

KREATRYX

K Notes

NETWORKS

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Manual for K-Notes

Why K-Notes?

Towards the end of preparation, a student has lost the time to revise all the chapters from his / her class notes / standard text books. This is the reason why K-Notes is specifically intended for Quick Revision and should not be considered as comprehensive study material.

What are K-Notes?

A 40 page or less notebook for each subject which contains all concepts covered in GATE Curriculum in a concise manner to aid a student in final stages of his/her preparation. It is highly useful for both the students as well as working professionals who are preparing for GATE as it comes handy while traveling long distances.

When do I start using K-Notes?

It is highly recommended to use K-Notes in the last 2 months before GATE Exam (November end onwards).

How do I use K-Notes?

Once you finish the entire K-Notes for a particular subject, you should practice the respective Subject Test / Mixed Question Bag containing questions from all the Chapters to make best use of it.

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Network Elements

Active & Passive Elements

If any elements absorb, dissipate, waste, convert electrical energy it is called as passive element.
Eg. Resistor, Inductor, Capacitor.
If any elements energize, deliver, give out, drive the electrical energy it is called as active element.
Eg. BJT, MOSFET.

Network Technologies

Node : It is a point of interconnection or junction between two or more components.
Branch : It is an elemental connection between two nodes.
Mesh: A mesh is a close path which should not have any further closed path in it.
Loop : All possible close path.

Ohm's law

At constant temperature and for uniform cross section of conductor.

$J = \sigma E$

σ= conductivity,

$\rho = \frac{1}{\sigma} =$ resistivity .

$V = IR$

$R = \frac{\rho l}{A}$

Circuit Symbol: 

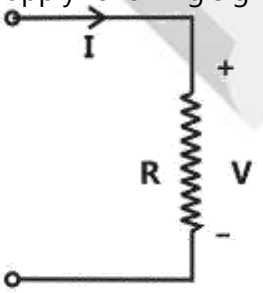
$l =$ lenght of conductor

A = Area of conductor.

Conductance of circuit elements is

$G = \frac{1}{R}$

Sign Convention
To apply ohm's law, we must apply following sign convention.



Short circuit & open circuit

Voltage across terminals of a short circuit is always zero, regardless of the value of current which could be any value. (R = 0)

The current through an open circuit is always zero, regardless of voltage across the terminals which could be any value. $(R = \infty)$

Power of resistor

$$P = VI = \frac{V^2}{R} = I^2R$$

Resistance always absorbs or dissipates power.

Kirchoff's laws

- Kirchoff's current law(KCL)

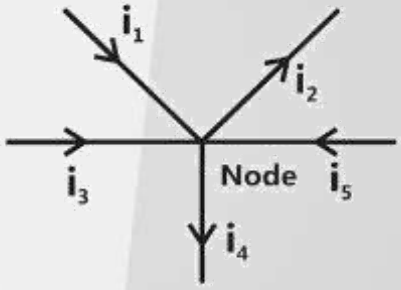
  It states that any instant the algebraic sum of current leaving any junction (or node) in a network is zero.

  In other words, current entering a node is equal to current leaving the node.

$$\sum_n i_n(t) = 0$$

$$\sum i_{entering} = \sum i_{leaving}$$

$$i_1 + i_3 + i_5 = i_2 + i_4$$



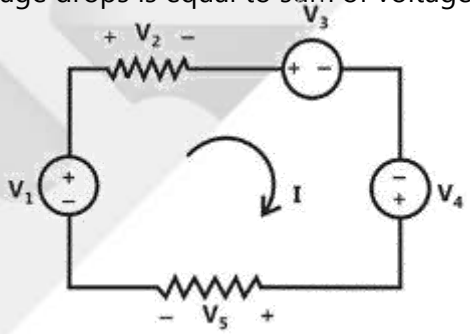
- Kirchoff's voltage law (KVL)

  It states that any instant the algebraic sum of the voltage around any closed path (or loop) within a network is zero. In other words, the sum of voltage drops is equal to sum of voltage rises.

$$\sum_n V_n(t) = 0$$

$$\sum V_{drop} = \sum V_{rise}$$

$$V_1 - V_2 - V_3 + V_4 - V_5 = 0$$



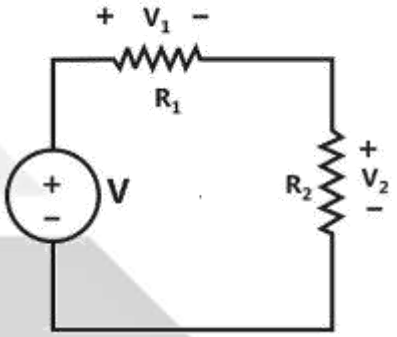
Series resistance or voltage division

Two or more circuit elements are connected in series means that current through all elements in same. If 'N' resistors, with resistance $R_1, R_2, ...R_n$ are connected in series

$R_{eq} = R_1 + R_2 + ...R_N$

$$V_1 = \frac{V(R_1)}{R_1 + R_2}$$

$$V_2 = \frac{V(R_2)}{R_1 + R_2}$$



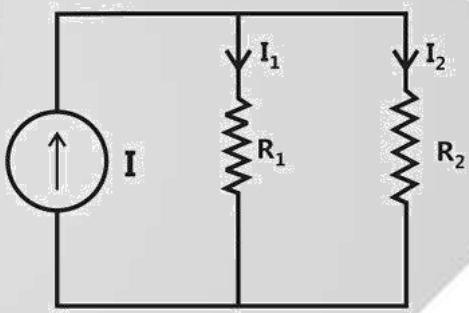
Parallel resistance or current division

Two or more circuit elements are connected in parallel means that voltage across all elements is same. If 'N' resistors are connected in parallel $R_1, R_2, ...R_N$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N}$$

$$I_1 = \frac{I(R_2)}{R_1 + R_2}$$

$$I_2 = \frac{I(R_1)}{R_1 + R_2}$$



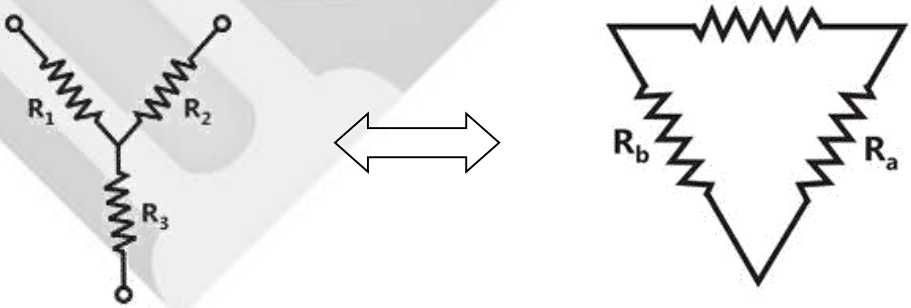
Star Delta Conversion

Start to Delta

$$R_a = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}$$

$$R_b = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_c = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$



Delta to start conversion

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Sources

Independent Voltage Source

An ideal independent voltage source maintains a specified voltage across its terminals. The voltage is independent of current flowing through it.

Independent current source

An ideal independent current source maintains a specified current to flow through it. The current through this is independent of voltage across it.

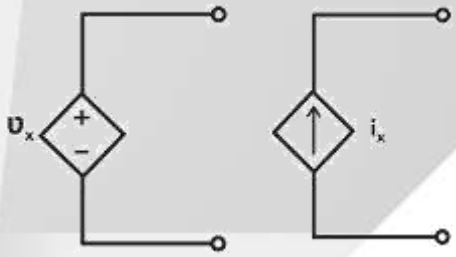
Dependent Source

Voltage controlled voltage source (VCVS) ; $V = AV_X$

Current controlled voltage source (CCVS) ; $V = Ai_X$

Voltage controlled current source (VCCS) ; $i = AV_x$

Current controlled current source (CCCS) ; $i = Ai_x$



Capacitor

A capacitor is a combination of a two conducting plates separated by a non-conducting material.

Capacitance is donated by 'C'

$$C = \frac{\varepsilon A}{d}$$

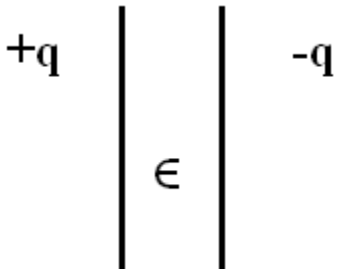
ϵ= Permittivity of medium

A = Area of Plates

D = distance between the plates.

Charge on Plates, Q = CV

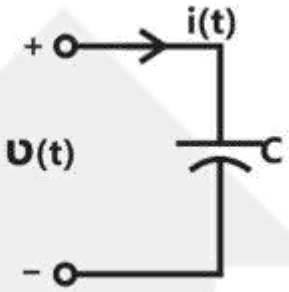
V = Potential difference between the plates.



Voltage Current relationship

$$i(t) = \frac{dq(t)}{dt}$$

$$i(t) = C\frac{dv(t)}{dt}$$



Sign Convention

Energy Stored in a capacitor

$$E = \frac{1}{2}cv^2(t) = \frac{\left(Q(t)\right)^2}{2C} = \frac{1}{2}Q(t)v(t)$$

Properties of ideal capacitor

- If voltage across capacitor is constant (dc) then current through capacitor is zero & it acts as open circuit.
- The voltage across capacitor must be continuous, if it as discontinuous, then $i = C\frac{dv(t)}{dt}$ is infinite, so voltage across capacitor cannot change instantaneously.
- A capacitor never dissipates energy, it only stores energy.

Capacitor in series & parallel

- In 'n' capacitors are connected in series $C_1, C_2, \ldots\ldots\ldots\ldots C_n$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots\ldots\ldots\ldots \frac{1}{C_n}}$$

- If 'n' capacitors are connected in parallel, $C_1, C_2, \ldots\ldots\ldots\ldots\ldots\ldots C_n$

$$C_{eq} = C_1 + C_2 + \ldots\ldots\ldots\ldots\ldots + C_n$$

In series connection, charge is same whereas in parallel connection voltage is same.

Inductor

It is a two terminals element consisting of winding of 'N' turns.

$L = \frac{\mu_0 \mu_r N^2 A}{l}$

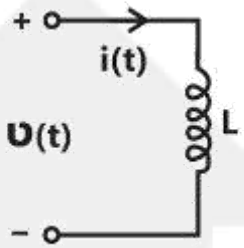
μ_0 = Permeability of free space

μ_r = relative Permeability

N = number of turns

A = area of cross section of coil

l = length of inductor



Current voltage relationship

$$v(t) = L\frac{di(t)}{dt}$$

L is constant, called as inductance

Energy Stored

$$E = \frac{1}{2}L\left(i(t)\right)^2$$

Like, Capacitor, inductor also stores energy but in electro-magnetic terms.

Series & parallel Connection

If 'n' inductors $L_1, L_2, \ldots\ldots\ldots\ldots, L_n$ are connected in series then

$$L_{eq} = L_1 + L_2 + \ldots\ldots\ldots\ldots + L_n$$

In 'n' inductors are connected in parallel $L_1, L_2, \ldots\ldots\ldots\ldots L_n$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots\ldots\ldots\ldots + \frac{1}{L_n}$$

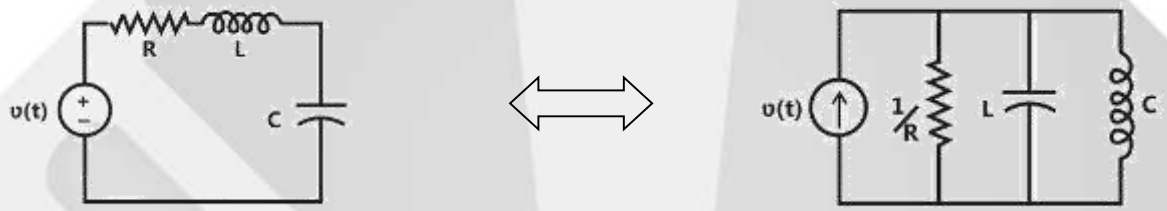
In series connection current in same, through all elements & in parallel connection voltage is same across all elements.

Duality

Two circuits N_1 & N_2 are called dual circuit if the branches KCL, KVL & branch v - i relationship becomes respectively KVL, KCL.

Dual Elements			
Resistance	RΩ	Conductance	G = RƱ
Capacitance	C F	Inductance	L = C H
Inductance	L H	Capacitance	C = L F
Open Circuit		Short Circuit	
Short Circuit		Open Circuit	
Voltage Source	V_S	Current Source	$I = V_S$
Current Source	I_S	Voltage Source	$V = I_S$
Series Connection		Parallel Connection	
Parallel Connection		Series Connection	

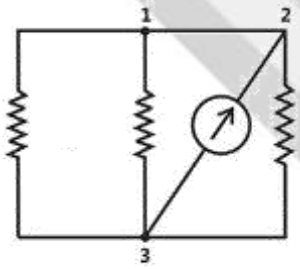
Eg.



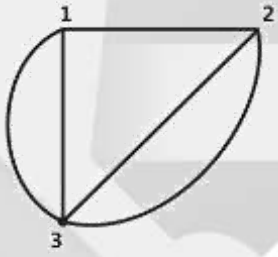
Graph Theory

Network Graph:

If all elements of a circuits are replaced a line segment between 2 end points called as nodes.



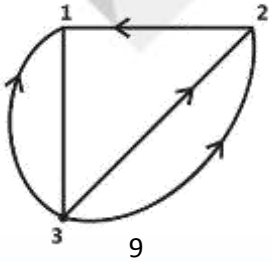
(a) Network



(b) Graph

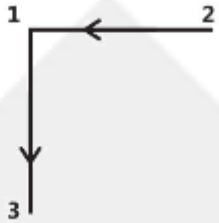
Directed Graphs:

If the branches of a graph has directions then it is called as a directed graph.



Sub graph

It consists of less or equal number of verticals (nodes) & edges, as in its complete graph.

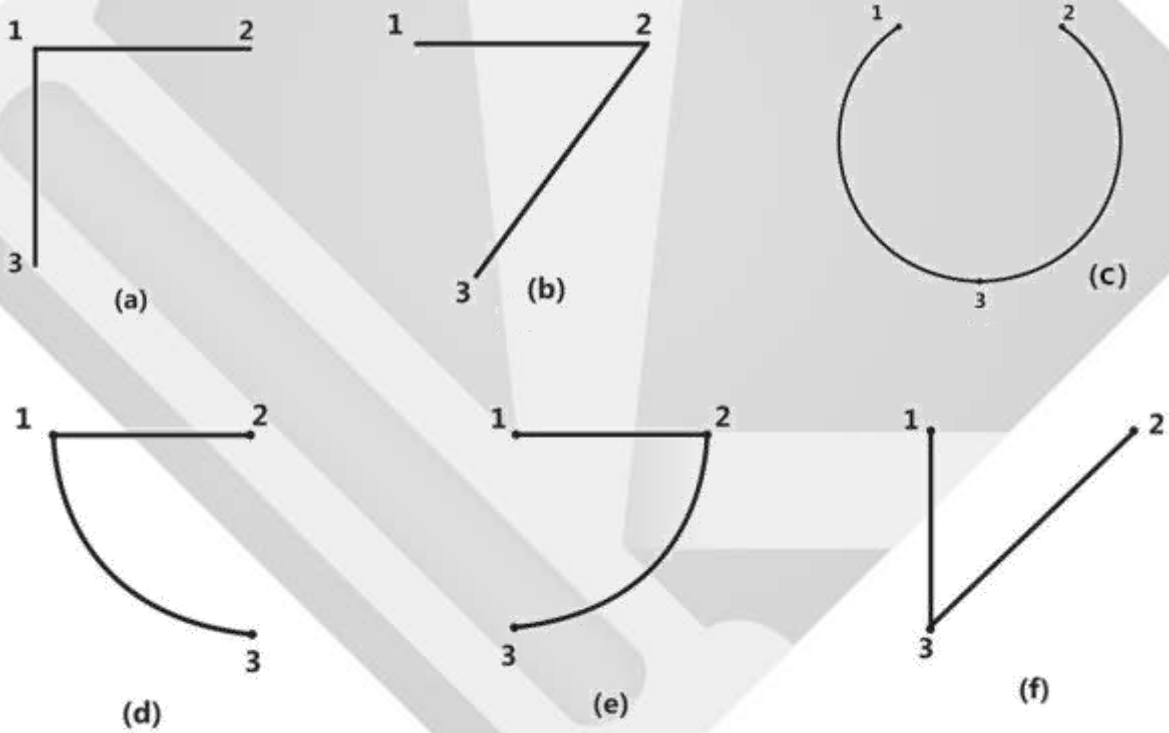


True & Co-tree

A connected sub-graph of a network which has its nodes same as original graph but does not contain any closed path is called tree of network.

A tree always has (n - 1) branches.

Eg. The following trees can be made from graph shown before.



The set of branches of a network which are remove to form a tree is called co-tree of graph.

Twigs & Links

The branches of a tree are called as its twigs & branches of a co-tree are called as chords or links.

Incidence Matrix

The dimension of incidence matrix is (nxb)

N = no. of nodes

B = no. of branches

It is represented by A

a_{ij} = + 1 , If j^{th} branch is oriented away from i^{th} node

a_{ij} = -1 , If j^{th} branch is oriented into i^{th} node.

a_{ij} = 0 , If j^{th} branch is not connected to i^{th} node

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & d & e & f \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 1 \\ -1 & -1 & 0 & +1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & -1 & 0 \end{bmatrix} \end{array}$$

If one of nodes is considered as ground & that particulars row is neglected while writing the incidence matrix, then it is reduced incidence matrix. Order $= (n-1) \times b$

Number of trees of any graph $= \det\left[[A_r][A_r]^T\right]$

A_r = reduced incidence matrix

Circuit Theorems

Linearity

A system is linear if it satisfies the following two properties.

1. Homogeneity Property

   It requires that if input is multiplied by constant hen output is multiplied by same constant.

   eg. V = IR

   is I becomes KI

   V' = KIR = KV

   So, resistance is a linear element & so are inductor & capacitor.

2. Additivity Property

   It requires that response to sum of inputs is sum of response to each input applied separately.

   $V_1 = I_1 R$

   $V_2 = I_2 R$

   If we apply $(I_1 + I_2)$

   We get $V_3 = (I_1 + I_2)R = V_1 + V_2$

Superposition

It states that, in any linear circuit containing multiple independent sources, the total current through or voltage across an element can be determined by algebraically adding the voltage or current due to each independent source acting alone with all other independent source set to zero.

Source Transformation

It states that as independent voltage source V_S in series with a resistance R is equivalent to independent current source $I_S = V_s / R$ in parallel with a resistance R.

Or

An independent current source I_S in parallel with a resistance R is equivalent to a dependent source $V_S = I_S R$ in series with a resistance R.



Thevenin's Theorem

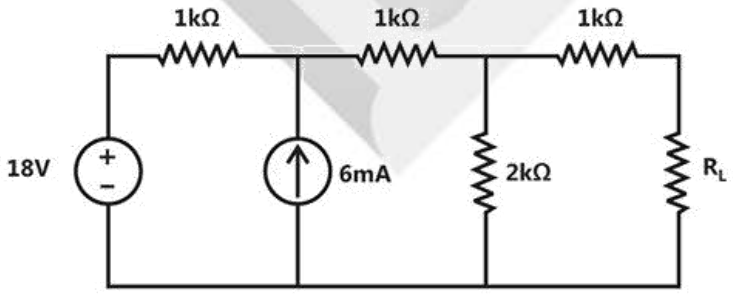
It states that any network composed of ideal voltage and current source, and of linear resistor, may be represented by an equivalent circuit consisting of an ideal voltage source V_{TH} in series with an equivalent resistance R_{TH}.

Methods to calculate thevein equivalent

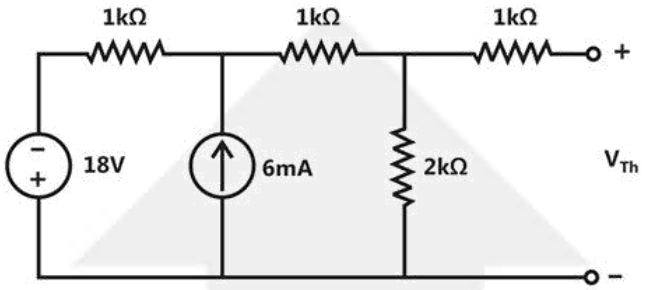
The therein voltage (V_{TH}) is equal to open circuit voltage across load terminals.

Therein resistance is input or equivalent resistance at open circuit terminals (load terminals) when all independent source are set to zero (voltage sources replaced by short circuit & current source by OC)

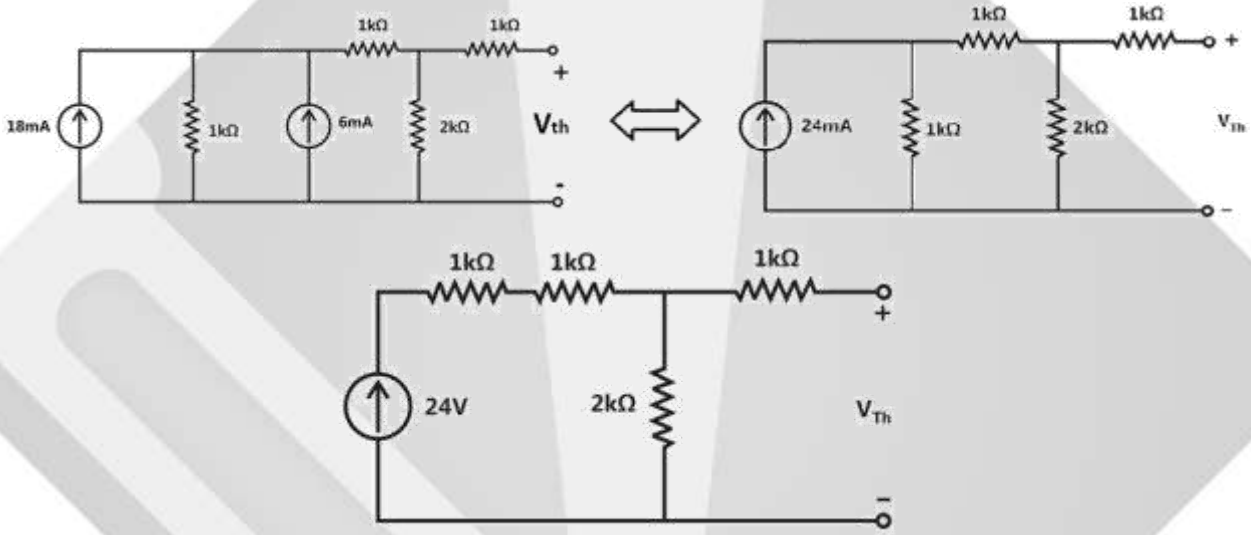
Case – 1 : Circuit with independent sources only



To calculate V_{TH}, open circuit of R_L



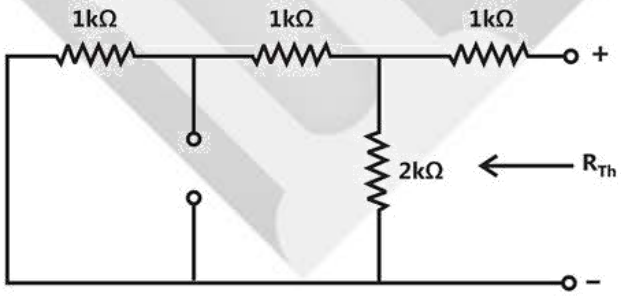
Using sources transformation



$$V_{TH} = \frac{2}{2+2} \times 24 = 12V$$

To calculate R_{th}

Short I & V sources & open 6mA source



$$R_{th} = 1 + 2 \,||\, (1+1) = 2k\Omega$$

Case – 1 : Circuit with both dependent & independent sources

Methodology 1:

- V_{Th} can be found in same way.
- For R_{TH} set all independent sources to zero.
- Remove load & put a test source V_{test} across its terminals, let current through test source is I_{test}.
- Thevenin resistance , $R_{TH} = \frac{V_{test}}{I_{test}}$
- This method is must if independent sources are absent.

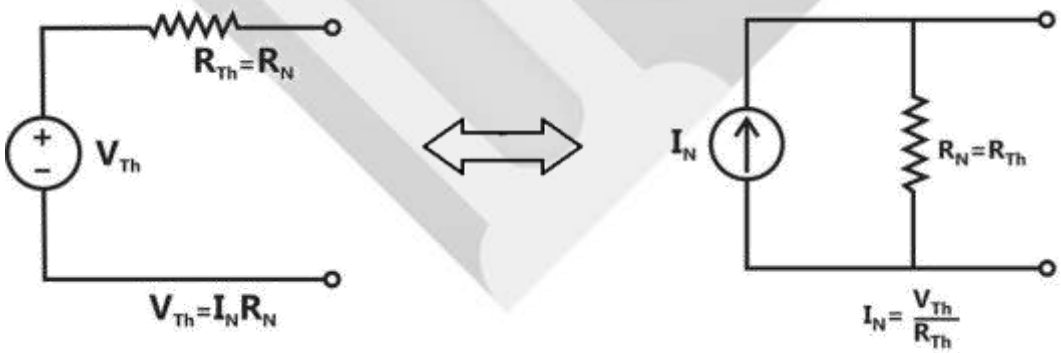
Methodology 2:

- V_{TH} is calculated in same way.
- For R_{TH} short circuit load terminals & leave independent sources as it is
- Obtain I_{SC} through load terminals.
- $R_{TH} = V_{OC}/I_{SC}$

Norton's Theorem

Any network composed ideal voltage & current sources, and of linear resistors, may be represented by an equivalent circuit consisting of an ideal current source I_N in parallel with an equivalent resistance R_N.

- $R_N = R_{TH}$
- To calculate I_N we short circuit load terminals & calculate short circuit current.
- Therein equivalent & Norton equivalent are dual of each other.



Maximum Power Transfer Theorem:

A load resistance R_L will receive maximum power from a circuit when load resistance is equal to Thevenin's/ Norton's resistance seen at load terminals.

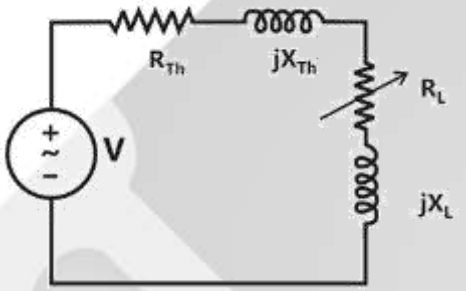
$R_L = R_{Th}$

In case of AC circuit, this condition translates to

$Z_L = Z_{th}^*$

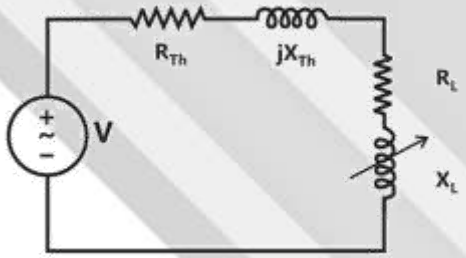
- But if load is resistive in AC circuit then

$R_L = |Z_{Th}|$

- 

For maximum power transfer

$R_L = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2}$

- 

For maximum power transfer

$X_L + X_{Th} = 0$

Transient Analysis

Time Constant :

It is the time required for the response to delay by a factor of $1/e$ or 36.8 % of its initial value.

It is represented by τ.

For a RC circuit

$\tau = RC$

For a RL circuit

$\tau = L/R$

R is the therein resistance across inductor or capacitor terminals.

General method of analysis

$$x(t) = x(\infty) + \left[x(t_0^+) - x(\infty)\right] e^{-(t-t_0)/\tau},\ t > 0$$

If switching is done at $t=t_0$

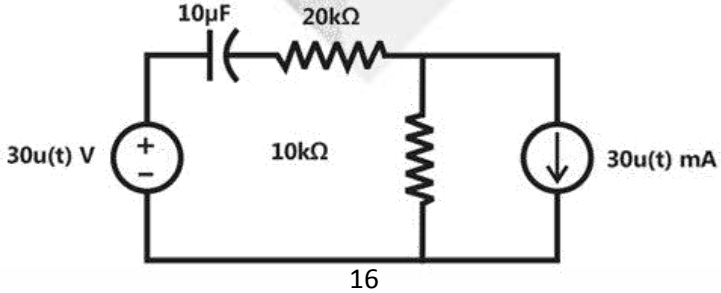
$x(t_0^+) =$ initial value of $x(t)$ at $t = t_0$

$x(\infty) =$ final value of $x(t)$ at $t = \infty$

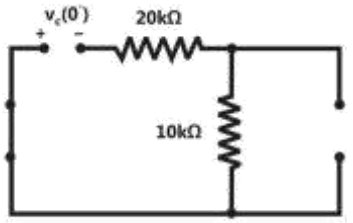
Algorithm

1. Choose any voltage & current in the circuit which has to be determined.
2. Assume circuit had reached steady state before switch was thrown at $t = t_0$. Draw the circuit at $t = t_0^-$ with capacitor replaced by open circuit and inductor replaced by short circuit. Solve for $v_C(t_0^-)$ & $i_L(t_0^-)$.
3. Voltage across capacitor and inductor current cannot change instantaneously.
   $V_C(t_0^+) = V_C(t_0^-) = V_C(t_0)$
   $i_L(t_0^+) = i_L(t_0^-) = i_L(t_0)$
4. Draw the circuit for $t = t^+$ with switches in new position. Replace a capacitor with a voltage source $V_C(t_0^-) = V_C(t_0^+)$ and inductor with a current source of value $i_L(t_0^+) = i_L(t_0^-)$. Solve for initial value of variable $x(t_0^+)$.
5. Draw the circuit for $t = \infty$, in a similar manner as step-2 and calculate $x(\infty)$.
   Calculate time constant of circuit
6. $\tau = R_{th} C$ or $\tau = L/R_{th}$
7. Substitute all value to calculate x(t).

Example
In the circuit shown below, $V_1(t)$ for t > 0 will be given as



Solution



Step 1 :

For t < 0

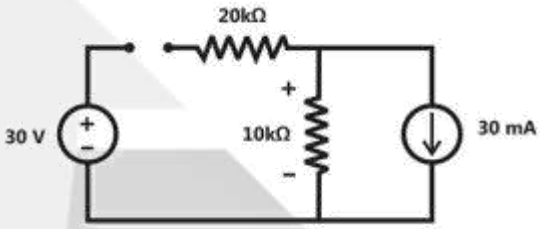
$30u(t) = 0 \;\; \& \;\; 3u(t) = 0$

$V_1(0^-) = 0V$

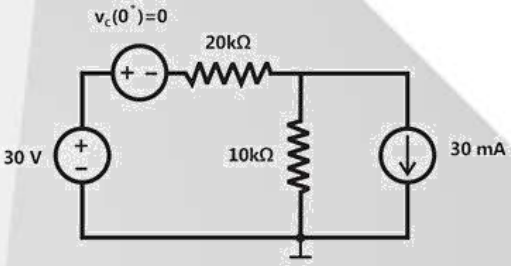
For $t = \infty$

$V_1(\infty) = -3mA \times 10k\Omega$

= -30 V



Step 2 :



At $t = 0^+$

$$\frac{V_1(0^+) - 30}{20k} + 3mA + \frac{V_1(0^+)}{10k} = 0$$

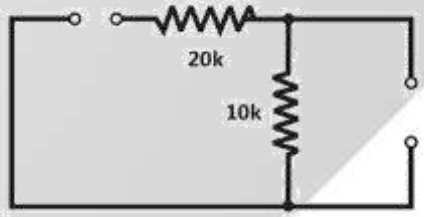
$$\frac{3}{20k} V_1(0^+) = -1.5mA$$

$$V_1(0^+) = -10V$$

$$V_1(t) = -30 + (-10 + 30)e^{-t/\tau}$$

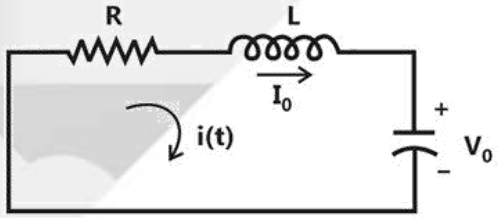
$$R_{TH} = 30k \;\; ; \;\; \tau = R_{TH}C = 0.3s$$

$$V_1(t) = \left(-30 + 20e^{-t/0.3}\right)u(t)V$$



Series RLC circuit

Without Source



$$V(0) = \frac{1}{C}\int_{-\infty}^{0} i(t)dt = V_0$$

$$i(o) = I_0$$

By KVL

$$Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int_{-\infty}^{t} i(t)dt = 0$$

Difference both sides

$$\frac{d^2i(t)}{dt^2} + \frac{R}{L}\frac{di(t)}{dt} + \frac{1}{LC}i(t) = 0$$

Substitute $i(t) = Ae^{st}$

$$Ae^{st}\left(S^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0 \quad \Rightarrow \quad S^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

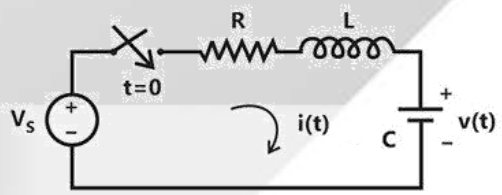
$$S_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}, \qquad S_2 = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - w_0^2} \;;\; \alpha = \frac{R}{2L} \;;\; w_0 = \frac{1}{\sqrt{LC}}$$

1. If $\alpha > w_0$ roots are real & unequal (over-damped)
   $i(t) = Ae^{s_1 t} + Be^{s_2 t}$
2. If $\alpha = w_0$, roots are real & equal (critically damped)
   $i(t) = (A + Bt)e^{-\alpha t}$
3. If $\alpha < w_0$, roots are complex conjugate (under-damped)
   $i(t) = e^{-\alpha t}(A\cos w_d t + B\sin w_d t)$
   $w_d = \sqrt{w_0^2 - \alpha^2}$

Calculate A & B using initial conditions.

With a Source



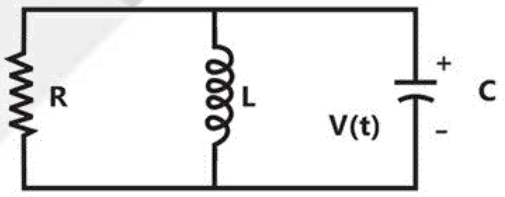
$v(t) = V_S + Ae^{s_1 t} + Be^{s_2 t}$ (Over-damped)

$v(t) = V_S(A + Bt)e^{-\alpha t}$ (Critically damped)

$v(t) = V_S(A\cos w_d t + B\sin w_d t)e^{-\alpha t}$ (under – damped)

Parallel RCL Circuit

Without Source



$$i(0) = \frac{1}{L}\int_{-\infty}^{0} v(t)dt$$

$v(0) = V_0$

By KCL

$$\frac{1}{R}v(t) + \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau + C\frac{dv(t)}{dt} = 0$$

Characteristics equation

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \quad ; \quad \alpha = \frac{1}{2RC} \ , \ w_0 = \frac{1}{\sqrt{LC}}$$

$$S_1, S_2 = -\alpha + \sqrt{\alpha^2 - w_0^2}$$

$$v(t) = Ae^{s_1 t} + Be^{S_2 t} \ \ (\text{over} - \text{damped})$$

$$v(t) = (A + Bt)e^{-\alpha t} \ \ (\text{critically damped})$$

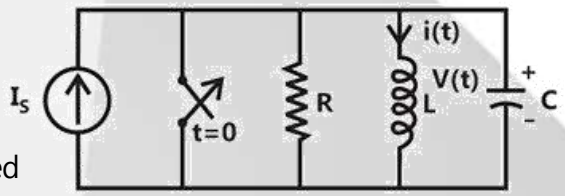
$$v(t) = e^{-\alpha t}(A\cos w_d t + B\sin w_d t) \ \ (\text{under} - \text{damped})$$

With a step input

$$i(t) = I_s + \left(Ae^{s_1 t} + Be^{S_2 t}\right) \ \text{Over} - \text{damped}$$

$$i(t) = I_s + (A + Bt)e^{-\alpha t} \ \text{Critically damped}$$

$$i(t) = I_s + (A\cos w_d t + B\sin w_d t)e^{-\alpha t} \ \text{Under} - \text{damped}$$



Steps:

1. Write differential equation that describe the circuit.
2. From differential equation model, construct characteristics equation & find roots.
3. Roots of characteristics equation determine the type of response over-damped, critically damped & under-damped.
4. Obtain the constant using initial conditions.

Sinusoidal steady state analysis

Lagging & Leading

We can compare the phases of two sinusoids provided that

- Both $V_1(t)$ & $V_2(t)$ are expressed in form of either sine function or cosine function.
- Both $V_1(t)$ & $V_2(t)$ are written with positive amplitude though they may not have same amplitude.

Both $V_1(t)$ & $V_2(t)$ have same frequency.

If $V_1(t) = A\sin wt$

$V_2(t) = B\sin(wt + \theta)\,;\ \theta > 0^0$

$V_2(t)$ leads $v_1(t)$ by an angle θ

$V_1(t)$ lags $v_2(t)$ by an angle θ

PHASORS

A phasor is complex number that represents the amplitude & phase angle information of a sinusoidal function.

$v(t) = V_m \sin(wt + \theta)$

Phasor representation, $\vec{V} = V_{rms}\angle\theta$

magnitude $= V_{rms}$

phase $= \theta$

Networks Elements

1. Resistor

   $\bar{V} = R\bar{I}$

2. Inductor

   $\bar{V} = (j\omega l)\bar{I}$

   $\omega = 2\pi f$; f = frequency of source

3. Capacitor

   $\bar{I} = (j\omega c)\bar{V}$

Impedance & Admittance

Impedance , $Z = \dfrac{\bar{V}}{\bar{I}} = \dfrac{V_{rms}}{I_{rms}}\angle(\theta_v - \theta_i)$

Unit of impedance $= \text{ohm}(\Omega)$

$Z = R + jX = |Z|\angle\theta$

R = resistive component

X = relative component

Phase angle, $\theta = \tan^{-1}\left(X/R\right)$

$$|Z| = \sqrt{R^2 + X^2}$$

For Resistor $Z_R = R$

For Inductor $Z_L = j\omega L$

For Capacitor $Z_C = \frac{1}{j\omega L} = -j/\omega c$

Inductive reactance, $X_L = \omega L$

Capacitive reactance, $X_L = \frac{-1}{\omega c}$

If X = 0, impedance is resistive; current & voltage are in same phase.

If X > 0, impedance is inductive; current lags voltage.

If X < 0, impedance is capacitive; current leads voltage.

Admittance, $Y = \frac{1}{Z} = G + jB$

G = Capacitance

B = Susceptance

Impedance in AC circuits behave like resistance in DC circuits and all the laws remain same like

- Series combination

$$Z_{eq} = Z_1 + Z_2 + \ldots\ldots\ldots\ldots + Z_n$$

- Parallel combination

$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \ldots\ldots\ldots\ldots\ldots + \frac{1}{Z_n}$$

$$Y_{eq} = Y_1 + Y_2 + \ldots\ldots\ldots + Y_n$$

- Star-delta conversion also remain same here.

Circuits analysis in AC domain

1. Identify the sinusoidal source & note the excitation frequency.
2. Convert source to phasor form.
3. Represent each circuit element by its impedance.
4. Solve circuits using circuit techniques (nodal analysis mesh analysis etc.)
5. All circuits' theorems are applicable here as well

Power analysis

Real Power

$P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

In a resistance

$P = \frac{V_{rms}^2}{R} = I_{rms}^2 R$

Complex Power

$S = V_{rms} I_{rms}^*$

$= V_{rms} I_{rms} \angle(\theta_v - \theta_i)$

Real part of $S = P = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$

Real part of $S = Q = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$

Reactive Power = Q

Q = 0 for resistive loads.

Q < 0 for capacitive loads $(\theta_v < \theta_i)$.

Q > 0 for inductive loads $(\theta_v > \theta_i)$

- If the current goes into an element, then it absorbs power and if current comes out it delivers power.
- Hence, a capacitor absorbs leading reactive power. We can also say it delivers lagging reactive power.
- Same way, inductor absorbs lagging reactive power & delivers leading reactive power.

Power Factor

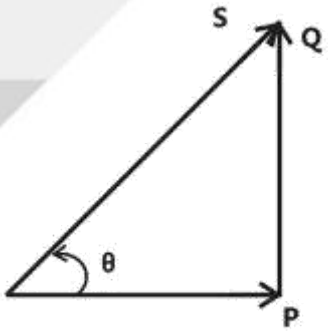
$pf = \frac{P}{|S|} = \cos(\theta_v - \theta_i)$

$\theta = \theta_v - \theta_i =$ power factor angle

If $\theta > 0$, power factor is lagging

If $\theta < 0$, power factor is leading

If $\theta = 0$, power factor is unity

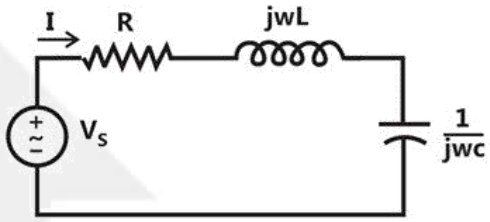


Power triangle

Resonance

Series resonance

For resonance



$$I_m(Z) = 0$$

$$Z = R + j\left(\omega L - 1/\omega C\right)$$

$$\omega L = 1/\omega C \;\Rightarrow\; \omega = \frac{1}{\sqrt{LC}}\text{rad/s}$$

The frequency at which impedance of circuits is purely resistive is called resonant frequency.

$$\omega_0 = \frac{1}{\sqrt{LC}}\text{rad/s}$$

At resonance

$$|I| = \frac{|V_S|}{R}$$

$$\bar{V}_R = \bar{I}R = \bar{V}_S$$

$$\bar{V}_L = j\omega_o L\bar{I} = j\omega_o L\left(\frac{\bar{V}_S}{R}\right)$$

$$\bar{V}_c = \frac{-j}{\omega_0 c}\bar{I} = \frac{-j}{\omega_o C}\left(\frac{\bar{V}_S}{R}\right)$$

At $\omega < \omega_0$; $|X_L| < |X_C|$, net reactance is capacitive so circuits operates at leading pf.

At $\omega = \omega_0$; $|X_L| = |X_C|$, net reactance is zero, so circuits operates at unity pf.

At $\omega > \omega_0$; $|X_L| > |X_C|$, net reactance is inductive, so circuits operates at lagging pf.

Bandwidth: range of frequency for which power delivered to R is half of power at resonance.

$$Bw = R/L$$

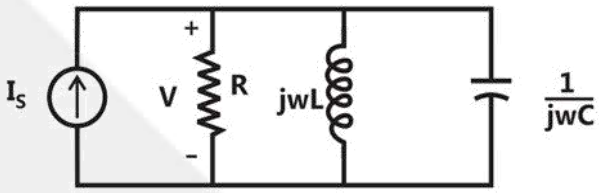
Quality factor

$$Q = \frac{\text{Reactive power}}{\text{Average power}} = \frac{\frac{1}{2}|I|^2|X_L|}{\frac{1}{2}|I|^2 R} = \frac{\frac{1}{2}|I|^2|X_C|}{\frac{1}{2}|I|^2 R}$$

$$Q = \frac{|X_L|}{R} = \frac{|X_C|}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Parallel Resonance

$$Y = 1/R + \frac{1}{j\omega L} + j\omega C$$



At resonance

$$I_m(Y) = 0$$

$$\omega_o = \frac{1}{\sqrt{LC}} \text{ rad/s}$$

At resonance $|V| = |I_S|R$

$$\bar{I}_R = \bar{I}_S$$

$$\bar{I}_L = \frac{\bar{V}}{j\omega_0 L} = \frac{\bar{I}_S R}{j\omega_0 L}$$

$$\bar{I}_C = \bar{V}(j\omega_0 C) = \bar{I}_S R(j\omega_0 C)$$

$$\text{Bandwidth} = \frac{1}{RC}$$

$$\text{Quality factor, } Q = \frac{\text{Reactive Power}}{\text{Average Power}}$$

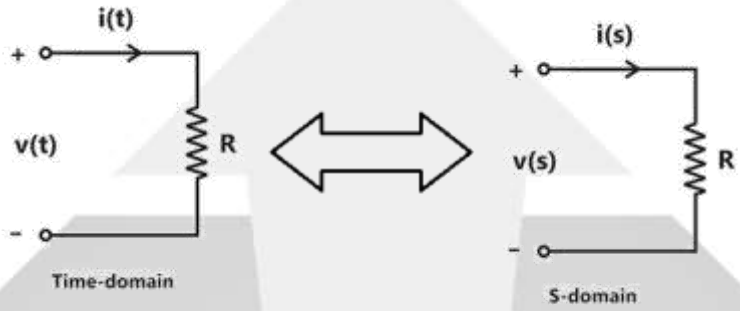
$$Q = \frac{\frac{|V|^2}{2|X_C|}}{\frac{|V|^2}{2R}} = \frac{\frac{|V|^2}{2|X_L|}}{\frac{|V|^2}{2R}} = \frac{R}{|X_L|} = \frac{R}{|X_C|} = R\sqrt{\frac{C}{L}}$$

Circuits analysis in Laplace domain

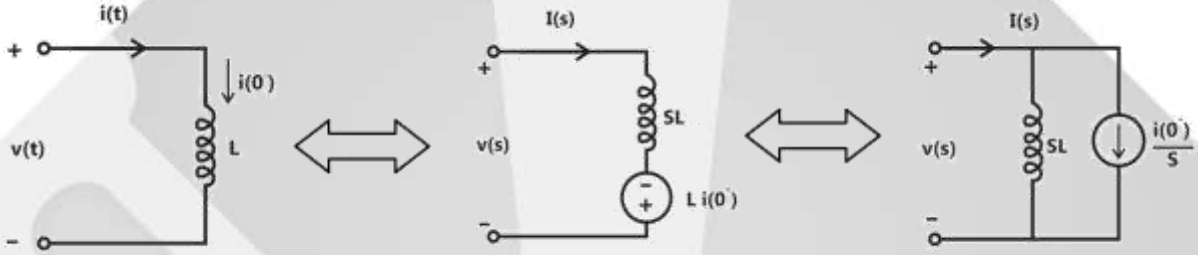
For basic of laplace transform, refer to signal & system k-notes. Laplace transform.

Resistor

$V(s) = RI(s)$

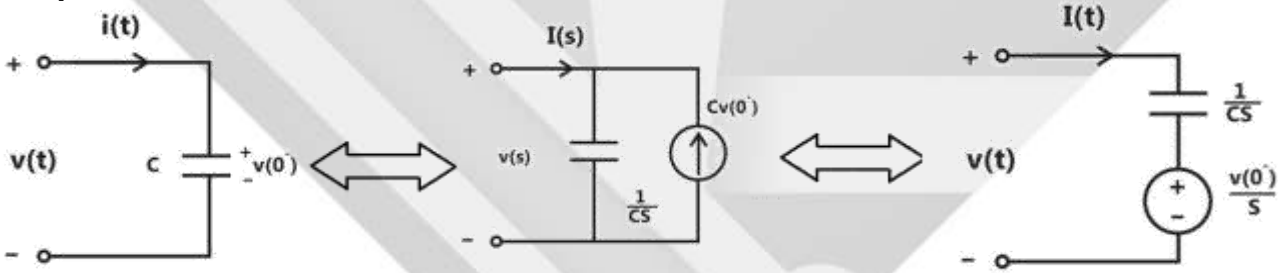


Inductor



$V(s) = sL\left(I(s)\right) - Li(0^-)$ Or $I(s) = \frac{V(s)}{sL} + \frac{i(0^-)}{s}$

Capacitor

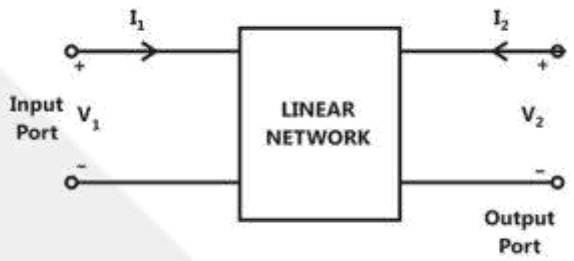


$I(s) = C\left[sV(s) - V(0^-)\right]$ Or $I(s) = \frac{1}{sC}I(s) + \frac{V(0^-)}{s}$

Methodology

1. Draw circuit in s-domain by substituting s-domain equivalent for each circuit element.
2. Apply circuit analysis to obtain desired voltage or current in s-domain.
3. Take inverse Laplace transform to convert voltage and current back in time-domain.

Two Port Network



Impedance Parameters

$V_1 = Z_{11}I_1 + Z_{12}I_2$

$V_2 = Z_{21}I_1 + Z_{22}I_2$

In matrix form $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ = open circuit input impedance

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ = open circuit transfer impedance form part 1 to part 2

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$ = open circuit transfer impedance form part 2 to part 1.

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$ = open circuit output impedance

Admittance parameters

$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

In matrix form,

$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$ = short circuit input admittance.

$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$ = short circuit transfer admittance from part 1 to part 2.

$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$ = short circuit transfer admittance from part 2 to part 1.

$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$ = short circuit output admittance.

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

Hybrid parameters

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

In matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ = short circuit input impedance.

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$ = open circuit reverse voltage gain.

$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$ = short circuit forward current gain.

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$ = open circuit output admittance.

Inverse hybrid parameters

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1}$$

Transmission parameters

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ = open circuits voltage ratio

$B = \left.\frac{-V_1}{I_2}\right|_{V_2=0}$ = negative short circuit transfer impedance.

$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$ = open circuit transfer admittance.

$D = -\left.\frac{I_1}{I_2}\right|_{V_2=0}$ = negative short circuit current ratio.

Symmetrical & Reciprocal N/w

For a reciprocal 2-part Network:

$Z_{12} = Z_{21}$

$Y_{12} = Y_{21}$

$h_{12} = -h_{21}$

$AD - BC = 1$

$g_{12} = -g_{21}$

For a symmetric 2-part network:

$Z_{11} = Z_{22}$

$Y_{11} = Y_{22}$

$h_{11}h_{22} - h_{21}h_{12} = 1$

$A = D$

$g_{11}g_{22} - g_{21}g_{12} = 1$

Interconnection of 2-part networks

- For a series connection of two networks N_a & N_b having z-parameters metric $[Z_a]$ & $[Z_b]$

  $[Z_{eq}] = [Z_a] + [Z_b]$

  Z – Parameter matrices are added.

- For a parallel connection of two networks N_a & N_b having y-parameter matrices $[y_a]$ & $[y_o]$

  $[y_{eq}] = [y_a] + [y_b]$

  Y – Parameter matrices are added.

- For a cascade connection of two networks N_a & N_b having transmission parameters matrices $[T_a]$ & $[T_b]$

  $[T_{eq}] = [T_a][T_b]$

  Transmission parameter matrices are multiplied.

Magnetically coupled circuits

If change in flux of one coil induces a voltage in second coil then both coils are said to be magnetically coupled.

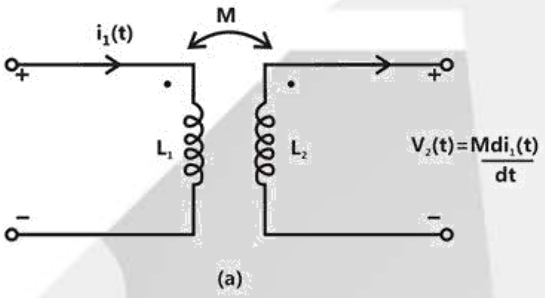
$$v_2(t) = M_{21}\frac{di_1(t)}{dt}$$

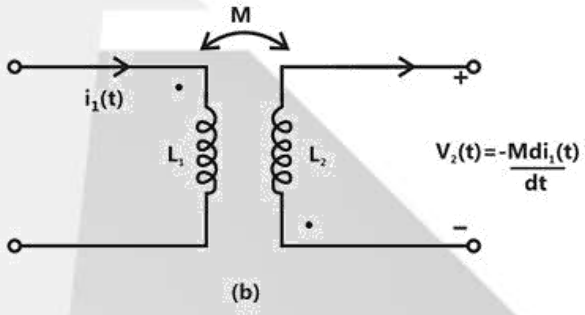
$$v_1(t) = M_{12}\frac{di_2(t)}{dt}$$

$$M_{12} = M_{21} = M$$

Dot convention

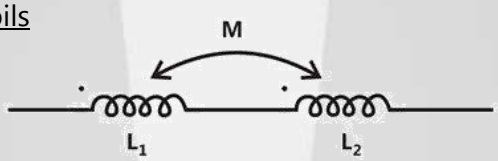
If a current enters the dotted terminals of one coil, then induced voltage in second coil has a positive voltage reference at dotted terminal of second coil.
If a current enters undotted terminals of one coil, then induced voltage n second coil as a positive voltage reference at undotted terminals of second coil.



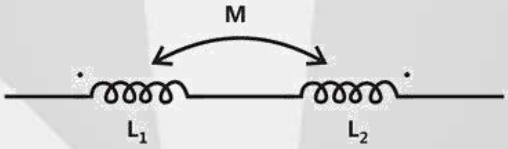


Series connection of coupled coils

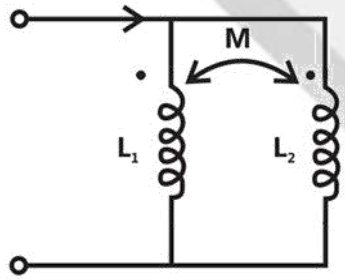
$$L_{eq} = L_1 + L_2 + 2M$$



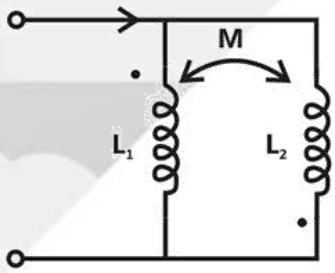
$$L_{eq} = L_1 + L_2 - 2M$$



Parallel connection of coupled coils



$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

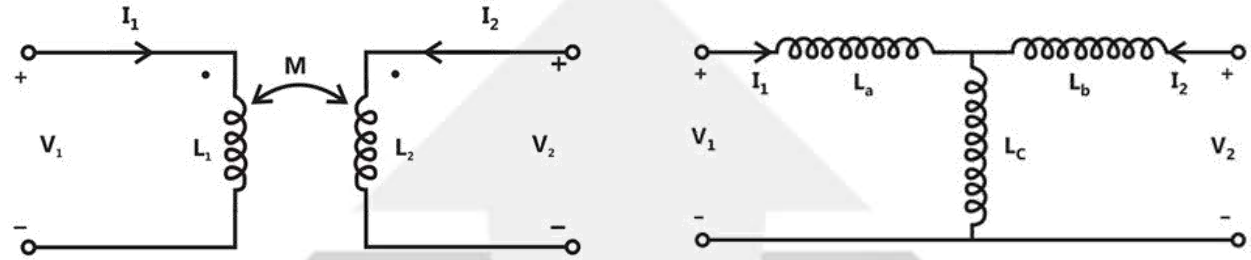


$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

$$\text{Coefficient of coupling } k = \frac{M}{\sqrt{L_1L_2}}$$

Equivalent circuits of linear transformers

T-equivalent circuit

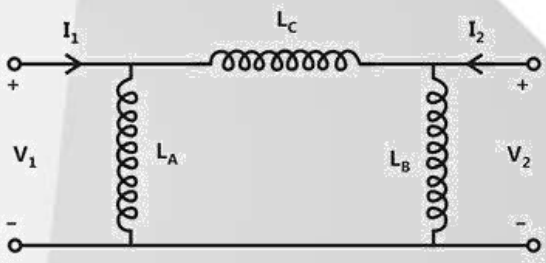


$L_a = L_1 - M$

$L_b = L_2 - M$

$L_C = M$

π- equivalent circuit



$$L_A = \frac{L_1L_2 - M^2}{L_2 - M}$$

$$L_B = \frac{L_1L_2 - M^2}{L_1 - M}$$

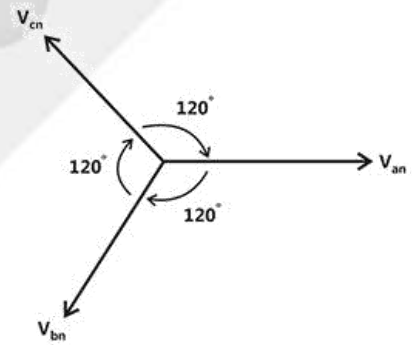
$$L_C = \frac{L_1L_2 - M^2}{M}$$

Three Phase Circuits

Balanced three phase system

A system in which all three voltage have equal voltage magnitude and are phase displaced by 120^0 with respect to each other.

Positive Sequence (abc)

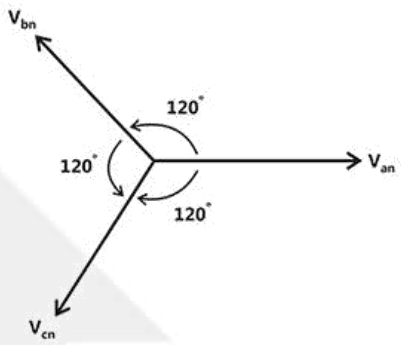


$$V_{an} = V_P \angle 0^0$$

$$V_{bn} = V_P \angle -120^0$$

$$V_{cn} = V_P \angle -240 = V_P \angle 120^0$$

Negative Sequence (acb)



$$V_{an} = V_P \angle 0^0$$

$$V_{cn} = V_P \angle -120^0$$

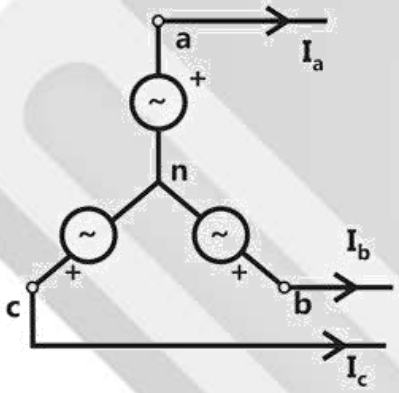
$$V_{bn} = V_P \angle -240^0 = V_P \angle 120^0$$

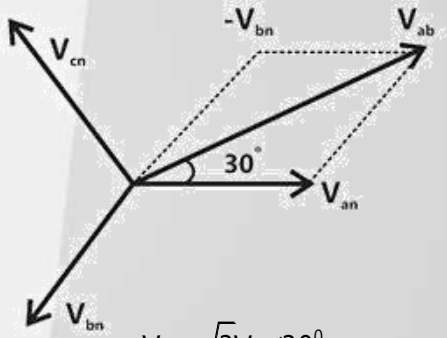
For both phase sequence,

$$V_{an} + V_{bn} + V_{cn}$$

Connections

Star Connection





$$V_L = \sqrt{3} V_P \angle 30^0$$

$$I_L = I_p$$

V_{ab} = line to line voltage or line voltage

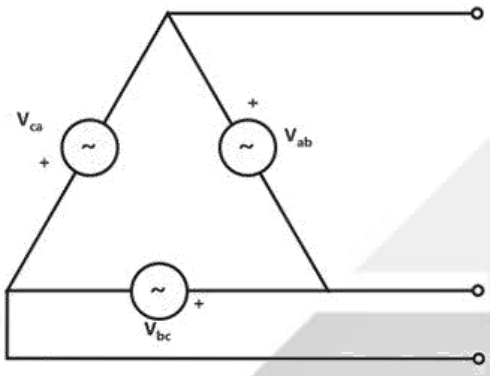
$$V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_P \angle 30^0$$

So, line voltage V_{ab}, V_{bc}, V_{ca} are $\sqrt{3}$ times the phase voltage & lead the respective phase voltage by 30^0.

Line currents I_a, I_b, I_c in this connection are equal to phase currents.

Phase current are the currents that flow in individual phases.

Delta Connection



$$V_L = V_P$$
$$I_L = \sqrt{3} I_P \angle -30^0$$

Line current is $\sqrt{3}$ times phase current & lags respective phase current by 30^0.

Power in a balanced $3-\phi$ system

$$P = \sqrt{3} V_L I_L \cos\phi = 3 V_P I_P \cos\phi$$
$$Q = \sqrt{3} V_L I_L \sin\phi = 3 V_P I_P \sin\phi$$
$$\bar{S} = \sqrt{3} \bar{V}_L \bar{I}_L^* = 3 \bar{V}_P \bar{I}_P^*$$

For power measurement, refer to electrical measurements k-notes, two wattmeter method.

Electrical & magnetic fields

Coulomb's law

Coulombs law states that magnitude of force between two point charges is directly proportional to square of distance between them & direction of force is along the line joining the charges.

$$\vec{F} = \frac{Q_1 Q_2}{4\pi \in |R|^2} \hat{a}_R$$

$\in = \in_o \in_r$; $\in_o = 8.854 \times 10^{-12}$ F/m = permittivity of free space

$\in_r$ = relative permittivity or dielectric constant.

Electric field intensity

$$E = \frac{F}{q} = \frac{Q}{4\pi \in |R|^2}\hat{a}_R$$

Electric field direction is away from a positive charge & towards negative charge.

Charge densities

1) Linear charge density
It is denoted by 'λ'. It is equal to charge per unit length.

$$\lambda = \frac{q}{l}(c/m)$$

2) Surface charge density
It is denoted by 'σ'. It is equal to charge per unit area.

$$\sigma = \frac{q}{A}\left(c/m^2\right)$$

3) Volume charge density
It is denoted by 'ρ'. It is equal to charge per unit volume.

$$\rho = \frac{q}{V}\left(c/m^3\right)$$

Electric field due to continuous charge distribution

1) Infinite line charge
Electric field intensity at a distance 'r' from a line charge of linear charge density λ

$$\vec{E} = \frac{\lambda}{2\pi \in_o r}\hat{a}_r$$

2) Infinite sheet charge
Electric field at a distance 'h' from an infinite charged sheet with charge density σ is

$$\vec{E} = \frac{\sigma}{2 \in}\hat{a}_n \;;\; \hat{a}_n = \text{Normal unit vector}$$

3) Conducting sphere
If a conducting sphere of radius 'R' is charged with a charge 'Q' then electric field.

$$E = \begin{cases} 0 & r < R \\ \dfrac{Q}{4\pi \in r^2} & r > R \end{cases}$$

Electric field inside conducting sphere is zero.

Electrical potential

The amount of work done in bringing a unit positive charge from infinity to a certain point in an electric field is called electric potential.

$$V_A = -\int_{\infty}^{A} \bar{E}.d\vec{L}$$

$$\bar{E} = -\nabla V$$

∇ = represent gradiant

For vector operations, refer engineering mathematics k-notes.

Electric Flux Density

$$\vec{D} = \in \vec{E}$$

Electrical flux $\Psi = \oint_S \vec{D}.d\vec{S}$

SI unit of electric flux is coulomb.

Gauss's law

It states that total electric flux through any closed surface is equal to charge enclosed by that surface.

$$\oint_S \vec{D}.d\vec{S} = \int_b \rho dV$$

By Gauss's Divergence theorem

$$\nabla.D = \rho$$

Magnetic flux Density

Magnetic flux per unit area is called magnetic flux density. It is a vector quantity and denoted by $\vec{B}$ & its unit is tesla (T).

Flux $\phi = \int \vec{B}.\, d\vec{S}$

Magnetic field intensity

Represented by $\vec{H}$.

$$\vec{B} = \mu\vec{H}$$

μ = permeability.

$\mu = \mu_o\mu_r$

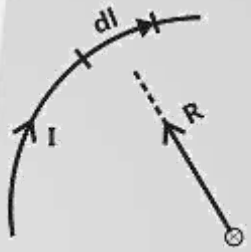
μ_r = relative permeability

μ_o = permeability of free space

$\mu_o = 4\pi \times 10^{-7}$ H/m

Biot – Savart's law

$$d\vec{H} = \frac{I}{4\pi|R|^2}\left(d\vec{L} \times \hat{a}_R\right)$$



Magnetic field due to infinite line current

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

ρ = perpendicular distance of point from line current.

$\hat{a}_\phi$ = Unit vector in cylindrical co-ordinates.

Ampere's Circuital law

It states that line integral of magnetic field intensity $\vec{H}$ around any closed path is exactly equal to net current enclosed by that path.

$$\oint \vec{H} . d\vec{L} = I_{enclosed}$$

$$\oint \vec{H} . d\vec{L} = \oint \vec{J} . d\vec{s}$$

By stokes theorem

$$\nabla \times \vec{H} = \vec{J}$$

Maxwell equations

1) $\oint \vec{E} \cdot d\vec{L} = \frac{-d}{dt} \iint \vec{B} \cdot d\vec{S}$ or $\nabla \times \vec{E} = \frac{-\partial \vec{B}}{\partial t}$

2) $\iint \vec{E} \cdot d\vec{S} = \frac{1}{\in} \iiint \rho dv$ or $\nabla \cdot \vec{E} = \rho / \in$

3) $\iint \vec{B} \cdot d\vec{S} = 0$ or $\nabla \cdot \vec{B} = 0$

4) $\oint \vec{B} \cdot d\vec{L} = \mu_0 \iint \vec{J} \cdot d\vec{s} + \mu_o \in_o \frac{d}{dt} \iint \vec{E} \cdot d\vec{s}$

or

$$\nabla \times \vec{B} = \mu_o \left(\vec{J} + \in_o \frac{\partial \vec{E}}{\partial t} \right)$$

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