6}{6} = -1$$

$$y = mx + c$$

$$0 = -1(6) + c$$

$$c = 6$$

$\therefore$ **$y = -x + 6$** is the equation of line CD

$$\text{b). distance} = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(0 - 6)^2 + (-2 - 6)^2}$$

$$= \sqrt{(-6)^2 + (-8)^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100} = \pm 10$$

$\therefore$ Distance of AB is **10 units**

29. a). The curve cuts the x – axis at points where $y = 0$. set y to 0

$$x(x - 2) = 0$$

$$x = 0 \text{ or } x - 2 = 0$$

$$x = 2$$

$\therefore$ The points where the curve cuts x – axis are **(0, 0) and (2, 0)**

b). $y = x(x - 2) = x^2 - 2x$. let $f(x) = x^2 - 2x$

$$f'(x) = 2x - 2 = 0, \quad 2x = 2 \quad x = 1,$$

substituting for x in $f(x)$, we have

$$f(1) = 2(1) - 2 = 2 - 2 = 0$$

$$y = 0 \text{ and } x = 1$$

$\therefore$ Minimum value is **(1, 0)**

$$c). \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-14 - k}{k - (-2)} = 2$$

$$\frac{-14 - k}{k + 2} = \frac{2}{1}$$

$$1(-14 - k) = 2(k + 2)$$

$$-14 - k = 2k + 4$$

$$2k + k = -14 - 4$$

$$3k = -18$$

$$\frac{3k}{3} = \frac{-18}{3} = -6$$

$\therefore$ The value of k is **-6**

$$30. \quad f(x) = \frac{2x+7}{8} \text{ and } g(x) = \frac{3x-6}{6}$$

$$a). \quad g(6) = \frac{3(6)-6}{6}$$

$$= \frac{18-6}{6}$$

$$= \frac{12}{6} = 2$$

$$\therefore g(6) = 2$$

$$b). \quad \text{let } y = \frac{2x+7}{8}$$

$$x = \frac{2y+7}{8}$$

$$8x = 2y + 7$$

$$2y = 8x - 7$$

$$\frac{2y}{2} = \frac{8x-7}{2}, y = \frac{8x-7}{2}$$

$$\therefore f^{-1}(x) = \frac{8x-7}{2}$$

c) x if $f(x) = g(x)$

$$\frac{2x+7}{8} = \frac{3x-6}{6}$$

$$6(2x + 7) = 8(3x - 6)$$

$$12x + 42 = 24x - 48$$

$$24x - 12x = 42 + 48$$

$$12x = 90$$

$$\frac{12x}{12} = \frac{90}{12} = 7\frac{1}{2}$$

$$\therefore x = 7\frac{1}{2}$$

31. a). Axis of symmetry is given by $x = -\frac{b}{2a}$

$$y = -x^2 - 2x + 3 \quad a = -1 \text{ and } b = -2$$

$$x = -\frac{(-2)}{2(-1)}$$

$$= -\frac{2}{2} = -1$$

$\therefore x = -1$ is the axis of symmetry of $y = -x^2 - 2x + 3$

b). let $f(x) = y = -x^2 - 2x + 3$

$$f'(x) = -2x - 2 = 0$$

$$-2x = 2$$

$$\frac{-2x}{-2} = \frac{2}{-2}$$

$$x = -1$$

$$y = -x^2 - 2x + 3$$

$$= -(-1)^2 - 2(-1) + 3$$

$$= -1 + 2 + 3 = 4$$

$$y = 4$$

$\therefore$ The coordinate of the turning point is $(-1, 4)$

NOTE: At turning points, the gradient is zero. Differentiating an equation gives the gradient at a certain point with a given value of x . to find the turning point, find the value of x where the gradient is zero. To find y , substitute the value of x into the original equation and evaluate. The two values (x, y) gives the coordinate of the turning point.

32. Equation of the straight line is $y + \frac{x}{7} = 1$, which can be re – written as

$$y = -\frac{1}{7}x + 1.$$

$m = -\frac{1}{7}$ since the lines are perpendicular, the product of their gradients is -1 .

$$\text{Gradient of } \perp \text{ line} = -1 \div -\frac{1}{7} = 7$$

$$y = mx + c$$

$$4 = 7(-4) + c$$

$$c = 4 + 28 = 32$$

$$c = 32 \text{ and } m = 7$$

$\therefore y = 7x + 32$ is the equation of the straight line passing through $(-4, 4)$ and

$$\perp \text{ to } y = -\frac{1}{7}x + 1.$$

CALCULUS

1. a). $y = x^3 - \frac{3}{2}x^2$, $\frac{dy}{dx} = 3x^2 - 3x$, $x = 2$

$$\frac{dy}{dx} = 3(2^2) - 3(2) = 12 - 6 = 6$$

$\therefore m_1 = 6$ *Tangent is perpendicular to the normal*

$$m_1 m_2 = -1 \quad m_2 = \frac{-1}{m_1}$$

$$= -\frac{1}{6} \text{ (Gradient of the normal).}$$

$$y = x^3 - \frac{3}{2}x^2 \text{ at } x = 2,$$

$$y = 2^3 - \frac{3}{2}2^2 = 8 - 6 = 2$$

$$y - y_1 = m_2(x - x_1)$$

$$y - 2 = -\frac{1}{6}(x - 2)$$

$$y = -\frac{1}{6}x + \frac{2}{6} + 2$$

$$y = -\frac{1}{6}x + \frac{14}{6}$$

$\therefore \mathbf{6y = -x + 14}$ is the equation of the normal.

b). $\frac{dy}{dx} = 0$ (stationary points)

$$3x^2 - 3x = 0, 3x(x - 1) = 0$$

$$3x = 0 \text{ or } x - 1 = 0$$

$$x = 0 \text{ or } x = 1$$

$$\text{for } x = 0 \quad y = 0^3 - \frac{3}{2}0^2 = 0 \text{ and}$$

$$\text{for } x = 1 \quad y = 1^3 - \frac{3}{2}1^2 = -\frac{1}{2}$$

$\therefore (0, 0)$ and $(1, -\frac{1}{2})$ are the stationary points.

$$2. \int_{-1}^3 (3x^2 - 2x) dx = \left[\frac{3x^3}{3} - \frac{2x^2}{2} \right]_{-1}^3$$

$$= [x^3 - x^2]_{-1}^3$$

$$= [3^3 - 3^2] - [(-1)^3 - (-1)^2]$$

$$= (27 - 9) - (-1 - 1)$$

$$= 18 - (-2)$$

$$= 18 + 2 = 20$$

$$\therefore \int_{-1}^3 (3x^2 - 2x) dx = 20$$

4. $y = x + \frac{4}{x}$, $\frac{dy}{dx} = 1 - \frac{4}{x^2}$ at $x = 4$

$$m_1 = 1 - \frac{4}{4^2} = 1 - \frac{4}{16} = \frac{3}{4}$$

$$m_1 m_2 = -1, m_2 = -1 \div \frac{3}{4} = -1 \times \frac{4}{3} = -\frac{4}{3}.$$

$$y = x + \frac{4}{x} = 4 + \frac{4}{4} = 5$$

$$y - y_1 = m_2(x - x_1)$$

$$y - 5 = -\frac{4}{3}(x - 4), \quad y = -\frac{4}{3}x + \frac{16}{3} + 5$$

$$y = -\frac{4}{3}x + \frac{31}{3}$$

$\therefore \mathbf{3y = -4x + 31}$ is the equation of the normal.

3 $y = 2x^3 - 3x^2 - 36x - 3$

$$\frac{dy}{dx} = 6x^2 - 6x - 36 = 0$$

$$6x^2 + 12x - 18x - 36 = 0$$

$$6x(x + 2) - 18(x + 2) = 0$$

$$(6x - 18)(x + 2) = 0$$

$$6x - 18 = 0 \text{ or } x + 2 = 0$$

$$\frac{6x}{6} = \frac{18}{6} = 3, \quad x = 3 \text{ or } x = -2$$

$$\text{when } x = -2$$

$$y = 2(-2)^3 - 3(-2)^2 - 36(-2) - 3 = 41$$

$$\text{when } x = 3$$

$$y = 2(3)^3 - 3(3)^2 - 36(3) - 3 = -84$$

$\therefore (-2, 41)$ and $(3, -84)$ are the coordinates on the curve.

$$\begin{aligned}
 5. \int_2^5 (3x^2 + 2)dx &= \left[\frac{3x^3}{3} + \frac{2x}{1} \right]_2^5 \\
 &= [x^3 + 2x] \\
 &= (5^3 + 2 \times 5) - (2^3 + 2 \times 2) \\
 &= (125 + 10) - (8 + 4) \\
 &= 135 - 12 = 123
 \end{aligned}$$

$$\therefore \int_2^5 (3x^2 + 2)dx = \mathbf{123}$$

$$\begin{aligned}
 6. \ y &= 2x^2 - 3x - 2 \\
 \frac{dy}{dx} &= 4x - 3 \text{ at } (3, 7),
 \end{aligned}$$

$$m_1 = 4(3) - 3 = 12 - 3 = 9$$

$$m_2 = -\frac{1}{m_1} = -\frac{1}{9}$$

$$y - y_1 = m_2(x - x_1)$$

$$y - 7 = -\frac{1}{9}(x - 3)$$

$$y = -\frac{1}{9}x + \frac{1}{3} + 7$$

$$y = -\frac{1}{9}x + \frac{22}{3}$$

$\therefore$ The equation of the normal to the curve is $\mathbf{9y = -x + 66}$.

$$7. \int_0^1 (x^2 - 2x - 3)dx$$

$$= \left[\frac{x^3}{3} - \frac{2x^2}{2} - 3x \right]_0^1$$

$$= \left[\frac{x^3}{3} - x^2 - 3x \right]_0^1$$

$$= \left(\frac{1}{3} - 1 - 3 \right) - \left(\frac{0}{3} - 0 - 3 \times 0 \right)$$

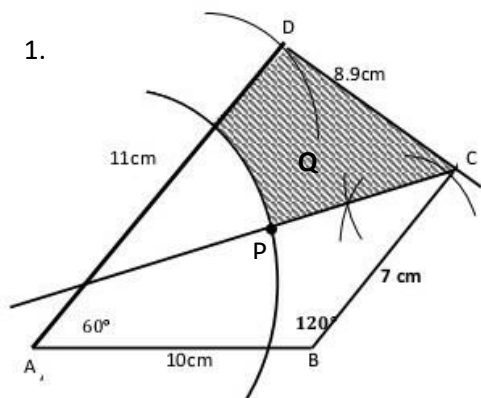
$$= \frac{1}{3} - 1 - 3 = \frac{1-3-9}{3}$$

$$= \frac{-11}{3} = -3\frac{2}{3}$$

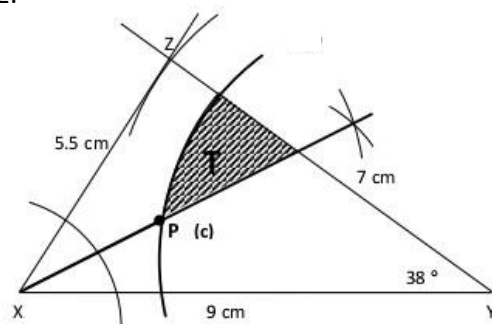
$$\therefore \int_0^1 (x^2 - 2x - 3)dx = -3\frac{2}{3}$$

CONSTRUCTION

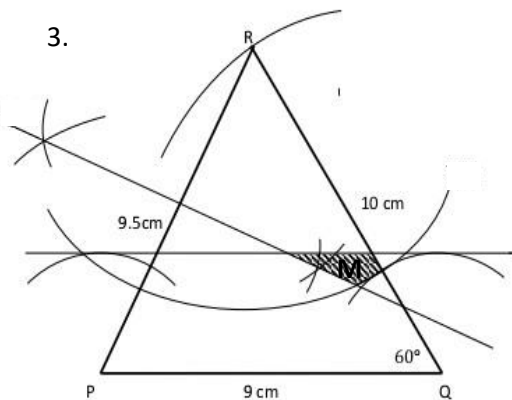
1.



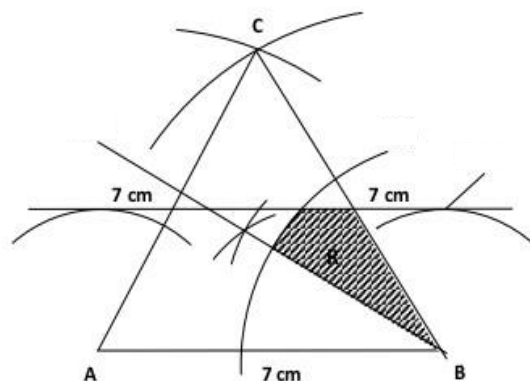
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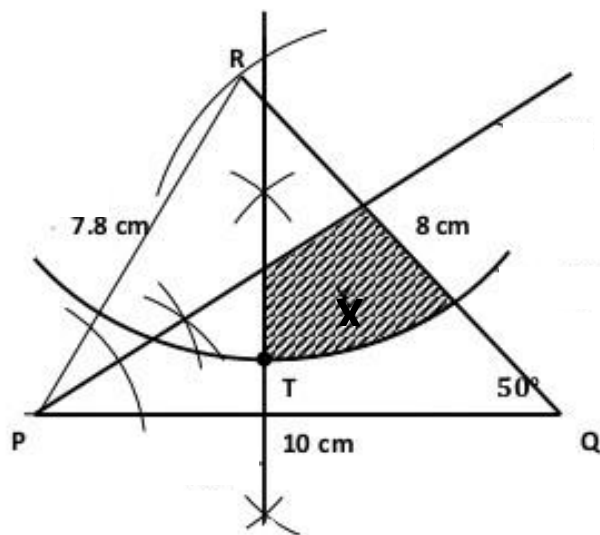
3.



4.

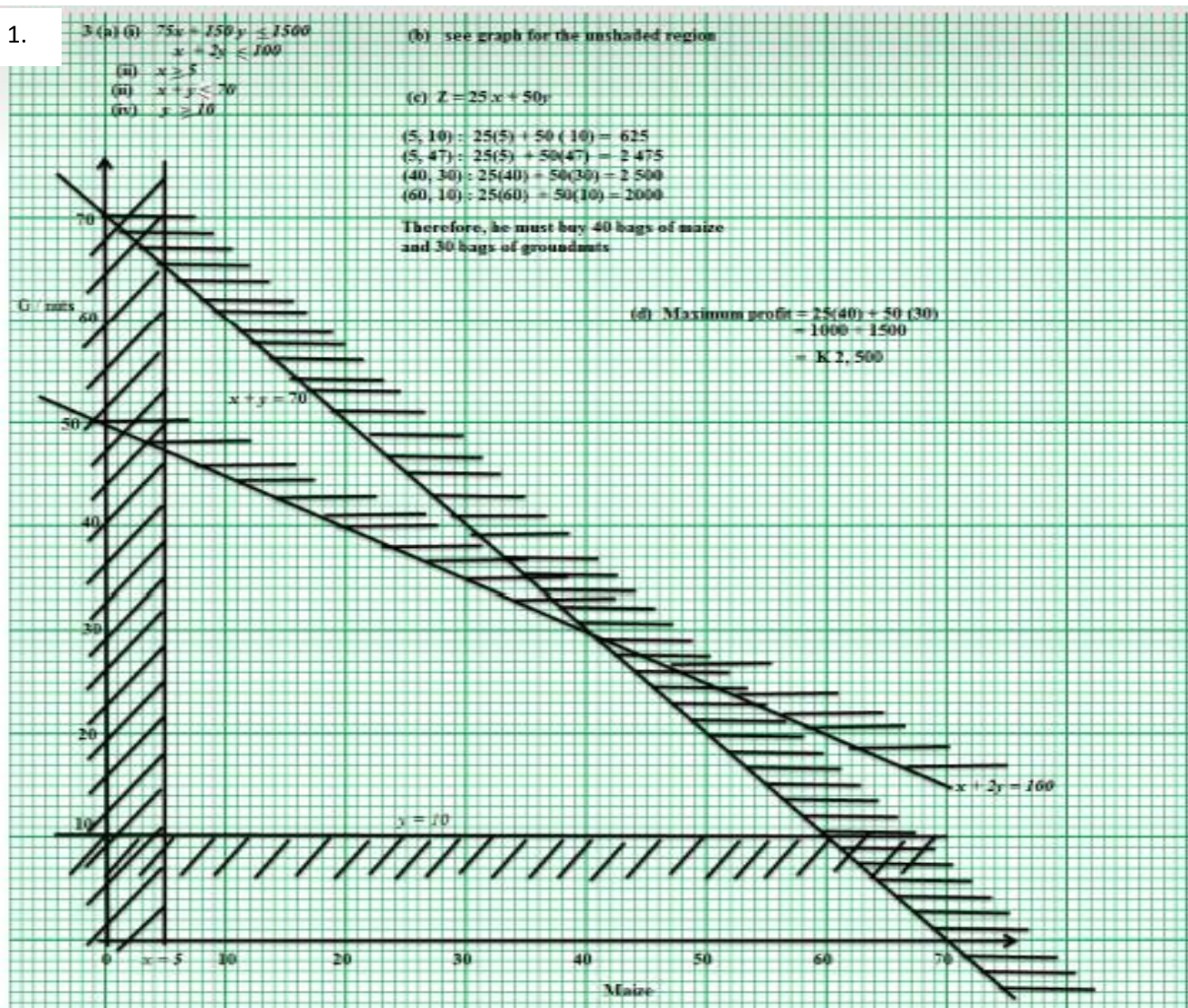


5.

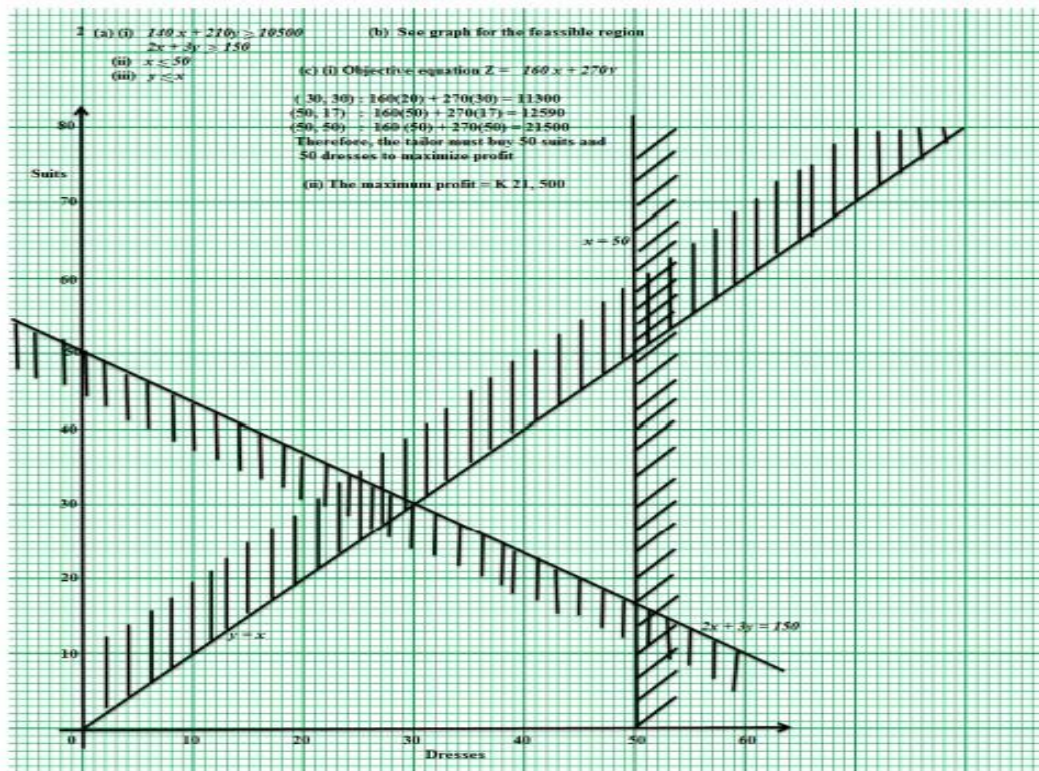


LINEAR PROGRAMMING

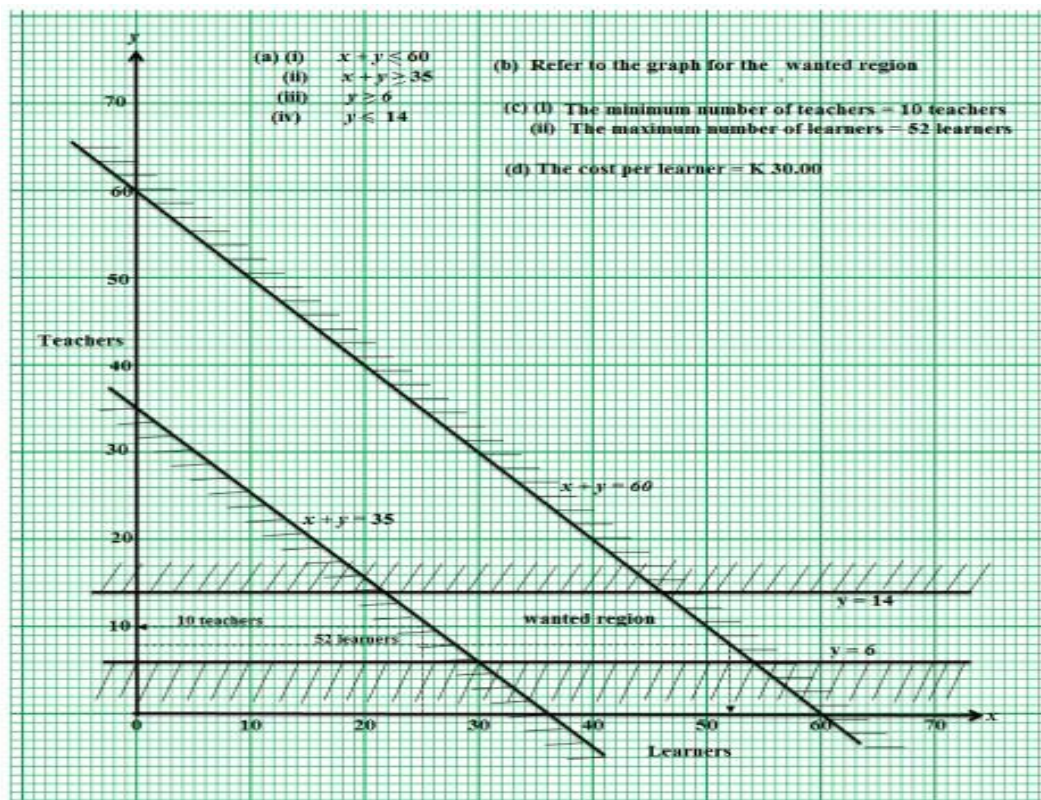
1.



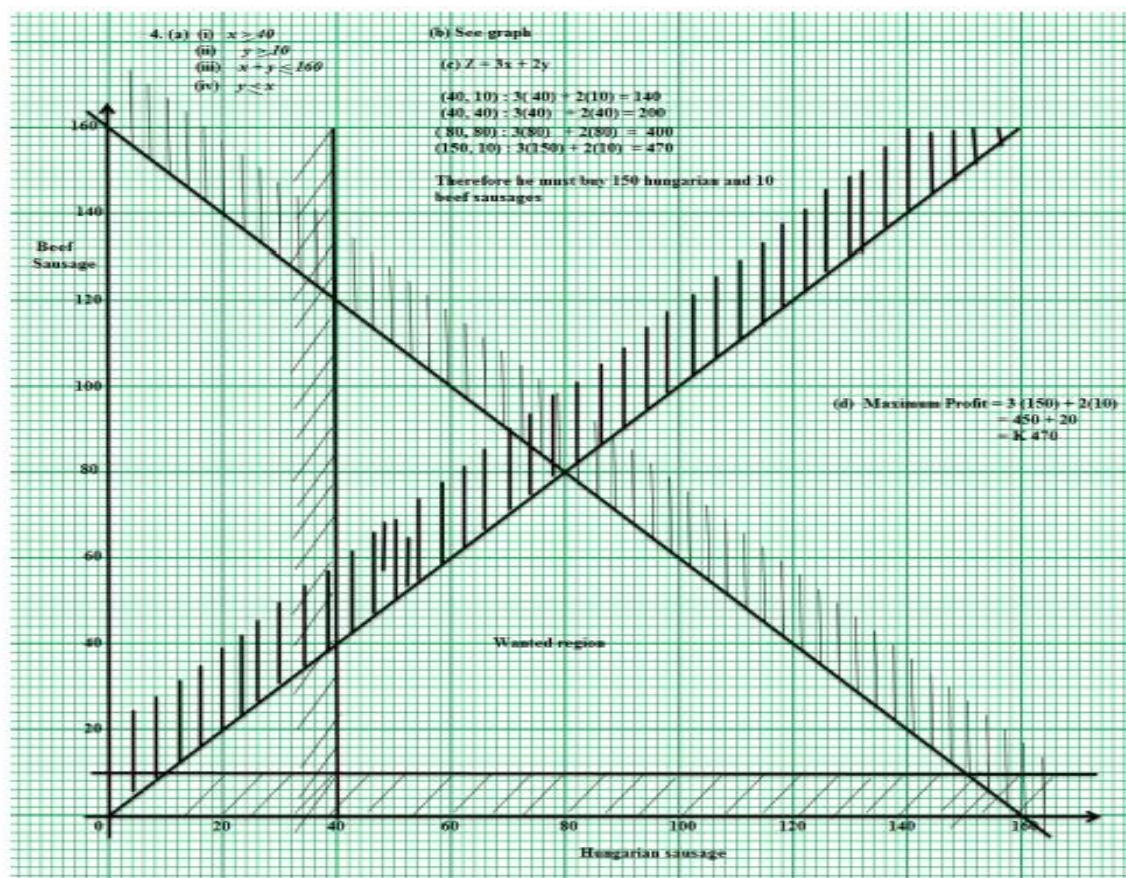
2.



3.



5.



6. a). Line 1 (0,0) and (5,5)

$$m_1 = \frac{5-0}{5-0} = 1$$

$$y - y_1 = m_1(x - x_1)$$

$$y - 0 = 1(x - 0)$$

$$y = x$$

Line 2 (0,1) and (7,0)

$$m_2 = \frac{0-1}{7-0} = -\frac{1}{7}$$

$$y - y_2 = m_2(x - x_2)$$

$$y - 1 = -\frac{1}{7}(x - 0)$$

$$y - 1 = \frac{-x}{7}$$

$$7y - 7 = -x$$

$$7y + x = 7$$

Line 3 (5,5) and (7,0)

$$m_3 = \frac{0-5}{7-5} = -\frac{5}{2}$$

$$y - y_3 = m_3(x - x_3)$$

$$y - 0 = -\frac{5}{2}(x - 7)$$

$$y = -\frac{5}{2}x + \frac{35}{2}$$

$$2y = -5x + 35$$

$$2y + 5x = 35$$

$\therefore$ The three inequalities defining un – shaded region R are

$$y - x \leq 0, 7y + x \geq 7 \text{ and } 2y + 5x \leq 35$$

b). To find the largest value get points on the boundary of the region and try them to find the largest value. The points are (1, 1), (5, 5) and (7, 0)

$$4x - 5y \text{ at } (1, 1) = 4(1) - 5(1) = 4 - 5 = -1$$

$$4x - 5y \text{ at } (5, 5) = 4(5) - 5(5) = 20 - 25 = -5 \text{ and}$$

$$4x - 5y \text{ at } (7, 0) = 4(7) - 5(0) = 28 - 0 = 28$$

$\therefore$ Largest value of $4x - 5y$ within the region is **28**

7. a). Line 1 (0, 20) and (40, 100)

$$m_1 = \frac{100-20}{40-0} = 2$$

$$y - y_1 = m_1(x - x_1)$$

$$y - 20 = 2(x - 0)$$

$$y - 20 = 2x$$

$$y = 2x + 20$$

$$y \leq 2x + 20$$

$$\text{Line 2, } y = 20$$

$$y \geq 20$$

$$\text{Line 3, } x = 110$$

$$x \leq 110$$

Line 4 (0, 150) and (150, 0)

$$m_4 = \frac{0 - 150}{150 - 0} = -1$$

$$y - y_4 = m_4(x - x_4)$$

$$y - 0 = -1(x - 150)$$

$$y = -x + 150$$

$$y + x \leq 150$$

$\therefore$ The other two inequalities defining region R are $x \leq 110$ and $y + x \leq 150$.

b). Chair = x and Table = y

i). **Profit = $100x + 60y$**

the four points surrounding region R are (0, 20), (40, 110), (110, 0) and (110, 40)

$$\text{Profit}_{(0,20)} = 100(0) + 60(20) = 0 + 1200 = 1200$$

$$\text{Profit}_{(40,110)} = 100(40) + 60(110) = 4000 + 6600 = 10600$$

$$\text{Profit}_{(110,0)} = 100(110) + 60(0) = 11000 + 0 = 11000$$

$$\text{Profit}_{(110,40)} = 100(110) + 60(40) = 11000 + 2400 = 13400$$

$\therefore$ The value of x and y which would give him maximum profit is **110** and **40** respectively.

ii). Maximum Profit = $100(110) + 60(40) = 11000 + 2400 = 13400$

$\therefore$ The carpenter would make a maximum profit of **K13 400.00**

TRIGONOMETRY

1. a) I). $\frac{KR}{\sin N} = \frac{KN}{\sin R}$
 $KR = \frac{KN \times \sin N}{\sin R}$
 $= \frac{80m \times \sin 60^\circ}{\sin 52^\circ}$
 $= \frac{80m \times 0.866025403}{0.788010753}$
 $= \frac{69.2820323m}{0.788010753}$
 $= 87.92016097m$
 $\therefore KR = \mathbf{87.9m}$

II). Area of $\triangle KNB$

$$A = \frac{1}{2}(KN)(NB) \sin N$$

$$= \frac{1}{2}(80)(50) \sin 60^\circ$$

$$= 2000 \sin 60^\circ$$

$$= 1732.050808$$

$$\therefore A = \mathbf{1732m^2}$$

b). $A = \frac{1}{2}bh$

$$= \frac{1}{2}(KN)(S.D)$$

$$S.D = \frac{2 \times A}{KN}$$

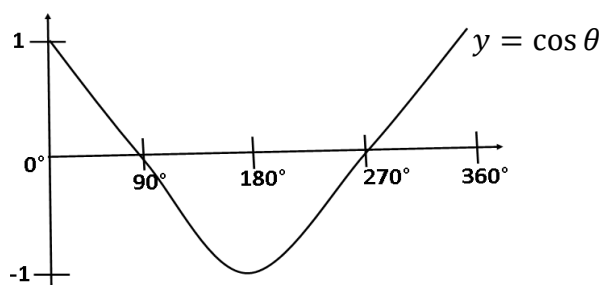
$$= \frac{2(3260)}{80} = \frac{6520}{80} = 81.5$$

$$\therefore S.D = \mathbf{81.5m}$$

shortest distance(S.D)

c). Graph of $y = \cos \theta$

θ	0°	90°	180°	270°	360°
$\cos \theta$	1	0	-1	0	1



2. a) I). $\frac{AC}{\sin B} = \frac{AB}{\sin C}$
 $AC = \frac{AB \times \sin B}{\sin C}$
 $= \frac{15 \times \sin 79^\circ}{\sin 40^\circ}$
 $= \frac{14.72440775}{0.642787609}$
 $= 22.90711198$

$$\therefore AC = \mathbf{22.9km}$$

II). $A = \frac{1}{2}(AC)(AB) \sin C$

$$\angle C = 180^\circ - (79^\circ + 40^\circ) = 61^\circ$$

$$A = \frac{1}{2} \times 15 \times 22.90711198 \sin 61^\circ$$

$$= 150.2625868$$

$$\therefore A = \mathbf{150.3km^2}$$

III). Shortest Distance (S.D)

$$A_\Delta = \frac{1}{2}(AC)(S.D)$$

$$S.D = \frac{2 \times A}{AC}$$

$$= \frac{2 \times 150.2625868}{22.90711198}$$

$$= 13.1192956$$

$$\therefore S.D = \mathbf{13.1km}$$

b). $\cos \theta = 0.937$

$$= \cos^{-1}(0.937)$$

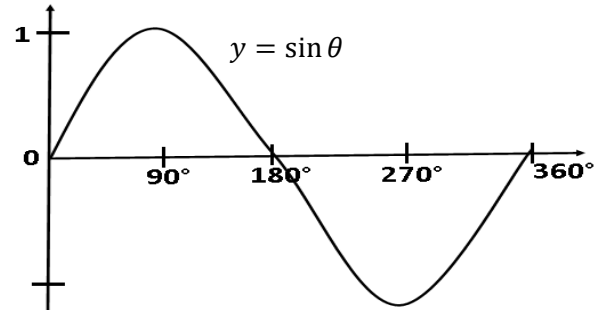
$$= 24.44690112 = 24.4$$

$$\theta = 360 - 24.4 = 339.6$$

$$\therefore \theta = 20.4^\circ \text{ and } \theta = 339.6^\circ$$

c). $y = \sin \theta$

θ	0°	90°	180°	270°	360°
$\sin \theta$	0	1	0	-1	0



3. a). I) $A = \frac{1}{2}(TH)(HS) \sin H$
 $= \frac{1}{2}(1.9m)(1.3m) \sin 130^\circ$
 $= 0.946064887$
 $\therefore A = 0.946km^2$

II) $TS^2 = HS^2 + TH^2 - 2(TS)(TH) \cos H$
 $= 1.3^2 + 1.9^2 - 2(1.3)(1.9) \cos 130^\circ$
 $= 5.3 - 4.94(-0.642787609)$
 $= 5.3 + 3.175370792$
 $= 8.745370792$
 $\sqrt{TS^2} = \sqrt{8.745370792} = \pm 2.911249009$
 $\therefore TS = 2.9km$

4. a). I). $\frac{PQ}{\sin R} = \frac{RQ}{\sin P}, PQ = \frac{RQ \times \sin R}{\sin P}$
 $\angle R = 180^\circ - (46^\circ + 36^\circ) = 98^\circ$
 $PQ = \frac{36.5 \times \sin 98^\circ}{\sin 46^\circ}$
 $= \frac{36.14478451}{0.7193398}$
 $= 50.24716343$
 $\therefore PQ = 50m$

III) Shortest distance (S.D) = $\frac{2 \times A}{TS}$
 $= \frac{2 \times 0.946064887}{2.911249009}$
 $= 0.649937455$

$$\therefore S.D = 0.65km$$

c). $\cos \theta = \frac{2}{3}$
 $\theta = \cos^{-1}\left(\frac{2}{3}\right)$
 $= \cos^{-1}(0.6666666666666666)$
 $= 48.1896851$
 $\therefore \theta = 48^\circ$

II). $A = \frac{1}{2}(PQ)(RQ) \sin Q$
 $= \frac{1}{2} \times 50.24716343 \times 36.5 \times \sin 36^\circ$
 $= 917.0107326 \times \sin 36^\circ$
 $= 539.0053848$
 $\therefore A = 539m^2$

$$\begin{aligned} \text{III) } S.D &= \frac{2 \times A}{PQ} = \frac{2 \times 539.0053848}{50.24716343} \\ &= \frac{1078.01077}{50.24716343} = 21.45416171 \\ \therefore \text{Shortest distance from} \\ R \text{ to } PQ &\text{ is } \mathbf{21.5m} \end{aligned}$$

$$\begin{aligned} \text{b). } \sin \theta &= 0.6792 \\ \theta &= \sin^{-1}(0.6792) = 42.7811598 \\ &= 42.78^\circ \\ \theta &= 180 - 42.78 = 137.22^\circ \\ \therefore \theta &= \mathbf{42.78^\circ \text{ and } 137.22^\circ} \end{aligned}$$

$$\begin{aligned} 5. \text{ a) I). } BP^2 &= BK^2 + KP^2 - 2(BK)(KP) \cos K \\ &= 3^2 + 5^2 - 2 \times 3 \times 5 \cos 110^\circ \\ &= 9 + 25 - 30(-0.342020143) \\ &= 34 + 10.2606043 = 44.2606043 \\ \sqrt{BP^2} &= \sqrt{44.2606043} = 6.652864368 \\ \therefore BP &= \mathbf{6.65km} \end{aligned}$$

$$\begin{aligned} \text{II). } A &= \frac{1}{2}(BK)(KP) \sin K \\ &= \frac{1}{2} \times 3 \times 5 \sin 110^\circ \\ &= 7.5 \sin 110^\circ \\ &= 7.047694656 \\ \therefore A &= \mathbf{7.05km^2} \end{aligned}$$

$$\begin{aligned} \text{III). } S.D &= \frac{2 \times A}{BP} = 2 \times \frac{7.047694656}{6.652864368} \\ &= 2 \times 1.059347413 \\ &= 2.118694826 \\ \therefore \text{Shortest distance from } K \text{ to } BP &\text{ is } \mathbf{2.12km.} \end{aligned}$$

$$\begin{aligned} \text{b). } \tan \theta &= 0.7 \\ \theta &= \tan^{-1}(0.7) \\ &= 34.9920202 \\ \therefore \theta &= \mathbf{35^\circ} \end{aligned}$$

$$\begin{aligned} 6. \text{ a). } LP^2 &= LM^2 + MP^2 - 2(LM)(MP) \cos M \\ \angle M &= 360^\circ - (105^\circ + 120^\circ) = 135^\circ \\ LP^2 &= 450^2 + 600^2 - 2 \times 450 \times 600 \cos 135^\circ \\ &= 202\,500 + 360\,000 - 540\,000(-0.70106781) \\ &= 562\,500 + 378.5766174 \\ &= 941\,076.6174 \end{aligned}$$

$$\begin{aligned} \text{b). } A &= \frac{1}{2}(LM)(MP) \sin M \\ &= \frac{1}{2}(450)(600) \sin 135^\circ \\ &= 135000 \sin 135^\circ \\ &= 95\,459.41546 \end{aligned}$$

$$\sqrt{LP^2} = \sqrt{941\,076.6174} = \pm 970.0910356$$

$$\therefore A = \mathbf{95\,459km^2}$$

$$\therefore LP = \mathbf{941km}$$

$$\text{c). } \angle MPL + \angle LMP + \angle MLP = 180^\circ, \angle MPL + 135^\circ + 30^\circ = 180^\circ$$

$$\angle MPL = 180^\circ - 165^\circ = \mathbf{15^\circ} \quad \therefore \angle MPL = \mathbf{15^\circ}$$

7. a). I) $BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos A$

$$\cos A = \frac{AB^2 + AC^2 - BC^2}{2(AB)(AC)}$$

$$= \frac{70^2 + 90^2 - 100^2}{2(70)(90)} = \frac{4900 + 8100 - 10000}{12600}$$

$$= \frac{3000}{12600} = 0.238095238$$

$$\angle A = \cos^{-1}(0.238095238)$$

$$= 76.225853$$

$$\therefore \angle BAC = 76^\circ$$

III). S.D = $2 \times \frac{A}{AC}$

$$= 2 \times \frac{3\,059.411708}{90} = 67.98692685$$

$$\therefore \text{Shortest distance from D to AC is } \mathbf{68m}$$

II) $A = \frac{1}{2}(AB)(AC) \sin A$

$$= \frac{1}{2} \times 70 \times 90 \sin 76^\circ$$

$$= 3150 \sin 76^\circ$$

$$= 3\,059.411708$$

$$\therefore A = \mathbf{3\,059m^2}$$

b). $\sin \theta = 0.766$

$$= \sin^{-1}(0.766)$$

$$= 49.99603866^\circ = 50^\circ$$

$$\theta = 180 - 50 = 130^\circ$$

$$\therefore \theta = \mathbf{50^\circ \text{ and } 130^\circ}$$

8. a) I). $BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos A$

$$\cos A = \frac{AB^2 + AC^2 - BC^2}{2(AB)(AC)}$$

$$= \frac{43^2 + 43^2 - 37^2}{2(43)(43)} = \frac{1\,849 + 1\,849 - 1\,369}{3\,698}$$

$$= \frac{2\,329}{3\,698} = 0.629799891$$

$$\angle A = \cos^{-1}(0.629799891)$$

$$= 50.96463954$$

$$\therefore \angle BAC = \mathbf{51^\circ}$$

b). $A = \frac{1}{2}(AB)(AC) \sin A$

$$= \frac{1}{2}(43)(43) \sin 51^\circ$$

$$= 924.5 \sin 51^\circ$$

$$= 718.4714414$$

$$\therefore A = \mathbf{718.5km^2}$$

II). $\angle ACB = \frac{1}{2}(180^\circ - 51^\circ)$

$$= \frac{1}{2} \times 129^\circ = 64.5^\circ$$

$$\therefore \angle ACB = \mathbf{64.5^\circ}$$

c). S.D = $2 \times \frac{A}{AB}$

$$= 2 \times \frac{718.5}{43}$$

$$= 2 \times 17.70930233$$

$$= 33.41860465$$

$$\therefore \text{The shortest distance from C to AB is } \mathbf{33km}$$

EARTH GEOMETRY

1. a). $A(60^\circ N, 51^\circ W)$ and $B(80^\circ S, 51^\circ W)$

b). Difference in longitudes between

$$C \text{ and } D = 81^\circ - 31^\circ = 50^\circ$$

$$\therefore \theta = 50^\circ$$

$$c). \text{Distance} = \frac{\vartheta}{360^\circ} \times 2\pi R \cos \theta$$

$$\theta = 80^\circ \text{ and } \vartheta = 81^\circ - 31^\circ = 50^\circ$$

$$\begin{aligned} CD &= \frac{50^\circ}{360^\circ} \times 2 \times 3.142 \\ &\quad \times 3437 \cos 80^\circ \\ &= 2999.73722 \cos 80^\circ \\ &= 520.8989021 \end{aligned}$$

$$\therefore CD = 521nm$$

$$d). \text{Total minutes} = 50 \times 4$$

$$= 200 \text{ minutes} = 3 \text{ hours } 20 \text{ minutes}$$

Now point C is to the west of point D,
thus time at C

$$\begin{aligned} &= 13:05 \text{ hours} - 03 \text{ hours } 20 \text{ minutes} \\ &= 09:45 \text{ hrs} \end{aligned}$$

$$\therefore \text{Time at C is } 09:45 \text{ hrs.}$$

2. a) Longitude A is 5°

b). Latitude of C + Latitude of A = 82°

$$\text{Latitude of C} = 82^\circ - 65^\circ = 17^\circ$$

$$\therefore \text{Latitude of C is } 17^\circ W$$

$$c). \text{I) Distance} = \frac{\theta}{360^\circ} \times 2\pi R, \theta = 82^\circ$$

$$\begin{aligned} AC &= \frac{82^\circ}{360^\circ} \times 2 \times 3.142 \times 3437 \\ &= 4919.569044 \end{aligned}$$

$$\therefore AC = 4920nm$$

$$\text{II) Distance} = \text{Distance} = \frac{\vartheta}{360^\circ} \times 2\pi R \cos \theta$$

$$\text{where, } \theta = 65^\circ \text{ and } \vartheta = 5^\circ + 45^\circ = 50^\circ$$

$$AB = \frac{50^\circ}{360^\circ} \times 2 \times 3.142 \times 3437 \cos 65^\circ$$

$$= 2999.7337222 \cos 65^\circ$$

$$= 1267.743731$$

$$\therefore AC = 1268nm$$

3. a). *difference in latitudes between C and D*

$$\theta = 50^\circ + 70^\circ = 120^\circ$$

$\therefore$ *difference in latitudes A and C is **120°***

b). $C = 2\pi R \cos \theta$

$$= 2 \times 3.142 \times 3437nm \times \cos 50^\circ$$

$$= 21598.108 \cos 50^\circ = 13882.99622$$

$\therefore$ *Length of the circle of latitude is **21 598nm***

c). $AD = \frac{\theta}{360^\circ} \times 2\pi R$

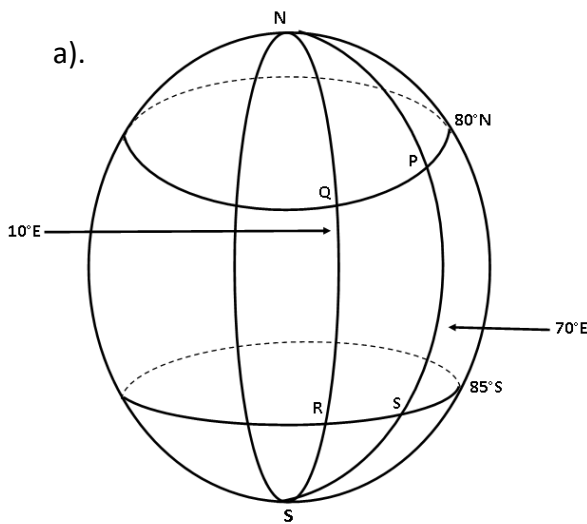
$$\theta = 50^\circ + 70^\circ = 120^\circ$$

$$AD = \frac{120^\circ}{360^\circ} \times 2 \times 3.142 \times 3437nm$$

$$= 7199.369333$$

$\therefore AD = \mathbf{7200nm}$

4.



b). i). $Distance\ QR = \frac{\theta}{360^\circ} \times 2\pi R \theta$

$$= 80^\circ + 85^\circ = 165^\circ$$

$$QR = \frac{165^\circ}{360^\circ} \times 2 \times 3.142 \times 3437nm$$

$$= 9899.132833nm$$

$\therefore$ *Distance QR = **9900nm***

ii). $C = 2\pi R \cos \theta$

$$= 2 \times 3.142 \times 3437nm \times \cos 85^\circ$$

$$= 21598.108nm \times \cos 85^\circ$$

$$= 1882.399145nm$$

$\therefore$ *Circumference of latitude 85°S is **1882nm***

5. a). *Difference in latitudes between W and Y*

$$\theta = 80^\circ + 30^\circ = 110^\circ$$

$\therefore \theta = \mathbf{110^\circ}$

b). i). $XZ = \frac{\theta}{360^\circ} \times 2\pi R$

$$= \frac{110^\circ}{360^\circ} \times 2 \times 3.142 \times 3437$$

$$= 6599.421889$$

$\therefore XZ = \mathbf{6600nm}$

ii). $YZ = \frac{\vartheta}{360^\circ} \times 2\pi R \cos \theta$

$$\theta = 30^\circ \text{ and } \vartheta = 15^\circ + 105^\circ = 120^\circ$$

$$YZ = \frac{120^\circ}{360^\circ} \times 2 \times 3.142 \times 3437nm \times \cos 30^\circ$$

$$= 7199.369333 \cos 30^\circ$$

$$= 6234.836734$$

$\therefore YZ = \mathbf{6235nm}$

$$\begin{aligned}
 6. \text{ a). } QR &= \frac{\vartheta}{360^\circ} \times 2\pi R \cos \theta \\
 \theta &= 30^\circ \text{ and } \vartheta = 80^\circ - 30^\circ = 50^\circ \\
 QR &= \frac{50^\circ}{360^\circ} \times 2 \times 3.142 \times 6370 \text{ km} \cos 30^\circ \\
 &= 5559.594444 \text{ km} \cos 30^\circ \\
 &= 4814.750024 \text{ km} \\
 \therefore QR &= \mathbf{4815 \text{ km}}
 \end{aligned}$$

$$\begin{aligned}
 \text{II). latitude of } T &= 20^\circ - 13^\circ = 7^\circ \\
 \therefore \text{Position of } T &\text{ is } \mathbf{T(20^\circ N, 7^\circ E)}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) i). } PT &= \frac{\vartheta}{360^\circ} \times 2\pi R \cos \theta, \theta = 20^\circ \text{ and} \\
 \vartheta &= \text{difference in angles between } P \text{ and } T \\
 \vartheta &= \frac{PT \times 360^\circ}{2\pi R \cos \theta} \\
 &= \frac{1232 \text{ km} \times 360^\circ}{2 \times 3.142 \times 6370 \text{ km} \cos 20^\circ} \\
 &= \frac{443520^\circ}{34666.20017} \\
 &= 12.79401832^\circ
 \end{aligned}$$

$\therefore$ Difference in angles between P and T is $\mathbf{13^\circ}$

$$\begin{aligned}
 7. \text{ a). } AC &= \frac{\theta}{360^\circ} \times 2\pi R \text{ where } \theta = 15^\circ + 35^\circ = 50^\circ \\
 AC &= \frac{50^\circ}{360^\circ} \times 2 \times 3.142 \times 6370 \text{ km} \\
 &= 5559.564444 \\
 \therefore AC &= \mathbf{5560 \text{ km}}
 \end{aligned}$$

$$\begin{aligned}
 \text{II). Longitude of } Q &= 70^\circ - 9.9^\circ = 60.1^\circ \\
 \therefore \text{Position of } Q &\text{ is } \mathbf{Q(35^\circ N, 60.1^\circ E)}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) i) } BQ &= \frac{\vartheta}{360^\circ} \times 2\pi R \cos \theta, \theta = 35^\circ \\
 \vartheta &= \frac{BQ \times 360^\circ}{2\pi R \cos \theta} \\
 &= \frac{900 \times 360^\circ}{2 \times 3.142 \times 6370 \cos 35^\circ} \\
 &= \frac{324000}{32789.90271} \\
 &= 9.881090616
 \end{aligned}$$

$\therefore$ The difference in longitudes is $\mathbf{9.9^\circ}$

8. a). $BC = \frac{\theta}{360^\circ} \times 2\pi R$

where $\theta = 20^\circ + 60^\circ = 80^\circ$

$$BC = \frac{80^\circ}{360^\circ} \times 2 \times 3.142 \times 3437$$
$$= \frac{1727848.64nm}{360} = 4799.579556$$

$\therefore BC = \mathbf{4800nm}$

b). $Speed = \frac{distance}{time}, time = 12hrs$

$$distance(CD) = \frac{\vartheta}{360^\circ} \times 2\pi R \cos\theta$$

$\vartheta = 20^\circ + 60^\circ = 80^\circ$ and $\theta = 60^\circ$

$$CD = \frac{80^\circ}{360^\circ} \times 2 \times 3.142 \times 3437 \cos 60^\circ$$

$$= 4799.557956 \cos 60^\circ$$

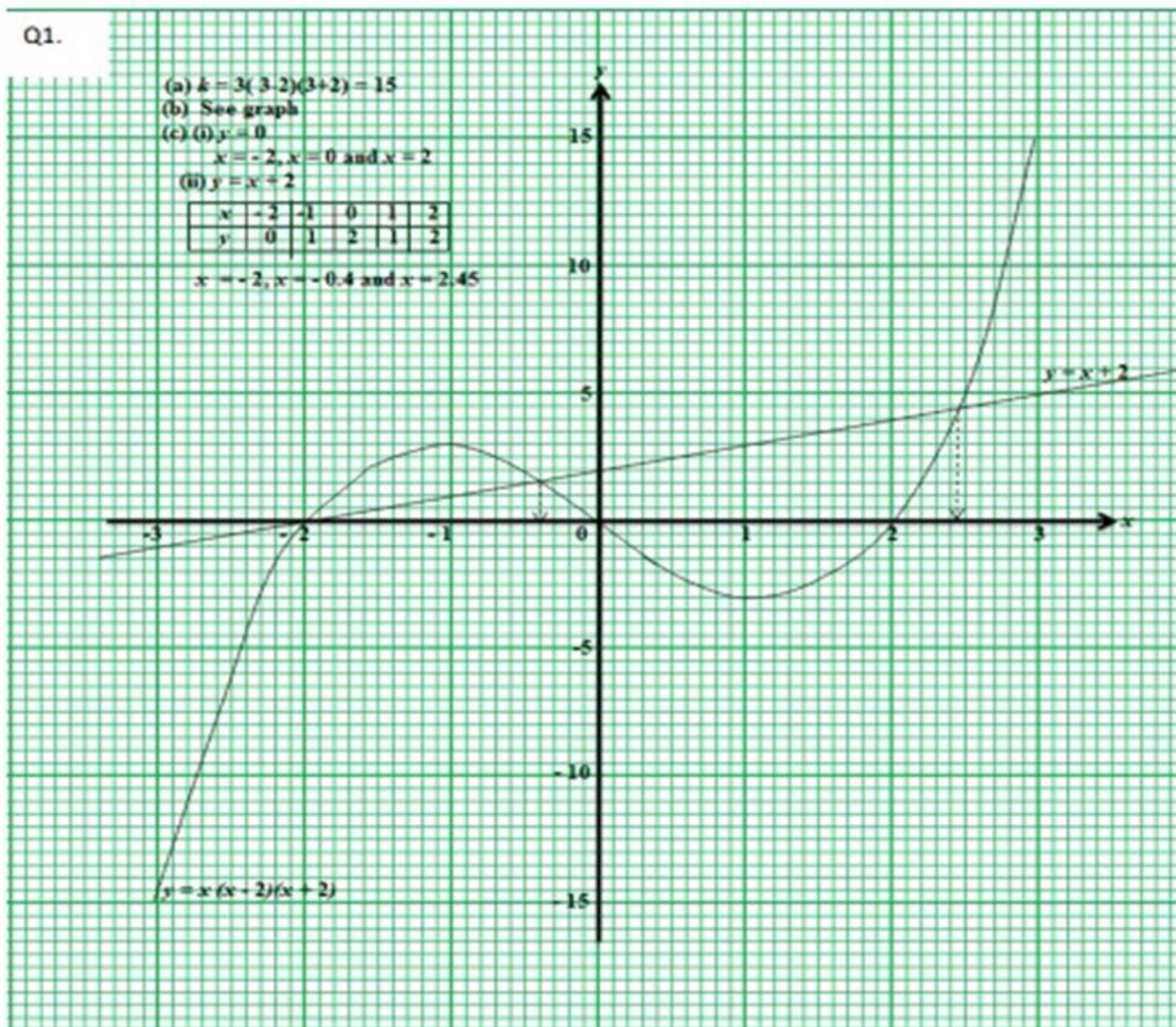
$$= 2399.789778nm$$

$$Speed = \frac{2399.789778nm}{12hrs}$$

$$= 199.9824815$$

$\therefore Speed = \mathbf{200knots}$

QUADRATIC FUNCTIONS



2. a). To estimate the value of the gradient on a curve, draw a straight line touching the curve and pick two points on the line and find the gradient using the two points. Note that a lot of points can be picked depending on the drawn line. In my case the line passes on (2,5) and (1, -6). this line touches the point (2,5).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 5}{1 - 2} = 11$$

$$\therefore m = 11$$

b) I). $y = x^3 + x^2 - 5x + 3$, where $x^3 + x^2 - 5x + 3 = 0$ $y = 0$.

y is zero at the points where the curve $y = x^3 + x^2 - 5x + 3$

touches or cuts the x – axis

$$\therefore x = -3 \text{ and } x = 1$$

II). Notice that the equation $y = 5x$ is a straight line. Draw the line $y = 5x$ on the same graph, the points where it cuts or meets the graph $y = x^3 + x^2 - 5x + 3$, are the solution to

$y = 5x$ or $y = x^3 + x^2 - 5x + 3 = 5x$. make the table of values as below.

x	0	1	5	-1
y	0	5	25	-5

$$\therefore x = -3.8, x = 0.3 \text{ and } x = 2.5$$

c). The given boundaries forms up two trapeziums.

$$A = \frac{1}{2}(a_1 + b_1)h_1 + \frac{1}{2}(a_2 + b_2)h_2$$

$$= \frac{1}{2}(8 + 9)1 + \frac{1}{2}(8 + 3)1 = 8.5 + 5.5 = 14$$

$$\therefore A = 14 \text{ square units}$$

3. a) I). $y = x^3 + 3x^2 - x - 3$, when $x^3 + 3x^2 - x - 3 = 0$, $y = 0$. And $y = 0$ on the points where the curve $x^3 + 3x^2 - x - 3$ cuts the x – axis

$$\therefore x = -3, x = -1 \text{ and } x = 1$$

$$\text{II). } x^3 + 3x^2 - x = 5 \text{ or } x^3 + 3x^2 - x - 5 = 0 \rightarrow (x^3 + 3x^2 - x - 3) - 2 = 0$$

But $y = x^3 + 3x^2 - x - 3$, replacing y for $x^3 + 3x^2 - x - 3$ we have $y - 2 = 0$ or $y = 2$.

Thus the solutions to the equation $x^3 + 3x^2 - x = 5$ are the points on the graph where $y = 2$

$$\therefore x = -2.6, x = -1.6 \text{ and } x = 1.15$$

b) I). Draw the straight line touching the point $(-3, 0)$ only and pick any other point from the same lie and find the gradient. In my case I pick $(-2, 8)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{-2 - (-3)} = 8$$

$$\therefore m = 8$$

$$\text{II). } A = \text{Area of a trapezium} = \frac{1}{2}(a + b)h = \frac{1}{2}(1 + 2.3)20 = 3.3(10) = 33$$

$$\therefore A = 33 \text{ square units}$$

4. a) I). $y = x^3 + x^2 - 12x$, when $x^3 + x^2 - 12x = 0$, $y = 0$. To find the solution, get the value of x where the function $y = x^3 + x^2 - 12x$ cuts the x -axis.

$$\therefore x = -4, x = 0 \text{ and } x = 3$$

II). For $y = x + 10$, make the table of values of $y = x + 10$ and draw the line on the same graph. The points where the line $y = x + 10$ cuts or meets or touches the curve $y = x^3 + x^2 - 12x$ is the solution to $y = x + 10$

x	-4	-2	0	3
y	6	8	10	13

$$\therefore x = -3.85, x = -0.75 \text{ and } x = 3.55$$

- b). I) Draw the straight line touching the point $(-3, 8)$ only and pick any other point from the same line and find the gradient. In my case I pick $(-2, 17)$

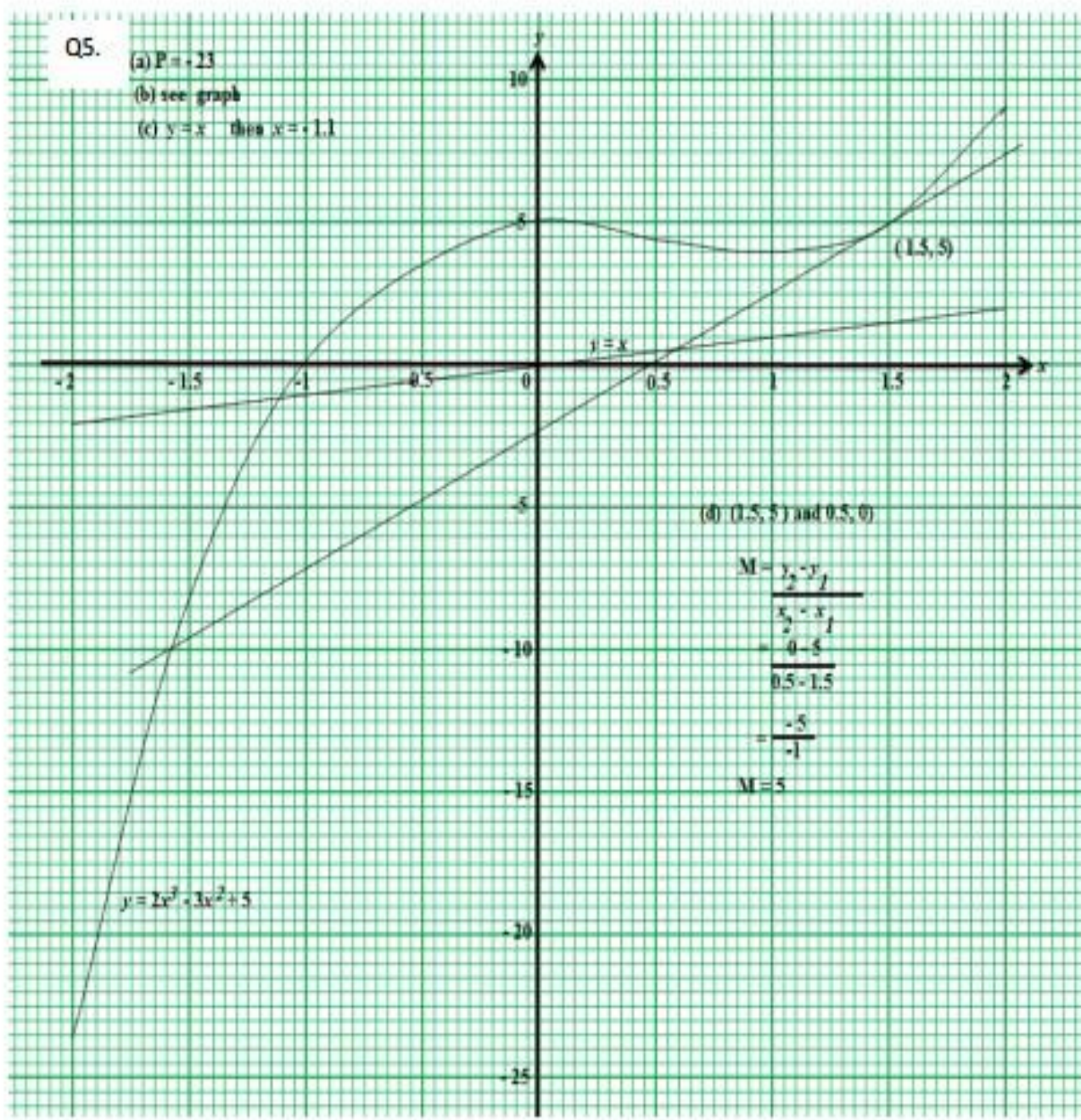
$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8 - 17}{-3 - (-2)} = 9$$

$$\therefore m = 9$$

II). Notice that there are two trapezium defining the required area. Thus $A_T = A_1 + A_2$

$$\begin{aligned} A &= \frac{1}{2}(a_1 + b_1)h_1 + \frac{1}{2}(a_2 + a_2)h_2 \\ &= \frac{1}{2}(28 + 30.5)1 + \frac{1}{2}(30.5 + 22)1 = 29.25 + 26.25 = 55.55 \end{aligned}$$

$$\therefore A = 56 \text{ square units}$$



STATISTICS

1. a).

Class marks	Mid-Point x	Frequency f	fx	x^2	fx^2
$0 < x \leq 10$	5	7	35	25	175
$10 < x \leq 20$	15	22	330	225	4950
$20 < x \leq 30$	25	28	700	625	17500
$30 < x \leq 40$	35	23	805	1225	28175
$40 < x \leq 50$	45	15	675	2025	30375
$50 < x \leq 60$	55	5	275	3035	15125
Totals		$\Sigma f = 100$	$\Sigma fx = 2820$		$\Sigma fx^2 = 96300$

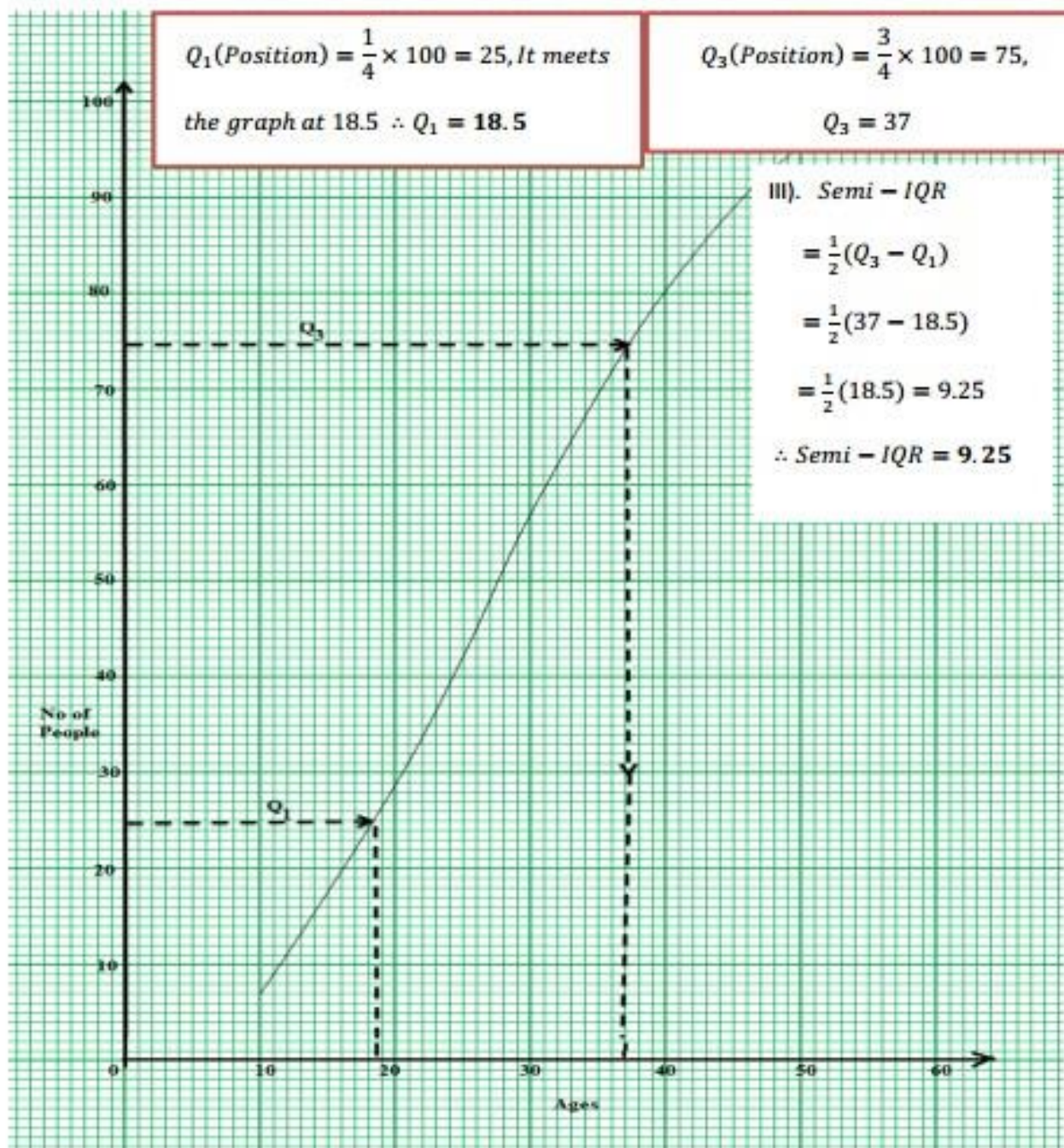
$$\begin{aligned}
 \text{Mean, } \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\
 &= \frac{2820}{100} \\
 &= \mathbf{28.2}
 \end{aligned}$$

$$\begin{aligned}
 SD &= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2} = \sqrt{\frac{96300}{100} - (28.2)^2} \\
 &= \sqrt{963 - 795.24} \\
 &= \sqrt{167.76} = \pm 12.95221989 \\
 \therefore SD &= \mathbf{12.95}
 \end{aligned}$$

b) I).

Age	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60
Number of people	7	29	57	80	95	100

II).



2. a)

Class marks	Mid-Point x	Frequency f	fx	x^2	fx^2
$25 < x \leq 30$	27.5	5	137.5	756.25	3781.25
$30 < x \leq 35$	32.5	4	130	1056.25	4225
$35 < x \leq 40$	37.5	7	262.5	1406.25	9843.75
$40 < x \leq 45$	42.5	11	467.5	1806.25	19868.75
$45 < x \leq 50$	47.5	12	570	2256.25	27075
$50 < x \leq 55$	52.5	8	420	2756.5	22050
$55 < x \leq 60$	57.5	1	57.5	3306.25	3306.25
Totals		$\Sigma f = 48$	$\Sigma fx = 2820$		$\Sigma fx^2 = 90150$

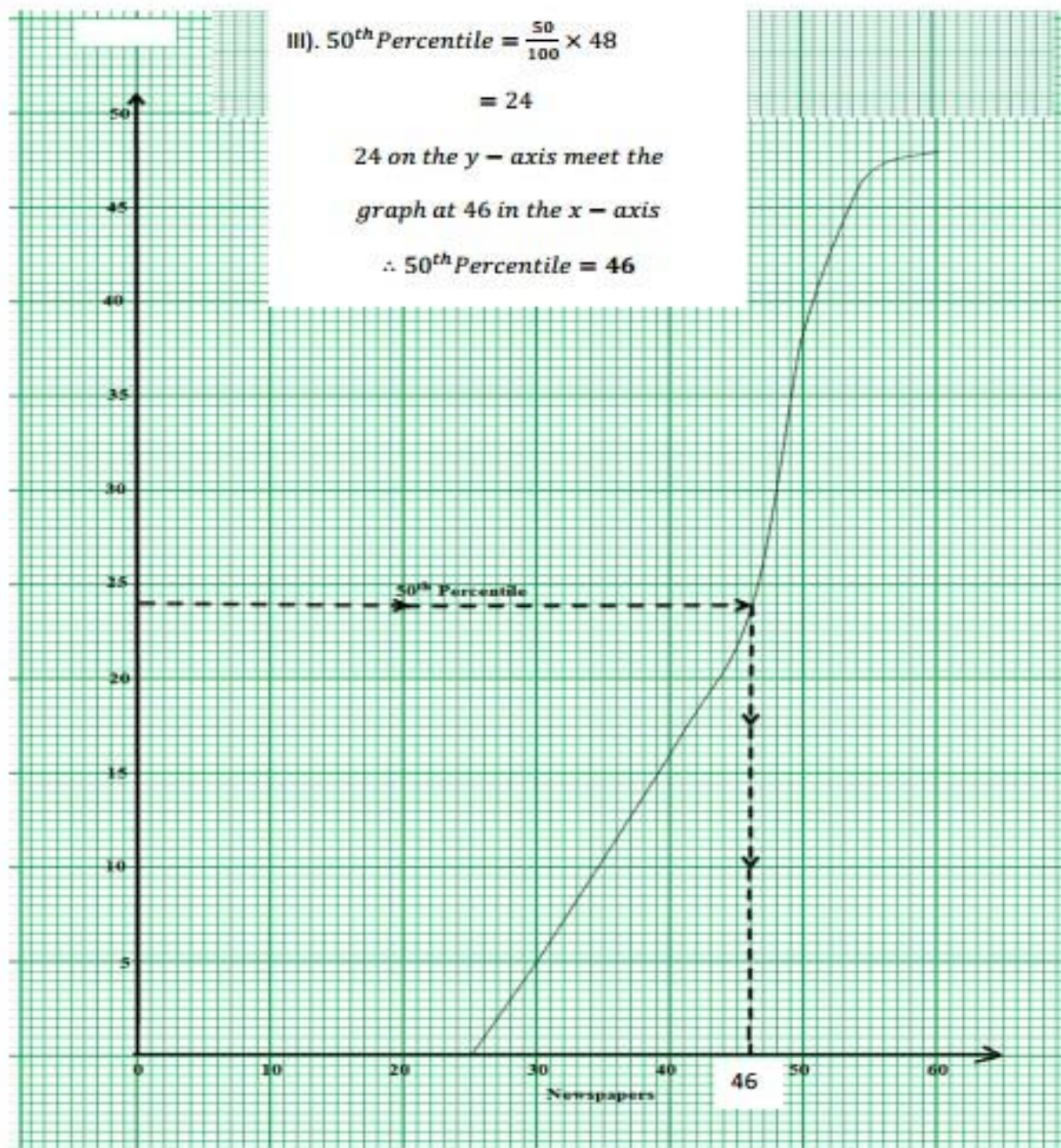
$$\begin{aligned}
 \text{Mean, } \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\
 &= \frac{2045}{48} \\
 &= \mathbf{42.6}
 \end{aligned}$$

$$\begin{aligned}
 SD &= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2} = \sqrt{\frac{90150}{48} - (42.6)^2} \\
 &= \sqrt{1878.125 - 1814.76} \\
 &= \sqrt{63.365} = \pm 7.960213565 \\
 \therefore SD &= \mathbf{7.96}
 \end{aligned}$$

b) I).

Copies of news paper	≤ 25	≤ 30	≤ 35	≤ 40	≤ 45	≤ 50	≤ 55	≤ 60
Number of Vendors	0	5	9	16	27	39	47	48

II).



3. a)

Class marks	Mid-Point x	Frequency f	fx	x^2	fx^2
$0 < x \leq 5$	2.5	13	32.5	6.25	81.25
$5 < x \leq 10$	7.5	27	205.5	52.5625	1419.1875
$10 < x \leq 15$	12.5	35	437.5	156.25	5468.75
$15 < x \leq 20$	17.5	16	280	306.25	4900
$20 < x \leq 25$	22.5	7	157.5	506.25	3543.75
$25 < x \leq 30$	27.5	2	55	756.25	1512.5
Totals		$\Sigma f = 100$	$\Sigma fx = 1168$		$\Sigma fx^2 = 16925$

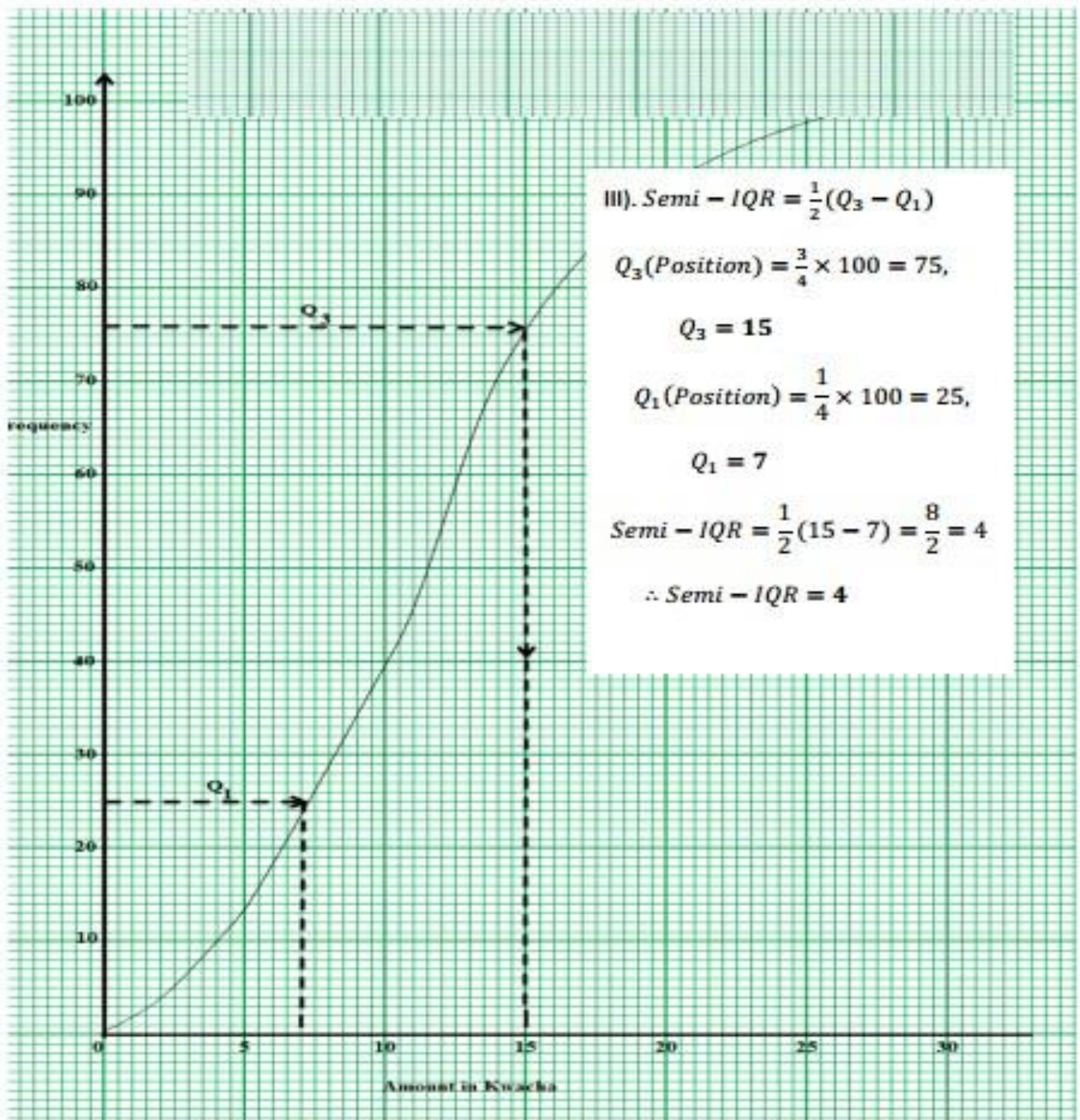
$$\begin{aligned}
 \text{Mean, } \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\
 &= \frac{1168}{100} \\
 &= \mathbf{11.68}
 \end{aligned}$$

$$\begin{aligned}
 SD &= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2} = \sqrt{\frac{16925}{100} - (11.68)^2} \\
 &= \sqrt{169.25 - 136.4224} \\
 &= \sqrt{32.832} = \pm 5.729919284 \\
 \therefore SD &= \mathbf{5.73}
 \end{aligned}$$

b) I).

Amount in Kwacha	≤ 0	≤ 5	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30
Cumulative Frequency	0	13	40	75	91	98	100

II).



4. a)

Class marks	Mid-Point x	Frequency f	fx	x^2	fx^2
$10 < x \leq 20$	15	2	30	225	450
$20 < x \leq 30$	25	10	250	625	6250
$30 < x \leq 40$	35	15	525	1225	18375
$40 < x \leq 50$	45	23	1038	2025	46575
$50 < x \leq 60$	55	30	1650	3025	90750
$60 < x \leq 70$	65	10	650	4425	42250
Totals		$\Sigma f = 90$	$\Sigma fx = 4140$		$\Sigma fx^2 = 204650$

$$\begin{aligned}
 \text{Mean, } \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\
 &= \frac{4140}{90} \\
 &= 48
 \end{aligned}$$

$$\begin{aligned}
 SD &= \sqrt{\frac{\Sigma fx^2}{\Sigma fx} - (\bar{x})^2} = \sqrt{\frac{204650}{90} - (48)^2} \\
 &= \sqrt{2273.89 - 2116} \\
 &= \sqrt{157.89} = \pm 12.5658 \\
 \therefore SD &= 12.57
 \end{aligned}$$

b) I).

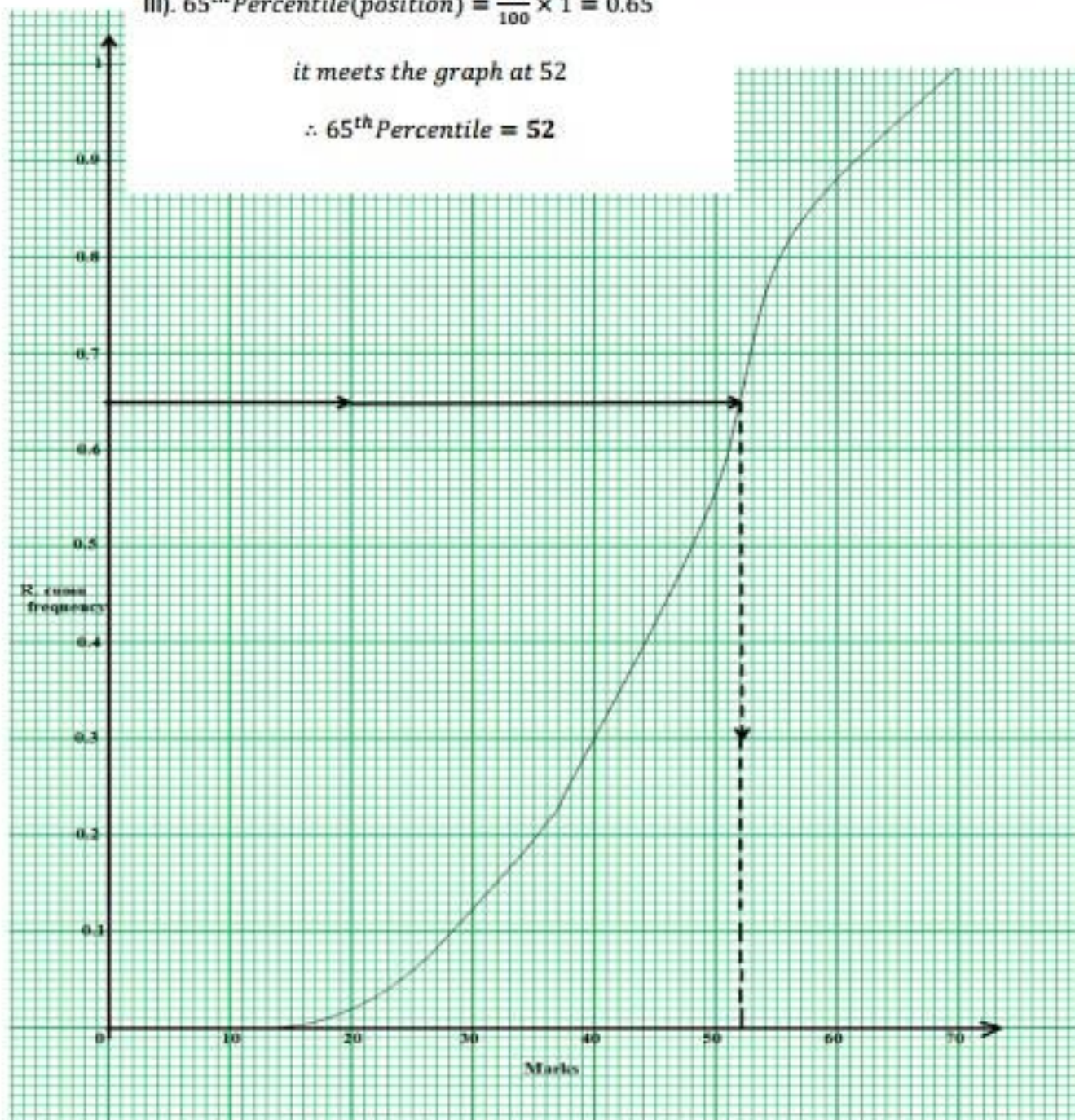
Marks(x)	≤ 10	≤ 20	≤ 30	≤ 40	≤ 50	≤ 60	≤ 70
Cumulative Frequency	0	2	12	27	50	80	90
Relative Cumulative Frequency	0	0.02	0.13	0.3	0.56	0.89	1

II).

III). $65^{\text{th}} \text{Percentile}(\text{position}) = \frac{65}{100} \times 1 = 0.65$

it meets the graph at 52

$\therefore 65^{\text{th}} \text{Percentile} = 52$



5. a)

Fruits x	Frequency f	fx	x^2	fx^2
2	1	2	4	4
3	5	15	9	45
4	4	16	16	64
5	6	30	25	150
6	10	60	36	360
7	16	112	49	784
8	18	144	64	1152
	$\Sigma f = 90$	$\Sigma fx = 379$		$\Sigma fx^2 = 2559$

$$\begin{aligned}
 \text{Mean}, \bar{x} &= \frac{\Sigma fx}{\Sigma f} \\
 &= \frac{370}{90} \\
 &= \mathbf{6.32}
 \end{aligned}$$

$$\begin{aligned}
 SD &= \sqrt{\frac{\Sigma fx^2}{\Sigma f} - (\bar{x})^2} = \sqrt{\frac{2559}{90} - (6.32)^2} \\
 &= \sqrt{42.65 - 39.9424} \\
 &= \sqrt{2.7076} = \pm 1.645478654 \\
 \therefore SD &= \mathbf{1.65}
 \end{aligned}$$

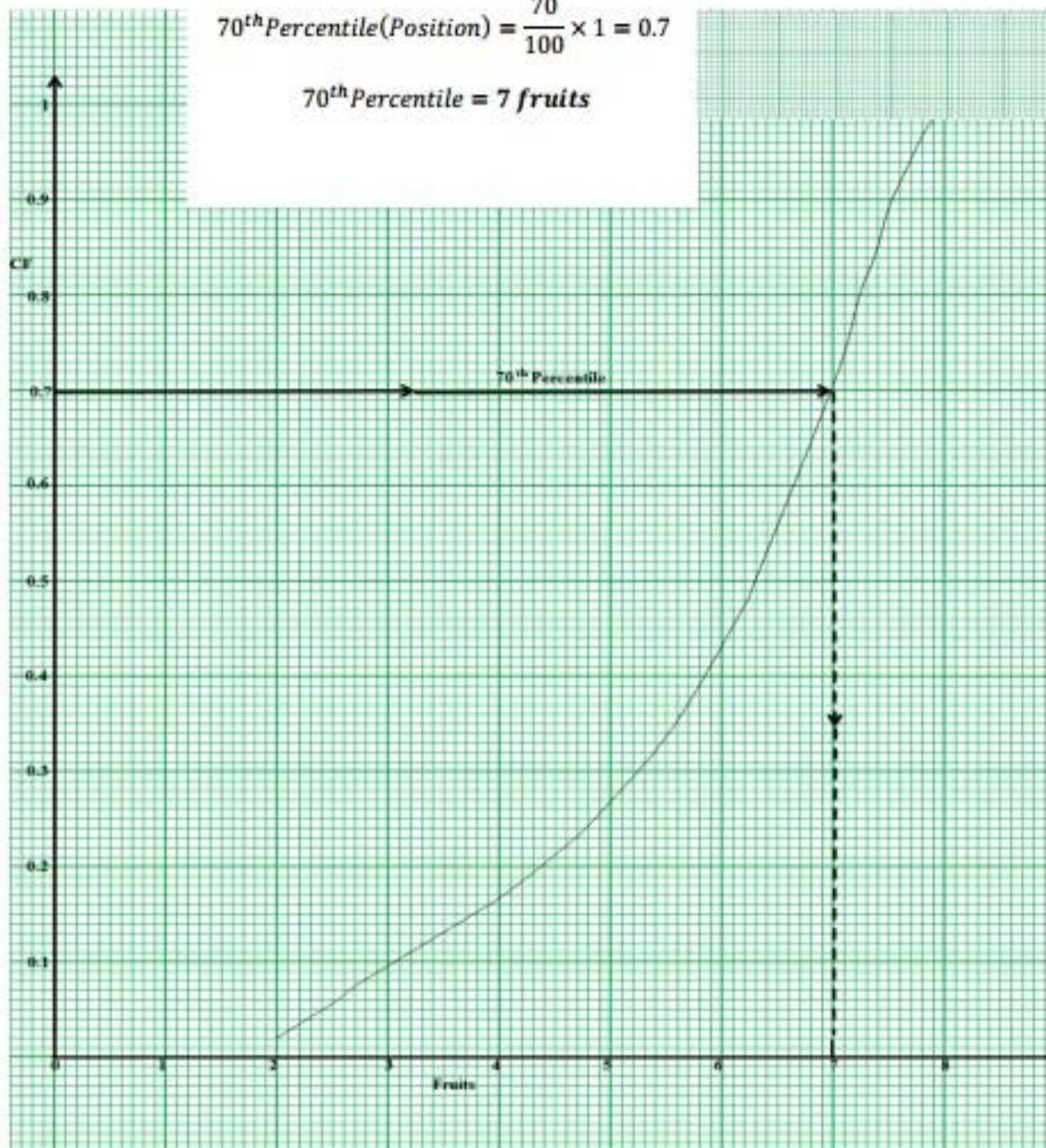
b) I).

Fruits per tree(x)	2	3	4	5	6	7	8
Cumulative Frequency	1	6	10	16	26	42	60
Relative Cumulative Frequency	0.02	0.1	0.17	0.27	0.43	0.7	1

II).

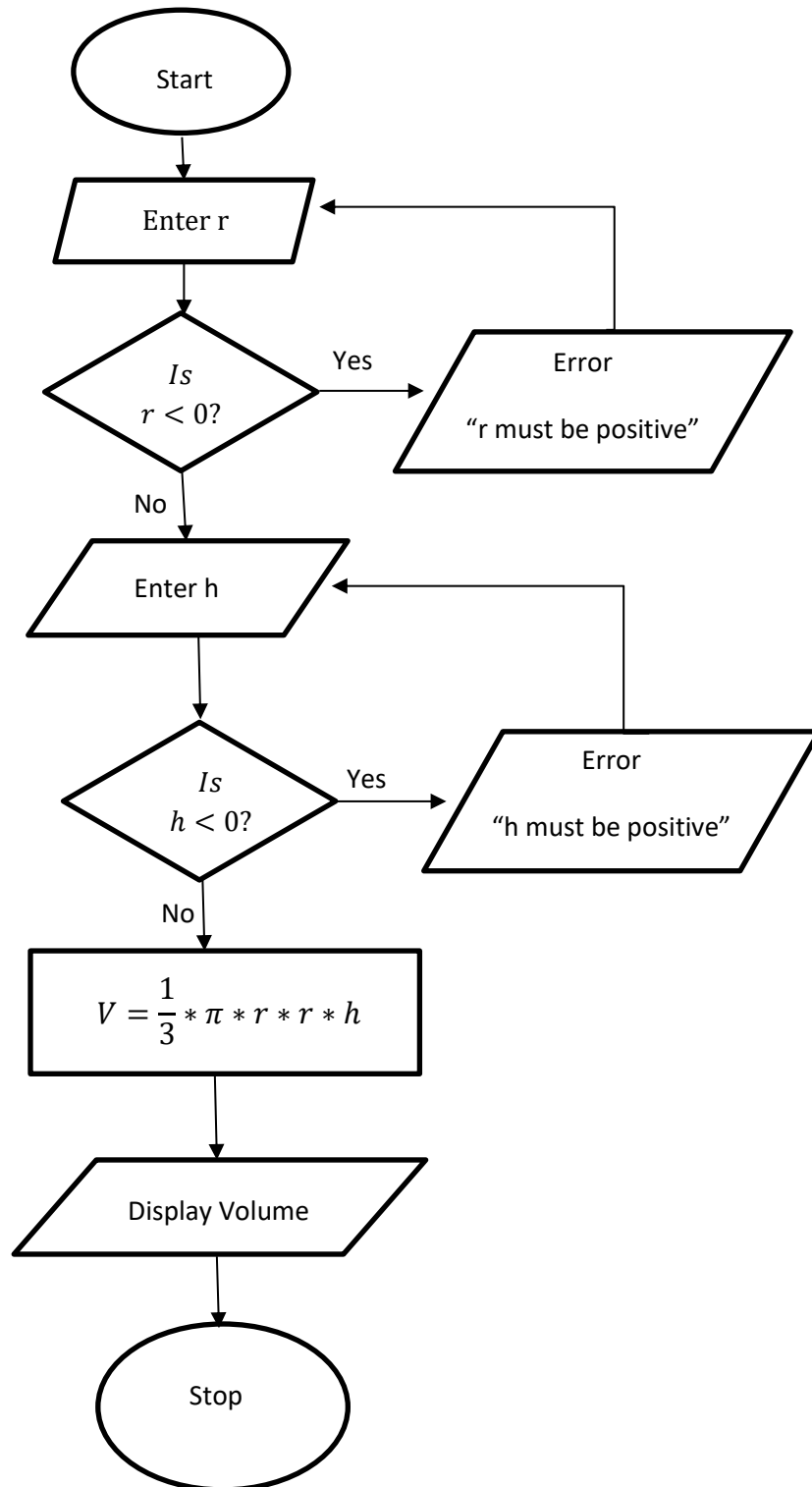
$$70^{\text{th}} \text{Percentile}(\text{Position}) = \frac{70}{100} \times 1 = 0.7$$

$70^{\text{th}} \text{Percentile} = 7 \text{ fruits}$

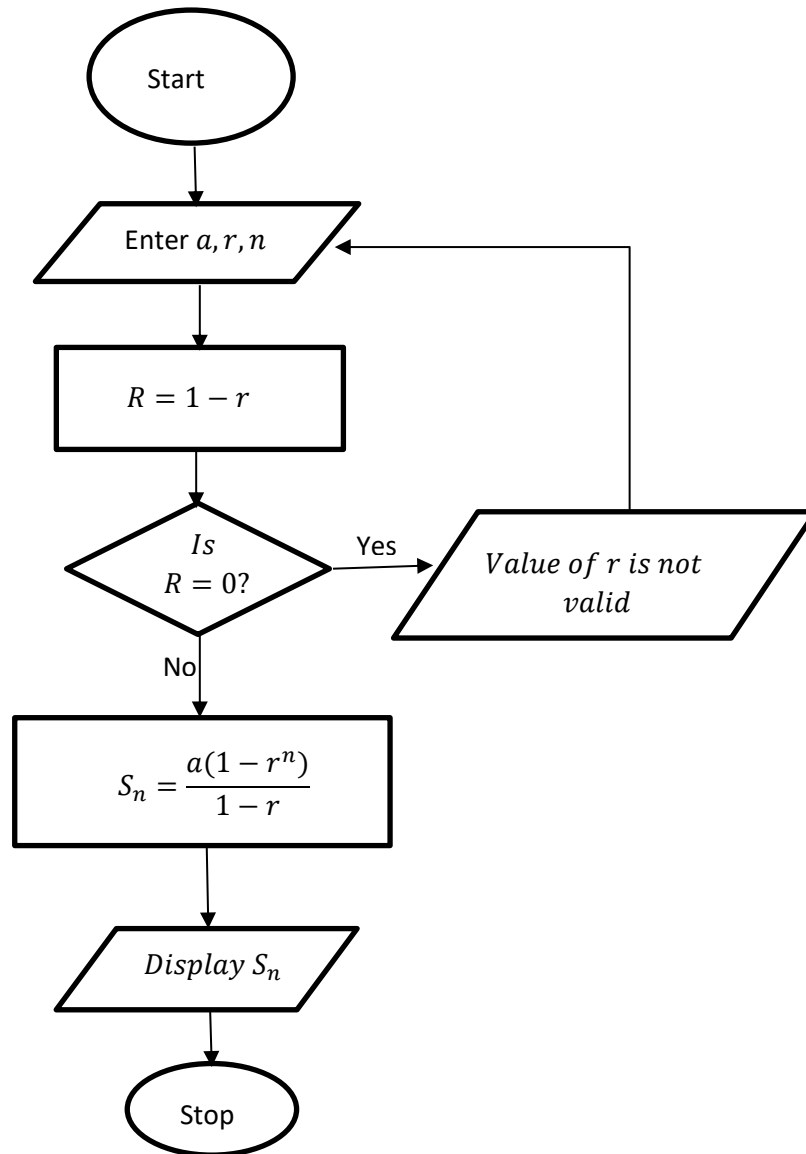


PSEUDO CODES AND FLOW CHARTS

1.



2.



3. Start

Enter a, r

If $|r| < 1$

Then sum to infinity = $\frac{a}{1-r}$

Else

End if

Display sum to infinity

Stop

4. Start

Enter radius

If radius < 0

Then display error message

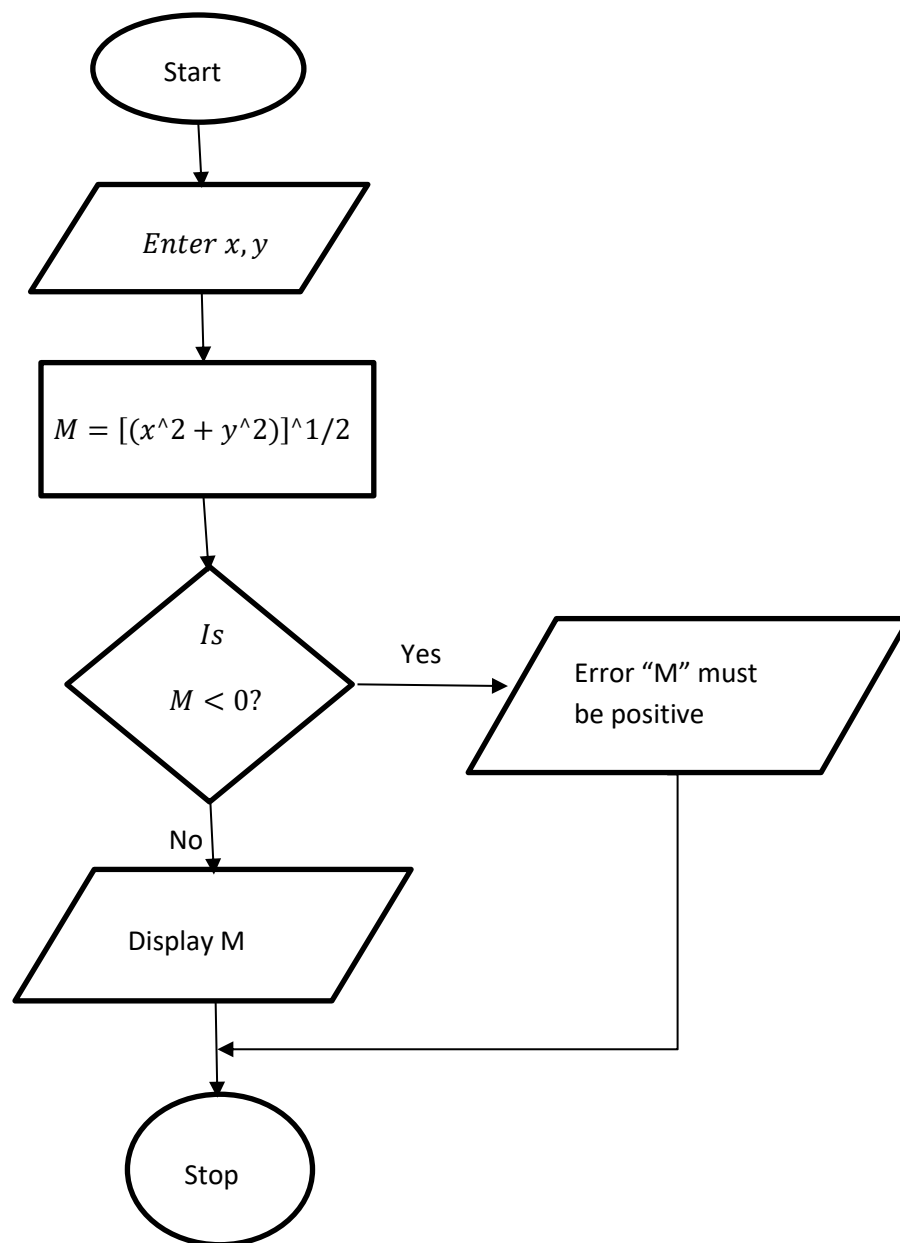
Else Area = $\frac{1}{2} * r * r \sin \theta$

End if

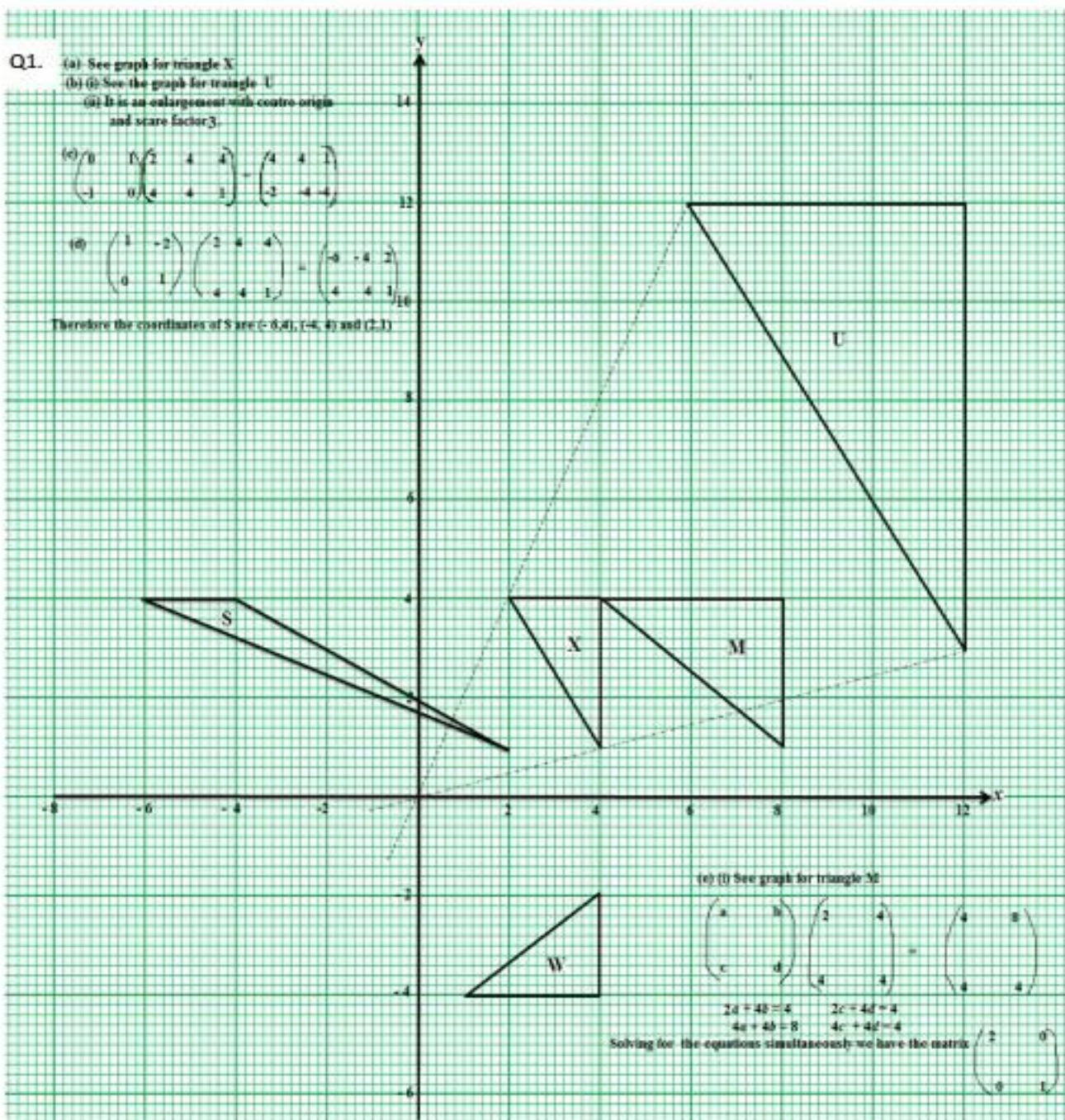
Display Area

Stop

5.



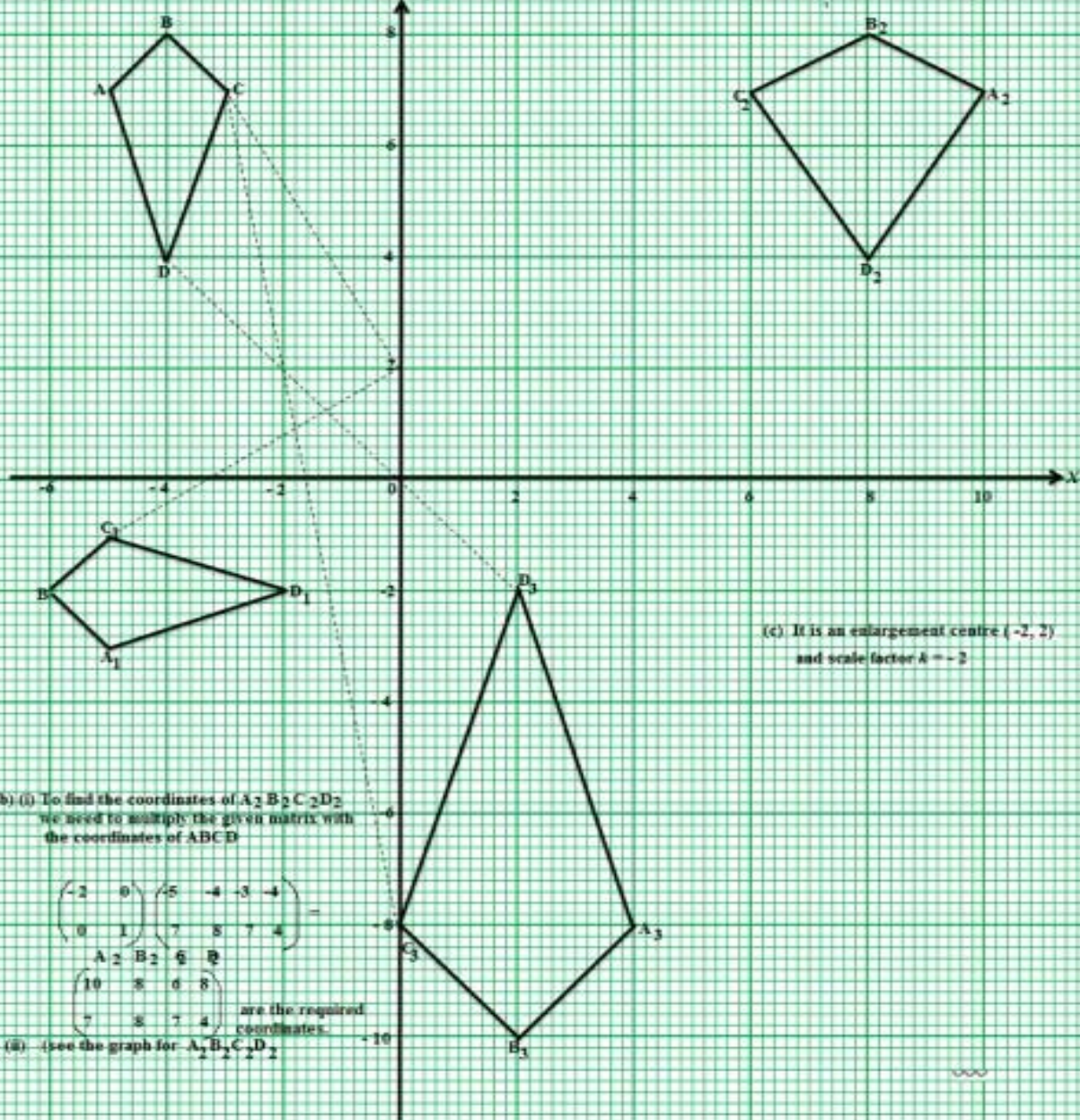
TRANSFORMATION



Q2

(a) See diagram for ABCD and its image $A_1B_1C_1D_1$

(ii) It is an anticlockwise rotation of 90° centre $(2,0)$



Q3.

(i) See the graph for both ABCD and $A_1B_1C_1D_1$

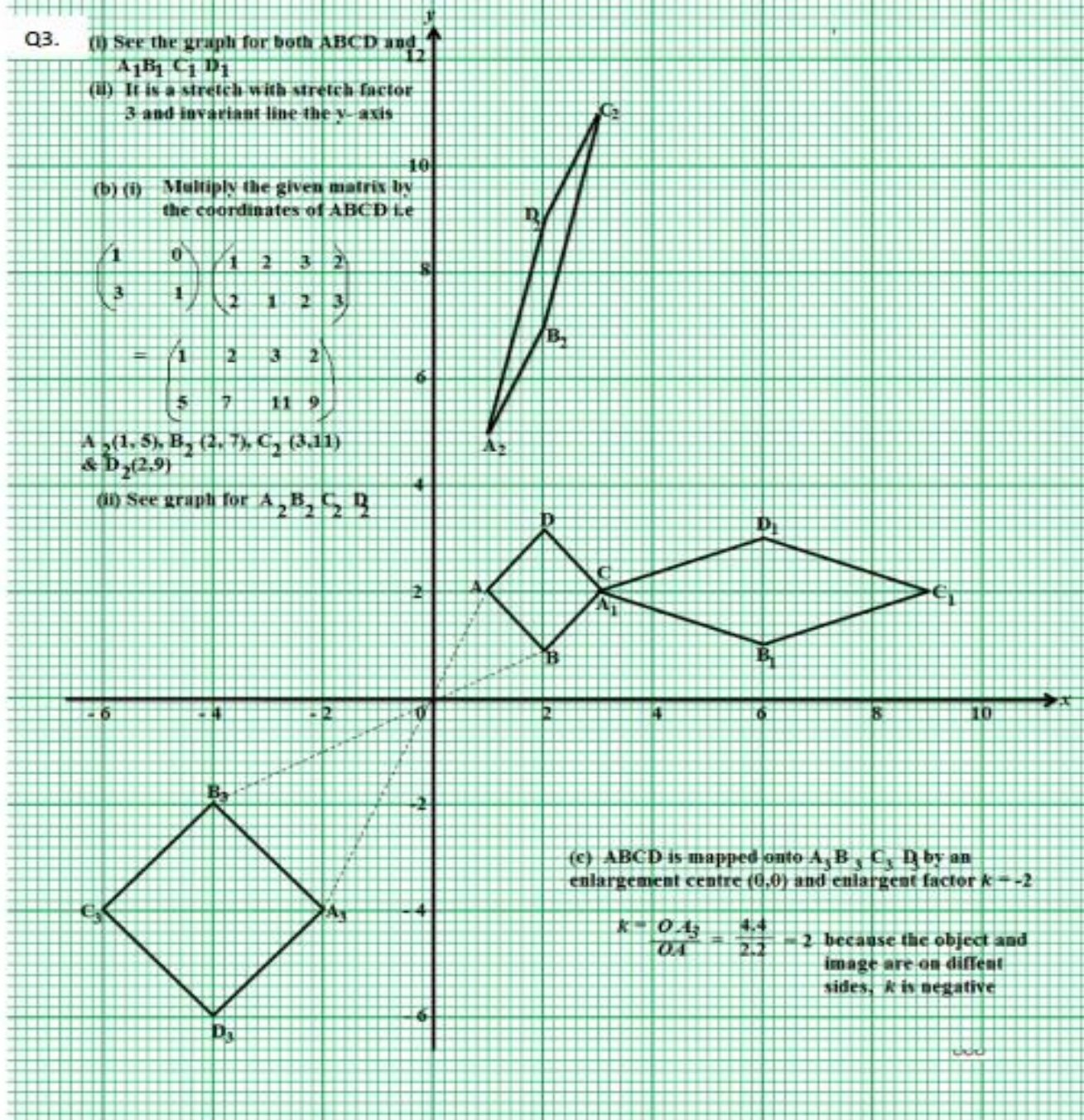
(ii) It is a stretch with stretch factor 3 and invariant line the y-axis

(b) (i) Multiply the given matrix by the coordinates of ABCD i.e

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 5 & 7 & 11 & 9 \end{pmatrix}$$

$A_2(1, 5), B_2(2, 7), C_2(3, 11)$
& $D_2(2, 9)$

(ii) See graph for $A_2B_2C_2D_2$



4. a). It is a clockwise rotation about the origin(0, 0).

b). It is an enlargement, Centre at the origin (0, 0) and a scale factor of 2

c). Let the matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, Pick the coordinates of P(in matrix form) times the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, and equate the product to the matrix made from coordinates of V as below.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 2 & 2 & 4 \\ 4 & 1 & 1 \end{pmatrix} = \begin{pmatrix} -4 & -4 & -8 \\ 4 & 1 & 1 \end{pmatrix}$$

After matrix operations, we have six equations. Solve them by any mean of your choice to find the values of a, b, c and d.

$$2a + 4b = -4 \dots \dots \dots (1)$$

$$2a + b = -4 \dots \dots \dots (ii)$$

$$2c + 4d = 4 \dots \dots \dots (iii)$$

$$2c + d = 1 \dots \dots \dots (iv)$$

$$4a + b = -8 \dots \dots \dots (v) \text{ and}$$

$$4c + d = 1 \dots \dots \dots (vi)$$

Confirm that

$$a = -2 \quad b = 0$$

$$c = 0 \text{ and } d = 1$$

$$\therefore \text{The required matrix is } \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$$

d). We get the coordinates of S, from the product of the given matrix and the coordinates of P.

$$\begin{aligned} & \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 & 4 \\ 4 & 1 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 2+0 & 2+0 & 4+0 \\ -4+1 & -4+4 & -8+1 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 2 & 4 \\ -3 & 0 & -7 \end{pmatrix} \end{aligned}$$

$\therefore$ The coordinates of S are (2, -3), (2, 0) and (4, -7).

5. a) I). Join any two corresponding points of the object and the image, the intersection point is the Centre of enlargement

$\therefore$ Centre of enlargement is (1,2).

$$\text{II). Scale factor} = \frac{\text{Length } A_1B_1}{\text{Length } AB} = \frac{4}{2} = 2$$

$$\therefore \text{Scale factor} = 2$$

b). Let the matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, Pick the coordinates of ABC (in matrix form) times the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, and equate the product to the matrix made from coordinates of $A_2B_2C_2$ as below.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 4 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 1 \\ 1 & 5 & 1 \end{pmatrix}$$

After matrix operations, we have six equations. Solve them by any mean of your choice to find the values of a, b, c and d .

$$\begin{aligned} a + 4b &= 1 \dots\dots (i) \\ 3a + 4b &= 3 \dots\dots (ii) \\ a + 5b &= 1 \dots\dots (iii) \\ c + 4d &= 1 \dots\dots (iv) \\ 3c + 4d &= 5 \dots\dots (v) \end{aligned}$$

Confirm that

$$a = 1 \quad b = 0$$

$$c = 2 \text{ and } d = -\frac{1}{4}$$

$$\therefore \text{The required matrix is } \begin{pmatrix} 1 & 0 \\ 2 & -\frac{1}{4} \end{pmatrix}$$

c). Triangle ABC is mapped onto triangle $A_3B_3C_3$ by an anticlockwise rotation of 90° about the origin $(0,0)$.

d). Compare $\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ you observe that the given matrix is enlarged -3 times in x – axis

$$\therefore \text{Scale factor} = -3$$

II). To find the coordinates of A_4B_4 and C_4 , Multiply the given matrix with the coordinates of triangle ABC.

$$\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 1 \\ 4 & 4 & 5 \end{pmatrix} = \begin{pmatrix} -3 & -9 & -3 \\ 4 & 4 & 5 \end{pmatrix}$$

$\therefore$ The coordinates of A_4B_4 and C_4 are $(-3, 4)$, $(-9, 4)$ and $(-3, 5)$ respectively.