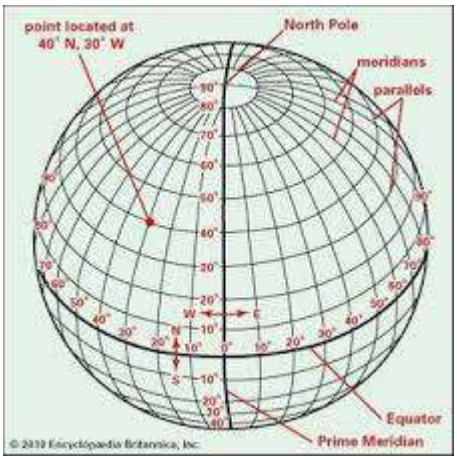
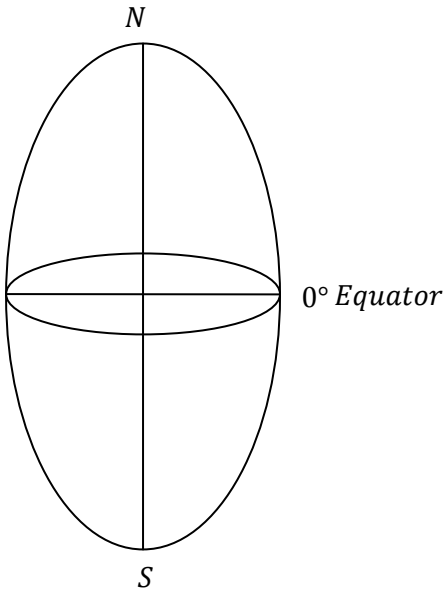
LATITUDE AND LONGITUDE (EARTH)

- The earth is spherical in shape but slightly flat at the poles.

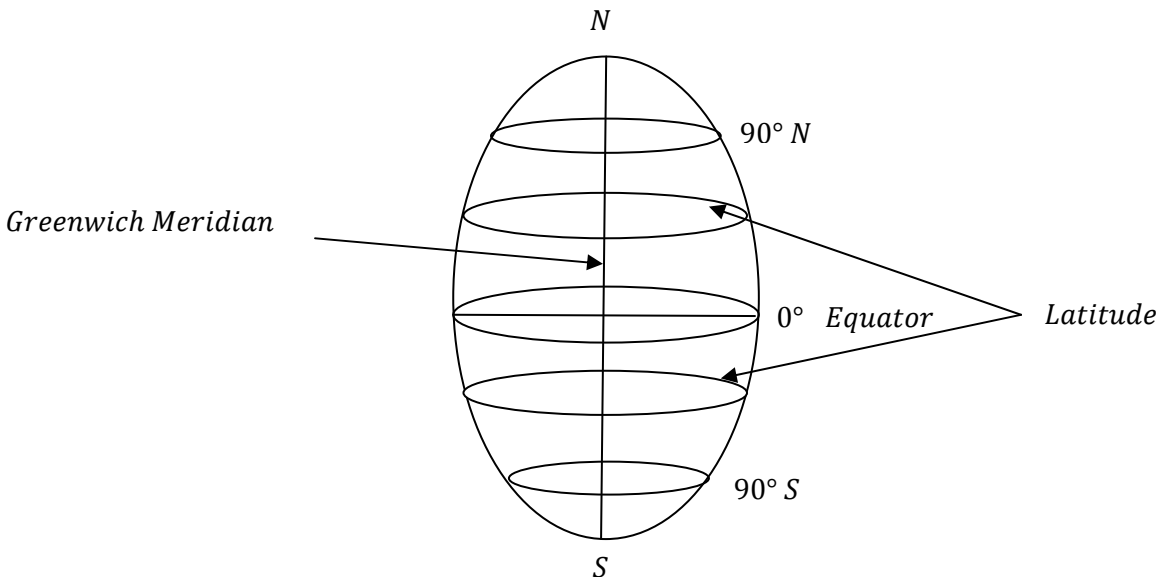


- In mathematical calculation and manipulation we do not draw the whole earth but we only draw its sketch as below.



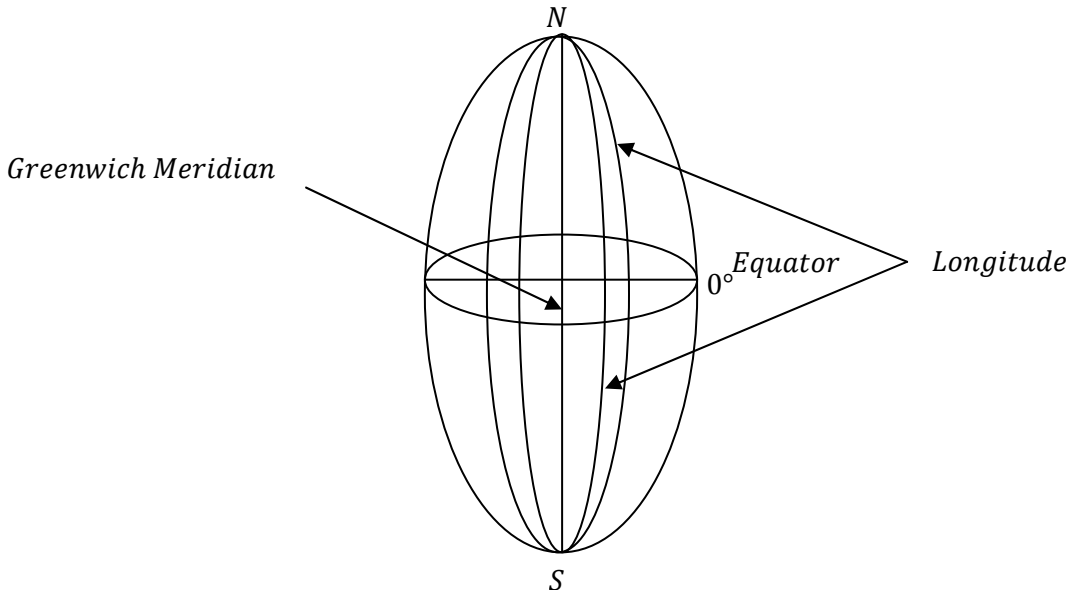
LATITUDE.

- These are imaginary line joining East to West.
- They range from $0° \; to \; 90°$ North or $0° \; to \; 90°$ South.
- The reference point while measuring their angle is the equator i.e. they are measured either clockwise or anticlockwise from the equator.



LONGITUDE.

- These are imaginary line joining North to South.
- They range from $0° \; to \; 180°$ East or $0° \; to \; 180°$ West.
- The reference point while measuring their angle is the Greenwich meridian.

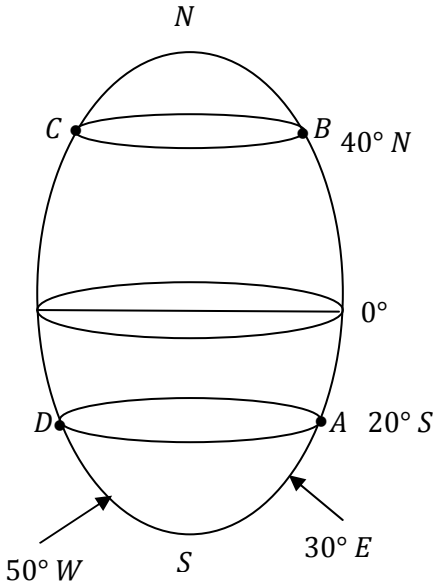


LOCATING THE POSITION OF A PLACE ON EARTH.

- It is given by;
  a) First Determining the latitude of a point.
  b) Second Determining the longitude of that same point.
  c) Giving the position in co-ordinate form i.e. $(latitude, longitude)$.

Example.

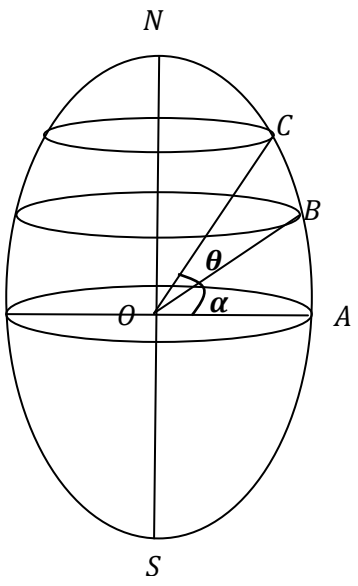
1. Determine the position of $A - D$ on the earth below.



i. $A(20°°S, 30°E)$
ii. $B(40°N, 30°E)$
iii. $C(40°N, 50°E)$
iv. $D(20°S, 50°W)$

LATITUDE DIFFERENCE.

- This is the difference between any two latitude.
- If latitudes are on the same longitude and in the same quadrant sphere in either east or west hemisphere, latitude difference is got by subtracting small latitude from larger latitude..
- The illustration below proves.



$< AOC = \alpha$

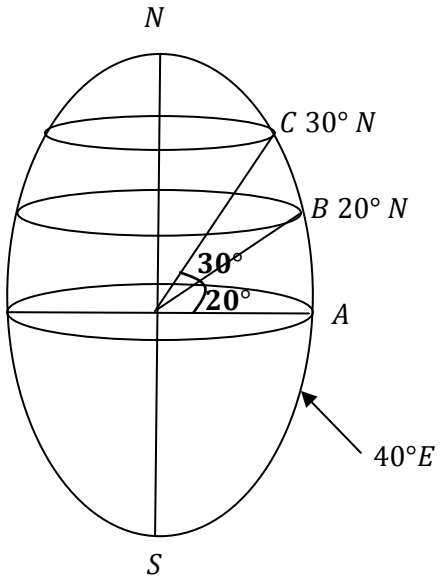
$< AOB = \theta$

$< BOC = \alpha - \theta$

Example.

Find the latitude difference of the following point.

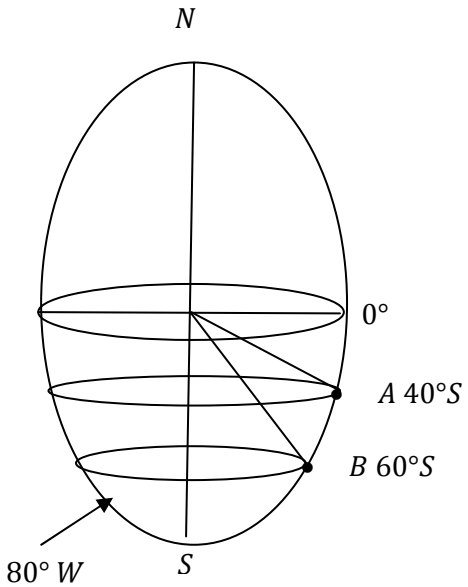
1. $A(30°N, 40°E)$
   $B(20°N, 40°E)$



$latitude\ difference = 30° - 20°$

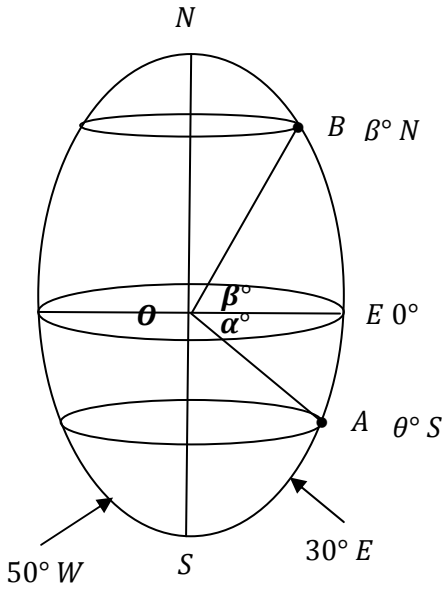
$= \mathbf{10°}$

2. $A(60°S, 80°W)$
   $B(40°S, 80°W)$



$$latitude\ difference = 60° - 40°$$
$$= \mathbf{20°}$$

- If latitude are on the same longitude but different quadrant sphere in the same hemisphere, latitude difference is got by adding their latitude..
- The illustration below proves.



$$< AOE = \alpha°$$

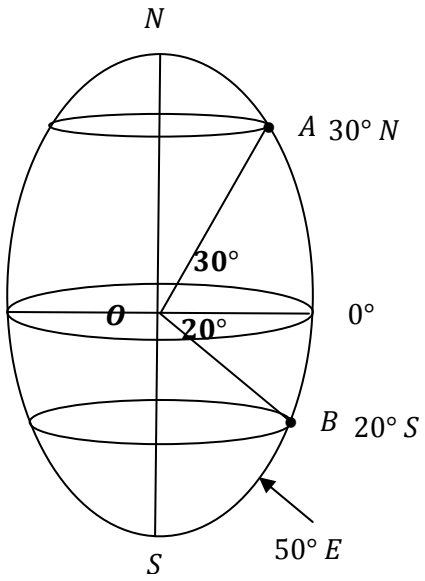
$$< EOB = \beta°$$

$$< AOB = \beta° + \alpha°$$

Examples.

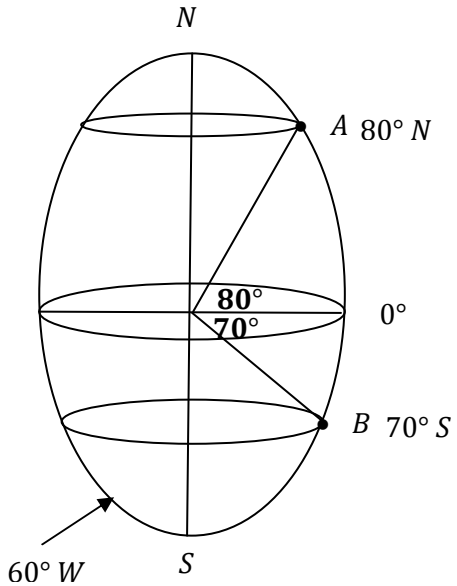
Find the latitude difference of the following.

1. $A(30°N, 50°E)$
   $B(20°S, 50°E)$



$$latitude\ difference = 30° + 20°$$
$$= \mathbf{50°}$$

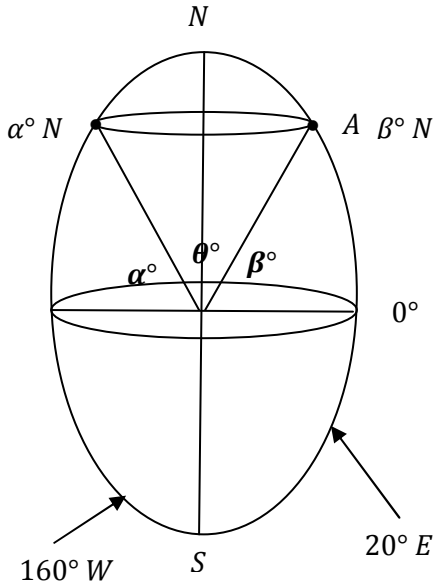
2. $A(80°N, 60°W)$
   $B(70°S, 60°W)$



$$latitude\ difference = 80° + 70°$$
$$= \mathbf{150°}$$

Note.

- Sometimes, latitude may be same/different but lying on different longitude in the same hemisphere either northern or southern hemisphere.
- In such case the sum of the longitude angle given is equal to 180° (**longitude/meridian are directly opposite**).
- To find the latitude difference, sum the latitude and subtract from 180°.
- The illustration below justify.



- To get angle θ

$$\theta = 180° - (\beta + \alpha)$$

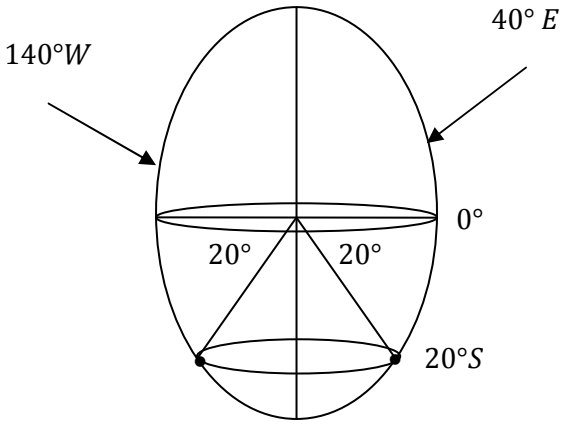
$$\boldsymbol{Latitude\ difference = 180° - \sum Latitude\ of\ the\ two\ points}$$

- The equator acts as a straight line, therefore applying the property of angles on a straight line we obtain the above equation.

Examples.

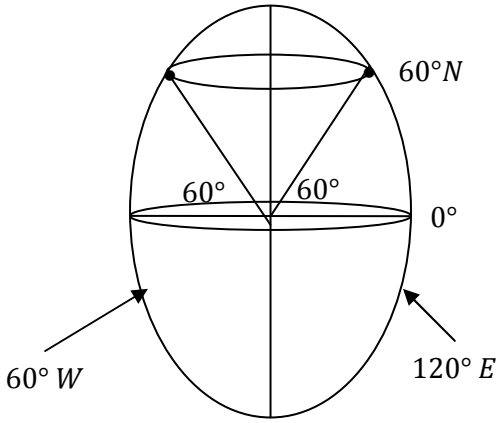
Find the latitude difference of the point below on the earth.

(a) $A(20°S, 40E)$
$B(20°S, 140°W)$



$$latitude\ difference = 180° - (20° + 20°)$$
$$= 180° - 40°$$
$$= \mathbf{140}°$$

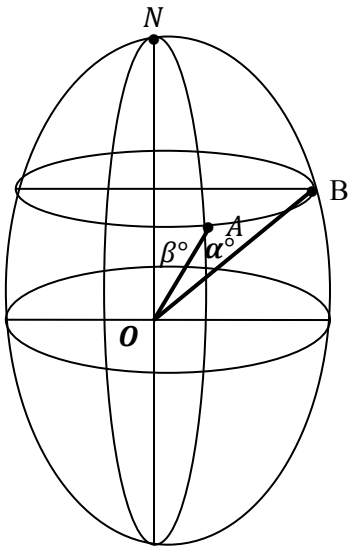
(b) $A(60°N, 120°E)$
$B(60°N, 60°W)$



$$latitude\ difference = 180° - (60° + 60°)$$
$$= 180° - 120°$$
$$= \mathbf{60}°$$

LONGITUDE DIFFERENCE.

- This is the difference between any two longitudes.
- If the longitudes are on the same latitude and on the same quadrant of the sphere, longitude difference is got by subtracting smaller longitude from the larger longitude.
- The illustration below justify.

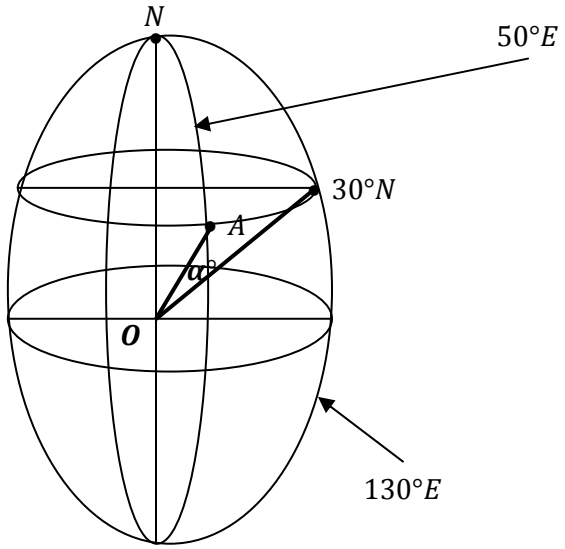


$$< BON = \alpha$$
$$< AON = \beta$$
$$< BOA = \alpha - \beta$$

Examples.

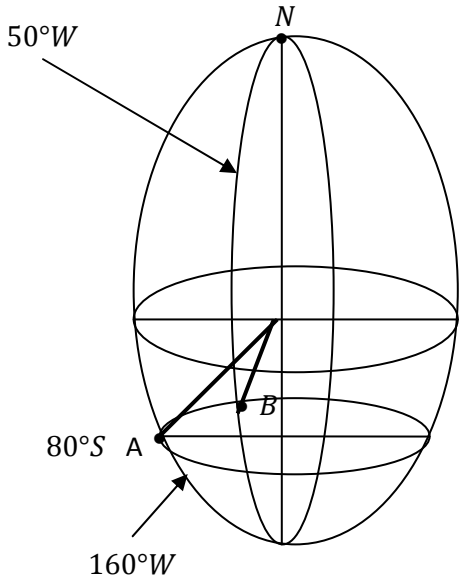
Determine the longitude difference of the following.

(a) $A(30°N, 50°E)$
$B(30°N, 130°E)$



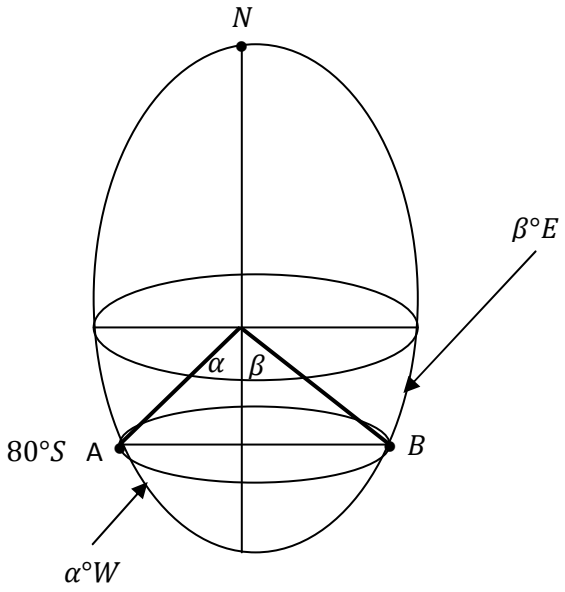
$$longitude\ difference = 130° - 50°$$
$$= \mathbf{80}°$$

(b) $A(80°S, 160°W)$
$B(80°S, 50°W)$



$$longitude\ difference = 160° - 50°$$
$$= \mathbf{110}°$$

- If the longitudes are lying on the same latitude but they are in the opposite side of Greenwich meridian and in the same hemisphere, longitude difference is got by getting the sum of the longitudes.
- The illustration below justifies the above statement.

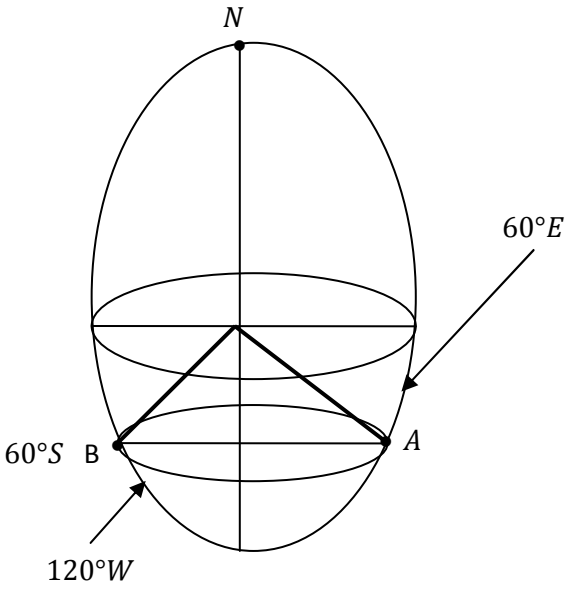


$$longitude\ difference = \beta + \alpha$$

Examples.

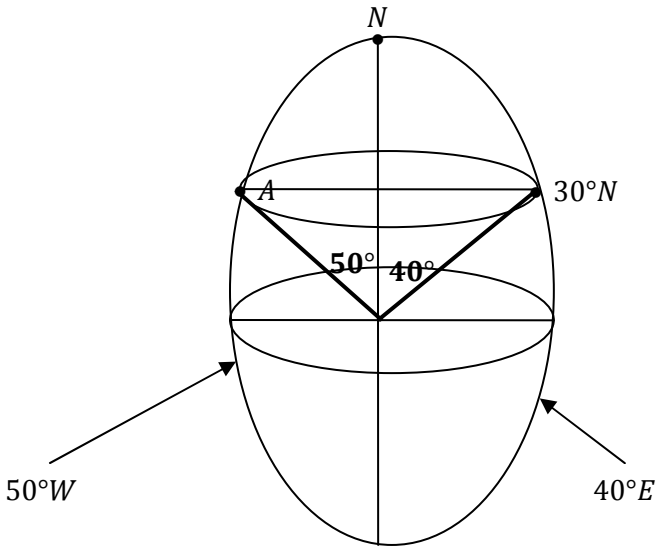
Find the longitude difference of the points below.

(a) $A(60°S, 120°W)$
$B(60°S, 60°E)$



$$longitude\ difference = 120° + 60°$$
$$= \mathbf{180°}$$

(b) $A(30°N, 40°E)$
$B(30°N, 50°W)$



$$longitude\ difference = 50° + 40°$$
$$= \mathbf{90°}$$

DISTANCE A LONG A GREAT CIRCLE IN KILOMETRES.

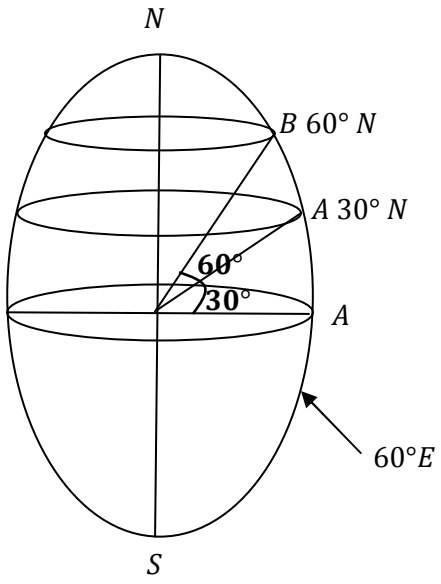
- Great circle is also called circles of longitude.
- These are circles whose radii are equivalent to the radius of the earth.
- The radius of the earth is constant i.e. $R = 6370\ km$.
- The formula used during calculation of the distance along the great circle is;

$$\frac{\theta}{360°} \times 2\pi R$$
$$= \frac{\theta}{180°}\pi R$$
$$= \frac{\theta}{360°}\pi D$$

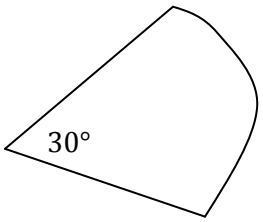
- Angle θ used in finding the distance along the great circle is the latitude difference.
  The circle of the equator is also a great circle because its radius is equal to the radius of the earth hence the same formula $\frac{\theta}{360°} \times 2\pi R$ is applied but in this case θ used is the longitude difference.
- Directly opposite meridians from great circles i.e. the Greenwich meridians 0° and the international Dateline $180°W$ form a great circle.
- If the sum of their longitude does not add up to 180°, the following should be observed;
  (a) The points will be lying on the same parallel of latitude e.g. $P(60°°N, 30°E)$ and $Q(60°N, 41°W)$.
  (b) The points will be lying on the same longitude if they are on the same side of Greenwich meridian e.g. $A(30°N, 60°E)$ and $B(60°N, 60°E)$.
  (c) All circles pass through the North pole and South pole of the great circles form great circle.
- Lines of longitude lie on great circles.

Examples.

1. Find the distance in km between $A(30°N, 60°E)$ and $B(60°N, 60°E)$. ($Take\ \pi = \frac{22}{7}\ and\ R = 6370\ km$)



$$latitude\ difference = 60° - 30°$$
$$= 30°$$

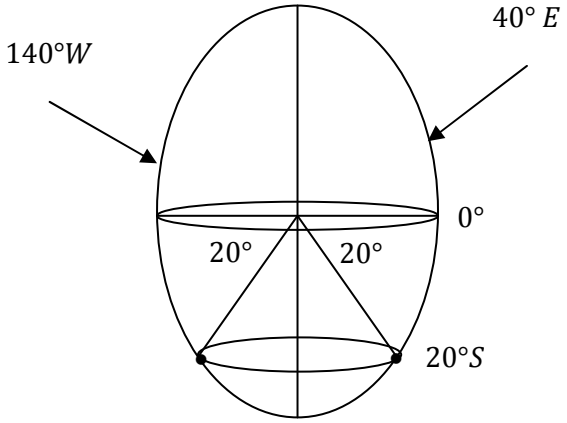


$$distance = \frac{\theta}{360°} \times 2\pi R$$

$$= 2 \times \frac{30°}{360°} \times \frac{22}{7} \times 6370$$

$$= \mathbf{3336.67\ km}$$

2. Find the distance between $A(20°S, 40°E)$ and $B(20°S, 140°W)$ via
   (a) South pole



$$latitude\ difference = 180° - (20° + 20°)$$
$$= 180° - 40°$$
$$= 140°$$

$$distance = \frac{\theta}{360°} \times 2\pi R$$

$$= 2 \times \frac{140°}{360°} \times \frac{22}{7} \times 6370$$

$$= \mathbf{15571.11\ km}$$

   (b) North pole

$$for\ north\ pole\ from\ the\ above\ diagram\ we\ find\ the\ reflex\ angle$$

$$= 360° - 140°°$$
$$= 220°$$

$$distance = \frac{\theta}{360°} \times 2\pi R$$

$$= 2 \times \frac{220°}{360°} \times \frac{22}{7} \times 6370$$

$$= \mathbf{24468.89\ km}$$

3. Find the distance between $A(0°, 30°E)$ and $B(0°S, 70°E)$

$$\text{longitude difference} = 70° - 30°$$
$$= 40°$$

$$\text{distance} = \frac{\theta}{360°} \times 2\pi R$$

$$= 2 \times \frac{40°}{360°} \times \frac{22}{7} \times 6370$$

$$= \mathbf{4448.89\ km}$$

4. Find the distance between $A(0°, 20°E)$ and $B(0°S, 20°W)$

$$longitude\ difference = 20° + 20°$$

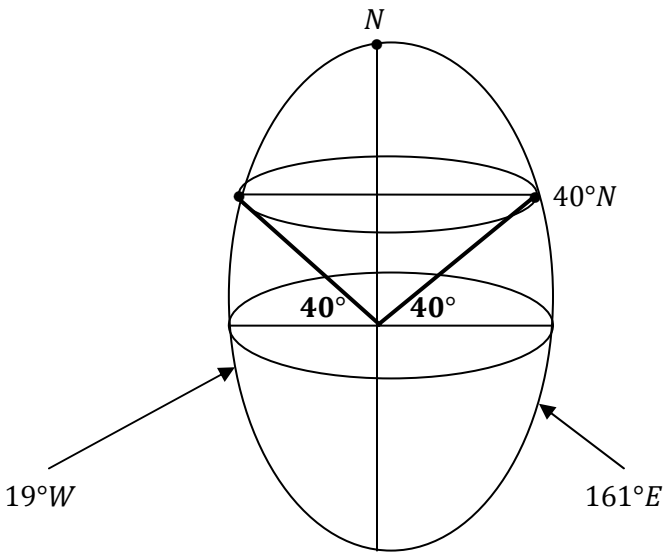
$$= 40°$$

$$distance = \frac{\theta}{360°} \times 2\pi R$$

$$= 2 \times \frac{40°}{360°} \times \frac{22}{7} \times 6370$$

$$= \mathbf{4448.89\ km}$$

5. Calculate the distance between points $A(40°N, 19°W)$ and $B(40°N, 161°E)$ on the globe via;
   (a) South pole



latitude difference $= 180° - (40° + 40°)$

$$= 180° - 80°$$

$$= 100°$$

From the above diagram, via south we will find the reflex angle

$$= 360° - 100°$$

$$= 260°$$

$$\text{distance} = \frac{\theta}{360°} \times 2\pi R$$

$$= 2 \times \frac{260°}{360°} \times \frac{22}{7} \times 6370$$

$$= \mathbf{28917.78\ km}$$

(b) North pole.

for the north pole the angle θ will remain the same 100°

$$\text{distance} = \frac{\theta}{360°} \times 2\pi R$$

$$= 2 \times \frac{100°}{360°} \times \frac{22}{7} \times 6370$$

$$= \mathbf{11122.22\ km}$$

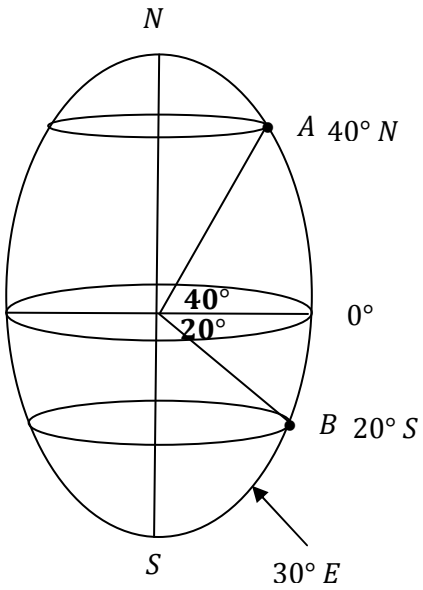
DSTANCE ALONG GREAT CIRCLE IN NEUTICAL MILE (nm)

- Nautical mile is the distance along circle which subtends one minutes at the centre of the earth.
- There are 360° in a circle and 60 $minutes$ in a degree.

1. $60\theta\ nm$ where
   $\theta - the\ latitude\ difference.$
2. $1\ nm = \frac{40040}{21600}\ km$
   $1.8537\ km$
3. $360° = 21600nm.$

Examples.

1. Find the distance between $A(40°N, 30°E)$ and $B(20°S, 30°E)$



latitude difference $= 40° + 20°$

$= 60°$

distance $= 60\theta$ nm

$= 60 \times 60$

$= \mathbf{3600\ nm}$

2. A spherical globe is of radius 20 cm. Find the shortest distance along its equator between longitudes $15°\ W\ and\ 120°\ E$.

$$longitude\ difference = 120° + 15°$$
$$= 135°$$
$$distance = \frac{135}{360} \times 2 \times \frac{22}{7} \times 20$$
$$= \mathbf{47.14\ cm}$$

FINDING THE POSITION OF A POINT ON GREAT CIRCLE GIVEN DISTANCE.

- The same formula $distance = \frac{\theta}{360°} \times 2\pi R$ for distance in kilometre and 60α for distance in nautical mile along great circle is used with correct substitution to get the value of unknown.
- Letting the new settlement of latitude to be unknown, apply the formula of latitude difference to get the unknown which is the position.
- During finding the position (latitude angle) when the angle got is negative then it is positive in the opposite side of the equator i.e. $-60°N = 60°S$.

Examples.

1. A point B is due east of A along equator. The distance between A and B is 1040 km. If the longitude of A is $15°E$, calculate the position of B.

let the longitude difference be x

$$\frac{x}{360} \times 2 \times \frac{22}{7} \times 6370 = 1040$$
$$x = \frac{1040 \times 360 \times 7}{2 \times 22 \times 6370}$$
$$= \frac{2620800}{280280}$$
$$= 9.35°$$

let the longitude of point B be y°E

$$\text{longitude difference} = y - 15° = 9.35°$$
$$y = 15° + 9.35$$
$$= 24.35°$$
$$\mathbf{B(0°, 24.34°E)}$$

2. A point A is $(40°N, 10°E)$. A point B is due south of A. Given that the distance between them is $3600nm$,determine;
   (a) The latitude of B

let the latitude difference = x

$$\text{distance} = 60\theta\ \text{nm}$$
$$60\theta = 3600$$
$$\theta = \frac{3600}{60}$$
$$\theta = 60°$$

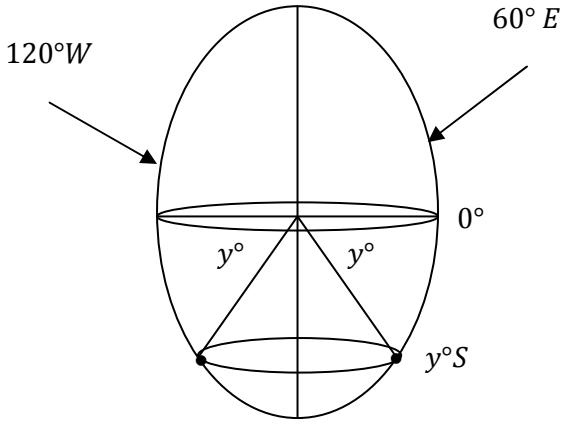
let the latitude of B be y.

$$40° + y = 60°$$
$$y = 60° - 40°$$
$$= \mathbf{20°S}$$

   (b) The position of B

$$\boldsymbol{B(20°S, 10°E)}$$

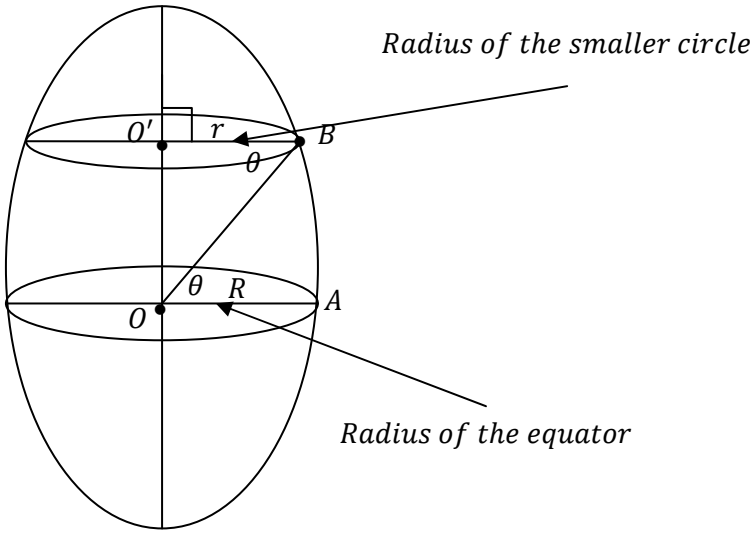
3. The figure below shows points P and Q on latitude $y°S$. They also lie on longitude $120°W$ *and* $60°E$ respectively. If the distance between them is $2890nm$. Calculate y



latitude difference $= 180° - 2y°$	$-2y = 48.17 - 180$
distance $= 60\theta$ nm	$2y = 131.83$
$2890 = 60(180° - 2y°)$	$\mathbf{y = 65.92°S}$
$180° - 2y = 48.17$	

DISTANCE ALONG SMALL CIRCLE IN KILOMETRE.

- Small circle is also called parallel of latitude.
- Radius of small circle is determined by $r = R\cos\theta$,
- The illustration below justifies the above formula.



OA = OB = R (Both are radii)	$\cos\theta = \frac{\text{adj}}{\text{hyp}}$
$OA \neq O'B$	Adj = r; hyp = R
⇨ O′B is a chord	$\cos\theta = \frac{r}{R}$
OA is a radius of the sphere.	$\mathbf{r = R\cos\theta}$ **(Hence shown)**
$< AOB =< OBO' = \theta$ (Alternating angle)	

Where;

- ✓ r −is the radius of small circle.
- ✓ R −Radius of the earth.
- ✓ $\theta = latitude\ angle\ along\ which\ longitudes\ are\ lying$

$$distance\ along\ small\ circle = \frac{\alpha}{360} \times 2\pi r$$

$$but\ r = R\cos\theta$$

$$now\ the\ general\ formula\ tends\ to;$$

$$distance\ along\ small\ circle = \frac{\alpha}{360} \times 2\pi R\cos\theta$$

$$Where\ \alpha = longitude\ difference$$

Examples.

1. Find the distance between the point $A(50°N, 60°E)$ and $B(50°N, 30°W)$

$$longitude\ diff = 60° + 30°$$

$$= 90°$$

$$distance\ along\ small\ circle = \frac{\alpha}{360} \times 2\pi R\cos\theta$$

$$distance = \frac{90°}{360°} \times 2 \times \frac{22}{7} \times\times 6370\cos 50°$$

$$= \mathbf{6434.30\ km}$$

2. Find the circumference of the tropic of cancer $(23\frac{1}{2}°N)$.

$$distance\ along\ small\ circle = \frac{\alpha}{360°} \times 2\pi R\cos\theta$$

$$\alpha = 0°\ Since\ there\ was\ no\ change\ on\ longitude\ hence\ longitude\ diff = 0°$$

$$distance = \frac{360°}{360°} \times 2 \times \frac{22}{7} \times\times 6370\cos(23\frac{1}{2})°$$

$$= \mathbf{36719.09\ km}$$

3. Find the circumference of the latitude $70°S$.

$$distance\ along\ small\ circle = \frac{\alpha}{360°} \times 2\pi R\cos\theta$$

$$\alpha = 0°\ Since\ there\ was\ no\ change\ on\ longitude\ hence\ longitude\ diff = 0°$$

$$distance = \frac{360°}{360°} \times 2 \times \frac{22}{7} \times\times 6370\cos 70°$$

$$= \mathbf{13694.49\ km}$$

DISTANCE ALONG SMALL CIRCLE IN NAUTICAL MILES.

- For the distance along small circle in nautical miles is given by the formula

$$distance\ along\ small\ circle = 60\alpha\cos\theta\ nm$$
$$where;;$$
$$\alpha = longitude\ difference$$
$$\theta = latitude\ along\ which\ the\ latitude\ are\ lying.$$

Examples.

1. Find the distance in nautical mile between $A(30°N, 40°E)$ and $B(30°N, 60°E)$

$$longitude\ diff = 60° - 40°$$
$$= 20°$$
$$distance\ along\ small\ circle = 60\alpha\cos\theta\ nm$$
$$= 60 \times 20\cos 30°$$
$$= \mathbf{1039.23\ nm}$$

FINDING THE POSITION OF A POINT ON A SMALL CIRCLE GIVEN DISTANCE.

- The same formula $distance = \frac{\theta}{360°} \times 2\pi R\cos\alpha$ for distance in kilometre and $60\theta\cos\alpha$ for distance in nautical mile along small circle is used with correct substitution to find the value of unknown.
- Letting the new settlement of longitude to be unknown, apply the formula of longitude difference to get the unknown which is the position.
- During finding the position (longitude angle) when the angle got is negative then it is positive in the opposite side i.e. $-60°E = 60°W$.

Examples.

1. A pilot leaves point $T(60°S, 10°E)$ and flies due East for a distance of 960 nm to point U. Determine the position of U. (3mks)

$$\text{distance} = 60\alpha\cos\theta$$
$$60\alpha\cos 60° = 960$$
$$\alpha = \frac{960}{60\cos 60°}$$
$$= 32°$$
$$\text{but } \alpha \text{ is latitude diff.} = 32°$$

$$\text{let the new latitude} = y$$
$$y - 10° = 32°$$
$$y = 32° + 10°$$
$$= 42°$$
$$\textbf{position is } (\mathbf{40°N, 42°E})$$

2. A plane starts at $(40°N, 30°E)$ and moves due east for 1000 nm. Find its new latitude and longitude.

$$\text{distance} = 60\alpha\cos\theta$$
$$60\alpha\cos 40° = 1000$$
$$\alpha = \frac{1000}{60\cos 40°}$$
$$= 22°$$
$$\text{but } \alpha \text{ is latitude diff.} = 22°$$

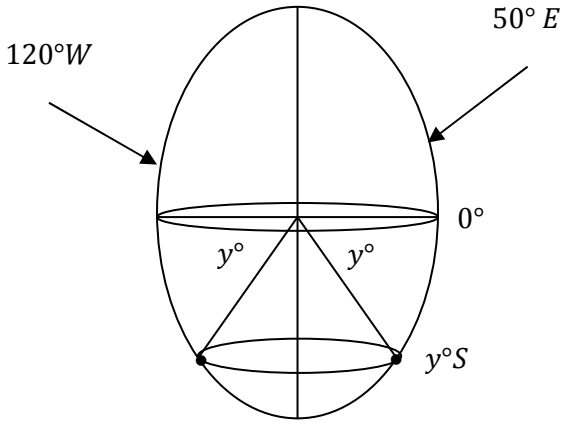
$$\text{let the new latitude} = y$$
$$y - 30° = 22°$$
$$y = 22° + 30°$$
$$= 52°$$
$$\textbf{position is } (\mathbf{40°N, 52°E})$$

3. A ship sails due west from $(30°S, 12°E)$ for 1500 nm. What is its new position.

$$\text{distance} = 60\alpha\cos\theta$$
$$60\alpha\cos 30° = 1500$$
$$\alpha = \frac{1500}{60\cos 30°}$$
$$= 29°$$
$$\text{but } \alpha \text{ is latitude diff.} = 29°$$

$$\text{let the new latitude} = y$$
$$y - 12° = 29°$$
$$y = 29° + 12°$$
$$= 41°$$
$$\textbf{position is } (\mathbf{40°N, 41°E})$$

4. The figure below shows points P and Q on latitude $y°S$. They also lie on longitude $120°W$ *and* $50°E$ respectively. If the distance between them is $2890nm$. Calculate y



$$\text{longitude difference} = 120° + 50°$$
$$\text{distance} = 60\theta\cos\theta \text{ nm}$$
$$2890 = 60(170°)\cos y$$

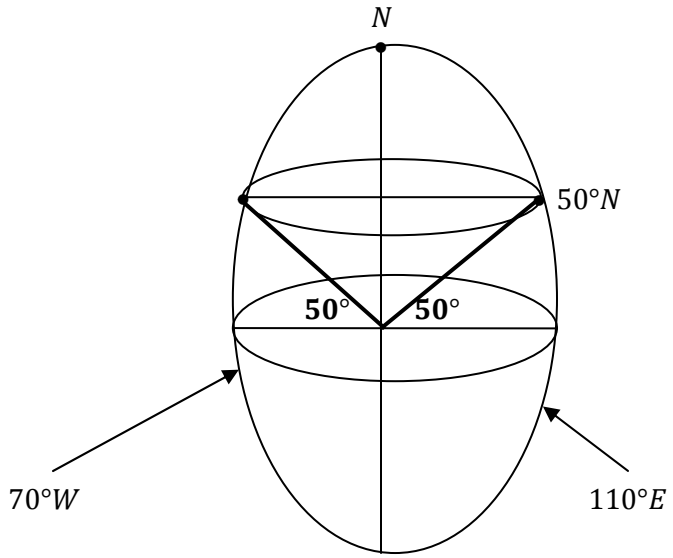
$$\cos y = \frac{2890}{10200}$$
$$\cos y = 0.2833333$$
$$y = \cos^{-1} 0.283333$$
$$= \mathbf{74°S}$$

SHORTEST DISTANCE ON THE EARTH.

- The shortest distance on the earth is the distance along the great circle.
- This can be demonstrated by taking two points on the same latitude and the two points being on great circle.
- Along the great circle, the latitude difference (θ) to give the shortest distance should be in the range $0° \leq \theta \leq 180°$
- If $\theta > 180°$, subtract it from $360°$.

Example.

Find the shortest distance between two points $P(50°N, 70°W)$ and $Q(50°N, 110°E)$ on the earth's surface in km.



for distance along great circle, we have

$$latitude\ difference = 180° - (50° + 50°)$$

$$= 180° - 120°$$

$$= 80°$$

$$distance = \frac{\theta}{360°} \times 2\pi R$$

$$= 2 \times \frac{80°}{360°} \times \frac{22}{7} \times 6370$$

$$= \mathbf{8897.78\ km}$$

for distance along small circle, we have

$$longitude\ diff = 110° + 70°$$

$$= 180°$$

$$distance\ along\ small\ circle = \frac{\alpha}{360} \times 2\pi R \cos\theta$$

$$distance = \frac{180°}{360°} \times 2 \times \frac{22}{7} \times\times 6370 \cos 50°$$

$$= \mathbf{12868.61\ km}$$

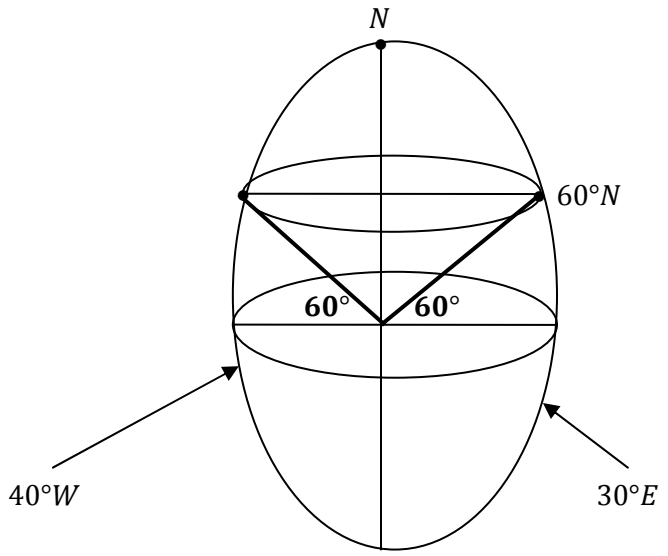
Comparing the distance got along great circle and along parallel of latitude, the distance along great circle is the shortest distance.

SHORTEST DISTANCE ON THE EARTH WHEN LONGITUDE IS NOT DIRECTLY OPPOSITE BUT LYING ON THE SAME LATITUDE.

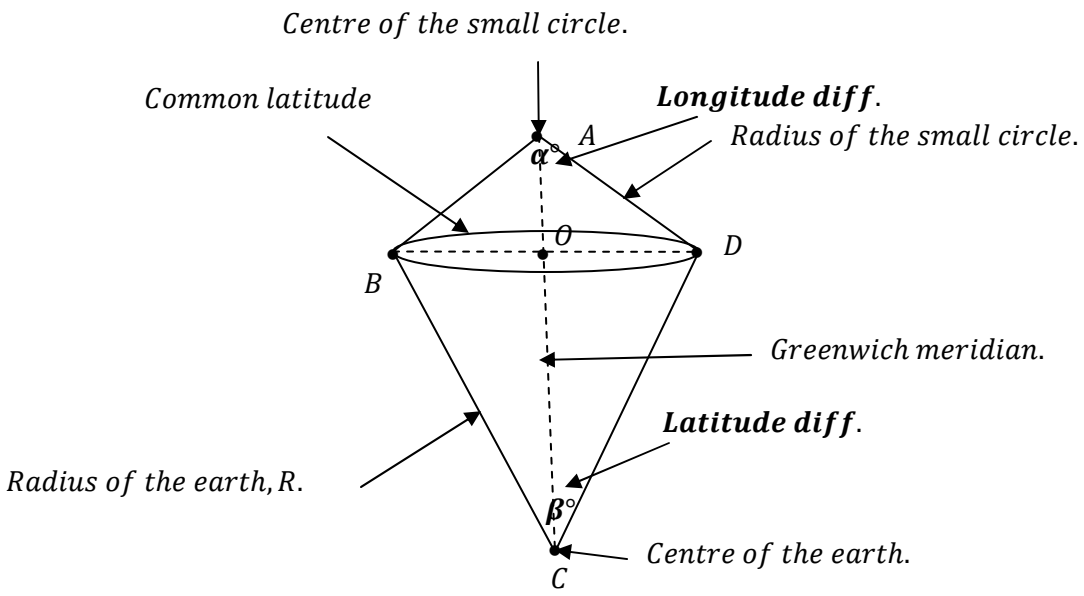
- Sometimes the sum of longitude given may not be directly opposite to each other but lying on the same latitude.
- To find the shortest distance i.e. distance along great circle on the surface of the earth, the following formula is used.

$$\text{shortest distance} = (\frac{\beta}{360^\circ} \times 2\pi R)\text{km}$$
$$= 60\beta\text{nm}$$
$$\text{where } \beta \text{ is the latitude difference}$$

- In this case, the method of calculating latitude different is not the same as we have calculating before i.e.
  - ✓ When longitudes are directly opposite to each other and lying on the same latitude.
  - ✓ When latitude are different and lying on the same longitude.
- Consider the figure below,



- The points are on the same latitude but longitudes not direct opposite to each other.
- In calculating longitude difference consider;

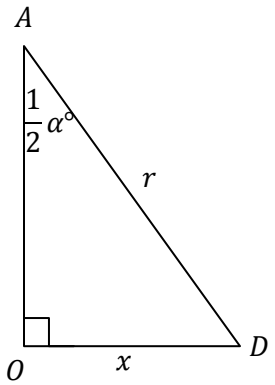


- If the longitudes are lying on the same latitude but they are in the opposite side of Greenwich meridian but in the same hemisphere, longitude difference is got by getting the sum of the longitudes.
- The figure formed above is in the form of a kite.

$$< OAD = \frac{1}{2}\alpha$$

$$< OCD = \frac{1}{2}\beta$$

Let OD be x cm



$$\sin\frac{1}{2}\alpha = \frac{x}{r}$$

$$x = r\sin\frac{1}{2}\alpha$$

but $r = R\cos\theta$

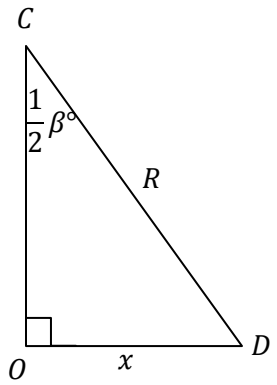
where ;

θ is the common latitude on which the longitude are lying.

α is the longitude difference.

now x reduces to;

$$x = R\sin\frac{1}{2}\alpha\cos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\text{eq. } 1$$



$$\sin\frac{1}{2}\beta = \frac{x}{R}$$

$$x = R\sin\frac{1}{2}\beta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\text{eq. } 2$$

now comparing eq. 1 and eq. 2 we have;

$$R\sin\frac{1}{2}\beta = R\sin\frac{1}{2}\alpha\cos\theta$$

Dividing through by R we have;

$$\mathbf{\sin\frac{1}{2}\beta = \sin\frac{1}{2}\alpha\cos\theta}$$

from the above equation β can be got which is now substituded to the formula.

Examples.

1. Calculate the shortest distance in km between towns $P(60°N, 40°W)$ and $Q(60°N, 30°E)$

Distance is along the great circle

$$= (\frac{\beta}{360°} \times 2\pi R)\text{km}$$

To get β i. e. the latitude difference we use the formula;

$$\sin\frac{1}{2}\beta = \sin\frac{1}{2}\alpha\cos\theta$$

where;

θ is the common latitude on which the longitude are lying.

α is the longitude difference

longitude difference $= 40° + 30°$

$= 70°$.

$$\frac{1}{2}\alpha = \frac{1}{2} \times 70°$$

$$= 35°$$

$$\sin\frac{1}{2}\beta = \sin 35°\cos 60°$$

$$\sin\frac{1}{2}\beta = 0.2867$$

$$\frac{1}{2}\beta = \sin^{-1} 0.2867$$

$$\frac{1}{2}\beta = 16.67°$$

$$\beta = 33.33°$$

now shortest distance

$$= \frac{33.33}{360} \times 2 \times \frac{22}{7} \times 6370$$

$$= \mathbf{3707.04\ km}$$

2. Calculate the shortest distance in km between towns $P(80°S, 110°E)$ and $Q(80°S, 150°W)$

Distance is along the great circle $= 60\beta$nm

To get β i. e. the latitude difference we use the formula;

$$\sin\frac{1}{2}\beta = \sin\frac{1}{2}\alpha\cos\theta$$

where;

θ is the common latitude on which the longitude are lying.

α is the longitude difference

longitude difference $= 150° + 110°$

$= 260°$.

angle that gives shortest distance

$= 360° - 260°$

$= 100°$

$$\frac{1}{2}\alpha = \frac{1}{2}\times 100°$$

$$= 50°$$

$$\sin\frac{1}{2}\beta = \sin 50°\cos 80°$$

$$\sin\frac{1}{2}\beta = 0.13302$$

$$\frac{1}{2}\beta = \sin^{-1} 0.13302$$

$$\frac{1}{2}\beta = 7.644°$$

$$\beta = 15.29°$$

now shortest distance $= 60\beta$

$= 60 \times 15.29$

$= \mathbf{917.31nm}$

3. Find the shortest distance between two points $P(50°N, 24°W)$ and $Q(50°N, 44°E)$ on the earth surface.

Distance is along the great circle

$$= (\frac{\beta}{360°}\times 2\pi R)\text{km}$$

To get β i. e. the latitude difference we use the formula;

$$\sin\frac{1}{2}\beta = \sin\frac{1}{2}\alpha\cos\theta$$

where;

θ is the common latitude on which the longitude are lying.

α is the longitude difference

longitude difference $= 24° + 44°$

$= 68°$.

$$\frac{1}{2}\alpha = \frac{1}{2}\times 68°$$

$$= 34°$$

$$\sin\frac{1}{2}\beta = \sin 34°\cos 50°$$

$$\sin\frac{1}{2}\beta = 0.2867$$

$$\frac{1}{2}\beta = \sin^{-1} 0.35944$$

$$\frac{1}{2}\beta = 21.07°$$

$$\beta = 42.132°$$

now shortest distance

$$= \frac{42.132}{360}\times 2\times\frac{22}{7}\times 6370$$

$= \mathbf{4686km}$

4. An object moves from points $A(30°N, 60°E)$ to $B(30°N, 80°W)$. Find the shortest distance it covered.

Distance is along the great circle

$$= (\frac{\beta}{360°} \times 2\pi R)\text{km}$$

To get β i. e. the latitude difference we use the formula;

$$\sin\frac{1}{2}\beta = \sin\frac{1}{2}\alpha \cos\theta$$

where;

θ is the common latitude on which the longitude are lying.

α is the longitude difference

longitude difference $= 60° + 80°$

$= 140°$.

$$\frac{1}{2}\alpha = \frac{1}{2} \times 140°$$

$$= 70°$$

$$\sin\frac{1}{2}\beta = \sin 70° \cos 30°$$

$$\sin\frac{1}{2}\beta = 0.8138$$

$$\frac{1}{2}\beta = \sin^{-1} 0.2867$$

$$\frac{1}{2}\beta = 54.47°$$

$$\beta = 108.94°$$

now shortest distance

$$= \frac{108.94}{360} \times 2 \times \frac{22}{7} \times 6370$$

$$= \mathbf{12116.25km}$$

5. A ship sails from points $A(80°N, 120°W)$ to $B(80°N, 40°E)$. Find the shortest distance it covered.

Distance is along the great circle

$$= (\frac{\beta}{360°} \times 2\pi R)\text{km}$$

To get β i. e. the latitude difference we use the formula;

$$\sin\frac{1}{2}\beta = \sin\frac{1}{2}\alpha \cos\theta$$

where;

θ is the common latitude on which the longitude are lying.

α is the longitude difference

longitude difference $= 120° + 40°$

$= 160°$.

$$\frac{1}{2}\alpha = \frac{1}{2} \times 160°$$

$$= 80°$$

$$\sin\frac{1}{2}\beta = \sin 80° \cos 80°$$

$$\sin\frac{1}{2}\beta = 0.1710$$

$$\frac{1}{2}\beta = \sin^{-1} 0.1710$$

$$\frac{1}{2}\beta = 9.8466°$$

$$\beta = 19.69°$$

now shortest distance

$$= \frac{19.69}{360} \times 2 \times \frac{22}{7} \times 6370$$

$$= \mathbf{2190.31km}$$

SPEED.

- Is the rate of change of distance with time.
- Speed on the earth are measured in km/hr and nm/hr $(knot)$

$$1\,nm/hr = 1knot$$

SPEED IN KM/HR AND KNOTS.

- To find the speed in km/hr the following steps are done;
  (a) Determine the distance in kilometer either along great circle or small circle.
  (b) Calculate the speed using the time given using the relation;

$$speed = \frac{distance}{time\ taken}$$

- To find the speed in nm/hr(knots) the same procedure above are taken only that the distance is calculated in nautical mile.

Example.

1. A ship sails West from $(20°S, 15°E)$ to $(20°S, 23°W)$ taking 37hrs. Find its speed;
   (a) In km/hr.

$$\text{longitude difference} = 23° + 15°$$
$$= 38°$$
$$\text{distance} = \frac{38}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 20$$
$$= 3971.56 \text{ km}$$
$$\text{speed} = \frac{\text{distance}}{\text{speed}}$$
$$= \frac{3971.56}{37}$$
$$= \mathbf{107.34\ km/hr}$$

   (b) In knots.

$$\text{longitude difference} = 23° + 15°$$

$$= 38°$$

$$\text{distance in nm} = 60\alpha \cos\theta$$

$$= 60 \times 38 \cos 20$$

$$= 2142.50$$

$$\text{speed} = \frac{2142.50}{37}$$

$$= \mathbf{57.91\ knots}$$

2. A plane take off from airport $P(0°, 40°W)$ and flies 1800 nautical miles due East to Q then 1800 nautical miles due South to R and finally 1800 nautical miles due West before landing at S.

(a) Find to the nearest degree the latitudes and longitudes of Q, R and S. (4mks)

from P to Q is along parallel of latitude.

let longitude diff be x

distance $= 60\alpha \cos\theta$

$60x \cos 0° = 1800$

$$x = \frac{1800}{60}$$

$= 30°$

let new longitude at Q be y°E

longitude diff $= 40° + y = 30°$

$y = 30° - 40°$

$= -10°E$

$= 10°W$

position Q(0°, 10°W)

from Q to R is along smaller circle

distance $= 60\theta$

let latitude diff be x

$60x = 1800$

$$x = \frac{1800}{60}$$

$= 30°$

let new latitude at R be y°S

latitude diff $= 0° + y = 30°$

$y = 30°$

position R(30°S, 10°W)

from R to S is along parallel of latitude.

let longitude diff be x

distance $= 60\alpha \cos\theta$

$60x \cos 30 = 1800$

$$x = \frac{1800}{60 \cos 30}$$

$= 35°$

let the new longitude at S be y°W

longitude diff $= y - 10° = 35°$

$y = 35° + 10°$

$= 45°W$

Position S(30°S, 45°W).

(b) If the total flight time is 16 hours, find the average speed in knots for the whole journey. (3mks)

$$total\ distance = 1800nm + 1800n + 1800nm$$

$$= 5400nm$$

$$speed = \frac{total\ distance}{total\ time\ taken}$$

$$= \frac{5400}{16}$$

$$\mathbf{337.5\ knots}.$$

(c) Find the time taken to fly from R to S, given that this was two hours shorter than the time taken from P to Q to R. (3mks)

let the time taken to fly from P to Q be xminutes

let the time taken to fly from Q ton R be y

total time taken to fly from P to Q to R $= (x + y)$hrs

but time taken to fly from R to S is shorter by 2 hours than the time taken to fly from P to Q to R

⇨ Time take to fly from R to S $= [(x + y) - 2]$hrs

tota time is expressed as;

$$[(x + y) + (x + y) - 2] = 16$$
$$x + y + x + y - 2 = 16$$
$$2x + 2y = 16 + 2$$
$$2x + 2y = 18$$
$$2(x + y) = 18$$
$$(x + y) = 9$$

buh $(x + y)$is the time taken to fly from P to Q to R

therefore time taken to fly from R to S $= 9 - 2$

$=$ **7minutes**.

TIME AND MERIDIANS.

- Time of a place is determined in relation to Greenwich mean time.
- The earth takes 24 hrs to make a single rotation of 360°.

therefore;

$$24\text{hrs} = 360°$$
$$1\text{hr} = \frac{360}{24}$$
$$= 15°$$

$$60\text{min} = 15°$$
$$1° = \frac{60\ \text{min}}{15}$$
$$= 4\text{min}.$$

- From the relation above i.e. $1° = 4\ min$, the following summary can be generalized.
  1. A point 1° to the East is 4 minutes ahead of the Greenwich mean time.
  2. A point 1° to the West is 4 minutes behind the Greenwich mean time.
- Therefore for calculation of the local time when one moves along latitude the above should be noted.

NOTE;

- Time taken along the longitude is not counted during local time calculations.

Examples.

1. Find the longitude of a plane for which the local time is 3:00 p.m. when it is mid-day at Greenwich.

converting to 24 hrs clock

$$\begin{array}{r} 1200 \\ 0300 \ + \\ \hline 1500 \end{array}$$

$\text{mid} - \text{day on Greenwich} = 1200\text{hrs}$

time difference between two points is

$$\begin{array}{r} 1500 \\ 1200 \ - \\ \hline 0300 \end{array}$$

$= 3\text{hrs}$

But greenwich is longitude 0°

$3\text{hrs} = 60 \times 3$

$= 180\text{min}$

$4\text{min} = 1$

$:.180\text{min}$

$$= \frac{180 \times 1}{4}$$

$= 45°$

comparing the two points above, the point is to the East of greenwich because the pointgiven is ahead in time compared to the initial point.

longitude of the point is $0° + 45°$

$= \mathbf{45°E}$.

2. Supposing the local time at the Greenwich meridian is 1200hrs, what is the local time at point x which is on longitude $40°E$.

$$\text{we find longitude diff.} = 40° - 0°$$
$$= 40°$$
$$\text{converting into minutes, we have}$$
$$40 \times 4$$
$$= 160 \text{ minutes}$$
$$= 2\text{hrs } 40 \text{ minutes}$$
$$\text{the time of this point is ahead of greenwich meridian hence the local time of the point is}$$
$$\begin{array}{r} 1200 \\ \underline{0240}\ + \\ 1440 \end{array}$$
$$= \mathbf{1440hrs}.$$

3. What is the longitude of a place M whose local time is 8:00 a.m. if the local time at the Greenwich meridian is noon.

$$time\ difference\ between\ two\ points\ is$$

$$\begin{array}{r} 1200 \\ \underline{0800}\ - \\ 0400 \end{array}$$

$$= 4hrs$$

$$But\ greenwich\ is\ longitude\ 0°$$

$$4hrs = 60 \times 4$$

$$= 240min$$

$$4min = 1$$

$$:.240min$$

$$= \frac{240 \times 1}{4}$$

$$= 60°$$

$$the\ time\ at\ M\ is\ behind\ that\ at\ greenich\ meridian.$$

$$so\ M\ must\ be\ to\ the\ West\ of\ greenwich\ meridian\ by\ 60°$$

$$\boldsymbol{longitude = 60°W}$$

4. If the local time at longitude $120°E$ is 3.00 p.m. What will be the local time at longitude $165°E$.

SAMPLE 10 MARKS QUESTIONS

1. P and Q are two airport on latitude $50°N$. P is at longitude $40°W$ and Q is 12865.10 km from P along parallel of latitude to the east. Take $\pi = \frac{22}{7}$ and $R = 6370\ km$.

   (a) Find the position of airport Q

$$\text{let long. diff be x}$$
$$\frac{x}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 50 = 12865.10$$
$$71.4922x = 12865.10$$
$$x = 180°$$
$$\text{let the new longitude at Q be y°E}$$
$$40° + y = 180°$$
$$y = 180° - 40°$$
$$= 140°E$$
$$Q(50°N, 140°E).$$

   (b) Calculate the distance from point P to Q a long a great circle.

$$\text{latitude diff.} = 180° - (50° + 50°)$$
$$= 180° - 100°$$
$$= 80°$$
$$\text{distance} = \frac{80}{360} \times 2 \times \frac{22}{7} \times 6370$$
$$= 8897.78 \text{ km.}$$

   (c) A pilot leaves port P at 8:30 a.m. flying at 750 knots to Q through the shortest distance. At what time did she arrive at Q.

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$
$$1\text{knot} = 1.853 \text{ km/hr}$$
$$\text{:.750 knots}$$
$$= \frac{750 \times 1.853}{1}$$
$$= 1445.34 \text{ km/hr}$$
$$\text{time} = \frac{8897.78}{1445.34}$$
$$= 6 \text{ hrs } 9 \text{ minutes}$$
$$\text{total time} = \begin{array}{r} 0830 \\ 0609 + \\ \hline 1409\text{hrs} \end{array}$$
$$= \mathbf{1409hrs}.$$

2. A plane leaves an airport $A(41.5°N, 36.4°W)$ at 10:00 a.m. and flies due north to airport T on latitude $53.2°N$ at an average speed 650km/hr. Calculate

(a) The distance covered by the plane.

$$\text{latitude difference} = 53.2° - 41.5° = 11.7°$$

$$\text{distance} = \frac{11.7}{360} \times 2 \times \frac{22}{7} \times 6370$$

$$= 1301.3 \text{ km.}$$

(b) After stopping for 30 minutes to refuel at T, the plane then flies due East to airport M, 2500 km from T. Find the position of M.

$$\text{let long. diff. be x}$$

$$\frac{x}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 53.2 = 2500$$

$$x = \frac{2500 \times 360 \times 7}{2 \times 22 \times 6370 \cos 53.2}$$

$$x = 37.5°$$

$$\text{let the new longitude at M be y°E}$$

$$36.4° + y = 37.5°$$

$$y = 37.5° - 36.4°$$

$$y = 1.1°E$$

$$1.1°E$$

$$\mathbf{M(53.2°N, 1.1°E)}.$$

(c) At what time did the plane arrive at M.

$$\text{longitude diff. from T to M} = 37.5°$$

$$= (37.5 \times 4)\text{minutes}$$

$$150 \text{ minutes}$$

$$= 2\text{hrs } 30 \text{ minutes.}$$

$$\text{time taken to fly from T to M} = \frac{2500}{650}$$

$$= 3\text{hrs } 50 \text{ minutes}$$

$$\text{total time taken by the plane}$$

$$\begin{array}{r} 0350 \\ 0230 + \\ \underline{0030} \\ 05110. \end{array}$$

$$= 05:110\text{hrs}$$

$$= 0650.$$

$$\text{time arrived;}$$

$$\begin{array}{r} 1000 \\ \underline{0650} + \\ 1650 \end{array}$$

$$\mathbf{1650hrs}$$

3. Points $D(0°, 24°E), E(0°, 21°W), F(60°S, 120°W)\ and\ G(60°S, 110°E)$ are marked in a globe representing the earth with radius 0.7 m. Take $\pi = \frac{22}{7}$

(a) Find the length of the arc DE. (3mks)

$$\text{it is along great circle.}$$

$$\text{longitude diff.} = 24° + 21°$$

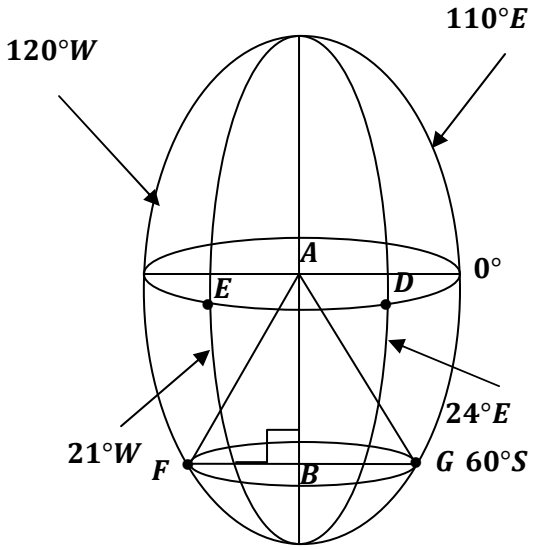
$$= 45°$$

$$\text{distance} = \frac{45}{360} \times 2 \times \frac{22}{7} \times 0.7$$

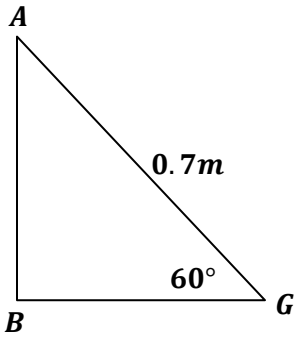
$$= 0.55\text{m.}$$

(b) If A is the centre of the latitude 0° and B is the centre of the latitude $60°S$, Find;

i. Length AB. (3mks)



$< AGB = 60°$.



$$\sin 60° = \frac{AB}{0.7}$$

$$AB = 0.7 \sin 60°$$

$$\boldsymbol{AB = 0.6062m}.$$

ii. The area of the major sector AFG. (4mks)

$$\text{area} = \frac{\beta}{360}\pi R^2$$

To get β i.e. the latitude difference we use the formula;

$$\sin\frac{1}{2}\beta = \sin\frac{1}{2}\alpha\cos\theta$$

where;

θ is the common latitude on which the longitude are lying.

α is the longitude difference

$$\text{longitude difference} = 120° + 110°$$

$$= 230°.$$

$$\frac{1}{2}\alpha = \frac{1}{2}\times 230°$$

$$= 115°$$

$$\sin\frac{1}{2}\beta = \sin 115°\cos 60°$$

$$\sin\frac{1}{2}\beta = 0.4532$$

$$\frac{1}{2}\beta = \sin^{-1} 0.4532$$

$$\frac{1}{2}\beta = 26.95°$$

$$\beta = 53.89°$$

for major sector we find reflex angle

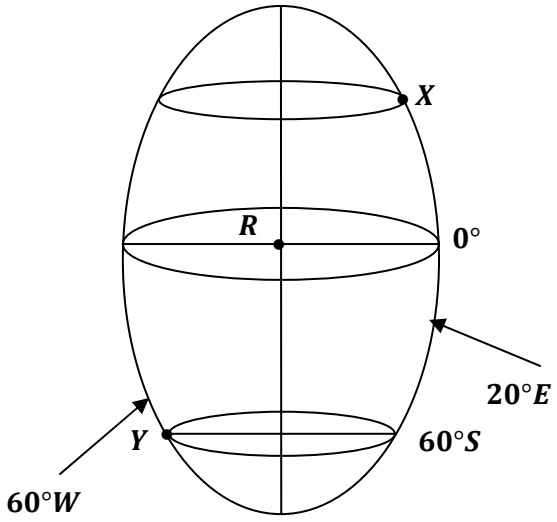
$$= 360° - 53.89°$$

$$= 306.11°$$

$$\text{area} = \frac{306.11}{360}\times\frac{22}{7}\times 0.7^2$$

$$= \mathbf{1.309m^2}.$$

4. Two airplanes P and Q left towns $X(60°N, 20°E)$ and $Y(60°S, 60°W)$ respectively at the same time and flew towards town R which lies on the equator and Greenwich meridian. P flew along the longitude then along the parallel of latitude, while Q flew along the parallel of latitude then along the longitude.
   (a) Given that the speed of P is 800 knots and that the two airplanes took the same time to cover the distance, calculate the speed of Q. (5mks)



distance for P
(60°N, 20°E) to (0°20°E)
latitude diff $= 60° + 0°$
$= 60°$
distance $= 60\theta$nm
60×60
$= 3600$nm
distasnce from $(0°, 20°E)$ to $(0°, 0°)$
longitude diff. $= 20° + 0°$
$= 20°$
distance $= 60\alpha \cos\theta$ nm
$= 60 \times 20 \cos 0$
$= 1200$nm
total distance $= (3600 + 1200)$
$= 4800$nm
distance for Q
from $(60°S, 60°W)$to $(60°S, 0°)$
longitude diff. $= 60° - 0°$
$= 60°$
distance $= 60\alpha \cos\theta$ nm
$= 60 \times 60 \cos 60$
$= 1800$nm
from $(60S, 0°)$to $(0°, 0°)$
latitude diff. $= 60° + 0°$
$= 60°$
distance $= 60\theta$nm
$= 60 \times 60$
$= 3600$nm
total distance $= 3600 + 1800$
$= 5400$nm
time taken by P $= \frac{4800}{800}$
$= 6$hrs
speed of Q $= \frac{5400}{6}$
$= 900$ knots.

(b) The pilot of P recorded his landing time as 2.30 p.m. What was the local time at X. (2mks)

time taken by P to land
longitude diff $= 20°$
$= (20 \times 4)$minutes
$= 80$minutes
$= 1$hr 20minutes
time taken to travel along latitude;
$T = \frac{D}{S} = \frac{1200}{800}$
$= 1$hr 30min
departure time without time loss or gain;

$$\begin{array}{r} 1430 \\ 0130 \; - \\ \hline 1300 \end{array}$$

departure time with time loss;

$$\begin{array}{r} 1300 \\ 0120 \; + \\ \hline 1420 \end{array}$$

$= \mathbf{2{:}\,20\ pm}$.

(c) Find the local time at Y when Q left Y, if Q landed four hour after P had landed. (3mks)

time at which Q landed;

$$\begin{array}{r} 1420 \\ 0400 \; + \\ \hline 1820\text{hrs} \end{array}$$

$= 1820$ hrs.
longitude diff $= 60°$
$= (60 \times 4)$minutes
$= 240$minutes
$= 4$hrs
total time taken by Q to fly;
time at Y $= 8.30$ a. m.
time taken by Q to fly along latitude

$T = \frac{D}{S} = \frac{1800}{900}$
$= 2$hrs
departure time for Q without time gain or loss

$$\begin{array}{r} 1820 \\ = 0200 \; - \\ \hline 1620 \end{array}$$

departure time including time gain

$$\begin{array}{r} 1620 \\ 0400 \\ \hline 1220 \end{array}$$

$= 1220$ hrs.

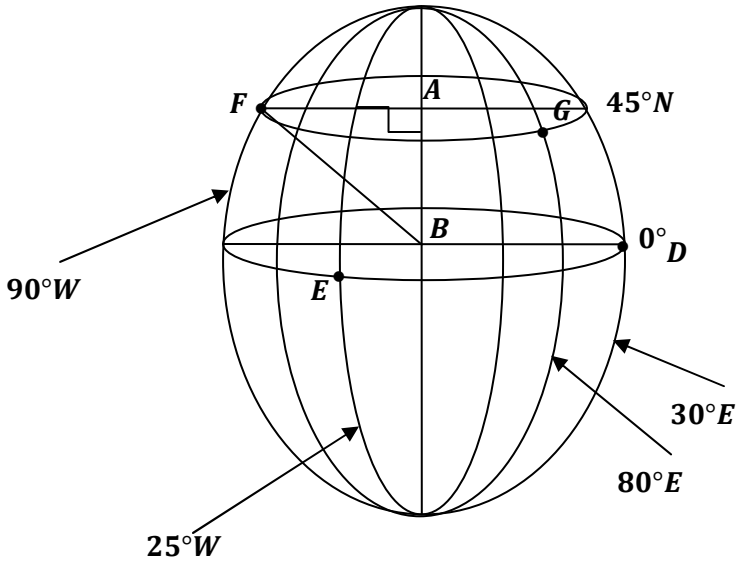
5. Points $D(0°, 30°E), E(0°, 25°W), F(45°N, 90°W)$ *and* $G(45°N, 80°E)$ are marked in a globe representing the earth with radius 28 m. Take $\pi = \frac{22}{7}$

(a) Find the length of the arc DE. (3mks)

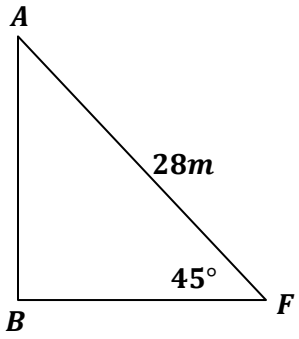
$it\ is\ along\ great\ circle.$
$longitude\ diff. = 30° + 25°$
$= 55°$
$distance = \frac{55}{360} \times 2 \times \frac{22}{7} \times 28$
$= 26.89m.$

(b) If A is the centre of the latitude $45°N$ and B is the centre of the latitude $O°$, Find;

i. Length AB. (3mks)



$< AGB = 60°$.



$$\sin 45° = \frac{AB}{28}$$

$$AB = 28 \sin 45°$$

$$\boldsymbol{AB = 19.80\ m.}$$

ii. The area of the major sector BFG. (4mks)

$$\text{area} = \frac{\beta}{360}\pi R^2$$

To get β i.e. the latitude difference we use the formula;

$$\sin\frac{1}{2}\beta = \sin\frac{1}{2}\alpha\cos\theta$$

where;

θ is the common latitude on which the longitude are lying.

α is the longitude difference

$$\text{longitude difference} = 90° + 80°$$

$$= 170°.$$

$$\frac{1}{2}\alpha = \frac{1}{2}\times 170°$$

$$= 85°$$

$$\sin\frac{1}{2}\beta = \sin 85°\cos 45°$$

$$\sin\frac{1}{2}\beta = 0.7044$$

$$\frac{1}{2}\beta = \sin^{-1} 0.7044$$

$$\frac{1}{2}\beta = 44.78°$$

$$\beta = 89.56°$$

for major sector we find reflex angle

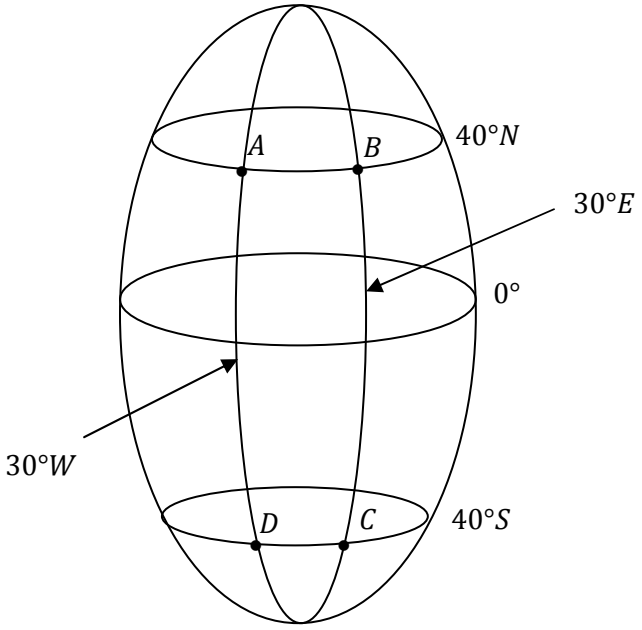
$$= 360° - 89.56°$$

$$= 270.44°$$

$$\text{area} = \frac{270.44}{360}\times\frac{22}{7}\times 28^2$$

$$= \mathbf{1850.98m^2}.$$

6. The figure below shows points on the earth surface.



(a) State the points of A,B,C and D in co-ordinate form. (3mks)

$A(40°N, 30°W)$
$B(40°N, 30°E)$
$C(40°S, 30°E)$
$D(40°S, 30°W)$.

(b) An aircraft flies from A to B along latitude $40°N$, B to C along longitude $30°E$, C to D along latitude $40°S$ and back to A. Calculate the total distance it covered. Take radius of the earth $R = 6371\ km\ and\ \pi = \frac{22}{7}$. (4mks)

distance AB

longitude diff. $= 30° + 30°$

$= 60°$

distance

$$= \frac{60}{360} \times \frac{22}{7} \times 2 \times 6371 \cos 40°$$

$= 5112.87$ km

distance BC

latitude diff. $= 40° + 40°$

$= 80°$

$$\text{distance} = \frac{80}{360} \times 2 \times \frac{22}{7} \times 6371$$

$= 8899.17$ km.

distance CD

longitude diff. $= 30° + 30°$

$= 60°$

distance

$$= \frac{60}{360} \times \frac{22}{7} \times 2 \times 6371 \cos 40°$$

$= 5112.87$ km

distance DA

latitude diff. $= 40° + 40°$

$= 80°$

$$\text{distance} = \frac{80}{360} \times 2 \times \frac{22}{7} \times 6371$$

$= 8899.17$ km

total distance

$= 5112.87 + 6699.17 + 5112.17 + 8899.17$

$= \mathbf{28024.08\ km}$.

(c) If the aircraft leaves A at 8.00a.m at a speed of $750\ km/hr$ to B. At what local time is it expected at B. (3mks)

distance AB

longitude diff. $= 30° + 30°$

$= 60°$

distance

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 40$$

$= 5112.87$ km

$$\text{time} = \frac{5112.87}{750}$$

$= 6$hrs 49minutes

the aircraft is moving to the East of A therefore the time is ahead

longitude diff. $= 60°$

$= (60° \times 4)$minutes

$= 240$minutes

$= 4$hrs

local time at B;

$$\begin{array}{r} 0800 \\ 0649 + \\ \underline{0400} \\ 1849\text{hrs} \end{array}$$

$= \mathbf{1849\ hrs}$.

7. Two towns A and B lie on the same parallel of latitude $60°N$. If the longitude of A and B are $42°W$ and $29°E$ respectively.

(a) Find the distance between A and B in nautical miles along the parallel of latitude. (2marks)

$$longitude\ diff. = 42° + 29°$$

$$= 71°$$

$$distance = 60\alpha \cos\theta\ nm$$

$$= 60 \times 71 \cos 60$$

$$= \mathbf{2130\ nm}.$$

(b) Find the local time at A if at B is 1.00pm. (2marks)

$$if\ 1° = 4minutes$$
$$:.71°$$
$$= 71 \times 4$$
$$= 284\ minutes$$
$$= 4hrs\ 44\ minutes$$
$$local\ time\ ar\ B;$$
$$1300\ _$$
$$\underline{0444\ _}$$
$$1260\ _$$
$$\underline{0444\ _}$$
$$0816hrs$$

$$\mathbf{8{:}16\ \boldsymbol{a.m.}}$$

(c) Find the shortest distance between A and B along the earth's surface in km ($Take\ \pi = \frac{22}{7}\ and\ R\ =\ 6370km$). (3marks)

Distance is along the great circle

$$= (\frac{\beta}{360°} \times 2\pi R)\text{km}$$

To get β i.e. the latitude difference we use the formula;

$$\sin\frac{1}{2}\beta = \sin\frac{1}{2}\alpha\cos\theta$$

where;

θ is the common latitude on which the longitude are lying.

α is the longitude difference

$$\text{longitude difference} = 42° + 29°$$
$$= 71°.$$
$$\frac{1}{2}\alpha = \frac{1}{2} \times 71°$$
$$= 35.5°$$
$$\sin\frac{1}{2}\beta = \sin 35.5° \cos 60°$$
$$\sin\frac{1}{2}\beta = 0.29035$$
$$\frac{1}{2}\beta = \sin^{-1} 0.29035$$
$$\frac{1}{2}\beta = 16.87899°$$
$$\beta = 33.758°$$

now shortest distance

$$= \frac{33.758}{360} \times 2 \times \frac{22}{7} \times 6370$$
$$= \mathbf{3754.64\ km}$$

(d) If C is another town due South of A and $10010km$ away from A, find the co-ordinates of C. (3marks)

$$\text{distance} = \frac{\theta}{360} \times 2\pi R$$
$$= \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 6370$$
$$\theta = \frac{10010 \times 7 \times 360}{2 \times 22 \times 6370}$$
$$= 90°$$

let the new latitude at C be y°S

$$60° + Y = 90°$$
$$Y = 90° - 60°$$
$$= 30°$$

$$\mathbf{C(30°S, 42°W)}.$$

8. A plane J flies from a point $K(40°N, 45°W)$ to a point $L(35°N, 45°W)$ and then onto a point $M\ (35°N, 135°E)$
   (a) Determine the distance from K to L in km. (Take R = 6370 km) (2marks)
   (b) Determine in nautical miles
      i. The shortest distance between L and M along the great circle (2marks)
      ii. The longest distance between L and M along small circle (to the nearest tens) (2marks)
   (c) Find the difference in time taken to the nearest minute when J flies along the shortest and longest routes if its speed is 420 knots in both cases. (4marks)

9. Two towns X and Y lie on the same latitude in the southern hemisphere. When it is 9.00am at X, the time at Y is 11.00am.
   (a) Given that the longitude of X is $12°E$, find the longitude of Y. (3marks)

   (b) A plane leaves X for Y and takes 2 hours to arrive at Y travelling at 600km/hr along a parallel of latitude. Find.

      i. The radius of circle of latitude on which towns X and Y lies. (3marks)
      ii. The positions of the two towns. (4marks)

10. The positions of airport P and Q are $(\ 60°N, 45°W)\ and\ (60°N, k°E)$ respectively. It takes a plane $5\ hrs$ to travel due East from P to Q at an average speed of 600 knots. By taking R = 6370km and $\pi = \frac{22}{7}$
    a) Calculate the value of k. (3marks)
    b) The local time at P is 10.45a.m. what is the local time at Q when the plane reaches there. (4marks)
    c) Find the distance PQ measured along a circle of latitude to the nearest km. (3marks)

11. P and Q are two airports on latitude $50°N$. P is at longitude $40°W$ Q is 12865.10 km from P along a parallel of latitude to the east.
    (a) Find the position of airport Q
    (b) Calculate the distance from point P to Q along a great circle.
    (c) A pilot leaves port P at 8:30 a.m. flying at 780knots to Q through the shortest distance. At what time did she arrive at Q. $(1knot = 1.853\ km/hr, R = 6370km, \pi = \frac{22}{7}$.

12. A plane leaves an airport $A(41.5°N, 36.4°W)$ at 10:00a.m and flies due north to airport T on latitude $53.2°N$ at an average speed of 650km/hr. Calculate;
    (a) The distance covered by the plane.
    (b) After stopping for 30 minutes to refuel at T, the plane then flies due east to airport M, 2500km from T. Find;
       i. The position of M.
       ii. At what time did the plane arrive at M.

13. $A(30°N, 20°W), B(30°N, 40°E), C(60°N, Y°E)$ and $D(X°N, Z°W)$ are on the surface of the earth. C is due north of B and D is due west of C and also due north of A.
    (a) Find the value of X,Y and Z. (3mks)
    (b) If the distances are measured along the circle of latitudes and meridians. Find the distance of C from A in kilometer along route ABC at a speed of 750km/hr and the other flies along the route ADC at a speed of 556km/hr. Which plane reaches C earlier and by what time? (7mks)

14. The position of two cities A and B are $A(30°S, 60°W)$ and $B(30°S, 25°E)$. Find;
    (a) The difference in longitude between A and B.
    (b) The distance between A and B along a circle of latitude;
        i. In nautical miles
        ii. In kilometres.
    (c) A city C is 300km due north of B;
        i. Find the latitude C.
        ii. State the position of C on the earth surface.

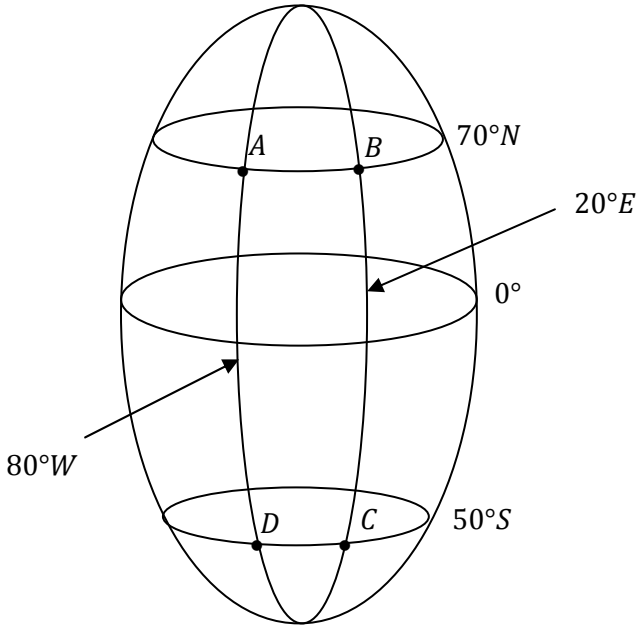
15. A plane leaves an airport $A(40°S, 36°W)$ at 9:00a.m and flies due north to airport $B(50°N, 36°W)$.
    (a) Find the distance covered by the plane in kilometers.
    (b) After stopping for 30 minutes to refuel at B the plane then flies due east to airport C, 3500km from B. Find;
        i. The position of C. (3mks)
        ii. The local time at C if its average speed from A to C is 1200km/hr. (4mks)

16. Two towns A and B lie on the same latitude in the northern hemisphere. When it is 8:00a.m. at A, the time at B is 11:00a.m
    (a) Given that the longitude of A is $15°E$, find the longitude of B. (3mks)
    (b) A plane leaves A for B and takes $3\frac{1}{2}hrs$ to arrive at B travelling along a parallel of latitude at 850km/hr. Find;
        i. The radius of the circle of latitude on which A and B lie. (4mks)
        ii. The latitude of two towns. (3mks)

17. Two aircrafts travel from the position $P(30°N, 130°W) to\ Q(50°N, 170°E)$ all leaving at 0845hrs and at the same speed of 500km/hr. Aircraft A travels due north to the position $(50°N, 130°W)$ and then along a parallel of latitude using the shortest route. Aircraft B travels along a parallel of latitude $(30°N, 170°E)$ and then due north.
    (a) If a third aircraft C had left a point $R(30°N, 10°W)$ at the same time as the two above aircrafts left P and flew via the shortest possible distance to point Q, calculate its position when the aircraft A was passing the longitude $180°W$. (5mks)
    (b) Calculate the arrival local time of the two aircrafts. (5mks)

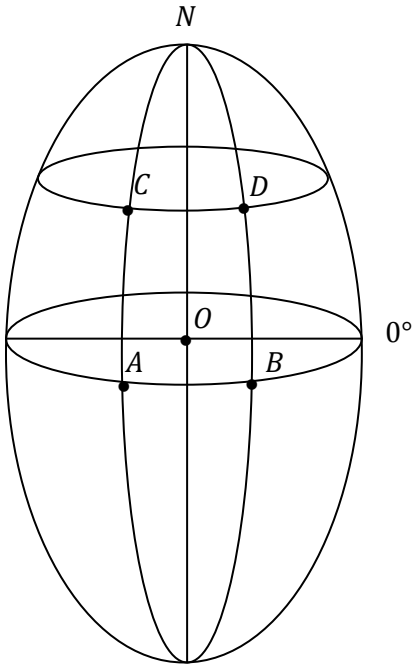
18. A ship leaves an island $(5°N, 45°E)$ and sails due east for 120 hours to another island. The average speed of the ship is 27knots.
    (a) Calculate the distance between the two islands;
        i. In nautical miles
        ii. In kilometers.
    (b) Calculate the speed of the ship in kilometers per hour.
    (c) Find the position of the second island

19. The figure below shows the globe of the earth.



(a) State the coordinates of point A. (2mks)
(b) Calculate in kilometers, the distance between A and C via B. (3mks)
(c) A plane travels due north from A for a distance of 420nm. Give the coordinates of its new position after covering the distance. (3mks)

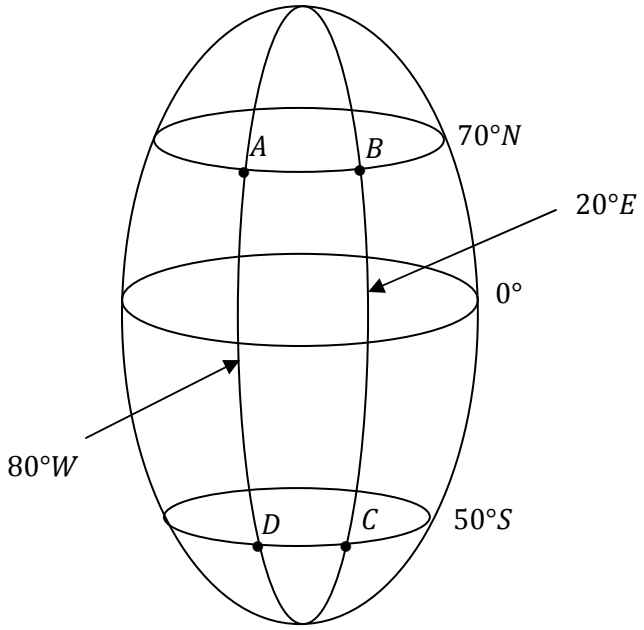
20. A,B,C and D are four points on the surface of the earth and N is the north pole.



(a) Calculate the distance, in nautical miles between $A(0°, 30°W)$ and $B(0°, 60°E)$, measured along the equator. (2mks)
(b) An aircraft flies from A to B, then to N and finally back to A. Find the total distance travelled in nautical miles. (2mks)
(c) Calculate the distance between $C(55°N, 30°W)$ and $D(55°N, 60°E)$ (2mks)

(d) Another aircraft travels from A due north to C then due east to D and finally due south to B. Find in nautical miles the total distance travelled, and total flying time, given that the aircraft flies at a steady speed of 160knots. (4mks)

21. Given that $A(70°N, 80°W), C(50°S, 70°E)$ as shown below.



(a) Calculate in kilometers the distance between A and C via B.
(b) A plane travels due north from A for a distance of 600nm. Give the coordinates of its new position after covering this distance.
(c) State the local time at A if the time at C is Monday 8:00a.m

22. The positions of two towns on the earth's surface are $A(40°S, 45°W)$ and $B(40°S, 135°E)$.
(a) Calculate the difference in distance between towns A and B along the parallel of latitude and along the great circle in nautical miles (nm).
(b) Two planes X and Y left town A at 8:00 a.m. flying at 750 knots each towards town B. If plane X flies along the parallel of latitude and plane Y along the great circle, determine the position of one of the planes when the other lands at town B.

23. P and Q are two towns on a model globe of radius 25cm. They both lie on the parallel of latitude $50°N$. P has longitude $90°W$ and Q has longitude $90°E$. A piece of string AB has one end a on P and the other end B just reaches Q when the string is stretched over the north pole.
(a) Calculate the length AB of the string.
(b) If instead the string with the end A still on P is laid along the parallel of latitude $50°N$ running east from P, calculate the longitude reached by end B.
(c) State the position reached by end B if the string was stretched along the great circle of point P towards the south pole if end A still remained on point P.

24. An aeroplane leaves town $A(83°N, 155°W)$ to town $B(40°N, 25°E)$ using the shortest route at a speed of 450knots.
(a) Calculate;
(b) From B the plane flies westwards along the latitude to town $C(40°N, 13°W)$, calculate the distance BC in kilometers.
(c) From town C, the plane took off at 3:10p.m. towards town $D(10°N, 13°W)$, at the same speed. At what time did the plane land at D.

25. Two aircrafts X and Y took off at the same time from an aeroplane at $P(84°S, 20°E)$. The aircraft X flew due west while Y flew due east.
    (a) Calculate the total length of the latitude in kilometers at which they are flying.
    (b) Determine the position of town Q that is situated north of the aeroplane P and whose distance from P is equal to 3 times the total length of the latitude above.
    (c) Given that the two aircrafts met at $(84°S, k°W)$ and the ratios of the speed X:Speed of Y$= 8:7$, find the value of k.

26. Three towns are located on earth's surface at coordinates $A(10°N, 35°E), B(30°S, 35°E)$ and $C(10°N, 25°W)$. An aeroplane P travelled from A to B using the shortest route possible. Another aeroplane Q left A when the local time was 2.30p.m. and travelled to C along a circle parallel to latitude.
    (a) Calculate;
        i. Shortest distance in kilometre travelled by aeroplane Q from A to B.
        ii. The distance in kilometre travelled by aeroplane Q between tows A to C.
    (b) If aeroplane Q took 8 hours 40 minutes to travel from town A to C, find the local time in C when aeroplane Q arrived.

27. Three towns are located on earth's surface at coordinates $A(15°N, 55°E), B(30°S, 45°E)$ and $C(15°N, 90°E)$. A jet fighter P flies from A to B in 6hours 40 minutes using the shortest route between the two towns. A similar jet fighter Q takes off from town A at the same time as P and flies to town C along the parallel of latitude.
    (a) Calculate the shortest distance between towns A and B in kilometre. (4mks)
    (b) Find the distance in kilometre travelled by jet Q between towns A and C. (4mks)
    (c) If the two jets fly at the same speed, determine to the nearest minutes, which jet reaches its destination earlier and by how long. (2mks)

28. The positions of two towns M and N on the earth's surface are $(38°N, 48°E)$ and $(38°N, 132°W)$respectively.
    (a) Find the difference in longitude between town M and N. (2mks)
    (b) Calculate;
        i. The distance in kilometre from M to N along a parallel of latitude. (2mks)
        ii. The shortest distance from M to N along a great circle. (3mks)
    (c) Another town P, is 840km east of town N and on the same latitude as town M and N. Find the longitude of P to the nearest whole number. (3mks)

29. The positions of two towns on the earth's surface are $A(30°S, 20°W)$ and $B(30°S, 80°E)$. Find;
    (a) The difference in longitude. (2mks)
    (b) The distance between the two towns along a parallel of latitude in;
        i. Kilometre. (2mks)
        ii. Nautical miles (nm) (2mks)

30. Three towns are located on earth's surface at coordinates $P(60°N, 35°W), Q(75°N, 145°E)$ and $R(60°N, 25°E)$. A commercial airliner A whose speed is 600km/hr flies from P to Q using the shortest route possible. A similar airliner B takes off from B at the same time as A and flies to town R along the parallel of latitude. Both airliners fly at the same speed. Giving your answer to the nearest minutes, determine which airliner arrives at its destination earlier and by how long. (10mks)

31. A plane flies from town $A(20°S, 42°W)$ along a parallel of latitude to town B on longitude $35°E$ at a speed of 280knots.
    (a) Calculate the distance covered in nautical mile. (2mks)
    (b) After 30 minutes rest at B, it flies due north at speed of 350km/hr to town C, on longitude $18°N$. Calculate the distance BC in kilometre, correct to two decimal places. (2mks)
    (c) It rest at C for 1 hour and then proceeds to town D on the same latitude due West. The journey from C to D takes 4 hours at a speed of 542.1 knots. Find the position of D. (3mks)
    (d) The plane took off at town A when the local time there was 6:30 a.m. on Tuesday. Determine the local time of town C and day when the plane landed there. (3mks)

32. Two cities $P(60°N, 40°E)$ and $Q(18°S, 40°E)$ are on the surface of the earth. A commercial plane flies from P to Q by the shortest route between the two cities.
    (a) Calculate the shortest distance between P and Q.
        i. In nautical miles. (2mks)
        ii. In kilometre. (2mks)
    (b) The speed of the plane is 410 knots. Calculate how long it takes to fly from P to Q. (3mks)
    (c) If the plane flew to town $R(18°S, 80°E)$, determine the distance from Q to R along the parallel of latitude in kilometre. (3mks)

33. A plane flying at 360 knots left airport $A(40°N, 30°W)$ and flew due south to airport $B(20°S, 30°W)$.
    (a) Calculate the distance covered by the plane in nautical miles.
    (b) After 20 minutes stop at B, the plane flew due East to airport $C(20°S, 60°E)$ at an average speed of 1000 knots. At what time did it reach C from A through B if it left airport A at 1935 hours

34. In a model of the earth surface whose radius is 21 cm, a boy marked three points $P(60°N, 55°W)$, $Q(20°S, 55°W)$ and $S(60°N, 125°E)$.
    (a) Calculate the distances in cm of;
        i. PQ
        ii. PS along the parallel of latitude.
        iii. The shortest distance between Q and S.
    (b) A plane leaves town $T(84°S, 10°E)$ and flies westwards for $6\frac{1}{2}\ hrs$ at a speed of 268 knots and reaches town $R(84°S, X°)$. Find the longitude X.

35. An aeroplane leaves town $A(75°N, 60°W)$ to town $B(55°N, 20°E)$ using the shortest route at a speed of 500 knots.
    (a) Calculate the distance from A to B in nautical miles.
    (b) Calculate the time taken to travel from town A to B.
    (c) From town B, the aeroplane flies along the latitude eastwards to town $C(55°N, 60°E)$. Calculate the distance in kilometre from B to C.
    (d) While at town C the pilot receives an emergency call from town $D(55°N. 36°E)$ at 1.20 p.m. Greenwich mean time. He took $2\frac{1}{2}\ hrs$ to get to D. At what time did he arrived at town D.

36. Two towns A and B lie on the same parallel of latitude $60°N$. If the longitude of A And B are $42°W$ and $29°E$ respectively.
    (a) Find the distance between A and B in nautical miles along the parallel of latitude.
    (b) Find the local time at A if at B is 1.00 p.m.
    (c) Find the shortest distance between A and B along the earth's surface in kilometre.
    (d) If C is another town due south of A and 10010 km away from A, find the coordinate of C.

37. Town P is at $30°S, 20°E$) and Q is $30°S, 30°W$). A plane leaves town P at 8:00 a.m. travelling at a speed of 850km/hr to town Q.
    (a) What time was it at Q when the plane leaving town Q.
    (b) How long does the plane take to town Q.
    (c) After reaching Q, the plane continues due west for a further one-hour and reaches town C. Give the coordinates of the position of C.

38. Two aircrafts A_1 and A_2 took off at the same time from an aero dome at $P(64°N, 20°E)$ and flies by the shortest route to another airport at $B(X°N, 144°W)$ covering a total distance of 4800nautical miles.
    (a) Find the latitude $X°N$
    (b) If instead the aircraft had flown along the latitude and then along the Meridian to point B, find how longer it would have flown in nautical miles.
    (c) If the aircraft arrived at airport B at 1846hrs local time, what was the local time at airport A at that time.

39. Points $P(30°N, 20°W), Q(30°N, 40°E), R(60°N, a°E)$ and $S(b°N, c°W)$ are four points on the surface of the Q and is due West of R and due north of P.
    (a) State the values of a, b and c.
    (b) Given that all distances are measured along parallel of latitudes or along meridians and in nautical miles, find the distances of R from P using two alternatives routes via Q and S.
    (c) Two pilots starts flying from P to R one along the route PQR at 400 knots and the other along PSR at 300 knots. Which one reaches R earlier and by how long?

40. The positions of two towns on the earth's surface are $A\ (40°S, 45°W)$ and $B\ (40°S, 135°E)$ Calculate:
    (a) The difference in distance between towns A and B along the parallel of latitude and along the great Circle (in nm) (4marks)
    (b) Two planes X and Y left town A at 8:00 a.m. flying at 758 knots each towards town B. If plane X flies along the parallel of latitude and plane Y along the great circle; then determine the position of Plane X when plane Y lands at town B (4marks)
    (c) What is the local time at town B when the second plane lands (2marks)

41. A globe representing the earth has a radius of 0.5m, points $A\ (0°, 10°W), B(0°, 35°E), P(60°N, 110°E)$ and $Q(60°N, 120°°W)$ are marked on the globe.
    (a) Find the length of arc AB, leaving your answer in terms of π. (3marks)
    (b) If O is the centre of the latitude $60°N$, find the area of the minor sector OPQ. (4marks)
    (c) If the local time at Q is 10.30 am on Monday. Determine the local time at P. (3marks)

42. Boat P leaves port $A(45°N, 50°W)$ and sails at an average speed of 10 knots. It sails due east along a parallel of latitude of latitude to $B(45°N, 42°W)$ and then sails due north to $C(48°N, 42°W)$. Another boat Q leaves $D(55°N10°W)$ at the same time as P leaves A. It sails due west and then due south to meet port P.

    (a) How long does it take boat P to reach point C? (4mks)
    (b) If boat Q sails at the same speed as boat P, how long does the boat take to reach the point.(4mks)
    (c) At what speed would boat Q have sailed to reach point C at the same time as boat P. (2mks)

43. A globe representing the earth has a radius of 0.5 m: points $A(0°, 10°W), B(0°, 35°E)$, $P(60°N, 110°E)$ and $Q(60°N, 120°W)$ are marked on the globe.
    (a) Find the length of arc, leaving in terms of π
    (b) If O is the centre of the latitude $60°N$, find the area of the minor sector OPQ.
    (c) If the local time at Q is 10:30 am. On Monday. Determine the local time at P.

44. An aeroplane flies from a point $P(60°N, 45°W)$ to a point $Q(60°N, 135°E)$. Given that the radius of the earth is 6370km,
    (a) Calculate the shortest distance between P and Q.
        i. In kilometre. (3mks)
        ii. In nautical miles. (1mk)
    (b) If the plane flew at a speed of 600 knots, how long did it take to move from P to Q? (2mks)
    (c) The plane left P at 10:00 a.m. on Monday. At what time did it arrive at Q if it travelled along a parallel of latitude at the same speed. (4mks)

45. An aircraft leaves town $P(30°S, 14°W)$ and moves directly east to town Q at a speed of 270 knots for 12 hours. Determine:
    (a) The distance moved in nautical miles.
    (b) The distance moved in kilometre. (Take 1nm=1.853km)
    (c) The position of town Q
    (d) The local time at Q if the local time P is 9:13 p.m. give the answer to the nearest minutes

46. A plane leaves airport $A(42°N, 35°W)$ at 9:00 a.m. and flies due north to airport B on latitude $75°N$.
    (a) Calculate the distance covered by the plane in;
        i. Kilometre
        ii. Nautical miles
    (b) After stopping at B for 40 minutes, the plane flew due east to an airport C 1500km from B. Find;
        i. The position of C. (3mks)
        ii. The local time at C when the plane landed there given that the plane maintained a constant speed of 500km/hr