

SUBJECT : MATHEMATICS

Topic Name: LCD and Differentiation

1. If $y = \tan^{-1} \left(\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right)$, $x^2 \leq 1$, then find $\frac{dy}{dx}$. [2015 4M]
2. The function $f(x) = \frac{x-1}{x(x^2-1)}$ is discontinuous at [2020 1M]
 - (A) exactly one point
 - (B) exactly two points
 - (C) exactly three points
 - (D) no point
3. Find the value of k , so that the function $f(x) = \begin{cases} kx^2 + 5 & \text{if } x \leq 1 \\ 2 & \text{if } x > 1 \end{cases}$ is continuous at $x = 1$. [2020 1M]
4. Differentiate $\sec^2(x^2)$ with respect to x^2 . [2020 1M]

Or

If $y = f(x^2)$ and $f'(x) = e^{\sqrt{x}}$, then find $\frac{dy}{dx}$. [2020 1M]
5. If $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$, find $f'\left(\frac{\pi}{3}\right)$. [2020 2M]

Or

Find $f'(x)$ if $f(x) = (\tan x)^{\tan x}$. [2020 2M]
6. If $\tan^{-1}\left(\frac{y}{x}\right) = \log \sqrt{x^2 + y^2}$, prove that $\frac{dy}{dx} = \frac{x+y}{x-y}$. [2020 4M]

Or

If $y = e^{\cos^{-1} x}$, $-1 < x < 1$, then show that [2020 4M]

$$(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - y^2 = 0.$$
7. If $y = \sin^{-1} x + \cos^{-1} x$, find $\frac{dy}{dx}$ [2019 1M]

8. If $(\sin x)^y = x + y$, find $\frac{dy}{dx}$. [2019 4M]
9. If $y = (\sec^{-1} x)^2$, $x > 0$ show that $x^2(x^2 - 1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} - 2 = 0$. [2019 4M]
10. Differentiate $\tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right)$ with respect to x . [2018 2M]
11. If $y = \sin(\sin x)$, prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$ [2018 4M]
12. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$. [2018 4M]

OR

- If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$. [2018 4M]
13. Determine the value of 'k' for which the following function is continuous at $x = 3$: [2017 1M]
- $$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$
14. If $x^y + y^x = a^b$, find $\frac{dy}{dx}$ [2017 4M]
- OR
- If $e^y(x+1) = 1$ then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ [2017 4M]
15. If $x = a \sin 2t(1 + \cos 2t)$ and $y = b \cos 2t(1 - \cos 2t)$, find the values of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$. [2016 4M]

OR

- If $y = x^x$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$. [2016 4M]
16. Find the value of p and q, for which $f(x) = \begin{cases} \frac{1 - \sin^3 x}{3 \cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ p, & \text{if } x = \frac{\pi}{2} \\ \frac{q(1 - \sin x)}{(\pi - 2x)^2}, & \text{if } x > \pi/2 \end{cases}$ is continuous at $x = \pi/2$ [2016 4M]
17. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$, if $x = ae^\theta(\sin \theta - \cos \theta)$ and $y = ae^\theta(\sin \theta + \cos \theta)$. [2014 4M]
18. If $y^x = e^{y-x}$, prove that $\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}$ [2013 4M]

19. Differentiate the following with respect to x : $\sin^{-1}\left(\frac{2^{x+1}.3^x}{1+(36)^x}\right)$ [2013 4M]

20. Find the value of k , for which $f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x < 1 \end{cases}$, is continuous at $x = 0$. [2013 4M]

OR

If $x = a \cos^3 \theta$ and $y = a \sin^3 \theta$, then find the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{6}$. [2013 4M]

21. If the function $f(x) = \begin{cases} 3x-8, & \text{if } x \leq 5 \\ 2k, & \text{if } x > 5 \end{cases}$ is continuous, then the value of k is: [2021 1M]

(a) $\frac{2}{7}$ (b) $\frac{7}{2}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$

22. The function $f(x) = [x]$, where $[x]$ is the greatest integer function that is less than or equal to x , is continuous at: [2021 1M]

(a) 4 (b) -2 (c) 1.5 (d) 1

23. If $y = \tan^{-1}(e^{2x})$, then $\frac{dy}{dx}$ is equal to: [2021 1M]

(a) $\frac{2e^{2x}}{1+e^{4x}}$ (b) $\frac{1}{1+e^{4x}}$
(c) $\frac{2}{e^{2x}+e^{-2x}}$ (d) $\frac{1}{e^{2x}-e^{-2x}}$

24. If $y = e^{-x}$, then $\frac{d^2y}{dx^2}$ is equal to: [2021 1M]

(a) $-y$ (b) y (c) x (d) $-x$

25. If $x = t^2 + 1$, $y = 2at$ then $\frac{d^2y}{dx^2}$ at $t = a$ is: [2021 1M]

(a) $-\frac{1}{a}$ (b) $-\frac{1}{2a^2}$ (c) $\frac{1}{2a^2}$ (d) 0

26. The function $f(x) = \begin{cases} x^2, & \text{for } x < 1 \\ 2-x, & \text{for } x \geq 1 \end{cases}$ is; [2021 1M]

- (a) not differentiable at $x = 1$
(b) differentiable at $x = 1$
(c) not continuous at $x = 1$
(d) neither continuous nor differentiable at $x = 1$



Answer Key

1. $-\frac{x}{\sqrt{1-x^4}}$
2. (C)
3. (-3)
4. $2.\sec^2 x^2.\tan x^2$ OR $2xe^x$.
5. $\frac{2}{3}$ OR $\tan x^{\tan x} \sec^2 x [1 + \log(\tan x)]$.
6. 0
7. (0)
8. $\frac{1 - (xy + y^2)\cot x}{(x + y)\log(\sin x) - 1}$
9. 0
10. $-\frac{1}{2}$
11. 0
12. $\frac{y - 4x^3 - 4xy^2}{4y^2 + 4x^2y - x}$ OR $-\frac{1}{\sqrt{3}}$
13. (12)
14. $-\frac{y^x \log y + yx^{y-1}}{x^y \log x + xy^{x-1}}$ OR
15. $\sqrt{3}\frac{b}{a}$ OR
16. (p=1/2 OR q=4)
17. (1)
18. $\frac{(1 + \log_e y)^2}{\log_e y}$
19. $\frac{2.6^x \log 6}{1 + (36)^x}$
20. -1 OR $\frac{32}{27a}$
21. $\frac{7}{2}$
22. The function $[x]$ is continuous at non-integer values.
Hence, $f(x)$ is continuous at 1.5
23. $\frac{2e^{2x}}{1 + e^{4x}}$
24. (y)
25. $-\frac{1}{2a^2}$
26. (1)