Chapter I

Introduction

1.1 What is Logic?

1.1.1 A Branch of Philosophy

What is Philosophy?

The word "philosophy" is derived from the Greek words and word-fragments:

Philia/ phile /philo—love of, affinity for, liking of

Sophia/ sophos / sophist *wisdom*

And this means love of wisdom.

Philosophy is the systematic inquiry into the principles and presuppositions of any field of study.

Branches of Philosophy

The main branches of philosophy are divided as to the nature of the questions asked in each area. The integrity of these divisions cannot be rigidly maintained, for one area overlaps into the others.

1. **Metaphysics** or **Ontology**: the study of what is *really* real. Metaphysics deals with the so-called first principles of the natural order and "the ultimate generalizations available to the human intellect." Specifically, ontology seeks to indentify and establish the relationships between the categories, if any, of the types of existent things.

What kinds of things exist? Do only particular things exist or do general things also exist? How is existence possible? Questions as to identity and change of objects—are you the same person you were as a baby? as of yesterday? as of a moment ago?

How do ideas exist if they have no size, shape, or color? (My idea of the Empire State Building is quite as "small" or as "large" as my idea of a book. *I.e.*, an idea is not extended in space.) What is space? What is time?

E.g., Consider the truths of mathematics: in what manner do geometric figures exist? Are points, lines, or planes real or not? Of what are they made?

What is spirit? or soul? or matter? space? Are they made up of the same sort of "stuff"? When, if ever,

are events necessary? Under what conditions are they possible?

2. **Epistemology**: the study of knowledge. In particular, epistemology is the study of the nature, scope, and limits of human knowledge.

Epistemology investigates the origin, structure, methods, and integrity of knowledge.

1. Consider the degree of truth of the statement, "The earth is round." Does its truth depend upon the context in which the statement is uttered?

3. Axiology or **Value theory**: the study of value; the investigation of its nature, criteria, and metaphysical status.

Some significant questions in axiology include the following:

Nature of value: is value a fulfillment of desire, a pleasure, a preference, a behavioral disposition, or simply a human interest of some kind?

Criteria of value: do objective or universal standards apply?

Status of value: how are values related to (scientific) facts? What ultimate worth, if any, do human values have?

Axiology is usually divided into two main parts.

3.1 **Ethics**: the study of values in human behavior or the study of moral problems: *e.g.*, (1) the rightness and wrongness of actions, (2) the kinds of things which are good or desirable, and (3) whether actions are blameworthy or praiseworthy.

3.2 Aesthetics: the study of value in the arts or the inquiry into feelings, judgments, or standards of beauty and related concepts. Philosophy of art is concerned with judgments of sense, taste, and emotion.

E.g., Is art an intellectual or representational activity? What would the realistic representations in pop art represent? Does art

represent sensible objects or ideal objects?

Is artistic value objective? Is it merely coincidental that many forms in architecture and painting seem to illustrate mathematical principles? Are there standards of taste?

Is there a clear distinction between art and reality?

4. Logic is a branch of philosophy that deals with arguments.

Logic is also the science that evaluates arguments.

This means that logic aims at criticizing the arguments of others and constructing arguments of our own.

1.1.3 Brief on History of Logic

Aristotle (384-322 B.C) is the ***father of logic***. His logic is called ***syllogistic logic***. He also catalogued a number of informal fallacies.

The major logician of the middle age was Peter Abelard (1079-1142). He refined Aristotle's logic and originated a theory of ***universals***

The Oxford philosopher William Occam (C.1285-1349) also contributed to the concept of ***Modal logic*** (involves possibilities such as possibility, necessity, belief and doubt) and ***metalanguage.***

G. W Leibniz (1646-1716) introduced ***symbolic logic*** , which later developed by A. DeMorgan, George Boole, W.S.Jevons and J. Venn

The British Philosopher J. S. Mill (1806-1873) initiated the revival of ***inductive logic***.

Gottlob Frege (1848-1925) founded modern Mathematical logic

Most recently logic has made a major contribution to technology by providing a conceptual foundation for the electronic circuitry of digital computers.

1.2 The Nature of Arguments

1.2.1 What is an Argument?

An **argument**, in logic, is a group of **statements**, one or more of which (the **premises**) are claimed to provide support for, or reasons to believe, one of the others (the **conclusion**).

Example:

Man is mortal.

Professor Kitaw is man.

Professor Kitaw is mortal.

Based on the logical relation between the premises and the conclusion arguments may be said good or **bad.**

1.2.2. Components of Argument

A **statement** is a sentence that has a **truth value** (which is either true or false).

A **proposition** is the meaning or information content of a statement. (we can interchangeably used it with a statement)

Example: Cairo is in Ethiopia.

Statements in an argument contain one or more **premises** and one and only one **conclusion.**

The premises are statements that set forth the *reasons* or *evidence* for the conclusion.

The conclusion is the statement that the evidence is *claimed to support* or *imply*. In other words, it is what follows from the evidence or premises.

Premises and conclusions may be identified in arguments from their indicators.

Example:

Some **conclusion Indicators**: therefore, whence, accordingly, we may conclude, as a result, so, thus, hence, entails that, consequently, it follows that, we may infer, implies that etc.

Some **premise indicators**:

Since, as indicated by, because, for, in that, may be inferred from, as, given that, seeing that, for the reason that, in as much as, owing to etc.

Inference is a **reasoning process** expressed by an argument.

In the loose sense inference is used interchangeably with argument.

An argument with one premise is also called an immediate inference.

Example:

All Japan Cell phones are original apparatuses.

Therefore, it is false that no Japan Cell phones are original apparatuses.

1.3 Recognizing Arguments

A passage may contain arguments if it purports to **prove** something, and not otherwise.

A passage which purport to prove something is only the one that fulfills the following two claims:

1.3.1 Factual & Inferential Claim

1. **Factual Claim:** at least one of the statements must claim to present evidence or reasons. (This mainly refers premises)
2. **Inferential Claim:** there must be a claim that something follows from the alleged evidence. (This mainly refers the logical relationship between premises and the conclusion)

The inferential claim may be **explicit** or **implicit.**

Explicit inferential claims are asserted by the premise or conclusion **indicator** words.

Implicit inferential claims exist if there is an inferential relationship between the statements in a passage but there are no indicator words.

Example:

The price reduction [seen with electronic calculator] is the result of a technological revolution. The calculator of 1960s used integrated electronic circuit that contained bout a dozed transistors or similar components on a single chip. Today, mass production chips, only a few millimeters square contain several thousand such components. (Boikess and Edelson, in Hurley)

To decide whether a passage contains an argument, we should look for three things:

1. Indicator words

2. The presence of inferential relationship between the statements

1.3.2 Typical kinds of Non-arguments

A. Passages lacking an inferential claim such as: warnings, statements of beliefs or opinions, loosely associated statements, report, expository passage, illustration,

B. Conditional statements

The reason is the **antecedent** and the **consequent** show a **causal** connection; not inferential relationship.

Example:

If Alemu studies hard, (antecedent) he will pass the exam. (consequence)

Note that:

A **single** conditional statement is **not** an argument.

A conditional statement may serve as **either** the **premise** or the **conclusion** (or both) of an argument.

Example:

If Alemu studies hard, he will pass the exam.

Alemu really studies hard.

Therefore, Alemu will pass the exam.

The inferential content of a conditional statement may be **re-expressed** to form an argument.

C. Explanations

The reason is statements in explanations intend to **shed light** on some accepted facts; but not they use accepted facts as a **claim to prove** a conclusion.

Just as arguments have premises and conclusions, so do explanations have **explanans** and **explanandum.**

Explanandum: is the statment that is to be explaind.

Explanans: is the statement or group of statements that purports to do the explaining.

As both express the outcome of a reasoning process, explanations and arguments have certain similarities, however, explanations do not claim to prove anything.

Many [particularly, scientific] explanations can be reexpressed to form arguments.

Example: (Hurley: 22-23)

The sky appears blue from the earth's surface because light rays from the sun are scattered by particles in the atmosphere. (**Explanation**)

Light rays from the sun are scattered by particles in the atmosphere.

Therefore, the sky appears blue from the earth's surface. (**Argument**)

Some passages may be interpreted as either explanations or Arguments or both.

Example: (Hurley: 23)

Women become intoxicated by drinking a smaller amount of alcohol than men, because men metabolize part of the alcohol before it reaches the bloodstream, whereas women do not.

Some indicators have a twofold meaning.

1.4 Types of Arguments

Based on the *strengths of an arguments inferential claim* arguments are divided in to **deductive** and **inductive**.

1.4.1. Deductive Argument

In deductive arguments the conclusion is claimed to follow **necessarily or absolutely** from the premises.

A deductive argument is an argument in which if the premises are assumed to be true, it is **impossible** for the conclusion to be false.

Example:

All Ethiopians love unity.

ILS students are Ethiopians.

Therefore, ILS students love unity.

1.4.2. Inductive Argument

In these arguments the conclusion is claimed to follow **only probably** from the premises.

It is an argument in which if the premises are assumed to be true, then based on that assumption it is **improbable** that the conclusion is false.

Example:

The vast majority if AU students are from Tigray region.

Engineering students are AU students.

Therefore, Engineering students are from Tigray region.

1.4.3. Three Factors to decide arguments as deductive or inductive

1. Special i**ndicator** words

Deductive indicators:

necessarily, certainly absolutely, definitely

Inductive indicators

Probably, improbable, plausible, likely, unlikely and reasonably to conclude

2. The actual strength of **inferential claim**

3. Form of Arguments

Typically Deductive Arguments:

Arguments based on Mathematics

Argument from definition

Categorical Syllogism (Contains words All, No, some)

Hypothetical syllogism (if-then)

Disjunctive syllogism (either-or)

Typically Inductive Arguments

Prediction

Argument from Analogy

(Vs. Arguments in geometry)

Inductive Generalization

Argument from Authority

Argument based on signs

A causal inference

N.B. Argument from science can be either inductive or deductive, depending on the circumstances.

N.B. Traditional conception of Deductive and inductive arguments that involves proceedings from particular to general and vice versa is not acceptable for modern logicians.

1.5 Evaluating Arguments

Based on the **factual** and **inferential claims** of the arguments.

In other words to evaluate an argument one has to answer the following two questions:

1. Do premises support the conclusion? (Inferential claim)

2. Are all the premises true? (Factual claim)

1.4.1 Evaluating Deductive Arguments

Deductive arguments will be evaluated for:

1. **Validity**
2. **Truthfulness**
3. **Soundness**

1. Validity Vs Invalidity

Validity is about whether premises support the conclusion.

There is no middle ground between valid and invalid arguments.

A. Valid Deductive Argument

It is the one in which the conclusion follows from the premises with **necessity or certainty**.

It is an argument in such that if the premises are **assumed true**, it is **impossible** for the conclusion to be **false**.

An argument can be valid regardless of the fact that the [content of the] premises are false (because truth or falsity is irrelevant for validity).

B. Invalid Deductive Argument

It is an argument such that if the premises are **assumed true**, it is **possible** for the conclusion to be **false**.

In these arguments the conclusion does **not** follow with **strict necessity** from the premises, even though it is claimed to.

It is an argument having **true premises** and a **false conclusion**

Example:

All cats are animals. **True**

Dogs are animals. **True**

Therefore, dogs are cats. **False**

Thus, the argument is **Invalid.**

2. Truth and Falsity

The truth and falsity of premises and conclusion is **irrelevant** to the question of validity except in one case: (Have true premises and a false conclusion), which is **invalid**. The reason is an argument with false premise and false conclusion could be valid. Similarly, an argument with true premise and true conclusion could be invalid

Example:

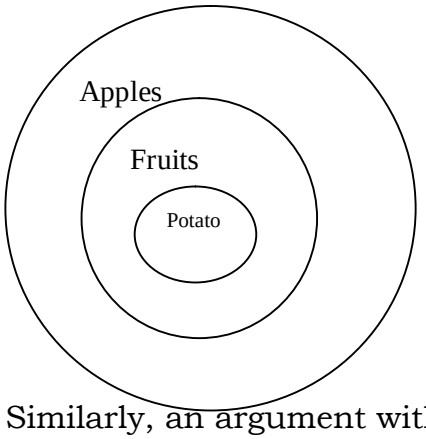
All fruits are apples. **False**

Potato is a fruit. **False**

Therefore, potato is an apple. **False**

(But it is Valid)

Look the pictorial representation



Similarly, an argument with true premise and true conclusion could be **invalid.**

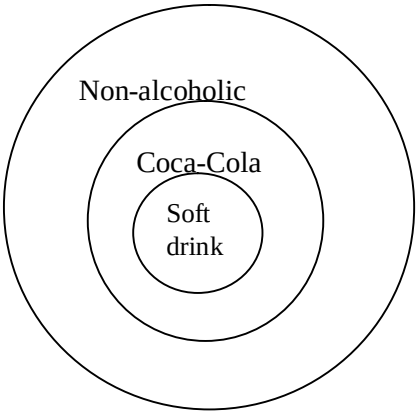
Example:

All Soft drinks are non-alcoholics. **T**

Coca-Cola is non-alcoholic. **T**

Therefore, Coca-Cola is Soft drink. **T**

Look at the following pictorial representation of the above argument.



3. Sound and Unsound Arguments

Truth and Falsity is **relevant** to evaluate arguments as **Sound** and **Unsound**.

Sound Deductive Arguments

A **sound deductive** argument is an argument that is **valid** and has **all true** premises.

The **only case** of a Sound Deductive argument is **true premises** and **true conclusions.**

Unsound Deductive Arguments

Regardless of their truth truth or falsity all **invalid** deductive arguments are **Unsound.**

Summary

The relationship among **Truthfulness, validity** and **soundness** is summarized as follows.

Pre.	Con.	Validity	Soundness
T	T	?	?
T	**F**	Invalid	Unsound
F	T	?	Unsound
F	F	?	Unsound

N.B The question mark (?) in the table indicates that based on the factual and/or the inferential claim the argument could be valid, invalid, sound or unsound.

N.B The case: True Premise & False Conclusion is impossible; if it happened, the argument is necessarily **invalid.**

1.4.2 Evaluating Inductive Arguments

To evaluate an inductive argument one has to answer the following two questions:

1. Do premises support the conclusion?
2. Are all the premises true?

Inductive arguments will be evaluated for

1. Strength
2. Truthfulness
3. Cogency

1.Strong and weak arguments:

Strength is the feature of inductive arguments

Strength of an inductive argument is determined by the degree of probability for the conclusion to come out from the premises.

The probability may be determined by various factors.

Example: The selection of a larger or smaller sample, the trustworthiness of the authorities or witnesses etc.

A. Strong Inductive Argument

It is an argument such that if the **premises** are **assumed true**, then, based on that assumption, it is **probable** that the **conclusion** is **true**.

Example:

Famine had been occurring in every decade for the last several consecutive decades in Ethiopia. **T**

Therefore, probably there will be famine in Ethiopia in the next decade. (**Probably True**)

If we assume the premises are true, it is probable for the above conclusion to be true. Indeed, the above premise is **actually true**. Therefore, it is a **Strong Inductive Argument.**

B. Weak Inductive Argument

It is an argument such that if the **premises** are **assumed true**, then, based on that assumption, it is **not probable** that the **conclusion** is **true**.

Example:

The dreams that Weyzero Debre repeatedly dreams sometimes occur directly. (**True**)

Weyzero Debre dreamt that there will be a government change in Ethiopia. (**True**)

Therefore, probably there will be a government change in Ethiopia. (**Probably False**)

If we assume the premises are true, it is **NOT** probable for the above conclusion to be true for it has the least probability (for her dreams occur directly only **occasionally**). Therefore, it is a **Weak Inductive Argument.**

2. Truth and Falsity

Truth and falsity are **irrelevant** in evaluating arguments as strong or weak.

The reason is inductive argument having **false premise** and a **probably false conclusion** could be **strong**.

Example:

All previous Soccer world cups were won by the Ethiopian team. (**False)**

Therefore, probably the next world cup will be won by the Ethiopian team. (**Probably False)**

If we assume the premises are true, it is probable for the above conclusion to be true. Therefore, this is a **Strong Inductive Argument.**

Similarly an inductive argument having **true premise** and a **probably true conclusion** could be **weak**.

Example:

A few Ethiopian leaders were warriors. (**True**)

Therefore, probably the next Ethiopian leader will be a man. (**Probably True**)

In fact both the premises and the conclusion is true, however the above argument is **weak** for the conclusion do not probably come from the premise.

3. Cogent and Uncogent Arguments

A **cogent argument** is is an inductive argument that is **strong** and has **all true premises**, if either

condition is missing, the argument is **uncogent**.

Summary

The relationship among **Truthfulness, strength** and **cogency** is summarized as follows.

Pre.	**Con.**	**Strength**	**Cogency**
T	probably T	?	?
T	probably **F**	Weak	Uncogent
F	probably T	?	Uncogent
F	probably F	?	Uncogent

Chapter II

Language: Meaning and Definition

2.1. Cognitive and Emotive Meaning

Cognitive Meanings: are terms or phrases in a language that conveys **information**.

Emotive Meanings: are terms or phrases in a language that expresses or evokes **feelings**.

Look at the meanings of the following three meanings of **Abortion**:

Abortion is an inhuman act of murdering innocent citizens which in most cases results in the death of the mother.

Since this passage contains terms like inhuman, murdering, and innocent, it evokes **negative feeling** and therefore is **Emotive.**

Abortion is a safe and systematic relief of women from unwanted pregnancy that might lead the woman to certain economic, medical, social or psychological complications.

Since this passage contains terms like safe and systematic relief it evokes **positive feeling** towards abortion and therefore is **Emotive.**

Abortion is a medical procedure in which conception or pregnancy is discontinued for certain reason.

Since this definition simply gives neutral information about abortion its meaning is **Cognitive**.

2.2 The Intension and Extension of Terms

Terms: A term is any word or arrangements of words that may serve as the s**ubject** of a statement. Terms consists of proper names, common names and descriptive phrases.

Example:

Minilik, dog, the first university of Ethiopia, those who are poor, etc

Intentional meaning: consists of qualities or attributes that the term **connotes**. (Connotation)

It implies sense or intention.

Example:

The connotation of the term cat consists of the attributes of being furry or having four legs, of moving in certain way, of emitting certain sounds, and so on.

Extensional Meaning: consists of members of the class that the term **denotes**. (Denotation)

Example:

The denotation of the term cat consists of the cats themselves - all the cats in the universe.

N.B. The meaning of connotation and denotation in logic is different from their use in grammar.

Conventional connotation is connotation of a term consists of the properties or attributes that the term commonly connotes to the members of the community who speaks the language in question. This helps to avoid confusion and decide on the appropriate connotation of a term.

Empty Extension is a form of extensional meaning (denotation) when the term (class) has **no** members.

This might happen for entities that have **perished (disappeared)** with the passage of time.

Example:

Unicorn, elf, dinosaur, current king of Ethiopia etc

Intension **determines** extension means connotation serves as the criterion for deciding what extension consists of.

Orders of Intension and Extension

1. Increasing Intension: when each term in the series (except the first) connotes **more attributes** (become **more specific**) than the one preceding it.

Example:

Living things, animals, insects, bee

2. Decreasing Intension: when each term in the series (except the first) connotes **less attributes** (become **more general**) than the one preceding it.

Example:

bee, insects, animals, Living things

3. Increasing extension: when each term in the series (except the first)

denotes a class having **more members** than the one preceding it.

Example:

bee, insects, animals, Living things

4. Decreasing Extension: when each term in the series (except the first) denotes a class having **fewer members** than the one preceding it.

Example:

Living things, animals, insects, bee

N.B Increasing Intension and **Decreasing Extension** on the one hand, **Decreasing Intension** and **Increasing extension,** on the other hand are the same.

2.3 Definitions and their Purposes

Pilosophers have conflicting views on the purpose of definition.

Example:

Plato- to explicate the meaning of certain eternal essence or forms

Modern logicians: to explicate the meaning of words.

Definition, for modern logicians, is a group of words that assign meaning for some word or group of words.

Definition consists of two parts:

1. **Definiendum :** the word or group of words that is supposed to be defined.
2. **Definiens:** the word or group of words that does the defining.

Definiens is not the meaning of the definiendum, but a symbol.

Other Philosophers: argue that since definition result in clarification of language, they provide a means for the discovery of deeper philosophical truths.

Pragmatic Approach: tries to explicate the purpose of definitions by discussing the various **kinds** of definitions.

1. Stipulative Definitions:

It assigns meaning for the **first** time.

It involves **coining** a new word or giving a **new meaning** for an old word.

Example: tigon and liger,አማርኛ

"**Tigon**" means an animal which is hybrid from male tiger and female lion.

We use stipulative definition when **new phenomena** occur or to set up secret **code**.
Due to its arbitrary nature, there can **not** be such a thing as true or false for Stipulative definition.

2. Lexical Definitions:

It is used to report the meaning that a word **already has** in a language.
They are **dictionary** definitions
They may be true or false
They have a purpose of eliminating **ambiguity**.
Example:
"Fiduciary" means a having to do with a confidence or trust; a person who holds something in trust.
An **Ambiguous word:** when the word can be interpreted as having two or more clearly distinct meanings in a given context.
Example: sound, right, bank,race, light

3. Précising Definitions
It gives precise definition of a word for specific situation/ discipline.
It is a definition which aims at reducing the **vagueness** of a word.
A **Vague word:** when there are borderline cases such that it is impossibele to tell whether the word applies to them or not.
Example: love, happiness, peace rich, fresh, poor, normal, blind, member, contract, equality, teacher, force, criminal, moment of death etc.
Example:
"Blind" means, for federal income tax purposes, either the inability to see better than 20/200 in the better eye with glasses or having a field of vision
Unlike Stipulative definition, a great deal of care must be taken in assigning meaning in Précising definition.

4. Theoretical Definitions

It provides a theoretical picture or characterization of the entity or entities denoted by definiendum.
Example: **scientific** words such as: light, force, mass, acceleration and **philosophical** words: good, mind God, change, idea, cause.
Example:
"**Sound**" means a compression wave in air or some other elastic medium

having a frequency ranging (for humans) from 20 to 20,000 vibrations per second.
Like stipulative definitions theoretical definitions are neither true nor false.

5. Persuasive Definitions

The purpose of this definition is to engender (create) a **favorable** or **unfavorable** attitude toward what is denoted by the definiendum.
It involves **emotionally charged** or **value laden** meaning to a word.
Example:
"Capitalism means" the economic system in which humanity is scarified to the wanton quest for money, and mutual understanding and respect are replaced by alienation, greed and selfishness.

2.4 Definitional Techniques
Two kinds:
A. An Extensional (Denotative) Definition

It is the one that assigns a meaning to a term **by indicating the members** of the class that the definiendum denotes.
They are chiefly used for producing **lexical** and **stipulative** definitions.
Three kinds of denotations:

1. Demonstrative (ostensive)
By **pointing** to one or all of the members of that class
Example:
"**Board"** means this or this and this and this (as you point it)

2. Enumerative Definitions
It assigns a meaning to a term by partially or completely **naming the members** of the **class** the term denotes.
Example:
"Actor" means a person such as Fikadu Teklemariam, Mulualem, Serawit, or Tilahun

3. Definition by subclass
It assigns a meaning to a term by partially or completely **naming subclasses** of the class denoted by the term.
Example: "Philosophy" means metaphysics, epistemology, logic, ethics, or aesthetics.

B. Intentional (connotative) Definitions

It is one that assigns meaning to a word by **indicating the qualities** or **attributes** that the word connotes. Four kinds:

1. Synonymous definition

It is one which the definiens is a **single word** that connotes the same attributes as the definirndum.

Example:

"Physician" means doctor

2. Etymological Definition

It assigns a meaning to a word by disclosing the word's **ancestry** in both in its own language and other language.

Example:

Logic, Philosophy, Orthodox, ሥነምግባር

"Orthodox" is originated from two Greek term *ortho*-straight or correct and *doxa*-belief or opinion.

3. Operational definition

It assigns a meaning to a word by specifying certain **experimental procedures** that determine whether or not the word applies to a certain thing.

Example:

A solution is acid if and only if litmus paper turns red when dipped in to it.

4. Definition by Genus and Difference

It is assigning a meaning to a term by identifying a **genus** term and one or more **difference**.

Example:

Ice means frozen water

(Species) (Difference) (Genus)

Criteria for Lexical Definitions

Rule 1. It should conform to the standards of **proper grammar**.

Example:

"cardiac" is like something to do with the heart. (improper grammar)

Rule 2. It should convey the **essential meaning** of the word being defined.

Example:
"Computer" is a machine which is capable of storing a great deal of information. (Not essential)

Rule 3. It should be neither too **broad** nor too **narrow**.

Example:
"Bird" means a warm blooded animal. (Broad)
"**Bird**" means any awarm blooded animal, feathered animal that can fly. (Narrow)

Rule 4.They should **not** be **circular**.

Example:
"Silence" means the state of being silent.

Rule 5. It should **not** be **negative** when it can be affirmative.

Example:
"Concord" means the absence of discord. (negative)

Rule 6. It should not be expressed in **figurative, obscure, vague,** or **ambiguous** languages.

Examples:
"**Architecture**" means frozen music. (Figurative)
"**Bunny**" means a mammalian of the family Leporidae of the order Lagomorpha whose young are born furless and blind. (Obscure)
"**Democracy**" means a kind of government where the people are in control.(vague)
"Triangle" means a figure composed of three straight lines in which all the angles are equal to 180˚. (ambigious)

Rule 7. It should avoid **affective** terminology.

Example:
"Government" means the **cruel** and **irresponsible** executive committee of the exploitation class (Incorrect)

Rule 8. It should indicate the context to which the definiens pertains. (Whenever the definiendum

is a word that means different things
in different contexts, a reference to
the context is important.)

Example:
"Strike" means (in baseball) a pitch
at which a batter swings and misses.

Chapter III

Informal Fallacies

3.1 Fallacies in General

Fallacy: is a **defect** in an argument that consists in something other than merely false premises.

A fallacy is a **bad** argument. Conversely, any bad argument is bad because either it contains a **fallacy** or because it has one or more **false premise** or both.

It usually involves either a **mistake in reasoning** or the **creation of some illusion** that makes a bad argument appear good.

The tricks of **fallacies** fool not only the listener but also the arguer.

Formal Fallacy: is the one that may be identified through mere inspection of the **form** or structure of the argument.
This will be studied in detail on the later chapters.

Example: Fallacy of affirming the consequence

K D

D____

K

If Mohammed Amin was killed in a plane crash, then Mohammed Amin is dead.
Mohammed Amin is dead.
Therefore, Mohammed Amin was killed in a plane crash.
(It is **fallacious**, because the argument has true premises and a false conclusion.)

Informal Fallacy: is the one that can be detected only through analysis of the **content** of the argument.
Example: Fallacy of Accident
Whoever thrusts a knife in to another person should be arrested. But surgeons precisely this when operating. Therefore surgeons should be arrested. (Hurley 129)

The fallacies in **most**, but **not all deductive** arguments may be identified through mere inspection of the **form**.

Example 1:

All factories are plants.

All plants are things that contain chlorophyll.

Therefore, all factories are things that contain chlorophyll.

(The **fallacious** nature of this argument cannot be identified through the mere inspection of the **form**, but through analysis of the **content**)

Example 2:

All valid deductive arguments with all true premises are **sound**.

All **sounds** can be measured in bel or decibel.

Therefore, all valid arguments can be measured in bel or decibel.

(**Fallacious**, because there is an **illusion** in the meaning of the word **sound**)

3.2 Classification of Informal Fallacies

Aristotle and other logicians classified informal fallacies in to many forms.

The major categories are fallacies of relevance, weak induction, presumption, ambiguity and grammatical analogy.

3.2.1 Fallacies of Relevance

1. Appeal to force (Argumentum ad Baculum)

Is also called (appeal to the stick)

It occurs whenever an arguer uses a **physical** or **psychological** threat (force), which is logically irrelevant to the subject matter of the conclusion, and is simply to convince its opponent.

Example:

A teacher to a student:

"You should attend the lecture regularly. If you miss a single lecture you will be dismissed from the university"

Example:

(Police inspector to suspected criminal)

You have committed the crime; if you do not accept this you will be tortured the whole night.

2. Appeal to Pity (Argumentum ad Misricordiam)

It occurs whenever an arguer attempts to convince the listener or reader by **evoke pity**.

Example:

A tax payer to judge: Your Honor, I admit that I declared thirteen children as dependents on my tax return, even through I have only two, and I realize that this was wrong. But if you find me guilty of tax evasion, my reputation will be ruined. I will probably lose my job, my poor wife will not be able to have the operation that she desperately needs, and my kids will starve. Surely I am not guilty. (Hurley: 118)

3. Appeal to the People

(Argumentum ad Populum)

3.1. Direct Approach:

Occurs when an arguer, addressing a large group of people, **excites the emotions** and enthusiasm of the **crowed** (arouse **mob mentality**) to win acceptance for his conclusion. Mob mentality usually created through mass media.

Example:

Communism is a political system where by men become the master of itself and Mr. Marx is a **fighter of communism**. Therefore, it is logical to accept Marxism.

2. Indirect Approach:

In this case the arguer directs his appeal not to the crowed as a whole but to one or more **individuals separately**, focusing up on some aspect of **relationship** to the **crowed**.

Three forms:

2.1. Bandwagon Argument

The idea is that **you will be left behind** or **left out of the group** if you do not accept it.

Example:

Of course, you have to vote the Social democrats, because majority of the people vote social democrats.

2.2. Appeal to Vanity

It attempts to persuade people by **associating the idea** with a certain celebrity who is **admired** and pursued.

Example:

You should buy Kangaroo foam because it is the mattress that Haile Gebresillsie uses.

2.3. Appeal to snobbery

It attempts to persuade the individual to accept the idea by making **feel** he/she is among the **few best**.

Example:

Hummer automobiles are not for every one but only for the few rich, if you qualify as one of the select few purchase it.

4. Argument against the person (Argumentum ad Hominem)

It involves two arguers and is occurred when one of the arguers attempts to convince the other not by presenting convincing arguments; but through attacking the person by mentioning its weaker sides.

Three forms:

4.1. The **ad hominem abusive**

It occurs when the second person responds the first person's argument by **verbally abusing** the first person.

Example:

Her argument for abortion is useless because she argued that way for she aborted three times.

4.2. The **ad hominem circumstantial**

It occurs when the second person responds the first person's argument by **discrediting** the opponent's argument by alluding (referring indirectly) to certain **circumstances** that affect the opponent.

Example:

The dean's argument for 70-30 policy is useless for he argues in that way simply because he is member of the ruling political party.

4.3. The **tu quoque ("you too")**

It begins when the second arguer attempts to convince by citing features in the life or **behavior** of the first arguer that **conflict** with his/her **conclusion**.

Example:

His son concluded that his father's argument against smoking is useless for his father smokes too.

5. Accident

It is committed when a **general rule** is **misapplied** to a **specific case** which was not intended to cover.

Example:

Property should be returned to its rightful owner. The drunker man who is starting a fight with his opponents at the pool table lent you his pistol, and now he wants it back.

Therefore, you should return it to him now. (Adapted from Hurely: 123)

6. Straw Man

It is committed when an arguer **distorts** an opponent's argument for the purpose of more **easily attacking** it.

Example:

Wr/t Meron's argument for college love affair is useless. Obviously this love affair leads the students to unwanted pregnancy. And unwanted pregnancy will definitely results in college drop out, medical and psychological complications. Therefore, her argument is useless.

7. Missing the Point (Ignoratio Elenchi)

It occurs when the premises of an argument supports one **particular conclusion,** often **vaguely related** to the correct conclusion, is drawn.

Example:

Democracy is a mock in most African countries. The only alternative is therefore, reestablishing the military government.

8. Red Herring

It is committed when the arguer **diverts** the **attention** of the reader or listener by **changing the subject** to some totally different issue.

Example:

Ato Belete is asked to report why he has been late last night. He started to talk about the amazing TV show he watched that night, and smartly managed to change the attention of the requester.

N.B. While both Red herring and Straw man fallacies proceed by generating a new set of premises, but Missing the point draws inappropriate conclusion from the original premise.

3.2.2 Fallacies of Weak Induction

They occur due to the **weak connection** between the premises and the conclusion.

9. Appeal to unqualified Authority (Argumentum

ad verecundiam)

It occurs when the cited authority or witness is not trustworthy.

Example:

Our mathematics professor said that Ethiopia had no written philosophy until C 20th. Therefore, it is true that Zerayacob's C17th philosophy can not be considered as a written Ethiopian Philosophy.

10. Appeal to Ignorance (ad Ignoratiam)

It is occurred when the arguer concludes something to be evident because **noting is known with certainty**, and vice versa.

Example:

Noting is known with certainty about the existence of devil. Therefore, devil does not exist.

11. Hasty Generalization (Converse Accident)

It is committed when a **specific case (sample not representative)** is applied to a **general rule** which was not intended to cover.

Example:

Last year three students of AU were found to be addicted to marijuana. Therefore, majority of the students of AU are addicted to the drug.

12. False cause

It occurs whenever the link between premises and conclusion depends on some **imaginary causal connection** that probably does not exist.

Three types:

12.1. Post hoc ergo prompter hoc

It occurs just because one event precedes another event and the arguer concluded that the preceding cause is the cause for the current effect ="**after this on account of this**"

Example:

During the past two months every Saturday a fox had crossed Ato Adefris on his way for hunting and he was unfortunate. Therefore, to be fortunate in his hunting task in the future Ato adefiris should change his route.

12.2. Non causa pro causa

It occurs when what is taken to be the **cause** of the something **is not really the cause** at all.

Example:

Best medical doctors are those who are paid salaries in excess of 10,000 Eth. Birr. Therefore, the best way to ensure Dr. Dereje will become the best medical doctor is to raise his salary to at least 10,000 Eth. Birr.

12.3. Oversimplified cause

It occurs when a **multitude of cause** is responsible for a certain effect but the arguer selects just **one** of these and represents it as if it is the **sole** (only) cause.

Example:

There are more laws on books today than ever before, and more crimes are being committed than ever before. Therefore, to reduce crime we must eliminate the laws. (Hurley 140)

13. Slippery slope

It is another variety of false cause. It occurs when the conclusion of an argument rests up on an **alleged chain reaction** and there is **no sufficient reason** (not likely to occur) to think that the chain of reaction will actually takes place.

14. Weak Analogy

It affects inductive argument from Analogy. It is committed when the **analogy is not strong** enough to support the conclusion. **Example:**

Object A has a, b, c and z qualities.
Object B has a, b, c, qualities.
Therefore, object b probably has quality z.

3.2.3 Fallacies of Presumption

Arguments containing these fallacies presume (guess) what they purport to prove.

Four kinds:

15. Begging the question (Petitio Principii)

To commit this fallacy some form of **phraseology** be used that tends to conceal the questionably true character of a key premise.

In other words, the arguer creates the **illusion** that premises provide adequate support by:

1. Leaving out key premise

(Ignoring the questionably true premise which is needed to make the argument valid.)

Example:

Murder is morally wrong. This being the case, it follows that abortion is morally wrong.

(It ignores the questionably true premise which is

"Abortion is murder")

2. Restating the conclusion as a premise

Example:

Anyone who does not have an innate ability could not be a great football player. Therefore, any football player has an innate ability. (The **questionable** [may be false] premise says the same thing as the conclusion)

3. Reasoning in a circle Example: Democracy is the best political system. That is why it contains the most profound insights. Thus it influences most countries in the world. Therefore, it is the best political system.

16. Complex Question

This fallacy consists in phrasing **two or more questions** in the form of a **single question** and a **single answer** is applies to both or all questions.

Example:

Have you stopped smoking?

[If the answer is yes, it means the person has been smoking. If no, it means the person at least has been smoking.]

17. False Dichotomy

False bifurcation or Either-or fallacy

The arguer presents **two unlikely options** as if they were the only ones available.

Example:

Either you borrow me the money or you don't deserve to be called my brother.

18. Suppressed Evidence

It is committed when the arguer ignores stronger evidence that supports a different conclusion.

It occurs when an **inductive** argument **ignores** some important piece of evidence that **outweighs** the presented evidence and entails a very **different** conclusion.

Example:

Typewriter repairmen have earned a good living in the past. Therefore, typewriter repairmen will earn a good living in the future.

[It ignores a piece of evidence that computers have almost completely replaced typewriters in recent years.]

3.2.4 Fallacies of Ambiguity

They arise from the use of **ambiguous** language in the premise or conclusion.

19. Equivocation

It occurs when the conclusion of an

argument depends on **a word** or **a phrase** being used in two different senses.

Example:

All valid deductive arguments with all true premises are **sound**.

All **sounds** can be measured in bel or decibel.

Therefore, all valid arguments can be measured in bel or decibel.

(**Fallacious**, because there is an **ambiguity** in the meaning of the *word* **sound** which has two different meanings in different senses)

You can also imagine arguments that contain words like *obtuse, law, right, fan,* etc.

20. Amphiboly

It occurs when an arguer misinterprets an **ambiguous statement** and then draws a conclusion based on the faulty interpretation.

The ambiguous statement is made by **someone other** that the arguer.

The ambiguity arises from, a mistake in **grammar** or **punctuation**.

Example:

Almaz told workinesh that she had made a mistake. It follow that Almaz has at least the courage to admit her own mistake.

Example:

"Kill him not leave him."

Based on this written message from his boss a solider should conclude that he is ordered to kill the man in question.

3.2.5. Fallacies of Grammatical Analogy

They are called grammatical analogy because they occur in arguments that are grammatically similar to other, non fallacious arguments.

21. Composition

It occurs when the arguer **erroneously** (wrongly) transfers an attribute from the **parts** of something onto the **whole**.

Example:

Each atom in this piece of chalk is invisible. Therefore, the chalk is invisible.

[The attribute invisible is wrongly transferred from the parts on to the whole.]

22. Division

It occurs when the arguer **erroneously** (wrongly) transfers an attribute from the **whole** of something onto the **parts**.

Example:

Salt is a nonpoisonous compound. Therefore, its component elements sodium and chlorine are nonpoisonous.

[The attribute nonpoisonous is wrongly transferred from the **whole** on to the **parts**.

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Syllogistic Logic

4.1 Definition of concepts

Syllogism is an argument with two premises and a conclusion.

4.2 Types of Syllogism

1. Categorical syllogism is a deductive argument consisting of three categorical propositions with exactly three shared terms, two terms per proposition.

Categorical Syllogism (Contains words All, No, some)

2. Hypothetical syllogism is a type of syllogism which is characterized by its use of conditional sentences as either a premise, or a conclusion , or both.-(*if-then*)

3. Disjunctive syllogism is a type of syllogism which is characterized by its use of either-or type statements.

4.3 Categorical proposition is a statement that relates two classes or categories. The **Two categories** of categorical syllogisms are **subject term** and **predicate term.**

Categorical propositions may be found either in **ordinary** or **standard** forms.

Compare the following pairs of propositions which are presented in **ordinary** and **standard forms** respectively.

All dogs bark.

All dogs are animals that bark.

Not a single horse is mule.

No horses are mules.

Some laptops are expensive.

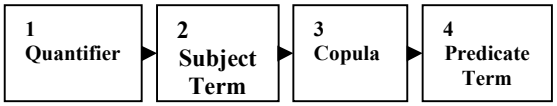
Some laptops are expensive computers.

Not everyone who preaches is a theologian.

Some preachers are not theologians.

4.3.1 Standard form

A categorical Proposition is in standard form when it contains the four components in the following order:



Component Parts

1. Quantifiers are words that show quantity.

Example: All, No Some

2. **Subject term** is a word or group of words that can serve as the

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3. Copula is a word or that connect the subject term and the predicate term.

Example: are

4. Predicate Term is a word or group of words that tell us something about the subject.

Example: animals

Types and codes

There are **four** types of categorical propositions which are codified in the first four vowel letters: **A, E, I** and **O.**

Example:

A **All** cats are mammals.

E **No** cats are mammals.

I **Some** cats are mammals.

O **Some** cats are *not* mammals.

4.3.2. Attributes of Categorical propositions

1. Quality

It refers to whether the proposition is in **affirmative** or in the **negative**.

A and **I** have **negative** attribute

E and **O** have **affirmative** attribute

2. Quantity

It refers to whether categorical proposition is either **universal** or **particular**.

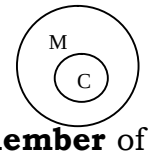
A and **E** have **universal** attribute

E and **O** have **Particular** attribute

3. Distribution

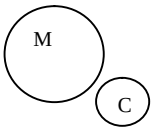
It refers to whether the proposition makes an assertion about **every member** of the class denoted by the **term**; otherwise it is undistributed.

All **cats (C)** are **mammals (M)**.



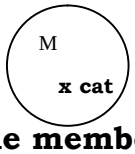
The proposition "**A**" asserts about **every member** of the **subject term** "cat"**.** Thus the **subject term** is **distributed** in proposition "**A**"

No **cats (C)** are **mammals (M)**.



The proposition "E" asserts about **every member** of the **subject term** "cat" and the **predicate term** "mammals"**.** Thus both the **subject** and **predicate terms** are **distributed** in proposition "**E**"

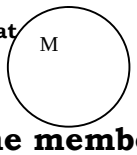
Some (at least one) ***cats (C)*** *are* ***mammals (M)****.*



The proposition "**I**" assert about **at least one member** of the **subject and the predicate terms.** Thus neither the **subject term** nor the **predicate term** is **distributed** in proposition "**I**"

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ot

x cat 

The proposition "**O**" assert about **at least one member** of the **subject term** but it asserts about **every member** of the predicate term**.** Thus the **predicate term** is **distributed** in proposition "**O**"

Consider the following table

Two mnemonic devices for distribution	
ðUnprepared Students Never Pass“ **U**niversals distribute **S**ubjects. **N**egatives distribute **P**redicates.	”Any Student Earning B’s Is Not On Probation“ **A** distributes **S**ubject. **E** distributes **B**oth. **I** distributes **N**either. **O** distributes **P**redicate

Summary

Type	Quantity	Quality	Distribution
A	**Universal**	**Affirmative**	**S**
E	**Universal**	**Negative**	**S & P**
I	**Particular**	**Affirmative**	**None**
O	**Particular**	**Negative**	**P**

4.3.3. Venn Diagrams

Venn diagram is an arrangement of *overlapping circles* such that each circle represents the *class* denoted by the term in categorical propositions.

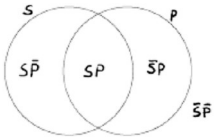
John Venn, the C19th logician, developed the system of diagrams.

Categorical propositions can be represented by Venn diagrams.

Steps

To represent Categorical propositions we should follow the three steps given below:

1. Represent the subject and predicate term with letters.
2. Draw overlapping circles in which the left hand represent the subject term, and the right hand- the predicate term.

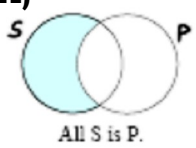


Area 1 ($S\bar{P}$)
Area 2 (SP)
Area 3 ($\bar{S}P$)

$S\bar{P}$ = S that are not P
SP = S that are P
$\bar{S}P$ = P that are not S

3. Indicate the various **areas** of the diagram whether they are **empty** (by **shading** the area) or contain at least one member of the class (by putting letter **x**).

Universal affirmative proposition

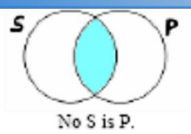
(A) 

All S is P.

"All s is p." means No members of S is outside P.

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(E) No S is P. means

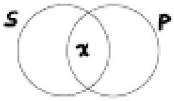


No S is P.

No members of S is inside P.

Particular affirmative proposition

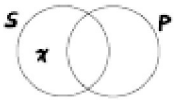
(I) Some S is P." means at least one S exists and that S is a P.



Some S is P.

Particular negative proposition

(O) Some S is not P." means at least one S exists and that S is not a P.



Some S is not P.

3.3.2 Immediate Inferences

Immediate Inferences are arguments with **single** premise and conclusion.

Venn Diagrams and **Square of oppositions,** as well as the rules of **conversion, obversion and contraposition** can be used to test the validity of these arguments.

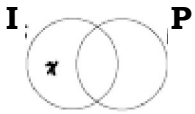
Let us practice to using **Venn Diagrams** to test the validity of an inference:

Example:

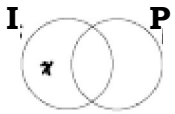
Some instructors are not professional teachers. Therefore, it is false that all instructors are professional teachers.

Step 1. Draw overlapping circles for both the premise and conclusion as we have seen earlier.

Step 2. Compare the two, if they are identical the argument is valid; if not it is invalid.



Premise

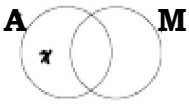


Conclusion

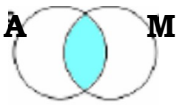
Since they are identical the argument is **valid.**

Example

It is false that that all animals are mammals. Therefore, no animals are mammals.



Premise



Conclusion

It is **invalid**, because they are not identical.

4.3.4. Square of Oppositions

It is an arrangement of lines that illustrates **logically** necessary **relations** among the four kinds of categorical propositions.

Types of squares

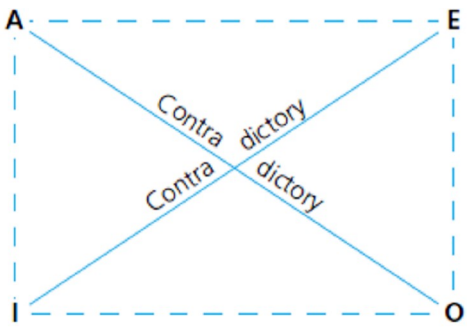
A. **Aristotelian (Traditional)**

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While the **modern square** is **neutral** about whether universal (A and E) propositions make claims about actually existing things, the **Aristotelian** interpretation assumes that the **subject term** of universal propositions denotes things that **actually exist**.

Because of this existential assumption the traditional square contains more relations than the modern square.

A. Modern square of Opposition



Modern square of opposition has **contradictory** relations. And they necessarily have **opposite** truth value.

Thus, if **A** is true **O** must be false.

If **I** is true **E** must be false.

Given the truth value of an **A** or **E** the **Corresponding** propositions **E** and **I** have logically **undetermined** truth value.

Given the truth value of an **I** or **E** propositions, **A** and **O** propositions have logically **undetermined** truth value

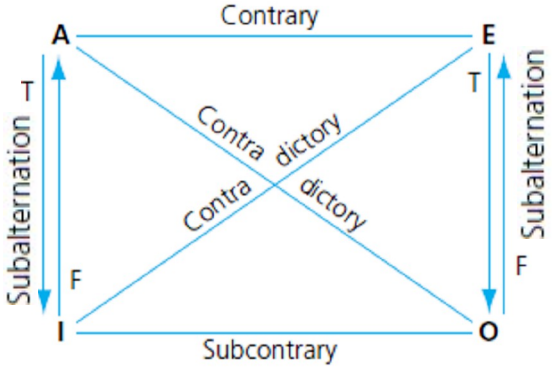
Thus, if **I** is given True, we cannot determine the truth value of the corresponding **A** or **O** propositions.

Example:

All camels are mammals. Therefore, it is false that some camels are not mammals.

Since the premise is an **A** proposition (which is given True) and the conclusion is an O proposition (which is given False), the argument is Valid

B. Traditional (Aristotelian) Square of opposition



There are four types of relations among the propositions according to Aristotelian interpretation.

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propositions.

Example

If A = T, O must be false

If A=T, E or I is undetermined

2. Contrary expresses only partial opposition. That is (at least one is false, not both true)

Example

If A= True, E must be False

If A= False, E is undetermined

3. Sub contrary expresses only partial opposition. That is (at least one is True, not both false)

Example

If I= True, O is undetermined

If I= False, O must be True

4. Subalteration expresses truth goes downward and falsity goes upward.

Example

If A= True, I = True

If A= False, I is undetermined

But:

If I= True, A is undetermined

If I= False, A=F

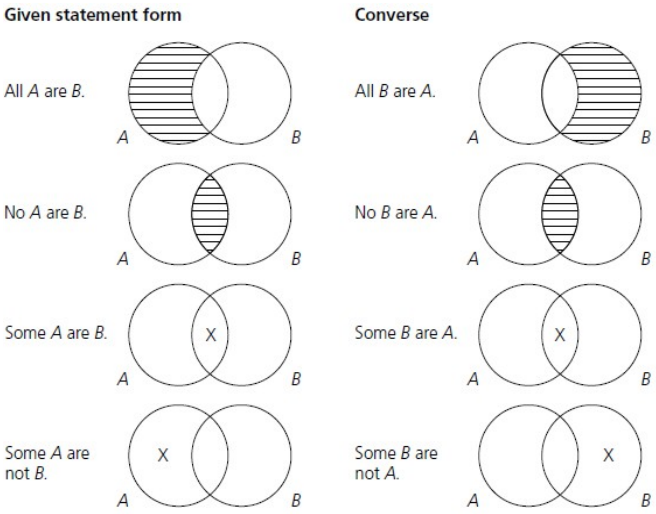
4.3.5. Categorical Operations (Conversion, Obversion, Contraposition)

1. Conversion switching the subject term with the predicate term.

Example

Proposition	Example	T/Value
A	All S are P.	Undetermined
Converse A	All P are S.	
E	No S are P.	Same
Converse E	No P are S.	
I	Some S are P.	Same
Converse I	Some P are S.	
O	Some S are not P.	Undetermined
Converse O	Some P are not S.	

Compare the Venn Diagrams



2. Obversion

It involves two steps:

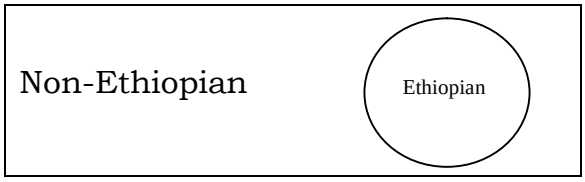
1. Changing the quality without changing the quantity
2. Replacing the predicate term with its term complement.

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group consisting of everything outside the class.

Example

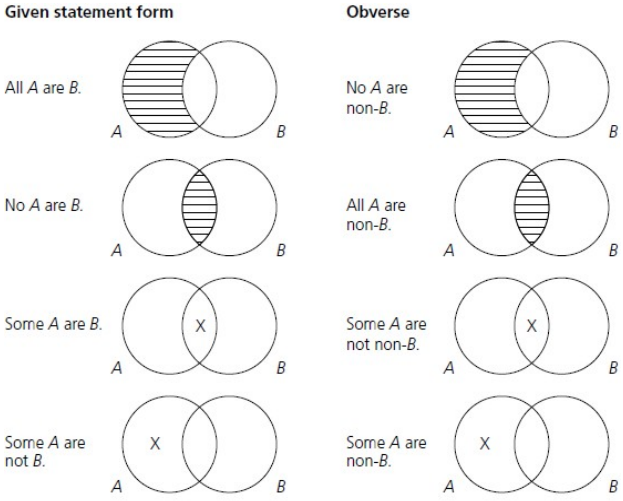
The complement of the class of Ethiopian is the group that includes people who are not Ethiopian.



Look at the following table

Proposition	Example	T/Value
A	All S are P.	Same
Obverse A	No S is non-P.	
E	No S are P.	Same
Obverse E	All S are non-P.	
I	Some S are P.	Same
Obverse I	Some S are not non-P.	
O	Some S are not P.	Same
Obverse O	Some S are non-P.	

Compare the Venn Diagrams



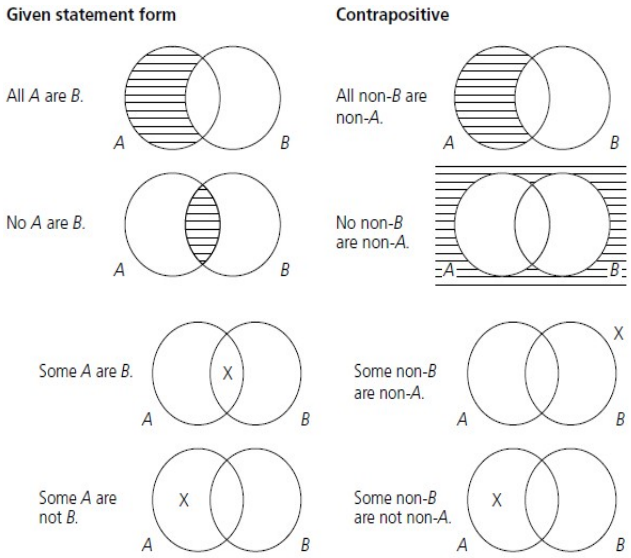
3. Contraposition

It involves two steps:

1. Converting the proposition
2. Replacing the subject and the predicate terms with their term complements

Proposition	Example	T/Value
A	All S are P.	Same
Contra pose A	All non-P are non-S.	
E	No S are P.	Undetermined
Contra pose E	No non-P are non-S.	
I	Some S are P.	Undetermined
Contra pose I	Some non-P are non-S.	
O	Some S are not P.	Same
Contra pose O	Some non-P are not non-S.	

Compare the Venn Diagrams



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ve may identify different formal fallacies

1. The fallacy of **illicit contrary** committed when we detect an argument depends on an incorrect application of the contrary relation.

Example:

Illicit contrary

It is false that all *A* are *B*.

Therefore, no *A* are *B*.

It is false that no *A* are *B*.

Therefore, all *A* are *B*.

2. The fallacy of **illicit subcontrary** committed when we detect an argument depends on an incorrect application of the subcontrary relation.

Example:

Illicit subcontrary

Some *A* are *B*.

Therefore, it is false that some *A* are not *B*.

Some *A* are not *B*.

Therefore, some *A* are *B*.

3. **Illicit subalternation.** depend on an illicit application subalternation

Example:

Illicit subalternation

Some *A* are not *B*.

Therefore, no *A* are *B*.

It is false that all *A* are *B*.

Therefore, it is false that some *A* are *B*.

4. Cases of the incorrect application of the contradictory relation are so infrequent that an "illicit contradictory" fallacy is not usually recognized.
5. **Existential fallacy,** is committed whenever contrary, subcontrary, and subalternation are used (in an otherwise correct way) on propositions about things that do not exist.

   In other words, if the traditional square of opposition is used with propositions about things that do not exist.

   Example:

All witches who fly on broomsticks are fearless women.

Therefore, some witches who fly on broomsticks are fearless women.

No wizards with magical powers are malevolent beings.

Therefore, it is false that all wizards with magical powers are malevolent beings.

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relation, and the second on an otherwise correct use of the contrary relation. If flying witches and magical wizards actually existed, both arguments would be valid. But since they do not exist, both arguments are invalid and commit the existential fallacy.

4.3.7. Representing Aristotelian standpoint I Venn diagram

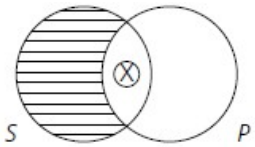
The difference between the Boolean standpoint and the Aristotelian standpoint concerns only universal (**A** and **E**) propositions.

The Boolean interpretation of these propositions makes no assumption that the subject term denotes actually existing things, whereas the Aristotelian interpretation does.

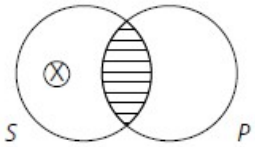
Therefore, if we are to construct a Venn diagram to represent the Aristotelian interpretation of such a statement, we need to introduce some symbol that represents this assumption of existence.

The symbol that we will introduce for this purpose is an "X" surrounded by a circle. Like the "X"s that we have used up until now, this circled "X" signifies that something exists in the area in which it is placed. However, the two symbols differ in that the uncircled "X" represents the positive claim of existence made by particular (**I** and **O**) propositions, whereas the circled "X" represents the assumption of existence made by universal (**A** and **E**) propositions. The Aristotelian interpretation of universal propositions may now be diagrammed as follows:

A: All *S* are *P*



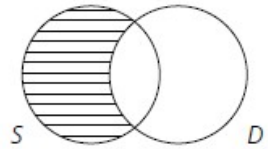
E: No *S* are *P*



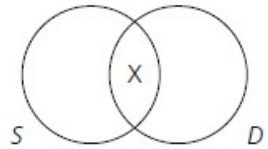
Example:

All sugar plum fairies are delicate creatures.
Therefore, some sugar plum fairies are delicate creatures. (Hurley 227)
First we test the argument from the Boolean standpoint:

All *S* are *D*.



Some *S* are *D*.



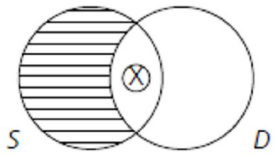
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om

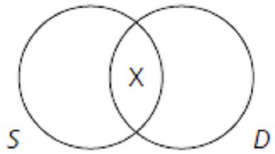
proceed to the second step and place a circled "X" in the unshaded part of the left-hand premise

circle:

All *S* are *D*.



Some *S* are *D*.



Thus this argument is Valid from the Aristotelian standpoint.

4.4. Categorical syllogism

Categorical syllogism is a deductive argument consisting of three categorical propositions with exactly three shared terms, two terms per proposition.

4.4.1. Basic Concepts

Major term – the term occurring in the **predicate** of the conclusion in a standard form categorical syllogism.

Minor term – the term occurring in the **subject** of the conclusion in a standard form categorical syllogism.

Middle term – the term occurring in **both** the major and minor premises of a standard form categorical syllogism, but not in the conclusion.

Major premise – the premise of a categorical syllogism that contains an instance of the **major term**.

Minor Premise – the premise of a categorical syllogism that contains an instance of the **minor term**.

Note: The major and minor premises are not determined by their placement in a categorical syllogism, but by terms that are contained within them. Both premises of a syllogism contain the middle term, but only the major premise and the conclusion contain the major term, and only the minor premise and the conclusion contain the minor term.

4.4.2 Standard Form of categorical Syllogism

A categorical syllogism is in **standard form** when it fulfills the following four conditions:

1. All the propositions must be in **standard form**.
2. Each term appears **twice** in the argument

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4. The **major** premise is listed **first**, the **minor** premise **second** and the **conclusion last**.

Example

Some disciples are preachers.
All disciples are Saints.
Therefore some saints are preachers.

3.4.2. Mood and Figure

All standard form categorical syllogisms can be described in terms of their ***mood*** and ***figure***.

Mood

The **mood** of a syllogism is represented by the three letters that represent the type of each proposition in the syllogism.

Example:

A standard form syllogism with three universal affirmative propositions has a mood of ***AAA.*** However, the mood of a syllogism does not fully characterize its form. For example consider these two arguments each of which has a mood of ***AAA.***

Example

All men are mortal.
All preachers are men.
All preachers are mortal.

Example

All Christians are men.
All preachers are men.
All preachers are Christians.

Both of these arguments have a mood of ***AAA,*** but they differ in how the **middle term** is placed. The first argument places the middle term in the subject of the major premise, and the predicate of the minor premise, but the second argument places the middle term in the predicate of both major and minor premises. So, although both have the same mood, they differ in form.

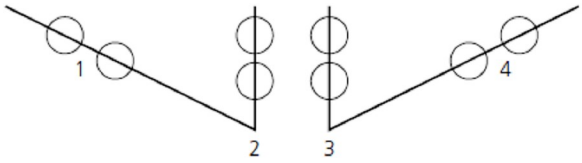
Figure

Figure refers to the **placement** of the middle term in the argument. There are **four** different figures that an argument can have as illustrated in this table:

M – *P*	*P* – ***M***
S – ***M***	*S* – ***M***
S – *P*	*S* – *P*
Figure 1	***Figure 2***
M – *P*	*P* – ***M***
M – *S*	***M*** – *S*
S – *P*	*S* – *P*
Figure 3	***Figure 4***

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The form of a standard form categorical syllogism can be represented by noting the argument's mood and figure.

In the two arguments given above the first argument is ***AAA*** in the first figure (or ***AAA-1***) and the second is of the form ***AAA*** in the second figure (***AAA-2***).

If we were to list all of the possible moods that an argument could have, we would find that **sixty four** moods are possible (4x4x4). Then if we add the figures to the number (64) arguments of the first figure, 64 for the second figure, and so on), we find that there are **256** possible argument forms which are possible (4x4x4x4). Most of these forms, however, are invalid forms.

The form of an argument is the most important aspect of an argument when considering

its validity, because validity and invalidity depend exclusively on the argument's form. In general if an argument's form is valid, the argument is valid, and if an argument's form is invalid, the argument will be invalid.

Example

All men are mortal.

All preachers are men.

All preachers are mortal.

This argument with the form ***AAA-1*** is a **valid** argument.

Example

All Christians are men.

All preachers are men.

All preachers are Christians.

This argument with the form ***AAA-2*** is **invalid**. Because any argument with an invalid form is an invalid argument, it is sometimes useful to draw analogies between an argument which is asserted as a proof and another argument of the same form with an obviously false conclusion.

Example:

All trees are plants.

All flowers are plaints.

Therefore all flowers are trees.

The fact that this argument has true premises and an obviously false conclusion proves that the argument

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invalid, and any argument of the form ***AAA-2*** is invalid. (See the table below)

Summary

Unconditionally Valid forms

Unconditionally valid			
Figure 1	Figure 2	Figure 3	Figure 4
AAA	EAE	IAI	AEE
EAE	AEE	AII	IAI
AII	EIO	OAO	EIO
EIO	AOO	EIO	

Conditionally Valid Forms

Conditionally valid				
Figure 1	Figure 2	Figure 3	Figure 4	Required condition
AAI EAO	AEO EAO		AEO	*S* exists
		AAI EAO	EAO	*M* exists
			AAI	*P* exists

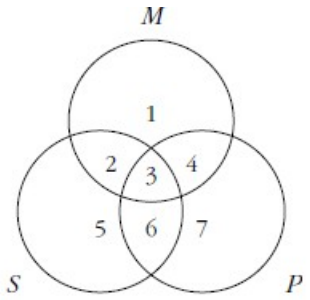
The reason for these additional valid forms is that the Aristotelian standpoint (when properly adopted) recognizes that the premises of a syllogism convey information about existence where as the Boolean stand point does not.

4.4.3. Venn Diagrams

Venn diagrams introduced by John Venn (1834-1923) provide one means for testing the validity of categorical syllogisms.

Venn diagram for categorical syllogisms is the extension of Venn diagrams for categorical propositions. (See section 4.5)

Diagramming a categorical syllogism requires the addition of a third circle in order to represent the three categories in the three terms of such syllogisms.



In the diagram above three circles are used to represent the three categories of a standard form categorical syllogism.

The letters label the terms of the syllogism and in this case the S, P, and M terms which stand for the **minor term** (the subject of the conclusion), the **major term** (the predicate of the conclusion) and the **middle term**.

Just as the two circle diagram represents more than two categories, the three circle diagram represents much more than three categories.

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S

(10) non-M, (11) S non-P, etc.

Such a diagram as that above says nothing about the categories represented. In order to represent the propositions of a syllogism we use shading to represent empty classes, and the letter “x” to represent where at least one member of a class exists.

Preliminary Pointers to represent categorical syllogisms with Venn diagrams

1. Marks (shading or placing "x") are entered only for the premises; no marks are made for the conclusion.

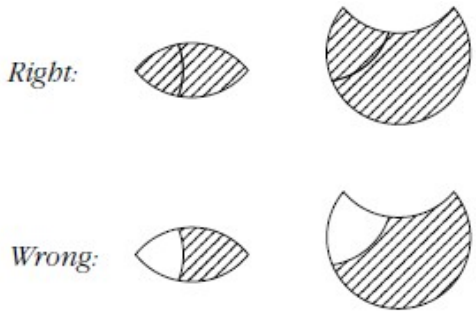
2. If the argument contains one universal premise, this premise should be entered first in the diagram. If there are two universal premises, either one can be done first.

3. When entering the information contained in a premise, one should concentrate on the circles corresponding to the two terms in the statement. While the third circle cannot be ignored altogether, it should be given only minimal attention.

4. When inspecting a complete diagram to see whether it supports a particular conclusion, one should remember that particular statements assert two things. "Some S are P." means "at least one S exists and that s is a P.";

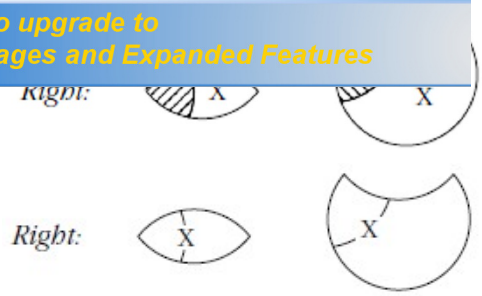
"Some S are not P." means "at least one S exists and that s is not a P."

5. When shading an area, one must be careful to shade all of the area in question.

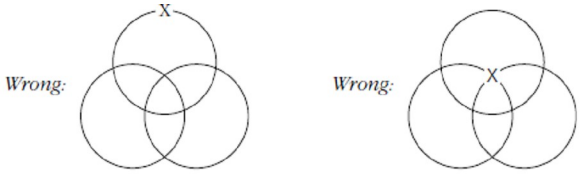


6. The area where an "x" goes is always initially divided in to two parts. If one of these two parts is already been shaded, the "x" goes in the unshaded part. If one of the two parts is not shaded, the "x goes on the line separating the two parts." This means that the "x" may be in either (or both) of the two areas- but it is not known which one.

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7. An "x" should never be placed in such a way that it dangles outside of the diagram, and it should never be placed on the intersection of two lines.

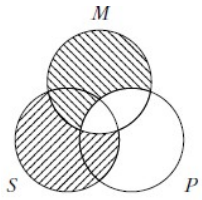


Let's note a few **examples** first involving only universal propositions.

Example 1

AAA-1. This argument has the following format:

All M are P.
<u>All S are M</u>.
All S are P.



First we shade the part of M that does not overlap P. This says that there are no members of M that exist outside of P. This represents the **major** premise of the argument.

Next we shade the part of S that does not overlap M. Again this represents that there are no members of S that are not M. This represents the **minor** premise.

Now we can ask if the premises of the proposition justify the conclusion, "All S are P," and we can see that it does. The only non-shaded part of S is within the P circle, and that represents the conclusion that all S are indeed P. Because we have shown with this example that the **AAA-1** syllogism is valid in this argument, we know that **AAA-1** syllogisms are valid in for all arguments where it occurs.

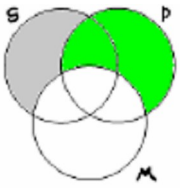
Note:

The position of the three circles may vary. In the above example the middle term was represented with the circle at the top, (recommended) however, in the following examples you will find the middle tem at the bottom. You can use either; but you need to be consistent.

Example 2

Now let us look at an argument of the form **AAA-2**, which has this form:

All P are M.
<u>All S are M.</u>
All S are P.



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ow, but to prevent confusion, it is a good place to start when it is allowed. We shade out all parts of P that are not part of M (green).

Next we represent the **minor premise** by shading all parts of S which are not M (grey). In this case part of S was already shaded when we represented the major premise.

Now we can see if the conclusion, "All S are P," is represented in the diagram, and we see that it is **not**. There is still a part of S that falls outside of P, so we are not justified in drawing the stated conclusion. The diagram represents categories in which the major premise and the minor premise are true, but the conclusion is false. No valid argument can have true premises and a false conclusion, so the argument format **AAA-2** is **invalid** wherever it is found.

It is important to note that proving that an argument is invalid is not the same thing as proving that the conclusion is false. It proves only that the truth of the premises do not justify the conclusion. Note this example **AAA-2** argument:

All Priests are Men.

All Saints are Men.

All Saints are Priests.

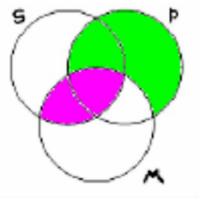
This argument has a true conclusion, but the conclusion is not proven from the premises given.

Example 3: AEE-2

All P are M.

No S are M.

No S are P.



We **start** as we did in the preceding example by shading all of P that is not a part of M (green). Again this represents the proposition that "All P are M."

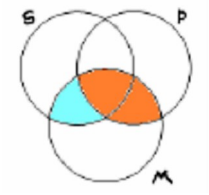
The **minor premise** states that no part of S is a part of M, so we shade out the part of S that intersects with the circle that represents M.

Now we can see if we can correctly infer the conclusion of the proposition, and it is easy to see that no part of S can be part of P, so the diagram proves that the argument is **valid**, and that the argument form **AEE-2** can be properly used with other arguments.

Example 4

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such an argument:



No M are P.
No M are S.
No S are P.

Starting with the **major premise** we shade out all of P that intersects with M (burnt orange).

Next we shade out the part of S that intersects with M to represent the minor premise (light blue).

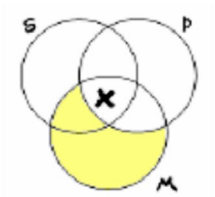
Then we note if the conclusion is represented in the diagram.

The conclusion says that no part of S could be part of P, but that is not represented in the diagram, so the syllogism is **invalid**.

A little extra care is necessary when diagramming categorical syllogisms with particular propositions within them. When doing so, diagram universal propositions before the particular propositions.

Example 5

Let's diagram an **AII-1** proposition first.



All M are P.
Some S are M.
Some S are P.

In this case the major premise is universal, and the minor premise is particular, so we start by diagramming the **major premise** which states that there is no M which is not P. To indicate this on the diagram we shade out all of M which is not P (yellow).

The **minor premise** indicates that there is at least one thing that is both S and M. This is indicated in the diagram by placing an "*x*" in the white space that is shared by S and M.

Don't place the "*x*" in the shaded area, because the other proposition says there is nothing there.

Now we can check to see if the conclusion is justified. The conclusion says that there is at least one thing that is both S and P, and we can see that the diagram confirms that conclusion. The argument is a **valid** argument.

Example 6

Let us see how an invalid argument with particular propositions appears in a diagram.

Consider this **AII-2** argument:

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Some S are P.

Again we **start** with the **universal proposition** (we would do this whether or not it was the major premise) which says that there is no P that is not M, so we shade out the part of P that is not M (grey).

The **minor premise** states that there is at least one S that is also an M. To indicate this we need to put an "*x*" in the white area that is shared by S and M, but should it go in the area within P or without P? The propositions do not tell us, so we avoid saying more than the propositions by placing the letter on the line between the two areas.

The **next** thing to do is to check to see if the diagram indicates that the conclusion has been proven true. The conclusion says that there is at least one S that is also a P, but the diagram does not say that! We cannot tell where the "*x*" should go relative to P, so the conclusion does not follow from the premises, and the argument form is **invalid**.

Example 7

This argument is invalid from the Boolean standpoint but valid from the Aristotelian.

No fighter pilots are tank commanders.

All fighter pilots are courageous individuals.

Therefore, some courageous individuals are not tank commanders.

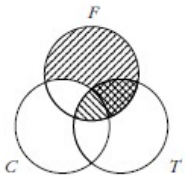
Step 1

We test the syllogism from the Boolean standpoint:

No *F* are *T*.

All *F* are *C*.

Some *C* are not *T*.



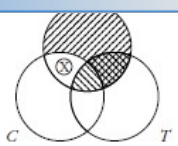
The conclusion asserts that there is an "X" that is inside the *C* circle and outside the *T* circle. Inspection of the diagram reveals no "X"s at all, so the syllogism is invalid from the Boolean standpoint.

Step 2

However, proceeding to step 2, we notice that the *F* circle is all shaded except for one area. Thus, we place a circled "X" in the one remaining area of the *F* circle: (assume F exists)

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All *F* are *C*.
Some *C* are not *T*.



Now the diagram indicates that the argument is valid, so we proceed to

step 3

We will determine whether the circled "X" represents something that actually exists. This is equivalent to determining whether *F* denotes something that exists. Since *F* stands for fighter pilots, which do exist, the circled "X" does represent something that exists. Thus, the syllogism is **valid** from the Aristotelian standpoint.

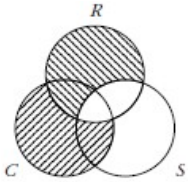
Example 2:

All reptiles are scaly animals.
All currently living dinosaurs are reptiles.
Therefore, some currently living dinosaurs are scaly animals.

Step 1

First we test the syllogism from the Boolean standpoint:

All *R* are *S*.
All *C* are *R*.
Some *C* are *S*.

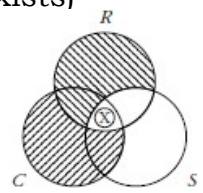


The conclusion asserts that there is an "X" in the area where the *C* and *S* circles overlap. Since the diagram contains no "X"s at all, the argument is **invalid** from the Boolean stand point.

Step 2

Proceeding to step 2, we notice that the *C* circle is all shaded except for one area. Thus, we place a circled "X" in the unshaded area of the *C* circle: (assume C exists)

All *R* are *S*.
All *C* are *R*.
Some *C* are *S*.



The diagram now indicates that the syllogism is valid.

Step 3

Now we proceed to the third step and determine whether the circled "X" stands for something that actually exists. In other words, we determine whether *C* denotes existing things. Returning to the original argument, we see that *C* stands for currently living dinosaurs, which do not exist. Thus, the circled "X" does not stand for something that actually exists, so the syllogism is **invalid**.

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in determining whether the circled "X" stands for something that exists, we always look to the Venn circle that is all shaded except for one area. If the term corresponding to that circle denotes existing things, then the circled "X" represents one of those things. In some diagrams, however, there may be two Venn circles that are all shaded except for one area, and each may contain a circled "X" in the unshaded area. In these cases we direct our attention only to the circled "X" needed to draw the conclusion. If that circled "X" stands for something that exists, the argument is valid; if not, it is invalid.

4.4.4 Rules and Fallacies

Today's logicians generally settle on **five rules**, and if any one of these rules is violated, a specific **formal fallacy** is committed and accordingly, the syllogism is **invalid**.

The first two rules are based on the concept of **distribution** and the last three are based on the concept of **quality** and **quantity**.

Rule 1: The middle term must be distributed at least once.

Fallacy: **Undistributed middle**

Example:

All animals are *living things*.

All plants are *living things*.

All plants are animals.

Rule 2: If a term is distributed in the conclusion, then it must be distributed in the premise.

Fallacy:

Illicit major

All professors are *scholars*.

Some lecturers are not professors.

Some professors are not *scholars*.

Illicit minor

All tigers are mammals.

All mammals are *animals*.

All *animals* are tigers.

Rule 3 Two negative premises are not allowed

Fallacy: Exclusive premise

Example:

No B are A.

Some B are **not** C.

Some C are not A.

Rule 4: A negative premise requires a negative conclusion, and a negative conclusion requires a negative premise.

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conclusion from a negative premise.

All A are B.

Some C are **not** B.

Some C are A.

Drawing a negative conclusion from affirmative premises.

All A are B.

All B are C.

Some A are **not** C.

Rule 5. If both premises are universal, the conclusion cannot be particular.

Fallacy: Existential fallacy

If a categorical syllogism breaks only Rule 5, it is valid from the Aristotelian standpoint but not from the Boolean standpoint. The nine syllogistic forms that fall into this category are those that are included in the "conditionally valid" list in Section 3.4.2. For each of these forms, the list specifies one term that must denote existing things before the syllogism will be valid.

Example:

All A are B. (A exists)

All C are A.

Some C are A.

Compare the Two examples

All mammals are animals.

All unicorns are mammals.

Some unicorns are animals.

Invalid (unicorns do not exist)

All mammals are animals.

All tigers are mammals.

Some tigers are animals.

Valid (because tigers exist)

Note

Both of the above syllogisms are invalid from the Boolean perspective. They commit Existential fallacy for their premises are universal and their conclusions are particular. However, based on the existential condition of the terms, the argument may be valid. Thus, in the above syllogisms (AAI-1) the argument is valid when the subject term tiger represents a class of terms which exists, but invalid when it contains the subject term unicorn which does not exist.

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Propositional Logic & Natural Deduction

5.1. Propositional Logic

Propositions are statements that are either true or false.

Propositional Logic, is also known as *sentential logic* and *statement logic*.

It is the branch of deductive logic that studies ways of *joining* and /or *modifying* entire propositions, statements or sentences to form more complicated propositions, statements or sentences, as well as the logical relationships and properties that are derived from these methods of combining or altering statements.

5.1.1. Symbols and Translation

All simple and compound statements, logical functions as well as arguments can be represented by symbols.

1. Simple Statement

Simple statement is one that does not contain any other statement as a component

Any upper case letter can be used to represent

Example:

ASTU is a model University. [A]

2. Compound Statement

A compound statement is one that contains one simple statement as a component

Can be represented by any upper case letter and a **logical operator**

Example:

Abel is a teacher **and** Rahel is a director.

It is not the case that Vampires are paranormal.

3. Logical Operators

These are symbols which represent logical functions.

A. Tilde (~) Negation

Not, it is not the case that, it is false that,

Example:

It is not the case that Walia Ibex is Ethiopian endemic animal. ~W

Walia Ibex is not Ethiopian endemic animal. ~W

It is not the case that Walia Ibex is Ethiopian endemic animal. ~W

These statements are all **negations.**

The **main operator** is a tilde.

~*B*

~ (*G* *H*)

~ [(*A* ≡ *F*) • (*C* ≡ *G*)]

B. Dot (•) Conjunction

And, yet, but, however, moreover, nevertheless, still, also, although, both

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director. A • R
Abel is a teacher but Rahel is a director. A • R
Abel is a teacher however Rahel is a director. A • R
These statements are all **conjunctions.**
The main operator is a dot.
$K \bullet \sim L$
$(E \vee F) \bullet (G \vee H)$
$[(R \quad T) \vee (S \quad U)] \bullet [(W \equiv X) \vee (Y \equiv Z)]$

C. Wedge (∨) Disjunction
Or, either…or, unless
Example
Logic is a common course or Hydraulics is a major course. L ∨ H
Either Logic is a common course or Hydraulics is a major course. L ∨ H
Unless Logic is a common course, Hydraulics is a major course. L ∨ H
Logic is a common course unless Hydraulics is a major course. L ∨ H
These statements are all **disjunctions.**
The main operator is a wedge.
$\sim C \vee \sim D$
$(F \bullet H) \vee (\sim K \bullet \sim L)$
$[S \bullet (T \quad U)] \vee [X \bullet (Y \equiv Z)]$

D. Horseshoe ( ) Implication
If, only if, given that, in case, provided that, on condition that, sufficient condition for, necessary condition for
Example:
If Gonder is a city, then Addis Ababa is a Mega city. G A
Gonder is a city if Addis Ababa is a Mega city. A G
Gonder is a city only if Addis Ababa is a Mega city. G A
Note:
To avoid mistakes in translating "if" and "only if" remember this rule: The statement that follows "if" is always the antecedent, and the statement that follows "only if" is always the consequent. Thus "*G* only if *A*" is translated *G* A, whereas "*G* if *A*" is translated A G.
These statements are all **conditionals** (material implications).
The main operator is a horseshoe.
$H \quad \sim J$
$(A \vee C) \quad (D \bullet E)$
$[K \vee (S \bullet \sim T)] \quad [\sim F \vee (M \bullet O)]$

E. Triple Bar ( ) equivalence
If and only if, sufficient and necessary condition for
Event *A* is said to be a **sufficient condition** for event *B* whenever the occurrence of *A* is all that is required for the occurrence of *B*. On the other hand, event *A* is said to be a **necessary condition** for event *B* whenever *B* cannot occur without the occurrence of *A*.
Place the statement that names the sufficient condition in the antecedent of the conditional and the statement that names the necessary condition in the consequent.
Note:

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be
le
in mind. Turning the "U" sideways creates *S* N, wherein *S* and *N* designate sufficient and necessary conditions, respectively.

Example

Abel is a teacher if and only if Rahel is a director.

Biconditional is logically equivalent to two conditionals

A R is logically equivalent to:

(A R) • (R A)

Do not confuse these three statement forms:

A if *B* *B* *A*

A only if *B* *A* *B*

A if and only if *B* *A*≡ *B*

These statements are all **biconditionals** (material equivalences).

The main operator is a triple bar.

M ≡ ~*T*

(*B* v *D*) ≡ (*A* • *C*)

[*K* v (*F* *I*)] ≡ [~*L* • (*G* v *H*)]

Parentheses, Brackets and Braces

Whenever more than two letters and operators appear in the translated statement, we will use one or more of them to separate.

Example

[*K* v (*F* *I*)] ≡ [~*L* • (*G* v *H*)]

Well-formed formulas (WFFs)

A well-formed formula is a *syntactically* correct arrangement of symbols.

5.1.2. Truth Functions

A. Function (Truth Function)

It is the truth value of a compound proposition expressed in terms of one or more logical operators.

B. Statement Variables

These are lower-case letters (*p, q, r, s*) that can stand for any statement (simple or compound).

Example:

A p

A B q

B v C r

C. Statement Form

It is an arrangement of statement variables and operators such that the uniform substitution of statements in place of the variables results in a statement.

Example

~*p* and *p* *q* are statement forms because substituting the statements *A* and *B* in place of *p* and *q*, respectively, results in the statements ~*A* and *A* *B*.

D. Truth Table

It is an arrangement of truth values that shows in every possible case how the truth value of a compound proposition is determined by the truth values of its simple components.

E. Truth Table for Compound Propositions

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Negation

p	$\sim p$
T	F
F	T

2. Truth Table for Conjunctions

Conjunction

p	q	$p \cdot q$
T	T	T
T	F	F
F	T	F
F	F	F

3. Truth Table for Disjunctions

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

4. Truth Table for Implications

Conditional
(material implication)

p	q	$p \supset q$
T	T	T
T	F	F
F	T	T
F	F	T

5. Truth Table for Bi-implications

Biconditional
(material equivalence)

p	q	$p \equiv q$
T	T	T
T	F	F
F	T	F
F	F	T

F. Computing the Truth Value for Longer Propositions

Steps

1. Write the truth values of the simple propositions immediately below the respective letters and bring the operators and parentheses down.
2. Use the truth values of simple propositions to compute the truth value of the longer proposition (follow the rules given below).

Rules to compute truth values

1. Individual letters representing simple propositions
2. Tildes immediately preceding individual letters
3. Operators joining letters or negated letters
4. Tildes immediately preceding parentheses
5. Operators joining parenthesis, or negated parenthesis with letters, parenthesis or negated parenthesis.
6. Tildes immediately preceding brackets

Example:

Step 1

$$[(D \equiv \sim A) \cdot \sim (C \cdot \sim B)] \equiv \sim [(A \supset \sim D) \vee (C \equiv E)]$$
$$\text{F T F T T T T F F T } \textcircled{F} \text{ F T T T F T T F F}$$

Step 2

$$\sim \{[(C \cdot \sim E) \supset \sim (A \cdot \sim B)] \text{ T } [\sim (B \vee D) \equiv (\sim C \vee E)]\}$$
$$\textcircled{F} \text{ T T T F T T T F F T T F T T F T F F}$$

G. Procedures for constructing Truth Table

Steps:

1. Determine the number of lines
   Use the formula $L = 2^n$ where "L" is number of lines and "n" the number of different simple propositions.
2. Divide the number of lines in half and assign true and half false.

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d
ow

the same procedure foe every simple proposition).

4. Duplicate the truth values
5. Compute the remaining column following the rules given above.

Example 1:

To construct a truth table for the following compound proposition $(A \vee \sim B)$ B follow the steps given in section 5.3.2.

Step 1.

$(A \vee \sim B) \supset B$

Step 2

$(A \vee \sim B) \supset B$
T
T
F
F

Step 3

$(A \vee \sim B) \supset B$	
T	T
T	F
F	T
F	F

Step 4

$(A \vee \sim B) \supset B$		
T	T	T
T	F	F
F	T	T
F	F	F

Step 5a

$(A \vee \sim B) \supset B$			
T	F	T	T
T	T	F	F
F	F	T	T
F	T	F	F

Step 5b

$(A \vee \sim B) \supset B$			
T	T	F	T
T	T	T	F
F	F	F	T
F	T	T	F

Step 5c

$(A \vee \sim B) \supset B$					
T	T	F	T	T	T
T	T	T	F	F	F
F	F	F	T	T	T
F	T	T	F	F	F

Example 2:

Construct a truth value for

$(C \bullet \sim D) \quad E$

Steps 1-3

$(C \bullet \sim D) \supset E$			
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Steps 4 & 5

$(C \bullet \sim D) \supset E$					
T	F	F	T	T	T
T	F	F	T	T	F
T	T	T	F	T	T
T	T	T	F	F	F
F	F	F	T	T	T
F	F	F	T	T	F
F	F	T	F	T	T
F	F	T	F	T	F

H. Classifying Statements

Column Under Main Operator	Classification
All True	**Tautologous (Logically true)**
All False	**Self-contradictory (logically false)**
At least one True, at least one false	**Contingent**

Examples:

A. Tautologous

$[(G \supset H] \bullet G] \supset H$						
T	T	T	T	T	T	T
T	F	F	F	T	T	F
F	T	T	F	F	T	T
F	T	F	F	F	T	F

B. Self-contradictory

$(G \vee H) \equiv (\sim G \bullet \sim H)$							
T	T	T	F	F	T	F	T
T	T	F	F	F	T	F	F
F	T	T	F	T	F	F	T
F	F	F	F	T	F	T	F

C. Contingent

$(A \vee \sim B) \supset B$					
T	T	F	T	T	T
T	T	T	F	F	F
F	F	F	T	T	T
F	T	T	F	F	F

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Operator	Relation
Same truth value each line	**Logically equivalent**
Opposite truth value each line	**Contradictory**
At least there is one line on which truth values are both true	**Consistent**
There is no line on which the truth values are both true	**Inconsistent**

Examples:

A. Logically Equivalent

K	⊃	L
T	T	T
T	F	F
F	T	T
F	T	F

~	L	⊃	~	K
F	T	T	F	T
T	F	F	F	T
F	T	T	T	F
T	F	T	T	F

B. Contradictory

K	⊃	L
T	T	T
T	F	F
F	T	T
F	T	F

K	•	~	L
T	F	F	T
T	T	T	F
F	F	F	T
F	F	T	F

C. Consistent

K	v	L
T	T	T
T	T	F
F	T	T
F	F	F

K	•	L
T	T	T
T	F	F
F	F	T
F	F	F

D. Inconsistent

K	≡	L
T	T	T
T	F	F
F	F	T
F	T	F

K	•	~	L
T	F	F	T
T	T	T	F
F	F	F	T
F	F	T	F

5.1.3. Truth Table for Arguments

Truth tables provide the standard technique for testing the validity of arguments in propositional logic.

Steps

1. Symbolize the arguments using letters to represent the simple propositions.
2. Write out the symbolized argument, placing a single slash between the premises and a double slash between the last premise and the conclusion.
3. Draw a truth table for the symbolized argument as if it were a proposition broken into parts, outlining the columns representing the premises and conclusion.
4. Look for a line in which all of the premises are true and the conclusion is false. If such a line exists, the argument is invalid; if not, it is valid.

Example: 1

If juvenile killers are as responsible for their crimes as adults, then execution is a justifiable punishment. Juvenile killers are not as responsible for their crimes as adults. Therefore, execution is not a justifiable punishment.

Step 1

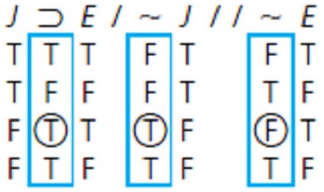
$$J \supset E$$
$$\sim J$$
$$\therefore \sim E$$

Step 2

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F T F T
F F F F

Step 3 & 4

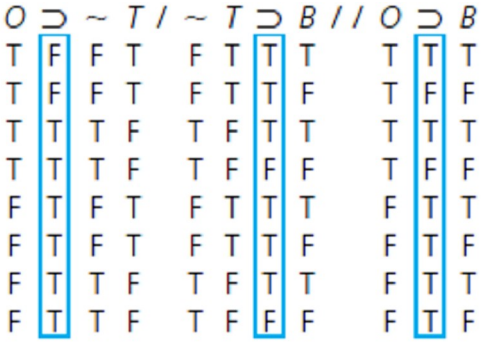


Inspection of the third line reveals that both of the premises are true and the conclusion is false. The argument is therefore invalid.

Example 2

If insider trading occurs, then investors will not trust the securities markets. If investors do not trust the securities markets, then business in general will suffer. Therefore, if insider trading occurs, then business in general will suffer.

Steps 1-5



Inspection of the truth table reveals that there is no line on which both premises are true and the conclusion is false. The argument is therefore valid.

Notes:

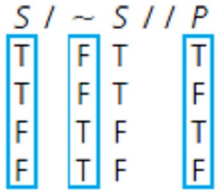
Any argument having **inconsistent premises** is valid regardless of what its conclusion may be.

Example:

The sky is blue.

The sky is not blue.

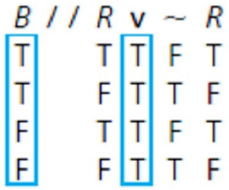
Therefore, Paris is the capital of France.



Any argument having a **tautologous conclusion** is valid regardless of what its premises may be.

Example:

Bern is the capital of Switzerland. Therefore, it is either raining or it is not raining.



5.1.4. Indirect Truth table for Arguments

Indirect truth tables provide a shorter and faster method for testing the validity of arguments than that provided by ordinary truth tables.

Because an argument with 5 propositions demands 32 lines in L= 2^n

Steps:

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t is it is possible for the premises to be true and the conclusion false.

2. Enter True under the main operator of the premises and False beneath the conclusion.

3. Derive the truth values of the separate components by working backward.

4. Inspect: if no contradiction is obtained in the process, this means that the argument is invalid. But if contradicts at any spot, it is valid.

Example:1

$\sim A \supset (B \vee C)$

$\sim B$

$C \supset A$

Step 1

$\sim A \supset (B \vee C) \;/\; \sim B \;//\; C \supset A$

$\quad\quad T \quad\quad\quad\quad T \quad\quad\quad F$

Step 2

$\sim A \supset (B \vee C) \;/\; \sim B \;//\; C \supset A$

$\quad\quad T \quad\quad\quad\quad T\ F \quad\quad T\ F\ F$

Step 3

$\sim A \supset (B \vee C) \;/\; \sim B \;//\; C \supset A$

$T\ F\ T\ \ F\ T\ T \quad\quad T\ F \quad\quad T\ F\ F$

Step 4

Our inspection witnesses that the truth values of the propositions perfectly fit to our assumption: the premises true and the conclusion false. The argument is therefore invalid.

Note:

In deriving the truth values of the propositions through indirect truth table, Sometimes a single row of truth values may not be sufficient to prove an argument **valid**.

If an indirect truth table requires more than one line, the method to be followed is this. Either select one of the premises and compute all of the ways it can be made true, or select the conclusion and compute all of the ways it can be made false. This selection should be dictated by the requirement of simplicity.

Example:

Step 1

$\sim A \supset B \;/\; B \supset A \;/\; A \supset \sim B \;//\; A \cdot \sim B$

$\quad T \quad\quad\quad T \quad\quad\quad T \quad\quad\quad\quad F$

Step 2 (Let's select the conclusion)

$\sim A \supset B \;/\; B \supset A \;/\; A \supset \sim B \;//\; A \cdot \sim B$

$\quad T \quad\quad\quad T \quad\quad\quad T \quad\quad\quad\quad T\ F\ F\ T$

$\quad T \quad\quad\quad T \quad\quad\quad T \quad\quad\quad\quad F\ F\ T\ F$

$\quad T \quad\quad\quad T \quad\quad\quad T \quad\quad\quad\quad F\ F\ F\ T$

Step 3

$\sim A \supset B \;/\; B \supset A \;/\; A \supset \sim B \;//\; A \cdot \sim B$

$\quad\quad T \quad\quad\quad\quad T \quad\quad\quad (T\ T\ F)\ T \quad\quad\quad\quad T\ F\ F\ T$

$(T\ F\ T\ F) \quad\quad\quad T \quad\quad\quad\quad T \quad\quad\quad\quad F\ F\ T\ F$

$\quad\quad T \quad\quad\quad\quad (T\ T\ F) \quad\quad\quad T \quad\quad\quad\quad F\ F\ F\ T$

Since a contradiction is obtained on each line, the argument is valid. If a contradiction had not been obtained on every line, the argument would, of course, be invalid, because it would be possible for the premises to be true and the conclusion false.

5.1.5. Argument Forms and Fallacies

An **argument form** is an arrangement of statement variables and operators such that the uniform replacement of the variables by statements results in an argument.

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A. Common Valid Argument Forms

A *valid* argument form is any argument form that satisfies the truth table test.

1. Disjunctive Syllogism

disjunctive syllogism (DS):

$$\begin{array}{l} p \vee q \\ \underline{\sim p} \\ q \end{array}$$

2. Hypothetical syllogism

$$\begin{array}{l} p \supset q \\ \underline{q \supset r} \\ p \supset r \end{array}$$

pure hypothetical syllogism (HS)

3. Asserting mode (MP)

modus ponens (MP):

$$\begin{array}{l} p \supset q \\ \underline{p} \\ q \end{array}$$

4.Denying Mode (MT)

modus tollens (MT):

$$\begin{array}{l} p \supset q \\ \underline{\sim q} \\ \sim p \end{array}$$

5. Constructive Dilemma (CD)

$$\begin{array}{l} (p \supset q) \cdot (r \supset s) \\ \underline{p \vee r} \\ q \vee s \end{array}$$

constructive dilemma (CD)

6.Destructive Dilemma (DD)

$$\begin{array}{l} (p \supset q) \cdot (r \supset s) \\ \underline{\sim q \vee \sim s} \\ \sim p \vee \sim r \end{array}$$

destructive dilemma (DD)

B. Common Invalid Argument forms

1.

$$\begin{array}{l} p \supset q \\ \underline{q} \\ p \end{array}$$

affirming the consequent (AC)

2.

$$\begin{array}{l} p \supset q \\ \underline{\sim p} \\ \sim q \end{array}$$

denying the antecedent (DA)

Procedures of using Argument Forms to test Validity

1. Symbolize the argument using upper case letters

2. Check if the symbolized argument fits the pattern of one of the forms.

Note:

Keep these points in mind:

1. The statement form $p \vee q$ is logically equivalent to $q \vee p$.

$$\begin{array}{l} A \vee B \\ \underline{\sim B} \\ A \end{array}$$ is similar to $$\begin{array}{l} B \vee A \\ \underline{\sim B} \\ A \end{array}$$

2. Negated letters, as well as non-negated letters, may be interpreted as substitution instances of the *p*, *q*, *r*, and *s* in the argument forms.

$$\begin{array}{l} \sim A \supset B \\ \underline{\sim A} \\ B \end{array}$$

When _*A* is substituted in the place of *p* and *B* in the place of *q*, the above argument will be seen to have the form of *modus ponens*.

3. The simple statement form *p* is logically equivalent to __*p*.

$$\begin{array}{l} A \supset \sim B \\ \underline{B} \\ \sim A \end{array}$$ is similar to $$\begin{array}{l} A \supset \sim B \\ \underline{\sim\sim B} \\ \sim A \end{array}$$

4. The order of the premises does not affect the form of an argument.

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$\sim B$

Modus Tollens

$$\begin{array}{l} B \supset C \\ A \supset B \\ \hline A \supset C \end{array}$$

Pure Hypothetical Syllogism

5.2. Natural Deduction in Propositional Logic

A. Natural deduction

It is a method for establishing the validity of propositional type arguments that is both simpler and more enlightening than the method of truth tables.

Natural deduction resembles the method used in geometry to derive theorems relating to lines and figures.

The method of natural deduction is thus equal in power to the truth table method as far as proving **validity** is concerned. However, since natural deduction cannot be used with any facility to prove **invalidity**, we still need the truth table method for that purpose.

Each step in a **logical proof** depends on a **rule of inference.**

B. Logical proof in Natural Deduction (ND)

It consists of a sequence of propositions, each of which is either a premise or is derived from preceding propositions by application of a rule of inference and the last of which is the conclusion of the original argument.

C. Rules of Inference

There are eighteen rules of inference in propositional logic.

Once we are supplied with all eighteen rules together with conditional proof, we can derive the conclusion of any valid argument in propositional logic.

5.2.1. Rules of Implication I

1. Modus Ponens (MP)

Modus ponens (MP):

$$\begin{array}{l} p \supset q \\ p \\ \hline q \end{array}$$

The following instances are also **MP**

$$\begin{array}{l} \sim F \supset (G \equiv H) \\ \sim F \\ \hline G \equiv H \end{array} \qquad \begin{array}{l} (A \vee B) \supset \sim (C \bullet D) \\ A \vee B \\ \hline \sim (C \bullet D) \end{array}$$

$$\begin{array}{l} K \bullet L \\ (K \bullet L) \supset [(R \supset S) \bullet (T \supset U)] \\ \hline (R \supset S) \bullet (T \supset U) \end{array}$$

2. Modus Tollens (MT)

$$\begin{array}{l} p \supset q \\ \sim q \\ \hline \sim p \end{array}$$

The following instances are also **MT**

$$\begin{array}{l} (D \vee F) \supset K \\ \sim K \\ \hline \sim (D \vee F) \end{array} \qquad \begin{array}{l} \sim G \supset \sim (M \vee N) \\ \sim\sim (M \vee N) \\ \hline \sim\sim G \end{array}$$

$$\begin{array}{l} \sim T \\ [(H \vee K) \bullet (L \vee N)] \supset T \\ \hline \sim [(H \vee K) \bullet (L \vee N)] \end{array}$$

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$$\frac{q \supset r}{p \supset r}$$

The following instances are also **HS**

$$\frac{A \supset (D \cdot F)}{\frac{(D \cdot F) \supset \sim H}{A \supset \sim H}} \qquad \frac{\sim M \supset (R \supset S)}{\frac{(C \vee K) \supset \sim M}{(C \vee K) \supset (R \supset S)}}$$

$$\frac{(L \supset N) \supset [(S \vee T) \cdot K]}{\frac{(C \equiv F) \supset (L \supset N)}{(C \equiv F) \supset [(S \vee T) \cdot K]}}$$

4. Disjunctive Syllogism (DS)

$$\frac{p \vee q}{\frac{\sim p}{q}}$$

The following instances are also **DS**

$$\frac{U \vee \sim(W \cdot X)}{\frac{\sim U}{\sim(W \cdot X)}} \qquad \frac{\sim(E \vee F)}{\frac{(E \vee F) \vee (N \supset K)}{N \supset K}}$$

$$\frac{\sim B \vee [(H \supset M) \cdot (S \supset T)]}{\frac{\sim\sim B}{(H \supset M) \cdot (S \supset T)}}$$

Proving Propositional Arguments

A. About Proof

Proof is a thought process thus**,** we should never write down a line in a proof unless we know why we are doing it and where it leads. Typically, good proofs are not produced haphazardly or by luck; rather, they are produced by organized logical thinking. Occasionally, of course, we may be baffled by an especially difficult proof, and random deductive steps noted on the side may be useful.

B. Steps to Proof

1. Symbolize the argument.
2. Give numbers for each premise separately
3. Write the conclusion to the right of the last premise, separated by a slash mark. (NB. Writing the conclusion does not mean proving the conclusion)
4. Derive the conclusion from the premises (write the justification fro each line you add to the immediate right.)

C. Strategies to Rules of Implication I

Strategy 1

Always begin by attempting to "find" the conclusion in the premises.

Strategy 2

If the conclusion contains a letter that appears in the consequent of a conditional statement in the premises, consider obtaining that letter via *modus ponens* **(MP)**

Strategy 3

If the conclusion contains a negated letter and that appears in the antecedent of a conditional statement in the premises, consider obtaining the negated letter via *modus tollens* **(MT)**

Strategy 4

If the conclusion is a conditional statement, consider obtaining it via hypothetical syllogism (**HS)**

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letter that appears in a disjunctive statement in the premises, consider obtaining that letter via disjunctive syllogism **(DS)**

D. Examples

Use the first four rules of inference to derive the conclusions of the following arguments:

Example 1

If Adama is a city then Bahirdar is a Mega city. Either Chancho is a town or Adama is a city. Adama is a city. It follows that, Bahirdar is a Mega city.

1. $A \supset B$
2. $C \vee A$
3. A / B
4. B 1, 3, MP

Follow the steps 1-4 and strategies 1& 2 above

Example 2

If Chancho is a town then Bahirdar is a Mega city. If Adama is a city then Bahirdar is a Mega City. But Bahirdar is not a mega city. Therefore, Adama is not a city.

1. $C \supset B$
2. $A \supset B$
3. $\sim B$ / $\sim A$
4. $\sim A$ 2, 3, MT

Follow the steps 1-4 and strategies 1 & 3 above

Example 3

1. $B \supset C$
2. $C \supset A$
3. $A \supset B$ / $A \supset C$
4. $A \supset C$ 1, 3, HS

Follow the steps 1-4 and strategies 1& 4 above

Example 4

1. $A \supset B$
2. $A \vee C$
3. $\sim A$ / C
4. C 2, 3, DS

Follow the steps 1-4 and strategies 1 & 5 above

Example 5

If the Aster wins the game, then Bekele will lose the medal. If the Aster does not win the game, then either Challa or Demeke will be fired. Bekele will not lose the medal. Furthermore, Challa will not be fired. Therefore, Demeke will be fired.

1. $A \supset B$
2. $\sim A \supset (C \vee D)$
3. $\sim B$
4. $\sim C$ / D
5. $\sim A$ 1, 3, MT
6. $C \vee D$ 2, 5, MP
7. D 4, 6, DS

Follow the steps 1-4 and the strategies 1,2,3,5 above

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5. Constructive Dilemma (CD)

$$\frac{(p \supset q) \bullet (r \supset s) \quad p \vee r}{q \vee s}$$

The following instances are also (**CD**)

$$\frac{\sim M \vee N \quad (\sim M \supset S) \bullet (N \supset \sim T)}{S \vee \sim T}$$

$$\frac{[(K \supset T) \supset (A \bullet B)] \bullet [(H \supset P) \supset (A \bullet C)] \quad (K \supset T) \vee (H \supset P)}{(A \bullet B) \vee (A \bullet C)}$$

6. Simplification (Simp)

$$\frac{p \bullet q}{p}$$

The following instances are also (**Simp**)

$$\frac{\sim F \bullet (U \equiv E)}{\sim F} \qquad \frac{(M \vee T) \bullet (S \supset R)}{(M \vee T)}$$

$$\frac{[(X \supset Z) \bullet M] \bullet (G \supset H)}{[(X \supset Z) \bullet M]}$$

7. Conjunction (Conj)

$$\frac{p \quad q}{p \bullet q}$$

The following instances are also (**Conj**)

$$\frac{\sim E \quad \sim G}{\sim E \bullet \sim G} \qquad \frac{C \supset M \quad D \supset N}{(C \supset M) \bullet (D \supset N)}$$

$$\frac{R \supset (H \bullet T) \quad K \supset (H \bullet O)}{[R \supset (H \bullet T)] \bullet [K \supset (H \bullet O)]}$$

8. Addition (Add)

$$\frac{p}{p \vee q}$$

The following instances are also (**Add**)

$$\frac{S}{S \vee \sim T} \qquad \frac{C \bullet D}{(C \bullet D) \vee (K \bullet \sim P)}$$

$$\frac{W \equiv Z}{(W \equiv Z) \vee [A \supset (M \supset O)]}$$

Strategies to the Rules of Implication II

Strategy 6

If the conclusion contains a letter that appears in a conjunctive statement in the premises, consider obtaining that letter via simplification:

Strategy 7

If the conclusion is a conjunctive statement, consider obtaining it via conjunction by first obtaining the individual conjuncts:

Strategy 8

If the conclusion is a disjunctive statement, consider obtaining it via constructive dilemma or addition:

Strategy 9

If the conclusion contains a letter not found in the premises, addition *must* be used to obtain that letter.

Examples

Use the above four rules of inference to derive the conclusions of the following arguments:

Example 1

1. $A \supset B$	
2. $C \bullet B$	
3. $C \supset A$	/ C
4. C	2, Simp

Follow the steps 1-4 and the strategies 1& 6 above

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2. B
3. $\sim C$ / $B \bullet \sim C$
4. $B \bullet \sim C$ 2, 3, Conj

Follow the steps 1-4 and the strategies 1& 7 above

Example 3

1. $(A \supset B) \bullet (C \supset D)$
2. $B \supset C$
3. $A \vee C$ / $B \vee D$
4. $B \vee D$ 1, 3, CD

Follow the steps 1-4 and the strategies 1& 8 above

Example 4

1. $A \vee C$
2. B
3. $C \supset D$ / $B \vee D$
4. $B \vee D$ 2, Add

Follow the steps 1-4 and the strategies 1& 9 above

5.2.3. Rules of Replacement I

The ten rules of replacement are stated in the form of logical equivalences. For this purpose, a new symbol consisting of four dots (::) will be used to designate logical equivalence.
Underlying the use of the rules of replacement is an **axiom of replacement,** which asserts that within the context of a proof, logically equivalent expressions may replace one another.

9. DeMorgan's Rule (DM)

DeMorgan's Rule (named after the 19th century logician Augustus DeMorgan). It indicates "Not both p and q" is logically equivalent to "Not p or not q," and that "Not either p or q" is logically equivalent to "Not p and not q."

$\sim(p \bullet q) :: (\sim p \vee \sim q)$
$\sim(p \vee q) :: (\sim p \bullet \sim q)$

10. Commutativity (Com)

The **commutativity rule** asserts that the meaning of a conjunction or disjunction is unaffected by the order in which the components are listed. In other words, the component statements may be commuted, or switched for one another, without affecting the meaning.

$(p \vee q) :: (q \vee p)$
$(p \bullet q) :: (q \bullet p)$

11. Associativity (Assoc)

The **associativity rule** states that the meaning of a conjunction or disjunction is unaffected by the placement of parentheses when the same operator is used throughout.

$[p \vee (q \vee r)] :: [(p \vee q) \vee r]$
$[p \bullet (q \bullet r)] :: [(p \bullet q) \bullet r]$

12. Distribution (Dist)

The **distribution rule**, like DeMorgan's Rule, pertains only to conjunction and disjunction. When a

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a
eses
or disjoined to a conjunctive statement in parentheses, the rule allows us to put that proposition together with each of the components inside the parentheses, and also to go in the reverse direction.

$[p \bullet (q \vee r)] :: [(p \bullet q) \vee (p \bullet r)]$
$[p \vee (q \bullet r)] :: [(p \vee q) \bullet (p \vee r)]$

13. Double negation (DN):

The **double negation rule** is fairly obvious. The rule states simply that pairs of tildes immediately adjacent to one another may be either deleted or introduced without affecting the meaning of the statement.

$p :: \sim\sim p$

Strategies to the Rules of Replacement I

Strategy 10:

Conjunction can be used to set up DeMorgan's Rule

1. $\sim A$	
2. $\sim B$	
3. $\sim A \bullet \sim B$	1, 2, Conj
4. $\sim(A \vee B)$	3, DM

Strategy 11:

Constructive dilemma can be used to set up DeMorgan's Rule:

1. $(A \supset \sim B) \bullet (C \supset \sim D)$	
2. $A \vee C$	
3. $\sim B \vee \sim D$	1, 2, CD
4. $\sim(B \bullet D)$	3, DM

Strategy 12:

Addition can be used to set up DeMorgan's Rule:

1. $\sim A$	
2. $\sim A \vee \sim B$	1, Add
3. $\sim(A \bullet B)$	2, DM

Strategy 13:

Distribution can be used in two ways to set up disjunctive syllogism:

1. $(A \vee B) \bullet (A \vee C)$	
2. $\sim A$	
3. $A \vee (B \bullet C)$	1, Dist
4. $B \bullet C$	2, 3, DS

1. $A \bullet (B \vee C)$	
2. $\sim(A \bullet B)$	
3. $(A \bullet B) \vee (A \bullet C)$	1, Dist
4. $A \bullet C$	2, 3, DS

Strategy 14:

Distribution can be used in two ways to set up simplification

1. $A \vee (B \bullet C)$	
2. $(A \vee B) \bullet (A \vee C)$	1, Dist
3. $A \vee B$	2, Simp

1. $(A \bullet B) \vee (A \bullet C)$	
2. $A \bullet (B \vee C)$	1, Dist
3. A	2, Simp

Strategy 15:

If inspection of the premises does not reveal how the conclusion should be derived, consider using the rules of replacement to "deconstruct" the conclusion.

Examples

Use the above rules of inference to derive the conclusions of the following arguments:

Example 1

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3. A	2, Simp
4. $\sim(B \bullet C)$	1, 3, MP
5. $\sim B \vee \sim C$	4, DM
6. $C \bullet A$	2, Com
7. C	6, Simp
8. $\sim\sim C$	7, DN
9. $\sim C \vee \sim B$	5, Com
10. $\sim B$	8, 9, DS

Example 2

1. $D \bullet (E \vee F)$	
2. $\sim D \vee \sim F$	/ $D \bullet E$
3. $(D \bullet E) \vee (D \bullet F)$	1, Dist
4. $\sim(D \bullet F)$	2, DM
5. $D \bullet E$	3, 4, Com, DS

Example 3

1. $K \supset (F \vee B)$	
2. $G \bullet K$	/ $(F \bullet G) \vee (B \bullet G)$
3. G	2, Simp
4. $K \bullet G$	2, Com
5. K	4, Simp
6. $F \vee B$	1, 5, MP
7. $G \bullet (F \vee B)$	3, 6, Conj
8. $(G \bullet F) \vee (G \bullet B)$	7, Dist
9. $(F \bullet G) \vee (B \bullet G)$	8, Com, Com

5.2.4. Rules of Replacement II

14. Transposition (Trans)

$(p \supset q) :: (\sim q \supset \sim p)$

15. Material implication (Impl)

$(p \supset q) :: (\sim p \vee q)$

16. Material equivalence (Equiv)

$(p \equiv q) :: [(p \supset q) \bullet (q \supset p)]$
$(p \equiv q) :: [(p \bullet q) \vee (\sim p \bullet \sim q)]$

17. Exportation (Exp)

$[(p \bullet q) \supset r] :: [p \supset (q \supset r)]$

18. Tautology (Taut)

$p :: (p \vee p)$
$p :: (p \bullet p)$

Strategies to the Rules of Replacement II

Strategy 16

Material implication can be used to set up hypothetical syllogism

1. $\sim A \vee B$	
2. $\sim B \vee C$	
3. $A \supset B$	1, Impl
4. $B \supset C$	2, Impl
5. $A \supset C$	3, 4, HS

Strategy 17:

Exportation can be used to set up *modus ponens*

1. $(A \bullet B) \supset C$	
2. A	
3. $A \supset (B \supset C)$	1, Exp
4. $B \supset C$	2, 3, MP

Strategy 18:

Exportation can be used to set up *modus tollens*

1. $A \supset (B \supset C)$	
2. $\sim C$	
3. $(A \bullet B) \supset C$	1, Exp
4. $\sim(A \bullet B)$	2, 3, MT

Strategy 19:

Addition can be used to set up material implication

1. A	
2. $A \vee \sim B$	1, Add
3. $\sim B \vee A$	2, Com
4. $B \supset A$	3, Impl

Strategy 20:

Transposition can be used to set up hypothetical syllogism:

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3. $B \supset C$	2, Trans
4. $A \supset C$	1, 3, HS

Strategy 21:

Transposition can be used to set up constructive dilemma

1. $(A \supset B) \bullet (C \supset D)$	
2. $\sim B \vee \sim D$	
3. $(\sim B \supset \sim A) \bullet (\sim D \supset \sim C)$	1, Trans, Trans
4. $\sim A \vee \sim C$	2, 3, CD

Strategy 22:

Constructive dilemma can be used to set up tautology

1. $(A \supset C) \bullet (B \supset C)$	
2. $A \vee B$	
3. $C \vee C$	1, 2, CD
4. C	3, Taut

Strategy 23:

Material implication can be used to set up tautology

1. $A \supset \sim A$	
2. $\sim A \vee \sim A$	1, Impl
3. $\sim A$	2, Taut

Strategy 24:

Material implication can be used to set up distribution

1. $A \supset (B \bullet C)$	
2. $\sim A \vee (B \bullet C)$	1, Impl
3. $(\sim A \vee B) \bullet (\sim A \vee C)$	2, Dist

Example 1

1. $F \supset G$	
2. $F \vee G$	/ G
3. $\sim F \supset G$	2, DN, Impl
4. $\sim G \supset F$	3, Trans, DN
5. $\sim G \supset G$	1, 4, HS
6. $G \vee G$	5, Impl, DN
7. G	6, Taut

Example 2

1. $J \supset (K \supset L)$	/ $K \supset (J \supset L)$
2. $(J \bullet K) \supset L$	1, Exp
3. $(K \bullet J) \supset L$	2, Com
4. $K \supset (J \supset L)$	3, Exp

Example 3

1. $M \supset N$	
2. $M \supset O$	/ $M \supset (N \bullet O)$
3. $\sim M \vee N$	1, Impl
4. $\sim M \vee O$	2, Impl
5. $(\sim M \vee N) \bullet (\sim M \vee O)$	3, 4, Conj
6. $\sim M \vee (N \bullet O)$	5, Dist
7. $M \supset (N \bullet O)$	6, Impl