Linear Programming Problem & Numerical Analysis

NOTE: If you have any doubt or written mistake, contact us at $199903932, $588044919

Linear Programming Problem...

* **General form of LPP** → Optimize

$$Z = c_1x_1 + c_2x_2 + \dots + c_nx_n \quad \text{— (1)}$$

subject to conditions —

$$\left.\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \,(\leq, \geq, =)\, b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \,(\leq, \geq, =)\, b_2 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{mn}x_n \,(\leq, \geq, =)\, b_m \end{aligned}\right\} \quad \text{— (2)}$$

$$x_1, x_2, \dots, x_n \geq 0 \quad \text{— (3)} \quad \text{(non-ve restriction)}$$

where $c_1, c_2, \dots, c_n$ are cost vectors.

$x_1, x_2, \dots, x_n$ are decision variables or structural variables.

and $b_1, b_2, b_3, \dots, b_n$ are required variables.

* **Solution of LPP** → The set of variables $x_1, x_2, \dots, x_n$ are called sol^n of given LPP if it satisfies inequality (2)

* **Feasible Sol^n** → Those sol^n's which satisfy the condition of feasibility i.e. non-ve restrictions (3) is called feasible sol^n.

* **Optimal Sol^n** → The feasible sol^n which optimize the given objective fn (1) is called optimal sol^n.

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✱ Methods to solve LPP →

1. Graphical Method → Write each inequality as eqⁿ.
   Plot the graph of each line.
   Justify the region according to constraints.
   Check whether it is bdd. or unbdd.
   For bdd. region, no feasible region ⇒ no solⁿ.

- If Optimal solⁿ of LPP exists, then it must exist atleast one of the vertex of feasible region.
- If feasible region is bdd., then optimal solⁿ must exist.

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Q- Minimize $Z = x_1 - x_2$ subject to

$2x_1 + x_2 \geq 2$ — (a) — (✱)

$3x_1 + 4x_2 \leq 12$ — (b)

$x_1, x_2 \geq 0$

The LPP has —

(a) no feasible solⁿ.

(b) unbdd. solⁿ.

(c) bdd. solⁿ. ✓

(d) none of these.

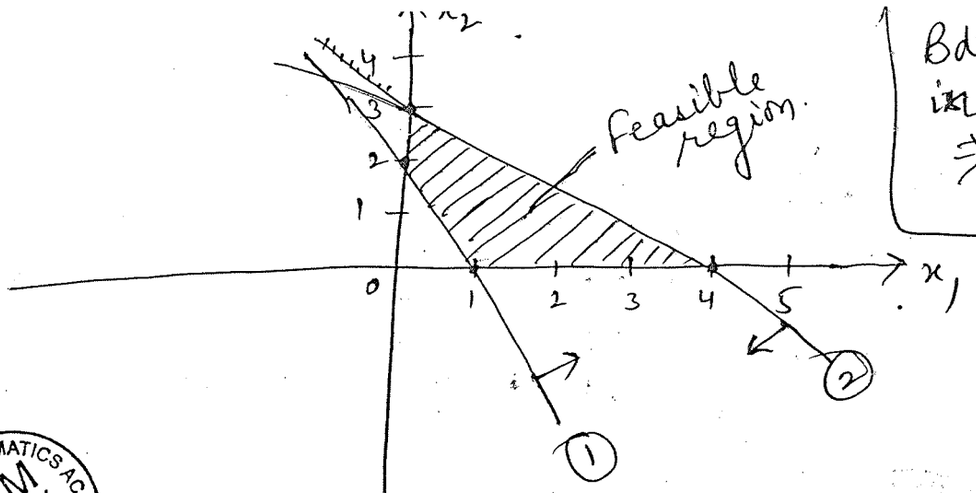
Solⁿ:- From (✱), we get —

$2x_1 + x_2 = 2$ — (1)

x_1	0	1
x_2	2	0

$3x_1 + 4x_2 = 12$ — (2)

x_1	0	4
x_2	3	0



Bdd. region in I quadrant ⇒ Feasible region

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for line ①, put (0,0) in (a) of ✱, we have

$0+0 \geq 2 \Rightarrow 0 \geq 2$, which is not true.

⇒ Region is away from the origin.

for line ②, we have -

$0+0 \leq 12 \Rightarrow 0 \leq 12$, which is true.

⇒ Region is towards origin.

Q- Maximize $Z = 4x_1 + 3x_2$ subject to

$$\left.\begin{aligned} x_1 + x_2 &\leq 1 \\ x_1 + 2x_2 &\geq 4 \end{aligned}\right\}$$

; $x_1 \geq 0, x_2 \geq 0$.

Then, the given problem has —

(a) unbdd. region

(b) no sol^n.

(c) no feasible sol^n ✓

(d) N.O.T.

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Sol^n: $x_1 + x_2 = 1$ — ①

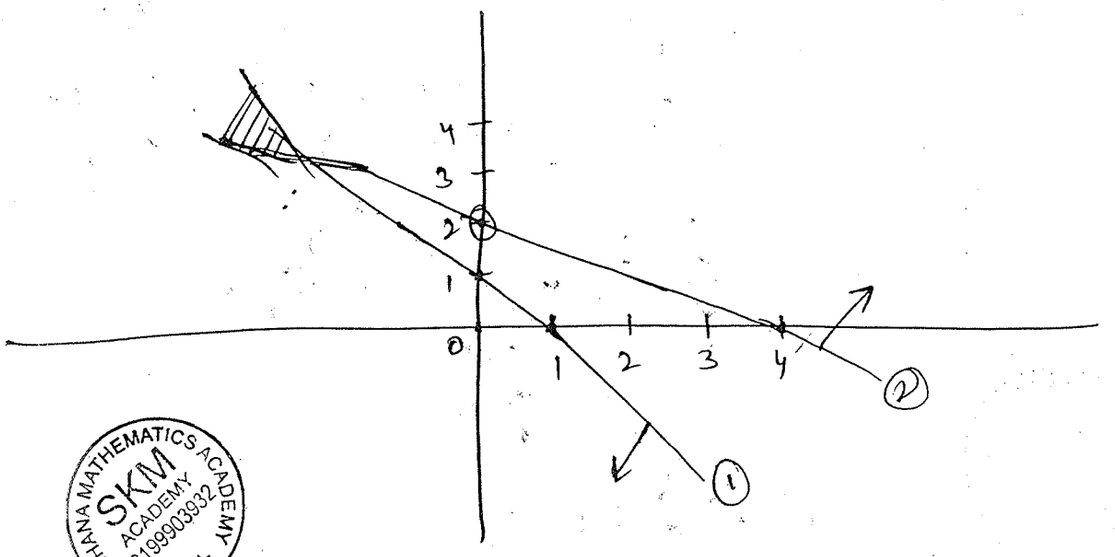
$x_1 + 2x_2 = 4$ — ②

①

x_1	0	1
x_2	1	0

②

x_1	0	4
x_2	2	0



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Q. Consider the LPP- Maximize $z = 3x_1 + 2x_2$

subject to $x_1 + x_2 \geq 5$
$4x_1 - x_2 \leq 15$
$4x_2 - x_1 \leq 15$

; $x_1, x_2 \geq 0$.

Then, which of the following is true?

(a) has no feasible sol^n.
(b) optimal value is 25. ✓
(c) optimal sol^n is $(x_1, x_2) = (5, 5)$ ✓
(d) optimal sol^n is $(x_1, x_2) = (5, 0)$.

Sol^n -

$x_1 + x_2 = 5$ — ①
$4x_1 - x_2 = 15$ — ②
$-x_1 + 4x_2 = 15$ — ③

①

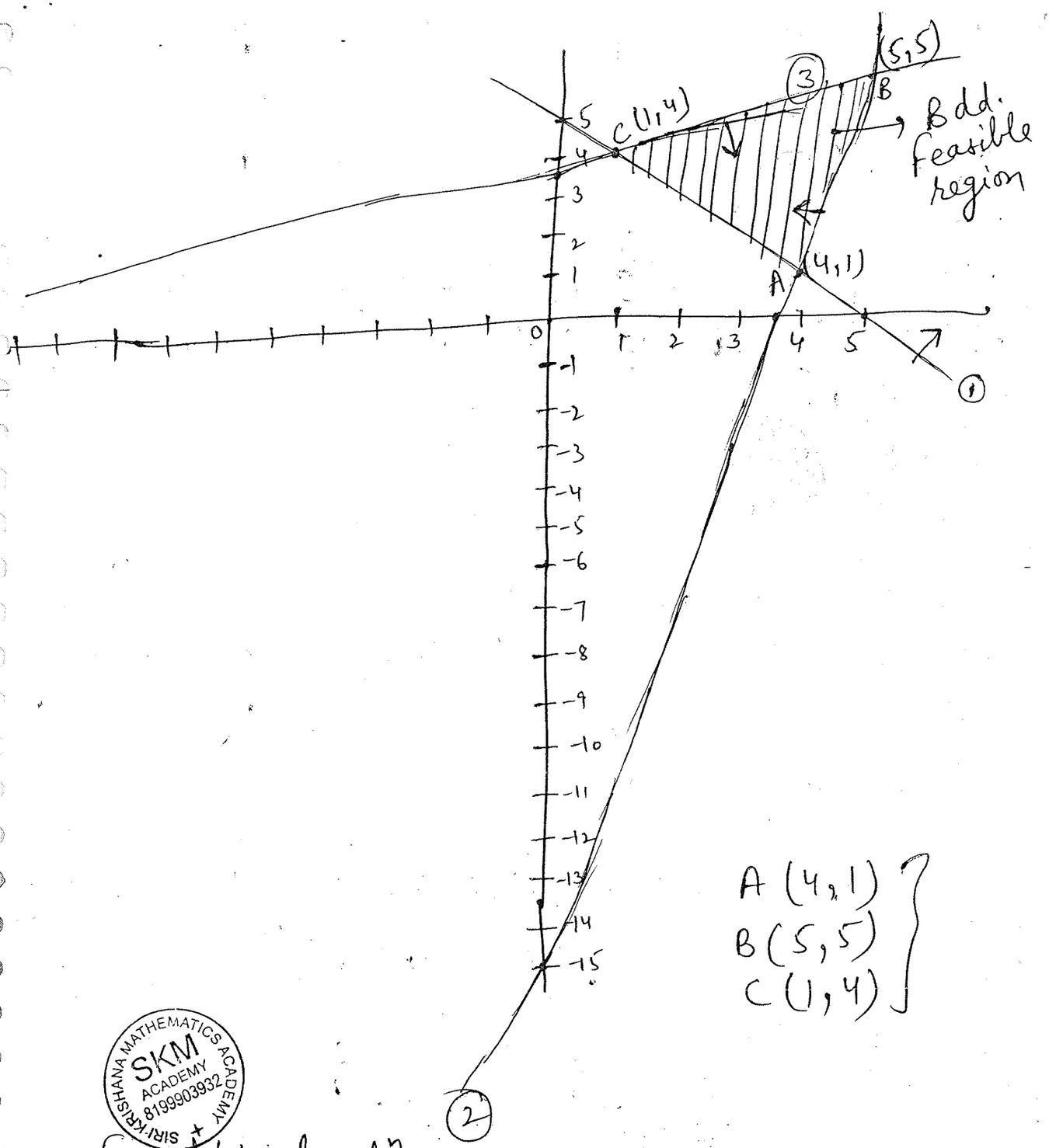
x_1	0	5
x_2	5	0

②

x_1	0	$15/4 = 3.75$
x_2	-15	0

③

x_1	0	-15
x_2	$\frac{15}{4}$	0



A (4,1)
B (5,5)
C (1,4)

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for optimal sol^n,

$Z_A = 3(4)+2(1) = 14$

$Z_B = 15+10 = 25$

$Z_C = 11$

} From here, maximum value will be optimal value & the corresponding point is optimal sol^n.

$\therefore$ Optimal value = 25 & Optimal sol^n is (5,5).

Min. value = 11.

✱ Result → 1. If LPP has optimal sol^n at more than one vertex, then LPP has infinitely many optimal sol^{n}s. They are either called alternative or multiple sol^{n}s.

2. If an LPP has alternative optimal sol^n, then objective fx^n line must be parallel to any one of the given constraints but the converse is not true.

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✱ Unbounded sol^n → If feasible sol^n are there, but we can't obtain the optimal sol^n, then the sol^n is called unbounded sol^n. e.g. Maximize $z = x_1$

If region is unbdd., then sol^n can be bdd. or unbdd. But if sol^n is unbdd., then region is unbdd.

① Maximize $Z = 6x_1 - 2x_2$

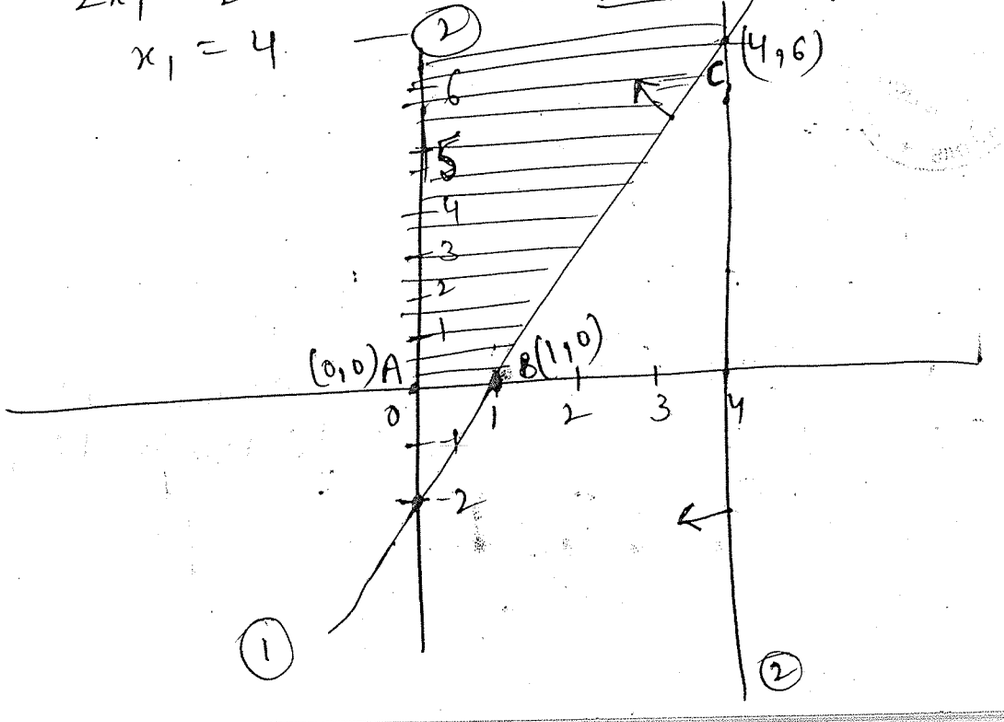
Subject to $2x_1 - x_2 \leq 2$
$x_1 \leq 4$
; $x_1, x_2 \geq 0$.

Sol^n

$2x_1 - x_2 = 2$ — ①

x_1	0	1
x_2	-2	0

$x_1 = 4$ — ②



$z_A = 0$

$z_B = 6$

$z_C = 24 - 12 = 12$. → Max.

Let us take $D(4,7)$.

$\Rightarrow z_D = 24 - 14 = 10$. → Value is decreasing.

$\Rightarrow$ Sol^n is bdd.

$\therefore$ We have max. value at $C(4,6)$.

$\Rightarrow$ Optimal sol^n is $(4,6)$ & optimal value is 12.

$\Rightarrow$ Bdd. optimal sol^n but region is unbdd.

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✱	Feasible region	Optimal Sol^n
(i)	Bounded	exist (bdd.)
(ii)	No feasible region	no optimal sol^n.
(iii)	unbdd. →	bdd. sol^n
		unbdd. sol^n

Dec'15

(1) Maximize $z = x_1 + \frac{5}{2}x_2$

subject to $5x_1 + 3x_2 \leq 15$

$-x_1 + x_2 \leq 1$

$2x_1 + 5x_2 \leq 10$

; $x_1, x_2 \geq 0$.

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Then the problem has —

(a) no feasible sol^n.

(b) infinitely many optimal sol^{n}s. ✓

(c) a unique optimal sol^n.

(d) an unbdd. sol^n.

Soln $5x_1 + 3x_2 = 15$ — (1)

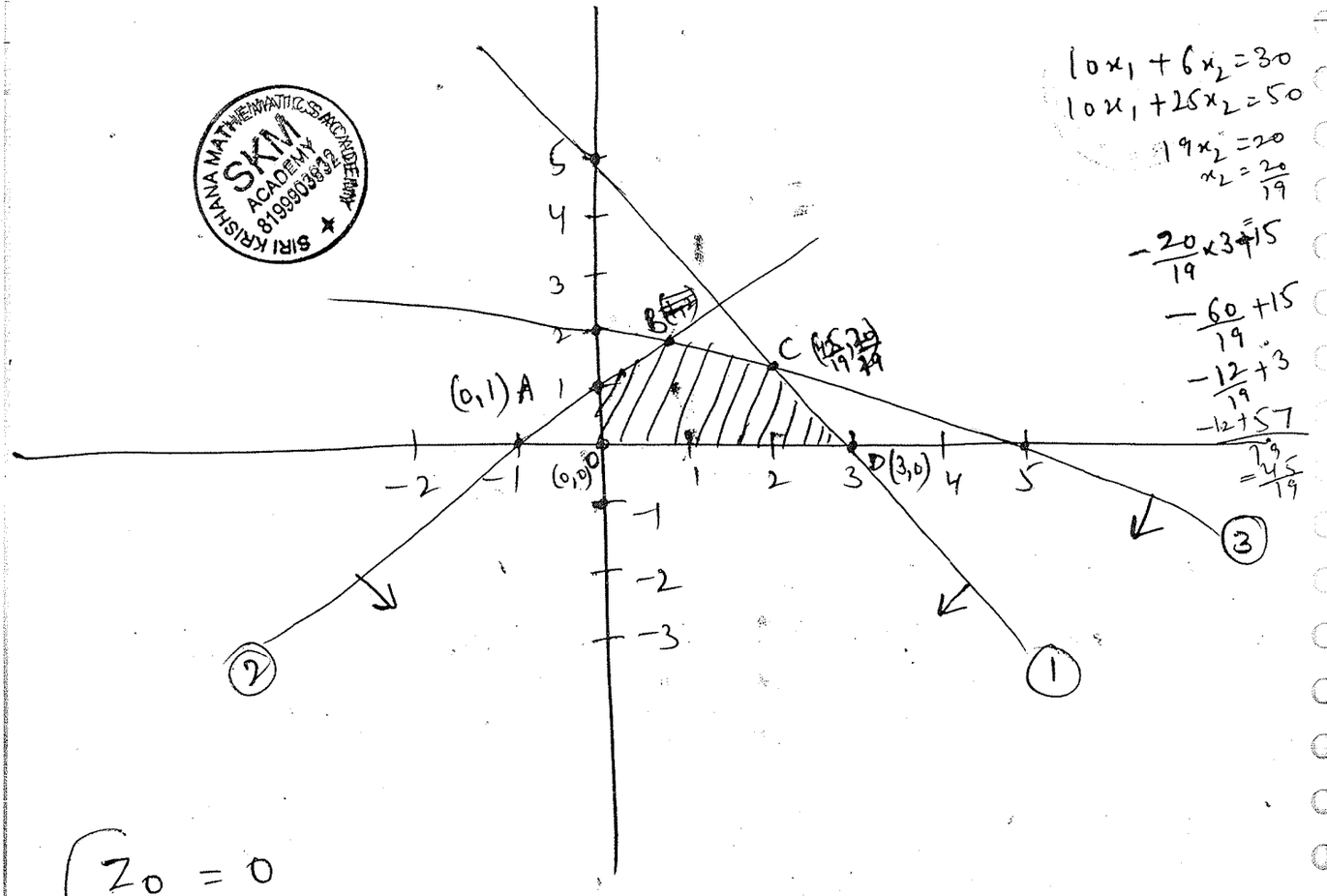
$-x_1 + x_2 = 1$ — (2)

$2x_1 + 5x_2 = 10$ — (3)

x_1	0	3
x_2	5	0

x_1	0	-1
x_2	1	0

x_1	0	5
x_2	2	0



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$10x_1 + 6x_2 = 30$

$10x_1 + 25x_2 = 50$

$19x_2 = 20$

$x_2 = \frac{20}{19}$

$-\frac{20}{19} \times 3 + 5$

$-\frac{60}{19} + 15$

$-\frac{12}{19} + 3$

$\frac{-12+57}{19} = \frac{45}{19}$

$Z_O = 0$

$Z_A = \frac{5}{2}$

$Z_B = \frac{5}{2}\ 1+5 = 6$

$Z_C = 2 + \frac{5}{4} = \frac{8+5}{4} = \frac{13}{4}$

$Z_D = 3.$

$O(0,0)$, $A(0,1)$, $B\left(\frac{5}{7}, \frac{12}{7}\right)$, $C\left(\frac{45}{19}, \frac{20}{19}\right)$, $D(3,0)$

$Z_0 = 0$

$Z_A = \frac{5}{2}$

$Z_B = \frac{5}{7} + \frac{5}{2} \cdot \frac{12}{7} = \frac{5}{7} + \frac{30}{7} = \frac{35}{7} = 5$

$Z_C = \frac{45}{19} + \frac{5}{2} \cdot \frac{20}{19} = \frac{45+50}{19} = \frac{95}{19} = 5$

$Z_D = 3$.

Here $Z_B = Z_C$.

$\Rightarrow$ Infinitely many optimal sol^{n}s.

To find these sol^{n}s,

$[x_1, x_2]$; x_1 & x_2 are points where optimal value is same.

$$L = \left\{ x : x = \lambda x_1 + (1-\lambda) x_2 \; ; \; 0 \le \lambda \le 1 \right\}.$$

This is set of optimal sol^{n}s.

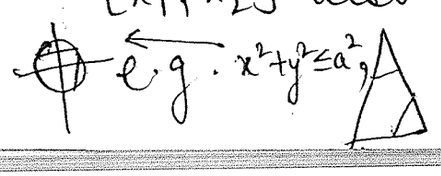
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Now here $x_1 = \left(\frac{5}{7}, \frac{12}{7}\right)$ & $x_2 = \left(\frac{45}{19}, \frac{20}{19}\right)$

$$\therefore L = \left\{ x : x = \lambda \left(\frac{5}{7}, \frac{12}{7}\right) + (1-\lambda)\left(\frac{45}{19}, \frac{20}{19}\right) ; 0 \le \lambda \le 1 \right\}$$

$$L = \left\{ x : x = \frac{5}{7}\lambda + (1-\lambda)\frac{45}{19}, \frac{12}{17} + (1-\lambda)\left(\frac{20}{19}\right) ; 0 \le \lambda \le 1 \right\}$$

✱ <u>Convex Set</u> → A set C is said to be convex set if for every $x_1, x_2 \in C$, the line segment $[x_1, x_2]$ also lies in C.



✱ Convex Combination →

If x is any vector & it is written as —

$$x = c_1 x_2 + c_2 x_2 + \cdots + c_n x_n.$$

i.e. L.C. of vectors $x_1, x_2, \to x_n$

s.t. $\sum_{i=1}^{n} c_i = 1.$

Then the linear combination is called convex combination.

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✱ Result → 1. Feasible region of LPP is always convex set.

2. Intersection of finite no. of convex sets are convex sets.

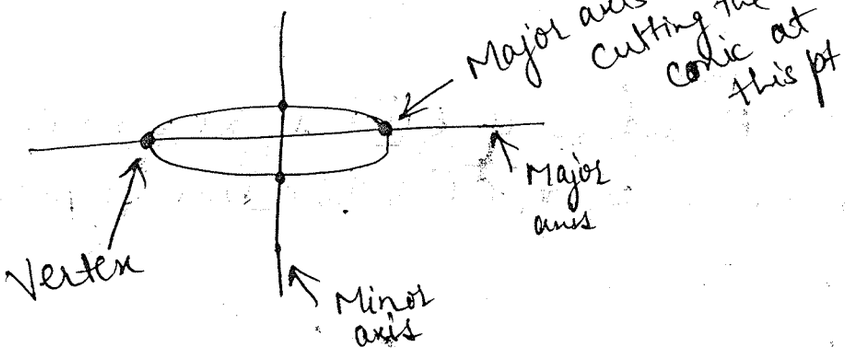
3. Union of two convex sets need not be convex.

4. Empty set is always convex set.

✱ Extreme Point → A pt. C of convex set S is called extreme point if it can't be written as convex combination of 2 points of set S.

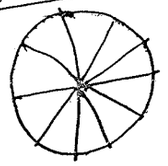
✱ Vertices → Where the axis of conic cuts the conic is called vertex of the conic.

Every vertex is extreme point but not conversely.



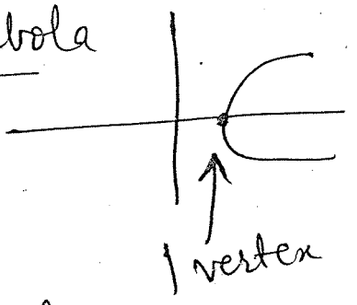
Here, all the boundary pts. are extreme pts.

Circle



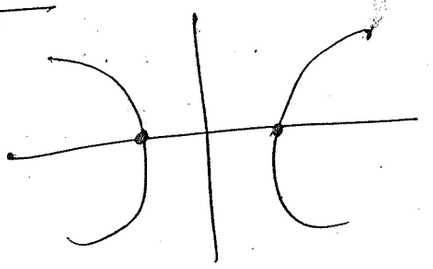
Infinite vertices.

Parabola



1 vertex

Hyperbola



2 vertices.

✱ Matrix Form of LPP →

$$Z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

$$= \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = CX$$

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$\Rightarrow Z = CX$

subject to $AX (\leq, \geq, =) b$

$X \geq 0$

where A is coefficient matrix $[A]_{m \times n}$,

$C = [c_1 \; c_2 \cdots c_n]$,

$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$.

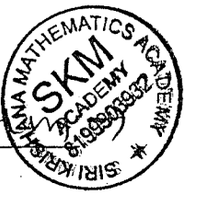
✱ Canonical form →

1. Maximization Canonical form → Objective fn" is of maximize type & constraits have ≤ inequality.

i.e. Max $Z = CX$

subject to $AX \leq b$

$X \geq 0$.



2. Minimization Canonical form

$\min Z = CX$

$AX \geq b$

$X \geq 0$

✱ Standard form of LPP →

Maximize $Z = CX$

subject to $AX = b$

& $X \geq 0$.

i $b \geq 0$.

- To convert in standard form →

(i) Objective fn" should be of maximize type. If not, then convert it into maximize type by multiplying it with (−1).

(ii) All b_i's should be greater than equal to zero. If not, then multiply them with (−1).

(iii) Write inequalities as eq"s by introducing surplus or slack variable.

(iv) All X_i's should be ≥ 0.

If $Z = x_1 + x_2 + x_3$

$\vdots$

$x_1, x_2 \geq 0$

We take $x_3 = x_3' - x_3''$; $x_3', x_3'' \geq 0$.

Then substitute in Z,

$Z = x_1 + x_2 + x_3' - x_3''$.

Now, Z is in 4 variables.

Q- Convert the LPP to standard form —

Min $Z = -x_1 + 2x_2 - 3x_3$

subject to $x_1 + x_2 - x_3 \geq -3$

$x_1 - x_3 = 4$

$2x_1 + 3x_2 + 4x_3 \leq -4$

; $x_1, x_3 \geq 0$.

Sol^n- Here x_2 is unrestricted.

∴ We will write,

$x_2 = x_2' - x_2''$; $x_2', x_2'' \geq 0$

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Now, we will convert given LPP to standard form-

Max $Z' = x_1 - 2x_2 + 3x_3$ {Multi. by (-1)}

$\Rightarrow Z' = x_1 - 2(x_2' - x_2'') + 3x_3$

$\Rightarrow Z = x_1 - 2x_2' + 2x_2'' + 3x_3$

subject to $-x_1 - x_2 + x_3 \leq 3$

$x_1 - x_3 = 4$

$-2x_1 - 3x_2 - 4x_3 \geq 4$

b_i's should be +ve i.e. 3, 4, 4 should be +ve

Now, converting into eq^{n}s,

$-x_1 - x_2 + x_3 + s_1 = 3$

$x_1 - x_3 = 4$

$-2x_1 - 3x_2 - 4x_3 - s_2 = 4$.

$s_1 \rightarrow$ slack var.

$s_2 \rightarrow$ surplus "

Here change x_2 to $x_2' - x_2''$ also

Q:- $Z_{max} = 3x_1 - 3x_2 + x_3$

Sub. to $x_1 - x_2 \leq -3$

$3x_1 + 2x_2 - x_3 \geq 4$

$-x_1 - x_2 - 4x_3 = -7$

; $x_1, x_2 \geq 0$.

Soln:- $x_3 = x_3' - x_3''$; $x_3', x_3'' \geq 0$.

$Z_{max} = 3x_1 - 3x_2 + x_3 \Rightarrow Z = 3x_1 - 3x_2 + x_3' - x_3''$

Sub. to $-x_1 + x_2 \geq 3$

$3x_1 + 2x_2 - x_3 \geq 4$

$x_1 + x_2 + 4x_3 = 7$

; ~~x_1, x_2~~

Converting into eq^{n}s, we have —

$-x_1 + x_2 - s_1 = 3$

$3x_1 + 2x_2 - x_3 - s_2 = 4$

$x_1 + x_2 + 4x_3 = 7$

$s_1, s_2 \rightarrow$ surplus var.

$\Rightarrow$

$-x_1 + x_2 - s_1 = 3$

$3x_1 + 2x_2 - x_3' + x_3'' - s_2 = 4$

$x_1 + x_2 + 4x_3' - 4x_3'' = 7$

; $x_1, x_2, x_3', x_3'', s_1, s_2 \geq 0$.

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* **Basic Sol^n** → Consider a system –

$$AX = b,$$

where A is $m \times n$ order matrix, $m \le n$ & rank $A = m$.

For any $m \times m$ non-singular matrix is choosen from A.

All $(n-m)$ variables not associated with the column of this matrix are set zero.

Then, the sol^n of resulting system is called basic sol^n & the variables involved are basic variables

Maximum no. of non-singular matrices is ${}^nC_m$.

⇒ Maximum no. of basic sol^{n}s $= {}^nC_m$.

Q. $3x_1 + 2x_2 + x_3 + 4x_4 = 6$

$6x_1 + x_2 + 2x_3 + 2x_4 = 12$

find all the basic sol^{n}s.

Sol^n Here, $m = 2$, $n = 4$.

$$A = \begin{bmatrix} 3 & 2 & 1 & 4 \\ 6 & 1 & 2 & 2 \end{bmatrix}$$

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Max. no. of non-singular matrices, hence basic sol^{n}s

$$= {}^4C_2 = \frac{4!}{2!\,2!} = 6.$$

(i) $x_3 = x_4 = 0$.

$$\begin{vmatrix} 3 & 2 \\ 6 & 1 \end{vmatrix} = 3 - 12 = -9 \neq 0$$

$3x_1 + 2x_2 = 6$ — (1)

$6x_1 + x_2 = 12$ — (2)

(1) × 2 − (2), we get

$3x_2 = 0 \Rightarrow x_2 = 0$

$\Rightarrow x_1 = 2.$

∴ Basic Sol^n is $(2,0,0,0)$

$x_1, x_2 \rightarrow$ Basic variables.

$x_3, x_4 \rightarrow$ Non basic variables.

(Max non-zero $= m = 2$)

(ii) $x_2 = x_4 = 0$.

$\begin{vmatrix} 3 & 1 \\ 6 & 2 \end{vmatrix} = 6 - 6 = 0$. → No Basic sol^n.

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(iii) $x_2 = x_3 = 0$

$\begin{vmatrix} 3 & 4 \\ 6 & 2 \end{vmatrix} = 6 - 12 = -6 \neq 0$.

$3x_1 + 4x_4 = 6$ — (3)

$6x_1 + 2x_4 = 12$ — (4)

(3) × 2 − (4), we get −

$6x_4 = 0$

$\Rightarrow x_4 = 0$

$\Rightarrow x_1 = 2$.

∴ Basic sol^n is $(2,0,0,0)$

(iv) $x_1 = x_2 = 0$

$\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} = 2 - 8 = -6 \neq 0$.

$x_3 + 4x_4 = 6$ — (5)

$2x_3 + 2x_4 = 12$ — (6)

2 × (5) − (6), we get −

$6x_4 = 0$

$\Rightarrow x_4 = 0$

$\Rightarrow x_3 = 6$

∴ Basic sol^n is $(0,0,6,0)$.

(v) $x_1 = x_3 = 0$

$\begin{vmatrix} 2 & 4 \\ 1 & 2 \end{vmatrix} = 4 - 4 = 0$ → No Basic sol^n.

(vi) $x_1 = x_4 = 0$.

$$\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 2 - 1 = 1 \neq 0.$$

$2x_2 + x_3 = 6$ — (7)

$x_2 + 2x_3 = 12$ — (8)

(7) − 2 × (8), we get

$-3x_3 = -18$

$\Rightarrow x_3 = 6$

$x_2 = 0$.

Basic sol^n is $(0, 0, 6, 0)$.

✱ Basic Feasible Sol^n → In Basic sol^n, all basic variables are ≥ 0, then sol^n is basic feasible sol^n.

✱ Degenerate or non-degenerate basic feasible sol^{n}s → A basic feasible sol^n is called degenerate basic feasible sol^n if any one basic variable is at zero level. Otherwise non-degenerate basic feasible sol^n.

✱ Results → 1. LPP has feasible sol^n iff it has basic feasible sol^n.

2. All the extreme points of ~~basic~~ feasible region are basic feasible sol^n & vice versa.

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3. No. of basic feasible sol^{n}s is always greater than equal to no. of extreme points.

4. In a basic sol^n, at most m variables can have non-zero values.

5. In a basic sol^n, at least $(n-m)$ variables can have zero value.

6. If optimal sol^n of LPP exists, then it must exist on atleast one basic feasible sol^n.

7. If all the basic feasible sol^{n}s are non-degenerate, then no. of basic feasible sol^{n}s is equal to no. of extreme points but not vice versa.

8. If no. of basic feasible sol^{n}s is greater than no. of extreme points, then LPP has degenerate basic feasible sol^{n}s but not vice versa.

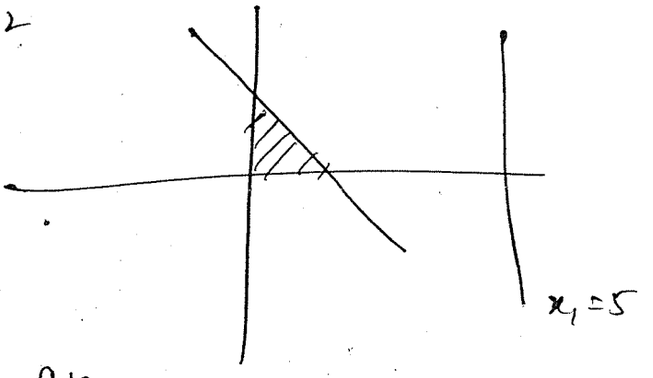
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✱ Redundent Constraints → The constraints which do not affect the ~~feasibility~~ of feasible region are called redundent constraints.

e.g. Max $Z = 2x_1 + 3x_2$

sub to $x_1 + x_2 \leq 1$

$x_1 \leq 5$



Whenever, a redundent constraint passes through an extreme pt., then corresponding to that pt, we always get ~~basic feasible sol^n~~ degenerate basic feasible sol^n.

✱ Duality →

Primal	Dual
Max $Z = CX$	Min $Z' = b^T W$
$AX \leq b$	$A^T W \geq C^T$
$X \geq 0$	$W \geq 0$
$C = (C_1, C_2, \dots, C_n)$	$b^T = (b_1\ b_2 \dots b_m)$
$b = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$, $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$	$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_m \end{bmatrix}$

Q. Construct the dual of the primal problem —

Maximize $z = 6x_1 + 17x_2 + 8x_3$

sub. to $x_1 - 2x_2 + x_3 \leq 3$

$-3x_1 + 2x_2 \leq 1$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$

Sol^n —

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -3 & 2 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -3 \\ -2 & 2 \\ 1 & 0 \end{bmatrix}$$

$$C = [6 \quad 17 \quad 8], \quad C^T = \begin{bmatrix} 6 \\ 17 \\ 8 \end{bmatrix}$$

$$b = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \quad b^T = [3 \quad 1]$$

~~$\Rightarrow$ Min $z' = -6x_1 - 17x_2 - 8x_3$~~

$\Rightarrow$ Min $z' = [3 \quad 1] \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = 3w_1 + w_2$

sub. to $\begin{bmatrix} 1 & -3 \\ -2 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \geq \begin{bmatrix} 6 \\ 17 \\ 8 \end{bmatrix}$

$w_1 - 3w_2 \geq 6$

$-2w_1 + 2w_2 \geq 17$

$w_1 \geq 8$

; $w_1, w_2 \geq 0.$

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Q. Maximize $z = 3x_1 + 5x_2$

Sub. to
$$\left.\begin{aligned} 6x_1 + 6x_2 &\le 55 \\ 8x_1 + 2x_2 &\le 30 \\ 5x_1 - 3x_2 &\le 18 \\ x_2 &\le 28 \end{aligned}\right\}$$

(A)

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Sol^n – Min $55w_1 + 30w_2 + 18w_3 + 28w_4$

Sub. to
$$\begin{aligned} 6w_1 + 8w_2 + 5w_3 &\ge 3 \\ 6w_1 + 2w_2 - 3w_3 + w_4 &\ge 5 \end{aligned}$$

✱ Result →

(A)

Primal	Dual
1. ith variable ≥ 0	ith ~~vario~~ constraint $\le$
2. " " ≤ 0	" $\ge$
3. ith variable unrestricted.	" $=$
4. jth constraint $\ge$	jth variable ≤ 0
5. " " $\le$	" " ≥ 0
6. " " $=$	" " unrestricted.

(B)

P \ D	Bdd	infeasible	unbdd.
Bdd	✓	×	×
Infeasible	×	✓	✓
unbdd.	×	✓	×

(C) Let $\hat{x}$ & $\hat{w}$ are feasible sol^{n}s of primal & dual problem respectively & $C\hat{x} = b^T\hat{w}$; i.e. $Z_{\hat{x}} = Z'_{\hat{w}}$.

Then, $\hat{x}$ & $\hat{w}$ are optimal sol^{n}s of primal &

dual problems respectively.

(D) If $\hat{x}$ & $\hat{w}$ are sol^{n}s of primal (maximize type) and dual (minimize type), then $C\hat{X} \leq b^T \hat{w}$,

i.e. $Z_{\hat{x}} \leq Z'_{\hat{w}}$.

(E)

P \ D	feasible	infeasible
feasible	has same optimal sol^n	P → unbdd.
infeasible	dual-unbdd.	possible.

Dec'16

Q Consider the LPP —

Maximize $z = 3x + 5y$.

sub. to $x + 5y \leq 10$
$2x + 2y \leq 5$
; $x \geq 0, y \geq 0$

Then —

(a) LPP does not admit any feasible sol^n.

(b) ∃ a unique optimal sol^n to LPP.

(c) ∃ a unique optimal sol^n to dual problem.

(d) The dual problem has an unbdd. sol^n.

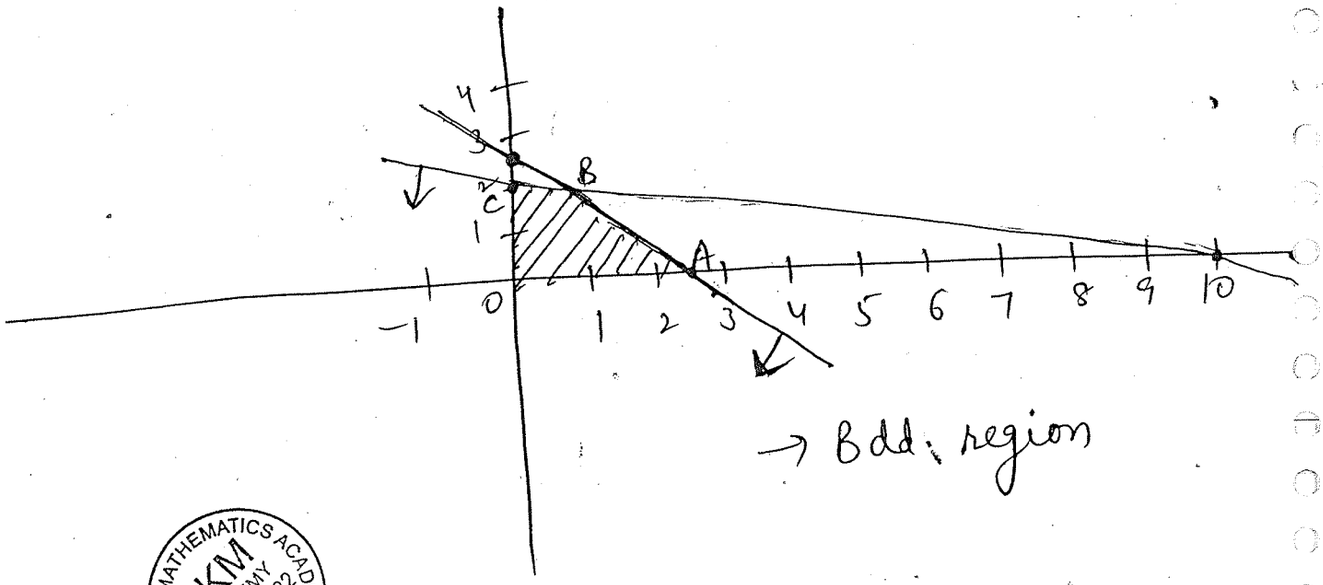
Sol^n — $x + 5y = 10$

$2x + 2y = 5$

x_1	0	10
x_2	2	0

x	0	5/2
y	2	0

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→ Bdd. region

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June '14

Q Consider the primal problem —

$$\text{Max } Z = -3x_1 + 2x_2$$

$$\text{Sub. to } \quad x_1 \leq 3$$

$$x_1 - x_2 \leq 0$$

$$x_1, x_2 \geq 0.$$

Which of the following statements are true?

(a) Primal problem has an optimal soln.

(b) " " " " unbdd. " . ✓

(c) Dual problem has an unbdd. soln.

(d) " " " no feasible soln. ✓

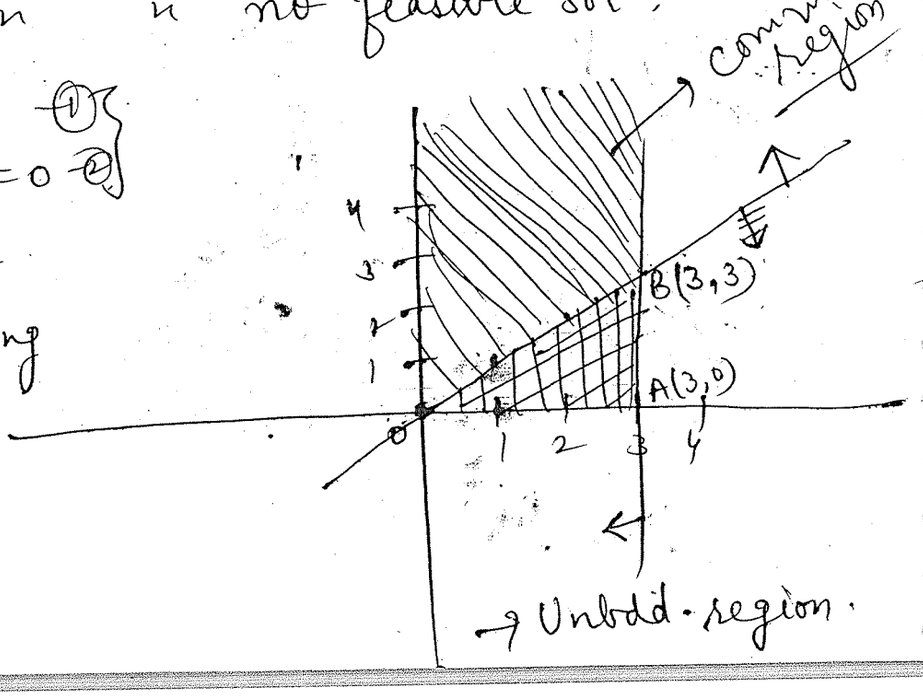
Soln — $x_1 = 3$ — (1)

$x_1 - x_2 = 0$ — (2)

$\Rightarrow x_1 = x_2$

Since (2) line is passing through $(0,0)$.

∴ We will take some other point say $(1,0)$



→ Unbdd. region.

$z_0 = 0$

$z_B = -9+6 = -3$.

~~Optimal value = 0.~~

No optimal value.

$\rightarrow$ Unbdd. sol^n. (Primal)

$\therefore$ Dual is infeasible.

$-9+6=-3$
$-9+8=-1$
$-9+10=1$

↑ing value

Q- Dec'12 A LPP maximize $c'X$, sub to $Ax \leq b$, $x \geq 0$ attains the optimum value at two distinct feasible sol^{n}s. Which of the following must be true?

(a) There must exist infinitely many feasible sol^{n}s where the optimal value is attained. ✓

(b) All feasible sol^{n}s must be optimal.

(c) The dual has unbdd. feasible region.

(d) Rank of $\begin{pmatrix} c' \\ A \end{pmatrix}$ = Rank A. ✓

Sol^n- If ∃ infinitely many optimal sol^{n}s, then objective fu^n line is parallel to ~~atleast~~ any one constraint line.

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Q- June'12 Consider the variable $x_1 \geq 0$ & $x_2 \geq 0$ satisfying the constraints -

$$x_1 + x_2 \geq 5$$
$$4x_1 - x_2 \leq 15$$
$$4x_2 - x_1 \leq 15.$$

Which of the following statements is/are correct?

(a) The maximum value of $3x_1 + 2x_2$ is 25.

(b) The minimum value of $3x_1 + 2x_2$ is 11.

(c) $3x_1 + 2x_2$ has no finite minimum.

(d) $3x_1 + 2x_2$ has no finite maximum.

Soln $x_1 + x_2 = 5$

(Q3- previous)

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Dec'14

Q- Consider the LPP — Min $z = -2x - 5y$

sub. to $3x + 4y \geq 5$

$x \geq 0, y \geq 0$.

(a) Set of feasible soln is empty.

(b) ✓ Set of feasible soln is non-empty but no optimal soln

(c) optimal value is attained at $\left(0, \frac{5}{4}\right)$.

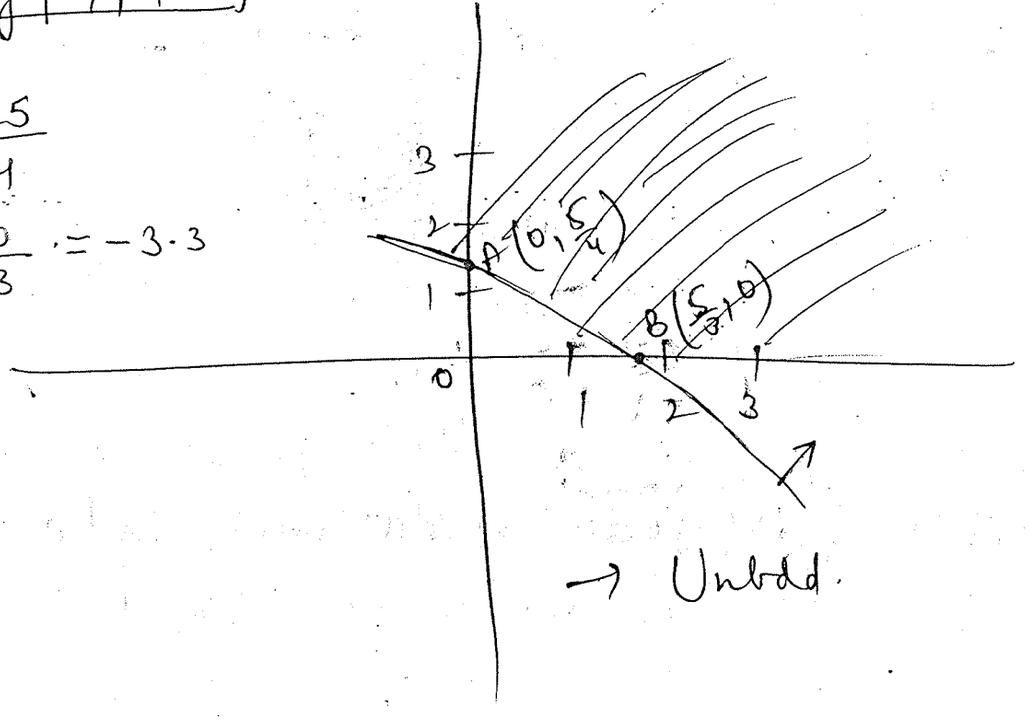
(d) " " " " " $\left(\frac{5}{3}, 0\right)$.

Soln- $3x + 4y = 5$.

x	0	5/3
y	5/4	0

$z_A = -\frac{25}{4}$

$z_B = -\frac{10}{3} = -3.3$



→ Unbdd.

Q. (June '16) Max $3x+4y$ s.t. $x \geq 0, y \geq 0, x \leq 3$

$\frac{1}{2}x + y \leq 4$, $x + y \leq 5$.

Which among the following are correct?

(a) The optimal value is 19.

(b) The optimal value is 18. ✓

(c) (3,2) is an extreme pt. of feasible region. ✓

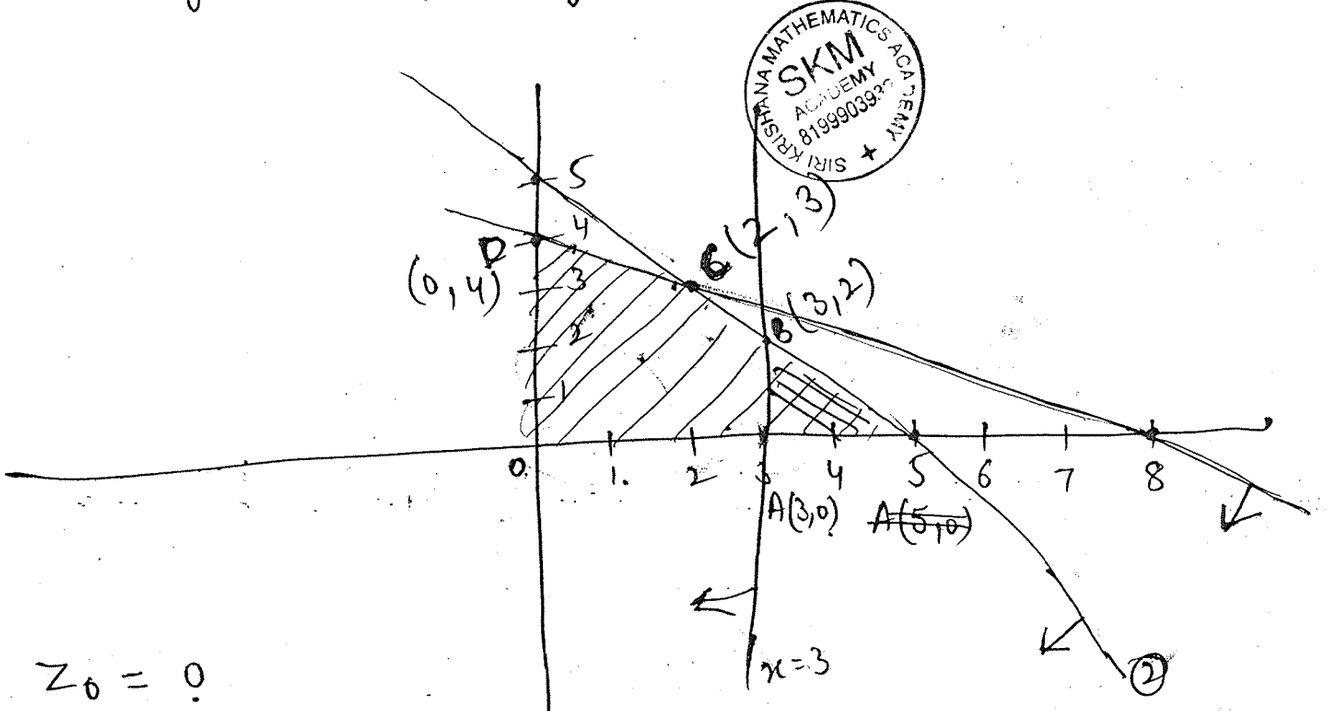
(d) $(3, \frac{5}{2})$ is an extreme pt. of feasible region.

Soln $\frac{1}{2}x + y = 4 \Rightarrow x + 2y = 8$ — (1)

$x + y = 5$ — (2)

x	0	8
y	4	0

x	0	5
y	5	0



$Z_O = 0$

$Z_A = 9$

$Z_B = 9 + 8 = 17$

$Z_C = 18 \rightarrow$ optimal value.

$Z_D = 16$

$\Rightarrow (2,3)$ is optimal soln.

Q. Dec'13 Consider the following LPP –

Max $Z = 2x_1 + 4x_2$

Sub to $x_1 + 2x_2 \leq 5$
$x_1 + x_2 \leq 3$
; $x_1, x_2 \geq 0$

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(a) An optimum soln is $(x_1, x_2) = (1, 2)$. ✓

(b) An " " " " $= (3, 1)$ ✓

(c) " " " " " $= (0, 2.5)$ ✓

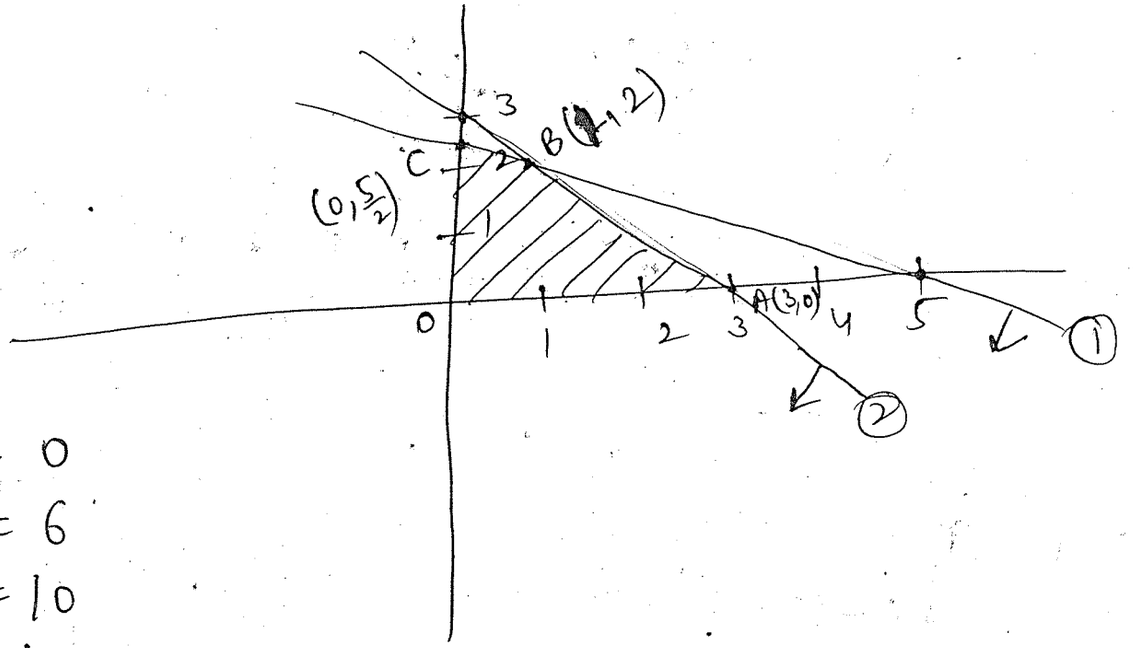
(d) The objective fxn is unbdd.

Soln $x_1 + 2x_2 = 5$

$x_1 + x_2 = 3$

x_1	0	5
x_2	5/2	0

x_1	0	3
x_2	3	0



$Z_0 = 0$

$Z_A = 6$

$Z_B = 10$

$Z_C = 10$.

2. Simplex Method →

(i) Convert / Formulate the LPP in standard form.

(ii) Find the initial basic feasible sol^n and construct simplex table.

(iii) Check the optimality of basic feasible sol^n.

For this, we find $\Delta_J = C_J - C_\beta Y_J$.

(a) If $\Delta_J \leq 0 \ \forall \ J$, then the sol^n under test is optimal.

~~(b)~~ (i) If ~~all~~ $\Delta_J < 0 \ \forall \ J$ (for all non basic variables), then the sol^n under test is unique optimal sol^n.

(ii) If $\Delta_J = 0$ for any basic variable, then other optimal sol^n's exist with the same value of Z.

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(iii) If corresponding to max. +ve Δ_J, the entries in the column Y_J are 0 or −ve, then the sol^n under test is unbdd.

(iv) If the value of atleast one artificial variable appearing in the basis is non-zero & the condition of optimality is satisfied, then the LPP has no feasible sol^n.

(b) If $\Delta_J > 0$ for any J, then the sol^n under test is not optimal. Then, we improve our sol^n by proceeding to next step.

(c) For improving basic feasible sol^n, we find the incoming ~~or~~ and outgoing vector.

(i) To find incoming vector, the incoming vector will be taken as a_k if $\Delta_k \geq \max \Delta_J$.

If maximum value of Δ_j occurs at more than one a_j, then any one of these may be taken as incoming vector.

(ii) ~~For~~ To find outgoing vector,

Outgoing vector β_r is taken corresponding to that value of r for which

$$\frac{x_{\beta r}}{y_{rk}} = \min\left\{\frac{x_{\beta i}}{y_{jk}} ; y_{jk} > 0\right\}.$$

If min is not unique, then we get degenerate basic feasible sol^n for which outgoing vector is taken ~~as~~ in different way.

(d) ~~(iii)~~ When $a_k (y_k)$ is incoming vector and $\beta_r (y_r)$ is outgoing vector, then the element $y_{rk} (a_{rk})$ is called key element or pivot element which is intersection of incoming vector and outgoing vector.

(e) ~~(iv)~~ Test improved basic feasible sol^n for optimality. Repeat as in step (iii) (initially) and repeat step (c) & (d) until optimal sol^n exists.

(f) In case of degeneracy, if the same min. ratio occurs at more than one value of j say $j_1, j_2, \ldots, j_n$-th, then

(i) To select outgoing vector, we proceed as —

Denote the column of unit matrix in proper

order

$\overline{y_1} = [1, 0, 0, \dots]$

$\overline{y_2} = [0, 1, 0, \dots]$

Compute $\min_{i \in I_1} \left\{ \frac{\text{element of } \overline{Y_i} \text{ in } i^{th} \text{ row}}{\text{element of } Y_k \text{ in } i^{th} \text{ row}} \right\}$

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where $I_1 = \{j_1, j_2, \dots \to j_n\}$

If this min is unique say $j = j_2 \Rightarrow j_2$ is outgoing vector.

If not unique, then we proceed with next Y_i.

June'17

Q- Consider the LPP—

Max $2y_1 + 3y_2 + 5y_3 + 4y_4$

Sub. to $y_1 + y_2 \le 1$

$y_2 + y_3 \le 1$

$y_3 + y_4 \le 1$

$y_4 + y_1 \le 1$

; $y_i \ge 0 \ \forall \ i = 1, 2, 3, 4.$

Then, the optimal value is —

(a) = 8.

(b) b/w 8 & 9.

(c) greater than or equal to 7.

(d) less than or equal to 7.

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Soln-

$y_1 + y_2 + s_1 = 1$

$y_2 + y_3 + 0 \cdot s_1 + s_2 = 1$

$y_3 + y_4 + 0 \cdot s_1 + 0 \cdot s_2 + s_3 = 1$

$y_4 + y_1 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 + s_4 = 1.$

	C_j	C_1	C_2	---	C_n	0	0	---	0	X_β
β C_β	X_β	Y_1 (a_1)	Y_2 (a_2)	---	Y_n (a_n)	Y_{n+1} (β_1)	Y_{n+2} (β_2)	---	Y_{n+m} (β_m)	Y_k
β_1 0										
β_2 0										
⋮ ⋮										
β_m 0										
$Z=C_\beta X_\beta$	Δ_J									

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* This is general table

Q continued ....

	C_j	2	3	5	4	0	0	0	0	X_β
β C_β	X_β	Y_1	Y_2	Y_3	Y_4	S_1	S_2	S_3	S_4	Y_k
						$\frac{S_1}{Y_1}$	$\frac{S_2}{Y_2}$	$\frac{S_3}{Y_3}$	$\frac{S_4}{Y_4}$	
S_1 0	1	1	1	0	0	1	0	0	0	—
S_2 0	1	0	1	1	0	0	1	0	0	1
S_3 0	1	0	0	1	1	0	0	1	0	1
S_4 0	1	1	0	0	1	0	0	0	1	—
$Z=C_\beta X_\beta$	Δ_J	2	3	5	4	0	0	0	0	1

Pivot element

Now, $\Delta_J = C_j - C_\beta Y_j$

$\Delta_J \geq 0 \Rightarrow$ Sol is not optimal.

Corresponding to Max. Δ_J, we get incoming vector.

$\min \frac{Y_k}{Y_3} = \left\{\frac{0}{1}, \frac{0}{1}\right\}$ → not unique

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$$\min \frac{y_2}{y_3} = \min\left\{\frac{1}{1}, \frac{0}{1}\right\} = 0$$

	C_j	2	3	5	4	0	0	0	0	X_β / y_2
β C_β	X_β	y_1 (a_1)	y_2 (a_2)	y_3 (a_3)	y_4 (a_4)	δ_1	δ_2	δ_3	δ_4	
β_1 0	1	1	1	0	0	1	0	0	0	1
Δ_2 0	0	0	1	0	−1	0	1	−1	0	0
y_3 5	1	0	0	1	1	0	0	1	0	—
δ_4 0	1	1	0	0	1	0	0	0	1	—
$Z = C_\beta X_\beta$	Δ_j	2	3	0	−1	0	0	−5	0	

		2	3	5	4	0	0	0	0	
	X_β	y_4								
Δ_1 0	1	1	0	0	1	1	−1	1	0	1
y_2 3	0	0	1	0	−1	0	1	−1	0	0
y_3 5	1	0	0	1	1	0	0	1	0	‡
δ_4 0	1	1	0	0	1	0	0	0	1	‡
		2	0	0	2	0	−3	−2	0	

	X_β									
Δ_1 0	1	1	0	0	0	1	0	0	0	1
y_4 4	0	0	−1	0	1	0	−1	1	0	—
y_3 5	1	0	1	1	0	0	1	0	0	—
δ_4 0	1	1	1	0	0	0	1	−1	1	1
		2	2	0	0	0	−1	−4		

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		X_β								$\frac{X_\beta}{Y_k}$
Δ_1	0	0	0	0	0	1	−1	1	1	7
Y_4	4	0	0	−1	0	1	0	−1	1	0
Y_3	5	1	0	1	1	0	0	1	0	—
Y_4	2	1	1	1	0	0	0	1	−1	1
			0	0	0	0	0	−3	−2	−1	

Optimal value = 7.

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Q. Minimize $Z = x_1 - 3x_2 + 2x_3$

Sub. to $3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

; $x_1, x_2, x_3 \geq 0$.

Sol^n Max $z' = -x_1 + 3x_2 - 2x_3$

sub to $3x_1 - x_2 + 2x_3 + \Delta_1 = 7$

$-2x_1 + 4x_2 + 0\cdot\Delta_1 + \Delta_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + 0\cdot\Delta_1 + 0\cdot\Delta_2 + \Delta_3 = 10$

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		C_j	−1	3	−2	0	0	0	$\frac{X_\beta}{Y_k}$
β	C_β	X_β	x_1	x_2	x_3	Δ_1	Δ_2	Δ_3	
Δ_1	0	7	3	−1	2	1	0	0	−7
Δ_2	0	12	−2	4	0	0	1	0	3
Δ_3	0	10	−4	3	8	0	0	1	10/3
$Z = C_\beta X_\beta$		Δ_J	−1	3	−2	0	0	0	

			−1	3	−2	0	0	0	$\frac{X_\beta}{Y_k}$
β	C_β	X_β	x_1	x_2	x_3	Δ_1	Δ_2	Δ_3	
x_2	3								
Δ_2	0								
Δ_3	0								

3. Two-phase Method → (It is used when all the inequalities are ≥ type & all b_i's are +ve.)

Phase-I → In phase-I, we remove the artificial variables from the basis and introduce other variables.

Phase-II → In phase-II, we use the sol^n of phase-I as the initial basis sol^n and apply simplex method to determine optimal sol^n.



Working Rules :- 1. Make all b_i positive. Convert each constraint as eq^n.

2. Obtain new objective fx^n z' by assigning -1 cost to each artificial variable & zero to all other.
   i.e. $z' = -$(sum of artificial variable).

3. Using simplex method, maximize the new objective fx^n subject to constraints of original problem.
   If original problem has a feasible sol^n, then this problem has an optimal sol^n.
   (i) If max $z' < 0$ & atleast one artificial variable appears in the optimal basis at +ve level, then given problem does not possess any feasible sol^n and so procedure terminated.
   (ii) If max $z' = 0$, then proceed to phase-II.

Phase-II →

1. Start with optimal basic feasible sol^n contained in the final simplex table of phase-I & using simplex method, maximize objective

fx^n z with the actual cost to the primal variable and zero cost to surplus variable.

Q- Min $z = x_1 + x_2$

sub to $2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$ } .

$x_1, x_2 \geq 0$.

Soln- Max $z' = -x_1 - x_2$

sub to $2x_1 + x_2 - s_1 + A_1 = 4$

$x_1 + 7x_2 - s_2 + A_2 = 7$ } — (1)

$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$.

Now, we will construct new objective fx^n.

Max $z'' = -A_1 - A_2$.

sub to (1)..

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		C_j	0	0	0	0	−1	−1	$\frac{X_\beta}{X_2}$
β	C_β	X_β	x_1	X_2	s_1	s_2	A_1	A_2	
A_1	−1	4	2	1	−1	0	1	0	4
A_2	−1	7	1	7	0	−1	0	1	1
		Δ_J	3	8	−1	−1	0	0	
		C_j	0	0	0	0	−1	−1	
β	C_β	X_β							
A_1	−1	3	$13/7$	0	−1	$1/7$	1	$-1/7$	$\frac{21}{13}$
X_2	0	1	$1/7$	1	0	$-1/7$	0	$1/7$	7
		Δ_J	$13/7$	0	−1	$1/7$	0	−ve	

x_1	0	$\frac{21}{13}$	1	0	$-7/13$	$\frac{1}{13}$	$\frac{7}{13}$	$\frac{-1}{13}$
x_2	0	$\frac{10}{13}$	0	1	$1/13$	$-2/13$	$-1/13$	$\frac{12}{13 \cdot 7}$
			0	0	0	0	-1	-1

All $\Delta_J \leq 0$

Max Δ_J

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Phase - II:

			-1	-1	0	0	
β	C_β	x_β	x_1	x_2	Δ_1	Δ_2	
x_1	-1	$\frac{21}{13}$	1	0	$-7/13$	$1/13$	
x_2	-1	$\frac{10}{13}$	0	1	$1/13$	$-2/13$	
		Δ_J	0	0	$-ve$	$-ve$	

All $\Delta_J \leq 0$

$x_1 = \frac{21}{13}$, $x_2 = \frac{10}{13}$

Sol^n is —

$z = -\frac{21}{13} - \frac{10}{13} = -\frac{31}{13}$.

$\therefore$ Min $z = \frac{31}{13}$.

Q- Max $z = 3x_1 + 2x_2 + x_3 + 4x_4$

Sub. to $4x_1 + 5x_2 + x_3 + 5x_4 = 5$

$2x_1 - 3x_2 - 4x_3 + 5x_4 = 7$

$x_1 + 4x_2 + 5x_3 - 4x_4 = 6$

$x_1, x_2, x_3, x_4 \geq 0.$



S.No.	PARTICULARS	Page	Date	Remarks

DATE : / 20
PAGE No

(Numerical Methods)

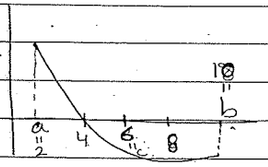
1. **Bisection method** :- (Mid-point formula / Bolzano method)

$f(x) = 0$

$f(a) > 0$; $f(b) < 0$

Root lies b/w a and b.

$c = \frac{a+b}{2}$



If $f(c) < 0$ then Root lies b/w a & c

& If $f(c) > 0$ then Root lies b/w c & b.

eg:- $f(2) > 0$ & $f(10) < 0$.

Root lies b/w 2 & 10.

$c = \frac{2+10}{2} = \frac{12}{2} = 6$ & $f(6) = -ve \Rightarrow$ Root lies in (6,6)

$c = \frac{2+6}{2} = \frac{8}{2} = 4$ & $f(4) = 0 \Rightarrow 4$ is the req. root

* If permissible error is ε then approximate no. of iteration is given by $\frac{b-a}{2^n} < \varepsilon$.

eg:- $\varepsilon = 0.01$ & Interval = (0,10)

i.e. $a = 0$, $b = 10$

no. of iter :- $\Rightarrow \frac{10}{2^n} \leq 0.01 \Rightarrow 2^n \geq 0.01$

$\Rightarrow 2^{-n} \leq 10^{-2}$.

* The minimum no. of iteration required for converging to a root in an interval (0,1) for a given ε then

ε :	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
n :	7	10	14	17	20	24
	3	4	3	3	3	4

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DATE : / 20
PAGE No

2. Regula falsi :- (Secant Method) → formula same as Regula falsi

$$x_{n+1} = \frac{x_{n-1} f_n - x_n f_{n-1}}{f_n - f_{n-1}}$$

where $f_n = f(x_n)$

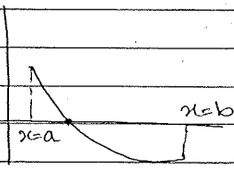
* Secant method does not give guarantee of the solution but Bisection & Newton Raphson gives.
* Bisection method is quite lengthy as compared to Regula falsi & Secant but gives accurate result than both of those methods.

* Radius / Rate of convergence :-

Bisection = 1
Regula falsi / Secant = 1.618 or $\frac{1+\sqrt{5}}{2}$

3. Newton Raphson :⇒

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



$x_n \Rightarrow$ closest point (root)

eg :- $f(x) = x^3 - x + 1$
$f(0) = 1$, $f(-1) = 1 > 0$, $f(-2) = -5 < 0$
Root lies b/w -2 and -1.
But the required root is nearest to $f(-1)$ (zero)
So $x_n = -1$

DATE : / 20
PAGE No

* Newton Raphson method is also called quadratic convergence.
* Its Radius / Rate of convergence is 2.

4. * Chebyshev method :-
For this method Rate of convergence is 3.

* If x_0 be the root of $f(x)=0$ with multiplicity p then iterative formula, $x_{n+1} = x_n - \frac{p f(x_n)}{f'(x_n)}$

and rate of convergence is also be 2.

Ques :⇒ Find iterative formula for the eqn $x = \sqrt{2}$.
Soln :⇒ $x = \sqrt{2} \Rightarrow x^2 = 2$
$\therefore f(x) = x^2 - 2 = 0$
Now by NR-method, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$= x_n - \left(\frac{x_n^2 - 2}{2x_n}\right) = \frac{2x_n^2 - x_n^2 + 2}{2x_n} = \frac{x_n^2 + 2}{2x_n} = \frac{x_n}{2} + \frac{1}{x_n}$$

V.V.Imp Ques :⇒ $x_{n+1} = \frac{x_n^2 + 2}{2x_n}$. This formula gives an iterative formula for an eqn. Find the eqn.

Soln :- let $x_n \to \alpha$ so $x_{n+1} \to \alpha$
Now by NR-method, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
we are given that $x_{n+1} = \frac{x_n^2 + 2}{2x_n}$

$$\Rightarrow \alpha = \frac{\alpha^2 + 2}{2\alpha} \Rightarrow 2\alpha^2 = \alpha^2 + 2 \Rightarrow 2\alpha^2 - \alpha^2 = 2$$

$\Rightarrow \alpha^2 = 2$ i.e. $x_n^2 = 2 \Rightarrow x_n^2 - 2 = 0$

So $f(x) = x^2 - 2$

★ Iterative formula for cube root :-

$x^3 = N$

where N is any natural no.

$f(x) = x^3 - N$

Now by NR method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= x_n - \frac{x_n^3 - N}{3x_n^2} = \frac{3x_n^3 - x_n^3 + N}{3x_n^2} = \frac{2x_n^3 + N}{3x_n^2}$

$\Rightarrow \boxed{x_{n+1} = \frac{2}{3}x_n + \frac{N}{3x_n^2}}$

Square root of 2 :-

Let $x^2 = 2 \Rightarrow f(x) = x^2 - 2 = 0$.

Now $x_{n+1} = \frac{x_n^2 + 2}{2x_n}$ [iterative formula for square root]

$f(0) = -2$, $f(1) = 0$, $f(1.5) = 2.25 - 2 = 0.25 > 0$

Root lies b/w 1 & 1.5 since 0 is nearer to $f(1.5)$

So $x_0 = 1.5$

Now $x_1 = \frac{x_0^2 + 2}{2x_0} = \frac{(1.5)^2 + 2}{2 \times 1.5} = \frac{2.25 + 2}{3} = \frac{4.25}{3} = 1.416$

$x_2 = \frac{x_1^2 + 2}{2x_1} = \frac{(1.416)^2 + 2}{2 \times 1.416} = \frac{2.005 + 2}{2.832} = \frac{4.005}{2.832} = 1.414$

$x_3 = \frac{x_2^2 + 2}{2x_2} = \frac{(1.414)^2 + 2}{2 \times 1.414} = \frac{1.999 + 2}{2.828} = \frac{3.999}{2.828} = 1.414$

So $x = 1.414$

$\Rightarrow \sqrt{2} = 1.414$ Ans.

★ Efficiency Formula :-

$\boxed{E = p^{1/n}}$

where E is efficiency, p is the order of convergence and n is the no. of functions and derivative in a step.

1. Efficiency for NR method :- order = 2, 1 f^n & 1 derivative of the f^n, so $n = 1+1 = 2$

Now $E = 2^{1/2} = 1.414$

2. For Regula Falsi :-

order = 1.618, $f^n = 1$, derivative = 0

So $n = 1 + 0 = 1$.

Now $E = (1.618)^1 = 1.618$ or 1.62

3. For chebyshew method :- $E = 1.44$

MOST IMP

★ Methods of Iteration :-

$\boxed{\begin{array}{l} x = \phi(x) \\ |\phi'(x)| < 1 \end{array}}$

eg :- $f(x) = x^2 + x - 2 = 0$

$x = 2 - x^2$ Here $\phi(x) = 2 - x^2$

or $x = \sqrt{2 - x}$ " $\phi(x) = \sqrt{2 - x}$

or $x(x+1) = 2 \Rightarrow x = \frac{2}{x+1}$ Here $\phi(x) = \frac{2}{x+1}$

$f(x) = x^2 + x - 2 = 0$

$f(0) = -2 < 0$, $f(1) = -1 < 0$, $f(2) = 4 > 0$

Root lies in the interval (1, 2)

(1) $x = 2 - x^2 = \phi(x)$

$\phi'(x) = -2x \Rightarrow |\phi'(x)| = |-2x| = 2x$ depends on x

(2) $\phi(x) = \sqrt{2-x}$, $\phi'(x) = \frac{-1}{2\sqrt{2-x}}$

$|\phi'(x)| = \frac{1}{2\sqrt{2-x}} < 1$ in $(1,2)$

Dec-2015 Ques:- Let $f(x) = ax + 100$, $a \in \mathbb{R}$. The iteration $x_{n+1} = f(x_n)$, $x \geq 0$ converges for

1) $a = 5$ 2) $a = 1$ 3) $a = 0$ 4) $a = 1.0$

Soln:- $f(x) = ax + 100$

$f'(x) = a$, $|f'(x)| < 1 \Rightarrow a < 1$ & $0.1 < 1$.

for convergence.

☆ Gauss Elimination :-

$$x + y + z = 2$$
$$2x - y + 2z = 4$$
$$x - y + 2z = 7$$

$$\begin{bmatrix} 1 & 1 & 1 & : 2 \\ (2)\ -1 & 2 & : 4 \\ (1)\ -1 & 2 & : 7 \end{bmatrix}$$

These should be zero after operating them

then find x, y, z

☆ Jacobi Method :-

$$\left.\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= b_1 \\ a_{21}x + a_{22}y + a_{23}z &= b_2 \\ a_{31}x + a_{32}y + a_{33}z &= b_3 \end{aligned}\right\} \quad \text{---(1)}$$

$$x_1 = \frac{1}{a_{11}}(b_1 - a_{12}y_0 - a_{13}z_0)$$

$$y_1 = \frac{1}{a_{22}}(b_2 - a_{21}x_0 - a_{23}z_0), \quad z_1 = \frac{1}{a_{33}}(b_3 - a_{31}x_0 - a_{32}y_0)$$

(x_0, y_0, z_0) → initial approximation

$$x_2 = \frac{1}{a_{11}}(b_1 - a_{12}y_1 - a_{13}z_1), \quad y_2 = \frac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}z_1)$$

... ...

GATE Imp ☆ Gauss Seidal :-

Soln of system (1) is found as follows:-

$$x_1 = \frac{1}{a_{11}}(b_1 - a_{12}y_0 - a_{13}z_0)$$

$$y_1 = \frac{1}{a_{22}}(b_2 - a_{21}x_1 - a_{23}z_0), \quad z_1 = \frac{1}{a_{33}}(b_3 - a_{31}x_1 - a_{32}y_1)$$

...

✳ Jacobi method is lengthy as compared to Gauss Seidal method.

these are not same. both have a little diff.

☆ LU-Decomposition (Crout / Dolittle decomposition) :⇒

$A = LU$, L = lower triangular matrix

U = Upper " "

$$L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \quad U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

✳ If l_{ii} is 1 $\forall i$ then the method is Dolittle.
& If u_{ii} is 1 $\forall i$ then the method is Crout.

Remark :⇒

This method fails if any of the diagonal elements l_{ii} or u_{ii} is zero.

LU decomposition method is guaranteed when matrix A is positive definite. (diag. > 0 & det. > 0)

DATE : / 20
PAGE No

System $AX=B$, $X = A^{-1}B$

$A = LU \Rightarrow$ Therefore $LUX = B$

Let $UX = Z \Rightarrow X = U^{-1}Z$ & $LZ = B \rightarrow Z = L^{-1}B$

$\therefore X = U^{-1}L^{-1}B$.

Cholesky Method :→ (Square root method)

$A = LL^T$, $L^T \rightarrow$ Transpose of lower triangular matrix

This method is applicable when matrix A is symmetric and positive definite.

* Entries of A should be real.

In Gauss Seidal method, system should be diagonally dominate. i.e.

$a_{11}x + a_{12}y + a_{13}z = b_1$

$a_{21}x + a_{22}y + a_{23}z = b_2$

$a_{31}x + a_{32}y + a_{33}z = b_3$

$|a_{11}| \geq |a_{12}| + |a_{13}|$

$|a_{22}| \geq |a_{21}| + |a_{23}|$

$|a_{33}| \geq |a_{31}| + |a_{32}|$

Finite Difference operators :→

1. Forward Difference operator (Δ) :→

$$\Delta f(x) = f(x+h) - f(x)$$

where h is length of interval or step length. Either h is given or taken to be 1.

2. Backward D.O. (∇) :→

$$\nabla f(x) = f(x) - f(x-h)$$

DATE : / 20
PAGE No

3. Shift operator (E) :-

$E f(x) = f(x+h)$, $h \rightarrow$ Step length

$E^{-1} f(x) = f(x-h)$

$E^2 f(x) = f(x-2h)$

4. Central D.O. (S) :→

$$S f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)$$

5. Average operator (μ) :→

$$\mu f(x) = \frac{1}{2}\left[f\left(x + \frac{h}{2}\right) + f\left(x - \frac{h}{2}\right)\right]$$

Relation b/w operators :→

1) $\Delta = E - 1$

2) $\nabla = 1 - E^{-1}$

3) $S = E^{1/2} - E^{-1/2}$

4) $\mu = \frac{1}{2}(E^{1/2} + E^{-1/2})$

5) $\mu = \sqrt{1 + \frac{S^2}{4}}$

6) $\Delta + \nabla = E - E^{-1}$

Interpolation :→

1. Lagrange's method :→

Suppose there are three given points x_0, x_1, x_2 & their corresponding values of function are $f(x_0), f(x_1), f(x_2)$. i.e.

x_0	$f(x_0)$
x_1	$f(x_1)$
x_2	$f(x_2)$

then

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

DATE : / 20
PAGE No

or $f(x) = \underbrace{l_0(x)}_{\text{fundamental Poly.}} f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$

eg :-

x	$y = f(x)$
0	0
1	1
2	4

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$= \frac{(x-1)(x-2)}{(-1)(-2)} \times 0 + \frac{(x-0)(x-2)}{1 \times (-1)} \times 1 + \frac{(x-0)(x-1)}{2 \times 1} \times 4$$

$$= 0 + -x^2 + 2x + 2x^2 - 2x = x^2$$

$\therefore f(x) = x^2$.

$f(x) = l_0(x) f(x_0) + l_1(x) f(x_1) + l_2(x) f(x_2)$

$l_0(x) + l_1(x) + l_2(x) = 1$

$l_0(x_0) = 1$, $l_0(x_1) = 0$, $l_1(x_0) = 0$, $l_1(x_1) = 1$.

$$l_i(x_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

★ Divided Difference :-

$$[x_0, x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \underset{x_1}{\Delta} f(x_0)$$

$$[x_0, x_1, x_2] = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0} = \underset{x_1, x_2}{\Delta^2} f(x_0)$$

DATE : / 20
PAGE No

eg: $f(x) = x^2$

$$[a, b, c] = \frac{f(b, c) - f(a, b)}{c - a}$$

$$f(b, c) = \frac{f(c) - f(b)}{c - b} = \frac{c^2 - b^2}{c - b} = c + b$$

$$f(a, b) = \frac{f(b) - f(a)}{b - a} = \frac{b^2 - a^2}{b - a} = b + a$$

$$[a, b, c] = \frac{c + b - (b + a)}{c - a} = \frac{c - a}{c - a} = 1$$

★ Newton Divided Difference formula :-

$$P(x) = f(x_0) + (x - x_0) \Delta f(x_0) + (x - x_0)(x - x_1) \Delta^2 f(x_0) + \dots$$

eg :

x	$y = f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
0	0	$\frac{1-0}{1-0} = 1$	$\frac{3-1}{2-0} = \frac{2}{2} = 1$
1	1	$\frac{4-1}{2-1} = 3$	
2	4		

$$P(x) = 0 + (x - 0)(1) + (x - 0)(x - 1)(1) = x + x^2 - x = x^2$$

For equal interval length → Newton Gregory Forward
→ Newton Gregory Backward

Forward Difference Table :-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0	y_0	$\Delta y_0 = y_1 - y_0$	$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$	$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$
x_1	y_1	$\Delta y_1 = y_2 - y_1$	$\Delta^2 y_1 = \Delta y_2 - \Delta y_1$	
x_2	y_2	$\Delta y_2 = y_3 - y_2$		
x_3	y_3			

DATE : / 20
PAGE No

Backward Difference Table :-

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
x_0	y_0			
x_1	y_1	$\nabla y_1 = y_1 - y_0$		
x_2	y_2	$\nabla y_2 = y_2 - y_1$	$\nabla^2 y_2 = \nabla y_2 - \nabla y_1$	
x_3	y_3	$\nabla y_3 = y_3 - y_2$	$\nabla^2 y_3 = \nabla y_3 - \nabla y_2$	$\nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$

Newton Gregory forward difference formula :-

$$f(x) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \dots$$

$u = \frac{x-a}{h}$; $a \rightarrow$ first term, $h \rightarrow$ interval length

or $f(x) = \sum_{i=0}^{n} u C_i \Delta^i f_0$

NG - Backward difference formula :-

$$f(x) = f(x_n) + u.\nabla f(x_n) + \frac{u(u+1)}{2!}\nabla^2 f(x_n) + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \dots$$

Newton forward formula is used when the desired value is near to the given starting values of x.

eg :-

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$
1	1	$\nabla y_1 = 26$	$\nabla^2 y_2 = 72$	$\nabla^3 y_3 = 48$
3	27	$\nabla y_2 = 98$	$\nabla^2 y_3 = 120$	
5	125	$\nabla y_3 = 218$		
7	343			

$y(6) = ?$

DATE : / 20
PAGE No

Soln :- $u = \frac{x-a}{h} = \frac{6-7}{2} = -\frac{1}{2}$ (Here $a \rightarrow$ last term)

$$y(6) = y(7) + \left(-\frac{1}{2}\right)\times 109 + \frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)}{2}\times 30 + \frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)}{3\times 2}\times 15$$

$$= 343 - 109 - 15 - 3 = 343 - 127 = 216$$

$\therefore y(6) = 216$ Ans.

ODE of 1st order :-

$\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$

1. Picard method.
2. Euler's method :-

$$y_{n+1} = y_n + hf(x_n, y_n)$$

This method is of order 1.

3. Euler's modified :-

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1}^{(1)} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_n, y_{n+1})\right]$$

$$y_{n+1}^{(2)} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_n, y_{n+1}^{(1)})\right] \dots$$

Ques :- $f(x,y) = x+y$, $y(1) = ?$

Soln :- $f(x,y) = x+y$, $h = 1/2$.

$$y_1 = 1 + \frac{1}{2}\times 1 = \frac{3}{2} = 1.5 \quad , \quad y_2 = 1 + \frac{1}{4}\left[1 + \frac{3}{2}\right] = 1 + \frac{1}{4}\times\frac{5}{2}$$

$$= 1 + \frac{5}{8} = \frac{13}{8}$$

DATE : / 20
PAGE No

$y_2 = y(x_2) = y(x+2h) = y(0+2\times\frac{1}{2}) = y(1)$

$\therefore y(1) = \frac{13}{8} = 1.6$

V.V.Imp 4. **RK-method :⇒** (Two stage explicite method)

(A) $y_{n+1} = y_n + w_1k_1 + w_2k_2$

where, $k_1 = hf(x_n, y_n)$, $k_2 = hf(x_n+\alpha h, y_n+\beta k_1)$

$w_1 + w_2 = 1$

$\alpha = \beta$, $w_2 = \frac{1}{2\alpha}$, $w_1 = 1 - \frac{1}{2\alpha}$

(B) I **RK-method of 1st order :-**

Same as Euler's method

II **RK-method of 2nd order :⇒**

$y_{n+1} = y_n + \frac{1}{4}(k_1 + 3k_2)$

$k_1 = hf(x_n, y_n)$, $k_2 = hf\left(x_n + \frac{2}{3}h, y_n + \frac{2}{3}k_1\right)$

III **RK-method of 3rd order :-**

$y_{n+1} = y_n + \frac{1}{8}(2k_1 + 3k_2 + 3k_3)$

$k_1 = hf(x_n, y_n)$, $k_2 = hf\left(x_n + \frac{2}{3}h, y_n + \frac{2}{3}k_1\right)$

$k_3 = hf\left(x_n + \frac{2}{3}h, y_n + \frac{2}{3}k_2\right)$

IV **RK-method of 4th order :-**

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where,

$k_1 = hf(x_n, y_n)$, $k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$

DATE : / 20
PAGE No

$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$, $k_4 = hf(x_n + h, y_n + k_3)$

2016 Ques :- consider the RK-method of the form

$y_{n+1} = y_n + ak_1 + bk_2$

$k_1 = hf(x_n, y_n)$, $k_2 = hf(x_n + \alpha h, y_n + \beta k)$

$y'(x,y) = f(x, y(x_0)) = y_0$

✓(A) $a = 1/2$, $b = 1/2$, $\alpha = 1$, $\beta = 1$

(B) $a = 1$, $b = 1$, $\alpha = 1/2$, $\beta = 1/2$

✓(C) $a = 1/4$, $b = 3/4$, $\alpha = 2/3$, $\beta = 2/3$

(D) $a = 3/4$, $b = 1/4$, $\alpha = 1$, $\beta = 1$

* **Numerical Integration :⇒**

1) Trapezoidal
2) Simpson 1/3
3) Simpson 3/8

\# length of sub-interval $= \frac{b-a}{n}$ (b → upper limit, a → lower limit, n → no. of subintervals)

1. **Trapezoidal rule**

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

8 subintervals.

$\int_a^b f(x)\,dx = \frac{h}{2}[(y_0+y_8) + 2(y_1+y_2+\ldots+y_7)]$

2. **Simpson 1/3 :⇒**

$\int_a^b f(x)\,dx = \frac{h}{3}[(y_0+y_8) + 2(y_2+y_4+y_6) + 4(y_1+y_3+y_5+y_7)]$

DATE : / 20
PAGE No

3 Simpson 3/8 :-

$$\int_a^b f(x)dx = \frac{3h}{8}\left[(y_0+y_8)+2(y_3+y_6)+3(y_1+y_2+y_4+y_5+y_7)\right]$$

Remarks :-

1) Trapezoidal rule is of order 1 and Simpson is of order 3 or we can say that Trapezoidal rule is exact for the poly. of degree ≤ 1 and Simpson is exact for the poly. of degree ≤ 3.

2) We can take any no of subintervals in Trapezoidal rule but in Simpson 1/3 there should be given no. of intervals. In Simpson 3/8 intervals should be multiple of 3.

3) Error in Trapezoidal rule :-

$$R_1 = \frac{-h^3}{12} f''(\eta) = -\frac{(b-a)^3}{12} f''(\eta) \quad \Big] \; h = b-a$$

$$R_2 = \frac{-h^5}{90} f^{IV}(\eta) = -\frac{(b-a)^5}{2880} f^{IV}(\eta) \quad \Big] \; h = \frac{b-a}{2}$$

$$R_3 = \frac{-3}{80} h^5 f^{IV}(\eta) \quad \Big] \; h = \frac{b-a}{3}$$

Imp

4) Trapezoidal rule :- $T = \frac{h}{2}[f(a)+f(b)]$

Simpson, $S_{1/3} = \frac{h}{3}[f_0+4f_1+f_2]$

$$= \frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

Simpson 3/8, $S_{3/8} = \frac{3h}{8}[f_0+3f_1+3f_2+f_3]$

DATE : / 20
PAGE No

ques :- Solve $\int_0^5 x^2\,dx$ by taking $n=5$.

Soln :- $h = \frac{5-0}{5} = 1$

$$T = \frac{1}{2}[(0+25)+2(1+4+9+16)] = \frac{1}{2}[25+2\times30]$$

$$= \frac{1}{2}[25+60] = \frac{85}{2} = 42.5$$

$$S_{1/3} = \frac{1}{3}[(0+25)+2(4+16)+4(1+9)] = \frac{1}{3}[25+40+40] = 35$$

$$S_{3/8} = \frac{3}{8}[25+2\times9+3(1+4+16)] = \frac{3}{8}[25+18+63]$$

$$= \frac{3\times106}{8_4} = \frac{159}{4}$$

ques :- use Euler's method for $u' = -2+u^2$, $u(0)=1$ on $[0,1]$ neglecting the round off errors. Determine the bound of errors.

Soln :- In Euler's method, $\boxed{\text{error} = \frac{h^2}{2}y''(\xi)}$

Convergence of Iterative method :-

1) A matrix A is said to be diagonally dominant if

$$|a_{ii}| \geq \sum_{\substack{j=1 \\ i\neq j}}^{n} |a_{ij}| \quad \text{---- (1)}$$

2) System $AX=B$ is diagonally dominant system if eqn (1) is satisfied and strict inequality is satisfied for atleast once i.

3) If coeff. matrix A is diagonally dominant then both Gauss Jacobi & Gauss Seidal converges for initial approx. This is the sufficient condition for the

convergence of both the methods.

4) If both the methods converge then Gauss Seidal method converges atleast two times faster than Jacobi method.

Imp 5) A necessary & sufficient condition for the convergence is $\rho(H) < 1$ where $\rho(H)$ is the spectral radius which means largest eigen value in modulus.

6) If iteration method converges then $r = -\log_{10}(\rho(H))$ is the rate of convergence for the iterative method.

* Set of eigen values → Spectrum.

Dec-2015 Ques:- $f(x) = \sqrt{x+3}$, $x \geq -3$. consider the iteration $x_{n+1} = f(x_n)$, $x_0 = 0$. The possible limits of iteration are

(A) -1 (B) 3 (C) 0 (D) $\sqrt{3+\sqrt{3+\sqrt{3+\dots}}}$

$x = \sqrt{3+x} = f(x)$

Ques:- The iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ for given $x_n \neq 0$ is an iteration of

1) fixed pt. iteration for $f(x) = x^2 - 2$
2) Newton method for $f(x) = x^2 - 2$
3) fixed pt. iteration for $f(x) = \frac{x^2+2}{2x}$
4) Newton method for $f(x) = x^2 + 2$

$x_n = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right) \Rightarrow 2x_n^2 = x_n^2 + 2 \Rightarrow x_n^2 = 2$

fixed pt., $x = x^2 - 2$ ×

$\frac{x^2+2}{2x} = x \Rightarrow x^2 + 2 = 2x^2 \Rightarrow x^2 = 2$.

Ques:- The following numerical formula is exact for all poly. of degree ≤ 3.

1) Simpson 1/3 2) Simpson 3/8
3) Trapezoidal 4) Gauss Legendre 2 pt. formula

Gauss Legendre 2 pt. formula is exact for poly. of degree ≤ 3.

? Ques:- Let $f: \mathbb{R} \to \mathbb{R}$ be a smooth fn with non vanishing derivative. The newton method for finding the root of $f(x) = 0$ is same as

1) fixed pt. iteration for the map $g(x) = x - \frac{f(x)}{f'(x)}$
2) Forward Euler method - unit step length for $\frac{dy}{dx} + \frac{f(y)}{f'(y)} = 0$
3) fixed pt. iteration for the map $g(x) = x + f(x)$
4) " " " " $g(x) = x - f(x)$

$x = x - \frac{f(x)}{f'(x)} \Rightarrow \frac{f(x)}{f'(x)} = 0 \Rightarrow \frac{(f'(x))^2 - f(x)f''(x)}{(f'(x))^2} = 0$

[* For Newton Raphson method, condition for convergence is $|f(x)f''(x)| < (f'(x))^2$]

$(f'(x))^2 - f(x)f''(x) = 0 \Rightarrow f(x)f''(x) = (f'(x))^2$

$\Rightarrow \left|\frac{f(x)f''(x)}{(f'(x))^2}\right| = 1$

$\Rightarrow |f(x)f''(x)| < (f'(x))^2$.

DATE : / 20
PAGE No

ques:- Legendre interpolation poly. passing through the pts. (1, 10), (2, −2), (3, 8)

1) $11x^2 - 45x + 38$
2) $11x^2 - 45x + 36$
3) $11x^2 - 45x + 30$
4) $11x^2 - 45x + 44$ ✓

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$y = 11x^2 - 45x + 38$

$10 = 11 - 45 + 38 \Rightarrow 10 = 49 - 45$ ✗

$10 = 11 - 45 + 36$ ✗

$10 = 11 - 45 + 30$ ✗

$10 = 11 - 45 + 44 \Rightarrow 10 = 11 - 1 \Rightarrow 10 = 10$ ✓

ques:- For an approx. computation of $1/\alpha$ based on Newton Raphson method where α is +ve real no. The f^n $f(x)$ in the iteration formula $x_{n+1} = f(x_n)$ has the form.

1) $x(1-\alpha x)$ 2) $x(2-\alpha x)$ 3) $x(1-\alpha)$ 4) $x(2+\alpha x)$

$x = x(1-\alpha x) \Rightarrow 1 - \alpha x = 1 \Rightarrow -\alpha x = 0$ ✗

$x = x(2-\alpha x) \Rightarrow 2 - \alpha x = 1 \Rightarrow -\alpha x = -1 \Rightarrow \alpha x = 1 \Rightarrow x = \frac{1}{\alpha}$

ques:- Let $P_n(x)$ be the Taylor Poly. of degree $n \geq 0$ for the f^n e^x about $x = 0$.

1) $\frac{x^n}{n!} e^{\xi}$, $0 < \xi < x$
2) $\frac{x^n}{(n+1)!} e^{\xi}$ 3) $\frac{x^{n+1}}{n!} e^{\xi}$ 4) $\frac{x^{n+1}}{(n+1)!} e^{\xi}$

$f(x) = f(\xi) + h f'(\xi) + \frac{h^2}{2!} f''(\xi) + \dots$

$e^x = e^{\xi} + h e^{\xi} + \frac{h^2}{2!} e^{\xi} + \dots + \frac{h^n}{n!} e^{\xi} + \frac{h^{n+1}}{(n+1)!} e^{\xi} + \dots$

DATE : / 20
PAGE No

→ for n degree poly. (n+1) term give error

ques:-

i :	0	1	2
x_i :	0	1	2
$f(x_i)$:	1	3	2

Find $f(0.5)$ using interpolation formula

(A) 1.234 (B) 1.832 (C) 2.301 (4) 2.375

Sol:- $u = \frac{x-a}{h} = \frac{x-0}{1} = \frac{0.5}{1} = \frac{1}{2}$

Using NG- forward formula :-

$f(x) = f(x_0) + u \Delta f(x_0) + \frac{u(u-1)}{2!} \Delta^2 f(x_0)$

$= 1 + \frac{1}{2} \times 2 + \frac{\frac{1}{2}\left(\frac{-1}{2}\right) \times (-3)}{2} = 2 + \frac{1}{8} \times 3 = 2 + \frac{3}{8}$

$= \frac{16+3}{8} = \frac{19}{8} = 2.375$

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ques:- The min. no. of iteration required to solve $x^3 - 4x + 10 = 0$ using bisection method with accuracy 10^{-3} is

(a) 12 (b) 10 (c) 9 (d) 15

ques:- $f_0'' = \frac{1}{h^2}(f_1 - 2f_0 + f_1) + e(x_j)$

error term is given by

1) $h f'(x_j)/2$ 2) $-h^2 f''(x_j)/6$
3) $h^3 f^{IV}(x_j)/12$ 4) $h^2 f^{IV}(x_j)/12$

DATE : / 20
PAGE No

NA (Booklet)

DATE : / 20
PAGE No

Dec-2015

Ques:45 :- $f(x) = ax + 100$ for $a \in \mathbb{R}$, $x_{n+1} = f(x_n)$, $n \geq 0$, $x_0 = 0$. converges for.

1) $a = 5$ 2) $a = 1$ 3) $a = 0.1$ 4) $a = 10$

$f(x) = ax + 100$

$f'(x) = a$

For convergence $|f'(x)| < 1$ i.e. $a < 1$ So $a = 0.1$.

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(96) $f(x) = \sqrt{x+3}$, $x \geq -3$ $x_{n+1} = f(x_n)$, $x_0 = 0$, $n \geq 0$.

possible limits of iteration :- ?

check by opt. $\sqrt{3+\sqrt{3+\sqrt{3+}}}$

let $x = \sqrt{3+\sqrt{3+\sqrt{3+}}}$

So $x = \sqrt{3+x} = f(x)$.

So Ans :- opt. (4) $\sqrt{3+\sqrt{3+\sqrt{3+}}}$

(101) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right) \Rightarrow x_{n+1} = \frac{1}{2}\left(\frac{x_n^2 + 2}{x_n}\right)$

By NR method, $x_{n+1} = x_n - f(x)$

let $x_n \to \alpha$, $x_{n+1} \to \alpha$.

$f(x) = x^2 - 2$, By NR-method $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$\Rightarrow x_{n+1} = x_n - \frac{x_n^2 - 2}{2x_n} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$ opt.(2)

opt.1 :- let $x = x^2 - 2$ X

opt.3. $x = \frac{x^2+2}{2x} \Rightarrow 2x^2 = x^2 + 2 \Rightarrow x^2 - 2 = 0$

$\Rightarrow f(x) = x^2 - 2 = 0$ ✓

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June-2014

Ques:44 $f : \mathbb{R} \to \mathbb{R}$, $f(x) = a_0 + a_1 x + a_2 x^2$.

$E_1 = \int_{-1}^{1} f(x)\,dx = [f(-1) + f(1)]$

$= \left[a_0 x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3}\right]_{-1}^{1} - [a_0 - a_1 + a_2 + a_0 + a_1 + a_2]$

DATE : / 20
PAGE No

$= a_0 + \frac{a_1}{2} + \frac{a_2}{3} + a_0 - \frac{a_1}{2} + \frac{a_2}{3} - 2a_0 + \frac{2a_2}{3}$

$= \frac{2a_1}{3} - 2a_2 - \frac{-4a_2}{3}$

$E_2 = \int_{-1}^{1} f(x)dx - \frac{1}{2}(f(-1) + 2f(0) + f(1))$

$= a_0 + \frac{a_1}{2} + \frac{a_2}{3} + a_0 - \frac{a_1}{2} + \frac{a_2}{3} - \frac{1}{2}[a_0 - a_1 + a_2 + 2a_0 + a_0 + a_1 + a_2]$

$= 2a_0 + \frac{2a_2}{3} - \frac{1}{2}[4a_0 + 2a_2] = 2a_0 + \frac{2a_2}{3} - 2a_0 - a_2$

$= -\frac{a_2}{3}$

$\therefore |E_1| = \left|\frac{4a_2}{3}\right| = 4\left|\frac{a_2}{3}\right| = 4|E_2|$

(101) opt. (2), (3), (4) (direct result)

Dec-2014 Ques:- 95

(101) $f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \dots$ (Taylor series)

opt. (i) $f'(x) = \frac{f(x+h) - f(x)}{h} = f'(x) + \frac{h}{2}f''(x) + \frac{h^2}{3!}f'''(x) + \dots$

X.

(Here 2nd term is of order 1)

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DATE : / 20
PAGE No

(101) opt (ii) X.

(iii) $f'(x) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

$= \frac{3f(x) - 4\left[f(x) - hf'(x) + \frac{h^2}{2!}f''(x) - \dots\right] + f(x) - 2hf'(x) + \frac{4h^2}{2!}f''(x) - \frac{8h^3}{3!}f'''(x) + \dots}{2h}$

$= \frac{3f(x) - 4f(x) + f(x) + 2hf'(x) + 4hf'(x) - 2hf'(x) + \frac{4h^2}{2!}f''(x) - \frac{4h^2}{2!}f''(x) + \frac{8h^3}{3!}f'''(x) - \frac{8h^3}{3!}f'''(x) + \dots}{2h}$

$= \frac{2hf'(x) - \frac{4h^3}{3!}f'''(x) + \dots}{2h} = f'(x) - \underbrace{\frac{2h^2}{3!}f'''(x) + \dots}_{\text{error term } O(h^2)}$

(iv) $f'(x) = \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h}$

$= \frac{-3f(x) + 4f(x) + 4hf'(x) + \frac{4h^2}{2!}f''(x) + \dots - f(x) - 2hf'(x) - \frac{4h^2}{2!}f''(x) - \frac{8h^3}{3!}f'''(x) + \dots}{2h}$

$= \frac{2hf'(x) - \frac{4h^3}{3!}f'''(x)}{2h} = f'(x) - \frac{2h^2}{3!}f'''(x) + \dots$

(iii), (iv)

June-2014 Ques:- $f(x) = ax + b$, $x_{n+1} = f(x_n)$

$f'(x) = a$

for convergence $|f'(x)| < 1$ i.e. $a < 1$.

(c) for $a \in [0, 1)$

DATE : / 20
PAGE No

(95) $g(x) = \sqrt{2+x}$, $x \geq -2$, $x_{n+1} = g(x_n)$, $x_0 = 1$

limits of iteration = ?

let $\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}} = x$, $\sqrt{2+x} < 2 \ \forall x$

$x_1 = g(x_0) = \sqrt{2+1} < 2$

$\Rightarrow x = \sqrt{2+x} = g(x)$ ... $x_2 = g(x_1) < 2$

Ans :- (a) $\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}$ (c) 2.

(98) Given iteration $x_{n+1} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$, $n \geq 0$, $x_0 \neq 0$

$x = \sqrt{2} \Rightarrow x^2 = 2$. $f(x) = x^2 - 2 = 0$.

By NR method, $x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 2}{2x_n}$

$= \frac{x_n^2 + 2}{2x_n} = \frac{1}{2}x_n + \frac{1}{x_n} = \frac{1}{2}\left(x_n + \frac{2}{x_n}\right)$

Thus the given iter. formula converges to $\sqrt{2}$

Since ROC for NR method is 2 so its ROC = 2

& this iter. is the newton's method for $x^2 - 2$

Ans :- (b) & (d)

Dec. 2012

Q105 :- 95 :- $g(x) = e^x$ is approx. by Taylor's poly. of degreen at the pt. $x = \frac{1}{2}$ on $[0, 1]$

Absolute error $\leq 10^{-2}$. $n = ?$

(100) $\int_0^1 f(x)\,dx = \sum_{i=1}^{n} f(m_i)\,h$

$= h f(m_1) + h f(m_2) + \cdots + h f(m_n)$

$= h[f(m_1) + f(m_2) + \cdots + f(m_n)]$

$= h\left[f\left(\frac{x_0+x_1}{2}\right) + f\left(\frac{x_1+x_2}{2}\right) + \cdots + f\left(\frac{x_n+x_{n+1}}{2}\right)\right]$

m_i = midpt. of $[x_{i-1}, x_i]$

DATE : / 20
PAGE No

(101) $x_{-1} = x_0 - h$, $x_1 = x_0 + h$, $h > 0$, $f(x_0) = f_0$

$f(x_j) = f_j$ for $j = -1, 1$, $f'(x_0) = f_0'$

$f_1 = f(x_1) = f(x_0+h) = f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \cdots$

$f_{-1} = f(x_{-1}) = f(x_0 - h) = f(x_0) - h f'(x_0) + \frac{h^2}{2!} f''(x_0) - \cdots$ (By Taylor series)

$\frac{f_1 - f_{-1}}{2h} = \frac{2hf'(x_0) + \frac{2h^3}{3!} f'''(x_0) + \cdots}{2h}$

$= f'(x_0) + \frac{h^2}{3!} f'''(x_0) + \cdots$

$f'(x_0) = \frac{f_1 - f_{-1}}{2h} - \frac{h^2}{6} f'''(x_0)$

Ans :- (C)

June 2013

Q102 :- 116 :- (a) $w_1 = 1/2 \neq w_2 = 1/2$, $\alpha = 1, \beta = 1$

$\alpha = \beta = 1$, $w_1 + w_2 = \frac{1}{2} + \frac{1}{2} = 1$, $w_2 = \frac{1}{2\alpha} = \frac{1}{2}$, $w_1 = 1 - \frac{1}{2\alpha} = \frac{1}{2}$ ✓

(b) $w_1 + w_2 = 2 + 1 \neq 1$ ×

(c) $w_1 + w_2 = \frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$, $\alpha = \beta = \frac{3}{4}$

$w_2 = \frac{1}{2\alpha} = \frac{1}{2 \times \frac{3}{4}} = \frac{2}{3}$, $w_1 = 1 - \frac{1}{2\alpha} = 1 - \frac{2}{3} = \frac{1}{3}$ ✓

(d) $w_1 + w_2 = \frac{3}{4} + \frac{1}{4} = 1$, $\alpha \neq \beta \neq \frac{3}{4}$, $w_2 = \frac{1}{2\alpha} = \frac{1}{2 \times 2} = \frac{1}{4}$

$w_1 = 1 - \frac{1}{4} = \frac{3}{4}$ ✓

Ans :- (a), (c), (d).

DATE : / 20
PAGE No

Dec-2012

Ques:-95. (a), (b), (c), (d)

(a) $x = \frac{x^2}{4} \Rightarrow x^2 - 4x \Rightarrow x^2 - 4x = 0$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 4x_n}{2x_n - 4} = \frac{x_n^2}{2(x_n - 2)}$ defined on $[a, 1]$.

Not by (b), (c), (d).

(100) $g(u) = u^3 - u - 1 = 0$

(a) $\Rightarrow u = u^3 - 1 = g(u)$

$g'(u) = 3u^2$

For convergence, $|g'(u)| < 1$ & $|3u^2| \not< 1$. ✗

(b) $g(u) = \sqrt{1+u^3}$

$g'(u) = \frac{1}{2\sqrt{1+u^3}} \cdot 3u^2$

$|g'(u)| = \left|\frac{3u^2}{2\sqrt{1+u^3}}\right| = \left|\frac{3(1.5)^2}{2\sqrt{1+(1.5)^3}}\right|$ [Put $u_0 = 1.5$]

$= \frac{3 \times 2.25}{2\sqrt{1+3.375}} = \frac{6.75}{2\sqrt{4.375}} = \frac{3.375}{\sqrt{4.375}} = \frac{3.375}{0.6615} = 5.10 \not< 1$

(c) $f(u) = 0 \Rightarrow u^3 - u - 1 = 0$

$u = 1.325$ (approx.)

(d) $f(0) = 0 - 0 - 1 = -1$, $f(1) = 1 - 1 - 1 = -ve$, $f(2) = 8 - 2 - 1 = +ve$

$\Rightarrow$ Root lies b/w 1 & 2.

(101) $p(t_i) = e^{t_i}$, $i = 1, 2, 3$

$t_1 = 0$, $t_2 = 0.5$, $t_3 = 1$

$p(t_1) = p(0) = e^0 = 1$

$p(t_2) = p(0.5) = e^{0.5} = 1.6487$, $p(t_3) = p(1) = e^1 = 2.7183$

x	y
0	1
0.5	1.6487 = 1.65
1	2.7183 = 2.72

or

x	y
0	1
0.5	$e^{1/2}$
1	e

DATE : / 20
PAGE No

By Lagrange's interpolation formula:-

$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$

$\therefore p(t) = \frac{(t-0.5)(t-1)}{(-0.5)(-1)} \times 1 + \frac{(t-0)(t-1)}{(0.5)(-0.5)} \times e^{1/2} + \frac{(t-0)(t-0.5)}{2 \times 1.5} e$

$= \frac{t^2 - 1.5t + 0.5}{0.5} - \frac{t^2 - t}{0.25} e^{1/2} + \frac{t^2 - 0.5t}{3} e$

$= L_1(t) + e^{1/2} L_2(t) + e L_3(t)$

Ans:- (a), (b), (d)

June-2012

Ques:-98 $f(x) = x^2 - x - 2$

$x = x^2 - 2$

or $x^2 = x + 2 \Rightarrow x \times x = x + 2 \Rightarrow x = \frac{x+2}{x} = 1 + \frac{2}{x}$

i.e. $x = 1 + \frac{2}{x}$

Ans:- (b)

(a) $x = g(x) \Rightarrow x = x - \frac{x^2 - x - 2}{m}$, $m \in [-a, a]$

$\Rightarrow \frac{-(x^2 - x - 2)}{m} = 0 \Rightarrow x^2 - x - 2 = 0$ ✗ a+m=0 not defined

(c) $x = x - \frac{x^2 - x - 2}{K}$, $K \neq 0$, $K \in \mathbb{R}$

$\frac{-(x^2 - x - 2)}{K} = 0 \Rightarrow x^2 - x - 2 = 0$ ✓

Ans:- (a), (b), (c)

Dec-2011

Ques:-96

x	1	1/2	3
y	3	-10	2

By Lagrange's interpolation formula,

DATE : / 20
PAGE No

$$f(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} f(x_2)$$

$$= \frac{(x-1/2)(x-3)}{(1-\frac{1}{2})(1-3)} \times 3 + \frac{(x-1)(x-3)}{(\frac{1}{2}-1)(\frac{1}{2}-3)}(-10) + \frac{(x-1)(x-1/2)}{(3-1)(3-1/2)} \times 2$$

$$= 3\frac{(x-1/2)(x-3)}{\frac{1}{2}x-2} - \frac{10}{-\frac{1}{2}}\frac{(x-1)(x-3)}{x-\frac{5}{2}} + \frac{2(x-1)(x-1/2)}{2\times\frac{5}{2}x}$$

$$= -3\left(x-\frac{1}{2}\right)(x-3) - \frac{2}{5}(x-1)(x-3) + \frac{2}{5}(x-1)\left(x-\frac{1}{2}\right)$$

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Ans:- (a)

(97) $A = \begin{pmatrix} 1.01 & 0.99 \\ 0.99 & 1.01 \end{pmatrix}$, $b = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$AX = B$, $x_1 = \begin{pmatrix} 1.01 \\ 1.01 \end{pmatrix}$, $x_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$

$X = A^{-1}B$.

$$A^{-1} = \frac{1}{0.04}\begin{bmatrix} 1.01 & -0.99 \\ -0.99 & 1.01 \end{bmatrix} = \begin{bmatrix} 25.25 & -24.75 \\ -24.75 & 25.25 \end{bmatrix}$$

$$x_0 = A^{-1}B = \begin{bmatrix} 25.25 & -24.75 \\ -24.75 & 25.25 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\therefore$ x_0 is the unique soln of the system $AX = B$.

Now $x_1 = Ax_0 - Ax_1 = b - Ax_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{bmatrix} 1.01 & 0.99 \\ 0.99 & 1.01 \end{bmatrix}\begin{bmatrix} 1.01 \\ 1.01 \end{bmatrix}$

$$= \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 2.02 \\ 2.02 \end{bmatrix} = \begin{bmatrix} -0.01 \\ -0.01 \end{bmatrix} = -\begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}$$

$$x_2 = b - Ax_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 1.01 & 0.99 \\ 0.99 & 1.01 \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 2.02 \\ 1.98 \end{bmatrix}$$

$$= \begin{bmatrix} -0.01 \\ 0.02 \end{bmatrix}$$

Ans:- (a), (d)

DATE : / 20
PAGE No

June-2011

Ques:-98 $g(x) = x - \frac{f(x)}{f'(x)}$

iter. formula for square root of 2, $x_{n+1} = \frac{x_n^2+2}{2x_n}$

$\therefore$ $x_1 = \frac{x_0^2+2}{2x_0} = \frac{2^2+2}{2\times 2}$ [Given $x_0 = 2$]

$= \frac{4+2}{4} = \frac{6}{4}\frac{3}{2} = \frac{3}{2} = 1.5$

$x_2 = \frac{x_1^2+2}{2x_1} = \frac{(1.5)^2+2}{2\times 1.5} = \frac{2.25+2}{3} = \frac{4.25}{3} = 1.416$

Ans:- (b), (c)

Sample paper-2011

Ques:-44 :- $\int_0^4 (2x^3 - 3x^2 + 5x + 1)\,dx$

$f(x) = 2x^3 - 3x^2 + 5x + 1$, let $h = 1$, $\left(h = \frac{b-a}{n} \Rightarrow n = \frac{b-a}{h} = \frac{4}{1} = 4.\right)$

$y_0 = f(x_0) = f(0) = 1$, $y_1 = f(x_1) = f(x_0+h) = f(1) = 5$

$y_2 = f(x_2) = f(x_0+2h) = f(2) = 16 - 12 + 10 + 1 = 15$

$y_3 = f(x_3) = f(x_0+3h) = f(3) = 54 - 27 + 15 + 1 = 43$

$y_4 = f(x_4) = f(x_0+4h) = f(4) = 128 - 48 + 20 + 1 = 101$

By Trapezoidal rule, $\int_0^4 f(x)dx = \frac{h}{2}\left[[f(x_0)+f(x_4)] + 2[f(x_1)+f(x_2)+f(x_3)]\right]$

$= \frac{1}{2}\left[(1+101) + 2(5+15+43)\right] = \frac{1}{2}\left[102 + 2\times 63\right]$

$= \frac{1}{2}[102+126) = \frac{228}{2} = 114$

By Simpson 1/3, $\int_0^4 f(x)dx = \frac{h}{3}\left[(y_0+y_4) + 2(y_2) + 4(y_1+y_3)\right]$

$= \frac{1}{3}\left[(1+101) + 2\times 15 + 4(5+43)\right]$.

$= \frac{1}{3}[102 + 30 + 4\times 48] = \frac{1}{3}[132 + 192] = \frac{1}{3}\times\overset{108}{324} = 108$

$$\int_0^4 (2x^3 - 3x^2 + 5x + 1)\,dx = \left[\frac{2x^4}{4\times 2} - \frac{3x^3}{3} + \frac{5x^2}{2} + x\right]_0^4$$

$$= 128 - 64 + \frac{5\times 16}{2} + 4 = 128 - 64 + 40 = 76$$

Ans:- (b)

Dec-2010

Ques:- (78) $f(x) = x^3 + 4x - 10$, $\varepsilon = 10^{-3}$

Ans:- (2) 10

(79) $f(x) = \cos x$

$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \dots$

$f(0+h) = f(h) = f(0) + h f'(0) + \frac{h^2}{2!} f''(0) + \frac{h^3}{3!} f'''(0) + \dots$

$h \to x$

$f(x) = 1 + x(-\sin 0) + \frac{x^2}{2!}(-\cos 0) + \frac{x^3}{3!}(\sin 0) + \frac{x^4}{4!}(\cos 0) + \frac{x^5}{5!}(-\sin 0) + \dots$

$\Rightarrow \cos x = 1 + 0 - \frac{x^2}{2!} + 0 + \frac{x^4}{4!} + 0 - \frac{x^6}{6!} + \dots$

$\Rightarrow \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

Third degree poly with zero = 0 is

$\cos x + \frac{x^2}{2} = 1 + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

or $\cos x + \frac{1}{2}x^2 = 1 + O(x^4)$.

Ans:- (3)

June-2010

Ques:- (85)

i	0	1	2
x_i	0	1	2
$f(x_i)$	1	3	2

$f(0.5) = ?$

By Newton's forward interpolation formula, we have

$f(x) = f(x_0) + u\Delta f(x_0) + \frac{u(u-1)}{2!}\Delta^2 f(x_0) + \dots$ ——— (1)

x_i	$f(x_i)$	$\Delta f(x_i)$	$\Delta^2 f(x_i)$
0	(1)	(2)	
1	3		(-3)
2	2	-1	

$u = \frac{x - a}{h} = \frac{0.5 - 0}{1} = 0.5$

$\therefore$ From (1), $f(x) = 1 + 0.5(2) + \frac{(0.5)(-0.5)}{2}(-3)$

$= 1 + 1 + 0.375 = 2.375$.

Ans:- (4)

(86) $f_i = f(x_i)$, $f_i' = f'(x_i)$, $x_i = x_0 + ih$, $i = 0, \pm 1$

$f_0'' = \frac{1}{h^2}(f_{-1} - 2f_0 + f_1) + e(\xi)$ ——— (1), $e(\xi) = ?$

$f_{-1} = f(x_{-1}) = f(x_0 - h) = f(x_0) - h f'(x_0) + \frac{h^2}{2!} f''(x_0) - \dots$

$2f_0 = 2f(x_0)$

$f_1 = f(x_0 + h) = f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \frac{h^3}{3!} f'''(x_0) + \dots$

$f_{-1} + 2f_0 + f_1 = f(x_0) - h f'(x_0) + \frac{h^2}{2!} f''(x_0) - \frac{h^3}{3!} f'''(x_0) + \dots - 2f(x_0) + f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \dots$

$= \frac{2h^2}{2!} f''(x_0) + \frac{2h^4}{4!} f^{IV}(x_0) + \dots$

$\frac{f_{-1} + 2f_0 + f_1}{h^2} = f''(x_0) + \frac{2}{4\times 3\times 2} h^2 f^{IV}(x_0) + \frac{2h^4}{6!} f^{VI}(x_0) + \dots$

$\Rightarrow f''(x_0) = \frac{1}{h^2}(f_{-1} - 2f_0 + f_1) - \frac{h^2}{12} f^{IV}(x_0)$

comparing it with eqn (1), we get $e(\xi) = -\frac{h^2}{12} f^{IV}(\xi)$

Ans:- (4)

Dec-2009

Ques:-93 $f(x) = e^x$

$f(x_0+h) = f(\frac{1}{2}+h) = f(\frac{1}{2}) + h f'(\frac{1}{2}) + \frac{h^2}{2!} f''(\frac{1}{2}) + \dots$

$= e^{1/2} + h e^{1/2} + \frac{h^2}{2!} e^{1/2} + \dots$

$= e^{1/2}\left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \dots\right)$

Ans:- (3)

June-2009

Ques:-46 $f(x) = e^x$

Ans:- (4)

(74) $x = x(1-\alpha x) \Rightarrow \alpha = 0$ ✗

$x = x(2-\alpha x) \Rightarrow 2 - \alpha x = 1 \Rightarrow \alpha x = 1 \Rightarrow x = 1/\alpha$ ✓

$x = x(1-\alpha) \Rightarrow 1 - \alpha = 0 \Rightarrow \alpha = 1$ ✗

$x = x(2+\alpha x) \Rightarrow 2 + \alpha x = 1 \Rightarrow \alpha x = -1 \Rightarrow x = -1/\alpha$ ✗

Model Paper

Ques:-2009 (85) pts (1,10), (2,-2), (3,8)

(1) $11x^2 - 45x + 38 = y$

$11 - 45 + 38 = 10 \Rightarrow 49 - 45 \neq 10$ ✗

(2) $11x^2 - 45x + 36 = y$

$11 - 45 + 36 \neq 10$ ✗

(3) $11x^2 - 45x + 30 = y \Rightarrow 11 - 45 + 30 = 10$ ✗

(4) $11x^2 - 45x + 44 = y$

$11 - 45 + 44 = 10$ ✓

Ans:- (4)

(86) Square root of a:- $x = \sqrt{a} \Rightarrow x^2 = a$

$f(x) = x^2 - a = 0$

By NR-method, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - a}{2x_n}$

$= \frac{1}{2} x_n + \frac{a}{2x_n} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$

Ans:- (2)

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