

Tutorial letter 201/1/2017

LINEAR ALGEBRA

MAT1503

Semester 1

Department of Mathematical Sciences

This tutorial letter contains solutions for assignment 01.

BARCODE

SEMESTER 1

ASSIGNMENT 01

Fixed closing date: 20 February 2017

Unique assignment number: 693486

Key: 1211212111

(a) 1

The trivial solution is always a solution of any homogeneous system.

(b) 2

The coefficient determinant is different from 0 {It is $1 \neq 0$ } and since the system is non-homogeneous, it has a (unique) solution, by Cramers rule.

Alternatively, solving the system by substitution yields $x = -2$, $y = 4$.

(c) $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix}$

$$\text{So } AB = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 7 & 4 \end{bmatrix}$$

$$\text{whilst } BA = \begin{bmatrix} -1 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 3 & 0 \end{bmatrix}$$

$$\neq AB$$

$$\text{But } AB = \begin{bmatrix} -1 & 2 \\ 7 & 4 \end{bmatrix} \text{ is true}$$

The option is 1

(d) $AB = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{2}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\text{Similarly } BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

$$\text{Thus } AB = BA$$

The option is 1

(e) 2

In (c) both AB and BA exist, but $AB \neq BA$.

(f) The determinant of this matrix is $4(\sin^2 \theta + \cos^2 \theta) = 4 \cdot 1 \neq 0$. Hence the matrix is invertible. 1 is the correct option.

(g) In this case $\det A^{-1}kA = k^4 \cdot \det A$.

The correct option is 2.

(h) $\det(A^{-1}A) = \det A^{-1} \cdot \det A$

$$= \frac{1}{\det A} \cdot \det A = 1$$

The correct option is 1.

(i) C is a 4×4 matrix and $\det(C) = -3$.

$$\begin{aligned} \text{Now } \det(2C) &= 2^4 \det C = 16(-3) \\ &= -48 \end{aligned}$$

The correct option is 1.

(j) First, the system is linear (Why?) and non-homogeneous(Why?). So it makes sense to talk about Cramer's rule. The determinant of the coefficient matrix is $\cos^2 \theta + \sin^2 \theta = 1 \neq 0$. Now Cramer's rule may be applied since the coefficient matrix's determinant is different from zero. Option 1 is correct.