

SUBJECT : MATHEMATICS

Topic Name: LCD and Differentiation

HINTS & SOLUTIONS

1.

Sol. Putting $x^2 = \cos \theta$, we get

$$\begin{aligned}
 y &= \tan^{-1} \left(\frac{\sqrt{1+\cos \theta} + \sqrt{1-\cos \theta}}{\sqrt{1+\cos \theta} - \sqrt{1-\cos \theta}} \right) \\
 &= \tan^{-1} \left(\frac{\cos \frac{\theta}{2} + \sin \frac{\theta}{2}}{\cos \frac{\theta}{2} - \sin \frac{\theta}{2}} \right) = \tan^{-1} \left(\frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} \right) \\
 y &= \frac{\pi}{4} + \frac{\theta}{2} = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2 \\
 \frac{dy}{dx} &= -\frac{1}{2} \frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{x}{\sqrt{1-x^4}}
 \end{aligned}$$

2.

Sol. Given, $f(x) = \frac{x-1}{x(x^2-1)}$

We can write the function as,

$$\Rightarrow f(x) = \frac{x-1}{x(x-1)(x+1)}$$

Here, the function is discontinuous if,

$$x(x-1)(x+1) = 0$$

$$x = 0 \text{ or } x - 1 = 0 \text{ or } x + 1 = 0$$

$$x = 0 \text{ or } x = 1 \text{ or } x = -1$$

Therefore, the function is discontinuous exactly at three points.

The correct option is C.

3.

Sol. For, $x > 1$, the value of $f(x)$, such that,

$$f(x)_{x \rightarrow 1^+} = 2$$

For, $x \leq 1$, the value of $f(x)$, such that,

$$f(x)_{x \rightarrow 1^+} = k(1)^2 + 5$$

$$f(x)_{x \rightarrow 1^+} = k + 5$$

For the function to be continuous

$$f(x)_{x \rightarrow 1^+} = f(x)_{x \rightarrow 1^-}$$

$$\Rightarrow 2 = k + 5$$

$$\Rightarrow k = -3$$

4.

Sol. We need to find

$$\frac{d(\sec^2(x^2))}{dx^2}$$

Let $x^2 = t$

$$\text{So, } \frac{d(\sec^2(t))}{dt}$$

$$= 2 \cdot \sec t \cdot (\sec t)'$$

$$= 2 \cdot \sec t \cdot \sec t \cdot \tan t$$

$$= 2 \cdot \sec^2 t \cdot \tan t$$

Putting $t = x^2$

$$= 2 \cdot \sec^2 x^2 \cdot \tan x^2$$

OrGiven, $y = f(x^2)$ Differentiating both sides w.r.t x , we get,

$$= \frac{dy}{dx} = f'(x^2) \cdot 2x$$

Also, given, $f'(x) = e^{\sqrt{x}}$.

$$f'(x^2) = e^{\sqrt{x^2}} = e^x$$

$$\therefore \frac{dy}{dx} = 2xe^x.$$

5.

Sol. Given, $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$ Using, $\sec x = \frac{1}{\cos x}$.

$$f(x) = \sqrt{\frac{\frac{1}{\cos x} - 1}{\frac{1}{\cos x} + 1}}$$

$$f(x) = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

$$f(x) = \sqrt{\frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}}}$$

Since, $1 - \cos x = 2 \sin^2 \frac{x}{2}$ and $1 + \cos x = 2 \cos^2 \frac{x}{2}$. So, we get,

$$f(x) = \frac{\sin \frac{x}{2}}{\cos \frac{x}{2}} = \tan \frac{x}{2}$$

Differentiating both sides w.r.t x , we get,

$$f'(x) = \frac{d}{dx} \left(\tan \frac{x}{2} \right)$$

$$f'(x) = \sec^2 \frac{x}{2} \cdot \frac{1}{2}$$

$$f'(x) = \frac{1}{2} \sec^2 \frac{x}{2}$$

$$\text{Therefore, } f' \left(\frac{\pi}{3} \right) = \frac{1}{2} \sec^2 \left(\frac{\pi}{2.3} \right)$$

$$\Rightarrow f' \left(\frac{\pi}{3} \right) = \frac{1}{2} \sec^2 \left(\frac{\pi}{6} \right)$$

$$\Rightarrow f' \left(\frac{\pi}{3} \right) = \frac{1}{2} \left(\frac{2}{\sqrt{3}} \right)^2$$

$$\Rightarrow f' \left(\frac{\pi}{3} \right) = \frac{2}{3}$$

Or

$$\text{Given, } f(x) = (\tan x)^{\tan x}$$

$$\text{Let, } f(x) = (\tan x)^{\tan x} = y$$

Taking log on both sides, we get,

$$\log y = \log(\tan x^{\tan x})$$

$$\Rightarrow \log y = \tan x \log(\tan x)$$

Differentiating both sides w.r.t x , we get,

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \tan x \cdot \frac{1}{\tan x} \cdot \sec^2 x + \log(\tan x) \cdot \sec^2 x$$

$$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \sec^2 x + \sec^2 x \log(\tan x)$$

$$\Rightarrow \frac{dy}{dx} = y [\sec^2 x + \sec^2 x \log(\tan x)]$$

$$\text{Since, } y = \tan x^{\tan x}.$$

$$\text{Therefore, } f'(x) = \tan x^{\tan x} \sec^2 x [1 + \log(\tan x)].$$

6.

$$\text{Sol. Given, } \tan^{-1} \left(\frac{y}{x} \right) = \log \sqrt{x^2 + y^2}$$

$$\Rightarrow \tan^{-1} \left(\frac{y}{x} \right) = \frac{1}{2} \log(x^2 + y^2)$$

Now, differentiating both sides w.r.t x ,

$$\Rightarrow \frac{1}{1 + \left(\frac{y}{x}\right)^2} \frac{d}{dx} \left(\frac{y}{x}\right) = \frac{1}{2} \cdot \frac{1}{x^2 + y^2} \cdot \frac{d}{dx} (x^2 + y^2)$$

$$\Rightarrow \frac{x^2}{x^2 + y^2} \cdot \left(\frac{x \frac{dy}{dx} - y}{x^2} \right) = \frac{1}{2} \cdot \frac{2 \left(x + y \frac{dy}{dx} \right)}{x^2 + y^2}$$

$$\Rightarrow x + y \frac{dy}{dx} = x \frac{dy}{dx} - y$$

$$\Rightarrow x \frac{dy}{dx} - y \frac{dy}{dx} = x + y$$

$$\Rightarrow \frac{dy}{dx} (x - y) = (x + y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{x + y}{x - y}$$

Hence, proved.

Or

Given curve is $y = e^{a \cos^{-1} x}$

Differentiating given curve,

$$y' = e^{a \cos^{-1} x} \cdot \frac{d}{dx} (a \cos^{-1} x)$$

$$y' = e^{a \cos^{-1} x} \cdot \frac{(-a)}{\sqrt{1-x^2}}$$

$$\Rightarrow y' = \frac{-ay}{\sqrt{1-x^2}} \quad \dots(1)$$

$$\left[\therefore y = e^{a \cos^{-1} x} \right]$$

On differentiating above equation again w.r.t x , we get

$$\Rightarrow y'' = \frac{-a \left(-ae^{a \cos^{-1} x} + \frac{x \cdot e^{a \cos^{-1} x}}{\sqrt{1-x^2}} \right)}{(1-x^2)}$$

$$\Rightarrow (1-x^2) y'' = -a \left(-ae^{a \cos^{-1} x} + \frac{x \cdot e^{a \cos^{-1} x}}{\sqrt{1-x^2}} \right)$$

$$\Rightarrow (1-x^2) y'' = a^2 e^{a \cos^{-1} x} - \frac{a \cdot e^{a \cos^{-1} x}}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2) y'' = a^2 y + xy'$$

[from (1)]

$$\Rightarrow (1-x^2) y'' - xy' - a^2 y = 0$$

$$\Rightarrow (1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$$

Hence, proved.

7.

Sol. $y = \sin^{-1} x + \cos^{-1} x$

$$\begin{aligned}\Rightarrow \frac{dy}{dx} &= \frac{d}{dx}(\sin^{-1} x + \cos^{-1} x) \\ &= \frac{d}{dx}(\sin^{-1} x) + \frac{d}{dx}(\cos^{-1} x) \\ &= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \\ &= 0\end{aligned}$$

Therefore, $\frac{dy}{dx} = 0$

8.

Sol. $(\sin x)^y = x + y$

Taking log both side we get,

$$\log(\sin x)^y = \log(x + y)$$

$$\Rightarrow y \log(\sin x) = \log(x + y) \quad \dots\dots(i)$$

Differentiate on both side w.r.t. x we get,

$$\log(\sin x) \cdot \frac{dy}{dx} + y \cdot \frac{d}{dx}[\log(\sin x)] = \frac{d}{dx}[\log(x + y)]$$

$$\Rightarrow \log(\sin x) \cdot \frac{dy}{dx} + y \cdot \frac{\cos x}{\sin x} = \frac{1}{(x + y)} \cdot \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} \left[\log(\sin x) - \frac{1}{(x + y)} \right] = \frac{1}{(x + y)} - y \cot x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - (xy + y^2) \cot x}{(x + y) \log(\sin x) - 1}$$

9.

Sol. $y = (\sec^{-1} x), x > 0$

$$\Rightarrow \frac{dy}{dx} = \sec^{-1} x \cdot \frac{d(\sec^{-1} x)}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \sec^{-1} x \cdot \frac{1}{x\sqrt{x^2 - 1}} \quad \dots\dots(i)$$

$$\Rightarrow \frac{d^2 y}{dx^2} = 2 \left[\frac{1}{x^2(x^2-1)} \right] + 2 \sec^{-1} x \left[\frac{-\sqrt{x^2-1} - x \left(\frac{2x}{2\sqrt{x^2-1}} \right)}{x^2(x^2-1)} \right]$$

$$\Rightarrow \frac{d^2 y}{dx^2} = 2 \left[\frac{1}{x^2(x^2-1)} \right] + 2 \sec^{-1} x \frac{1}{x\sqrt{x^2-1}} \left[\frac{x(1-2x^2)}{x^2(x^2-1)} \right] \dots\dots\dots(ii)$$

$$\frac{d^2 y}{dx^2} = 2 \left[\frac{1}{x^2(x^2-1)} \right] + \frac{dy}{dx} \left[\frac{x(1-2x^2)}{x^2(x^2-1)} \right]$$

$$\Rightarrow x^2(x^2-1) \frac{d^2 y}{dx^2} + (2x^3 - x) \frac{dy}{dx} - 2 = 0$$

10.

Sol. Let $y = \tan^{-1} \left(\frac{1 + \cos x}{\sin x} \right)$

$$\therefore y = \tan^{-1} \left(\frac{2 \cos^2 x/2}{2 \sin x/2 \cos x/2} \right)$$

$$y = \tan^{-1} \left(\cot x/2 \right)$$

$$y = \tan^{-1} \left(\tan \left(\frac{\pi}{2} - \frac{x}{2} \right) \right)$$

$$\therefore y = \frac{\pi}{2} - \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\pi}{2} - \frac{x}{2} \right) = -\frac{1}{2}$$

11.

Sol. $Y = \sin(\sin x) \dots\dots\dots(1)$

$$\frac{dy}{dx} = \cos(\sin x) \times \cos x \Rightarrow \frac{dy}{\cos x} = \cos(\sin x) \dots\dots\dots(2)$$

$$\frac{d^2 y}{dx^2} = -\cos(\sin x) \times \sin x - \cos x \sin(\sin x) \cos x \dots\dots\dots(3)$$

Put (1) and (2) in (3)

$$\frac{d^2 y}{dx^2} = - \left(\frac{dy}{\cos x} \right) \times \sin x - y \cos^2 x$$

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} \tan x + y \cos^2 x = 0$$

12:

Sol. Given:

$$(x^2 + y^2)^2 = xy$$

$$x^4 + y^4 + 2x^2y^2 = xy$$

Diff. w.r.t. x we get,

$$4x^3 + 4y^3 \frac{dy}{dx} + 2 \left(2x^2y \frac{dy}{dx} + 2xy^2 \right) = \left(x \frac{dy}{dx} + y \right)$$

$$4y^3 \frac{dy}{dx} + 4x^2y \frac{dy}{dx} - x \frac{dy}{dx} = y - 4x^3 - 4xy^2$$

$$\frac{dy}{dx} (4y^2 + 4x^2y - x) = y - 4x^3 - 4xy^2$$

$$\frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4y^2 + 4x^2y - x}$$

OR

$$y = a(1 - \cos 2\theta), \quad \frac{dy}{d\theta} = a(0 - 2\sin 2\theta)$$

$$\therefore \frac{dy}{d\theta} = -2a \sin 2\theta$$

$$x = a(2\theta - \sin 2\theta), \quad \frac{dx}{d\theta} = a(2 - 2\cos 2\theta)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2a \sin 2\theta}{2a[1 - \cos 2\theta]} \quad \left(\because \frac{dx}{d\theta} \neq 0 \right)$$

$$\Rightarrow \frac{-2 \sin \theta \cos \theta}{2 \sin^2 \theta} = -\cot \theta$$

$$\therefore \frac{dy}{dx} = -\cot \left(\frac{\pi}{3} \right) = -\frac{1}{\sqrt{3}}$$

13.

Sol. $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} \frac{(x+3)^2 - 36}{x-3}$

$$= \lim_{x \rightarrow 3} \frac{(x+3-6)(x+3+6)}{(x-3)}$$

$$= 12$$

given that $f(x)$ is continuous at $x = 3$

$$\therefore \lim_{x \rightarrow 3} f(x) = f(3)$$

$$\Rightarrow k = 12$$

14.

Sol. We have $x^y + y^x = a^b$

Differentiating w.r.t. x , we get $\frac{d}{dx}(x^y) + \frac{d}{dx}(y^x) = 0$ (1)

Let $u = x^y \therefore \log u = y \log x$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}; \Rightarrow \frac{du}{dx} = u \left(\frac{y}{x} + \log x \frac{dy}{dx} \right)$$

$$\text{or } \frac{d}{dx}(x^y) = x^y \left(\frac{y}{x} + \log x \frac{dy}{dx} \right) \quad \dots (2)$$

Let $v = y^x \therefore \log v = x \log y$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = x \cdot \frac{1}{y} \frac{dy}{dx} + \log y \cdot 1; \Rightarrow \frac{dv}{dx} = v \left(\frac{x}{y} \frac{dy}{dx} + \log y \right)$$

$$\text{or } \frac{d}{dx}(y^x) = y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) \quad \dots (3)$$

Using (2) and (3) in (1),

$$\text{We get. } x^y \left(\frac{y}{x} + \log x \frac{dy}{dx} \right) + y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right) = 0. \quad \dots (4)$$

$$\Rightarrow (x^y \log x + xy^{x-1}) \frac{dy}{dx} = -(y^x \log y + yx^{y-1}) \text{ or } \frac{dy}{dx} = -\frac{y^x \log y + yx^{y-1}}{x^y \log x + xy^{x-1}}$$

OR

Let $e^y (x+1) = 1$

$$e^y (1) + (x+1) e^y \frac{dy}{dx} = 0$$

$$\Rightarrow (x+1) \frac{dy}{dx} + 1 = 0 \quad \dots (1)$$

Again differentiating w.r.t. x ,

$$\therefore (x+1) \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx} \right) \cdot 1 = 0$$

$$\frac{d^2 y}{dx^2} = -\frac{\frac{dy}{dx}}{(x+1)}$$

$$\frac{d^2 y}{dx^2} = -\frac{dy}{dx} \cdot \frac{dy}{dx} \quad [\text{equation (1)}]$$

$$\frac{d^2 y}{dx^2} = \left(\frac{dy}{dx} \right)^2$$

15.

Sol. Here $x = a \left(\sin 2t + \frac{1}{2} \sin 4t \right), y = b (\cos 2t - \cos^2 2t)$

$$\frac{dx}{dt} = 2a[\cos 2t + \cos 4t], \frac{dy}{dt} = 2b[-\sin 2t + 2\cos 2t \sin 2t] = 2b[\sin 4t - \sin 2t]$$

$$\frac{dy}{dx} = \frac{b}{a} \left[\frac{\sin 4t - \sin 2t}{\cos 4t + \cos 2t} \right]$$

$$\left. \frac{dy}{dx} \right|_{t=\frac{\pi}{4}} = \frac{b}{a} \text{ and } \left. \frac{dy}{dx} \right|_{t=\frac{\pi}{3}} = \sqrt{3} \frac{b}{a}$$

OR

$$y = x^x \Rightarrow \log y = x \log x$$

$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = (1 + \log x)$$

$$\Rightarrow \frac{1}{y} \frac{d^2 y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx} \right)^2 - \frac{y}{x} = 0$$

16.

$$\text{Sol. LHL} = \lim_{x \rightarrow \frac{\pi}{2}^-} \frac{(1 - \sin x)(1 + \sin x + \sin^2 x)}{3(1 - \sin x)(1 + \sin x)}$$

$$= \frac{1}{2}$$

$$\therefore p = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \rightarrow \frac{\pi}{2}^+} \frac{q(1 - \sin x)}{(\pi - 2x)^2} = \lim_{h \rightarrow 0} \frac{q(1 - \cosh)}{(2h)^2} \text{ where } x - \frac{\pi}{2} = h$$

$$= \lim_{h \rightarrow 0} \frac{2q \sin^2 \frac{h}{2}}{4 \cdot \frac{h^2}{4}} = \frac{q}{8}$$

$$\therefore \frac{q}{8} = \frac{1}{2} \Rightarrow q = 4$$

17.

$$\text{Sol. } y = ae^\theta (\sin \theta + \cos \theta)$$

$$x = ae^\theta (\sin \theta - \cos \theta)$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} \text{ (Applying parametric differentiation) ... (1)}$$

$$\text{Now, } \frac{dy}{d\theta} = ae^\theta (\cos \theta - \sin \theta) + ae^\theta (\sin \theta + \cos \theta)$$

$$= 2ae^\theta (\cos \theta) \text{ (Applying product Rule)}$$

$$\frac{dx}{d\theta} = ae^\theta (\cos \theta + \sin \theta) + ae^\theta (\sin \theta - \cos \theta)$$

$$= 2ae^{\theta} (\sin \theta)$$

Substituting the values of $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$ in (1)

$$\frac{dy}{dx} = \frac{2ae^{\theta} \cos \theta}{2ae^{\theta} \sin \theta} = \cot \theta$$

Now $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$

$$[\cot \theta]_{\theta=\pi/4} = \cot \frac{\pi}{4} = 1.$$

18.

Sol. $y^x = e^{y-x}$

Taking log both side we get,

$$\Rightarrow x \log_e y = y - x \quad \dots(1)$$

Differentiating w.r.t.x

$$\Rightarrow \log_e y + x \frac{1}{y} \frac{dy}{dx} = \frac{dy}{dx} - 1$$

$$\Rightarrow \log_e y + 1 = \frac{dy}{dx} \left(1 - \frac{x}{y} \right) \left\{ \text{form (1)} \frac{x}{y} = \frac{1}{1 + \log_e y} \right\}$$

$$\Rightarrow \log_e y + 1 = \frac{dy}{dx} \left(1 - \frac{1}{1 + \log_e y} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1 + \log_e y)^2}{\log_e y}$$

19.

Sol. $y = \sin^{-1} \left[\frac{2^{x+1} \cdot 3^x}{1 + (36)^x} \right]$

$$y = \sin^{-1} \left[\frac{2^x \cdot 2 \cdot 3^x}{1 + (36)^x} \right]$$

$$y = \sin^{-1} \left[\frac{2 \cdot (6)^x}{1 + (6)^{2x}} \right]$$

$$y = 2 \tan^{-1} (6)^x$$

$$\frac{dy}{dx} = \frac{2}{1 + (6)^{2x}} \cdot 6^x \log 6$$

$$\frac{dy}{dx} = \frac{2 \cdot 6^x \log 6}{1 + (36)^x}$$

20.

Sol. $f(0) = \lim_{x \rightarrow 0} f(x)$

$$\Rightarrow \frac{0+1}{0-1} = \lim_{x \rightarrow 0} \left(\frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \right)$$

$$\Rightarrow -1 = \lim_{x \rightarrow 0} \left(\frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \right) \left(\frac{\sqrt{1+kx} + \sqrt{1-kx}}{\sqrt{1+kx} + \sqrt{1-kx}} \right)$$

$$\Rightarrow -1 = \lim_{x \rightarrow 0} \frac{2k}{\sqrt{1+kx} + \sqrt{1-kx}}$$

$$\Rightarrow k = -1$$

OR

$$x = a \cos^3 \theta \text{ and } y = a \sin^3 \theta$$

$$\frac{dx}{d\theta} = -3a \cos^2 \theta \sin \theta \text{ and } \frac{dy}{d\theta} = 3a \sin^2 \theta \cos \theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$$\Rightarrow \frac{dy}{dx} = -\tan \theta$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\sec^2 \theta \frac{1}{(-3a \cos^2 \theta \sin \theta)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{3a} \sec^4 \theta \csc \theta$$

$$\Rightarrow \left(\frac{d^2y}{dx^2} \right)_{\theta = \frac{\pi}{6}} = \frac{1}{3a} \left(\frac{2}{\sqrt{3}} \right)^4 \cdot 2 = \frac{32}{27a}$$

21.

Sol. $\lim_{x \rightarrow 5} f(x) = \lim_{x \rightarrow 5} 2k$

$$\Rightarrow 2k = 15-8$$

$$\Rightarrow k = \frac{7}{2}$$

22.

Sol. The function $[x]$ is continuous at non-integer values.

Hence, $f(x)$ is continuous at 1.5

23.

Sol. Given: $y = \tan^{-1}(e^{2x})$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{1+(e^{2x})^2} \times \frac{d}{dx}(e^{2x})$$

$$\Rightarrow \frac{dy}{dx} = \frac{2e^{2x}}{1+e^{4x}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2e^{2x}}{1+e^{4x}}$$

24.

Sol. Given: $y = e^{-x}$

$$\Rightarrow \frac{dy}{dx} = -e^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = e^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = y$$

25.

Sol. Given: $x = t^2 + 1$, $y = 2at$

$$\Rightarrow \frac{dx}{dt} = 2t \text{ and } \frac{dy}{dt} = 2a$$

$$\Rightarrow \frac{dy}{dx} = \frac{a}{t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{a}{t^2} \times \frac{dt}{dx}$$

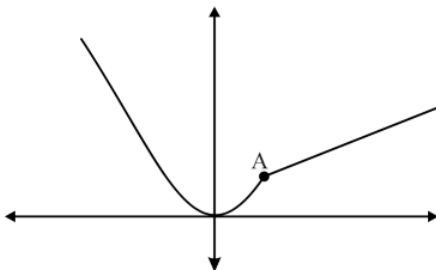
$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{a}{2t^3}$$

$$\Rightarrow \left. \frac{d^2y}{dx^2} \right|_{t=a} = -\frac{1}{2a^2}$$

26.

Sol. Given: $f(x) = \begin{cases} x^2, & \text{for } x < 1 \\ 2-x, & \text{for } x \geq 1 \end{cases}$

Graph of this function is



From the graph, At the point A(1, 1), we can't draw a tangent.

Hence, $f(x)$ is not differentiable at $x = 1$.

