

Lecture Outline 3:

8. Sources of return from investing in a fixed-rate bond

Bonds have three sources of return: 1) receipts of the promised coupon and principal payments on the scheduled dates, 2) reinvestment of coupon payments, and 3) potential capital gains or losses on the sale of the bond prior to maturity. Now, when a bond is purchased at a premium or a discount, it adds another aspect to the rate of return. Through amortization, the bond's carrying value reaches par value at maturity. The carrying value is the purchase price plus (minus) the amortized amount of the discount (premium) if the bond is purchased at a discount (premium).

The investment *horizon* is at the heart of understanding interest rate risk and return. There are two offsetting types of interest rate risk that affect the bond investor: coupon reinvestment risk and market price risk. The future value of reinvested coupon payments (and in a portfolio, the principal on bonds that mature before the horizon date) increases when the interest rates go up and decreases when rates go down. The sale price on a bond that matures after the horizon date (and thus needs to be sold) decreases when interest rates go up and increases when rates go down. Coupon reinvestment risk matters more when the investor has a long term horizon relative to the time to maturity of the bond. Market price risk matters more when the investor has a short-term horizon relative to the time-to-maturity. Therefore, two investors holding the same bond (or bond portfolio) can have different exposures to interest rate risk if they have different investment horizons.

9. Macaulay, Modified, Approximate, Money and Effective duration

Macaulay Duration (chapter 9 on your textbook): Earliest measure of duration, based on time. Because Macaulay duration is based on the expected cash flows for an option-free bond, it is not appropriate estimate of the price sensitivity of bonds with embedded options.

$$D = \frac{\sum_{t=1}^N CF_t \times DF_t \times t}{\sum_{t=1}^N CF_t \times DF_t} = \frac{\sum_{t=1}^N PV_t \times t}{\sum_{t=1}^N PV_t}$$

Modified duration (chapter 9 on your textbook) is derived from Macaulay duration and offers a slight improvement over Macaulay duration in that it takes the current YTM into account. It is appropriate to do so because duration decreases as YTM increases. If Macaulay Duration is known, the modified duration:

$$D = \text{Macaulay Duration} / (1 + \text{YTM})$$

If Macaulay duration is not known, then it can be approximated this way:

$$\text{ApproxModDuration: } (PV_-) - (PV_+) / 2 * (\Delta \text{Yield}) * (PV_0)$$

Money (or in the U.S., Dollar) Duration: Modified duration is a measure of the *percentage* price change of a bond given a change in its yield-to-maturity. A related statistic is money duration. The money duration of a bond is a measure of the price *change* in units of the currency in which the bond is denominated. The money duration can be stated per 100 of par value or in terms of the actual position size of the bond in the portfolio:

$$\text{MoneyDur} = \text{AnnModDur} * \text{Price of the bond (Ideally full price)}$$

Another version of money duration is the price value of basis point for the bond.

$$PVBP = (PV_-) - (PV_+) / 2$$

Effective Duration: Effective duration of a bond is the sensitivity of the bond's price to a change in a benchmark yield curve. The formula is:

$$\text{EffDuration} = (PV_-) - (PV_+) / 2 * (\Delta \text{Curve}) * (PV_0)$$

Notice the difference between approximate modified duration and effective duration. It is only in the denominator. And in concept! Modified duration is a yield duration statistic in that it measures interest rate risk in terms of a change in the bond's own yield to maturity (ΔYield). Effective duration is a curve duration statistic in that it measures interest rate risk in terms of a change in the benchmark yield curve (ΔCurve). Effective duration (on Bloomberg, the effective duration is called the "OAS duration") is the most appropriate interest rate risk measure for bonds with embedded options.

Suggested exercises:

An investor purchases a nine-year, 7% annual coupon payment bond at a price equal to par value. After the bond is purchased and before the first coupon is received, interest rates increase to 8%. The investor sells the bond after five years. Assume that the interest rates remain unchanged at 8% over the five-year holding period:

1. Per 100 of par value, the future value of the reinvested coupon payments at the end of the holding period is closest to:
 - a. 35.00
 - b. 40.26
 - c. **41.07**
2. The capital gain/loss per 100 of par value resulting from the sale of the bond at the end of the five-year holding period is closest to:
 - a. Loss of 8.45
 - b. **Loss of 3.31**
 - c. Gain of 2.75
3. Assuming that all coupons are reinvested over the holding period, the investor's five-year horizon yield is closest to:
 - a. 5.66%
 - b. 6.62%
 - c. **7.12%**

4. An investor buys a three-year bond with a 5% coupon rate paid annually. The bond, with a yield-to-maturity of 3%, is purchased at a price of 105.657223 per 100 of par value. Assuming a 5-basis point change in yield to maturity, the bond's approximate modified duration is closest to:
- 2.78**
 - 2.86
 - 5.56

5. Using the information below, which bond has the greatest money duration per 100 of par value assuming annual coupon payment and no accrued interest?

Bond	Time-to-Maturity	Price per 100 of par value	Coupon	Yield-to-Maturity	Modified Duration
A	6 years	85.0	2.0%	4.95%	5.42
B	10 years	80.0	2.4%	4.99%	8.44
C	15 years	85.78	5.0%	5.00%	7.54

- Bond A
- Bond B**
- Bond C

Money duration= Bond Price (ideally full price)*MD

Bond A = $85 * 5.42\% = \$4.607$

Bond B = $80 * 8.44\% = \$6.752$

Bond C = $85.78 * 7.54\% = \$6.467$

6. A bond with exactly nine years remaining until maturity offers a 3% coupon rate with annual coupons. The bond, with a yield to maturity of 5%, is priced at 85.784357 per 100 of par value. The estimated price value of a basis point for the bond is closest to:

- 0.0086
- 0.0648**
- 0.1295

$PVBP = (PV - PV_t) / 2$

$P_{@4.99} =$

$N = 9$

$PMT = 100 * 3\% = 3$

$1/y = 4.99$

$FV = 100$

$PV = 85.84913$

$P_{@5.01}$

$i/y = 5.01$

$PV = 85.71964$

$85.84913 - 85.71964 / 2 = \mathbf{0.06475}$

7. A Canadian pension fund manager seeks to measure the sensitivity of her pension liabilities to market interest rate changes. The manager determines the present value of the liabilities under three interest rate scenarios: a base rate of 7%, a 100 basis point

increase in rates up to 8%, and a 100 basis point drop in rates down to 6%. The results of the manager's analysis are presented below:

Interest Rate Assumption	Present Value of Liabilities
6%	CAD 510.1 million
7%	CAD 455.4 million
8%	CAD 373.6 million

The effective duration of the pension's fund liabilities is closest to:

- a. 1.49
- b. **14.99**
- c. 29.97

$$\text{Effe. Duration} = \frac{PV - PV_t}{2 \times \text{Curve} \times PV_0}$$

$$\text{Effe. Duration} = \frac{510.1 - 373.6}{2 \times 0.01 \times 455.4\text{m}} = \mathbf{14.986}$$

8. Which of the following statements about duration is correct? A bond's:

- a. effective duration is a measure of yield duration
- b. modified duration is a measure of curve duration
- c. **modified duration cannot be larger than its Macaulay duration**

9. The interest rate risk of a fixed-rate bond with an embedded call option is best measured by:

- a. **effective duration**
- b. modified duration
- c. Macaulay duration

10. A bond portfolio consists of the following three fixed-rate bonds. Assume annual coupon payments and no accrued interest on the bonds. Prices are per 100 of par value.

Bond	Maturity	Market Value	Price	Coupon	Yield-to-Maturity	Modified Duration
A	6 years	170,000	85.0	2.0%	4.95%	5.42
B	10 years	120,000	80.0	2.4%	4.99%	8.44
C	15 years	100,000	100.0	5.0%	5.00%	10.38

The bond portfolio's modified duration is closest to:

- a. **7.62**
- b. 8.08
- c. 8.20

$$\text{Portfolio's MD} = 170\text{k}/390\text{k} \times 5.42\% + 120\text{k}/390\text{k} \times 8.44\% + 100\text{k}/390\text{k} \times 10.38\% = \mathbf{7.621\%}$$

11. A limitation of calculating a bond portfolio's duration as the weighted average of the yield durations of the individual bonds that compose the portfolio is that it:

- a. assumes a parallel shift to the yield curve.
- b. is less accurate when the yield curve is less steeply sloped.
- c. **is not applicable to portfolios that have bonds with embedded options.**

