UNIVERSITY EXAMINATIONS

UNIVERSITEITSEKSAMENS

UNISA university of south africa

MAT1503

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LINEAR ALGEBRA

Duration 2 Hours

100 Marks

EXAMINERS

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SECOND DR ZI ALI

Closed book examination

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This paper consists of 4 pages

THE USE OF A POCKET CALCULATOR IS NOT PERMITTED.

Answer all the questions

There is a total of 100 marks

QUESTION 1

(a) Describe the properties of a matrix in reduced row-echelon form (4)

(b) Solve the following system using Gauss elimination

$$\begin{cases} x - 2y + 3z &= 9 \\ -x + 3y &= -4 \\ 2x - 5y + 5z &= 17 \end{cases}$$

(6)

(c) Let

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

and find A^3 (5)

(d) Given

$$B = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ -6 & 2 & 3 \end{bmatrix}$$

Using row operations, find B^{-1} and verify that $BB^{-1} = B^{-1}B = I_3$, where I_3 is the 3×3 identity matrix (10)

[25]

[TURN OVER]

QUESTION 2

(a) Given

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$$

find $adj\,(A)$ (5)

(b) Let $k = -2$

$$B = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & 4 & 5 \end{bmatrix}$$

find $\det(B)$ and $\det(kB)$ and compare them (7)

(c) Let

$$C = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$$

and show that $\det(CD) = \det(C)\det(D)$ (8)

(d) Given the following system of equations

$$\begin{cases} -x + 2y - 3z = 1 \\ 2x + z = 0 \\ 3x - 4y + 4z = 2 \end{cases}$$

apply Cramer's rule to find x (5)

[25]

QUESTION 3

(a) Consider the vectors $\underline{u} = (1, 0, 4)$ and $\underline{v} = (1, 1, 0)$

(i) Determine the orthogonal projection $proj_{\underline{u}}\underline{v}$ (4)

(ii) Calculate the area of the parallelogram bounded by $\underline{v}$ and $\underline{u}$ (6)

(iii) Determine the perimeter of the parallelogram bounded by $\underline{v}$ and $\underline{u}$ (5)

(b) Find an equation of the plane containing the point $(0, 1, 1)$ and perpendicular to the line passing through the points $(2, 1, 0)$ and $(1, -1, 0)$ (6)

[TURN OVER]

(c) Determine whether the point $(-2, -10, 8)$ lies on the plane whose parametric equations are $x = 2 - t, y = 2 - 3t, z = 4 + t$ (4)

[25]

QUESTION 4

(a) Use De Moivre's theorem to express $\sin 4\theta$ in terms of powers of $\sin\theta$ and $\cos\theta$ (10)

(b) Determine the cube roots of -8 in polar form (10)

(c) Let a and b be real numbers such that $(a - ib)^3 = 8$ where $a \neq 0$ and $b \neq 0$

(i) Prove that $a^3 = 8 + 3ab^2$ (2)

(ii) Verify that $b^2 = 3a^2$ (3)

[25]

TOTAL [100]

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